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Testing asymmetry in financial time series

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1 Introduction

Financial time series and their statistical modeling has been studied in depth in the last few decades, and the huge amount of work in this area has led to a quite general consensus on some empirical features known as *stylized facts*. Non-normality of financial returns, excess of kurtosis, havy tails and clustering effects are examples of *stylized facts*. However, there are some statistical characteristics that are still disputable both because empirical findings are not univocal and because the tools to detect them correctly are relatively recent.
One of the questionable features of financial time series is skewness of the unconditional distribution of returns[1]. Although some authors found or assumed relevant asymmetries in the return distributions (e.g. Kim and White (2004), Engle and Patterson (2001), Cont (2001), Chen *et al.* (2001)), others (e.g. Premaratne and Bera (2005), Peiró (2004)) are more doubtful about the pervasive presence of skewness in returns and believe that, in many cases, it is due to the use of unsuitable measurement tools.
However, relatively little work has been done to detect skewness with respect to other characteristics. This is curious, considering that skewness, besides being important from a statistical point of view, is also relevant from a financial one because it may be considered as a further measure of risk. For example, Kim and White (2003) stress that, if investors prefer right-skewed portfolios then, for equal variance, one should expect a "skew premium" to reward investors willing to invest in left-skewed portfolios. With respect to optimal portfolio allocation, Chunhachinda

[1]Note that here we are not referring to the asymmetrical effects that negative and positive returns can have on volatility, but to the symmetry of the marginal distribution of returns.

et al. (1997) showed that it can change considerably if higher than second moments are considered in selection. Along the same lines, Jondeau and Rockinger (2004) measured the advantages of using a strategy based on high-order moments. Other examples of the economic and financial importance of asymmetry are given by Peiró (2004).
In view of the importance attributed to symmetry in the literature, we believe it is of interest on one hand to go deeper into this point and, on the other, to have available statistical tests that can correctly identify the presence of asymmetry in data.
Over the years, various measures of sample skewness have been proposed and studied (e.g. Kim and White, 2004; Joanes and Gill, 1998). However, most of the empirical and theoretical works regarding financial markets have used the conventional measure of skewness given by the standardized third moment

$$S = \frac{\mu_3}{\mu_2^{3/2}}, \tag{1}$$

where μ_j is the $j-$th central moment. It is well-known (e.g. Kendall and Stuart, 1969) that the estimate $\hat{S}$ of S, obtained by replacing the corresponding sample moments in (1), under the hypothesis $S = 0$, has a gaussian asymptotic distribution which allows symmetry to be tested. However, the variance of this distribution depends crucially on hypotheses of gaussianity and independence of data. Several authors (e.g. Bai and Ng 2005; Premaratne and Bera 2005, Peiró 1999, 2004; Lupi and Ordine 2001) have noted that the assumptions of gaussianity and independence are not realistic in several contexts, including that of financial returns. Some of these authors have also shown how the variance of the asymptotic distribution of the sample skewness coefficient changes when one or more of these assumptions are relaxed.
Within this context, the aim of the present work was to examine the problem of asymmetry in financial time series, starting from a comparative analysis of various existing tests. In order to overcome some of their limitations, a bootstrap test is proposed, and its performance was studied by means of Monte Carlo simulations. The tests were the applied to 72 real time series.

2 Testing for skewness

This section briefly reviews some symmetry tests proposed in the literature, based on the standardized third moment in order to highlight their adavantages and disadvantages.
When data are generated by an i.i.d. gaussian process, it is well-known (see, for example Kendall and Stuart, 1969) that, asymptotically,

$$\sqrt{\frac{n}{6}}\hat{S} \xrightarrow{d} N(0,1). \tag{2}$$

Thus, for practical purposes, we can consider the relationship $\hat{S} \sim N(0, \sqrt{\frac{6}{n}})$ for testing symmetry.

Although this limiting distribution has been widely used in several contexts, and often in the analysis of financial data, it is clear that its applicative framework cannot be generalized and that, in particular, it cannot be extended to time series. In this regard, it is curious to note that, several types of software, even those created for applications to dependent data, make use of distribution (2) for their analyses. For example, when implementing the Jarque and Bera test, which in turn is based on the distribution of $\hat{S}$, the distribution implied by (2) is used by *EViews*, the *Finmetrics* module of S+, the G@rch module of OX, and the library *tseries* di R.
When data are correlated, Lomnicki (1961) proved that, for gaussian generator processes that can be written in a moving average form such as $y_t = \theta(L)\varepsilon_t$, con $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$, then asymptotically

$$\sqrt{\frac{n}{6}} \left(\sum_{j=-\infty}^{\infty} \rho_j^3 \right)^{-1/2} \hat{S} \xrightarrow{d} N(0,1), \tag{3}$$

