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Review of recent advances in local approaches applied to pre-stressed components under fatigue loading

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Abstract

Fatigue strength of mechanical components in the high cycle regime depends on both the applied loading and the intensity of any residual stress field induced by either non-homogeneous plastic deformation or the solidification of a local portion of material due to welding operations. In presence of geometric variations that are amenable to being modelled as a sharp V-notch, the residual stress distribution near the notch tip is singular and follows the same form as the solution obtained by Williams in 1952 where the intensity of the asymptotic stress field is quantified by the notch stress intensity factor (NSIF). However, the residual stress varies during fatigue loading and a stable value may be reached. Numerical models have been developed for the calculation of the residual NSIFs and their variation under fatigue loading. Taking advantage of these models, new local approaches have also been recently developed which are able to predict the fatigue strength of pre-stressed notched components. The present paper provides a brief review of such recent advances.

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1. Introduction

Following the early work by Williams (1952), a large number of studies have been reported that have evaluated the effect of local stress fields on the static and fatigue strength of mechanical components which contain a geometric variation modelled as sharp V-notch. For instance, the fatigue strength of welded joints can be quantified in terms of

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Notch Stress Intensity Factors (NSIFs) (e.g. Atzori and Meneghetti (2001), Atzori et al. (1999), Lazzarin and Livieri (2001), Lazzarin and Tovo (1998)) or via local Strain Energy Density (SED) averaged over a control volume of radius R_c (e.g. Lazzarin and Zambardi (2001), Livieri and Lazzarin (2005), Berto and Lazzarin (2009)). In all these studies, the residual stress effect on fatigue strength of welded joints is included through reference curves that are derived from a large body of experimental data. This simplification is necessary because of the difficulties in quantifying the intensity and distribution of residual stress near the weld toe either experimentally or using numerical models. A further complication is linked to the dependence of residual stress on welding parameters, joint geometry, clamping conditions, number of applied load cycles and the level remotely applied stress, and hence there is very significant variability in residual stress for welds made under nominally similar conditions.

Even using such sophisticated techniques as synchrotron X-ray and neutron radiation sources where a significant number of measurements can be obtained in the time allocated for an experiment (typically 2-5 days), sample-to-sample variation can make drawing generalised conclusions rather difficult. If however, sample experimental data can be used to calibrate numerical or analytical models that are capable of capturing the evolution of the as-welded and load-modified residual stress field near the most likely crack initiation sites, then advances in knowledge and understanding are possible. The present authors believe that first published work in which the asymptotic nature of the residual stress near the weld toe was shown is that by Ferro et al. (2006). Their work described in detail the effect of stationary and transient thermal loads on thermal and residual stress fields near the tip of a V-notch. It was shown that both the thermal and residual stress fields near a V-notch tip are singular; the singularity degree, which depends on the V-notch opening angle, matches either the elastic (Williams (1952)) or the elastic-plastic solution (Hutchinson (1968), Rice and Rosengren (1968)), depending on the magnitude of the thermal loads and clamping conditions. Further in-depth investigations followed that first analysis. The influence of clamping conditions and phase transformation effects (transformation plasticity (Leblond and Deveaux (1989)), specific volume change) on residual stress distributions were investigated in work by Ferro (2012) and Ferro and Petrone (2009). In particular, it is worth mentioning that phase transformations affect the sign of residual asymptotic stress field so that, according to the material to be welded, a stress-relief heat treatment may either enhance or decrease the fatigue strength of the joint.

In order to evaluate the influence of residual stress on fatigue strength of welded joints, the calculation of NSIFs for as-welded joints is insufficient. During cyclic load, a redistribution/relaxation of the residual stress is observed due to the effects of plasticity. Some analytical work has indicated that the redistribution primarily occurs during the initial loading cycles and that it then remains stable during successive load cycles (Ferro et al. (2016)). However, in other experimental work residual stresses have been observed to continue relaxing (depending on the level of plasticity experienced at the crack tip) over a considerable number of load cycles and also, that the direction of the principal strains can also change during load cycling (Asquith et al. (2007)). As with many other aspects of residual stresses and their relaxation it remains hard to predict a priori exactly what might happen during subsequent load cycling of a structure containing residual stresses, although this does seem likely to be strongly influenced by the acuity of the notch, with sharp V-notches probably redistributing the stress and stabilising after very few applied load cycles. The redistribution effect particularly has to be considered in the low-cycle regime while it can be neglected in the high-cycle regime where the redistribution of residual stresses induced by plastic effects is negligible (*small scale yielding* hypothesis) (Ferro et al. (2016)). When the residual and stationary NSIF is calculated using a reliable numerical model, the residual asymptotic stress field can be treated as analogous to a ‘*mean stress*’ field as described in work by Ferro (2014). The present work reviews the most recent advances in this field for sharp V-shaped notches.

