Planning Real-time Energy Efficient Trajectories for a Two Degrees of Freedom Balanced Serial Manipulator

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Abstract. The recent climate crisis and energy price rises are indicators of the necessity of a paradigm shift in the way we think industry. A more sustainable manufacturing industry is needed to tackle nowadays' challenges. Practical solutions may be either using cleaner energy sources or reducing the overall energy consumption. Efficient use of robotic systems has been identified as a promising approach, for instance re-shaping robot trajectories ensuring reduced energy consumptions without penalizing throughput at the same time. The problem is herein addressed for a particular class of manipulators, i.e. those that exhibit linear dynamics. In this case, the optimal control problem can be solved in closed form, giving the optimal solution in terms of minimum energy. The solution is obtained for a two degrees of freedom planar balanced manipulator and compared to a reference standard law. Results show a significant reduction of the energy consumption. Since the solution is obtained analytically, the computational burden allows real-time applicability.

Keywords: SDG9 · Energy Saving · Optimal Control · Robotic Manipulator

1 Introduction

The recent energy price rises and the governments' goals for sustainable growth suggest efforts on developing strategies for reducing energy expenditure (EE) in the manufacturing industry. In this scenario, both academia and industry are working to reduce the impact of energy consumption of robotic sources without affecting the throughput. For example, in [6, 5] lighter parts were designed to obtain a more efficient manipulator. Other approaches exploit elastic elements: for instance, in [2, 10, 7] a proper spring system was developed to balance gravity-related effects. Elastic elements can be introduced and tuned to obtain an efficient conversion between kinetic and potential energy, as in [9]. Software solutions have the advantage of easier implementation. Point-to-point (PTP) motions are a typical application. Previous works have exploited the tuning of given laws to obtain energy-efficient trajectories, such as [4, 3, 1]. Furthermore, not only the motion law has influence on the EE but also the location in the workspace of the given task. For example, in [11] a novel performance index was developed, able to predict the energy expenditure of a task based on the inertia ellipsoid. A more general solution for the problem of finding the minimum-energy trajectory for PTP motions lies in optimal control. On the other hand, a closed-form solution is typically difficult to obtain since the dynamics of manipulators are, in general, nonlinear. Yet, there exist particular cases that exhibit linear dynamics. That is the case, for example, of a balanced planar two Degree-of-Freedom (DoF) manipulator equipped with revolute joints. In this work, a methodology to derive optimal (in terms of minimum EE) trajectories for PTP motions is presented. This is developed for a 2 DoF balanced planar manipulator. The result is obtained using Hamilton's canonical equations.

The methodology can be applied for any manipulator that exhibits linear dynamics, for instance, cartesian robots. The aforementioned planar manipulator was chosen to show the applicability of the method also in presence of coupled dynamics.

The remainder of this paper is organized as follows. The formulation of the problem is provided in Section 2. Section 3 presents algorithms for real-time computation of energy-optimal trajectories. Numerical results in Section 4 demonstrate the benefits of the presented methodology. Conclusions are in Section 5.

2 Problem statement

2.1 Electro-mechanical model

Considering a manipulator equipped with DC motors (extension to permanent magnet synchronous motors is straightforward using the DC-equivalent model), the electrical power consumption, for each motor, can be computed as:

$$\mathcal{P}_j(t) = v_{a,j}(t) i_{a,j}(t) \tag{1}$$

where P_j is the absorbed electrical power, $v_{a,j}$ is the armature tension and $i_{a,j}$ is the armature current for the j-th motor. Based on the equivalent circuit model, the tension is made up of two terms, one related to the copper losses and one to the back electromotive force:

$$v_{a,j}(t) = R_{a,j} i_{a,j}(t) + L_{a,j} \frac{d\, i_{a,j}(t)}{dt} + k_{v,j}\, \dot{\vartheta}_j \tag{2}$$

where $R_{a,j}$ and $L_{a,j}$ are the resistance and inductance of the armature of the j-th motor. $k_{v,j}$ is the back electromotive force (EMF) constant and $\dot{\vartheta}_j$ is the angular velocity of the j-th motor. The inductive term of the armature tension is not included since it is negligible with respect to the other two terms. In general, the armature current and the torque τ delivered by each motor are related by the torque constant $i_a = \tau / k_t$. As known, numerically the torque constant is equal

