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Control of Second Order Processes with Dead Time: the Predictive PID Solutions

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Abstract:
Dead times affect many industry processes and are mainly caused by the time required to transport mass, energy or information. In process with dead times the performance of classical PID controllers may be significantly decreased, especially when the dead times are large and higher than the dominant time constant of the process. Several solutions have been presented over the years to improve the control in such cases. The paper contributes in this direction by presenting an extension to second order stable processes of the predictive PI controller introduced by Hägglund in 1996 for first order processes. Both real and complex poles cases are considered. The solutions are derived in special forms in which the classical PID controller is maintained and a new linear block, which just requires one additional parameter, is inserted. In this way, the flexibility of the PID is conserved and control performances improve. For the sake of coherence with respect to the Hägglund's controller, the proposed solutions are called Predictive PID controllers (PPID). Simulation examples show the good performance of the PPID controllers.

Keywords: PID; SOPDT; Predictive.

1. INTRODUCTION

Dead times are found in many industry processes and they are mainly caused by the time required to transport mass, energy or information, but they can also be caused by processing time or by the accumulation of time lags in a number of simple dynamic systems connected in series. Moreover, dead-times are also used to compensate for model reduction where high-order systems are represented by low-order models with delays, Visioli [2006]. For processes exhibiting dead time, every action executed in the manipulated variable of the process will only affect the controlled variable after the process dead time. Dead times produce a decrease in the system phase and also give rise to a non-rational transfer function of the system, making them more difficult to analyse and control, Normey-Rico and Camacho [2007]. For example, in process with dead times the performance of PID (Proportional Integral Derivative) controllers (i.e. standard regulators) may by significantly decrease, especially when the dead times are large and higher than the dominant time constant of the process, Hägglund [1996].

Because of these characteristics, dead-time control problems have attracted the attention of engineers and researchers who have developed dead-time compensators. Among them, the most famous is the Smith's predictor which, undoubtedly, constitutes a milestone, Smith [1957]. However, standard Smith's predictor requires a model of the process and time consuming identification experiments are required to design the final scheme. This fact represents the main obstacle to its effective use in the industry.

For these reasons, over the years, various alternative compensators have been proposed in order to avoid the system identification task. In particular, Hägglund [1996] introduced a scheme called Predictive Proportional-Integral controller (PPI) for First Order Plus Dead Time (FOPDT) processes. Comparing with Smith's solution, the PPI does not require process identification and its tuning procedure is easier: user has only to set three parameters (the proportional gain, the integral time of a PI controller and the dead time). The flexibility available in the PPI has enabled its big spread over the past 20 years and its robustness has been analysed by several authors. Some focuses on robust PPI tuning procedures have been considered in Ingimundarson and Hägglund [2000], Ingimundarson and Hägglund [2001] while Normey-Rico et al. [1997] proposed a filtered version that improves PPI reliability. The influence of bad estimation of the process dead time on the PPI performances and certain techniques to overcome this kind of problem are presented in Veronesi [2003, 2011].

In the above cited literature, the PPI controller is derived for FOPDT process and its goodness for other kind of processes is not proved. In this sense, using an equivalent representation of the Smith's predictor, Airikka [2011] derived some PPI extensions for integral processes and models with coincident time constants. However, despite their significance, they not clearly extend the PPI approach to Second Order Plus Dead Time (SOPDT) processes.

Based on these considerations, the aim of this paper is to fill this gap by presenting a new dead time compensator for SOPDT processes. For the sake of coherence with respect to Hägglund's PPI, the proposed solution is called

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Predictive PID compensator (PPID). Process models with both real and complex stable poles are considered. The resulting schemes include a classical PID controller and an additional linear block. Only four parameters have be tuned: three of them are the PID parameters, the fourth is the process dead time. Therefore, the meaning of the adjustable parameters is preserved making the PPID suitable for applications. The paper is outlined as follow: in Section 2, PI and PID laws are presented, while the PPI is derived in a different way with respect to the original Hägglund's computation. This helps to understand notations and modus operandi adopted in the subsequent PPID formulation. In Section 3, PPID is derived for SOPDT processes with real poles and complex ones. Section 4 shows simulation results while Section 5 gives some concluding remarks.

