# Modelling and forecasting hourly spot electricity prices: some preliminary results 

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#### Abstract

This paper focuses on the day-ahead forecasting performance of some models for hourly electricity spot prices. In particular, our first aim is to investigate the forecasting performance of the model proposed by Huisman et al. (2007) to model hourly power prices in a panel framework. Secondly, we want to extend the model specifying a GARCH structure to take into account a possible conditional heteroskedasticity. A further extension concerns the introduction of cross-lagged correlations into the model through a VAR-type specification. Finally a non linear Markov switching structure is considered to take into account the occurrence of price spikes. We consider price data from different european electricity power markets and obtain forecasts for one-day-ahead up to one-week-ahead horizons. For forecast comparisons we use the RMSPE, the MAE and the Diebold and Mariano test. Results show that some improvement can be obtained by GARCH specification in the one-day ahead prediction exercise and by the VAR and MS models in longer prediction horizons.


Keywords: Spot electricity-prices; Vector Autoregression; Seemingly Unrelated Regression Equations; GARCH; Regime-Switching; Forecasting.

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#### Abstract

This paper focuses on the day-ahead forecasting performance of some models for hourly electricity spot prices. In particular, our first aim is to investigate the forecasting performance of the model proposed by Huisman et al. (2007) to model hourly power prices in a panel framework. Secondly, we want to extend the model specifying a GARCH structure to take into account a possible conditional heteroskedasticity. A further extension concerns the introduction of cross-lagged correlations into the model through a VAR-type specification. Finally a non linear Markov switching structure is considered to take into account the occurrence of price spikes. We consider price data from different european electricity power markets and obtain forecasts for one-day-ahead up to one-week-ahead horizons. For forecast comparisons we use the RMSPE, the MAE and the Diebold and Mariano test. Results show that some improvement can be obtained by GARCH specification in the one-day ahead prediction exercise and by the VAR and MS models in longer prediction horizons.


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## 1 Introduction

The recent deregulation of the market for electric power in many parts of world has greatly incentivated the study of methods for electricity price modelling and forecasting. However electricity as a commodity possesses certain special features not shared by other commodities, since there is no available technique to store or to inventory economically electricity once generated. Thus modelling electricity price behaviour is not a simple matter because it is not possible to transfer standard fi-
nancial time series models to wholesale electricity prices.
The peculiar characteristics of electricity prices have induced researchers to develop different special electricity price models (see, for example, Lucia and Schwartz (2002), Deng (2000), Escribano et al. (2002), Huisman and Mahieu (2003), de Jong and Huisman (2002), Knittel and Roberts (2005), Geman and Roncoroni (2006), Weron (2006), Koopman et al. (2007)). However the majority of the papers concentrates on daily average data and, even if the dynamics of daily average prices are extremely important, models developed for daily average prices cannot directly be applied to describe the dynamics of hourly prices. Using hourly prices instead of daily prices makes it possible to explain the different patterns of prices over the day. Moreover, we have to take into account that the day-ahead spot markets typically consist of 24 hourly (or 48 semi-hourly) auctions that take place simultaneously one day in advance. That is, in the day-ahead market every transaction for each hour of the next day is programmed. This means that hourly prices for next day delivery are determined at the same time.
In a recent paper, Huisman et al. (2007) propose to model hourly power prices in a panel framework. They consider a very simple model where each hour is modeled by a univariate $\mathrm{AR}(1)$ model and the error term is assumed to be independent over the days but cross-sectional covariance between the hours is allowed. However, they do not consider forecasting issues in their work and some typical features of electricity prices, such as conditional heteroskedasticity and the occurrence of spikes, are not taken into account.
In this paper our first aim is to investigate the forecasting performance of the Huisman et al. (2007) panel model. Secondly, we extend the model specifying a GARCH structure to take into account a possible conditional heteroskedasticity. A further extension concerns the introduction of cross-lagged correlations into the model through a VAR-type specification. Finally a non linear Markov switching structure is considered to take into account the occurrence of price spikes.
To evaluate the forecasting performances of the previous models we consider price data from different european electricity power market (i.e. Powernext, EEAX, OMEL) and obtain forecasts for one-day-ahead up to one-week-ahead horizons. Finally, for forecast comparisons we use the Root Mean Square Prediction Error (RMSPE), the Mean Absolute Prediction Error (MAE) and the Diebold and Mariano test.
Results show that some improvement can be obtained by model with GARCH specification in the one-day ahead prediction exercise and by the VAR and MS models in longer prediction horizons.
The plan of the remainder of this paper is as follow. In Section 2 we briefly introduce the models we will use. Section 3 describes the datasets. Section 4 presents the estimation and forecasting results. Conclusions are reported in the last section.

## 2 Modelling spot electricity prices

In this section we briefly recall the models for the dynamics of the spot electricity prices that we have used in this work. As discussed above, we have to take into account that hourly prices within a day behave cross-sectionally and hourly dynamics over days behave according time-series properties (Huisman et al., 2007).
To introduce the model, let $p_{h}(t)$ be the natural logarithm of the day-ahead price observed on day $t$ for the delivery of one MW electricity in hour $h$ of the next day $t+1$.
We describe the behaviour of the (log-) spot prices in terms of two types of components. The first one is a totally predictable deterministic component that take into account regularities in the evolution of prices, such as a deterministic trend and periodic behaviour. The second component is stochastic and will be assumed to follow some stochastic process. That is:

$$
\begin{equation*}
p_{h}(t)=f(t)+y_{h}(t) \quad h=1,2, \ldots, 24 . \tag{1}
\end{equation*}
$$

As usual in literature, the deterministic component, $f(t)$, (common to all hours) ${ }^{1}$, consists of a mean price level, $\mu_{0}$ and six dummies variables, $I^{d}(t)$, to take into account that price levels are different for different days of the week ( $\mathrm{d}=1$ corresponds with Thursday , d=2 corresponds with Friday, ..., d=6 corresponds with Monday and Wednesday is the baseline). Furthermore, one dummy variable, $I^{c}(t)$, is considered to take into account calendar effects (particular holidays and other special days). The seasonal fluctuations (annual cycle), due to the use artificial light and heating in winter and to the air conditioning in summer, is modeled by a spline. ${ }^{2}$ We also added a deterministic linear time trend. The expression for the deterministic component becomes:

$$
\begin{equation*}
f(t)=\mu_{0}+\sum_{d=1}^{6} \beta_{d} I^{d}(t)+\gamma I^{c}(t)+\alpha t+S(t) \tag{2}
\end{equation*}
$$

where $S(t)$ is a cubic spline.
The stochastic part, $y_{h}(t)$, may account electricity spot prices features such as mean reversion, extreme price volatility, abrupt and unanticipated extreme changes known as jumps or spikes. To take into account these features, we model the stochastic component, $y_{h}(t)$, with the following different processes.

### 2.1 Basic AR-GARCH models

From an empirical point of view, one key aspect of electricity spot prices is a nonconstant conditional variance (heteroskedasticity). In particular, spot prices present a strong dependence of the variability of the series on its own past. In literature some studies (see, for example, Garcia et al. (2005) and Weron (2006) and the reference

[^0]therein) have shown that GARCH model outperforms ARIMA model when volatility and price spikes are present. Moreover, in the majority of the cases, simple AR(1) is sufficient to model the conditional mean and simple $\operatorname{GARCH}(1,1)$ is sufficient to model conditional variance. For this reason the basic model that we assume as benchmark is the $\operatorname{AR}(1)-\operatorname{GARCH}(1,1)$ model:
\[

$$
\begin{array}{r}
y_{h}(t)=\phi_{h} y_{h}(t-1)+\sigma_{h}(t) \epsilon_{h}(t)  \tag{3}\\
\sigma_{h}^{2}(t)=\omega_{h}+\alpha_{h} \epsilon_{h}^{2}(t-1)+\beta_{h} \sigma_{h}^{2}(t-1) \\
h=1, \ldots, 24 ; \quad t=1, \ldots, T
\end{array}
$$
\]

where $\epsilon_{h}(t)$ is distributed as $i . i . d . N(0,1)$ and the coefficients have to satisfy $\omega_{h}>0$, $\alpha_{h}, \beta_{h} \geq 0$ and $\alpha_{h}+\beta_{h}<1$ in order to ensure that the conditional variance is strictly positive.
The GARCH model is especially interesting as it comes to interval forecasts for future spot prices, even if the power market literature has rather focused on point forecasts.

### 2.2 SUR-GARCH models

The seemingly unrelated regression (SUR) model due to Theil (1961) and Zellner (1962) is the unrestricted system of linear regression equations of type:

$$
\begin{equation*}
y_{h}(t)=\phi_{h} y_{h}(t-1)+\epsilon_{h}(t), \quad h=1, \ldots, 24 ; \quad t=1, \ldots, T \tag{4}
\end{equation*}
$$

where the error terms $\epsilon_{h}(t)$ are assumed to be contemporaneously correlated across equations (hours) but independent over the days, that is $E\left[\epsilon_{h}(t) \epsilon_{j}(s)\right]=\sigma_{h j}$ for $t=s ; 0$ otherwise. In this model, the price for delivery in hour $h$ in day $t$ depends on the price for that hour in the previous day but not on the price in the previous hour. Allowing for cross-sectional correlation is important because the information set used for setting the price of delivery in hour $h$ is the same as the information set used to set the price for delivery in a different hour $j$. The information set is constant within the day and updates over the days (Huisman et al. (2007)). The parameters in the model are estimated using the Seemingly Unrelated Regressions (SUR) method. SUR estimates the parameters of the 24 hourly time series taking into account contemporaneus correlations in the errors across the time series. To take into account heteroskedasticity we model the error term with a $\operatorname{GARCH}(1,1)$ model.

### 2.3 VAR-GARCH models

The vector autoregression (VAR) model is one of the most successful, flexible, and easy to use models for the analysis of multivariate time series. It is a natural extension of the univariate autoregressive model to dynamic multivariate time series. The VAR model has proven to be especially useful for describing the dynamic behavior of economic and financial time series and for forecasting. It often provides superior forecasts to those from univariate time series models and elaborate theory-based simultaneous equations models.

Let $\boldsymbol{Y}(t)=\left(y_{1}(t), y_{2}(t), \ldots, y_{n}(t)\right)^{\prime}$ denote an $(n \times 1)$ vector of time series. The basic $p-$ lag $\operatorname{VAR}(\mathrm{p})$ model has the form:

$$
\begin{equation*}
\boldsymbol{Y}(t)=\boldsymbol{c}+\boldsymbol{\Pi}_{\mathbf{1}} \boldsymbol{Y}(t-1)+\boldsymbol{\Pi}_{\mathbf{2}} \boldsymbol{Y}(t-2)+\ldots+\boldsymbol{\Pi}_{\boldsymbol{p}} \boldsymbol{Y}(t-p)+\boldsymbol{\epsilon}(t) \quad t=1, \ldots, T \tag{5}
\end{equation*}
$$

where $\boldsymbol{\Pi}_{\boldsymbol{i}}$ are $(n \times n)$ matrices of coefficients and $\boldsymbol{\epsilon}(t)$ is a $(n \times 1)$ vector of unobservable zero mean white noise processes with time invariant covariance matrix $\boldsymbol{\Sigma}$. Notice that each equation has the same regressors, that is lagged values of $\boldsymbol{Y}(t)$. Hence, the $\operatorname{VAR}(\mathrm{p})$ model is just a seemingly unrelated regression (SUR) model with lagged variables and deterministic terms as common regressors.
To take into account heteroskedasticity we model the error terms with a $\operatorname{GARCH}(1,1)$ model.