where ρ_j is the autocorrelation coefficient at lag j.
However, for non-gaussian data, and specifically for data the distribution of which is leptokurtic or platikurtic, previous results no longer hold good either in the dependent or independent case. In particular, for leptokurtic distributions the variance of the test statistics is underestimated and leads to rejection of the null hypothesis of symmetry too often, whereas the opposite occurs for platikurtic distributions, making the test too conservative with respect to the hypothesis of symmetry. Leaving for the moment normality, Premaratne and Bera (2005), exploiting a result of Godfrey and Orme (1991), derived the distribution of $\hat{S}$ under the hypothesis of symmetry for i.i.d. but not necessaily gaussian data. In particular, assuming the existence of moments up to the sixth, they showed that asymptotically $V_1^{-1/2}(\hat{S}) \xrightarrow{d} N(0,1)$ with

$$V_1 = \frac{1}{n}\left(9 + \mu_6\mu_2^{-3} - 6\mu_4\mu_2^{-2}\right) \tag{4}$$

and μ_j j-th central moment. In this case, therefore, the variance of the distribution of $\hat{S}$ depends on the second, fourth and sixth moments.
In their work, Premaratne and Bera (2005) show by Monte Carlo simulations that their test works properly for i.i.d. data but apply it to real time series without any simulation. Recently, Bai and Ng (2005) derived the limiting distribution of $\hat{S}$ in the more general case of dependent data, not necessarily gaussian, and under an arbitrary skewness coefficient S. Assuming the existence of the sixth moment and some mixing conditions which guarantee that the central limit theorem holds for the 4×1 vector series $W_t = [Y_t - \mu, (Y_t - \mu)^2 - \sigma^2, (Y_t - \mu)^3 - \mu_3, (Y_t - \mu)^4 - \mu_4]$, they found that, under the hypothesis $S = 0$, $V_2^{-1/2}(\hat{S}) \xrightarrow{d} N(0,1)$ with

$$V_2 = \frac{1}{n}\frac{\alpha\Gamma\alpha'}{\sigma^6}, \tag{5}$$

where $\alpha = [1, -3\sigma^2]$ and Γ is the 2×2 matrix defined as $\Gamma = \lim_{n\to\infty} nE(\bar{Z}\bar{Z}')$, with $\bar{Z}$ sample mean of

$$Z_t = \begin{bmatrix} (Y_t - \mu)^3 \\ (Y_t - \mu) \end{bmatrix}. \tag{6}$$

In this framework, the serial dependence in Y_t is explained through Γ, which represents the spectral density matrix of Z_t at frequency 0. It is not difficult to show that, in the independent case, the Bai and Ng test reduces to that of Premaratne and Bera which is, thus, a particular case of the former.

Both Bai and Ng's and Premaratne and Bera's test have the drawback of requiring the existence of the sixth moment. This means, for example, that they cannot be applied to $t-$Student distributions, t_ν, with $\nu \leq 6$, because clearly only moments of orders less than the degree of freedom exist.
This fact, which is not particularly important in some contexts, becomes very important in the case of financial time series, since they have leptokurtic and heavy tail marginal distribution and, therefore, the existence of high-order moments can not taken for granted and should generally be verified. Instead, in real applications is quite common to estimate models which do not admit the sixth moment. An example is given by a common GARCH(1,1) model with $t-$Student innovations. Table 1 lists the results of parameter estimation of such a model for four cases in which conditional distributions do not have the sixth moment. In addition, if we consider that the marginal distribution has higher kurtosis than the conditional this problem is clearly one which can influence several real financial time series. This consideration is in line with the findings of Chen (2001), which in an empirical study investigated the moment conditions of daily excess returns of twelve major stock indices and found that all the returns have finite third moments but not finite sixth moments. Other authors who showed that the existence of the sixth moment is too restrictive for economic and financial data are Jansen and de Vries (1991), Loretan and Phillips (1994), and de Lima (1997).

Series	Period	ω	α	β	ν
Motorola	01/03/95 - 09/02/01	$1.8 * 10^{-4}$	0.055	0.925	5.4
Pepsi	01/03/95 - 09/02/01	$4.0 * 10^{-6}$	0.041	0.943	5.9
3M	10/01/99 – 01/10/04	$2.1 * 10^{-6}$	0.036	0.956	5.2
SEAT pg	22/09/98 – 10/01/04	$6.1 * 10^{-6}$	0.0800	0.911	4.9

Table 1: Estimates of a GARCH(1,1) model with t-Student innnovations for some real time series. Parameters α, β and ν are all significant at 5% level. Parameter ν represents degrees of freedom.