2. PPI CONTROLLER FOR FOPDT PROCESSES

For sake of clarity, in this section PI and PID laws are given and PPI scheme is analytically derived using a direct synthesis approach.

2.1 Standard Regulators

We consider the classical unitary feedback loop, Fig. 1, where C is the controller, P represents the process, s denotes the Laplace variable, r is the set-point, y is the system output, e represents the error while u is the process input. It is assumed that $C(s)$ is a standard regulator, in a first instance. The concept of PID controller is certainly well-known to the reader. However, it is useful to recall both the PI and PID parallel forms which are used in the rest of the paper.

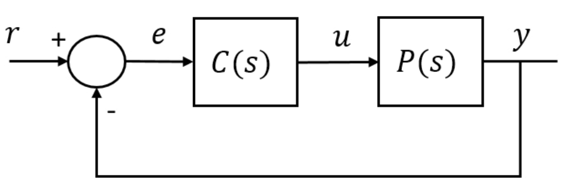

Fig. 1. feedback control scheme.

$$U_{PI}(s) = K_P\left(1 + \frac{1}{sT_I}\right)E(s), \tag{1}$$

$$U_{PID}(s) = K_P\left(1 + \frac{1}{sT_I} + sT_D\right)E(s). \tag{2}$$

In (1)-(2), the parameters K_P, T_I, and T_D are called proportional gain, integral time and derivative time, respectively. In order to improve the derivative performance in presence of noise and to ensure PID controller feasibility, it is a common choice to replace the term sT_D by the filtered version:

$$sT_d \to \frac{sT_D}{\left(1 + s\frac{T_D}{N}\right)}, \tag{3}$$

in which usually $N \in [8, 20]$, Astrom and Hagglund [2006]. A typical choice is $N = 15$.

2.2 PPI Controller

The most commonly used model to describe the dynamics of a process is the FOPDT model (4).

$$P(s) = \frac{\mu}{1 + sT}e^{-s\theta}. \tag{4}$$

By proper choice of static gain μ, time constant T, and the dead time θ, this model can adequately represent the main dynamics of many industrial processes. On the other hand, generally the FOPDT parameters are only partially known or it can be very difficult or time consuming to infer.

According to the feedback control scheme, Fig. 1, the transfer function $F(s)$, between the reference r and the output y, results:

$$F(s) = \frac{Y(s)}{R(s)} = \frac{C(s)P(s)}{1 + C(s)P(s)}. \tag{5}$$

Therefore, the controller is given by:

$$C(s) = \frac{U(s)}{E(s)} = \frac{F(s)}{P(s)\left[1 - F(s)\right]}. \tag{6}$$

By means of an analytical approach, we set the behaviour of the feedback system (such that the control design specifications are guaranteed) and then we derive the controller $C(s)$. To do this, $F(s)$ is set to be a FOPDT:

$$F(s) = \frac{1}{1 + s\alpha T}e^{-s\theta}, \tag{7}$$

where α is a degree of freedom, which denotes the ratio between desired and original process time constant, while θ is the dead time. Choosing small α entails fast output response (strong control action). Conversely, by increasing α the output dynamic becomes slower (smooth control action). It is common practice to set $\alpha = 1$, that is, the desired time constant equals the process time constant, Veronesi [2011]. Finally, by arranging (6), (7), and by defining:

$$K_P = \frac{1}{\mu\alpha}, \qquad T_I = T, \tag{8}$$

the following control law is derived:

$$U(s) = K_P\left(1 + \frac{1}{sT_I}\right)E(s) - \frac{1}{s\alpha T_I}\left(1 - e^{-s\theta}\right)U(s). \tag{9}$$

In the time domain (9) becomes:

$$u(t) = K_P e(t) + \frac{K_P}{T_I}\int_0^t e(\tau)d\tau - \frac{1}{\alpha T_I}\int_0^t \left[u(\tau) - u(\tau - \theta)\right]d\tau. \tag{10}$$

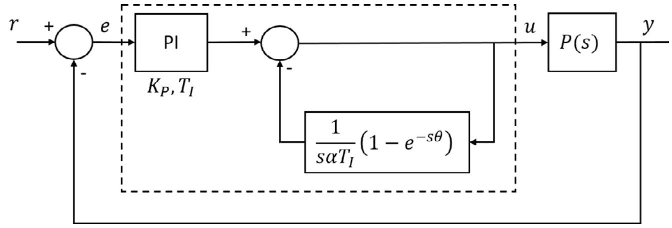

Fig. 2. PPI controller.

