

Sede Amministrativa: Università degli Studi di Padova
Dipartimento di Filosofia

SCUOLA DI DOTTORATO DI RICERCA IN FILOSOFIA
INDIRIZZO: FILOSOFIA TEORETICA E PRATICA
CICLO XXII

Vagueness and Identity. A Granular Approach

Direttore della Scuola: Ch.mo Prof. Giovanni Fiaschi

Coordinatrice d'indirizzo: Ch.ma Prof.ssa Francesca Menegoni

Supervisore: Prof. Massimiliano Carrara

Dottoranda: Silvia Gaio

Abstract in English

In this research work I take into account the relation of indistinguishability. This relation seems to be *prima facie* reflexive, symmetric and transitive; in short, an equivalence relation. However, there are some cases where the relation under consideration fails to be transitive. In this thesis I will discuss two of those cases: vagueness of gradable adjectives and count nouns, and identity criteria involving perceptual phenomena. My research attempts to answer the following question: how is it possible to communicate and to make meaningful judgments using vague terms and non-transitive identity criteria?

This thesis presents an analysis of vagueness and identity criteria that shows that speakers always consider the elements which vague expressions refer to or whose names compose identity statements within a context and from a certain level of precision.

I also attempt to provide a formal treatment of vague adjectives and count nouns on the one hand, and of identity criteria on the other. I employ two key concepts in both the treatments: context dependence and granularity.

Abstract in Italiano

Il filo conduttore della ricerca che viene qui presentata è la relazione di indistinguibilità. Questa relazione sembra essere, *prima facie*, riflessiva, simmetrica e transitiva, dunque, una relazione d'equivalenza. Tuttavia, ci sono dei casi in cui la relazione di indistinguibilità non risulta transitiva. In questa tesi prendo in considerazione due di questi casi: la vaghezza degli aggettivi gradualmente e dei sostantivi numerabili, e criteri d'identità la cui determinazione coinvolge dei fenomeni percettivi. La mia ricerca vuole rispondere alla domanda: come è possibile comunicare efficacemente ed esprimere degli enunciati sensati usando termini vaghi e criteri d'identità non transitivi?

Questa tesi presenta un'analisi che mostra come i parlanti di un linguaggio naturale quale l'inglese considerino sempre all'interno di un contesto e secondo un certo livello di precisione gli elementi a cui i termini vaghi si riferiscono o i cui nomi compongono enunciati d'identità.

Nella tesi cerco anche di offrire una trattazione formale degli aggettivi e dei sostantivi numerabili vaghi da una parte, e dei criteri d'identità dall'altra. In entrambe le trattazioni formali faccio uso delle nozioni di dipendenza contestuale e granularità.

Contents

1	Introduction	9
2	Vagueness. A Problem across Linguistics and Philosophy	13
2.1	Sketching the problem of vagueness	13
2.2	Matter of perception or matter of fact	15
2.3	Characteristics of vague expressions	18
2.4	Sorites paradox	20
2.5	Approaches to Sorites: an overview	24
2.5.1	Semantic approach	24
2.5.2	Epistemic approach	29
2.5.3	Ontological approach	30
2.5.4	Contextualist approach	31
2.6	Formal models for vague expressions	34
3	A Contextualist Account for Gradable Adjectives	37
3.1	Polar gradable adjectives	37
3.2	Theories for gradable adjectives	40
3.2.1	Degrees	40
3.2.2	Intervals	42
3.2.3	Tropes	45
3.2.4	Any alternative?	47
3.3	A model for polar adjectives	48
3.3.1	Aim	48
3.3.2	Theoretical background	49
3.3.3	Language and interpretation	50
3.3.4	Context structures and weak orderings	52
3.3.5	Van Benthem's constraints	54
3.3.6	Comparative relation	55
3.3.7	Granular levels	55
3.3.8	A problem and a refinement of the model	58
3.3.9	Example of an application of the model	63

3.3.10	Context structures and semi-orders	64
3.4	Results and remarks	69
3.4.1	Further results	69
3.4.2	Proof of (A)	71
3.4.3	Proof of (D)	72
3.4.4	Theoretical remark	74
3.5	Open problems	75
3.5.1	About infinity	76
3.5.2	About polarity	76
3.5.3	About the notion of granularity	77
3.5.4	Granular levels and precisifications	78
3.5.5	A brief comparison with the degree-based approach	79
4	Vague Count Nouns and Sortal Concepts	81
4.1	Introduction	81
4.2	Linguistic considerations about sortal terms	83
4.2.1	Sortal logics	88
4.3	Philosophical considerations about sortal concepts	89
4.3.1	On the debate about ontological realism	95
4.3.2	Strawson	96
4.3.3	Lowe	97
4.3.4	Lombard	98
4.3.5	Wiggins	100
4.4	Sortal vagueness vs. ontic vagueness	103
4.4.1	Vague objects	105
4.4.2	Vagueness of singular terms vs. sortal vagueness	110
4.4.3	A formal treatment of vague sortal terms	111
4.5	Appendix: a DL account for vague medical concepts	114
4.5.1	A study case of vague sortals: diseases	115
4.5.2	Sepsis: its characterization in Rough DL	115
4.5.3	Problematic issues	117
5	Towards a Logical Adequacy of Identity Criteria	121
5.1	Introduction	121
5.2	The identification role of ICs	124
5.2.1	Ontological, epistemic and semantic functions of ICs	128
5.3	Formal formulations of ICs	133
5.3.1	Requirements for R	135
5.4	Logical adequacy of ICs	138
5.4.1	Overlapping approach: foundational issues	142
5.4.2	Overlapping approximations	143

5.4.3	Quantitative vs. qualitative approach	145
5.4.4	Possible refinements	146
5.5	Approximating identity conditions. A granular proposal	149
5.5.1	Problematic issues	150
5.5.2	Contexts and levels of granularity	152
5.5.3	Granular models	153
5.5.4	Example	155
5.5.5	Objections and replies	155
5.5.6	Summing up	157
6	Conclusion	159
	Bibliography	161

Acknowledgements

I warmly thank Frank Veltman and Robert van Rooij (ILLC, Universiteit van Amsterdam), who carefully supervised my research on gradable adjectives. The main ideas for the formal model described in Chapter 3 came up from our fruitful discussions.

I would like to thank all the PhD students and young researchers I met during those three years in Padua and in Amsterdam and during some conferences abroad. The discussions with them helped me clarifying my thoughts about vagueness and identity criteria; they let me discover (and join) a vital and young logic-philosophical community.

More than anyone else, though, I am deeply grateful to my supervisor, Massimiliano Carrara: since my MA in Philosophy he has taken care of my training in Logic and Philosophy. I felt like he took me by hand and (very) patiently made me take my first, uncertain steps in the philosophical research. Thank you so much for everything you have taught to me.

Life is [...] a trap for logicians.
It looks just a little more mathematical and regular than it is;
its exactitude is obvious, but its inexactitude is hidden;
its wildness lies in wait.

(G. K. Chesterton, *The Paradoxes of Christianity*)

Chapter 1

Introduction

In this research work I take into account the relation of indistinguishability. As natural language speakers we make use of such a relation on several occasions in everyday life. We use it, for instance, to compare objects with respect to some properties, and to express the result of such a comparison, e.g. if two children are indistinguishable with respect to their height, we will say that they are equally tall. We also use the relation of indistinguishability to make identity judgments: if we see two pieces of cloth and we cannot distinguish the color of one from the color of the other, we will say that the two pieces of cloth have the *same* color.

The relation of indistinguishability is taken to be reflexive (any individual is indistinguishable from itself) and symmetric (given individuals x and y , if x is indistinguishable from y , then y is also indistinguishable from x). Furthermore, we tend to think of the relation of indistinguishability as transitive: if x is indistinguishable from y and y from z , then x is indistinguishable from z . In short, we tend to believe that the relation of indistinguishability is an equivalence relation: the relation seems to be *prima facie* reflexive, symmetric and transitive.

However, if we look at how things are in the world, we can reckon that there are some cases where the relation under consideration fails to be transitive. In this thesis I will discuss two of those cases: vagueness of gradable adjectives and count nouns, and identity criteria involving perceptual phenomena. The goal is to provide a formal treatment of gradable adjectives and count nouns on the one hand, and of identity criteria on the other, taking into account the logical problems related to the relation of indistinguishability (i.e. transitivity failure).

In the second chapter I will try to characterize the problem of vagueness in natural language. One of the features of vague expressions is that they give rise to the so-called Sorites paradox. In that paradox the relation of indistin-

guishability plays a central role and it turns out not to be transitive. I will also briefly present and discuss the most well-known philosophical theories relating to vagueness. Showing some shortcomings in those philosophical approaches, I will present my research work as an attempt to discuss the problem of vagueness both from a linguistic and a philosophical perspective.

In the third chapter I will present a model to formally represent the behavior of a class of vague linguistic expressions: relative gradable adjectives like ‘tall’, ‘young’ and ‘fast’. The leading questions behind the research on gradable adjectives are: How can we effectively communicate by using vague adjectives? What is the logic behind our use of them? The model developed is thought of as an attempt to answer those questions.

In the fourth chapter another class of linguistic expressions will be presented - that of count nouns. The philosophically interesting notion associated with the lexical category of count nouns is the notion of *sortal concepts*. I will offer some general thoughts on count nouns from a linguistic perspective, and on the correspondent sortal concepts from a philosophical perspective. I will then focus on an aspect related to the model presented in chapter 3. In that model, adjectives are represented by functions whose domains are sets denoted by count nouns. However, some count nouns are vague: how can they be represented in the model? I will suggest a formal treatment of vague count nouns to be integrated in the model for gradable adjectives in order to improve the model itself.

In the fifth chapter I will present the second case where the relation of indistinguishability fails to be transitive: identity criteria associated with sortals, i.e. with count nouns. In the philosophical literature some suggestions have been given in order to overcome the transitivity problems of the relation of indistinguishability for identity criteria. I will first present those approaches and then improve one of them by adding some further ingredients. The main objective of the chapter is to offer a (tentative) formal treatment of identity criteria.

There are two key concepts that are common to the models developed for treating vague adjectives and for representing identity conditions: context dependence and granularity.

Consider context dependence. The notion of context plays a central role when we have to establish whether or not two or more objects are distinguishable under some aspect. As will be shown in the thesis, the fact that some objects appear indistinguishable to us depends on the context within which we observe them. If we have, for instance, to compare the color of two pieces of cloth while looking at them, we could recognize the two objects as indistinguishable in color. But if we compare them with some other pieces of cloth with a similar color, maybe we can recognize a difference in shade.

In this thesis I will try to underline how contexts influence our use both of gradable adjectives and identity criteria for count nouns. Therefore, I will take context dependence to be a basic ingredient for a formal treatment both of vague expressions and of identity criteria.

Granularity is the second key concept. You can observe the world under several perspectives that differ from one another for the standard of precision adopted in each of them. For instance, if you want to draw the map of Italy, you can take a coarse point of view and draw Italy as boot-shaped. But taking a less coarse perspective, you can add some further details and draw the shore line more precisely, and the finer the perspective is, the more the inlets are detected and drawn. Those informal considerations from more or less refined points of view towards the world are captured by the notion of granularity. When you use gradable adjectives and when you make identity judgments you look at elements in a context under a certain perspective, or point of view, that can be more or less refined. In my thesis I will underline that the perspective under which you observe and compare elements in a context also influences your statements about those elements. In the formal treatment of gradable adjectives and identity criteria, the standards of precision of observations are called *granular levels* and are ordered from the coarsest to the finest.

What is the advantage of employing the notions of context dependence and granularity to account for vague expressions and identity criteria? My research is based on an analysis of how natural language speakers use vague expressions to communicate and how they use identity criteria to make identity statements. Such an analysis shows that in those cases speakers always consider the elements which vague expressions refer to or whose names enter into identity statements within a context and from a certain level of precision. If the context or the granular level varies, then, additionally, the truth value of statements containing vague expressions or the identity relation varies. Contexts and granular levels influence the semantics of vague expressions and of identity statements. So, the advantage of employing the notions of context dependence and granularity in a formal model is that such a model is able to reflect (at least, in part) the way natural language speakers *use* vague expressions and identity criteria.

Chapter 2

Vagueness. A Problem across Linguistics and Philosophy

The aim of this chapter is to introduce the problem of vagueness. I will try to characterize the linguistic phenomenon and to briefly sketch and discuss the main philosophical approaches to it. In the end of the chapter I try to draw the theoretical background of the model for vague expressions that will be presented in Chapter 3.

2.1 Sketching the problem of vagueness

Vagueness is a phenomenon that arises in ordinary language and involves several lexical categories¹. It is not easy to give a precise definition of what

¹Russell in [96] even argues for the thesis that all the words in a ordinary language are vague, even the words of pure logic such as logical connectives. ‘Or’ and ‘and’ seem at first glance to have a precise meaning. That a sentence containing ‘or’ or ‘and’ is true or false depends on the truth value of the terms that are connected by the logical connectives. Nevertheless, if such truth-bearers are not precise, then the truth value of sentences that connect them will not be precise either. Russell’s argument is the following:

P1 Non-logical words are vague.

P2 Truth and falsehood, as concepts applied to propositions containing non-logical words, are vague too.

P3 Propositions containing non-logical words are the substructure on which logical propositions (containing logical connectives) are built.

C Logical propositions become vague through the vagueness of truth and falsehood of the propositions containing non-logical words.

There are of course some objections that can be raised. One is the following: vague non-logical words, when connected one to each other by logical connectives, give rise to vague

vagueness is. What I will then present here is what vagueness phenomena look like.