2. Asymptotic residual stress field

Before any model is developed that can quantify the influence of residual stresses on fatigue strength of pre-stressed notched components, it is necessary to first consider the distribution of residual stress near a ‘*geometric singularity*’. Consider the problem of the elastic equilibrium in the presence of a V-shaped notch with an opening angle 2β (Fig. 1).

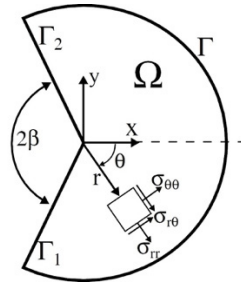


Fig. 1. Domain Ω for the sharp V-notch problem.

If the material is homogeneous and isotropic, under the hypothesis of linear, thermo-elastic theory and plane-strain conditions, the equations representing the stress field near the V-notch, are independent of the thermal terms and match the solution obtained by Williams (1952) (Ferro et al. (2006)). Whatever load type is applied (e.g. thermal or mechanical), under a linear-elastic hypothesis and for plane-stress or plane-strain conditions, the induced stress field near the notch tip (considering only to the first term of the Williams solution and mode I of V-notch opening), is described by the following asymptotic equation:

$$\sigma_{ij}(\theta) = \frac{K_I^{th,m}}{r^{1-\lambda_1}} g_{ij}(\theta) \quad (i, j = r, \theta) \tag{1}$$

where $g_{ij}(\theta)$ are the angular functions, λ_1 is the first eigenvalue obtained from Eq. (2),

$$\lambda \sin(2\beta) + \sin(2\beta\lambda) = 0 \tag{2}$$

and $K_I^{th,m}$ is the NSIF due to a thermal (*th*) or mechanical (*m*) symmetrical load (opening mode I). According to Gross and Mendelson's definition (1972):

$$K_I^{th,m} = \sqrt{2\pi} \lim_{r \rightarrow 0} r^{1-\lambda_1} \sigma_{\theta\theta} \quad (r, \theta = 0) \tag{3}$$

The first eigenvalue depends only by the V-notch angle (2β) and varies in the range between 0.5 and 1. The eigenvalue is 0.5 in the crack case ($2\beta=0$), and increases to 0.674 and 0.757 when the notch opening angles are equal to 135 and 150 degrees, respectively. By simulating the solidification of a fusion zone (FZ) near the tips of a double V-notched plate ($2\beta = 135^\circ$), the asymptotic nature of residual stresses is revealed (Fig. 1).

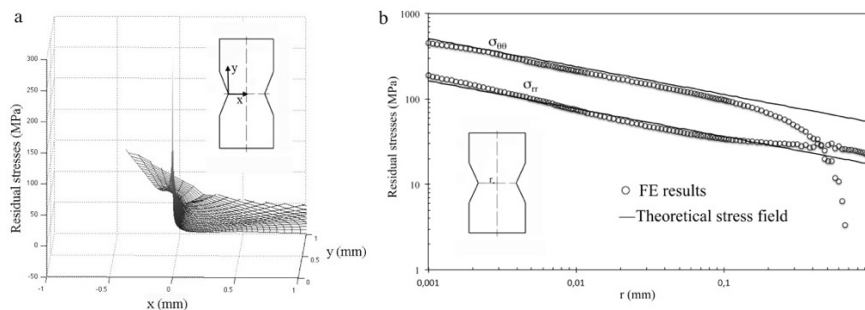


Fig. 2. (a) In-plane distribution of residual stresses (radial component, σ_r) near the notch tip; (b) Tensile residual stresses along the bisector of the

V-notch ($2\beta=135^\circ$), ($K_{I1}^{res}=135 \text{ MPamm}^{0.3264}$) (material: ASTM 11 SA 516, free edges) (Ferro, (2014)).