In (9) a low-pass filter replaces the prediction term of the standard PID law (i.e. the derivative term). For this reason, the formulation is called Predictive PI controller (PPI, Hägglund [1996]) and its scheme is represented in Fig. 2. The first part of PPI law (9) represents a PI controller in the classical parallel form in which the proportional gain and the integral time depend on the process via static gain and time constant. Since these parameters are not known (otherwise, Smith's predictor can be designed), the user has to tune the Predictive PI acting on K_P, T_I, and θ by trial and error procedure (like in a standard regulator) until the output behaviour satisfies the desired performances. In some sense, when the target behaviour is obtained, the process is also identified because its parameters are related to the PI parameters via the (8) inverse relationships:

$$\mu = \frac{1}{\alpha K_P}, \qquad T = T_I. \tag{11}$$

Obviously, this statement holds if the the process is well approximated by a FOPDT model.

3. PPID CONTROLLER FOR SOPDT PROCESSES

In this section we extend the analysis to SOPDT processes.

3.1 SOPDT with Real Poles

Let is consider the SOPDT stable process:

$$P(s) = \frac{\mu}{(1+sT_1)(1+sT_2)} e^{-s\theta}, \tag{12}$$

where $T_1 > 0$ and $T_2 > 0$ denote the time constants, which are assumed unknown, as well as the static gain μ and dead time θ. Without loss of generality we assume $T_1 \geq T_2$.

In order to derive the dead time compensator $C(s)$, the following SOPDT transfer function is imposed between reference r and output y, Fig. 1:

$$F(s) = \frac{1}{(1+s\alpha_1 T_1)(1+s\alpha_2 T_2)} e^{-s\theta}. \tag{13}$$

Similarly to the PPI case, α_1 and α_2 represent the ratios between the desired and the original process time constants. We can act on these parameters to regulate the output characteristics and to modify the closed loop poles ratio. Small values for α_1 and α_2 cause fast tracking dynamic and strong control action, while the closed loop system exhibits slow dynamics when α_1 and α_2 decrease. If $\alpha_1 = \alpha_2$, poles distance in the closed loop case is maintained unchanged with respect to the original process.

By means of (6) and (12) (13) the $C(s)$ results as follows:

$$C(s) = \frac{U(s)}{E(s)} = \frac{1}{\mu} \frac{(1+sT_1)(1+sT_2)}{(1+s\alpha_1 T_1)(1+s\alpha_2 T_2) - e^{-s\theta}}, \tag{14}$$

and the Laplace transform of the control signal $u(t)$ can be expressed as:

$$U(s) = \frac{1}{\mu}\frac{1}{\alpha_1\alpha_2}\left(1+\frac{1}{sT_1}\right)\left(1+\frac{1}{sT_2}\right)E(s) - \frac{1}{s^2}\frac{1}{\alpha_1 T_1} \cdot \frac{1}{\alpha_2 T_2}\left[1+s(\alpha_1 T_1+\alpha_2 T_2)-e^{-s\theta}\right]U(s). \tag{15}$$

The transfer functions at the second member of (15) is proper; in particular, the *auto-dependency* form the control signal is only related to past values. This ensures the feasibility of the control law (15). From (15), after some re-arrangements, the control action results as follows:

$$U(s) = \frac{1}{\mu}\frac{1}{\alpha_1\alpha_2}\frac{T_1+T_2}{T_1T_2}\left[1+\frac{1}{(T_1+T_2)s}+s\frac{T_1T_2}{T_1+T_2}\right] \cdot E(s)\frac{1}{s} - \frac{1}{s^2}\frac{1}{\alpha_1T_1\alpha_2T_2}\left[1+s(\alpha_1T_1+\alpha_2T_2) - e^{-s\theta}\right]U(s). \tag{16}$$