Consider a British fellow named John who is 173 cm tall. Suppose an agent, referring to him, says:

(1) John is tall.

How can we evaluate (1)? Is it true or not? Our indecision about the truth value of (1) might be considered *prima facie* to derive from some sort of ignorance or from some kind of linguistic imprecision of the use of ‘tall’. ‘Tall’ is said to be a vague term, since the indecision about the truth value of (1) depends somehow on the meaning (in a wide sense) of ‘tall’.

In the case you think that the indecision derives from some kind of ignorance, you may claim, for example, that we do not know something concerning the discourse itself. That means, we do not know if our statements involving adjectives like ‘tall’ are true or false because we do not know if ‘tall’ can be applied to some cases (the so-called *borderline* cases, as we will see later on). However, suppose we are acquainted with the knowledge of the exact measure of John’s height and compare it with statistics for the group of people he belongs to, for example to British fellows. So, we know some relevant facts of the matter (John’s height compared to the average of British men) but we might still not be able to say if John is tall or not. Does our ignorance lie on the fact that we do not know if the adjective ‘tall’ is used appropriately in (1)? If so, it seems that the extension of predicate ‘tall’ is not fixed in a clear way because we are not sure about what the predicate applies to. It seems that we can say then that John is not clearly tall, nor clearly not tall. When we utter a sentence like (1), we use a predicate whose meaning is not well defined. If you consider, then, that the indecision about the semantic value of (1) is due to some linguistic imprecision, then you may endorse a view on vagueness close to Bosch’s one (see Bosch [11]), according to which vagueness is a case of incomplete definition.

Even if we have precise pieces of information about what the world looks like, we are still ignorant of something, namely, of the truth value of (1). Since, according to Tarski’s schema, “John is tall ” is true if and only if John is tall, and we do not know if “John is tall ” is true, it follows that we do not know if John is tall. Formally speaking:

propositions, but logical connectives are by themselves not vague. The source of vagueness is still natural language, and not logic itself, so it is wrong to say that logical connectives *are* vague.

$True^\ulcorner \phi^\urcorner \leftrightarrow \phi$

$\neg K(True^\ulcorner \phi^\urcorner)$

Then: $\neg K(\phi)$.

So, our ignorance does not just depend on a linguistics imprecision concerning the use of a term. The problem is epistemic too and concerns the fact that we do not know how many centimetres are enough to say that an individual is tall. We are not able to say where the cut-off point between tall and short individuals lies, if such exists.

However, it seems that we cannot avoid using vague predicates. Most people are not aware that vague words create semantic difficulties (See Parikh [82], p. 249). Nevertheless, we communicate through and by vague predicates. But what do we communicate? Which kind of information are we able to convey?

Barker [3] sketches an interesting theory about what we communicate while uttering a vague expression. If someone asserts (1), the audience can implicate (as a sort of Gricean implicature) that the standard of tallness for her speaking community is not greater than the degree of John's tallness. Basically, what the speaker does by uttering (1) is update the communicative context. Barker assigns to (1) a metalinguistic use (under some circumstances): the speaker asserting (1) communicates how she uses the word 'tall' appropriately. Barker assigns to the speaker herself the intention to use the vague expression to convey such a metalinguistic information. However, it is plausible to think that in some cases the speaker could not have a conscious intention of conveying some information. Nevertheless, also in such cases the result of her utterance of (1) for the hearer is that whatever the context is, the hearer will update her own information states: she gets the information that for the speaker the standard of tallness is at least as high as John's tallness.

2.2 Matter of perception or matter of fact

Consider now the adjective 'red'. Imagine a wall that is red on the left and progressively becomes orange on the right. So, there is a progressive series of color chips. Each chip differs from the next one by an imperceptible change in hue, that is, there is a difference, but we as human beings, with our perceptive apparatus, are not able to see it. So, if you take the first chip from the left side and then the second one, you do not notice any difference, so you do

infer that, if the first chip is red, the second is red as well. And so on, taking into consideration only two chips next one another, applying this reasoning we would come up saying that also the last chip, on the right side, is red. But we perceive it as clearly orange. What happens? We cannot distinguish two contiguous color chips, but we can distinguish the first from the last chip in the series of parts of the wall. The problem we are facing concerns the relation of indistinguishability: we have a series of color chips, each of them is indistinguishable from the contiguous ones; however, we can distinguish the color of the first chip from the last one. That means that the relation of indistinguishability is not transitive, because the property of being red is not maintained in the series. This is a form of the Sorites paradox (see sections 2.3 and 2.4) that involves a series of questions on perception. We have to consider “the difference between being red (a question of fact) and seeming red (a question of perception)” (Barker [4], p. 3 (online version); see also Raffman [93]). There are precise measurement devices that can determine whether an object is red, for any object you pick. Conversely, our perceptual ability is not great enough to determine with precision whether an object is red.

In case our perception is not precise enough to determine the application of a color predicate, we could stipulate that an expert could decide which chips are red, and which ones are orange. The color expert can use instruments to measure the precise amount of hue that there is in a part of our wall. In such a case, it seems that the problems of the speakers concerning the applicability of the color-predicates can be solved thanks to an appeal to an expert’s knowledge. If philosophers and linguists usually think of vagueness as a problem to be solved in some theory on natural language, this objection, that I will call *expert objection*, is against such a standard position. A thesis that a supporter of the expert objection may take on his own is that color predicates like ‘red’ are not observational.

There are some possible replies to the expert objection. I rehearse those by R. Parikh [83] (See especially pp. 522-523). First of all, if we have an instrument that can discriminate differences greater than a certain measure e , then we can still produce a paradox for differences between two objects smaller than e . Secondly, saying that a color predicate is not observational seems to go against our intuitions, since we decide whether an object has a color by looking at it. So, even though we can develop a non-observational theory about color-predicates, we nonetheless need a theory about how we *use* such predicates. Even if there is a community of experts that are able to determine the exact applicability of some predicate, in our normal use of language we cannot always ask an expert to judge whether a wall is red. Speakers use ‘red’ according to their color perception, and not to any precise

scale of hues or similar devices. Moreover, what Parikh stresses is that we learn the use of this kind of predicates by ostension, since it is not possible to give them definitions as we do for some other adjectives, for instance ‘bachelor’. We can learn what ‘bachelor’ means only by looking at a definition: ‘bachelor = not married man’. As soon as we learn that, there are not any doubts about when it applies or not. But we cannot do the same for ‘red’. We need to *see* some red things and someone telling us that those object are red.

Imagine a mother showing her child red objects, in order to make it learn what ‘red’ means. She cannot, though, discuss with it the colors of the objects it will see during its life. So, it might be that the child fixes an extension of ‘red’ that has much in common with its mothers words extension, but is not exactly the same. Let $X = \{o_1, \dots, o_k\}$ be the set of objects described by the mother as red and $Y = \{o'_1, \dots, o'_k\}$ the set of objects described by her as other than red. The child learns about R , the set of red objects, the following:

- $X \subseteq R$: the set of objects described by the mother as red is a subset of the set of red things, i.e. there can be red things other than those described by the mother;
- $R \cap Y = \emptyset$: the set of objects described as other than red have no element in common with R .

However, those two conditions might be satisfied by more than one set R . For instance, X might be the smallest set of red objects R , while Y^- , the complement of Y , the largest set R . Now, the child probably will not fix X nor Y^- as the right set R , since the child will recognise other objects than those collected by X that are red, and some examples of non-red objects in the set Y^- . The child will choose some set similar to X as the extension of ‘red’, but nothing guarantees that X will be exactly the child’s exact interpretation of ‘red’. This theoretical argument seems to be confirmed by an experiment based on the Munsell color chart. Its technical details can be seen in Parikh [83], p. 524. I report only the main result: We have disagreement about category boundaries between colors. Even if we arrive at a definition of ‘red’, it does not mean that we already have such a definition and that it governs our use of the word. As Parikh states (Parikh [83], p. 525):

if we were to *now* define the color red as light of wavelength 6,000 angstrom units, that would not explain how Shakespeare was using the word and why we are able to understand him.

2.3 Characteristics of vague expressions

Up to now I mentioned only vague adjectives. They are probably the most widely discussed example of vague expressions. However, there are other interesting kinds of linguistic expressions that are considered vague as well:

1. count nouns ('mountain', 'teenager', ...)
2. adverbs ('quickly', 'sadly', ...)
3. quantifiers ('almost', 'most', 'many', ...)
4. modifiers ('very', 'clearly', ...)
5. relations ('be friend of...', ...)
6. proper names of entities ('Mont Blanc', 'Sahara', ...) and definite descriptions ('Teseo's ship', ...).

In this dissertation I focus on vague adjectives (Chapter 3) and count nouns (Chapter 4) and I will mention the problem of vagueness of proper names (Chapter 4).

After having seen possible examples of vague expressions, let us try now to characterize vagueness. In order to do that, we need to have a good understand of the concept of vagueness that is used in the philosophical and linguistic literature.

Three features are attributed to vague expressions (I follow Keefe-Smith [46] and Keefe [45]):

1. **Borderline cases:** these are cases where it is not clear whether the vague linguistic expression applies. Some people are borderline tall when it is not clear whether or not they are tall, and a person is a borderline case of child when it is not clear if she is still a child or a teenager, and so on. The indeterminacy concerning the applicability of vague expressions to some cases may allow us to think that the correspondent sentences are neither true nor false. For instance, if John is a borderline case of tall man, sentence (1) turns out to be neither true nor false. This contrasts with the principle of bivalence. As we will see, some theories of vagueness vow to do away with such a principle.
2. **(Apparently) no sharp boundaries:** on a scale of heights, what is the exact measure that makes a person tall? What is the amount

of time or hearth-bites that makes a child become a teenager? With respect to a model-theoretical representation of predicates, having no sharp boundaries means lacking definite extensions. This is closely related to the presence of borderline cases. The indecisiveness about borderline cases makes it difficult to draw a sharp line between the extension and the anti-extension of vague predicates. But there to be no sharp boundary between the elements that are P , with P a vague predicate, and the elements that are not P does not coincide with there being a region of borderline cases. If such a region were sharply bounded, then P would have a sharp boundary; namely, its borderline cases constitute its boundaries. However, fuzzy boundaries affect borderline regions too, so the existence of fuzzy boundaries for P is not exactly the same as the existence of borderline cases of the applicability of P ². The presence of blurry boundaries must be distinguished from the presence of borderline cases for a further reason: consider the predicate ‘child*’³. By definition, the extension of ‘child*’ is the set of persons that are younger than sixteen years old, while its counter-extension is the set of persons that are older than eighteen. In such a case, the persons between sixteen and eighteen years old are borderline cases of the predicate ‘child*’. But that predicate and its counter-extension have sharp boundaries. That means, there are cases where borderline cases do not imply the presence of fuzzy boundaries, and the other way around.

3. **Sorites paradox:** vague expressions are susceptible to Sorites paradoxes, known also as “little-by-little arguments”. Soritical arguments will be considered in detail in the following section.

If an expression shows features 1-3, it can be called vague. Expressions that have only one or two of features 1-3 are not properly considered vague. In fact, taking those features into account, we can now consider what vagueness has to be distinguished from:

- **Ambiguity:** terms like ‘bank’ have two different main meanings. Both meanings, moreover, can be considered vague, but vagueness does not

²The fuzziness involving the boundaries of the region of borderline cases is closely related to the problem of higher-order vagueness. For my present purposes, though, I will not deeply analyze this problem.

³The example is taken from Sainsbury [99]. Consider also Sainsbury [97], where the author argues for the thesis that borderline cases are not enough to mark a linguistic expressions as vague. On the contrary, according to him, the absence of sharp boundaries is what marks a linguistic expression as vague.

concern the indecision over fixing one of those meanings as the intended one. That is just ambiguity. You can disambiguate ‘bank’ by fixing a context and looking at the objects the word refers to according to the speaker’s intentions in that context. Given an ambiguous term, for each context of use we can individuate the right referent for it, that is, we can individuate which of the two (or more) meanings the speaker attributes to it. This is not the case of vagueness: a vague word does not have multiple meanings. Consider the predicate ‘being tall’: it can have different extensions in different contexts, but not because it has different meanings. It is always related to the height of some individual. The adjective ‘tall’ has the meaning of ‘great in vertical dimension’ (see, for instance, the definition given in the lexical ontology WordNet). It can have different extensions in different contexts, but its definitional meaning does not change (see also Prinz [91], paragraph 5).

- **Underspecificity:** underspecified terms are not adequately informative for the purposes of the discourse. Consider the following expression as an example of underspecificity:

“ x is a natural number greater than 40”.

The subject of the sentence, x , varies on natural numbers greater than 40, and it does not refer to a certain, well-determined number. The predicate ‘being a natural number greater than 40’ can then be satisfied by different individuals, and nevertheless it has no fuzzy boundaries, no borderline cases, and does not give rise to a Sorites paradox.

- **Mere context-dependence** (usually regarding predicates corresponding to adjectives): that a predicate has different extensions in different contexts is not enough to say that such a predicate is vague. Take a vague predicate (e.g. ‘being tall’) and fix a context (the set of basketball players): the extension of the predicate in that context might be still vague. So, in order to characterize vagueness, it does not suffice to say that a predicate is vague *because* its extension varies from context to context.