### 2.4 Markov-Switching models

The underlying idea behind the Markov-Switching (MS) model (Hamilton, 1989) is that the observed stochastic behaviour of a specific time series could be modeled by two (or more) separate regimes with different underlyng processes. In other words the parameters of the underlying process may change for a certain period of time and then fall back to their original structure. For each regime one can define separate and independet different underlying price processes. The switching mechanism between the states is assumed to be governed by an unobserved random variable. In particular, the MS regimes are defined by the exogenous state of a Markov chain that governs the transition from one state to another.
Formally, a Markov Switching model of order $p(\mathrm{MS}(\mathrm{p}))$ with two regime can be defined as:

$$
y(t)= \begin{cases}\alpha_{1}+\sum_{i=1}^{p} \phi_{1, i} y(t-i)+\epsilon_{1}(t) & \text { if } s(t)=1  \tag{6}\\ \alpha_{2}+\sum_{i=1}^{p} \phi_{2, i} y(t-i)+\epsilon_{2}(t) & \text { if } s(t)=2\end{cases}
$$

where $\epsilon_{i}(t) \sim \operatorname{IID}\left(0, \sigma_{i}^{2}\right)$ indipendent of each other, and $s(t)$ assumes values in 1,2 . The state variable $s(t)$ is unobservable, and we assume that it is governed by a first order Markov chain with transition probabilities:

$$
P=\left[\begin{array}{ll}
p_{11} & p_{12} \\
p_{21} & p_{22}
\end{array}\right]
$$

where $p_{i j}=P\left(s_{t}=j \mid s_{t-1}=i\right)$ and $p_{11}+p_{12}=p_{21}+p_{22}=1$.
Following, for example, Ethier and Mount (1998), we could assume that the spot price display different behaviour at each point in time, depending on the regime 1 (base regime) or 2 (spike regime). Therefore our model is:

$$
y_{h}(t)= \begin{cases}\alpha_{1}+\phi_{1, h} y_{h}(t-1)+\epsilon_{1, h}(t) & \text { if } s(t)=1  \tag{7}\\ \alpha_{2}+\phi_{2, h} y_{h}(t-1)+\epsilon_{2, h}(t) & \text { if } s(t)=2\end{cases}
$$

for $h=1,2, \ldots, 24$.

## 3 Electricity data sets

For the empirical analyis we collect hourly time series of spot electricity prices registered in three European electricity markets: Powernext (France), EXAA (Austria)
and OMEL (Spain). These datasets are chosen because they offer freely accessible high quality electricity price data. ${ }^{3}$

## 1. Power Paris Exchange: Powernext

Powernext started operating on 27 November 2001. The system for fixing the day-ahead spot price is the one developed by NordPool.
Our dataset concerns French national hourly electricity spot prices from November 27, 2001 until April 21, 2009. This hourly time series consists of 2703 daily (or 64872 hourly) observations. The period November 27, 2001 - April 21, 2008 is used for calibration, the last year (April 22, 2008 - April 21, 2009) is used for forecasting.

## 2. Energy Exchange Austria: EXAA

EXAA started operating on 19 March 2002 after the full liberalization of the Austrian electricity market on 1 October 2001. Today, EXAA has become established as a European market for energy products. It was launched with 12 market participants, and at present, EXAA spot trading includes more than 60 electricity traders from over 14 countries on the Energy Spot Market.
The dataset we used concerns Austrian national hourly electricity spot prices from March 22, 2002 up to November 30, 2009. This hourly time series consists of 2812 daily (or 67488 hourly) observations. The period March 22, 2002 November 30, 2008 is used for the purpose of calibration.

## 3. Operador del Mercado Iberico de Energia - Polo Español: OMEL

 The Spanish electricity market began operation in January 1998, with dayahead trading.The dataset we used concerns Spanish national hourly electricity spot prices from November 1, 2001 up to October 30, 2009. This hourly time series consists of 2922 daily (or 70128 hourly) observations. The period November 1, 2008 October 30, 2009 is used for forecasting.

Some researchers (see, for example, Karakatsani and Bunn (2008) and Bosco et al. (2009)) prefer to remove weekend days and Bank holidays from dataset because these days show a different profile. On the contrary, we think that the information in these days is important and so we consider in our analysis all days of week. To take into account that for different days of the week and for Bank holidays price levels are different we consider in our models opportune dummies variables (like, for example, Huisman et al. (2007) and de Jong and Huisman (2002)).

Each day of all dataset consists of 24 periods: period 1 is defined as 00:00-01:00am and similarly the other periods up to 24 (23:00-00:00pm).

[^1]
## 4 Empirical Results

In this section we apply to the three electricity spot prices series the models we described in section 2.

### 4.1 Preliminary analysis

Figure 1 report the graphics of the hourly prices for the three electricity spot markets (in order Powernext, Exaa and Omel) togheter with its autocorrelation functions (for 30 days).
Figure 2 reports the graphics of the daily prices for the three electricity spot markets togheter with its autocorrelation functions (for 30 days). The series of daily prices is the series of the mean of each day (mean of 24 houlry observations).
The Figure 3 displays the hourly mean prices throughout the week (panel (a)) and the hourly mean prices separately for working days and weekend days (panel(b)). We can see that during weekend days the electricity prices are lower than in the working days. Moreover, we can see that during the central hours of the day the prices are higher than during the night, reflecting human activities. In particular we observe two points of very high prices (peak hour) at about 11:00-12:00am and 18:00-20:00pm.
These plots show:

1. strong seasonal movements within the day and the week;
2. presence of volatility cluster;
3. presence of multiple spikes (or jumps);
4. different price levels during the weekdays.

Moreover, we can see that while the behaviour of French and Austrian markets is very similar, the Spanish market is a little bit different. The Spanish market shows a higher degree of persistence, fewer spikes and a different behaviour between the days of the week. This probably occurs because of different Spanish climate ( Spain is warmer than France and Austria consequently the weather is different).

### 4.2 The deterministic component of the model

In order to implement the general model previously described by (1), we need to specify the deterministic time function $f(t)$. This function tries to capture any relevant predictable component of the electricity spot prices behaviour arising from genuine regularities along time. To estimate the deterministic component we use the formula (2) explained in section 2. We exclude Wednesday from the exogenous variables to prevent from multicollinearity. In table 1 we report the results of the parameter estimates for the deterministic component ${ }^{4}$.
As we can see from the table 1, in weekend days and holidays the level of prices

[^2]

Figure 1: (a) Hourly prices for the electricity spot markets; (b) Empirical autocorrelation function of hourly electricity spot prices (Powernext, Exaa, Omel)


Figure 2: (a) Daily prices for the electricity spot market; (b) Empirical autocorrelation function of daily electricity spot prices (Powernext, Exaa, Omel)


Figure 3: (a) Hourly average patterns for each day of the week. First day: Sunday ; (b) Hourly average patterns for working days and weekend days: red=working days, blue=Saturday, green=Sunday. (Powernext, Exaa, Omel)

Table 1: Estimation results for the deterministic component

| Powernext | Coefficients | Estimate | SE | t-value |
| :---: | :---: | :---: | :---: | :---: |
|  | Intercept | $2.745 \mathrm{e}+00$ | $3.398 \mathrm{e}-02$ | 80.764 |
|  | Monday | -1.007e-01 | $7.702 \mathrm{e}-03$ | -13.079 |
|  | Tuesday | 1.501e-02 | $7.692 \mathrm{e}-03$ | 1.951 |
|  | Thursday | -2.489e-03 | $7.703 \mathrm{e}-03$ | -0.323 |
|  | Friday | -5.705e-02 | 7.697e-03 | -7.412 |
|  | Saturday | -2.491e-01 | 7.697e-03 | -32.362 |
|  | Sunday | -5.338e-01 | $7.700 \mathrm{e}-03$ | -69.325 |
|  | Holiday | -3.825e-01 | $1.278 \mathrm{e}-02$ | -29.931 |
|  | Trend | $1.696 \mathrm{e}-05$ | $1.101 \mathrm{e}-07$ | 154.075 |
| Exaa | Coefficients | Estimate | SE | t-value |
|  | Intercept | $2.601 \mathrm{e}+00$ | 63.741e-02 | 69.515 |
|  | Monday | -8.788e-02 | $8.371 \mathrm{e}-03$ | -10.498 |
|  | Tuesday | $9.097 \mathrm{e}-03$ | $8.367 \mathrm{e}-03$ | 1.087 |
|  | Thursday | -2.046e-02 | $8.374 \mathrm{e}-03$ | -2.443 |
|  | Friday | -6.380e-02 | $8.367 \mathrm{e}-03$ | -7.625 |
|  | Saturday | -2.778e-01 | $8.368 \mathrm{e}-03$ | -33.196 |
|  | Sunday | -6.116e-01 | $8.372 \mathrm{e}-03$ | -73.051 |
|  | Holiday | -3.430e-01 | $1.408 \mathrm{e}-02$ | -24.351 |
|  | Trend | 1.148e-05 | $1.151 \mathrm{e}-07$ | 99.758 |
| Omel | Coefficients | Estimate | SE | t-value |
|  | Intercept | $3.162 \mathrm{e}+00$ | $2.611 \mathrm{e}-02$ | 121.118 |
|  | Monday | -3.336e-03 | $5.949 \mathrm{e}-03$ | -0.561 |
|  | Tuesday | $1.158 \mathrm{e}-03$ | $5.944 \mathrm{e}-03$ | 0.195 |
|  | Thursday | $1.030 \mathrm{e}-02$ | $5.942 \mathrm{e}-03$ | 1.733 |
|  | Friday | 1.988e-03 | $5.943 \mathrm{e}-03$ | 0.335 |
|  | Saturday | -8.921e-02 | $5.941 \mathrm{e}-03$ | -15.015 |
|  | Sunday | -2.058e-01 | 5.944e-03 | -34.617 |
|  | Holiday | -1.481e-01 | $9.903 \mathrm{e}-03$ | -14.960 |
|  | Trend | $8.075 \mathrm{e}-06$ | 7.902e-08 | 102.183 |

is significantly lesser than the baseline day (Wednesday) thus confirming the preliminary analysis. This effect is a bit weaker for the Spanish market. With respect to the other days, for the French market Tuesday and Thursday are not significantly different from Wednesday (maybe because they are near Wednesday) while Monday and Friday present a smaller level mean than Wednesday (maybe because these days are near weekend); for the Austrian market Tuesday is not significantly different from Wednesday while the other days present a smaller level mean than Wednesday; finally for the Spanish market every day is not significantly different from Wednesday. Figure 4 reports the grafics of spot electricity (log-)prices togheter with the estimation of the deterministic components.


Figure 4: In black: Hourly prices for the electricity spot market. In red: estimation of deterministic component (Powernext, Exaa, Omel).

### 4.3 Estimation

After eliminated from the series of log-prices, $p(t)$, the deterministic component, we concentrated on the stochastic component of the series. Recall that we consider our datasets as panel of 24 observations observed longitudinally. Thus $y_{h}(t)$ is the stochastic component of the log-price, $p_{h}(t)$, observed on day $t$ for the delivery of one MW electricity in hour $h$ of the next day $t+1$, $(h=1,2, \cdots, 24$.) Section 6 reports all the tables containing the parameter estimates of the proposed models.