The first part of the second side of (16) contains the *parallel form* of a PID, in which the PID parameters are defined as follows:

$$K_P = \frac{1}{\mu}\frac{1}{\alpha_1\alpha_2}\frac{T_1+T_2}{T_1T_2}, \tag{17}$$

$$T_I = T_1 + T_2, \tag{18}$$

$$T_D = \frac{T_1T_2}{(T_1+T_2)}. \tag{19}$$

By using these parameters, (16) becomes:

$$U(s) = K_P\left(1+\frac{1}{sT_I}+sT_D\right)E(s)\frac{1}{s} - \frac{1}{s^2}\frac{1}{\alpha_1T_1\alpha_2T_2}\left\{1+s\left[\alpha_1 f_1(T_I,T_D) +\alpha_2 f_2(T_I,T_D)\right] - e^{-s\theta}\right\}U(s), \tag{20}$$

where f_1 and f_2 are defined as follows:

$$f_1(T_I,T_D) = \frac{T_I+\sqrt{T_I^2-4T_IT_D}}{2}, \tag{21}$$

$$f_2(T_I,T_D) = \frac{2T_IT_D}{T_I+\sqrt{T_I^2-4T_IT_D}}, \tag{22}$$

with $T_I \geq 4T_D$. It is worth noticing that, in (20) the explicit dependency from the unknown process parameters has been removed.

Since (20) appears as an extension of the PPI law (9) for SOPDT processes, we call this formulation Predictive PID controller (PPID), Fig. 3. For the practical implementation of the PID a pole needs to be added to the derivative part.

We can tune the PPID acting on K_P, T_I, T_D, and θ parameters by trial and error procedure until the process output satisfies the desired behaviour. When the target behaviour is obtained, also the process has been identified because its parameters are related to the PID constants via the (17)-(19) inverse relationships:

$$\mu = \frac{1}{\alpha_1 \alpha_2 K_P T_D}, \tag{23}$$

$$T_1 = \frac{T_I + \sqrt{T_I^2 - 4T_I T_D}}{2}, \tag{24}$$

$$T_2 = \frac{2T_I T_D}{T_I + \sqrt{T_I^2 - 4T_I T_D}}. \tag{25}$$

These considerations hold if the real poles SOPDT assumption for the process is satisfied.

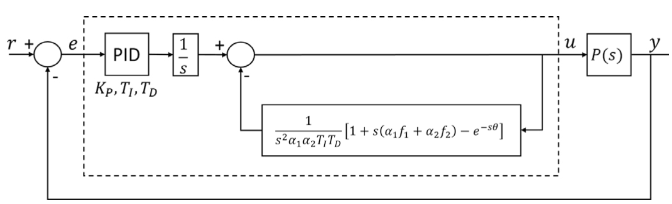

Fig. 3. PPID controller for second order processes with dead time and stable real poles.

3.2 SOPDT with Complex Poles

We consider the SOPDT process as follows:

$$P(s) = \frac{\mu\omega^2}{s^2 + 2\xi\omega s + \omega^2} e^{-s\theta}, \tag{26}$$

where the poles are complex-conjugate pairs, such that the process is stable. Static gain μ, dead time θ, natural frequency $\omega > 0$, and the damping coefficient, $0 < \xi < 1$, are not necessarily known. In order to design the Predictive PID controller for this process, we set the desired reference-output relationship as follows:

$$F(s) = \frac{\alpha_1^2\omega^2}{s^2 + 2\alpha_1\alpha_2\xi\omega s + \alpha_1^2\omega^2} e^{-s\theta}, \tag{27}$$

where $\alpha_1 > 0$ represents the ratio between desired and process natural frequency, while $\alpha_2 >$ is the ratio between desired and original damping coefficient. By tuning these parameters we can obtain the desired closed-loop dynamics (e.g. the desired settling time and overshoot for the process output). Values for α_1 and α_2 greater than one have to been set in order to reduce the settling time and the overshoot of the process output.