2.4 Sorites paradox

We have seen some conceptual difficulties in considering a series of objects $a_1 \dots a_{100}$ (in our examples, parts of a not evenly painted wall or individuals

ordered with respect to their height) that differ from one another only by little. The difference is so small that it is not enough to distinguish objects that are contiguous in the series. The crucial point is that small changes do not seem to bring any big consequence. Nevertheless, we might notice a very big difference between a_1 and a_{100} , even though between any a_n and a_{n+1} there is no big difference. From such a tension between small changes and big consequences the so called Sorites paradox arises. The paradox is also called ‘the paradox of the heap’, but it is usually referred to by the name ‘Sorites’, that comes from the Greek name ‘soros’ that means, precisely, ‘heap’. The first formulation of the paradox is attributed to Eubulides of Miletus (belonging to the Megarian logic school, IV. BC), to whom also the first version of the Liar paradox is attributed.

The usual formulation of the argument consists of two premises and a conclusion:

1. One grain of sand does not make a heap;
2. adding a single grain of sand to a collection of grains of sand that does not make a heap, does not turn that collection to a heap.

If we assume both the premises, then no matter how many grains we have, we never obtain a heap.

The paradox works also in the other way around. Consider the following premises:

1. 10000 grains of sand make a heap;
2. taking a single grain of sand off from a heap, that is still a heap.

So, if we start with a 10000-grains heap and take one by one 9999 grains off, we will get a heap made by only one grain; but that is against our intuitions: a single grain does not make a heap.

There are at least three ways to formalize the Sorites paradox⁴:

- Consider a series of objects o_0, \dots, o_n and a predicate F applying to o_0 , but not to o_n . We have:

Premise 1 $F(x_1)$

⁴For the considerations about the Sorites paradox I refer to the first chapter of Keefe [45].

Premise 2 $\forall i : F(x_i) \rightarrow F(x_{i+1})$

Conclusion $F(x_n)$

If we interpret F as the predicate *short*, and x_1, \dots, x_n as a series of persons that differ one from the next one for one millimeter with respect to height, and such that x_1 is 150 cm tall and x_n 200 cm tall, the conclusion is false. We would never agree that an individual who is 200 cm tall is short. It must be mentioned also that the second premise is the so-called *inductive step* and here it is formulated by a universal quantification. The universal quantification is given up in the second way of formalization.

- Replace the universally quantified inductive premise with a sequence of conditional premises of the type $F(x_i) \rightarrow F(x_{i+1})$, that is:

$$F(x_1) \rightarrow F(x_2)$$

$$F(x_2) \rightarrow F(x_3)$$

$$F(x_3) \rightarrow F(x_4)$$

...

- It is not necessary to express the inductive premise with a conditional. We can substitute the sequence of conditional premises as follows:

$$\neg(F(x_1) \wedge \neg F(x_2))$$

$$\neg(F(x_1) \wedge \neg F(x_3))$$

$$\neg(F(x_3) \wedge \neg F(x_4))$$

...

Or, in alternative to the sequence, a unified quantified premise:

$$\forall i, \neg(F(x_i) \wedge \neg F(x_{i+1}))$$

Why do we say that the Sorites argument (in one of its forms) is a paradox? The argument has apparently true premises, the inference rules seem to be well applied, but the conclusion we get is regarded as false (or, at least, counter-intuitive). Since we are not up to accept the conclusion, it seems that there is something wrong, i.e. some mistake occurs either in the premises or in the reasoning. To find such a mistake is doing what is called the *diagnosis* of the paradox⁵. Paradoxes are considered as a kind of pathology that appears in a language (in the case of the Sorites paradox, the language is

⁵For the terminology used in the philosophical discussion on paradoxes I refer to Bradley Armour-Garb [1].

any natural one). A diagnosis for a paradox has to show what makes the argument at issue paradoxical. Possible kinds of diagnoses are the following (see Armour-Garb [1], p. 116 and Keefe [45], pp. 19-20.):

1. There is a mistake in the reasoning, that is, in the use of inference rules;
2. One of the premises is not true;
3. The conclusion is only apparently false;
4. There is no mistake in the reasoning, nor in the premises, and the conclusion is actually false: so, some concept involved in the premises is either incoherent or has limited applicability. For instance, in the case of Sorites paradox, the vague predicate involved might turn out to be incoherent, whenever we accept premises, reasoning and conclusion of the argument.

Mostly, the proposed solutions for the Sorites paradox focused on the second diagnosis. The inductive premise (in one of its possible formulations) is the target usually chosen. For example, the epistemic theories deny the inductive premise and state there is a i such that $F(x_i) \wedge \neg F(x_{i+1})$, even if we do not know which i is.

A rather extreme (usually called *nihilist*) position supported by Peter Unger embraces the fourth kind of diagnoses: there is no mistake in the argument and the problem is the incoherence of some predicates. There is no extension at all for vague predicates, because they are intrinsically incoherent (see Unger [114]).

However, Unger's view is not the unique way to take if you accept the soundness of the Sorites argument. The problem can be seen from a pragmatic point of view. Dummett [24], for instance, argues that the *use* of (at least some) observational predicates (like color predicates) is inconsistent. Usually, we face situations where we are able to use observational predicates without difficulties and there is no inconsistency in those cases. Some problems arise when there is a Sorites series. In such a case, vague predicates are too coarse to be fruitfully used. We need more precise tools to describe the situation and detect the differences between objects. As already mentioned, a problem underlying the Sorites paradox is the transitivity failure of the relation of indistinguishability. Consider the case of the painted wall again. According to our perceptual apparatus, we see each pair of contiguous parts of the wall as having the same color. The first part is red, and since the second is indistinguishable from the first, we conclude that the second part is red as well. We go on in this way, but at some moment we are no longer

up to accept that the wall is still red, because we perceive it orange. So, the relation of indistinguishability is not transitive: given three parts of the wall x , y and z , if x and y are indistinguishable, and y and z too, you cannot infer that x and z are indistinguishable too. A pragmatic solution on the basis of Dummett's observations was developed by Veltman and Muskens [120]. Considerations of pragmatic nature like Dummett's ones are also the theoretical basis of the model proposed in Chapter 3.

2.5 Approaches to Sorites: an overview

Theories of vagueness usually aim to provide an account of borderline cases, to explain why vague expressions (apparently) have no sharp boundaries, to supply a logic and semantics to vague languages, and to deal with the Sorites paradox. Some theories deal only with some, but not all, those aspects. In this section I focus on the main approaches in the philosophical literature that deal with all of them and that, in particular, offer a solution to the Sorites paradox.

There are four approaches to vagueness, and in each of them different kinds of solutions are developed. The four approaches are: semantic, epistemic, ontological, contextualist (contextualist theories actually can be categorized as semantic or epistemic on the basis of the kind of explanation they offer to the problem of vagueness). I briefly sketch those accounts in the following lines. A more detailed description and comparison of the theories can be found in Keefe and Smith [47], Graff Fara and Williamson [35], and Paganini [81].

2.5.1 Semantic approach

According to the semantic approach, vagueness is due to the way the natural language is related to the world. When we have a sentence involving a vague predicate applied to a borderline case, we are not willing to give a determinate truth value to that sentence. Consider again sentence (1): "John is tall". Suppose that John is a borderline case of tall man. Supporters of the semantic approach claim that there is no fact of the matter that can determine whether or not the application of the vague predicate 'tall' to John is correct. The predicate itself does not have a well-defined extension. According to the semantic approach, there is some kind of semantic indeterminacy: our indecision in applying a vague predicate is due to some indeterminacy of the natural language. As a consequence of that, some theories refuse the principle of bivalence: it is not the case that every sentence

is either true or false. I consider here two of those theories: degree theory and supervaluationism.

Degree theory : it appeals to a kind of logic that allows for degrees of truth value, called Fuzzy Logic⁶. A predicate is no longer a function from objects to the set $\{0, 1\}$ (with 1 standing for *true* and 0 for *false*), but a function from objects to the values in the unit interval $[0,1]$. So, an object might be 0.2-red, and not either red or not red. Such a continuous series of degrees of truth is theoretically supported by the observation that the Sorites Paradox is developed on a continuous chain of cases.

The degree of truth of a compound formula is given by the degrees of truth of its components. Consider the constraints for the evaluations of negation, disjunction, conjunction and conditional:

$$\mathcal{V}(\neg\phi) = 1 - \mathcal{V}(\phi)$$

$$\mathcal{V}(\phi \vee \psi) = \max(\mathcal{V}(\phi), \mathcal{V}(\psi))$$

$$\mathcal{V}(\phi \wedge \psi) = \min(\mathcal{V}(\phi), \mathcal{V}(\psi))$$

$$\mathcal{V}(\phi \rightarrow \psi) = \begin{cases} 1 & \text{when } \mathcal{V}(\psi) \geq \mathcal{V}(\phi) \\ 1 - \{\mathcal{V}(\phi) - \mathcal{V}(\psi)\} & \text{when } \mathcal{V}(\phi) > \mathcal{V}(\psi) \end{cases}$$

The constraint for the conditional can be intuitively read in this way: if the antecedent has a higher degree of truth than the consequent, the conditional cannot be entirely true. Hence, given a conditional in a Sorites argument:

$$F(x_n) \rightarrow F(x_{n+1}),$$

if x_n is a borderline case of F , the antecedent has a degree of truth less than 1 and greater than 0, and higher than the consequent. So,

⁶It has to be noted, though, that in linguistic literature degree-based theories developed for vague adjectives do not make use of Fuzzy Logic. While Fuzzy Logic provides a scale of degrees of truth values ranging from 0 to 1, a linguistic degree-based theory considers adjectives as functions from objects to degrees on a scale. Such a scale does not consist of truth values degrees and, moreover, the range is from 0 to infinity rather than from 0 to 1. Degree-based theories for vague adjectives will be treated in next chapter and not in the present chapter, since they do not present a solution to the Sorites paradox.

the conditional is not entirely true. Call $F(x_0), F(x_1), \dots, F(x_n)$ the sentences obtained applying F to all the elements of a Sorites series. Intuitively, the degree of truth decreases as we go on along the Sorites series. This seems to be plausible; it also reflects the intuition of the gradual passage from F to $\neg F$ without any sharp boundary.

This theory presents some problems, though. First of all, one of the reasons to adopt Fuzzy Logic is to be closer to speakers' intuitions about the truth value of sentences containing vague expressions. It seems that a standard two-valued approach is not good because it cannot provide an agreement about the applicability of a predicate to some objects. But a plural valued approach like Fuzzy Logic does not seem to produce a better agreement. Consider a situation where it is lightly raining. Since it might seem not completely true to assert "It rains", if we were to take such an approach seriously, we could claim "It rains at 0.45 degree". However, such a claim is certainly not intuitive or natural at all. Secondly, it is not clear where these degrees come from and how we can state that an object is exactly 0.33-red and not 0.34-red. A third problem and, in my opinion, the most serious, arises from the semantics given for Fuzzy Logic. Recall the constraints for negation, conjunction and disjunction stated above. Consider an object o that is red to some extent, such that the sentence " o is red" has truth-degree of 0.5. But then, also its negation, " o is not red", will have degree equal to 0.5. If we take now the conjunction of the two sentences, we get something we do not want, that is: " o is red and is not red" has degree 0.5, and not 0 as we would expect, since it is contradictory. That means, if we get rid of bivalence and accept a degree-based theory of truth, then we can get unwanted results, such as a contradictory statement that is not (completely) false.

Supervaluationism : it defines precisifications, that is, partial assignments of truth values to statements. Formally, given a language \mathcal{L} of predicate logic, a supermodel \mathcal{M} for \mathcal{L} is an ordered pair $\langle \mathcal{D}, \mathcal{J} \rangle$ where

1. \mathcal{D} is the domain of \mathcal{M} , namely a non empty-set;
2. \mathcal{J} is a non empty set of partial interpretations such that
 - each $\mathcal{I} \in \mathcal{J}$ assigns to each constant a an element d of \mathcal{D} such that $\mathcal{I}(a) = d$;
 - each $\mathcal{I} \in \mathcal{J}$ assigns to each n-ary predicate P a partial function $\mathcal{I}(P)$ from \mathcal{D}^n into $0, 1$. This partial function is what is called *interpretation*.

The sentences that are true (or false) in a model have to remain true (or false) when the model is refined.

Consider what a refinement relation between models consists of. Take two models $\mathcal{M} = \langle \mathcal{D}, \mathcal{I} \rangle$ and $\mathcal{M}' = \langle \mathcal{D}, \mathcal{I}' \rangle$. They both have the same domain, but the interpretation function varies.

Definition 1 \mathcal{M}' is a refinement of \mathcal{M} iff:

1. If P is n -ary and $\mathcal{I}(P)(\langle d_0 \dots d_n \rangle) = 1$, then $\mathcal{I}'(P)(\langle d_0 \dots d_n \rangle) = 1$;
2. If P is n -ary and $\mathcal{I}(P)(\langle d_0 \dots d_n \rangle) = 0$, then $\mathcal{I}'(P)(\langle d_0 \dots d_n \rangle) = 0$.

The definition intuitively tells us that if we take an interpretation function $\mathcal{I}'(P)$ that is a refinement of $\mathcal{I}(P)$, $\mathcal{I}'(P)$ preserves the same truths and the same falsities as $\mathcal{I}(P)$, and maps at least one more element from \mathcal{D}^n into 0, 1. In the philosophical literature on vagueness a model \mathcal{M}' that is a refinement of \mathcal{M} is called a *sharpening* or *precisification* of \mathcal{M} .