### 4.4 Forecasting

In this section, we consider the forecasting performances provided by the four different models we described in section 2 . The comparison in conducted in four subsamples which reflect the four season: January (for winter), May (for spring), August (for summer) and November (for autumn). The aim is to understand if the forecasting accuracy of these different models depends from the step ahead and from a particular period (season) of the year. Moreover, we want to see if one of the considered models outperforms the others in terms of predictive performace. Forecasting performances will be compared both in terms of descriptive error statistics and in term of test.
Predictions for different lead times, $l=1,2, \ldots, 7$ (where a lead time of 7 corresponds to a week) are obtained using a rolling origin procedure that updates the forecasting origin and produces forecasts up to 7 steps-ahead from each new origin. The forecasting performances of the four considered models are then evaluated according to Mean Square Error (MSE) and Mean Absolute Error (MAE):

$$
\begin{aligned}
& M S E_{h}(l)=\frac{1}{T} \sum_{t=1}^{T}\left(P_{h}(t)-\hat{P}_{h}(t)\right)^{2} \\
& M A E_{h}(l)=\frac{1}{T} \sum_{t=1}^{T}\left|P_{h}(t)-\hat{P}_{h}(t)\right|
\end{aligned}
$$

where T is the size of the forecasting period, $P_{h}(t)$ is the real price at time $t$ for hour $h, \hat{P}_{h}(t)^{5}$ is our forecast at time $t$ for hour $h, l$ is the step of forecasting and $l=1,2, \cdots, 7$. Moreover, we consider the Diebold and Mariano test for equal predictive accuracy (Diebold and Mariano, 1995).
In this work we report only results for Powernext, since preliminary results about the other two markets are rather similar. Table 2 summarize the one up to seven step ahead forecasting performance of the four models estimated.

Some general comments can be summarize as follows:

- It seems that globally no model outperforms the other
- It seems that in January and August VAR-GARCH and MS models have the better performance

[^3]Table 2: Powernext: Prediction error statistics (F.P. = Forecasting Period. In bold the better statistics)

| F.P. | Step | AR-GARCH |  | SUR-GARCH |  | VAR-GARCH |  | MS |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE |
| January | 1 | 0.076 | 0.225 | 0.022 | 0.118 | 0.017 | 0.101 | 0.006 | 0.059 |
|  | 2 | 0.070 | 0.211 | 0.075 | 0.213 | 0.054 | 0.179 | 0.056 | 0.194 |
|  | 3 | 0.056 | 0.187 | 0.101 | 0.244 | 0.073 | 0.219 | 0.070 | 0.222 |
|  | 4 | 0.074 | 0.212 | 0.111 | 0.259 | 0.062 | 0.194 | 0.074 | 0.208 |
|  | 5 | 0.073 | 0.207 | 0.109 | 0.256 | 0.049 | 0.178 | 0.063 | 0.199 |
|  | 6 | 0.064 | 0.192 | 0.135 | 0.267 | 0.056 | 0.184 | 0.071 | 0.211 |
|  | 7 | 0.068 | 0.213 | 0.153 | 0.274 | 0.056 | 0.185 | 0.073 | 0.214 |
| May | 1 | 0.117 | 0.225 | 0.073 | 0.238 | 0.036 | 0.155 | 0.019 | 0.118 |
|  | 2 | 0.148 | 0.294 | 0.176 | 0.383 | 0.080 | 0.224 | 0.112 | 0.265 |
|  | 3 | 0.180 | 0.323 | 0.213 | 0.414 | 0.112 | 0.259 | 0.140 | 0.308 |
|  | 4 | 0.182 | 0.319 | 0.206 | 0.399 | 0.092 | 0.237 | 0.127 | 0.291 |
|  | 5 | 0.258 | 0.367 | 0.224 | 0.402 | 0.141 | 0.288 | 0.188 | 0.339 |
|  | 6 | 0.442 | 0.472 | 0.328 | 0.460 | 0.272 | 0.386 | 0.331 | 0.437 |
|  | 7 | 0.481 | 0.494 | 0.345 | 0.470 | 0.296 | 0.409 | 0.350 | 0.454 |
| August | 1 | 0.021 | 0.105 | 0.128 | 0.294 | 0.355 | 0.404 | 0.034 | 0.132 |
|  | 2 | 0.167 | 0.284 | 0.323 | 0.462 | 0.342 | 0.397 | 0.181 | 0.296 |
|  | 3 | 0.215 | 0.339 | 0.406 | 0.521 | 0.303 | 0.378 | 0.230 | 0.356 |
|  | 4 | 0.239 | 0.368 | 0.448 | 0.549 | 0.283 | 0.365 | 0.252 | 0.381 |
|  | 5 | 0.317 | 0.426 | 0.550 | 0.599 | 0.307 | 0.381 | 0.333 | 0.429 |
|  | 6 | 0.320 | 0.451 | 0.607 | 0.624 | 0.310 | 0.382 | 0.318 | 0.447 |
|  | 7 | 0.393 | 0.484 | 0.720 | 0.655 | 0.369 | 0.419 | 0.383 | 0.473 |
| November | 1 | 0.230 | 0.261 | 0.075 | 0.229 | 0.032 | 0.122 | 0.029 | 0.114 |
|  | 2 | 0.277 | 0.326 | 0.221 | 0.335 | 0.104 | 0.204 | 0.131 | 0.209 |
|  | 3 | 0.351 | 0.364 | 0.314 | 0.404 | 0.154 | 0.234 | 0.201 | 0.267 |
|  | 4 | 0.404 | 0.389 | 0.397 | 0.423 | 0.193 | 0.261 | 0.248 | 0.302 |
|  | 5 | 0.497 | 0.424 | 0.494 | 0.470 | 0.263 | 0.307 | 0.326 | 0.339 |
|  | 6 | 0.418 | 0.400 | 0.465 | 0.441 | 0.202 | 0.282 | 0.265 | 0.329 |
|  | 7 | 0.407 | 0.391 | 0.479 | 0.444 | 0.188 | 0.268 | 0.259 | 0.320 |

- The AR-GARCH model yielded (partially) the best prediction only in January and August
- The performance of SUR-GARCH model is rather poor. This model always give the worse results
- The Diebold-Mariano test (not reported) highlights that differences in forecasting performance are mostly significative.


## 5 Conclusions

- We want to use GARCH component of the model to construct interval forecasts for future spot prices, even if the power market literature has rather focused on point forecasts.
- Since different forecasting models capture different features of spot prices dynamics, no model provides superior performance in forecasting. We could to use a forecasting approach based on combination of forecast (as in Nan et al. (2009))
- Since MS models seem to be a useful tool to forecast electricity spot prices, it would be interesting to develop a multivariate MS model, eventually with time varying parameters.


## References

Bosco, B., L. Parisio, M. Pelagatti, and F. Baldi (2009). Long-run relations in euripean electricity prices. Journal of Applied Econometrics (DOI: 10.1002/jae.1095).
de Jong, C. and R. Huisman (2002). Option formulas for mean-reverting power prices with spikes. Report Series ERS-2002-96-F\&A, Erasmus Research Institute of Management (ERIM).

Deng, S. (2000). Stochastic models of energy commodity prices and their applications: Mean-reversion with jumps and spikes. Power Working Paper 2000-73, University of California Energy Institute.

Diebold, F. and R. Mariano (1995). Comparing predictive accuracy. Journal of Business and Economic Statistics 13, 253-263.

Escribano, A., J. Pena, and P. Villaplana (2002). Modeling electricity prices: International evidence. Working Paper 02-27, Universidad Carlos III de Madrid.

Ethier, R. and T. Mount (1998). Estimating the volatility of spot prices in restructured electricity markets and the implications for option values. Working Paper 98-31, Munich personal RePEc Archive.

Garcia, R., J. Contreras, M. van Akkeren, and J. Garcia (2005). A garch forecasting model to predict day-ahead electricity prices. IEEE Transactions on Power Systems 20(2), 867-874.

Geman, H. and A. Roncoroni (2006). Understanding the fine structure of electricity prices. The Journal of Business 79(3), 12251261.

Hamilton, J. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. Econometrica 57, 357-384.

Huisman, R., C. Huurman, and R. Mahieu (2007). Hourly electricity prices in dayahead markets. Energy Economics 29, 240-248.

Huisman, R. and R. Mahieu (2003). Regime jumps in electricity prices. Energy Economics 25, 425-434.

Karakatsani, N. V. and D. W. Bunn (2008). Forecasting electricity prices: The impact of fundamentals and time-varying coefficients. International Journal of Forecasting 24(4), 764-785.

Knittel, C. and M. Roberts (2005). An empirical examination of restructured electricity prices. Energy Economics 27, 791-817.

Koopman, S., M. Ooms, and M. Carnero (2007). Periodic seasonal reg-arfimagarch models for daily electricity spot prices. Journal of the American Statistical Association 102(477), 16-27.

Lucia, J. J. and E. S. Schwartz (2002). Electricity prices and power derivatives: Evidence from the nordic power exchange. Review of Derivatives Research 5(1), 5-50.

Nan, F., S. Bordignon, and F. Lisi (2009). Forecasting spot electricity prices through combination of forecasts. Proceedings of S.Co. 2009, 6th Conference, Complex Models and Computational Intensive Methods for Estimation and Prediction, Milano, 14-16 September 2009. Maggioli Ed., Santarcangelo di Romagna (RN)., 311316.

Theil, H. (1961). Economic Forecasts and Policy. North Holland, Amsterdam.
Weron, R. (2006). Modelling and Forecasting Electricity Loads and Prices: A Statistical Approach. Wiley, Chichester.

Zellner, A. (1962). An efficient method of estimating seemingly unrelated regressions and tests of aggregation bias. Journal of the American Statistical Association 5\%, 500-509.

## 6 Appendix

This section contains the tables with the parameter estimates of the models proposed in subsection 4.3 .