By means of simple calculations we obtain:

$$C(s) = \frac{U(s)}{E(s)} = \frac{\alpha_1^2\omega^2\left(s^2 + 2\xi\omega s + \omega^2\right)}{\mu\omega^2\left[s^2 + 2\alpha_1\alpha_2\xi\omega s + \alpha_1^2\omega^2\left(1 - e^{-s\theta}\right)\right]}, \tag{28}$$

from which:

$$\frac{\mu}{\alpha_1^2 s}\left[s^2 + 2\alpha_1\alpha_2\xi\omega s + \alpha_1^2\omega^2\left(1 - e^{-s\theta}\right)\right] U(s)$$
$$= 2\xi\omega\left[1 + \frac{1}{2\xi\omega^{-1}s} + (2\xi\omega)^{-1} s\right] E(s), \tag{29}$$

that leads to the PPID expression for SOPDT process with stable complex poles:

$$U(s) = K_p\left(1 + \frac{1}{T_I s} + T_D s\right) E(s)$$
$$- \frac{T_D}{\alpha_1\alpha_2 s}\left[s^2 + \frac{\alpha_1^2}{T_I T_D}\left(1 - e^{-s\theta}\right)\right] U(s). \tag{30}$$

Due to the improper nature of the second term in (30), at least two high frequency poles need to be added: a possible choice is to set two real (stable) coincident poles in $-T_D/N$, Fig 4. The same consideration is needed for the derivative part of the standard regulator.

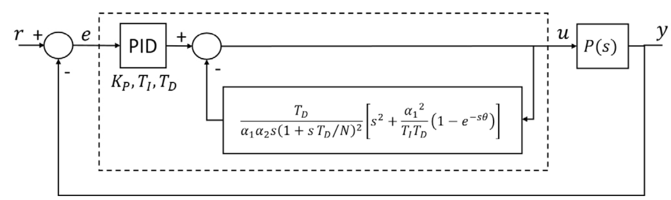

Fig. 4. PPID controller for SOPDT processes with stable complex poles.

It can be proved that in (30), the process parameters and the PID constants are related via the following equations:

$$\mu = \frac{\alpha_1}{K_P\alpha_2}, \qquad \omega^2 = \frac{1}{T_I T_D}, \qquad \xi = \frac{1}{2}\sqrt{\frac{T_I}{T_d}}, \tag{31}$$

making the PID tuning and the process identification available at the same time.

4. SIMULATION EXAMPLES

We consider the following process transfer functions:

$$P_1(s) = \frac{3}{(1+5s)(1+s)} e^{-10s}, \tag{32}$$

$$P_2(s) = \frac{3}{(1+5s)(1+4s)} e^{-10s}, \tag{33}$$

$$P_3(s) = \frac{3 \cdot 0.45^2}{s^2 + 2 \cdot 0.36 \cdot 0.45s + 0.45^2} e^{-10s}. \tag{34}$$

Table 1. Simulation Performance: ITSE.

Process	SIMC-PID	PPI	PPID
P_1	15.2	9.4	9.3
P_2	20.3	38.2	22.6
P_3, $\alpha_1 = \alpha 2 = 1$	105.5	89.5	14.3
P_3, $\alpha_1 = 1.20$, $\alpha_2 = 2.35$	37.1	44.8	3.3

Each of them is delay dominant, that is the dead time (set to $\theta = 10[s]$) is greater than the dominant time constant. The static gain is $\mu = 3$. $P_1(s)$ is a stable SOPDT process with real poles and two quite different time constants. $P_2(s)$ is a stable SOPDT, with real poles but its time constants are closely with one another. $P_3(s)$ refers to a stable SOPDT process with complex poles: the natural frequency is 0.45 [rad/s] and the damping factor is 0.36.

For all the process transfer functions we have compared the performance of PID, PPI, and PPID controllers for the unitary reference signal with step time 10[s]. Table 1 summarizes the performances of the different controllers in terms of the Integral Time Squared Error (ITSE) index.

The PID controller has been tuned via the SIMC rule, Skogestad [2003]. The PPI controller has been designed to obtain FOPDT dynamics in the process output with the same dead time of the original process. In the case of real poles processes (P_1 and P_2) the time constant for the closed loop transfer function has been set to the process dominant one, which corresponds to use $\alpha = 1$ in the PPI law (9). For the complex pole case (P_3), PPI constants have been changed until acceptable responses have been obtained. According to the paradigm proposed in the paper, the PPID has been designed to obtain the SOPDT closed loop dynamics with the same dead time of the original process. For stable real poles systems, the scheme of Fig. 3 has been used, while the structure of Fig. 4 has been reserved to the SOPDT process with stable complex poles. As expected, PPID and PPI controllers have similar performances when the process has two quite different time constants, that is when a FOPDT process may represent the plant with good approximation, Fig. 5. Significant differences arise when the process has two similar poles, Fig. 6: in this case, the PPID guarantees better performances than the PPI. Also the simply PID controller tuned via the SIMC rule seems satisfactory in this case but it requires a strong control action during the transient, Fig. 7 and the reduction of the ITSE index is negligible with respect to the PPID one (Table 1).