Other constraints are the following:

- for all $\mathcal{I}, \mathcal{I}' \in \mathcal{J}$ and each individual constant a : $\mathcal{I}(a) = \mathcal{I}'(a)$
- there is a $\mathcal{I}_0 \in \mathcal{J}$ such that for all $\mathcal{I} \in \mathcal{J}$: \mathcal{I} is a refinement of \mathcal{I}_0
- for all $\mathcal{I} \in \mathcal{J}$ there is a $\mathcal{I}' \in \mathcal{J}$ such that \mathcal{I}' is a refinement (precisification) of \mathcal{I}

In such a framework, sentences that are true in all the precisifications are said to be *supertrue* (hence, the name *supervaluationism*). Sentences that are false in all the precisifications are said to be *superfalse*. But sentences describing borderline cases for some vague predicate are true with regard to some precisifications, and false regarding to others. They are said to be neither true nor false, that is, to be lacking in truth values. Supervaluationism is then committed to a non-bivalent logic and specifically a logic that admits truth-value gaps. Such gaps are meant to capture the idea of the indeterminate application of vague predicates to borderline cases.

Logical truths turn out to be supertrue, while falsities (contradictions) are superfalse. Here we get then an improvement over Fuzzy Logic: a sentence like “ o is red and is not red” will be false in all the specifications, and “ o is red or is not red” will be true in all of them. One thing to notice is that if “ o is red” is true in some precisifications, false in others, also “ o is not red” is true in some precisifications, false in

others. In such a case “*o* is red and is not red” remains false in all the precisifications, while “*o* is red or is not red” remains true in all of them. That means, supervaluationist semantics is not truth-functional: the truth value of a compound sentence is not determined by truth values of its components.

What solution for the Sorites paradox is provided by supervaluationism? The supervaluationist logical machinery makes one of the conditional premises neither true nor false (consider sofar the second version of the Sorites paradox). Take for instance the conditional $F(x_i) \rightarrow F(x_{i+1})$. There are some precisifications of the interpretation of F that make both antecedent and consequent true, some precisifications that make both of them false, and one precisification that makes the antecedent true, but the consequent false. The Sorites reasoning is then blocked if in the series there is a conditional premise with those semantic features.

The problem that we have with supervaluationist theories is what is called *higher-order vagueness*. There is no distinction between borderline and not borderline cases. Another problematic issue for supervaluationist theories is that they refuse the truth of the induction premise. But if this means that they negate it, then they should accept that there is a i such that $F(x_i)$ and $\neg F(x_i + 1)$, as the epistemicist does, as we will see later. However, they do not accept that F has a clearly determined extension. In other words, the following formula turns out to be true in all precisifications:

$$\text{(SB)} \quad \exists n(F(x_i) \wedge \neg F(x_{i+1})).$$

(SB) claims is that there is a sharp boundary between F and $\neg F$. Even if (SB) is supertrue, supervaluationism keeps it true that there is no particular number satisfying the existential quantification in (SB). That means, it is supertrue that there is a n $F(x_n)$ and $\neg F(x_{n+1})$, but the formula is neither true nor false for any n you consider. That seems a counter-intuitive result (see Olin [80]).

There is another semantic theory, that does not discuss the principle of bivalence, nor refuse the inductive premise. On the contrary, it assumes that the Sorites argument is sound: the premises are true, the reasoning right. The conclusion, then, is also true, but it sounds wrong to us: that happens because the vague expressions involved in the argument are incoherent. So, basically, the whole Sorites argument is sound because, trivially, there is no

such a ordinary thing like a heap. I have already mentioned this theory: it is Unger's nihilist theory (see Unger [114]).

Some other philosophers carry on a non-classical reasoning. Consider borderline cases of predicate 'tall': they are borderline cases both of 'tall' and 'not-tall'. If John is such a borderline case, (1) is both true and false. In other words, instead of speaking of truth-value gaps, such an approach considers truth-value gluts. Some of its supporters are dialetheists or paraconsistent logicians: Graham Priest, JC Beall and Marc Colyvan (see, for example, Beall and Colyvan [5]).

2.5.2 Epistemic approach

Roy Sorensen and Timothy Williamson are the most well-known supporters of epistemic theories for vagueness⁷. Briefly sketched, the main thesis is that vague predicates have determined extensions and the sentences containing them are either true or false. The problem is that we do not know where the border of the extension is, nor the right truth value. That is, vagueness is due to our ignorance, not to our use of language.

The diagnosis of the Sorites paradox provided by the epistemic theory detects the problem in the inductive premise (premises). One instance of the universal inductive premise, or equivalently one of the conditional premises, is considered as not true. We do not know which specific instance this is, because our knowledge is limited, but the only way to get out of the paradox is to admit the existence of such a false instance.

In this approach logic is still classical and the principle of bivalence valid. We can consider the burden of Williamson's work to show that classical logic can still be considered as the logic of natural language. Nevertheless, the question that arises is: why should we keep classical logic for a theory that dogmatically says that there are cut-off points, without us being able to know them?

Moreover, another objection that can be raised against the epistemic view comes directly from its core. The statements about the existence of unknowable and mind-independent truths sound implausible: if there are unknown truths about the use of some words, how can we generally know what we are saying? According to Williamson, the theoretical understanding of words come together with linguistic practice. But I cannot see how this can justify the claim that we know some mind-independent truths, while we do not some others (relative to borderline cases), and moreover, that the

⁷To give some samples of their work, see their outstanding monographs on vagueness: Williamson [127], Sorensen [107].

latter are not knowable at all.

2.5.3 Ontological approach

The ontological approach supports a rather extreme ontological thesis, since it claims that vagueness is not a problem of our language, nor of our cognitive states, but it is a feature of the world (see van Inwagen [116], Tye [113]). The problem is brought to ontology. The vagueness of our language reflects the vagueness of states of affairs. Russell's criticism towards an ontological view of vagueness became famous. Russell [96] attributed a fallacy of verbalism to ontological theories of vagueness. According to him, it is mistaken to attribute a property of language (vagueness) to the world. In fact, in his words:

Vagueness and precision alike are characteristics which can only belong to a representation, of which language is an example.

So, vagueness and precision are properties of words and cannot be considered as properties of things.

An ontological approach is usually invoked in the issues concerning the identity relation (see in particular: Parsons [84] and Parsons-Woodruff [85], [128]). In general, though, the claim of the ontological approach to vagueness is that some objects (like Mount Everest) are vague because they lack of determinate boundaries. In other terms, the referents of singular terms are considered vague. But there are also predicates that seem to generate the same problem. Take the predicate 'cloud' and one of its instances: how can we determine the exact boundaries of the object under consideration? It seems that the evanescent nature of clouds prevent us from finding their boundaries. But can we think of ontological vagueness when we consider predicates like 'tall', 'red', ...? What does it mean to say that the vagueness of such predicates has an ontological nature? The idea is not that those predicates correspond to vague properties existing in the world, as though these were objects. The idea is rather the following: a sentence containing a vague predicate applied to some borderline case corresponds to a state of affairs. It is such a state of affairs that happens to be vague, according to the ontological approach to vagueness. Put in different terms, such an approach claims that there is no determinate fact of the matter whether a borderline case of tall man has the property of *being tall*, the latter conceived as something you can find in the world. The corresponding proposition is thought of as lacking truth value.

2.5.4 Contextualist approach

There is another way of approaching vagueness, that is, to consider the context-dependence a central feature of the vagueness phenomena. Far from identifying vagueness with mere context dependence (since, as we have seen, the two phenomena are not the same), what is underlined is that vague predicates are *also* context-dependent. Contextualism focuses more on predicates than on other linguistic expressions. But then, all the debate concentrates especially on vague predicates. Many theorists from Philosophy and Linguistics support contextualism, following Hans Kamp and Peter Bosch. I cautiously avoided identifying vagueness with context dependence. However, next chapter is concerned with a specific kind of vague linguistic expression, that is, adjectives. Many of them are context-dependent as their extension varies from context to context. So, *context-dependence* is recognized to be a feature of some predicates. For instance, the predicate ‘being tall’ applied to the context of human beings has a different extension than in the domain of equatorial trees. Far from saying that vagueness of adjectives like ‘tall’ is solved by saying that their meaning depends on the context taken into account, a contextualist approach to vague predicates wants to include also a treatment of context dependence in a theory of vagueness.

According to Delia Graff Fara, many contextualist theorists (both from Linguistics and Philosophy) have the goal of providing an answer to the question she calls *Psychological Question*:

If the universally generalized Sorites sentence is not true, why were we so inclined to accept it in the first place? In other words, what is it about vague predicates that makes them seem tolerant, and hence boundaryless to us? (Fara [27], p. 50).

The context dependence of vague predicates consists of two factors (I refer here to Barker [4]):

- dependence on a contextual standard: Bill is tall if he is tall at least to some degree d , where d is a “threshold for tallness provided by the context” (Barker [4], p. 6 - on-line version). Such a threshold for ‘tall’ can be thought of as the measure that is the average of the heights of the individuals in the context. This is not enough, though, to find a threshold. Consider the following example (given by Fara [27], p. 55-56): suppose we want to state the standard of height for basketball players. Suppose that all the tall basketball players are killed under some tragic circumstances. Now, the average height relevantly decreases. But in such a case, it does not seem to be true to say that

the tallest surviving basketball player is tall for a basketball player. So, to make an average is not always intuitively enough to find the threshold. Consider also a second example, given again by Fara. By some bizarre coincidence, all the basketball players happen to be also golfers. Now, it seems rather unfair to say that who is tall for a golfer is tall for a basketball player (Fara [27], p. 56). We have to consider the *typical* height for individuals of some specific kind. I doubt there can be exact criteria to do that: we have to trust our intuitions on the specific kind of things we compare. And here we get on to the second feature:

- reference to a comparison class: the threshold is often provided with respect to a set of objects and the predicate at issue is judged with respect to such a set that is called *comparison class*. The denotation of a predicate is then relativized to a class of objects; to give an example, ‘tall’ can be relativized to ‘basketball players’. In sentences containing vague predicates, the comparison classes can also be specified (“Maria is tall for an Italian woman”). However, ‘tall for an Italian woman’ is equally as vague as ‘tall’. We can also use the vague adjective in an attributive function: “Maria is a tall Italian woman”.

The relativization of predicates to a comparison class brings us to think that adjectives like ‘tall’ are not intersective, that is, the extension of ‘tall man’ is not given by the intersection between the extension of ‘tall’ and ‘man’. Namely, consider the case if John is a tall man and a basketball player. If we intersected the extensions of ‘tall’, ‘man’ and ‘basketball player’, we would get that John is also a tall basketball player, but this is a conclusion we cannot make, if our information is just that John is a man, tall, and plays basketball. Namely, he might well be a tall man, but a short basketball player.

But what should we consider as comparison classes? For example, we cannot say “Bill is tall for the objects in this room”: it is unnatural to say that there is a standard, a typical height among the objects present in a room. That means, we have no notion of what ‘being tall’ for the things in the room means, nor of what the typical degree of tallness for such set of objects must be. Fara [27] claims that comparison classes have to be *natural kinds*. Whether one object considered in a certain natural kind has a property depends on the typical standard of that natural kind, and we have some sort of idea of what such a degree is. But, as we have already noticed, the notion of natural kind refers to our intuitions, and is not strongly characterized. A further objection

to the theory of natural kinds can also be the following: consider a room containing only two distinct objects, a chair and a cupboard. It seems plausible to claim something like “the cupboard is tall for the objects in this room”: we are comparing the tallness of the chair with the tallness of the cupboard and that seems correct. Even if there are objections to the theory of natural kinds, an advantage of the latter is that you can accurately predict that when someone says “John is tall”, she means that John is tall compared with the natural kind of men, for instance. In such a case, it would be unnatural to think that the speaker aimed to compare John with the objects in a room. So, at least for sentences where a comparison class is not explicitly mentioned, it is useful to refer to natural kinds to make the comparison class explicit.

The two factors detected by Barker [4] are not enough to characterize context-dependence. There is a second sense of context-dependence that has to do with the domain of the discourse. Suppose you are a teacher and have a class of eight-year-old children. All of them are part of the natural kind of children, but still you do not want to compare each of them with all the children in the world belonging to the same natural kind, or with all eight-year-old children in the world, but rather only with the children in the class. Put otherwise, you want to determine who is tall among the children restricting the comparison class to your class. It might happen that some children that are classified as tall through such a comparison are not tall if considered in the natural kind of (eight-years-old) children (or some tall (eight-years-old children) might turn out to be short in the context of the class).

You can compare objects in a Sorites series taking them in larger or smaller contexts. Dummett made some interesting remarks about observational predicates. If you consider each time only a single pair of objects in a Sorites series generated by an observational predicate (like, for instance, ‘red’), one of which is next to the other in the series and they are not observably distinguishable, you accept each sentence of the type $F(x_i) \rightarrow F(x_{i+1})$. But still you do not accept all the sentences taken together, that is, considering all the objects at once. Such an idea to consider in a different way the pairs of objects of a Sorites series and the series itself as a whole has been formalised by Veltman and Muskens [120].

At first glance, this kind of contextual solution to the Sorites paradox seems to be peculiarly faithful with respect to the intuition that in natural language we give meaning to sentences considering them within a context and with respect to the observation that the relation of indistinguishability fails to be transitive. It is common practice to consider the meaning of

predicates with respect to the individuals in a specific context, that does not always coincide, though, with a comparison class. I spent more words in sketching the contextualist approach than the other ones because contexts play a central role in my proposal for vague adjectives.

2.6 Formal models for vague expressions

The approaches mentioned in this chapter provide a general treatment for vague expressions. The theories they propose do not make distinctions between adjectives, nouns and adverbs that suffer for vagueness: all of those expressions are equally treated in the theory. My point is that vague expressions cannot be treated at the same rate: we need to look also at the features of each linguistic expression taken into account.