3 A bootstrap test of skewness

To bypass the problem of the existence of the moments, in this section we propose a bootstrap test which only requires the existence of moments up to the third. This is the minimum requirement for the asymmetry coefficient to exist. The procedure involves a first filtering phase through ARMA models, in order to account for possible linear dependence in the data. On the whole, however, the algorithm can be made completely automatic.
The basic idea of the procedure is to use observed data to obtain a distribution in

such a way that it is symmetric, and use it to calculate critical values. The procedure for testing the hypothesis system $H_0 : S = 0$, $H_1 : S \neq 0$ is the following:

1. Given a time series $\{y_t\}, t = 1, ..., n$, fit a suitable ARMA(p, q) model. Orders p and q can be chosen by automatic criteria, i.e. those of Akaike or Schwarz. Let e_t be the series of the residuals of the model, i.e. $e_t = y_t - \hat{y}_t$.
2. For the series e_t calculate $\hat{S}_e$.
3. Define $e_t^* =\mid e_t - me \mid$, where me is the median of e_t and $\mid \cdot \mid$ denotes the absolute value.
4. Generate the bootstrap series

$$\tilde{e}_t = me + e_t^* * z_t \qquad\qquad t = 1, 2, ..., n$$

where e_t^* is sampled with replacement from the empirical distribution of e^*, and z_t is such that $P(z_t = -1) = P(z_t = 1) = 1/2$. The series $\tilde{e}_t$ represents a symmetrized version of e_t.

5. For the series $\tilde{e}_t$ calculate the skewness coefficient $\hat{S}$.
6. Repeat steps 4) and 5) M times, with large M, yielding M bootstrap replications $\tilde{e}_t^{(i)}$ and the corresponding estimates $\hat{S}^{(i)}$ for $i = 1, ..., M$.
7. Consider the bootstrap distribution of $\hat{S}$ obatined through M estimates $\hat{S}^{(i)}$ and find quantiles $\hat{S}_{\alpha/2}$ and $\hat{S}_{1-\alpha/2}$.
8. Accept H_0 at level α if $\hat{S}_{\alpha/2} \leq \hat{S} \leq \hat{S}_{1-\alpha/2}$.

At the end of the algorithm, the following note is appropriate. In step 4) the data were handled as if they were independent, whereas, in general, they are not. However, this simplification is less strong it might appear at first glance. After some algebra, it is possible to show that an estimate of Γ in (5) is given by

$$\hat{\Gamma} = \begin{bmatrix} m_6 + \sum_{j=1}^{n-1} \hat{\gamma}_{y^3}(j) & m_4 + \sum_{j=1}^{n-1} \hat{\gamma}_{y^3,y}(j) \\ \\ m_4 + \sum_{j=1}^{n-1} \hat{\gamma}_{y^3,y}(j) & m_2 + \sum_{j=1}^{n-1} \hat{\gamma}_{y}(j) \end{bmatrix}$$

where m_j is an estimate of the $j-$th central moment, $\gamma_{y^r}(j)$ is the autocovariance of y_t^r, and $\gamma_{y^3,y}(j)$ is the cross-covariance between y_t^3 and y_t. This means that the only dynamic quantities that enter variance V_2 are the autocorrelations of y_t, those of y_t^3 and the cross-covariance between y_t and y_t^3. The other quantities, i.e. the second, fourth and sixth moments, are not dynamic features.

Since the autocorrelation of y_t has already been accounted for in step 1), we only neglect the cross-correlation between third moments and the correlation between first and third moment. When these quantities are not important or even absent, the bootstrap distribution can be considered a good approximation of the true distribution.

4 Validation

After describing the procedure for the bootstrap test we now must validate it, and Monte Carlo simulations are used to study the real level and power of the test and to compare them with those of other tests. For the bootstrap test, the orders of model at step 1) are chosen by minimizing the Schwarz criterion. In addition, the bootstrap distribution of $\hat{S}$ is obtained using $M = 10000$ replications; in some pilot analyses, increasing M to 25000 did not change the results in any particular way.
Lastly, in all simulations, bilateral tests at level $\alpha = 10\%$, 5% and 1% are carried out.
The data are generated by processes (DGP) unlike the dependence structure and the characteristics of marginal distributions. The 18 processes and their coefficients of asymmetry and kurtosis are listed in the Appendix.
For each generator process, evaluation of the real level and power of the test is based on 2000 Monte Carlo replications of length $n = 100$, 200 and 500. When working with financial time series, these values correspond to series of very short, short, and medium lengths.
The analyses are divided into three parts: i) for independent data, comparison with test performance based on (2), called ASS (*asymptotic sample skewness*), of the Premaratne and Bera (BP), Bai and Ng (BN) and Bootstrap (BTP) test; 2) the same analyses are conducted on dependent data; 3) some applications to real time series are also considered.
For full comparisons, all the results of the simulations refer to the application of the tests on the same time series.