Table 3: Powernext: $\mathrm{AR}(1)-\mathrm{GARCH}(1,1)$ parameter estimates (standard error)

| Hour | $c$ | $\phi_{h}$ | $\omega_{h}$ | $\alpha_{h}$ | $\beta_{h}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1 | $-0.098(0.008)$ | $0.615(0.019)$ | $0.088(0.008)$ | $0.037(0.007)$ | $0.000(0.052)$ |
| 2 | $-0.129(0.009)$ | $0.655(0.018)$ | $0.115(0.006)$ | $0.217(0.039)$ | $0.000(0.030)$ |
| 3 | $-0.146(0.009)$ | $0.683(0.016)$ | $0.109(0.007)$ | $0.312(0.056)$ | $0.000(0.037)$ |
| 4 | $-0.211(0.014)$ | $0.626(0.020)$ | $0.021(0.003)$ | $0.076(0.012)$ | $0.824(0.024)$ |
| 5 | $-0.250(0.016)$ | $0.619(0.022)$ | $0.022(0.004)$ | $0.237(0.028)$ | $0.726(0.032)$ |
| 6 | $-0.139(0.011)$ | $0.708(0.019)$ | $0.123(0.003)$ | $0.209(0.030)$ | $0.000(0.002)$ |
| 7 | $-0.096(0.010)$ | $0.574(0.022)$ | $0.008(0.001)$ | $0.093(0.012)$ | $0.882(0.009)$ |
| 8 | $-0.009(0.009)$ | $0.507(0.022)$ | $0.010(0.002)$ | $0.085(0.013)$ | $0.872(0.019)$ |
| 9 | $0.065(0.007)$ | $0.594(0.019)$ | $0.005(0.001)$ | $0.079(0.010)$ | $0.881(0.016)$ |
| 10 | $0.062(0.006)$ | $0.741(0.018)$ | $0.004(0.001)$ | $0.187(0.022)$ | $0.797(0.024)$ |
| 11 | $0.055(0.006)$ | $0.796(0.016)$ | $0.003(0.001)$ | $0.134(0.016)$ | $0.840(0.019)$ |
| 12 | $0.060(0.006)$ | $0.800(0.015)$ | $0.002(0.000)$ | $0.131(0.015)$ | $0.851(0.0 .15)$ |
| 13 | $0.055(0.006)$ | $0.809(0.014)$ | $0.002(0.000)$ | $0.127(0.015)$ | $0.849(0.017)$ |
| 14 | $0.053(0.005)$ | $0.784(0.015)$ | $0.002(0.000)$ | $0.128(0.016)$ | $0.851(0.018)$ |
| 15 | $0.045(0.005)$ | $0.781(0.016)$ | $0.003(0.001)$ | $0.158(0.019)$ | $0.811(0.022)$ |
| 16 | $0.038(0.005)$ | $0.748(0.018)$ | $0.004(0.001)$ | $0.137(0.018)$ | $0.827(0.020)$ |
| 17 | $0.036(0.005)$ | $0.698(0.019)$ | $0.005(0.001)$ | $0.153(0.019)$ | $0.802(0.021)$ |
| 18 | $0.038(0.006)$ | $0.764(0.017)$ | $0.004(0.001)$ | $0.103(0.017)$ | $0.837(0.026)$ |
| 19 | $0.040(0.006)$ | $0.794(0.016)$ | $0.002(0.000)$ | $0.109(0.015)$ | $0.863(0.016)$ |
| 20 | $0.047(0.005)$ | $0.783(0.016)$ | $0.003(0.001)$ | $0.083(0.013)$ | $0.871(0.020)$ |
| 21 | $0.026(0.004)$ | $0.823(0.014)$ | $0.033(0.002)$ | $0.419(0.040)$ | $0.036(0.027))$ |
| 22 | $0.014(0.004)$ | $0.781(0.014)$ | $0.038(0.002)$ | $0.342(0.039)$ | $0.002(0.031)$ |
| 23 | $0.012(0.006)$ | $0.724(0.017)$ | $0.073(0.001)$ | $0.073(0.017)$ | $0.000(0.029)$ |
| 24 | $-0.017(0.006)$ | $0.717(0.018)$ | $0.079(0.001)$ | $0.072(0.018)$ | $0.000(0.036)$ |

Table 4: Exaa: $\operatorname{AR}(1)-\operatorname{GARCH}(1,1)$ parameter estimates (standard error)

| Hour | $c$ | $\phi_{h}$ | $\omega_{h}$ | $\alpha_{h}$ | $\beta_{h}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1 | $-0.050(0.005)$ | $0.724(0.014)$ | $0.001(0.000)$ | $0.012(0.004)$ | $0.097(0.011)$ |
| 2 | $-0.093(0.007)$ | $0.746(0.014)$ | $0.004(0.001)$ | $0.065(0.012)$ | $0.878(0.029)$ |
| 3 | $-0.121(0.008)$ | $0.753(0.014)$ | $0.046(0.005)$ | $0.245(0.031)$ | $0.192(0.076)$ |
| 4 | $-0.134(0.009)$ | $0.761(0.015)$ | $0.010(0.004)$ | $0.118(0.022)$ | $0.775(0.067)$ |
| 5 | $-0.117(0.008)$ | $0.796(0.013)$ | $0.032(0.004)$ | $0.332(0.037)$ | $0.291(0.059)$ |
| 6 | $-0.070(0.007)$ | $0.816(0.015)$ | $0.003(0.000)$ | $0.152(0.014)$ | $0.840(0.013)$ |
| 7 | $-0.096(0.009)$ | $0.564(0.021)$ | $0.011(0.002)$ | $0.239(0.022)$ | $0.773(0.016)$ |
| 8 | $0.044(0.008)$ | $0.536(0.019)$ | $0.004(0.001)$ | $0.152(0.016)$ | $0.849(0.014)$ |
| 9 | $0.072(0.007)$ | $0.729(0.017)$ | $0.001(0.000)$ | $0.174(0.015)$ | $0.859(0.009)$ |
| 10 | $0.048(0.005)$ | $0.847(0.013)$ | $0.001(0.000)$ | $0.169(0.019)$ | $0.818(0.018)$ |
| 11 | $0.047(0.005)$ | $0.873(0.011)$ | $0.002(0.000)$ | $0.185(0.019)$ | $0.779(0.019)$ |
| 12 | $0.051(0.006)$ | $0.893(0.011)$ | $0.002(0.000)$ | $0.324(0.027)$ | $0.761(0.015)$ |
| 13 | $0.050(0.005)$ | $0.861(0.012)$ | $0.003(0.000)$ | $0.179(0.023)$ | $0.767(0.027)$ |
| 14 | $0.047(0.005)$ | $0.851(0.012)$ | $0.002(0.000)$ | $0.163(0.020)$ | $0.808(0.020)$ |
| 15 | $0.040(0.005)$ | $0.839(0.013)$ | $0.002(0.001)$ | $0.191(0.022)$ | $0.798(0.020)$ |
| 16 | $0.037(0.004)$ | $0.829(0.013)$ | $0.001(0.000)$ | $0.136(0.017)$ | $0.847(0.017)$ |
| 17 | $0.029(0.004)$ | $0.848(0.013)$ | $0.001(0.000)$ | $0.141(0.017)$ | $0.841(0.017)$ |
| 18 | $0.028(0.004)$ | $0.884(0.011)$ | $0.002(0.000)$ | $0.134(0.017)$ | $0.833(0.019)$ |
| 19 | $0.038(0.005)$ | $0.868(0.012)$ | $0.002(0.000)$ | $0.094(0.014)$ | $0.859(0.022)$ |
| 20 | $0.042(0.005)$ | $0.859(0.012)$ | $0.029(0.003)$ | $0.232(0.031)$ | $0.044(0.064)$ |
| 21 | $0.037(0.004)$ | $0.848(0.013)$ | $0.029(0.002)$ | $0.239(0.038)$ | $0.000(0.066))$ |
| 22 | $0.025(0.004)$ | $0.808(0.014)$ | $0.035(0.003)$ | $0.182(0.036)$ | $0.000(0.077)$ |
| 23 | $0.014(0.004)$ | $0.801(0.016)$ | $0.035(0.002)$ | $0.565(0.054)$ | $0.000(0.024)$ |
| 24 | $-0.023(0.005)$ | $0.786(0.016)$ | $0.035(0.002)$ | $0.547(0.049)$ | $0.005(0.009)$ |

Table 5: Omel: $\operatorname{AR}(1)-\operatorname{GARCH}(1,1)$ parameter estimates (standard error)

| Hour | $c$ | $\phi_{h}$ | $\omega_{h}$ | $\alpha_{h}$ | $\beta_{h}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1 | $0.003(0.004)$ | $0.785(0.013)$ | $0.001(0.000)$ | $0.063(0.010)$ | $0.912(0.015)$ |
| 2 | $-0.023(0.004)$ | $0.819(0.012)$ | $0.001(0.000)$ | $0.063(0.010)$ | $0.907(0.017)$ |
| 3 | $-0.037(0.004)$ | $0.797(0.013)$ | $0.004(0.001)$ | $0.292(0.028)$ | $0.704(0.029)$ |
| 4 | $-0.042(0.004)$ | $0.862(0.010)$ | $0.016(0.002)$ | $0.239(0.031)$ | $0.345(0.059)$ |
| 5 | $-0.045(0.005)$ | $0.863(0.011)$ | $0.003(0.000)$ | $0.107(0.014)$ | $0.828(0.021)$ |
| 6 | $-0.040(0.004)$ | $0.883(0.009)$ | $0.002(0.000)$ | $0.125(0.015)$ | $0.809(0.023)$ |
| 7 | $-0.017(0.003)$ | $0.919(0.009)$ | $0.002(0.000)$ | $0.159(0.023)$ | $0.779(0.032)$ |
| 8 | $-0.005(0.003)$ | $0.867(0.012)$ | $0.001(0.000)$ | $0.142(0.013)$ | $0.859(0.011)$ |
| 9 | $0.019(0.004)$ | $0.803(0.014)$ | $0.000(0.000)$ | $0.114(0.009)$ | $0.897(0.007)$ |
| 10 | $0.034(0.005)$ | $0.806(0.013)$ | $0.000(0.000)$ | $0.113(0.011)$ | $0.892(0.009)$ |
| 11 | $0.052(0.005)$ | $0.802(0.014)$ | $0.000(0.000)$ | $0.080(0.009)$ | $0.923(0.007)$ |
| 12 | $0.056(0.005)$ | $0.804(0.013)$ | $0.000(0.000)$ | $0.058(0.007)$ | $0.943(0.006)$ |
| 13 | $0.055(0.005)$ | $0.821(0.012)$ | $0.000(0.000)$ | $0.054(0.007)$ | $0.946(0.006)$ |
| 14 | $0.041(0.005)$ | $0.846(0.011)$ | $0.000(0.000)$ | $0.055(0.007)$ | $0.943(0.006)$ |
| 15 | $0.021(0.004)$ | $0.874(0.011)$ | $0.001(0.000)$ | $0.102(0.014)$ | $0.879(0.017)$ |
| 16 | $0.023(0.004)$ | $0.849(0.012)$ | $0.000(0.000)$ | $0.069(0.009)$ | $0.927(0.009)$ |
| 17 | $0.024(0.004)$ | $0.842(0.012)$ | $0.000(0.000)$ | $0.076(0.007)$ | $0.929(0.006)$ |
| 18 | $0.037(0.004)$ | $0.824(0.012)$ | $0.000(0.000)$ | $0.041(0.005)$ | $0.960(0.004)$ |
| 19 | $0.038(0.005)$ | $0.849(0.011)$ | $0.000(0.000)$ | $0.039(0.005)$ | $0.959(0.005)$ |
| 20 | $0.027(0.004)$ | $0.885(0.011)$ | $0.001(0.000)$ | $0.074(0.011)$ | $0.908(0.014)$ |
| 21 | $0.029(0.004)$ | $0.874(0.011)$ | $0.002(0.000)$ | $0.074(0.011)$ | $0.884(0.018))$ |
| 22 | $0.047(0.005)$ | $0.834(0.013)$ | $0.002(0.000)$ | $0.085(0.013)$ | $0.877(0.018)$ |
| 23 | $0.052(0.006)$ | $0.765(0.014)$ | $0.001(0.000)$ | $0.058(0.010)$ | $0.924(0.013)$ |
| 24 | $0.007(0.004)$ | $0.833(0.012)$ | $0.038(0.003)$ | $0.310(0.035)$ | $0.000(0.005)$ |

