ERRATA - CORRIGE

Correction to my Paper: « Pontryagin Type Dualities Over Commutative Rings » (*).

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There is a mistake in the proof of Lemma 5.8 of [B] due to the application of Lemma 2.1 in [S] which is incorrect because of a misprint.

Thus in the main theorem the statement: « Δ_1: $\mathfrak{D}(E) \to \mathfrak{C}(E)$ is a duality », has to be replaced by: « Δ_1: $\mathfrak{D}(E) \to \mathfrak{C}(E)$ is a good duality », which means that Δ_1 is a duality and E is topologically quasi-injective in a strong sense (s.q.i.).

So the correct version of the theorem is:

THEOREM. – *Let $E \in CMR$ be a faithful module. The following are equivalent*:

(a) Δ_1: $\mathfrak{D}(E) \to \mathfrak{C}(E)$ *is a good duality.*

(b) E *has properties* P_1), P_2), P_3).

(c) *If* $P = \mathrm{Chom}_Z(E, \boldsymbol{K})$ ($\boldsymbol{K}$ denotes the compact group of complex numbers of modulo 1), P *is a projective and finitely generated R-module with endomorphism ring isomorphic to R.*

Moreover, if any of the previous condition holds, then

1) $\mathfrak{D}(E) = \textit{Mod-}R$;

2) $\mathfrak{C}(E) = CMR$;

and, if Γ denotes the Pontryagin duality between Mod-R and CMR, then:

3) $\Delta_1(M) = \Gamma(M \underset{R}{\otimes} P)$, *for every* $M \in \mathfrak{D}(E)$;

4) $\Delta_2(M) = \mathrm{Hom}_R(P, \Gamma(M))$, *for evergy* $M \in \mathfrak{C}(E)$;

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that is the duality Δ_E is the composition of the equivalence $-\underset{R}{\otimes} P$: *Mod-R* $\to$ *Mod-R with the Pontryagin duality.*

PROOF. – The equivalence of (*b*) and (*c*) is proved in Theor. 4.6; the implication (*b*) $\Rightarrow$ (*a*) is proved by Theor. 4.9 and 2.7.

The implication (*a*) $\Rightarrow$ (*b*) is proved by the results in § 5 where in each statement the hypothesis « Δ_E is a duality » has to be replaced by « Δ_E is a good duality ».

With this change Lemma 5.8 can be proved in the following way:

LEMMA 5.8. – *Let Δ_E be a good duality. Then* $\operatorname{Im} T = \operatorname{Gen}(P)$, $\operatorname{Gen}(P) = \overline{\operatorname{Gen}}(P)$ *and P is a flat R-module.*

PROOF. – Since E is s.q.i., E^n is s.q.i. by Lemma 2.5 [B]. This means that for each closed submodule B of E^n, $E^n/B \in \mathcal{C}(E)$.

Now, if L is a submodule of P^n, $\Gamma_1(L)$ is topologically isomorphic to $E^n/L^{\perp}$ thus, $L \approx \Gamma_2(\Gamma_1(L)) \approx \Gamma_2(E^n/L^{\perp}) \in \Gamma(\mathcal{C}(E))$ which is $\operatorname{Gen}(P)$ by Lemma 5.7 [B]. We then get that P generates every submodule of P^n and thus, by Lemma 1.4 [ZH], P is a flat R-module and $\operatorname{Gen}(P) = \overline{\operatorname{Gen}}(P)$.

REFERENCES

[B] S. BAZZONI, *Pontryagin type dualities over commutative rings*, Ann. Mat. Pura Appl., **121** (1979), pp. 373-385.

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