4.1 Independent data

To study the real level of the tests in the i.i.d. case, four symmetrical distributions are considered, S1, S2, S3 and S4. They are: standard normal, t-Student with seven degrees of freedom; Beta(2,2) and a distribution belonging to the Generalized Lambda family, which was also considered by Bai and Ng (2005). This family contains symmetrical and asymmetrical distributions which can be generated in terms of the inverse of the cumulative distribution function $F^{-1}(u) = \lambda_1 + \left[u^{\lambda_3} - (1-u)^{\lambda_4}\right]/\lambda_2$, $0 < u < 1$ (see, for example, Karian and Dudewicz (2000)).
To evaluate the adequacy of the nominal and real levels a binomial, the following hypothesis system was verified by a binomial test

$$\begin{aligned} H_0 &: p = p_0 \\ H_1 &: p \neq p_0 \end{aligned} \tag{7}$$

with $p = 0.1,\ 0.05,\ 0.01$, depending on significance level. The results (Table 4) indicate that only ASS in the gaussian case gives real levels statistically equal to the nominal ones for all three values of n. For gaussian data, the other three tests give real levels not significant different from the nominal ones only for $n = 500$. For $n = 200$ and 100 the real levels are lower but on the whole satisfactory at levels 10% and 5%, slightly less at 1%. For non-gaussian but symmetrical distributions with

leptokurtosis, the ASS test rejects the hypothesis of symmetry too often. The higher the kurtosis, the higher the real level. Instead, when the distribution is platikurtic, the test is too conservative and the null hypothesis is almost never rejected. In both these situations, nominal and real sizes are very different.
With regard to BOOT, PB and BN tests, the results of Table 4 clearly face leptokurtosis and platikurtosis correctly and have similar real levels. In addition, although the null hypothesis of system (7) is not always acceptable, real levels are comparable with nominal ones, particularly as n grows. The only exception is the 1% level for which all three test have much smaller real levels.
Note also that, in the i.i.d. case, PB and BN are the same and provide almost identical results. Analyses concerning the power of the tests were conducted on five asymmetrical distributions (A1, A2,...,A5) with different degrees of kurtosis. In this case distributions were Beta(2,1), two Generalized Lambda, Skew Normal, and Skew t. The last two distribution families were introduced by Azzalini (1985, 1986). The general framework of the experiment is identical to that for study of test levels.
The results of simulations are given in Table 5. As expected, power grows with series length, intensity of asymmetry and nominal test level.
For the reasons described above ASS test always has the greatest power, and the largest differences are found between ASS and the other three tests. Differences between BTP, PB and BN are smaller. To assess their significance, a binomial test was applied to the test powers. In this case, the binomial test concerns couples of powers p_1 and p_2 and the examined hypothesis system is

$$\begin{aligned} H_0 &: p_1 \le p_2 \\ H_1 &: p_1 > p_2. \end{aligned} \tag{8}$$

where, conventionally, it is assumed that p_1 is always the larger of the two powers in question.
The hypothesis system (8) is verified only for the powers of BTP and BN, BP being a particular case of BN. In Table 5, the asterisk means that the null hypothesis is rejected at 5% level and the circle denotes the same conclusion at 1% level. For example, if we consider the power of BTP for $A6$, at a nominal level of 5% and for $n = 500$, the circle means that the power 78.8 is significantly greater, at 1% level, than the value 71.2% reached by the BN test.
This kind of analysis shows that in two out of the five processes, the power of BTP is significantly greater than that of BN; the opposite, is true only in a few isolated cases. In more detail, it indicates that, when kurtosis is very high, BTP has more power.

4.2 Dependent data

If we wish to apply the tests to financial time series, we must to study their behaviors and performances under more general assumptions. We therefore concentrate on data with some dependence structure.
As the BP test is based on the independence hypothesis, hereafter only BTP and BN will be examined and compared. For the same reason, the results of the ASS