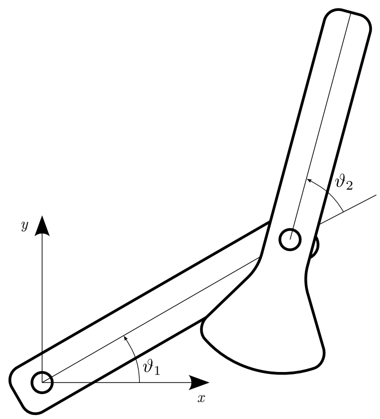

Fig. 1. Two Degree-of-Freedom balanced planar manipulator.

to the back EMF constant $k_t = k_v$. By substituting the current with the torque divided by the torque constant in (1), the following expression is obtained:

$$\mathcal{P}_j = \frac{R_{a,j}}{k_{t,j}^2\, k_{r,j}^2}\tau_j^2 + \tau_j\dot{\vartheta}_j \tag{3}$$

where $k_{r,j}$ is the ratio of the j-th gearbox. The efficiency of the gearbox is considered $\eta = 1$ for simplicity, but the derivation is straightforward with $\eta < 1$.

Referring to a two degrees of freedom planar manipulator equipped with revolute joints, some conditions are needed to exhibit linear dynamics. For example, a possible configuration is with the second link balanced (its center-of-mass is coincident with the second revolute joint) in the horizontal plane. A schematic is depicted in Fig. 1. The dynamics are:

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = \begin{bmatrix} I_1 + m_1\ell_1^2 + m_2 a_1^2 & I_2\\ I_2 & I_2\end{bmatrix}\begin{bmatrix}\ddot{\vartheta}_1\\ \ddot{\vartheta}_2\end{bmatrix} + \begin{bmatrix}F_1 & 0\\ 0 & F_2\end{bmatrix}\begin{bmatrix}\dot{\vartheta}_1\\ \dot{\vartheta}_2\end{bmatrix} \tag{4}$$

where I_j and F_j are, respectively, the total barycentric mass moment of inertia and viscous friction coefficient of the $j-th$ link. m_2 is the mass of the second link and a_1 the length of the first link, according to the Denavit-Hartenberg convention. From here onwards, the symbol $I_1^* = I_1 + m_1\ell_1^2 + m_2 a_1^2$ will be used for the sake of compactness. Substituting (4) into (3) gives the following expression for the powers required:

$$\begin{cases} \mathcal{P}_1 = \dfrac{R_{a,1}}{k_{t,1}^2\, k_{r,1}^2}\left(I_1^*\ddot{\vartheta}_1 + I_2\ddot{\vartheta}_2 + F_{m,1}\dot{\vartheta}_1\right)^2 + \left(I_1^*\ddot{\vartheta}_1 + I_2\ddot{\vartheta}_2 + F_{m,1}\dot{\vartheta}_1\right)\dot{\vartheta}_1\\ \mathcal{P}_2 = \dfrac{R_{a,2}}{k_{t,2}^2\, k_{r,2}^2}\left(I_2\ddot{\vartheta}_1 + I_2\ddot{\vartheta}_2 + F_{m,2}\dot{\vartheta}_2\right)^2 + \left(I_2\ddot{\vartheta}_1 + I_2\ddot{\vartheta}_2 + F_{m,2}\dot{\vartheta}_2\right)\dot{\vartheta}_2 \end{cases} \tag{5}$$

Clearly, the total power consumption is the sum of the power consumed by each motor.

2.2 Trajectory requirements

The focus of this work is on PTP motion with fixed final time t_f. This means that the trajectory has to satisfy the following boundary conditions:

$$\begin{cases} \vartheta_1(0) = \vartheta_{in,1} \\ \vartheta_1(t_f) = \vartheta_{fin,1} \\ \vartheta_2(0) = \vartheta_{in,2} \\ \vartheta_2(t_f) = \vartheta_{fin,2} \end{cases} \quad \begin{cases} \dot{\vartheta}_1(0) = 0 \\ \dot{\vartheta}_1(t_f) = 0 \\ \dot{\vartheta}_2(0) = 0 \\ \dot{\vartheta}_2(t_f) = 0 \end{cases} \tag{6}$$

where $\vartheta_{in,j}$ and $\vartheta_{fin,j}$ are the initial and final values of the angular position, respectively, for the j-th motor.