2.1. Influence of phase transformation on residual stress field

Further investigations were carried out in order to evaluate the influence of phase transformations on residual stresses. During solidification and cooling of a multi-phase material, the variation of the specific volume and the ‘transformation plasticity’ (Leblond and Deveaux (1989)) associated with phase transformation, influence the thermal and residual stresses induced by thermal loads. It was shown that such effects are sufficiently large that numerical models of the welding process must take into account phase transformation effects in calculating the thermal and residual stress field (Ferro et al. (2006)). When the scale of observation is focused on about one tenth of the notch depth, it was observed that phase transformation changes the sign of residual stresses compared to the results obtained in a simplified mono-phase material (Ferro (2012)). This implication is that stress relief heat treatments may decrease the fatigue strength in the high-cycle regime when the sign of the asymptotic residual stress is negative. Fig. 2 shows the asymptotic residual stress fields (θ component) along the bisector of the V-shaped weld toe calculated by taking into account only the volume changes that occur during phase transformation. As a general rule it was found that as-welded mono-phase materials, such as austenitic or ferritic stainless steels, are characterized by a compressive singular residual stress field, while a multi-phase material such as carbon steel shows a tensile asymptotic residual stress field (under free-edge clamping conditions).

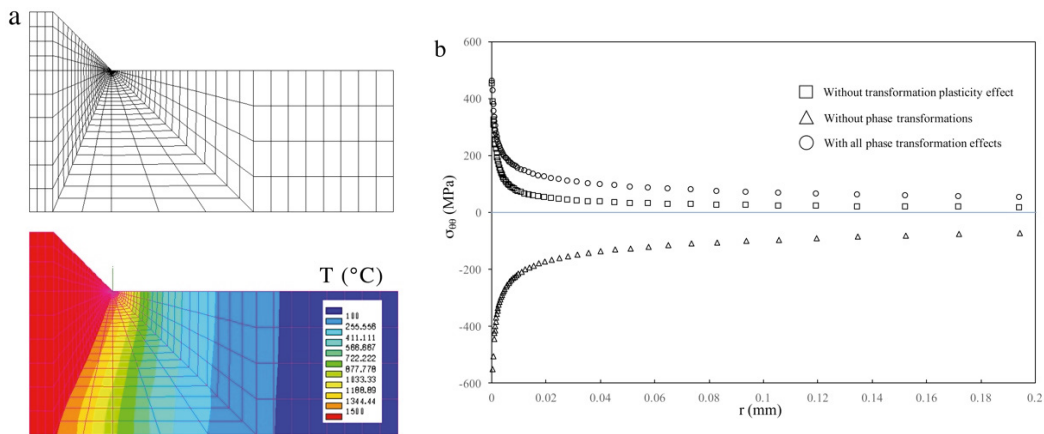


Fig. 3. (a) Mesh of the numerical model and fusion zone dimension and shape (butt-welded joint); (b) Phase transformation effects on residual stresses along the bisector of the V-notch ($2\beta=135^\circ$) (material: ASTM 11 SA 516, free edges).

2.2. Residual stresses redistribution

It is well known that residual stresses redistribute during cyclic load due to plasticity effects. However, for sharp notches such redistribution is completed after very few cycles and a residual NSIF (R-NSIF) stationary value is reached. Ferro (2014) found that the residual stress redistribution under fatigue loading is negligible in the high-cycle regime since the zone that experiences plastic deformation is restricted to about one tenth of the zone dominated by the elastic asymptotic residual stress distribution (*small scale yielding hypothesis*). Stress redistribution will increase, however, as the remotely applied stress increases. At high stress amplitudes, plasticity ‘erases’ the pre-existing residual stress field so that there is little difference between the fatigue strength of a stress-relieved joint and that of an as-welded joint. This means that the superposition property can be applied only in the high-cycle regime where the experimental results show that fatigue strength is sensitive to pre-existing residual stresses. In this case the R-NSIF

can be summed to give the stress-induced NSIF as shown in Fig. 4a. It should be noted that in that case, residual stresses are negative (single-phase material, AA 6063) and therefore decrease the maximum transverse stress field (Fig. 4a). Fig. 4b shows that at high values of the remotely applied stress amplitude, plasticity effects make the maximum transverse stress field almost insensitive to the pre-existing residual stress field.