test are reported for the sake of comparison but are not discussed.
When data are serially correlated, the distribution of $\hat{S}$ changes. If autocorrelation is neglected test performances can be seriously affected. The effect of autocorrelation is shown in Table 6, which lists the real levels of the tests for a gaussian AR(1), with parameter ϕ. Here, the marginal distribution of the data is gaussian and thus symmetric. When the correlation structure is weak, it has no particular effects on the tests. However, when it becomes stronger, if not explained, it leads to a real level which is definitely greater than the nominal one and thus appears to cause asymmetry even where there is none.
Note that, in the case of $\phi = 0.9$, pointed out by Bai and Ng (2005) as problematic, BTP again gives quite satisfactory results and significantly better than those of BN. Thus, in applications it is important to account for dependence and, in particular, for correlation. Conversely, it is also interesting to note that low levels of correlation do not have dramatic consequences on test performances.
Analyses of the real levels of dependent data were conducted on data generated from six models with different dependence structures and degrees of kurtosis (S5, S6,...,S10). In particular, data were generated by AR(1) and ARMA(1,1) models, with gaussian and t-Student innovations, which have a linear dependence structure. They also have marginal distributions which are symmetric but leptokurtic in the non-gaussian case. The family of GARCH processes was then used, because they produce uncorrelated, but not independent, data. The marginal distributions of the considered GARCH models are symmetric and, also in the gaussian case, leptokurtic. In order to have distributions with higher kurtosis, we also considered models with t-Student innovations. Another reason for interest in the GARCH models is their extensive use on the finance literature.
To evaluate the real dimension of the tests and to compare their powers, hypothesis systems (7) and (8) were again considered within the same framework of Section 4.1. The results are shown in Table 7. As for the independent case, the hypothesis of equal nominal and real levels cannot always be accepted, but the real levels are satisfactory, on average, at nominal levels of 10% and 5%, but are much lower at the nominal level of 1%. Note that, for very high levels of kurtosis, as in S10, BTP provides real levels more similar to nominal ones.
With regard to test power, our study was based on a bilinear model and two GARCH models with A4 and A5 innovations, called A6, A7 and A8. They produce uncorrelated data with asymmetric marginal distributions and different levels of kurtosis; all of them have in common not too high asymmetry and quite high kurtosis. Since power also depends on asymmetry intensity, it is clear that if very asymmetrical distributions are chosen very high power can be reached. In this work, instead, we prefer to consider processes with not too asymmetrical marginal distributions, in order to verify performances in relatively more difficult situations. In this sense, here the main interest lies in comparing the powers of the various test, more than the powers themselves. Table 8 shows that, in all three analyzed cases, BTP has significantly more power than BN. In general, as expected, power grows with n and with level test and asymmetry.

5 Empirical applications

After verifying that the bootstrap test works properly and comparing it, tis section applies compares the ASS, BTP and BN tests in 72 real daily financial time series. Again, the Premaratne and Bera test is not considered because it is a particular case of BN. Instead, the ASS test higlights the differing results which may be obtained.
The time series describe the returns of 30 stocks belonging to the Dow-Jones index, 30 belonging to the MIB30 index (the Italian stock index of the most higly capitalized firms) and 12 well-known international stock indexes (Dow-Jones, S&P500, Nasdaq100, Nikkei, FTSE100, SMI, CAC40, DAX, Mibtel, MIB30, Midex, Hang-Seng). The data refer to different periods, but most of them concern the interval January 1999 – October 2004. The lengths of the series range between $n = 575$ and $n = 4982$.
Since some series clearly have outliers, these were removed and replaced with the means of the previous data. Outliers were detected by visual inspection but all of them were at least 20 times the standard deviation of the data. Since previous analyses had shown that none of the proposed tests works well at a level of 1%, here we consider only the usual 5% level, which seems to be more reliable. The comparison was only made in terms of acceptance or rejection of the null hypothesis of symmetry.
As expected, ASS rejects the hypothesis symmetry very often - in 51 cases out of 72 (for 20 MIB30 stocks, 23 Dow-Jones stocks, and 8 stock indexes). The number of rejections for the bootstrap test and BN is much smaller: the former rejects the symmetry in 8 cases (6 MIB30 stocks, 1 Dow-Jones stock, and 1 index). Instead, BN rejects H_0 in 4 cases of MIB30 stocks, 1 Dow-Jones stock and 1 index; i.e. 6 series out of 72.
It is interesting to note that only for 23 series out of 72 do BTP and ASS reach the same conclusions about the presence or otherwise of asymmetry in the data. Table 2 illustrates some of the moste representative cases.
Conversely, there is very good agreement between BTP and BN, not surprising, as the performance of these two tests do not differ dramatically. However, there are three cases (3M, Seat Pagine Gialle, ST Microlectronics) in which the hypothesis of symmetry is rejected by BTP but not by BN (Table 2). It is interesting to note that two of these three cases were precisely those considered in Section 2 as examples of time series whose distributions may not have the sixth moment (see Table 1).

6 Conclusion

In this work describes some symmetry tests and connected problems when applied to financial time series. Since one of these problems is the existence of the sixth moment, a bootstrap test requiring only the existence of the moments up to the third is proposed. The procedure leads to a test that approximates the true distribution of $\hat{S}$, the sample skewness coefficient neglecting some correlations between high order moments. The test is very simple and intuitive and gives good results for dependent data and non-gaussian distributions. Its performance in term of real level and power were compared through Monte Carlo simulations considering several

Serie	$\hat{S}$	ASS	BTP	BN
3M	0.45	S	S	NS
Intel	-0.41	S	NS	NS
Banca Intesa	0.24	S	S	S
Capitalia	0.50	S	NS	NS
Seat Pagine Gialle	0.78	S	S	NS
ST Microelectronics	0.20	S	S	NS
Tim	0.23	S	S	S
Ftse100	-0.26	S	S	S
Nasdaq100	0.28	S	NS	NS
Mibtel	-0.46	S	NS	NS

Table 2: Results of the tests at the level of 5% on some real time series. S=Significant; NS=Not Significant.