2.3 Problem formulation

Since the goal is to minimize the overall energy expenditure, the integral of power has to be minimized. To do so, the problem is rewritten in state-space form, i.e. $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t)$. Let $[\vartheta_1 \ \dot{\vartheta}_1 \ \vartheta_2 \ \dot{\vartheta}_2]^\intercal = [x_1 \ x_2 \ x_3 \ x_4]^\intercal = \mathbf{x}$ and $[\ddot{\vartheta}_1 \ \ddot{\vartheta}_2]^\intercal = [u_1 \ u_2]^\intercal = \mathbf{u}$. The dynamics of the system is as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = u_1 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = u_2 \end{cases} \tag{7}$$

By substituting the symbols of the state-space form, the power required can be rewritten as:

$$\mathcal{P} = \pi_1 x_2^2 + \pi_2 x_4^2 + \pi_3 u_1^2 + \pi_4 u_2^2 + \pi_5 x_2 u_1 + \pi_6 x_2 u_2 + \pi_7 x_4 u_1 + \pi_8 x_4 u_2 + \pi_9 u_1 u_2 \tag{8}$$

where $\mathcal{P} = \mathcal{P}_1 + \mathcal{P}_2$ and the parameters π_j are readily obtained from (5). Hence, the optimal control problem at hand is:

$$\begin{aligned} \underset{u \in \mathcal{U}}{\text{minimize}} \quad & E = \int_0^{t_f} \mathcal{P}(\mathbf{x}, \mathbf{u}, t) \, dt \\ \text{subject to} \quad & (6) \\ & \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t) \end{aligned} \tag{9}$$

where t_f is the prescribed final time and $\mathcal{U}$ is the space of the possible controls according to the boundary conditions (6). In this work no constraints, such as torque and velocity limits, are imposed other than the boundary conditions.

Let t_{min} be the minimum time, according to the constraints, to perform the task. In this work, it is assumed that the assigned final time t_f is strictly greater than t_{min}. If $t_f < t_{min}$, there is clearly no solution. If $t_f = t_{min}$ the solution is the minimum time solution.

3 Solution

The problem stated in (9) can be solved using Pontryagin's maximum principle. The Hamiltonian of the system is:

$$\mathcal{H} = \mathcal{P} + \mathbf{p}^{\intercal}\mathbf{f} \tag{10}$$

where $\mathcal{P}$ is given by (8), $\mathbf{f}$ by (7) and $\mathbf{p} = [p_1\ p_2\ p_3\ p_4]^{\intercal}$ is the vector of co-states. The problem can be solved imposing the boundary conditions given by (6) since for a fixed final-time problem with n states, we can apply $2n$ boundary conditions. The necessary conditions for strong extrema are provided by Hamilton Canonical Equations. The stationary conditions are:

$$\frac{\partial \mathcal{H}}{\partial u_1} = 2\pi_3 u_1 + \pi_5 x_2 + \pi_7 x_4 + \pi_9 u_2 = 0 \tag{11}$$

$$\frac{\partial \mathcal{H}}{\partial u_2} = 2\pi_4 u_2 + \pi_6 x_2 + \pi_8 x_4 + \pi_9 u_1 = 0 \tag{12}$$

while the co-state equations are:

$$\frac{\partial \mathcal{H}}{\partial x_1} = 0 = -\dot{p}_1 \longrightarrow p_1 = \text{const} \tag{13}$$

$$\frac{\partial \mathcal{H}}{\partial x_2} = 2\pi_1 x_2 + \pi_5 u_1 + \pi_6 u_2 = -\dot{p}_2 \tag{14}$$

$$\frac{\partial \mathcal{H}}{\partial x_3} = 0 = -\dot{p}_3 \longrightarrow p_3 = \text{const} \tag{15}$$

$$\frac{\partial \mathcal{H}}{\partial x_4} = 2\pi_1 x_2 + \pi_5 u_1 + \pi_6 u_2 = -\dot{p}_4 \tag{16}$$

By deriving (11) and (12) with respect to time, isolating $\dot{p}_2$ and $\dot{p}_4$ and substituting, respectively, into (14) and (16), it is possible to write them in the following form:

$$\dot{\mathbf{y}} = \mathbf{A}\mathbf{y} + \mathbf{b} \rightarrow \begin{bmatrix} \dot{x}_2 \\ \dot{x}_4 \\ \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ \alpha_5 & \alpha_6 & \alpha_7 & \alpha_8 \end{bmatrix} \begin{bmatrix} x_2 \\ x_4 \\ u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \alpha_9 p_1 + \alpha_{10} p_3 \\ \alpha_{11} p_1 + \alpha_{12} p_3 \end{bmatrix} \tag{17}$$