Table 6: Powernext: SUR-GARCH $(1,1)$ parameter estimates (standard error)

| Hour | $c$ | $\phi_{h}$ | $\omega_{h}$ | $\alpha_{h}$ | $\beta_{h}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1 | $-0.195(0.008)$ | $0.214(0.009)$ | $0.098(0.020)$ | $0.121(0.019)$ | $0.787(0.033)$ |
| 2 | $-0.283(0.009)$ | $0.202(0.009)$ | $0.135(0.036)$ | $0.125(0.030)$ | $0.755(0.054)$ |
| 3 | $-0.351(0.010)$ | $0.218(0.009)$ | $0.744(0.019)$ | $0.312(0.039)$ | $0.000(0.196)$ |
| 4 | $-0.464(0.012)$ | $0.226(0.011)$ | $0.060(0.016)$ | $0.063(0.012)$ | $0.879(0.024)$ |
| 5 | $-0.529(0.014)$ | $0.244(0.012)$ | $0.028(0.007)$ | $0.062(0.009)$ | $0.912(0.013)$ |
| 6 | $-0.387(0.010)$ | $0.181(0.009)$ | $0.071(0.013)$ | $0.097(0.014)$ | $0.834(0.020)$ |
| 7 | $-0.239(0.010)$ | $0.196(0.011)$ | $0.055(0.013)$ | $0.072(0.012)$ | $0.876(0.020)$ |
| 8 | $-0.025(0.009)$ | $0.188(0.011)$ | $0.117(0.028)$ | $0.119(0.019)$ | $0.772(0.040)$ |
| 9 | $0.118(0.008)$ | $0.217(0.009)$ | $0.031(0.008)$ | $0.046(0.008)$ | $0.927(0.010)$ |
| 10 | $0.206(0.008)$ | $0.200(0.008)$ | $0.062(0.014)$ | $0.065(0.013)$ | $0.877(0.022)$ |
| 11 | $0.238(0.007)$ | $0.223(0.007)$ | $0.083(0.027)$ | $0.061(0.014)$ | $0.856(0.039)$ |
| 12 | $0.266(0.008)$ | $0.247(0.008)$ | $0.221(0.040)$ | $0.178(0.031)$ | $0.631(0.050)$ |
| 13 | $0.236(0.007)$ | $0.242(0.007)$ | $0.402(0.093)$ | $0.303(0.070)$ | $0.357(0.129)$ |
| 14 | $0.204(0.007)$ | $0.212(0.007)$ | $0.106(0.019)$ | $0.190(0.025)$ | $0.725(0.033)$ |
| 15 | $0.160(0.007)$ | $0.211(0.007)$ | $0.056(0.014)$ | $0.042(0.009)$ | $0.903(0.020)$ |
| 16 | $0.115(0.007)$ | $0.196(0.008)$ | $0.027(0.007)$ | $0.072(0.012)$ | $0.909(0.014)$ |
| 17 | $0.081(0.007)$ | $0.282(0.009)$ | $0.145(0.020)$ | $0.284(0.033)$ | $0.627(0.031)$ |
| 18 | $0.106(0.006)$ | $0.395(0.009)$ | $0.045(0.011)$ | $0.056(0.010)$ | $0.902(0.017)$ |
| 19 | $0.149(0.007)$ | $0.438(0.009)$ | $0.089(0.020)$ | $0.049(0.010)$ | $0.864(0.025)$ |
| 20 | $0.141(0.006)$ | $0.419(0.008)$ | $0.044(0.010)$ | $0.055(0.009)$ | $0.902(0.016)$ |
| 21 | $0.099(0.006)$ | $0.363(0.009)$ | $0.067(0.022)$ | $0.063(0.015)$ | $0.874(0.032)$ |
| 22 | $0.044(0.006)$ | $0.358(0.009)$ | $0.055(0.011)$ | $0.091(0.016)$ | $0.863(0.020)$ |
| 23 | $0.037(0.007)$ | $0.274(0.009)$ | $0.065(0.017)$ | $0.084(0.015)$ | $0.857(0.025)$ |
| 24 | $-0.004(0.007)$ | $0.258(0.009)$ | $0.059(0.014)$ | $0.109(0.017)$ | $0.837(0.026)$ |
|  |  |  |  |  |  |

Table 7: Exaa: SUR-GARCH $(1,1)$ parameter estimates (standard error)

| Hour | $c$ | $\phi_{h}$ | $\omega_{h}$ | $\alpha_{h}$ | $\beta_{h}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1 | $-0.126(0.006)$ | $0.327(0.007)$ | $0.045(0.009)$ | $0.100(0.012)$ | $0.860(0.016)$ |
| 2 | $-0.257(0.007)$ | $0.290(0.007)$ | $0.041(0.010)$ | $0.164(0.019)$ | $0.829(0.021)$ |
| 3 | $-0.358(0.008)$ | $0.262(0.008)$ | $0.572(0.043)$ | $0.510(0.046)$ | $0.036(0.041)$ |
| 4 | $-0.421(0.009)$ | $0.271(0.008)$ | $0.023(0.006)$ | $0.072(0.011)$ | $0.908(0.013)$ |
| 5 | $-0.399(0.009)$ | $0.291(0.008)$ | $0.028(0.006)$ | $0.077(0.009)$ | $0.903(0.010)$ |
| 6 | $-0.277(0.008)$ | $0.284(0.009)$ | $0.047(0.010)$ | $0.091(0.013)$ | $0.862(0.019)$ |
| 7 | $-0.181(0.010)$ | $0.264(0.010)$ | $0.096(0.021)$ | $0.109(0.022)$ | $0.803(0.036)$ |
| 8 | $-0.047(0.009)$ | $0.324(0.008)$ | $0.046(0.011)$ | $0.074(0.013)$ | $0.882(0.020)$ |
| 9 | $0.180(0.008)$ | $0.210(0.011)$ | $0.025(0.005)$ | $0.078(0.011)$ | $0.899(0.012)$ |
| 10 | $0.215(0.006)$ | $0.339(0.006)$ | $0.160(0.029)$ | $0.404(0.047)$ | $0.505(0.054)$ |
| 11 | $0.254(0.006)$ | $0.356(0.006)$ | $0.048(0.014)$ | $0.121(0.018)$ | $0.837(0.027)$ |
| 12 | $0.303(0.008)$ | $0.397(0.008)$ | $0.051(0.010)$ | $0.148(0.016)$ | $0.809(0.019)$ |
| 13 | $0.252(0.006)$ | $0.349(0.006)$ | $0.022(0.005)$ | $0.121(0.016)$ | $0.871(0.014)$ |
| 14 | $0.211(0.006)$ | $0.348(0.006)$ | $0.079(0.018)$ | $0.142(0.024)$ | $0.795(0.034)$ |
| 15 | $0.170(0.006)$ | $0.330(0.007)$ | $0.556(0.046)$ | $0.319(0.071)$ | $0.039(0.054)$ |
| 16 | $0.129(0.006)$ | $0.353(0.006)$ | $0.018(0.005)$ | $0.052(0.009)$ | $0.931(0.011)$ |
| 17 | $0.103(0.005)$ | $0.435(0.006)$ | $0.048(0.010)$ | $0.107(0.014)$ | $0.851(0.017)$ |
| 18 | $0.118(0.005)$ | $0.557(0.007)$ | $0.145(0.014)$ | $0.292(0.037)$ | $0.699(0.019)$ |
| 19 | $0.136(0.005)$ | $0.572(0.007)$ | $0.027(0.006)$ | $0.098(0.013)$ | $0.883(0.013)$ |
| 20 | $0.125(0.005)$ | $0.548(0.007)$ | $0.016(0.004)$ | $0.083(0.010)$ | $0.907(0.010)$ |
| 21 | $0.099(0.004)$ | $0.535(0.007)$ | $0.046(0.008)$ | $0.145(0.016)$ | $0.822(0.017)$ |
| 22 | $0.059(0.005)$ | $0.491(0.007)$ | $0.048(0.011)$ | $0.125(0.019)$ | $0.832(0.023)$ |
| 23 | $0.054(0.005)$ | $0.446(0.008)$ | $0.033(0.007)$ | $0.105(0.013)$ | $0.867(0.014)$ |
| 24 | $-0.046(0.005)$ | $0.443(0.008)$ | $0.055(0.009)$ | $0.136(0.016)$ | $0.813(0.020)$ |

Table 8: Omel: SUR-GARCH $(1,1)$ parameter estimates (standard error)

| Hour | $c$ | $\phi_{h}$ | $\omega_{h}$ | $\alpha_{h}$ | $\beta_{h}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1 | $0.005(0.005)$ | $0.420(0.009)$ | $0.006(0.003)$ | $0.075(0.009)$ | $0.923(0.009)$ |
| 2 | $-0.083(0.005)$ | $0.379(0.007)$ | $0.007(0.003)$ | $0.058(0.006)$ | $0.937(0.006)$ |
| 3 | $-0.155(0.006)$ | $0.362(0.009)$ | $0.033(0.009)$ | $0.116(0.015)$ | $0.857(0.018)$ |
| 4 | $-0.168(0.005)$ | $0.406(0.007)$ | $0.022(0.005)$ | $0.108(0.012)$ | $0.857(0.013)$ |
| 5 | $-0.197(0.006)$ | $0.398(0.007)$ | $0.029(0.009)$ | $0.094(0.015)$ | $0.877(0.022)$ |
| 6 | $-0.191(0.005)$ | $0.418(0.006)$ | $0.035(0.009)$ | $0.135(0.019)$ | $0.835(0.024)$ |
| 7 | $-0.134(0.005)$ | $0.403(0.007)$ | $0.038(0.011)$ | $0.135(0.019)$ | $0.818(0.030)$ |
| 8 | $-0.078(0.006)$ | $0.313(0.008)$ | $0.032(0.007)$ | $0.121(0.014)$ | $0.846(0.017)$ |
| 9 | $0.022(0.008)$ | $0.206(0.009)$ | $0.005(0.002)$ | $0.049(0.006)$ | $0.946(0.006)$ |
| 10 | $0.061(0.007)$ | $0.234(0.008)$ | $0.006(0.002)$ | $0.092(0.009)$ | $0.908(0.007)$ |
| 11 | $0.130(0.006)$ | $0.249(0.007)$ | $0.021(0.006)$ | $0.090(0.012)$ | $0.891(0.013)$ |
| 12 | $0.153(0.006)$ | $0.272(0.007)$ | $0.033(0.012)$ | $0.118(0.020)$ | $0.856(0.029)$ |
| 13 | $0.159(0.006)$ | $0.289(0.007)$ | $0.011(0.003)$ | $0.061(0.008)$ | $0.929(0.009)$ |
| 14 | $0.138(0.006)$ | $0.289(0.006)$ | $0.011(0.004)$ | $0.071(0.009)$ | $0.920(0.010)$ |
| 15 | $0.077(0.006)$ | $0.271(0.007)$ | $0.004(0.002)$ | $0.067(0.007)$ | $0.936(0.006)$ |
| 16 | $0.065(0.006)$ | $0.246(0.007)$ | $0.022(0.006)$ | $0.056(0.008)$ | $0.921(0.012)$ |
| 17 | $0.062(0.007)$ | $0.239(0.008)$ | $0.023(0.007)$ | $0.103(0.013)$ | $0.877(0.018)$ |
| 18 | $0.080(0.006)$ | $0.312(0.007)$ | $0.012(0.004)$ | $0.046(0.037)$ | $0.944(0.010)$ |
| 19 | $0.089(0.005)$ | $0.499(0.007)$ | $0.007(0.002)$ | $0.067(0.007)$ | $0.929(0.007)$ |
| 20 | $0.099(0.005)$ | $0.540(0.007)$ | $0.007(0.003)$ | $0.069(0.009)$ | $0.928(0.009)$ |
| 21 | $0.095(0.005)$ | $0.583(0.007)$ | $0.017(0.004)$ | $0.078(0.009)$ | $0.907(0.010)$ |
| 22 | $0.118(0.005)$ | $0.550(0.007)$ | $0.020(0.006)$ | $0.074(0.010)$ | $0.906(0.013)$ |
| 23 | $0.092(0.006)$ | $0.451(0.008)$ | $0.009(0.003)$ | $0.070(0.008)$ | $0.922(0.008)$ |
| 24 | $0.030(0.005)$ | $0.469(0.008)$ | $0.037(0.008)$ | $0.109(0.014)$ | $0.854(0.019)$ |