For examples, the phenomenon of vagueness of adjectives like ‘tall’, ‘old’ and ‘cheap’ is related to the fact that such expressions are context-dependent and that it is possible to express, for instance, how much tall an individual is and that one is taller than another, that is, make a comparison with respect to the property represented by the adjective. Moreover, the meaning of an adjective like ‘tall’ is related to the meaning of an antonym, say ‘short’. Compare now vague count nouns. A count noun like ‘mountain’ does not present the same features as gradable adjectives: for instance, you cannot compare mountains with respect of the property of ‘being a mountain’. It is unfair to say that a certain landform is “more mountain” than another. Moreover, the meaning of ‘mountain’ is not related to any “antonym” count noun. So, a theory for vague expressions cannot avoid to take into account the different semantic features presented by vague expressions.

In the present research work I do not intend to offer an approach to vagueness that is alternative to the epistemic, semantic, ontological or contextualist ones. What I rather wish to do is provide a formal model that can represent the semantic features of two types of linguistic expression, included the failure of transitivity in the Sorites paradox: relative gradable adjectives and count nouns.

In Chapter 3 I present a model for vague gradable adjectives. In order to be able to approach the problem of vagueness of this kind of linguistic expression it seems to be prerequisite work to understand how natural language speakers use this kind of adjectives. First of all, the linguistic features of vague adjectives will be underlined. Secondly, the most well-known linguistic theories for vague adjectives are presented, and the problematic assumptions underlying these theories analyzed. An alternative view on gradable adjectives is then proposed, that attempts to explain how we use these adjectives.

tives in unproblematic cases, taking into account two other factors related to vagueness: the notion of *comparison class* and the notion of *granularity*. The last section of the chapter collects some further formal results and theoretical remarks.

Chapter 4 takes into account a different kind of vague expressions: count nouns, that philosophers relate to sortal concepts. I will analyze, first, the linguistic features of vague count nouns and, second, discuss some theoretical problems that their correspondent concepts present. I will then show a way to represent vague count nouns in the model for gradable adjectives.

Summing roughly up, given an expression like ‘high mountain’, in Chapter 3 you will find a formal treatment of the semantic behavior of the adjective ‘tall’ and in Chapter 4 a formal treatment of the behavior of the count noun ‘mountain’. The two linguistic expressions are treated in a different way since they present different linguistic features connected with the vagueness phenomenon.

Chapter 3

A Contextualist Account for Gradable Adjectives

In this chapter, I wish to present the features of a class of linguistic expressions, the so-called gradable polar adjectives, discuss the most prominent and well-known linguistic theories that account for them and, finally, present an alternative account for them.

3.1 Polar gradable adjectives

Consider adjectives such as ‘tall’, ‘long’ and ‘expensive’. Those are called *gradable* adjectives because they have the following features:

- they can occur in a predicative position, that is, after verbs such as ‘be’, ‘become’, ‘seem’, ...;
- they can be preceded by degree modifiers such as ‘very’, ‘clearly’, ...;
- they can be made into comparatives and superlatives.

Most of them express some properties that can be measured on a scale of size or value.

On the other hand, examples of non-gradable adjectives are ‘married’, ‘female’ and ‘bachelor’. They can occur in a predicative position too. However, if they are modified by degree adverbs, the effect you get is just an emphasis (consider, for instance the effect you get with a sentence like “It is very married”). Furthermore, non-gradable adjectives cannot be made into comparatives nor superlatives. It does not make any sense, for example, to utter something like:

(a) *Mary is more female than Lucy

nor

(b) *Mary is the most female in the country.

If someone utters (a) or (b), she is using ‘female’ for some specific pragmatic purpose, that is not the ordinary use of the adjective.

Two theories account for the differences between gradable and non-gradable adjectives (see Kennedy [50]):

- The domains of gradable adjectives are ordered according to some property that is usually measurable and allows grading (such properties usually correspond to dimensions). For example, the domain of ‘tall’ is weakly or partially ordered according to the property of *height*; the domain of ‘warm’ according to the property of *temperature*.
- According to the theory of adjectives developed by Klein [54], that recalls the theory by Kamp [44], non-gradable adjectives are represented by complete functions from individuals to truth values, while gradable adjectives are represented by partial functions from individuals to truth values. That is, in the latter case, the functions can give value 0, 1 or no value at all for some objects in their domains.

Gradable adjectives, moreover, can be distinguished between relative and absolute (see Kennedy [53]):

Absolute Adjectives They have positive forms that relate objects to maximal or minimal degrees, and are not affected by the Sorites paradox, nor do they have borderline cases. They differ, though, from the non-gradable ones, since they demonstrably have all the features of gradable adjectives: absolute gradable adjectives can acceptably form comparatives and superlatives, can be modified by some degree modifiers, and can occur in a predicative position. Consider some examples. ‘Wet’ requires its argument to have a *minimal* degree of the property it describes (adjectives such as ‘wet’ or ‘open’ are called *minimum standard* absolute adjectives). The polar counterparts of ‘wet’ and ‘open’ are respectively ‘dry’ and ‘closed’. As they require their argument to possess a *maximal* degree of the property in question, they are called *maximum standard* absolute adjectives. Consider the following examples to verify the acceptability of comparatives raised by absolute adjectives, and their sensitivity to degree-modifiers:

“The platinum is less impure than the gold”;
“The table is wetter than the floor”;
“The door is closed enough to keep out the light”.

Relative Adjectives Examples of relative gradable adjectives are: “tall”, “big”, “expensive”. The features of relative gradable adjectives in their positive form are:

- Context-sensitivity: the extension of the predicates generated by relative adjectives changes from context to context. This means also that a sentence containing a relative gradable adjective can get a different truth value depending on the context of utterance. For example, sentence (1)
(1) John is tall,
can be true in the comparison class of men, but false in the comparison class of basketball players. Context-sensitivity can be thought of also as the problem of the shifting standards from context to context (see Chapter 2).
- Borderline cases: there are cases where it is difficult to determine whether an adjective can be attributed to some object. And moreover, there is no clear sharp boundary between a positive and a negative polar relative adjectives, for instance between ‘tall’ and ‘short’¹.
- Sorites-sensitivity: every relative gradable adjective can give rise to a Sorites paradox.

It seems clear that relative gradable adjectives turn out to have all the characteristics of vague expressions. Among the whole class of adjectives, then, the ones that generate vague predicates are the relative gradable adjectives. I will thus focus on this class of adjectives.

Sentences containing relative gradable adjectives can get different truth values in different contexts. In order to determine the truth value of “ x is ϕ ”, with ϕ a relative gradable adjective, we have to determine the meaning of x , the features of the utterance context and make a “judgment of whether x counts as ϕ in that context” (Kennedy [52], p. 34.). A semantic analysis of relative gradable adjectives will then make such a judgment possible giving to the sentence a definite interpretation and at the same time ensuring difference of interpretations across contexts.

Most of (or probably all) relative gradable adjectives have polar counterparts. Adjectives of that kind can be classified as positive or negative.

¹The distinction between positive and negative adjectives will be introduced soon.

Such a classification is based on some empirical characteristics demonstrated by the adjectives themselves. Measure phrases can be associated with positive adjectives, but not with negative ones (you can say “John is 178 cm tall” but not “John is 178 cm short”). Negative ones allow downward entailments, while positive ones allow upward entailments. Consider, for instance, the pair ‘safe’/‘dangerous’, such that ‘safe’ is negative, ‘dangerous’ positive. From “It is dangerous to drive in Paris” you infer “It is dangerous to drive fast in Paris” (and also “It is dangerous to drive slow in Paris”) but not the reverse, and from “It is safe to drive fast in Des Moines” (or from “It is safe to drive slowly in Des Moines”) you infer “It is safe to drive in Des Moines”, but not the reverse. Examples of polar pairs are: ‘tall’/‘short’, ‘expensive’/‘cheap’, ‘big’/‘small’, ‘clever’/‘stupid’.

Just in order to capture some intuitions about how positive and negative polar adjectives are related, consider what the degree-based theory says (see following section): usually relative gradable adjectives are measurable, therefore they are related to a scale of degrees. Degrees of positive adjectives range from the lower to the upper end of a scale, while the degrees of negative adjectives range from the upper to the lower end of a scale.

3.2 Theories for gradable adjectives

Consider now the most well-known accounts for gradable adjectives: (i) degree-based theories, (ii) interval theory (as a kind of degree-based theory), (iii) trope theory, and (iv) an alternative that I wish to propose.

3.2.1 Degrees

One approach to gradable adjectives is to analyze them as relations or functions from objects to degrees on a scale. We have an abstract representation (*scale*) that is a set of elements under a total ordering. Each of those elements is a *degree*. So, when we have a sentence like “ x is ϕ ”, this is true iff the degree to which x is ϕ is at least as great as the degree on the same scale that represents the standard of ϕ -ness. Comparatives seem to get a simple treatment within a degree-approach. Comparatives define ordering relations between degrees on some scale. A sentence like

(2) John is taller than Mary

is analyzed as follows:

$$(3) \exists d([d > \iota d'.tall(Mary, d')] \wedge [tall(John, d)]).$$

Take an antonymous pair of adjectives, like ‘tall’ and ‘short’: they both define relations (or functions) on the same scale, that is, the scale of height. The ordering relations they give rise to are reversed. Let ϕ_{pos} and ϕ_{neg} be respectively the positive and negative polar adjectives associated with a scale S . Assume ϕ_{pos} denotes a relation between objects and $\langle S, <_{\delta} \rangle$ and ϕ_{neg} a relation between objects and $\langle S, >_{\delta} \rangle$. The set of positive degrees and the set of negative degrees on S stand in dual relation and, since there is a bijection between the two sets (namely, the identity function), they are isomorphic (in other terms, positive degrees are the same objects as negative degrees). So, for all degrees $d_1, d_2 \in S$, the following holds:

$$(4) d_1 >_{\phi_{pos}} d_2 \Leftrightarrow d_2 >_{\phi_{neg}} d_1$$

Therefore, (2) and (5) are equivalent:

$$(5) \text{ Mary is shorter than John.}$$

The first formalizations of such an approach go back to Seuren [102] and Cresswell [18]². The intuition behind the degree-based approach is well expressed by Cresswell:

when we make comparisons we have in mind points on a scale.
(Cresswell [18], p. 266.)

A degree-based account is not able, though, to explain the anomaly of the so-called cross-polar phenomenon. One instance of this phenomenon is (6):

$$(6) * \text{ John is taller than Mary is short.}$$

Consider (6). The degrees of tallness are the same objects as the degrees of shortness. (6) is true whenever the degree of John’s height exceeds the degree of Mary’s height on a scale of height. The undesirable result is that (6) turns out to be interpretable and, even worse, logically equivalent to (2) and (5), which are not anomalous but perfectly acceptable.

²For a general discussion, see Klein [55].

3.2.2 Intervals

Kennedy in [52], [49], [51] criticises the degree-based approach because of its incapacity to explain the cross-polar anomaly as in (6). He proposes another approach, based on degrees not taken as points, but as intervals, or extents, on a scale. His theory is based on the works by Seuren [103], von Stechow [121], Löbner [61].

Focusing on the problem of cross-polar anomaly, the initial assumption Kennedy makes is (Kennedy [52], p. 51):

Comparatives are semantically well-formed only if they define ordering relations between the same sort of degrees: between positive degrees, between negative degrees, or between degrees that measure divergence from a reference point.

What Kennedy considers necessary to assume in an approach to polar adjectives is to make a sortal distinction between positive and negative degrees. Put otherwise, the set of positive degrees must be different from the one of negative degrees. But what he wants to ensure at the same time is the equivalence between (2) and (5), respecting then one of our strongest intuitions about polar adjectives.

Pairs of antonyms convey the same kind of information about an object: for instance, both ‘tall’ and ‘short’ convey some information about an object’s height. What differs is the perspective from which they consider the projection of any object on some scale. A positive adjective has a ‘down-up’ perspective towards an object x , negative adjective an ‘up-down’ perspective towards the same x , so to speak.

Kennedy defines a scale S as a linearly ordered, infinite set of points. Each scale represents some type of measurement indicated: height, length, weight, etc. A degree d is not defined as a point on S , but as a convex, nonempty subset of S , in the same way as an interval in a linearly ordered set of points p_1, p_2, \dots, p_n is usually defined, and it is called *extent*³:

$$\forall p_1, p_2 \in d \forall p_3 \in S [p_1 < p_3 < p_2 \rightarrow p_3 \in d].$$

Given a scale S , define the set of positive degrees $POS(S)$ and the set of negative degrees $NEG(S)$ as follows:

$$POS(S) = \{d \subseteq S \mid \exists p_1 \in d \forall p_2 \in S [p_2 \leq p_1 \rightarrow p_2 \in d]\}$$

³The terminology and the basic idea go back to Seuren [103].

$$NEG(S) = \{d \subseteq S \mid \exists p_1 \in d \forall p_2 \in S [p_1 \leq p_2 \rightarrow p_2 \in d]\}$$

As a consequence of this definition, $POS(S)$ and $NEG(S)$ are disjoint.

Let $pos_S(x)$ be the positive projection of an object x on S and $neg_S(x)$ the negative projection of x on S . They are ordered sets. Define now MAX and MIN as functions from ordered sets to their maximal and minimal element, respectively. The relation between $pos_S(x)$ and $neg_S(x)$ is the following:

$$\text{MAX}(pos_S(x)) = \text{MIN}(neg_S(x)).$$

That means, given a pair of antonyms, the maximal point of the interval identified by the positive adjective coincides with the minimal point of the interval identified by the negative adjective. So, the positive and negative projections of x on a scale S are complementary intervals on S .

Gradable adjectives are thought of as functions from objects to intervals. More precisely, positive adjectives denote functions from objects to positive intervals and negative adjectives functions from objects to negative intervals. Two antonyms, then, have the same domain but different ranges; they map the same objects onto complementary regions of the same scale.