It is worth noticing that p_1 and p_3 are unknown but constant. The parameters α_j are readily obtained from (11), (12), (14) and (16). The solution of (17) is the sum of two terms, a particular solution and the solution of the homogeneous associate system. In mathematical terms $\mathbf{y} = \mathbf{y}_{\text{st}} + \mathbf{y}_{\text{om}}$. The particular solution is straightforward by considering $\mathbf{y}_{\text{st}} = \text{const}$:

$$0 = \mathbf{A}\mathbf{y}_{\text{st}} + \mathbf{b} \rightarrow \mathbf{y}_{\text{st}} = -\mathbf{A}^{-1}\mathbf{b} = \mathbf{R}\,[p_1\ p_3]^{\intercal} \tag{18}$$

where in the last step the co-state variables are isolated using the 4×2 matrix $\mathbf{R}$. The solution of the associate homogeneous system is:

$$\mathbf{y}_{\text{om}}(t) = c_1\mathbf{v_1}e^{\lambda_1 t} + c_2\mathbf{v_2}e^{\lambda_2 t} + c_3\mathbf{v_3}e^{\lambda_3 t} + c_4\mathbf{v_4}e^{\lambda_4 t} \tag{19}$$

where c_j-s are integration constants, $\mathbf{v_j}$-s and λ_j-s are, respectively, the eigenvectors and eigenvalues of $\mathbf{A}$. Once (18) and (19) are obtained, the final solution will be the sum of the two:

$$\mathbf{y} = c_1\mathbf{v_1}e^{\lambda_1 t} + c_2\mathbf{v_2}e^{\lambda_2 t} + c_3\mathbf{v_3}e^{\lambda_3 t} + c_4\mathbf{v_4}e^{\lambda_4 t} + \mathbf{R}\,[p_1\ p_3]^\intercal \tag{20}$$

The integration of $\mathbf{y}$ is necessary to obtain the position law. In particular, integrating the first two terms of $\mathbf{y}$ gives:

$$x_1(t) = \int \{\mathbf{y}\}_1 dt = \sum_{k=1}^{4} \frac{1}{\lambda_k} c_k e^{\lambda_k t}\{\mathbf{v_k}\}_1 + r_{11}p_1 t + r_{12}p_3 t + c_5 \tag{21}$$

$$x_3(t) = \int \{\mathbf{y}\}_2 dt = \sum_{k=1}^{4} \frac{1}{\lambda_k} c_k e^{\lambda_k t}\{\mathbf{v_k}\}_2 + r_{21}p_1 t + r_{22}p_3 t + c_6 \tag{22}$$

where $\{(\cdot)\}_n$ means the *n-th* component of $(\cdot)$, and r_{ij} is the element in the *i-th* row and *j*-th column of $\mathbf{R}$. Using the first two terms of vector $\mathbf{y}$ defined as in (20) and the position laws given by (21) and (22) it is possible to impose the boundary condition stated in (6). In particular the following 8×8 linear system in the unknowns $[c_1, \ldots, c_6,\ p_1,\ p_3]$ has to be solved:

$$\begin{bmatrix} \mathbf{V} & \mathbf{0_2} & \mathbf{V} \\ \mathbf{V\Lambda} & \mathbf{0_2} & \mathbf{V} \\ \mathbf{V\Gamma} & \mathbf{1_2} & \mathbf{0_2} \\ \mathbf{V\Lambda\Gamma} & \mathbf{1_2} & t_f\mathbf{V} \end{bmatrix} \begin{bmatrix} c_1 \\ \vdots \\ c_6 \\ p_1 \\ p_3 \end{bmatrix} = \begin{bmatrix} \mathbf{0_{4,1}} \\ \vartheta_{in,1} \\ \vartheta_{in,2} \\ \vartheta_{fin,1} \\ \vartheta_{fin,2} \end{bmatrix} \tag{23}$$

where $\mathbf{V}$ is the matrix of the eigenvectors of $\mathbf{A}$, $\mathbf{\Lambda} = \mathrm{diag}(e^{\lambda_1 t_f},\ \ldots\ , e^{\lambda_4 t_f})$ and $\mathbf{\Gamma} = \mathrm{diag}(1/\lambda_1, \ldots, 1/\lambda_4)$. With $\mathbf{0_{n,m}}$ and $\mathbf{1_{n,m}}$ are indicated the $n \times m$ null and the identity matrix[1]. Once the constants are obtained, the solution is completely known using equation (21) and (22). By deriving them, also velocity and acceleration laws are known.