generator processes. Results suggest that the test works well.
Regarding asymmetry in returns distribution, first of it should be noted that results referring to ASS are not reliable. Analyses of BTP and BN indicate that skewness is not pervasive in financial time series and that, when it is present, it seems to be the exception more than the rule.
Another practical indication emerging from this study is that all tests provide unsatisfactory results at low levels (i.e., 1%) and that, when leptokurtosis occurs, they tend to be conservative with respect to the null hypothesis.
At the standard 5% level, simulations indicate that BTP is slightly more powerful than BN.
Lastly, we believe that asymmetry in financial time series is a topic which should be studied in more depth, by means of both tests and models. At the same time, more accurate empirical exploration of series at different frequencies and, in paticular, at intradaily frequence, would be appropriate.

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Appendix

Symmetric model for i.i.d. data

- S1: $N(0,1)$;
- S2: t_7;
- S3: $Beta(2,2)$;
- S4: $F^{-1}(u) = \lambda_1 + \left[u^{\lambda_3} - (1-u)^{\lambda_4}\right]/\lambda_2$ with $\lambda_1 = 0$, $\lambda_2 = -1$, $\lambda_3 = -0.24$ $\lambda_4 = -0.24$, $u \sim U(0,1)$.

Asymmetric model for i.i.d. data:

- A1: $Beta(2,1)$;
- A2: $Skew\ Normal(0,1,-2)$;
- A3: $Skew\ t(0,1,-2,10)$;
- A4: $F^{-1}(u) = \lambda_1 + \left[u^{\lambda_3} - (1-u)^{\lambda_4}\right]/\lambda_2$ with $\lambda_1 = 0$, $\lambda_2 = -1$, $\lambda_3 = -0.0075$ $\lambda_4 = -0.03$, $u \sim U(0,1)$;
- A5: $F^{-1}(u) = \lambda_1 + \left[u^{\lambda_3} - (1-u)^{\lambda_4}\right]/\lambda_2$ with $\lambda_1 = 0$, $\lambda_2 = -1$, $\lambda_3 = -0.1$ $\lambda_4 = -0.18$.

Symmetrica models for dependent data:

- S5: $y_t = 0.7y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0,1)$;
- S6: $y_t = 0.7y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim t_7$;
- S7: $y_t = 0.7y_{t-1} + \varepsilon_t - 0.6\varepsilon_{t-1}, \quad \varepsilon_t \sim N(0,1)$;
- S8: $y_t = 0.7y_{t-1} + \varepsilon_t - 0.6\varepsilon_{t-1}, \quad \varepsilon_t \sim t_7$;
- S9: $y_t = \varepsilon_t, \quad \varepsilon_t \mid I_{t-1} \sim N(0,\sigma_t^2), \quad \sigma_t^2 = 0.2 + 0.3\ \varepsilon_{t-1}^2 + 0.6\ \sigma_{t-1}^2$;
- S10: $y_t = \varepsilon_t, \quad \varepsilon_t \mid I_{t-1} \sim t_7(0,\sigma_t^2), \quad \sigma_t^2 = 0.2 + 0.3\ \varepsilon_{t-1}^2 + 0.6\ \sigma_{t-1}^2$.

Asymmetric models for dependent data

- A6: $y_t = 0.6\ y_{t-1}\ \varepsilon_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0,1)$;
- A7: GARCH$(1,1)$ with $A4$ innovations and $\sigma_t^2 = 0.2 + 0.3\ \varepsilon_{t-1}^2 + 0.6\ \sigma_{t-1}^2$;
- A8: GARCH$(1,1)$ with $A5$ innovations and $\sigma_t^2 = 0.2 + 0.3\ \varepsilon_{t-1}^2 + 0.6\ \sigma_{t-1}^2$.

DGP	S	K	PGD	S	K
S1	0	3	A1	-0.56	2.4
S2	0	5	A2	-0.45	3.3
S3	0	2.14	A3	-0.86	5.01
S4	0	37.5	A4	1.51	7.4
S5	0	3	A5	1.98	19.4
S6	0	3.7	A6	1.10	9.64
S7	0	3	A7	1.52	7.44
S8	0	3.7	A8	2.00	19.6
S9	0	10			
S10	0	160.5			

Table 3: Asymmetry (S) and kurtosis (K) coefficients of DGP's. The values of S and K are the mean of 5000 coefficient estimates on series of length $n = 10000$ generated from the different processes.