The interval-based theory developed by Kennedy gets the same positive results as the degree-based ones. For instance, (2) and (5) turn out to be logically equivalent. Nevertheless, Kennedy's theory is also able to overcome the difficulties the degree-based approach presents. First of all, cross-polar anomalies are not acceptable because *tallness* and *shortness* are not comparable, since they are different sorts of objects. This explanation is accepted in the interval-based approach thanks to the idea that the sets of positive and negative extents are disjoint. Informally speaking, (6) is true iff there is an extent that properly includes the extent of Mary's shortness, and John is tall to that extent. Kennedy's account imposes a restriction: the extent argument of a positive adjective must be a positive extent, and positive extents can include only positive extents. Similarly for negative adjectives: their extent arguments must be negative, and negative extents can include only negative extents. So, in order for (6) to be true, the argument of the positive adjective 'tall' has to be a negative extent. Such a restriction shows that there is a sortal mismatch, and (6) turns out to be anomalous.

There are four main objections that can be raised against the degree-based approach as well as the interval approach: the first concerns the ontological commitments of the approach, the second the multidimensional aspects of some adjectives, the third involves some cognitive aspects, and the fourth is about the interpretation of the positive form of adjectives.

1. What kind of objects are degrees? Does their use necessarily lead one to some ontological commitments? What is in doubt here is why we should assume a class of abstract objects. If we add a scale of degrees to our ontology, we have to justify it and say why we need to assume the existence of abstract objects. The ontology we get by adding scales of degrees is quite large, especially if we take a real-valued scale as the scale of degrees. The question is: is it necessary to have such a large ontology and admit infinite abstract objects to account for vague adjectives?

The same question can also be raised for Kennedy's theory: what kind of objects are intervals? Kennedy is aware of this ontological question and claims to address a similar question in Kennedy, [51]. In that paper Kennedy's train of thoughts seems to be the following: if you want to give the right interpretation to gradable adjectives, you need to replace degrees with intervals. But Kennedy does not explain why you need a class of abstract objects in order to account for vague adjectives. He seems to recognize this problem, though, and in footnote 3 he refers to another article by himself (Kennedy [50]), where he tries to show why approaches that do not make use of measure theories fail. Kennedy [50] takes into account the theories that account for gradable adjectives by analyzing them in terms of partial functions. According to Kennedy, those theories do a good job of explaining most of the semantic properties of gradable adjectives, but are not able to explain the behavior of antonymous adjectives in comparatives (neither the anomalies, nor the normal uses). For this reason, that is, showing that the alternative theories that are on the market at the moment fail to grasp some phenomena, he argues for the necessity of an interval-based approach. However, this kind of argument does not address the ontological problem directly. To the question of why we need abstract objects he replies: we need abstract entities because all the other alternatives given by now fail to grasp some phenomena that an interval-based theory does. But this is an *ad hoc* argument. That intervals work well is not a *sufficient* reason to make people believe in abstract things.

2. A problem arises also in the treatment of comparatives: adjectives like 'white', 'beautiful', 'wise' and 'happy', are different from 'tall', 'long' and 'expensive'. The second group of adjectives might be characterized as one-dimensional: the extension of each of these adjectives is determined by only one measurable aspect, and the comparative that is made from a one-dimensional adjective is not underspecified. As far as multi-dimensional adjectives are concerned, that is, the adjectives

of the first group, their extension depends upon more than one dimension and therefore might have more than one antonym. Moreover, the comparatives they built seem to suffer of underspecification. Take for instance ‘clever’: x might be clever with respect to some ability, and we cannot usually numerically measure the amount of intelligence a person has, taking into account all the dimensions. Consider the comparative: Mary is cleverer than John.

With respect to which ability is Mary cleverer than John? We might think of an ordering for each property that determines someone’s cleverness. Klein [54] suggests that whoever utters a sentence like “Mary is cleverer than John” has already fixed a dimension, without mentioning it. But it is not clear in sentences like that which property is involved.

3. When children learn to use relative gradable adjectives, they are taught that an individual is tall and another is short while comparing those individuals between them or within a class of individuals that differ from them. A child does not measure the difference between the individuals she sees, nor has she any clue about what a centimeter is, but despite all this she can learn and properly use ‘tall’ and ‘short’. So, it seems that we are able to use relative gradable adjectives without the notion of measurement. On such a notion the degree-based theory is based. But if we do not need to refer to measures to use vague adjectives, why should we use measures to model our use? Can we do the same without measures and degrees?
4. Consider the behavior of positive and comparative forms of adjectives according to the degree and interval approaches: take, for instance, ‘tall’. First, the meaning of the expression ‘tall to degree d ’ is determined. Then, the comparative ‘taller than’ is defined over ‘tall to degree d ’. Finally, the meaning of the positive form ‘tall’ is defined over the meaning of the comparative. The dependence of positive form from the comparative form is controversial. Most linguists tend to prefer a different treatment of the positive form of adjectives, namely to take that as a primitive (function or relation) and define the comparative form on it.

3.2.3 Tropes

An alternative approach to the degree- and interval-based approach for comparatives is presented by Moltmann [78] and especially [77]. The central notion of her proposal is what philosophers have ended up to call *tropes*.

Tropes are thought of as concrete objects which adjective nominalisations refer to, and which are actually compared. For instance, the nominalisation of ‘tall’ is ‘tallness’ and ‘tallness’ is a trope. Hence, (1) is understood as follows:

(7) John’s tallness exceeds Mary’s tallness.

Let f be a function that maps a property and an object to the trope corresponding to that property, relative to a world w and a time i . (1) will be analyzed as follows:

(8) $f(\text{John}, [\text{tall}], w, i) > f(\text{Mary}, [\text{tall}], w, i)$.

According to Moltmann, the advantages of a trope-based account are mainly the following:

- No abstract and hardly characterisable entities are invoked. Tropes are referents of adjective nominalisations and the speakers themselves refer to them.
- Two tropes of the same type can be ordered (John’s tallness $>$ Mary’s tallness, for instance) by an exceed-relation. The ordering depends on the entities involved themselves.
- Incommensurability is naturally accounted. For example, that height cannot be compared with happiness follows directly. Nevertheless, the theory allows comparison between height and width, because they can be viewed as tropes of the same sort.

A problem for this account is the treatment of polar adjectives. If (2) and (5) are equivalent, then John’s tallness and John’s shortness has the same trope as referent. But then we have the following wrong inference:

(A1) John’s tallness exceeds Mary’s tallness
 (A2) Mary’s shortness exceeds John’s shortness
 (A3) Mary’s shortness = Mary’s tallness
 (A4) John’s shortness = John’s tallness
 \Rightarrow (C1) John’s shortness exceeds Mary’s tallness
 \Rightarrow (C2) John’s shortness exceeds Mary’s shortness.

And also:

\Rightarrow (C3) John’s shortness exceeds John’s tallness.

Conclusion (C2) contradicts assumption (A2), while (C3) contradicts (A4).

In order to solve this problem Moltmann proposes to consider the ordering among tropes as imposed by the concepts of the adjectives in question. She

supports a trope-based account also to answer the question on which way the property expressed by the adjective imposes the ordering. Properties are construed in such a way that they resemble tropes,

more precisely in terms of functions mapping indices to sets of possible tropes that resemble each other. A gradable property, moreover, will be construed as a function mapping indices to ordered sets of tropes [...] If we call tropes as conceived on the standard view *standard tropes*, then the referents of adjective nominalizations should be entities that are standard tropes ‘insofar as’ they instantiate that property, and this means tropes whose properties should all be based on them playing the role of instances of the property in question. Thus, besides standard another kind of trope is needed for the semantics of nominalizations. The latter should fulfil two conditions that distinguish them from standard tropes: First, only one exceed-relation should be applicable to them. Second, John’s weakness should be distinct from the entity that is John’s strength. (Moltmann [77], pp. 19-20 (online version)).

The trope-based approach does not seem very convincing either, especially for the not elegant (and unclear) way to accommodate polarity of gradable adjectives in the theory. Moreover, even if Moltmann claims that there is no ontological assumption of hardly characterizable entities, she refers to properties as entities. Again, we have to assume something else than just adjectives. Even if it might be simpler to understand what tropes are, why again should we assume more entities than what we want to explain? That means, we have objects that have properties, but why should we consider these properties as objects too?

3.2.4 Any alternative?

In the degree and interval approaches considered so far all the properties are assumed to be measurable. However, is this assumption necessary? Furthermore, the trope theory does not solve the problems of abstractions that it wants to address and its assumptions are highly objectionable, as I have briefly sketched.

Now, is it possible to have an alternative theory that explains how we use relative gradable adjectives without assuming degrees, nor intervals, nor tropes? In the following chapter I try to develop an alternative theory that accounts for relative gradable adjectives, both in their positive and comparative forms. The goal is to define the meaning of the comparative over the

meaning of the positive form of the adjective. That means, first we determine the meaning of ‘tall’, then we define the meaning of ‘taller than’ by using comparison classes and constraints on the behavior of the adjective functions in comparison classes. The meaning of ‘tall to degree d ’, useful for measure phrases, could be then defined over the meaning of the comparative form. To do that, measurement theory can be used. But I do not go into that problem. What I am mostly concerned with, in this research, is how to get the meaning of the comparative form from the positive form of adjectives. Such an attempt goes back to Kamp [44] and Klein [54], and takes van Rooij’s suggestions as reference point (see van Rooij [118], [117]).

3.3 A model for polar adjectives

3.3.1 Aim

The aim of this section is to account for polar relative gradable adjectives such as ‘tall’/‘short’, ‘big’/‘small’, and so on.

Each gradable adjective comes with a polar counterpart. Some adjectives can form a polar pair with more than one adjective. For example, ‘short’ can form a pair with ‘tall’ and another with ‘long’. In the formalization given below, the meaning of ‘short’ in the pair ‘short’/‘tall’ will be considered different than the meaning of ‘short’ in ‘short’/‘long’. Moreover, there are one- and multi-dimensional adjectives. While ‘tall’ is uniquely used to refer to the distance from the top to the bottom of an object, ‘clever’ can be used to refer to some feature of cleverness that an individual endorses. Here a model for the linguistic use of one-dimensional adjectives, and not for multi-dimensional ones, is proposed.

When English native speakers have to judge on non-borderline cases, the use of adjectives such as ‘tall’/‘short’ is not problematic. For example, consider the set of men and a subset of it containing three individuals, John, Bill and Marc. John is 190 cm tall, Bill is 188 cm, Marc 160 cm. Speakers that see the three men do not know their precise height, but can observe that John and Bill do not relevantly differ in height, as well as they can detect a big difference between John-Bill and Marc. So, if the agents have to describe the height of John, Bill and Marc, they will naturally say that John and Bill are tall, Marc is short. The natural intuition seems to be this: when there is not a big difference between two objects with respect to some property represented by an adjective, we can appropriately attribute the same adjective to both the objects, but if there is a relevant difference between them, then we describe them using a pair of polar adjectives. In the

first case, though, if we have a soritical series we can get into trouble: indistinguishability fails to be transitive. However, in the second case, that is, when there is a relevant difference between the individuals under judgment, we do not have difficulty to use gradable adjectives. The intuition is that observations made for the unproblematic case cast light on the problematic ones. This section presents a model that describes the computational operations underlying speakers' decision about applying gradable adjectives to both problematic and unproblematic cases.

After some initial theoretical observations, will I define a language \mathcal{L} ; then a model is built up and applied to relative gradable adjectives. Adjectives are taken as primitive choice functions and the comparative relation is defined over the positive form of adjectives. Two ways of ordering elements in the domain are considered: weak orders and semi-orders.

3.3.2 Theoretical background

In this account, vagueness is considered to be a feature of some expressions of natural language and is related to context dependence and granularity.

As we have seen, context dependence is a feature of relative gradable adjectives. For instance, the predicate 'tall' applied to the context of human beings has a different extension than the same predicate applied to the domain of equatorial trees. However, also within the domain of human beings there might be sub-contexts that influence the interpretation of the predicate. For example, in the context where we consider only Dutch women the predicate has a different extension than in the context where we consider Japanese women. Moreover, suppose an individual named Bill is four years old and 130 cm tall. We can say that he is tall as a child, but short as a human being. In fact, if we compare Bill with the individuals of the set of all human beings, his height turns out to be below the height-average. However, considered as a child, he is quite tall. So, the extensions of vague predicates depend on the valuations that are made each time in a specific comparison class (or natural kind, following Fara [27]). But within a comparison class, like the class of children, we can be interested in a more restricted context. For example, the set of children of the first grade in some primary school in Padua.

Nevertheless, context dependence is not sufficient for explaining the vagueness of predicates. Given a context, that is, a set of individuals, and a vague predicate P , there are several ways to consider the differences between the individuals in the context. We can look at them assuming different standards of precision. In fact, the grain size varies from context to context and the grain size we choose often depends on our interest or our actual purpose

(see Hobbs [42] and Mani [73]). As the grain size changes, we may cover different things under the same label or split meanings in a more refined way. This phenomenon is called *granularity*. Different levels of granularity can be thought of as different standards of precision. Let us try to understand what that means.