The case of complex poles (transfer function P_3) is depicted in Fig. 8 where the PPID has been tuned to guarantee a closed loop response with the same natural frequency and damping factor of the process ones ($\alpha_1 = \alpha_2 = 1$). By setting $\alpha_1 = 1.20$, $\alpha_2 = 2.35$ a response with low overshoot and reduced settling time can be obtained, Fig. 9.

5. CONCLUSIONS

This paper extends the Predictive PI controller, which has been introduced by Hägglund in 1996, to SOPDT processes, frequent in the practice. Both real and complex stable poles systems have been considered; correspondingly, two schemes called PPID controllers, have been derived. The benefits of the proposed solutions have been demonstrated through numerical simulations.

Since there is the possibility to tune manually PPID parameters, without time-consuming process identification experiments, PPID schemes have the same advantages of the PID. With respect to the classical PID, only one additional parameter has to be tuned: the dead time of the process. Also for this parameter, trial and error procedure can be adopted. Moreover, the paper partially relaxes Airikka [2011] assumptions, that consider processes with coincident time constants.

Extension to higher order processes with dead time remains an open challenge. The modus operandi adopted in this paper does not permit to find a solution for processes with order higher than two. For these transfer functions, there is no one to one correspondence between PID and process parameters. Therefore, the knowledge of some process time constants is needed in order to derive the *PID plus compensator* form: this knowledge is a standard request for the Smith's predictor design, but PPI and PPID have been introduced exactly to avoid it.

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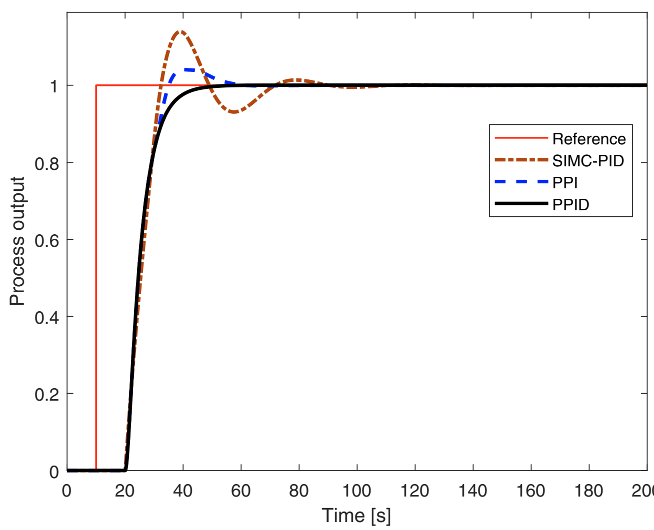

Fig. 5. P_1 process, closed loop step responses: PID, PPI, and PPID controllers.

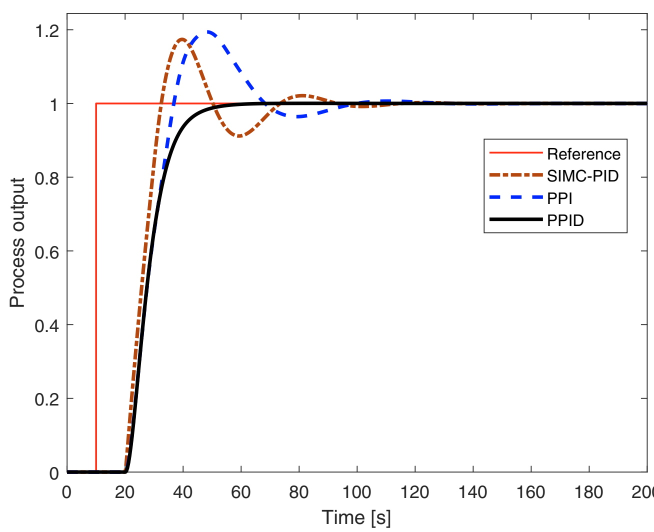

Fig. 6. P_2 process, closed loop step response: PID, PPI, and PPID controllers.