Table 9: Powernext: VAR-GARCH $(1,1)$ parameter estimates (standard error)

| Hour | $c$ | $\phi_{h}$ | $\omega_{h}$ | $\alpha_{h}$ | $\beta_{h}$ | other hours |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $-0.156(0.018)$ | $0.280(0.052)$ | $0.115(0.042)$ | $0.071(0.021)$ | $0.820(0.056)$ | $3,6,7,9,11,12,17,22,23,24$ |
| 2 | $-0.235(0.019)$ | $0.229(0.070)$ | $0.065(0.011)$ | $0.164(0.021)$ | $0.800(0.020)$ | $3,6,7,9,10,12,17,23$ |
| 3 | $-0.305(0.020)$ | $0.328(0.070)$ | $0.055(0.010)$ | $0.103(0.014)$ | $0.855(0.016)$ | $7,8,9,10,11,12,22,23,24$ |
| 4 | $-0.354(0.024)$ | $0.091(0.035)$ | $0.075(0.010)$ | $0.293(0.026)$ | $0.703(0.020)$ | $3,5,6,7,11,17,22,23,24$ |
| 5 | $-0.446(0.026)$ | $0.304(0.031)$ | $0.105(0.020)$ | $0.167(0.026)$ | $0.758(0.033)$ | $2,3,9,11,12,16,17,22,23,24$ |
| 6 | $-0.396(0.021)$ | $0.145(0.045)$ | $0.059(0.012)$ | $0.099(0.014)$ | $0.850(0.020)$ | $2,3,7,9,13,22,23$ |
| 7 | $-0.342(0.023)$ | $0.156(0.038)$ | $0.052(0.012)$ | $0.043(0.009)$ | $0.906(0.016)$ | $1,6,9,10,13,14,20,22,24$ |
| 8 | $-0.205(0.022)$ | $0.091(0.039)$ | $0.337(0.135)$ | $0.000(0.000)$ | $0.663(0.131)$ | $1,4,6,9,10,13,19,22$ |
| 9 | $-0.024(0.017)$ | $0.234(0.048)$ | $0.052(0.014)$ | $0.062(0.011)$ | $0.891(0.021)$ | $2,3,6,7,12,13,19,20,22,24$ |
| 10 | $0.078(0.016)$ | $-0.123(0.069$ | $0.054(0.013)$ | $0.097(0.014)$ | $0.853(0.022)$ | $2,3,7,9,12,18,19,22$ |
| 11 | $0.123(0.015)$ | $0.093(0.082)$ | $0.042(0.009)$ | $0.069(0.011)$ | $0.895(0.016)$ | $2,3,7,9,10,12,17,18,19,22$ |
| 12 | $0.153(0.016)$ | $0.449(0.061)$ | $0.030(0.006)$ | $0.089(0.011)$ | $0.879(0.014)$ | $2,37,10,17,18,22$ |
| 13 | $0.145(0.014)$ | $0.195(0.074)$ | $0.038(0.012)$ | $0.062(0.012)$ | $0.901(0.021)$ | $3,7,10,12,16,17,22$ |
| 14 | $0.111(0.014)$ | $0.014(0.089)$ | $0.066(0.022)$ | $0.114(0.028)$ | $0.831(0.042)$ | $3,7,10,12,16,17,19,22$ |
| 15 | $0.057(0.014)$ | $0.196(0.083)$ | $0.035(0.006)$ | $0.045(0.007)$ | $0.922(0.010)$ | $3,7,10,12,16,17,19,22,24$ |
| 16 | $0.002(0.015)$ | $0.215(0.067)$ | $0.054(0.013)$ | $0.114(0.016)$ | $0.842(0.024)$ | $3,7,10,12,17,19,22$ |
| 17 | $-0.035(0.015)$ | $0.159(0.043)$ | $0.059(0.013)$ | $0.167(0.019)$ | $0.799(0.022)$ | $1,5,10,12,19,22$ |
| 18 | $0.016(0.014)$ | $0.382(0.045)$ | $0.093(0.044)$ | $0.052(0.019)$ | $0.854(0.060)$ | $1,3,7,19,22$ |
| 19 | $0.109(0.014)$ | $0.497(0.047)$ | $0.181(0.040)$ | $0.069(0.016)$ | $0.754(0.049)$ | $3,17,18,21$ |
| 20 | $0.128(0.013)$ | $0.475(0.042)$ | $0.099(0.022)$ | $0.090(0.016)$ | $0.819(0.031)$ | $1,3,11,13,17,18,21$ |
| 21 | $0.088(0.013)$ | $0.218(0.042)$ | $0.137(0.034)$ | $0.121(0.022)$ | $0.742(0.052)$ | $1,3,5,6,7,11,12,20,22,24$ |
| 22 | $0.058(0.013)$ | $0.421(0.047)$ | $0.063(0.022)$ | $0.041(0.012)$ | $0.897(0.030)$ | $1,3,5,6,7,16$ |
| 23 | $0.047(0.016)$ | $0.098(0.068)$ | $0.047(0.010)$ | $0.071(0.011)$ | $0.884(0.017)$ | $1,3,6,7,11,13,22$ |
| 24 | $-0.032(0.016)$ | $0.170(0.061)$ | $0.157(0.074)$ | $0.071(0.011)$ | $0.884(0.017)$ | $1,3,6,7,11,12,22$ |

Table 10: Exaa: VAR-GARCH $(1,1)$ parameter estimates (standard error)

| Hour | $c$ | $\phi_{h}$ | $\omega_{h}$ | $\alpha_{h}$ | $\beta_{h}$ | other hours |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $-0.017(0.015)$ | $0.668(0.049)$ | $0.014(0.004)$ | $0.082(0.011)$ | $0.906(0.013)$ | $6,8,10,11,12,17,20,22$ |
| 2 | $-0.107(0.017)$ | $0.478(0.066)$ | $0.010(0.003)$ | $0.092(0.009)$ | $0.906(0.009)$ | $1,6,8,10,12,17,20,22,23,24$ |
| 3 | $-0.181(0.020)$ | $-0.071(0.069)$ | $0.015(0.005)$ | $0.084(0.012)$ | $0.904(0.014)$ | $1,2,6,10,12,16,20,23,24$ |
| 4 | $-0.238(0.021)$ | $0.314(0.064)$ | $0.016(0.005)$ | $0.079(0.015)$ | $0.907(0.017)$ | $1,2,3,6,7,10,12,21,23,24$ |
| 5 | $-0.235(0.021)$ | $0.052(0.062)$ | $0.007(0.002)$ | $0.057(0.007)$ | $0.939(0.007)$ | $1,2,3,4,6,7,8,10,21,23,24$ |
| 6 | $-0.224(0.020)$ | $0.303(0.043)$ | $0.008(0.003)$ | $0.071(0.008)$ | $0.926(0.008)$ | $1,2,3,4,7,8,12,14,21,22,23,24$ |
| 7 | $-0.290(0.030)$ | $0.489(0.040)$ | $0.011(0.004)$ | $0.070(0.011)$ | $0.922(0.013)$ | $1,3,4,8,9,10,11,14,17,22,23,24$ |
| 8 | $-0.132(0.026)$ | $-0.210(0.050)$ | $0.060(0.011)$ | $0.138(0.017)$ | $0.809(0.022)$ | $1,7,9,10,11,14,20,21,22,23,24$ |
| 9 | $-0.030(0.022)$ | $0.059(0.032)$ | $0.005(0.002)$ | $0.048(0.008)$ | $0.949(0.008)$ | $1,2,3,16,18,19,22$ |
| 10 | $0.059(0.014)$ | $0.069(0.069)$ | $0.007(0.003)$ | $0.076(0.009)$ | $0.923(0.009)$ | $6,12,14,16,20,22,23$ |
| 11 | $0.113(0.014)$ | $0.167(0.083)$ | $0.028(0.007)$ | $0.086(0.012)$ | $0.884(0.016)$ | $6,12,16,22,23$ |
| 12 | $0.178(0.019)$ | $0.617(0.041)$ | $0.020(0.006)$ | $0.086(0.012)$ | $0.899(0.015)$ | $4,5,6,7,8,22$ |
| 13 | $0.132(0.014)$ | $0.116(0.086)$ | $0.016(0.005)$ | $0.054(0.008)$ | $0.930(0.010)$ | $1,6,12,16,21,22,23$ |
| 14 | $0.077(0.014)$ | $0.293(0.086)$ | $0.010(0.003)$ | $0.106(0.012)$ | $0.891(0.011)$ | $6,12,16,22,23$ |
| 15 | $0.026(0.014)$ | $0.115(0.074)$ | $0.010(0.003)$ | $0.064(0.009)$ | $0.929(0.010)$ | $1,2,6,12,14,16,23$ |
| 16 | $-0.008(0.014)$ | $0.356(0.080)$ | $0.051(0.014)$ | $0.092(0.015)$ | $0.856(0.026)$ | $2,6,10,12,22,23$ |
| 17 | $-0.006(0.014)$ | $0.203(0.064)$ | $0.004(0.002)$ | $0.041(0.006)$ | $0.956(0.006)$ | $6,10,12,15,18,22,23$ |
| 18 | $0.044(0.014)$ | $0.762(0.057)$ | $0.013(0.004)$ | $0.067(0.009)$ | $0.921(0.010)$ | $1,6,8,10,15,19,23,24$ |
| 19 | $0.105(0.014)$ | $0.486(0.061)$ | $0.006(0.002)$ | $0.074(0.008)$ | $0.924(0.008)$ | $1,6,8,11,12,14,15,18,20,22,23,24$ |
| 20 | $0.121(0.014)$ | $0.460(0.053)$ | $0.012(0.004)$ | $0.102(0.012)$ | $0.897(0.012)$ | $1,6,8,10,11,12,14,21,22,23,24$ |
| 21 | $0.133(0.012)$ | $0.474(0.054)$ | $0.017(0.005)$ | $0.085(0.011)$ | $0.899(0.012)$ | $1,8,10,11,12,14,18,19,2023,24$ |
| 22 | $0.115(0.013)$ | $0.139(0.054)$ | $0.029(0.009)$ | $0.091(0.014)$ | $0.881(0.020)$ | $1,6,7,8,10,11,12,14,15,17,18,19,21,23,24$ |
| 23 | $0.155(0.016)$ | $0.360(0.060)$ | $0.008(0.002)$ | $0.048(0.005)$ | $0.946(0.005)$ | $1,2,6,8,9,10,11,12,15,18,19,20,24$ |
| 24 | $0.064(0.016)$ | $0.171(0.054)$ | $0.003(0.001)$ | $0.044(0.006)$ | $0.955(0.005)$ | $1,6,8,10,12,14,18,19,20$ |