Take the pair of polar adjectives ‘tall’ and ‘short’ and the example about John, Bill and Marc stated in section 3.2.1. Formally, we have a set $o = \{j, b, m\}$, with j, b, m standing for John, Bill, Marc, respectively. If we look at o from a very coarse point of view, no difference among its elements is detected: Bill, John and Marc are equally tall. A coarse point of view can be given, for example, by a distant point of observation or by some specific purpose (for example, if we have to enlist men shorter than 160 cm, we do not discriminate between the men we have to rule out: they are all equally tall as they all exceed the cut-off point). From a less coarse point of view, that is, using a finer grain size to discriminate differences, we might say that John and Bill are equally tall, while Marc is short. We can then establish a comparative relation: John (as well as Bill) is taller than Marc. However, with an even finer grain size we can perfectly distinguish the height of all the three men and say that John is taller than Bill and Marc, and Bill taller than Marc. Now, the same ordering between the elements is provided by two models that differ in the extension of ‘tall’ and ‘short’. According to model 1, both John and Bill are tall and Marc short, according to model 2, only John is tall and both Bill and Marc short. But our intuition is that only model 1 is correct. We want then to find some constraints to rule out models like 2 that do not respect our intuitions.

The phenomenon of vagueness seems then to be captured (also) by the idea of granularity: some words are vague because the degree of specification of their meaning varies.

3.3.3 Language and interpretation

To give a model to account for gradable adjectives, let us fix first a language and then an interpretation for it.

Language

Let \mathcal{L} be a formal language through which we can represent English expressions. \mathcal{L} consists of:

- countably many individual constant symbols (that represent proper names: ‘John’, ‘Mary’, ‘Sue’,...): $\bar{j}, \bar{m}, \bar{s}, \dots$

- countably many individual variable symbols: x, y, z, \dots
- countably many monadic predicates (representing common nouns like ‘pig’, ‘man’, ‘winner’): A, B, C, \dots
- countably many functions (representing adjectives): P_1, P_2, \dots, P_m , standing for ‘tall’, ‘big’, ‘fat’, ... We will later define the polar counterparts of such functions: $\overline{P}_1, \overline{P}_2, \dots, \overline{P}_m$, standing for ‘short’, ‘small’, ‘thin’, ...
- usual logical connectives with identity, quantifiers.

The set of terms and the set of formulas are defined in the standard way. The set of terms consists of:

1. individual constant symbols $\overline{j}, \overline{m}, \overline{s}, \dots$,
2. individual variable symbols x, t, z, \dots
3. if t_1, \dots, t_{a_i} are terms, then $P_i(t_1, \dots, t_{a_i})$ is a term (for $1 \leq i \leq m$).

Formulas are defined as follows:

1. If t_1, t_2 are terms, then $A(t_1), B(t_1), \dots$ are formulas;
2. If t_1, t_2 are terms, then $t_1 = t_2$ is a formula;
3. If ϕ, ψ are formulas, then $\phi \square \psi$ is a formula, where \square is one of the usual logical connectives;
4. If ϕ is a formula, then $\neg \phi$ is a formula;
5. If ϕ is a formula, then $\forall x_i \phi, \exists x_i \phi$ are formulas.

Interpretation of \mathcal{L}

First of all, assume a fixed and finite domain \mathcal{D} of objects. The strategy now is to define first comparison classes, and then make adjectival functions ranging over them.

Monadic predicates select some objects of \mathcal{D} . Their extensions are called *comparison classes*. I am assuming here that it is always possible to give a precise extension for each monadic predicate. For example, I do not consider for now the problem concerning the extension of ‘child’, that is known to be a vague predicate too. I do not face this kind of problems now because I am concerned only with polar vague adjectives. So far and for the sake of

simplicity, I assume comparison classes to be sets with precise boundaries. In Chapter 4 I will discuss and revise this assumption.

Let I be an interpretation function. $I(A)$ is a subset of the domain \mathcal{D} and is a comparison class, for A a monadic predicate. For the sake of simplicity, call $I(A)$: s . Let CC be the set of all comparison classes s .

So, first we have comparison classes, then we can apply the functions to the domain of each of those comparison classes. In this way, we can predict that an individual x is P in some comparison class s , \bar{P} in another comparison class s' : for example, when Bill is said to be tall as a child, but short as a man, it means that in the comparison class of children, the individual standing for Bill is within the extension of ‘tall’, while in the set of men, it is not. The distinction in the formal language between monadic predicates representing count nouns and predicates representing adjectives reflects the linguistic distinction between sortal predication and characterizing predication (see Strawson [109] and next chapter). The metaphysical counterparts of these two types of predications are sortal universals and characterizing universals. Individuals are always considered as falling under a sortal universal, while characterizing universals (adjectives) presuppose that the individuals are already distinguished by sortal universals⁴.

Nevertheless, as we saw, we need more than comparison classes in order to evaluate the assignment of an adjective to an individual. We need contexts. A context o is defined as a subset of a comparison class or, put otherwise, the set of all contexts in a comparison class s is defined as the powerset of s :

Definition 2 *Let O_s be the set of all contexts in some comparison class s : $O_s = \wp(s)$.*

3.3.4 Context structures and weak orderings

In this section I will first define contexts structures and then, given some cross-contextual constraints, the comparative relation. The ordering relation between the elements of each comparison class obtained will turn out to be a weak ordering. More constraints will be given in order to rule out the context structures that do not make a right attribution of gradable adjectives to elements of contexts.

As Luce [72] himself highlights, the non-transitivity of indifference relations reflects human inability to discriminate with precision among things that do not differ much from one another. Even if Luce’s considerations refer to indifference relations generated by preference orders, they perfectly fit

⁴This distinction will be analyzed more extensively in Chapter 4.

our problem with vague predicates too. We cannot make precise distinctions between two objects with respect to some observable property. That is why we get into trouble with Sorites series. Consider how Luce [72] expresses a version of the Sorites paradox for preference orders:

A person may be indifferent between 100 and 101 grains of sugar in his coffee, indifferent between 101 and 102, ..., and indifferent between 4999 and 5000. If indifference were transitive he would be indifferent between 100 and 5000 grains, and this is probably false.

Even if we are not able to discriminate between close quantities, if we have some more precise standard of precision or a better way of measurement, can we detect more differences between the elements we consider. That is nothing else than the concept of granularity, as we saw above.

According to different standards of precision, we can have different models that represent a different behavior of the adjectival functions. As already mentioned, adopting a coarse standard of precision we could regard all the individuals of a context as having all the same property K , while adopting a finer standard of precision we can distinguish some difference among the individuals with respect to the same property. Let us see how this idea can be formally represented.

Let $M = \langle \mathcal{D}, ICC, P \rangle$ be a fixed model, or context structure. \mathcal{D} is the whole domain, ICC the set of comparison classes, P a choice function that maps the individuals of context o to $P(o)$. The value of function P is then a set, $P(o)$. $P(o)$ is a subset of o , more precisely, the subset that contains the elements that are P . For instance, if T stands for ‘tall’, given a context of individuals o , $T(o)$ individuates the set of all tall individuals in o .

Then, let us define the polar counterpart \bar{P} of P as a function that applies to the elements in a context to which P does not apply:

Definition 3 $\bar{P}(o) = \{x \in o : x \notin P(o)\}$.

P and \bar{P} are then considered as contradictories⁵.

I want to make possible that the meaning of an adjective changes over contexts: some object that has a property in a context might not have that property in an enlarged domain.

How to account for this cross-contextual change of meaning?

⁵A further development of the model would be to treat P and \bar{P} as contraries, and not contradictories, allowing, in this way, for partial functions.

First of all, P can be considered as a choice function that takes elements from some finite set of options o : $P(o)$ is then a subset of o . Some constraints can be put on such a function in order to make it behave in a different way in each set (context).

3.3.5 Van Benthem's constraints

Consider the cross-contextual constraints that van Benthem introduced in [115]. They are based on the concept of difference pair (DP):

Definition 4 *Two elements x and y form a difference pair DP in a context o iff x is in the extension of P and y in the extension of \bar{P} , that is:*

$$\langle x, y \rangle \in DP(o) \text{ iff}_{def} x \in P(o) \text{ and } y \in \bar{P}(o).$$

The first constraint is the so-called **Upward Difference** (UD):

(UD) Let $\langle e, e' \rangle$ be a difference pair in a context o . In each context o' such that $o \subseteq o'$, there exist different pairs.

Put otherwise, if in a context o one element is tall, another short, (UD) makes sure that all the supersets of o will contain at least one element that is tall and one that is short. Those elements are not necessarily e and e' . Consider the following example: o contains an individual e that is 200 cm tall and an individual e' that is 180 cm tall, and other individuals whose height is somewhat between 180 and 200 cm. We can state that e and e' form a difference pair in o : e is the tallest and e' the shortest, so, comparing one with the other, e is tall and e' short, i.e. $\langle e, e' \rangle \in DP(o)$. But now, take a context o' that have all the same individuals as o , plus an individual e'' that is 150 cm tall. In this new context o' we would probably make a different consideration about the application of 'tall' to e and e' . Comparing the objects in the context, we would say no longer that e is tall and e' short, but rather that e and e' are tall, and e'' short. So, we detect some other difference pairs, for instance, $\langle e, e'' \rangle \in DP(o')$.

Van Benthem proposes two other constraints. Consider the following, called **No Reversal**:

(NR) Let $\langle e, e' \rangle$ be a difference pair in a context o . There is no context o' such that $\langle e', e \rangle \in DP(o')$.

(NR) says that, if in a context o one element e is tall and another e' short, in any other context o' the reverse cannot be the case. For instance, if in o e is tall and e' short, in a different context o' e and e' can be both tall, or short, but it can never be the case that e' is tall and e short.

The third constraint is **Downward Difference**:

(DD) Let $\langle e, e' \rangle$ be a difference pair in a context o . In each context o' such that $o' \subseteq o$ and that it includes e and e' , there exist some difference pairs

If e is tall and e' short in a large context o , in a smaller context o' containing e and e' there will be difference pairs too.

3.3.6 Comparative relation

Given the three constraints (NR), (UD), (DD), we can define first the comparative relation $>_P$ (to read: “more P than”)⁶:

Definition 5 $x >_P y$ iff $x \in P(\{x, y\}) \wedge y \notin P(\{x, y\})$.

The relation $>$ is defined with respect to a predicate P and gives rise to a *weak order*. A weak order is a structure $\langle I, R \rangle$ with R a binary relation on I that is irreflexive, transitive and almost-connected:

(IR) $\forall x : \neg R(x, x)$

(TR) $\forall x, y, z : (R(x, y) \wedge R(y, z)) \rightarrow R(x, z)$

(AC) $\forall x, y, z : R(x, y) \rightarrow (R(x, z) \vee R(z, y))$

Define now the relations ‘being as P as’ (i.e. the similarity relation \sim_P) and ‘being at least as P as’ (\geq_P) as follows, respectively:

Definition 6 $x \sim_P y$ iff_{def} it is not the case that $x >_P y$ nor $y >_P x$.

Definition 7 $x \geq_P y$ iff_{def} $x >_P y$ or $x \sim_P y$.

3.3.7 Granular levels

From what has been discussed in the previous section, we can make another consideration. The conditions for comparatives do not uniquely determine the behavior of property P across comparative classes. Let M be a context structure of type $\langle \mathcal{D}, I_{CC}, P \rangle$. Different context structures can give rise to different $>_P$ orderings for the same set of contexts. That means that some

⁶I will consider here only direct comparatives, not indirect ones, as in the example: “Compared to Mary, John is tall”. Different considerations are needed to account for the semantics of this kind of comparison. To give an example of a treatment of direct vs. indirect comparison, see Kennedy [48].

context structures detect more differences between the elements in the contexts than other context structures; this fact corresponds to the intuition of granularity. However, different context structures can have the same level of granularity.

Take a context $o \in O_s$. Since any context structure provides us with an equivalence relation \sim_P , equivalence classes partitioning the context o are obtained. Equivalence classes are groups of objects, that according to that specific context structure turn out to be indistinguishable:

Definition 8 *Let $e \in o$. Define the equivalence class of e under \sim_P as follows:*

$$[e]_{\sim_P} =_{def} \{x \in o : x \sim_P e\}.$$

Now we can define also a comparative relation $>^*_P$ between equivalence classes that allow us to say that a group of elements is taller than another, as follows:

Definition 9 $[x]_{\sim_P} >^*_P [y]_{\sim_P}$ *iff*_{def} $\exists x \in [x]_{\sim_P} \forall y \in [y]_{\sim_P} (x >_P y)$.

Given two equivalence classes $[x]_{\sim_P}$ and $[y]_{\sim_P}$, $[x]_{\sim_P} >^*_P [y]_{\sim_P}$ holds if and only if there is an element of $[x]_{\sim_P}$ such that is more P than the elements of $[y]_{\sim_P}$. For instance, let T stand for ‘tall’ and consider two equivalence class of individuals, $[e]_{\sim_T}$ and $[e']_{\sim_T}$. If you show that one element of $[e]_{\sim_T}$ is taller than all the elements of $[e']_{\sim_T}$, then you can conclude that the elements of $[e]_{\sim_T}$ are taller than the elements of $[e']_{\sim_T}$, in short: $[x]_{\sim_P} >^*_P [y]_{\sim_P}$.

Different context structures can give rise to different partitions, and therefore to different ordering between objects. Consider the following example.

Example 1. Given some comparison class s , let $o \in O_s$ be a context with three elements: $o = \{a, b, c\}$. Consider a function T , representing the adjective *tall*. We can have the following orderings for o :

- $a \sim_T b \sim_T c$. The context structures modeling such an ordering only give rise to one equivalence class; all the elements are considered equal with respect to T , so we cannot have a distinction between objects that are T and objects that are \bar{T} . The context structures giving such an ordering are the coarsest ones.
- $a >_T b >_T c$. The context structures that model such an ordering give rise to three partitions: $[a]_{\sim_T}$, $[b]_{\sim_T}$, $[c]_{\sim_T}$. Each object is different from the others with respect to T . The context structures giving such an ordering are the most fine-grained.