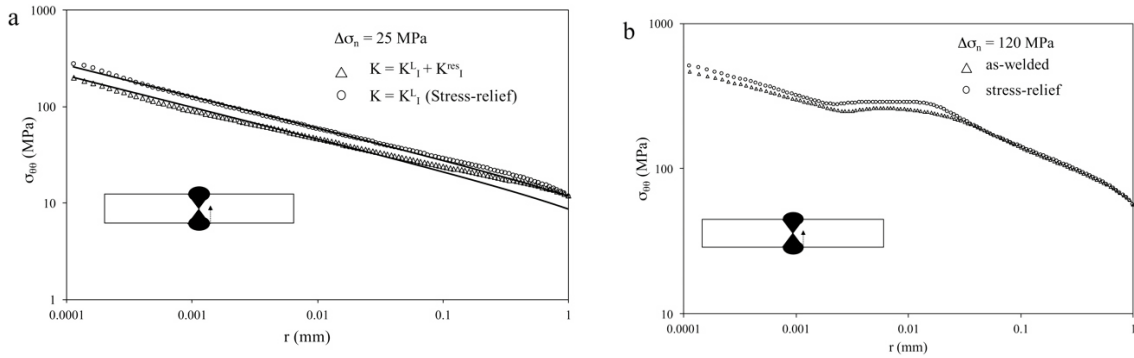


Fig. 4. Maximum transverse stress field of stress-relief and as-welded joint after ten cycles at different values of the remotely applied stress amplitude ($\Delta\sigma_n$) (a) $\Delta\sigma_n = 25$ MPa; (b) $\Delta\sigma_n = 120$ MPa; $2\beta = 135^\circ$, $K^L =$ NSIF induced by load, $K^{res} =$ NSIF induced by the solidification of the FZ: AA 6063, free edges (Ferro et al. 2016).

3. Quantification of the influence of residual stresses on fatigue strength of welded joints

On the basis of the developments outlined above, a model was developed by Ferro (2014) that quantifies the influence of residual stresses on fatigue strength of either welded joints or pre-stressed notched components, and it has been experimentally validated (Ferro et al. (2016)). The model uses the concept of strain energy density (SED) averaged over a control volume of radius R_C (e.g. Lazzarin and Zambardi (2001), Livieri and Lazzarin (2005), Berto and Lazzarin (2009)), which under plane strain conditions and mode I loading takes the form:

$$\bar{W} = \frac{e_I}{E} \left[\frac{K_I^{th,m}}{R_C^{1-\lambda_I}} \right]^2 \tag{4}$$

In Eq. (4), the parameter e_I depends on V-notch opening angle (2β), Poisson’s ratio (ν) of the material and failure hypothesis adopted. For the Beltrami failure hypothesis (total strain energy density) and plane strain conditions, with $\nu = 0.34$ (aluminum alloy AA 6063) and $2\beta = 135^\circ$, e_I is equal to 0.111. The control radius (R_C) is a material characteristic length that for Al-alloy welded joints was found to be equal to 0.12 mm (Livieri and Lazzarin (2005), Berto and Lazzarin (2009)). As residual stresses have the effect of modifying the local load ratio (R), the following relationships hold true:

$$\left. \begin{aligned} \Delta K &= K_{I_{max}}^m - K_{I_{min}}^m \\ R &= \frac{K_{I_{min}}^m + K_I^{res}}{K_{I_{max}}^m + K_I^{res}} \\ R &= \frac{K_{I_{min}}^m}{K_{I_{max}}^m} \end{aligned} \right\} K_{I_{min}}^m + K_I^{res} > 0 \tag{5}$$

$$\Delta K = \left. \begin{matrix} K_{I_{max}}^m + K_I^{res} \\ R = 0 \end{matrix} \right\} K_{I_{min}}^m + K_I^{res} \leq 0 \tag{6}$$

where K_I^{res} is the R-NSIF which characterizes the residual stress field. R^m and R correspond to the local load ratio of the nominal applied and real cycle experienced, respectively. Starting from Eqs (4-6), for $R = 0$, the following equation is obtained (full analytical details are published in Ferro (2014)):

$$\Delta \sigma_n = \frac{R_c^{1-\lambda_I} \left[\frac{E}{e_I} \left(\frac{C}{N} \right)^{1/z} \right]^{1/2}}{k_I t^{1-\lambda_I}} - \frac{K_I^{res}}{k_I t^{1-\lambda_I}} \tag{7}$$

where $\Delta \sigma_n (= \sigma_{n,max})$ is the nominal stress amplitude. Similarly, for $R > 0$ the following relationship is obtained:

$$\sigma_{max}^m = \frac{\left((K_I^{res})^2 + \frac{(1+R^m)}{(1-R^m)} \frac{E}{e_I} \left(\frac{C}{N} \right)^{1/z} \right)^{1/2}}{k_I t^{1-\lambda_I} (1+R^m)} - \frac{K_I^{res}}{k_I t^{1-\lambda_I} (1+R^m)} \tag{8}$$

where C is a constant and z is the slope of the fatigue data expressed in terms of local strain energy density experimentally calculated ($z = \text{Log}(N_{D_1} / N_{D_2}) / \text{Log}(\Delta \bar{W}_{D_2} / \Delta \bar{W}_{D_1})$), subscripts D_1 and D_2 indicate two points of the curve $\Delta \bar{W}(N)$; k_I is a non-dimensional coefficient, analogous to the shape functions of cracked components calculated using the following equation:

$$K_I^m = k_I \sigma_n t^{1-\lambda_I} \tag{9}$$

where σ_n is the remotely applied stress, and t is a geometrical parameter of the plate, according to Lazzarin and Tovo (1998). Eqs. (7) and (8) are applied in high-cycle fatigue regime where the redistribution of the pre-existing residual stresses is considered negligible (Ferro (2014)).

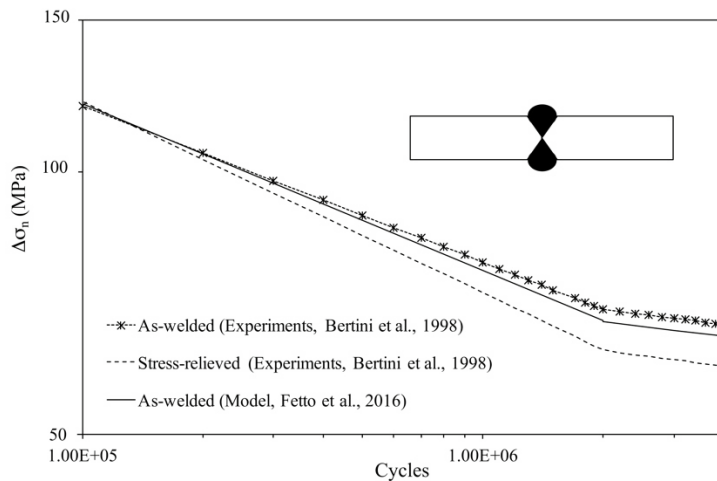


Fig. 5. Residual stress influence on fatigue strength of the AA 6063 butt-welded joint predicted by Eq. (7) (Ferro et al. (2016))

This model was validated using experimental results obtained for the fatigue strength of butt-welded AA 6063

joints in both the stress-relieved and as-welded condition (Bertini et al. (1998)), and the analysis was validated in Ferro et al. (2016). Under the condition $K_{I_{min}}^m + K_I^{res} \leq 0$, Fig. 5 shows prediction obtained using the present model Eq. (7) for the fatigue resistance of the stress-relieved and as-welded components. Due to the negative value of the R-NSIF, an improvement in the fatigue strength for as-welded joints is observed both experimentally and predicted by the model, compared with the fatigue strength of the stress-relieved specimens. It is worth mentioning that in this model the fatigue strength of as-welded joints in the low-cycle regime was set equal to that of stress-relieved specimens according to the redistribution/relaxation induced by high remotely applied stress amplitudes (Fig. 4b).

4. Conclusions

This paper has outlined the development that has occurred since 2006 of a model for the asymptotic residual stress fields in notched components that quantifies the influence of residual stresses on the fatigue strength of welded joints. Such asymptotic residual stress fields were found to be strongly influenced by mechanical constraints, geometry, process parameters and material. In particular, the sign of the residual stress field depends on phase transformation effects such as volume changes and transformation plasticity. Such effects cannot therefore be neglected in any reliable numerical model of the welding process. Furthermore, residual stresses redistribute during subsequent fatigue loading because of the plastic deformation that occurs near the weld toe at high applied stress amplitudes. However, when such remotely applied stress amplitudes are low, stress redistribution is not expected and the superposition principle can be applied. In such conditions, the residual stress field near the notch tip has the same effect as a superimposed mean stress and its influence on fatigue strength can be quantified through the model proposed in this paper.

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