PGD (S,K)	ASS 10%	ASS 5%	ASS 1%	BTP 10%	BTP 5%	BTP 1%	PB 10%	PB 5%	PB 1%	BN 10%	BN 5%	BN 1%
$S1$ $(0, 3)$												
n=100	8.8^*	4.1^*	1.1^*	7.1	2.1	0.2	8.5°	3.8°	0.4°	8.8^*	3.5	0.3
n=200	9.0^*	4.1^*	0.8^*	9^*	3.1	0.2	9.8^*	3.6	0.5°	9.8^*	3.8°	0.4°
n=500	9.8^*	4.5^*	0.9^*	9.8^*	4.1^*	0.5^*	10^*	4.2^*	0.6^*	9.8^*	4.3^*	0.6^*
$S2$ $(0, 5)$												
n=100	37.7	29.5	18.4	9.7^*	2.5	0.1	10.6^*	4.1^*	0.4°	10.5^*	4°	0.2
n=200	42	34.2	23.4	9.2^*	3.2	0.4°	9.6^*	3.8°	0.4°	9.7^*	3.6	0.2
n=500	46.3	38.8	27.2	10.1^*	4.2^*	0.1	9.9^*	4.3^*	0.2	9.8^*	4.5^*	0.2
$S3$ $(0, 2.14)$												
n=100	0.6	0.1	0	8.7^*	3.7°	0.3	9.5^*	4.4^*	0.5^*	9.4^*	4.1^*	0.4°
n=200	0.5	0.1	0	8.9^*	3.8°	0.5^*	9.2^*	4.2^*	1.1^*	9.1^*	4.1^*	0.8^*
n=500	0.4	0.1	0	10.2^*	5.1^*	0.7^*	10.4^*	5.1^*	0.7^*	10.2^*	4.9^*	0.6^*
$S4$ $(0, 37.5)$												
n=100	68.5	63.3	53.5	7.5	2.2	0	7.8	3.1	0.2	8.0	3.0	0.2
n=200	76.8	72.2	64.8	8.7^*	2.4	0	7.3	2.2	0	7.0	2.1	0
n=500	82.8	79.5	73.9	8.8^*	2.9°	0.2	6.9	2.1	0.1	6.9	2.0	0.1

Table 4: Independent data: real test levels based on 2000 Monte Carlo replications. The two numbers under the process name are the asymmetry (S) and kurtosis (K) coefficients. The asterisk (*) means that the null hypothesis of the system (7) is accepted at 5% level while the circle ($^\circ$) denotes the same conclusion 1%.

PGD (S, K)	ASS 10%	ASS 5%	ASS 1%	BTP 10%	BTP 5%	BTP 1%	PB 10%	PB 5%	PB 1%	BN 10%	BN 5%	BN 1%
$A1$ $(-0.56, 2.4)$												
n=100	83.6	68.7	30.8	96.9	91.9	60.2	97.2	93.8	75.2	96.9	92.8	68.3°
n=200	99.4	98	84.8	100	99.9	98.9	100	99.9	99.3	100	99.8	99
n=500	100	100	100	100	100	100	100	100	100	100	100	100
$A2$ $(-0.67, 3.3)$												
n=100	50.5	38.5	20	46.5	23.9	1.8	48.6	30.5	7.0	48.1	29.6°	5.8°
n=200	80.8	70.7	48.4	79.7	63.5	23.9	80.3	66	29.1	79.9	65.1	27.1*
n=500	99.2	98	92.9	99.2	97.5	85.5	99.2	97.8	87.4	99.2	97.8	86.8
$A3$ $(-0.86, 5)$												
n=100	82.1	75	58.7	63	36.2	3.5	62.3	40.8	10.7	62.4	38.8	9.4°
n=200	97	95	87.6	87.8	72.1	30.2	85.1	70.5	34	85.5	69.8	32.2
n=500	100	100	99.7	99.4	96.5	84.5	98.5	95.6	84	98.5	95.6	83.6
$A4$ $(1.51, 7.4)$												
n=100	99.4	98.6	95	89°	68.7°	13.8	83.6	67.6	30.3	83	64.8	26.8°
n=200	100	100	100	98.3°	92°	62.6°	94	85.7	60	93.8	84.7	57.3
n=500	100	100	100	99.9°	98.7°	93.1°	98.9	96.8	89.5	98.9	96.7	89.2
$A5$ $(2, 19.4)$												
n=100	88.5	85.2	78.5	44	21.1	1.8	44.8	26.1	5.4	44.5	24.9	4.2
n=200	97.2	96.4	94.1	71.8°	51.1°	13.1	66.4	45.5	15.1	66.5	45.2	13.8
n=500	99.9	99.8	99.6	91°	78.8°	49.2°	84.1	71.5	42.4	83.9	71.2	41.6

Table 5: Independent data: test powers based on 2000 Monte Carlo replications. The two numbers under the process name are the asymmetry (S) and kurtosis (K) coefficients. The asterisk (*) means that the null hypothesis of the system (8) is rejected at 5% level while the circle ($^\circ$) denotes the same conclusion 1%.