4 Results

In this section, numerical examples are provided to show the benefits of the method. First, the solution is compared to a standard law, such as the trapezoidal symmetric velocity law with a 20% acceleration ratio (amount of time with nonzero acceleration over total time t_f). The data used for the comparison are reported in Tab. 1. The energy consumed in the proposed case is 13.1 % less than in the standard case (6.43 J vs 7.40 J).

Moreover, the proposed solution is benchmarked with the one provided by a general open source numerical optimal control toolbox, ICLOCS2 [8]. In Fig. 2 a comparison between the proposed solution, the one generated by ICLOCS and the standard trapezoidal law is reported.

[1] If only one subscript is used, the matrix is considered square.

Table 1. Manipulator data used in the simulations.

Parameter	I_j	F_j	$R_{a,j}$	$k_{t,j}$	$k_{r,j}$	m_j	a_j	ℓ_j	$\vartheta_{in,j}$	$\vartheta_{fin,j}$
Unit	kg m^2	N m s/ rad	Ω	N m/A	-	kg	m	m	rad	rad
$j = 1$	0.21	$2 \cdot 10^{-3}$	1.0	0.2	50	2.0	0.175	0.0875	0	0
$j = 2$	0.0675	$2 \cdot 10^{-3}$	1.0	0.7	50	4.0	0.225	0	$\pi/2$	$\pi/2$

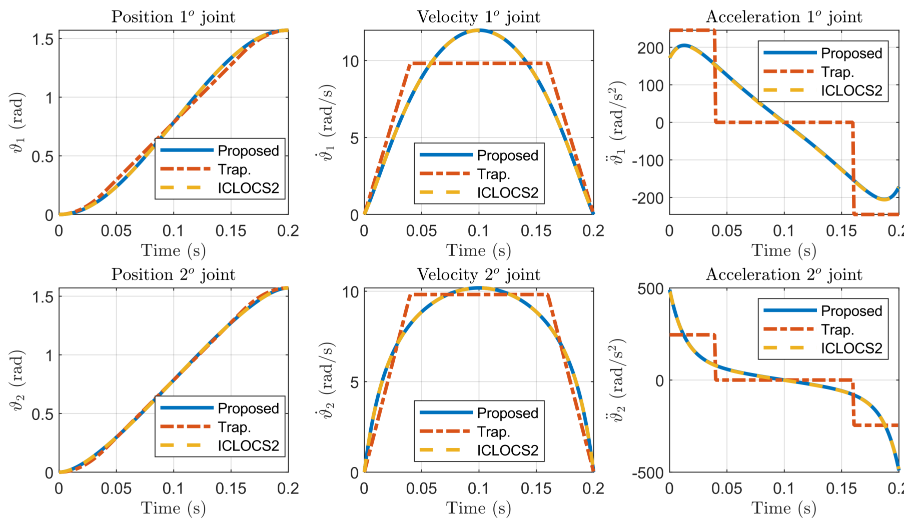

Fig. 2. The proposed solution is reported in blue. The trapezoidal law is reported in dotted orange line. The benchmark (ICLOCS2) solution is reported in yellow.

Due to the numerical nature of the ICLOCS solution, the required solving time is higher than for the herein proposed analytical solution. In particular, the computational time required by ICLOCS is 3 seconds against the 0.008 seconds of the proposed method (data obtained using a laptop equipped with a Ryzen 5 4500U and 8 GB of RAM). By comparing such figures with the task time t_f, the proposed method is deemed real-time capable for this application (0.008 s $< t_f$), that is an important benefit with respect to ICLOCS.

5 Conclusions

Reducing the energy expenditure in the manufacturing industry is important to face nowadays challenges. This paper presented a method for deriving optimal trajectories for point-to-point motions in terms of minimum energy expenditure for a two degrees of freedom balanced planar manipulator. This is achieved using Hamilton's canonical equations, and can be applied to all manipulators that exhibit linear dynamics. The validity of the proposed methodology is demonstrated through numerical results, showing its potential for reducing energy consumption and its real-time capability. Overall, the method provides a viable solution that can be implemented to reduce energy expenditure in robotic systems without affecting throughput.

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