Table 11: Omel: VAR-GARCH $(1,1)$ parameter estimates (standard error)

| Hour | $c$ | $\phi_{h}$ | $\omega_{h}$ | $\alpha_{h}$ | $\beta_{h}$ | other hours |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $-0.050(0.010)$ | $0.193(0.033)$ | $0.014(0.004)$ | $0.082(0.011)$ | $0.906(0.013)$ | $2,3,9,10,12,23,24$ |
| 2 | $-0.094(0.009)$ | $0.295(0.046)$ | $0.010(0.003)$ | $0.092(0.009)$ | $0.906(0.009)$ | $4,9,10,11,17,20,23,24$ |
| 3 | $-0.102(0.012)$ | $0.083(0.033)$ | $0.015(0.005)$ | $0.084(0.012)$ | $0.904(0.014)$ | $1,2,5,7,12,15,16,17,20,23,24$ |
| 4 | $-0.121(0.009)$ | $0.310(0.065)$ | $0.016(0.005)$ | $0.079(0.015)$ | $0.907(0.017)$ | $1,2,6,7,8,9,11,17,23,24$ |
| 5 | $-0.130(0.009)$ | $0.021(0.076)$ | $0.007(0.002)$ | $0.057(0.007)$ | $0.939(0.007)$ | $1,6,7,8,9,17,23,24$ |
| 6 | $-0.107(0.009)$ | $0.767(0.064)$ | $0.008(0.003)$ | $0.071(0.008)$ | $0.926(0.008)$ | $1,5,7,8,9,17,23,24$ |
| 7 | $-0.069(0.008)$ | $0.480(0.040)$ | $0.011(0.004)$ | $0.070(0.011)$ | $0.922(0.013)$ | $1,5,6,8,9,10,16,22,24$ |
| 8 | $-0.033(0.011)$ | $0.310(0.040)$ | $0.060(0.011)$ | $0.138(0.017)$ | $0.809(0.022)$ | $1,3,4,5,6,7,9,15,17,18,22,24$ |
| 9 | $-0.008(0.016)$ | $-0.159(0.042)$ | $0.005(0.002)$ | $0.048(0.008)$ | $0.949(0.008)$ | $1,3,5,6,7,8,10,15,17,19,20,22,23$ |
| 10 | $0.033(0.012)$ | $0.137(0.049)$ | $0.007(0.003)$ | $0.076(0.009)$ | $0.923(0.009)$ | $1,3,5,6,7,9,15,17,18,20,22,23,24$ |
| 11 | $0.079(0.011)$ | $-0.045(0.064)$ | $0.028(0.007)$ | $0.086(0.012)$ | $0.884(0.016)$ | $1,3,7,9,10,17,20,22,23$ |
| 12 | $0.105(0.010)$ | $0.040(0.083)$ | $0.020(0.006)$ | $0.086(0.012)$ | $0.899(0.015)$ | $1,3,7,9,10,11,13,17,20,23$ |
| 13 | $0.124(0.010)$ | $0.256(0.082)$ | $0.016(0.005)$ | $0.054(0.008)$ | $0.930(0.010)$ | $1,3,7,9,10,11,19,20,23$ |
| 14 | $0.108(0.010)$ | $0.183(0.072)$ | $0.010(0.003)$ | $0.106(0.012)$ | $0.891(0.011)$ | $1,3,7,9,18,19,23$ |
| 15 | $0.050(0.011)$ | $0.305(0.064)$ | $0.010(0.003)$ | $0.064(0.009)$ | $0.929(0.010)$ | $3,7,9,19,23$ |
| 16 | $0.049(0.011)$ | $0.187(0.088)$ | $0.051(0.014)$ | $0.092(0.015)$ | $0.856(0.026)$ | $2,3,4,7,9,10,11,15,18,19,21,23$ |
| 17 | $0.062(0.012)$ | $-0.074(0.065)$ | $0.004(0.002)$ | $0.041(0.006)$ | $0.956(0.006)$ | $2,3,4,6,7,9,10,11,15,18,19,23,24$ |
| 18 | $0.069(0.012)$ | $0.267(0.067)$ | $0.013(0.004)$ | $0.067(0.009)$ | $0.921(0.010)$ | $2,3,4,7,9,10,11,19,23,24$ |
| 19 | $0.073(0.011)$ | $0.692(0.044)$ | $0.006(0.002)$ | $0.074(0.008)$ | $0.924(0.008)$ | $3,7,9,20,21,23$ |
| 20 | $0.008(0.001)$ | $0.054(0.004)$ | $0.012(0.004)$ | $0.102(0.012)$ | $0.897(0.012)$ | $3,4,6,7,9,13,18,19,21,24$ |
| 21 | $0.066(0.010)$ | $0.638(0.037)$ | $0.017(0.005)$ | $0.085(0.011)$ | $0.899(0.012)$ | $3,4,7,9,13,14,18,20,22$ |
| 22 | $0.094(0.011)$ | $0.574(0.036)$ | $0.029(0.009)$ | $0.091(0.014)$ | $0.881(0.020)$ | $3,9,12,14,15,18,19,20,21$ |
| 23 | $0.060(0.011)$ | $0.248(0.042)$ | $0.008(0.002)$ | $0.048(0.005)$ | $0.946(0.005)$ | $1,3,9,10,12,14,18,19,20$ |
| 24 | $0.005(0.011)$ | $0.483(0.034)$ | $0.003(0.001)$ | $0.044(0.006)$ | $0.955(0.005)$ | $1,2,3,4,5,11,14,18,19,23$ |

Table 12: Powernext: Markov Switching parameter estimates (standard error).

| Hour | $\alpha_{1, h}$ | $\phi_{1, h}$ | $\sigma_{1, h}$ | $p_{11}$ |
| :---: | :---: | :---: | :---: | :---: |
|  | $\alpha_{2, h}$ | $\phi_{2, h}$ | $\sigma_{2, h}$ | $p_{22}$ |
| 1 | -0.057 (0.014) | 0.732 (0.033) | 0.398 (0.012) | 0.99 |
|  | -0.226 (0.018) | 0.223 (0.031) | 0.278 (0.009) | 0.99 |
| 2 | -0.065 (0.014) | 0.708 (0.024) | 0.249 (0.012) | 0.90 |
|  | -0.588 (0.079) | 0.126 (0.071) | 0.672 (0.051) | 0.42 |
| 3 | -0.111 (0.011) | 0.705 (0.017) | 0.281 (0.007) | 0.95 |
|  | -0.764 (0.105) | 0.060 (0.084) | 0.831 (0.067) | 0.44 |
| 4 | -0.181 (0.015) | 0.669 (0.019) | 0.329 (0.010) | 0.98 |
|  | -1.258 (0.239) | 0.023 (0.106) | 0.142 (0.204) | 0.41 |
| 5 | -0.205 (0.014) | 0.686 (0.017) | 0.337 (0.007) | 0.98 |
|  | -1.254 (0.179) | 0.015 (0.090) | 1.276 (0.099) | 0.66 |
| 6 | -0.097 (0.010) | 0.749 (0.018) | 0.225 (0.008) | 0.94 |
|  | -0.776 (0.085) | 0.089 (0.065) | 0.816 (0.060) | 0.56 |
| 7 | -0.028 (0.005) | 0.839 (0.017) | 0.144 (0.005) | 0.80 |
|  | -0.424 (0.034) | 0.194 (0.038) | 0.653 (0.019) | 0.64 |
| 8 | 0.103 (0.009) | 0.136 (0.019) | 0.209 (0.008) | 0.75 |
|  | -0.154 (0.021) | 0.828 (0.057) | 0.525 (0.017) | 0.65 |
| 9 | 0.073 (0.016) | 0.425 (0.033) | 0.486 (0.013) | 0.68 |
|  | -0.005 (0.007) | 0.895 (0.021) | 0.127 (0.005) | 0.74 |
| 10 | 0.159 (0.024) | 0.458 (0.039) | 0.520 (0.021) | 0.71 |
|  | 0.026 (0.006) | 0.862 (0.016) | 0.148 (0.006) | 0.89 |
| 11 | 0.182 (0.023) | 0.526 (0.036) | 0.464 (0.017) | 0.78 |
|  | 0.027 (0.000) | 0.887 (0.014) | 0.135 (0.005) | 0.91 |
| 12 | 0.236 (0.031) | 0.533 (0.039) | 0.531 (0.023) | 0.81 |
|  | 0.041 (0.006) | 0.852 (0.014) | 0.156 (0.005) | 0.94 |
| 13 | 0.185 (0.024) | 0.565 (0.036) | 0.429 (0.017) | 0.98 |
|  | 0.034 (0.005) | 0.866 (0.014) | 0.141 (0.004) | 0.86 |
| 14 | 0.158 (0.022) | 0.551 (0.036) | 0.428 (0.016) | 0.84 |
|  | 0.028 (0.005) | 0.861 (0.015) | 0.142 (0.004) | 0.94 |
| 15 | 0.128 (0.021) | 0.525 (0.038) | 0.450 (0.018) | 0.74 |
|  | 0.016 (0.005) | 0.872 (0.014) | 0.139 (0.005) | 0.90 |
| 16 | 0.089 (0.018) | 0.511 (0.035) | 0.443 (0.015) | 0.67 |
|  | 0.004 (0.004) | 0.876 (0.014) | 0.129 (0.005) | 0.84 |
| 17 | 0.059 (0.016) | 0.494 (0.035) | 0.434 (0.014) | 0.68 |
|  | 0.001 (0.005) | 0.879 (0.016) | 0.125 (0.005) | 0.83 |
| 18 | 0.082 (0.017) | 0.650 (0.032) | 0.408 (0.015) | 0.78 |
|  | 0.017 (0.005) | 0.859 (0.015) | 0.138 (0.004) | 0.90 |
| 19 | 0.138 (0.024) | 0.686 (0.032) | 0.447 (0.018) | 0.85 |
|  | 0.027 (0.005) | 0.859 (0.014) | 0.149 (0.004) | 0.95 |
| 20 | 0.144 (0.028) | 0.633 (0.045) | 0.430 (0.025) | 0.71 |
|  | 0.032 (0.005) | 0.839 (0.015) | 0.161 (0.005) | 0.94 |
| 21 | 0.106 (0.010) | 0.521 (0.022) | 0.250 (0.005) | 0.99 |
|  | -0.155 (0.014) | 0.206 (0.021) | 0.245 (0.007) | 0.99 |
| 22 | 0.029 (0.015) | 0.438 (0.025) | 0.235 (0.005) | 0.99 |
|  | -0.228 (0.012) | 0.193 (0.017) | 0.253 (0.007) | 0.98 |
| 23 | -0.023 (0.010) | 0.166 (0.022) | 0.224 (0.005) | 0.99 |
|  | -0.193 (0.017) | 0.845 (0.032) | 0.366 (0.010) | 0.98 |
| 24 | -0.107 (0.010) | 0.269 (0.023) | 0.239 (0.005) | 0.99 |
|  | -0.324 (0.024) | 0.918 (0.054) | 0.476 (0.022) | 0.96 |

Table 13: Exaa: Markov Switching parameter estimates (standard error).