PGD n=200	ASS 10%	ASS 5%	ASS 1%	BTP 10%	BTP 5%	BTP 1%	PB 10%	PB 5%	PB 1%	BN 10%	BN 5%	BN 1%
$\phi = 0.1$	9.3*	4.7*	0.9*	9.7*	4.2*	0.2	10.3*	4.3*	0.5°	9.9*	4.2*	0.5°
$\phi = 0.2$	10.2*	5.5*	1.1*	9.5*	4.1*	0.2	10.4*	4.2*	0.4°	10.4*	3.9°	0.4°
$\phi = 0.4$	11.1°	6.4	1.5°	9.0*	4.0°	0.3	11.5°	5.6*	0.9*	9.2*	4.2*	0.4°
$\phi = 0.7$	20.4	13.2	4.5	9.5*	3.7°	0.4°	23.9	15.2	4.8	10.2*	3.8°	0.2
$\phi = 0.9$	35.2	27.2	15.2	8.9*	3.0	0.2	47.2	37.5	22.6	6.5	1.4	0

Table 6: Real test levels for the process $y_t = \phi y_{t-1} + \varepsilon_t$. The symbols have the same meaning that in Table 4.

PGD (S, K)	ASS 10%	ASS 5%	ASS 1%	BTP 10%	BTP 5%	BTP 1%	BN 10%	BN 5%	BN 1%
$S5$ $(0, 3)$									
n=100	15.5	9.6	3	8.8^*	3	0.1	7.4	2.2	0.1
n=200	20.3	13.5	4.2	9.7^*	3.6	0.3	10.3^*	3.7	0.2
n=500	22.1	14.6	5.7	9.9^*	4.9^*	0.9^*	9.3^*	3.8°	0.4°
$S6$ $(0, 3.7)$									
n=100	27.8	20.2	11.3	7	2.5	0.1	8.4°	2.5	0.1
n=200	32.9	26.2	14.2	8.9^*	2.9	0.2	9.6^*	3.3	0.2
n=500	41.1	32.9	20.3	9.7^*	3.5	0.4°	10.4^*	4.7^*	0.4°
$S7$ $(0, 3)$									
n=100	22.4	14.8	5.7	9^*	2.8	0.1	6.2	1.4	0
n=200	25.1	17.4	9.4	9.2^*	3.4	0.2	8.6°	2.6	0
n=500	30.2	21.9	10.7	8.8^*	4.1^*	0.6^*	10.3^*	3.8°	0.2
$S8$ $(0, 3.5)$									
n=100	30.9	22.8	13	8.1	2.6	0	7	1.8	0
n=200	35.8	27.6	16.4	9.6^*	3.5	0.2	7.1	2.3	0.1
n=500	42.2	34.1	21.7	8.9^*	3.7°	0.3	9.3^*	3.5	0.2
$S9$ $(0, 10)$									
n=100	23.5	16.9	9.4	7.7	2.3	0.2	8.2	2.9	0.2
n=200	37.5	30.4	19.9	10.1^*	3.6	0.4°	8.5°	3.4	0.4°
n=500	50.3	43	31.9	10.1^*	4.2^*	0.6	8.0	2.8	0.3
$S10$ $(0, 141)$									
n=100	47.3	40.4	29.9	7.1	1.7	0	7.5	2.4	0.2
n=200	63.5	58	49.2	9.6^*	3.6	0.2	7.8	2.0	0.1
n=500	78.8	75.8	69.7	8.4°	3.5	0.2	5.8	1.8	0.1

Table 7: Dependent data: real test levels based on 2000 Monte Carlo replications. Under the process name the asymmetry (S) and kurtosis (K) coefficients. The symbols have the same meaning that in Table 4.

PGD $-(S, K)$	ASS 10%	ASS 5%	ASS 1%	BTP 10%	BTP 5%	BTP 1%	BN 10%	BN 5%	BN 1%
$A6$ $(1.1, 9.6)$									
n=100	67.8	61.4	48.6	38.7°	17.3°	0.8	30.2	13	1.2
n=200	89.9	86.7	77.9	67°	45.6°	9.8°	50.8	28.5	4.8
n=500	98.8	98.3	97	88.5°	78.1°	48.2°	78.2	63.5	28.5
$A7$ $(1.5, 7.5)$									
n=100	99.2	98.5	95	89.1°	66.5	16.3	83.5	64.1	24.9°
n=200	100	100	99.9	98°	92.5°	64.6°	93.9	85.6	58.7
n=500	100	100	100	99.9	98.9°	93.8°	99.2	96.9	89.6
$A8$ $(2, 19.6)$									
n=100	86.8	84.2	75.9	44*	20.6	1.1	40.1	21.6	5.2°
n=200	96.9	96.2	93.9	70.1°	47.7°	12	60.3	40.9	12.4
n=500	99.8	99.7	99.6	92°	81.1°	49.5°	85.5	72.9	42.1

Table 8: Dependent data: test powers based on 2000 Monte Carlo replications. Under the process name the asymmetry (S) and kurtosis (K) coefficients. The symbols have the same meaning that in Table 5.