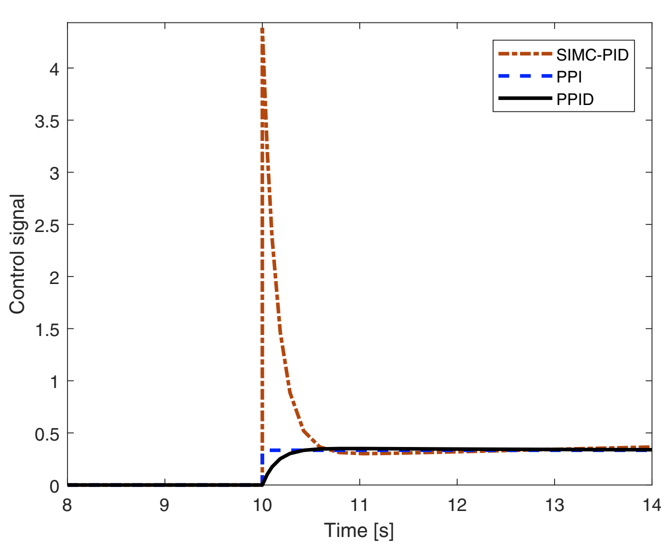

Fig. 7. P_2 process, control signals.

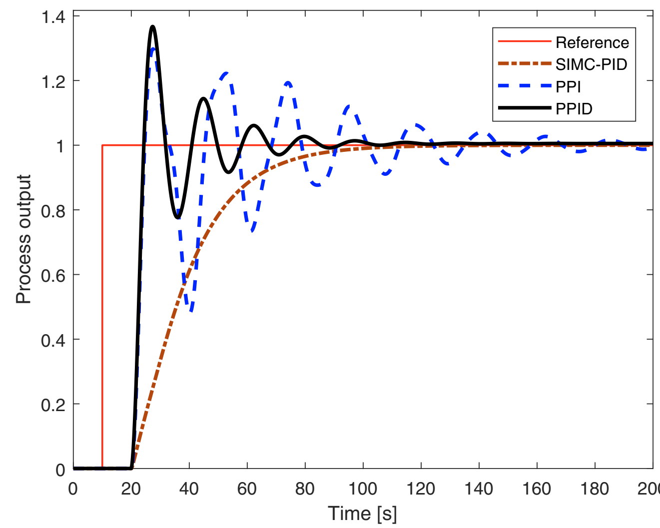

Fig. 8. P_3 process, closed loop step response: PID, PPI, and PPID (with $\alpha_1 = \alpha_2 = 1$) controllers.

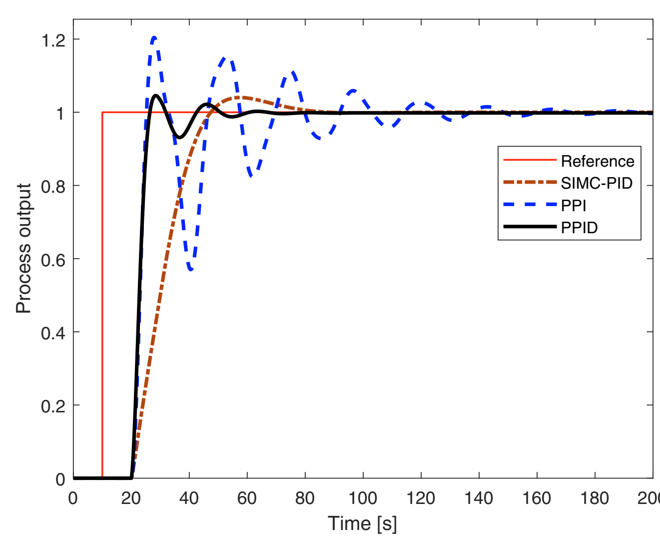

Fig. 9. P_3 process, closed loop step response: PID, PPI, and PPID (with $\alpha_1 = 1.20$, $\alpha_2 = 2.35$) controllers.