| Hour | $\begin{aligned} & \alpha_{1, h} \\ & \alpha_{2, h} \end{aligned}$ | $\begin{aligned} & \overline{\phi_{1, h}} \\ & \phi_{2, h} \end{aligned}$ | $\begin{aligned} & \sigma_{1, h} \\ & \sigma_{2, h} \end{aligned}$ | $\begin{aligned} & p_{11} \\ & p_{22} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | -0.079 (0.008) | 0.779 (0.023) | 0.189 (0.002) | 0.92 |
|  | -0.058 (0.004) | 0.636 (0.013) | 0.228 (0.003) | 0.97 |
| 2 | -0.041 (0.010) | 0.779 (0.024) | 0.290 (0.007) | 0.99 |
|  | -0.277 (0.020) | 0.482 (0.031) | 0.225 (0.006) | 1.00 |
| 3 | -0.110 (0.008) | 0.760 (0.014) | 0.252 (0.004) | 0.99 |
|  | -0.990 (0.253) | 0.284 (0.132) | 1.096 (0.131) | 0.70 |
| 4 | -0.114 (0.009) | 0.783 (0.014) | 0.240 (0.005) | 0.98 |
|  | -0.804 (0.140) | 0.327 (0.086) | 0.857 (0.065) | 0.71 |
| 5 | -0.100 (0.008) | 0.803 (0.013) | 0.223 (0.005) | 0.98 |
|  | -1.020 (0.176) | 0.233 (0.096) | 1.053 (0.091) | 0.67 |
| 6 | -0.030 (0.005) | 0.871 (0.012) | 0.138 (0.004) | 0.91 |
|  | -0.506 (0.045) | 0.292 (0.047) | 0.586 (0.023) | 0.64 |
| 7 | -0.475 (0.059) | 0.857 (0.082) | 0.721 (0.037) | 0.37 |
|  | -0.059 (0.008) | 0.259 (0.015) | 0.259 (0.007) | 0.86 |
| 8 | -0.255 (0.019) | 0.291 (0.040) | 0.419 (0.015) | 0.38 |
|  | 0.186 (0.009) | 0.191 (0.016) | 0.251 (0.007) | 0.58 |
| 9 | -0.007 (0.007) | 0.932 (0.016) | 0.128 (0.006) | 0.77 |
|  | 0.125 (0.020) | 0.284 (0.035) | 0.541 (0.016) | 0.61 |
| 10 | 0.015 (0.005) | 0.938 (0.012) | 0.105 (0.004) | 0.88 |
|  | 0.132 (0.014) | 0.616 (0.029) | 0.324 (0.010) | 0.82 |
| 11 | 0.028 (0.005) | 0.912 (0.011) | 0.120 (0.003) | 0.95 |
|  | 0.193 (0.022) | 0.624 (0.032) | 0.350 (0.013) | 0.87 |
| 12 | 0.053 (0.006) | 0.877 (0.012) | 0.146 (0.004) | 0.97 |
|  | 0.387 (0.044) | 0.479 (0.044) | 0.569 (0.024) | 0.85 |
| 13 | 0.036 (0.005) | 0.899 (0.011) | 0.128 (0.003) | 0.96 |
|  | 0.172 (0.022) | 0.637 (0.034) | 0.344 (0.013) | 0.88 |
| 14 | 0.031 (0.005) | 0.894 (0.012) | 0.129 (0.003) | 0.97 |
|  | 0.126 (0.017) | 0.660 (0.029) | 0.328 (0.011) | 0.93 |
| 15 | 0.017 (0.006) | 0.906 (0.015) | 0.127 (0.005) | 0.91 |
|  | 0.119 (0.017) | 0.578 (0.036) | 0.362 (0.014) | 0.81 |
| 16 | 0.021 (0.005) | 0.904 (0.014) | 0.131 (0.005) | 0.97 |
|  | 0.064 (0.011) | 0.642 (0.025) | 0.304 (0.008) | 0.97 |
| 17 | 0.016 (0.004) | 0.905 (0.014) | 0.121 (0.005) | 0.97 |
|  | 0.058 (0.011) | 0.690 (0.026) | 0.296 (0.009) | 0.95 |
| 18 | 0.023 (0.004) | 0.887 (0.011) | 0.125 (0.003) | 0.96 |
|  | 0.091 (0.018) | 0.791 (0.026) | 0.339 (0.012) | 0.90 |
| 19 | 0.034 (0.005) | 0.859 (0.012) | 0.150 (0.003) | 0.98 |
|  | 0.156 (0.031) | 0.767 (0.034) | 0.364 (0.017) | 0.88 |
| 20 | 0.096 (0.013) | 0.762 (0.023) | 0.247 (0.008) | 0.72 |
|  | -0.006 (0.006) | 0.889 (0.025) | 0.097 (0.011) | 0.60 |
| 21 | -0.010 (0.004) | 0.976 (0.012) | 0.067 (0.004) | 0.58 |
|  | 0.074 (0.008) | 0.714 (0.020) | 0.226 (0.005) | 0.79 |
| 22 | -0.008 (0.004) | 0.955 (0.013) | 0.069 (0.003) | 0.59 |
|  | 0.055 (0.008) | 0.660 (0.021) | 0.248 (0.005) | 0.77 |
| 23 | -0.002 (0.004) | 0.964 (0.014) | 0.071 (0.004) | 0.60 |
|  | 0.066 (0.009) | 0.563 (0.023) | 0.304 (0.006) | 0.78 |
| 24 | -0.003 (0.005) | 0.970 (0.015) | 0.073 (0.004) | 0.60 |
|  | -0.018 (0.008) | 0.609 (0.021) | 0.298 (0.006) | 0.79 |

Table 14: Omel: Markov Switching parameter estimates (standard error).

| Hour | $\begin{aligned} & \alpha_{1, h} \\ & \alpha_{2, h} \end{aligned}$ | $\begin{aligned} & \phi_{1, h} \\ & \phi_{2, h} \end{aligned}$ | $\begin{aligned} & \sigma_{1, h} \\ & \sigma_{2, h} \end{aligned}$ | $\begin{aligned} & p_{11} \\ & p_{22} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.002 (0.005) | 0.915 (0.016) | 0.125 (0.007) | 0.78 |
|  | 0.007 (0.010) | 0.659 (0.028) | 0.296 (0.009) | 0.76 |
| 2 | 0.024 (0.004) | 0.996 (0.012) | 0.076 (0.006) | 0.55 |
|  | -0.054 (0.008) | 0.690 (0.023) | 0.254 (0.006) | 0.70 |
| 3 | -0.032 (0.004) | 0.853 (0.011) | 0.174 (0.003) | 0.99 |
|  | -0.534 (0.159) | 0.349 (0.108) | 1.056 (0.101) | 0.76 |
| 4 | -0.026 (0.006) | 0.884 (0.012) | 0.145 (0.005) | 0.94 |
|  | -0.151 (0.030) | 0.689 (0.044) | 0.371 (0.027) | 0.64 |
| 5 | -0.027 (0.005) | 0.890 (0.011) | 0.140 (0.004) | 0.95 |
|  | -0.161 (0.030) | 0.742 (0.037) | 0.385 (0.022) | 0.70 |
| 6 | -0.021 (0.005) | 0.913 (0.010) | 0.126 (0.003) | 0.95 |
|  | -0.141 (0.031) | 0.795 (0.036) | 0.383 (0.023) | 0.65 |
| 7 | -0.013 (0.003) | 0.933 (0.008) | 0.107 (0.003) | 0.96 |
|  | -0.106 (0.028) | 0.725 (0.048) | 0.352 (0.023) | 0.73 |
| 8 | -0.067 (0.009) | 0.760 (0.022) | 0.292 (0.005) | 0.99 |
|  | 0.139 (0.006) | 0.322 (0.015) | 0.132 (0.005) | 0.98 |
| 9 | -0.056 (0.010) | 0.735 (0.023) | 0.386 (0.007) | 0.99 |
|  | 0.245 (0.007) | 0.120 (0.013) | 0.157 (0.005) | 0.97 |
| 10 | 0.007 (0.004) | 0.912 (0.012) | 0.120 (0.005) | 0.84 |
|  | -0.007 (0.017) | 0.447 (0.034) | 0.463 (0.015) | 0.69 |
| 11 | 0.010 (0.004) | 0.940 (0.010) | 0.101 (0.004) | 0.79 |
|  | 0.050 (0.012) | 0.511 (0.030) | 0.359 (0.010) | 0.72 |
| 12 | 0.014 (0.004) | 0.950 (0.010) | 0.088 (0.004) | 0.77 |
|  | 0.059 (0.010) | 0.589 (0.026) | 0.315 (0.008) | 0.76 |
| 13 | 0.015 (0.004) | 0.952 (0.010) | 0.087 (0.004) | 0.79 |
|  | 0.057 (0.010) | 0.643 (0.025) | 0.303 (0.007) | 0.79 |
| 14 | 0.017 (0.004) | 0.948 (0.009) | 0.088 (0.004) | 0.80 |
|  | 0.034 (0.010) | 0.687 (0.025) | 0.301 (0.008) | 0.75 |
| 15 | 0.017 (0.004) | 0.899 (0.010) | 0.142 (0.004) | 0.96 |
|  | -0.050 (0.025) | 0.609 (0.051) | 0.414 (0.024) | 0.82 |
| 16 | 0.013 (0.004) | 0.887 (0.010) | 0.149 (0.004) | 0.95 |
|  | -0.035 (0.029) | 0.486 (0.026) | 0.486 (0.026) | 0.70 |
| 17 | 0.015 (0.004) | 0.862 (0.011) | 0.170 (0.005) | 0.96 |
|  | -0.077 (0.047) | 0.382 (0.064) | 0.671 (0.041) | 0.67 |
| 18 | 0.010 (0.004) | 0.932 (0.013) | 0.109 (0.008) | 0.77 |
|  | 0.023 (0.013) | 0.557 (0.032) | 0.367 (0.013) | 0.66 |
| 19 | 0.014 (0.004) | 0.946 (0.010) | 0.092 (0.005) | 0.81 |
|  | 0.038 (0.009) | 0.693 (0.024) | 0.287 (0.007) | 0.82 |
| 20 | 0.015 (0.004) | 0.946 (0.009) | 0.120 (0.003) | 0.97 |
|  | 0.058 (0.010) | 0.594 (0.034) | 0.261 (0.007) | 0.95 |
| 21 | 0.021 (0.004) | 0.924 (0.010) | 0.129 (0.005) | 0.91 |
|  | 0.059 (0.015) | 0.644 (0.044) | 0.296 (0.012) | 0.76 |
| 22 | 0.033 (0.005) | 0.890 (0.014) | 0.143 (0.007) | 0.91 |
|  | 0.071 (0.017) | 0.665 (0.038) | 0.335 (0.017) | 0.79 |
| 23 | 0.032 (0.005) | 0.902 (0.017) | 0.123 (0.008) | 0.78 |
|  | 0.038 (0.011) | 0.639 (0.027) | 0.331 (0.012) | 0.74 |
| 24 | 0.029 (0.005) | 0.933 (0.016) | 0.109 (0.008) | 0.66 |
|  | -0.001 (0.010) | 0.657 (0.027) | 0.299 (0.009) | 0.68 |

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[^0]:    ${ }^{1}$ We also considered a deterministic component different for each hour, but empirical evidence based on the behaviour of spot prices led us to discard it.
    ${ }^{2}$ Other functions, like, for example, sinusoidal function or a constant piece-wise or step function using dummy variables, were considered. The results do not change substantially.

[^1]:    ${ }^{3}$ All these datasets are freely available at the following web sites: www.powernext.com; en.exaa.at; www.omel.es/frames/en/index_eng.jsp

[^2]:    ${ }^{4}$ We do not report the results for the spline's parameters but they are significantly different from zero

[^3]:    ${ }^{5}$ Indeed, $\hat{P}_{h}(t)=\exp \left(\hat{y}_{h}(t)+f(t)\right)$ where $\hat{y}_{h}(t)$ is the forecast of the stochastic component of log-price and $f(t)$ is the deterministic component.