- Between the coarsest and the finest orderings, there can be a third ordering: either $a \sim_T b >_T c$ or $a >_T b \sim_T c$. If a context structure gives rise to the first ordering, it gives two partitions: $[a]_{\sim_T}$ and $[c]_{\sim_T}$. If it gives rise to the second ordering, then it again gives two partitions, but different ones: $[a]_{\sim_T}$ and $[b]_{\sim_T}$.

We can partially order the context structures from the coarsest to the finest with respect to any context $o \in O_s$. For the sake of simplicity, consider a function P and let $[x]$ be an abbreviation for $[x]_{\sim_P}$. First, we need to define the relation \geq^c between cardinality of sets:

Definition 10 *Given some context o , for all the equivalence classes $[x]$ in M and all the equivalence classes $[y]$ in M' , $|[y]^{M'}| \geq^c |[x]^M|$ iff the number of equivalence classes in M' $[y]^{M'}$ is greater than or equal to the number of equivalence classes in M $[x]^M$.*

Now we can define the following relation (*finer than*) between context structures:

Definition 11 *M' is finer than M iff the number of equivalence classes of M' is larger than the number of equivalence classes of M , that is:*

$$M' \leq^{**} M \text{ iff } |[y]^{M'}| \geq^c |[x]^M|.$$

However, we are more interested in a refinement relation defined as follows:

Definition 12 *M' is a refinement of M iff each equivalence classes in M' is a (not necessarily proper) subset of an equivalence class in M , that is:*

$$M' \sqsubseteq M, \text{ iff } \forall [y]^{M'} \exists [x]^M : [y]^{M'} \subseteq [x]^M.$$

Just for completeness, I want to mention that the two refinement relations are interrelated. From definitions 11 and 12 follows:

$$\forall M (M' \sqsubseteq M \rightarrow M' \leq^{**} M).$$

A natural constraint on the refinement ordering between context structures is that the ordering between individuals given in the most fine-grained context structure cannot be reversed in the coarse-grained ones. Formally:

(RfM) $\forall x, y, z \in s$: if $M' \models x \geq_P y \wedge y \geq_P z$ and $M \models x \sim_P z$, then $M \models x \sim_P y \wedge y \sim_P z$.

Consider again Example 1. We can claim that a context structure that models $a \sim_T b >_T c$ and one that models $a >_T b \sim_T c$ are both finer than a context structure that models $a \sim_T b \sim_T c$, but coarser than one that models $a >_T b >_T c$. However, if we want to consider a refinement ordering between all those context structures, we start with the coarsest one, that is, one that models $a \sim_T b \sim_T c$. Then, we take either a context structure that models $a \sim_T b >_T c$ or one that models $a >_T b \sim_T c$ (they are alternative and incompatible; none of them is a refinement of the other). Finally, the finest context structure for o is one that models $a >_T b >_T c$ and it is the refinement of either the context structure that models $a \sim_T b >_T c$ or the one that models $a >_T b \sim_T c$. Put otherwise, we can also say that once we have a coarse context structure, we can refine it and get several context structures belonging to different granular levels.

3.3.8 A problem and a refinement of the model

An important observation has to be made at this point. For contexts with more than two equivalence classes, even if two context structures give rise to the same $>_P$ ordering, they might have different functions of type P and \bar{P} . In general, for contexts with more than two equivalence classes there are at least two context structures that give rise to the same ordering.

Consider some examples of context structures for the context $o_1 = \{a, b, e, f\}$ of the comparison class of men. Let T stand for ‘tall’ and suppose that the situation is the following: a represents an individual who is 200 cm tall, b an individual who is 190 cm tall, e an individual who is 170 cm tall and f an individual who is 160 cm tall. Consider context structures that order o_1 as follows: $a >_P b >_P e >_P f$ and that agree on the behavior of P on the pairs $\{a, b\}$, $\{b, e\}$, $\{e, f\}$. That means the following: taking the context $\{a, b\}$, which is a subset of o_1 , all the context structures agree about the following: a is tall and b is short. The same holds for $\{b, e\}$ and $\{e, f\}$. The context structures we are talking about are the following:

$$M_1 \models T(o_1) = \{a\}, \bar{T}(o_1) = \{b, e, f\}$$

$$M_2 \models T(o_1) = \{a, b\}, \bar{T}(o_1) = \{e, f\}$$

$$M_3 \models T(o_1) = \{a, b, e\}, \bar{T}(o_1) = \{f\}$$

It is not desirable to have all those context structures. Some of them should be ruled out, exactly those ones that predict wrongly which elements are P and which are \bar{P} . For instance, according to M_1 a is tall and b, e, f

short; but can we accept in the context o_1 that b , who is 190 cm tall, is short? If we take only a and b , we could say that, but if we consider a and b together with e and f , who are much shorter, we would not accept what M_1 says. In the situation outlined, then, we would not find difficulties in determining whom is tall and whom is short. The strategy now is to start analyzing an unproblematic case like that, trying to understand why we do not find difficulties in using vague adjectives in that case. What follows is an attempt to show what process underlies our use of vague adjectives in unproblematic cases, and to formally represent it.

To rule out the context structures that do not fit our intuitions, the suggestion is the following: given a context $o \in O_s$, consider also the context o^s , defined as follows:

Definition 13 $o^s =_{def} \{z \in s \mid \exists x, y \in o : x \geq_P z \geq_P y\}$.

I try to clarify the matter. Any context o contains some elements of the comparison class s . Given a set of context structures that give rise to the same ordering for all contexts in O_s , when we consider some context $o \in O_s$ we consider also the elements in the domain of s that are ‘in-between’ the elements of o , given the order raised by the set of context structures considered. In the example given above about o_1 , we can say that there are some elements in the comparison class of men whose height is in-between the heights of the elements a, b, e, f .

So, defining o_s we assume that there are enough elements ‘in-between’ the elements of o . But how can we guarantee this? I try to explain the problem in the following lines.

The intuition is the following. When we consider contexts, we look at real objects. Namely, when we use gradable adjectives we want to judge on some situation in the world. However, to correctly use gradable adjectives to describe objects, we can think to add possible objects that are ‘in-between’ the real objects according to the comparative relation. That means, if John and Bill are men that relevantly differ along their height, we say that John is tall and Bill short because there *might* be other men, whose height is different from their heights and is less than John’s but more than Bill’s. This intuition goes together with the fact that all relative gradable adjectives suffer for the Sorites Paradox: the crucial point of Sorites paradox is that we have a series of objects, such that there are small differences between every two objects that are contiguous in the series. We want to model vague relative adjectives that give rise to Sorites series: so, we need to assume that we *can* have a domain of individuals that are “equally distributed” with respect to a property, and such that each element of any comparison class s is indistinguishable from at

least two other from an observational point of view. Put otherwise, what we want is each set to have possible objects that form a Sorites series, and each real object to correspond to one of the possible objects of the domain. So, we need one restriction concerning the domain of individuals of each comparison class.

Now, a condition must be imposed to each comparison class. Take a comparison class s : each element of s is observably indistinguishable from at least two other elements. It might happen only for two elements that each of them is indistinguishable from another element⁷. So, every comparison class s comes with an indistinguishable relation \approx_{s_P} with respect to a property P , such that:

$$\text{(SC)} \quad \exists u \exists v (u \neq v \wedge \forall x ((x \neq u \wedge x \neq v) \rightarrow \exists y \exists z (x \approx_{s_P} y \wedge x \approx_{s_P} z \wedge \neg (y \approx_{s_P} z))) \wedge \exists z (z \neq u \wedge u \approx_{s_P} z) \wedge \exists z (z \neq v \wedge v \approx_{s_P} z)).$$

The point is the following: by the primitive relation \approx_{s_P} and the constraint (SC) we require comparison classes to contain elements that are indistinguishable from at least two other elements (with two exceptions). We do not impose any order to the elements (real or possible) of the comparison classes. The ordering between elements is given by comparative relations of kind $>_P$ in each context structure, as it has been already shown.

At this point, we have to redefine context structures. Let $M = \langle \mathcal{D}, I_{CC}, P, \approx_{s_P} \rangle$ be a fixed model, or context structure. \mathcal{D} is the whole domain, I_{CC} the set of comparison classes, P a function that maps the individuals of contexts $o \in O_s$ into $P(o)$, \approx_{s_P} the indistinguishability relation with respect to a $s \in I_{CC}$ and P .

There is an important observation to make at this point. It concerns the difference between \sim_P and \approx_P . They are not the same relation of indistinguishability. While \approx_P is given primitively to get Sorites series for each comparison class s , \sim_P is defined on the behavior of P and \bar{P} . Moreover, \sim_P is an equivalence relation, but \approx_P is not: it is reflexive, symmetric, but not necessarily transitive. This reflects our theoretical considerations about the failure of transitivity of the relation of indistinguishability in Sorites series. As we have seen, in Sorites series the relation of indistinguishability that holds between any two contiguous elements with respect to some property is not transitive: if you consider the first and the last element of the series, if the first has a property P and any two elements in the series are indistinguishable with respect to P , it does not necessarily follow that also the last element of the series has P . So, by (SC) we assume comparison classes to

⁷These last two elements are predicted to be the minimal and the maximal element of the set of the individuals when ordered.

have elements that form a Sorites series. Moreover, while \approx_P is used only to ensure that we have enough individuals, that is, they form a soritical series, \sim_P is defined on the choice function P within a specific context structure. Each context structure is a way to look at comparison classes and their contexts, and to partition each context. That means, according to each context structure we can detect some differences, and make finer and finer partitions also in soritical contexts, as the the grain size gets finer and finer.

Now, so far I assume that the contexts of type o^s as defined above are elements of O_s :

(Q) $\forall o : o^s \in O_s$.

It has to be noticed that o^s are Sorites series; namely, they contain a sequence of elements that satisfy (SC). By (Q) any context structure $M = \langle \mathcal{D}, ICC, P, \approx_{sP} \rangle$ gives an ordering to *all* the contexts $o \in O_s$. So, each M that has a function P that satisfies van Benthem's constraints, gives rise to a weak ordering also in contexts of type o^s . We will consider later, in section 3.2.10, what happens if we get rid of (Q).

Since the comparative relation $>_P$ gives rise to weak orders and weak orders partition all the contexts into equivalence classes, also the contexts of type o^s are partitioned into equivalence classes. In such a way, we get that all comparison classes are weakly ordered. Furthermore, if each context structure orders the elements of the subsets (i.e. the contexts) of a comparison class, the whole comparison class gets an ordering of the same type (weak) within the same context structure. That means, for each $s \in ICC$, we have structures of type $\langle s, >_P \rangle$.

Assume now that the extensions of each predicate P and of their complements \bar{P} in o^s , for all $o \in O_s$, have to be as follows:

(EX) $|\{[x]_{\sim_P} \in P(o^s)\}| = |\{[x]_{\sim_P} \in \bar{P}(o^s)\}| \pm 1$

Informally, (EX) says that, given a context o^s , half of the elements (or, better, equivalence classes) of o^s are P and half are \bar{P} . (EX) is used to draw a boundary of the extension of P in a Sorites series and so excludes borderline case. I assume (EX) because it is so far the only way for me to account for our intuitions about M_1, M_2, M_3 at the beginning of this section. The reason will get clear in the following lines.

Now, given a context $o \in O_s$ and a set of context structures (like M_1, M_2, M_3) that give rise to the same ordering, we accept the context structures that make the following formula true:

(R) $\forall x \in o : x \in P(o)$ iff $x \in P(o^s)$.

(R) is the constraint that restricts the set of context structures: it rules out the context structures that do not correctly predict our use of vague adjectives. (R) tells us that if some element x (or equivalence class $[x]$) is in the range of function P for the context o^s , then x (or $[x]$) must be in the range of P for the context o . To make the point clear, consider again the example mentioned in the beginning of this section.

The situation we want to formally represent is the following: in the comparison class of men there is a context $o_1 = \{a, b, e, f\}$ where a represents an individual who is 200 cm tall, b an individual who is 190 cm tall, e an individual who is 170 cm tall and f an individual who is 160 cm tall. Who is tall and who is short in o_1 ? I take into consideration function T (standing for ‘tall’).

Now, let o_1^s contain all the elements that are in-between a, b, e, f . We get a series of elements whose height ranges from a ’s height (200 cm) and f ’s height (160 cm). By (EX) we get that half of the elements of o_1^s are in $T(o_1^s)$ and half in $\bar{T}(o_1^s)$. More precisely, the elements whose heights is between 200 and 180 cm turn out to be in $T(o_1^s)$ and the individuals whose height is between 180 and 160 cm in $\bar{T}(o_1^s)$. Then, we have: $a, b \in T(o_1^s)$ and $e, f \in \bar{T}(o_1^s)$, that is, a and b are tall in the fulfilled context o_1^s , e and f short.

Consider again the context structures we had at the beginning:

$$M_1 \models T(o_1) = \{a\}, \bar{T}(o_1) = \{b, e, f\}$$

$$M_2 \models T(o_1) = \{a, b\}, \bar{T}(o_1) = \{e, f\}$$

$$M_3 \models T(o_1) = \{a, b, e\}, \bar{T}(o_1) = \{f\}$$

Applying (R) as a restriction on M_1, \dots, M_3 , we get only M_2 as acceptable model.

M_1 is ruled out because b is mapped into the set $\bar{T}(o_1)$, while our constraint (R) wants it to be mapped into $T(o_1)$. Informally speaking, the extension of $T(o_1)$ given by the model M_1 contains ‘too few’ equivalence classes.

M_3 is ruled out for the opposite reason: the extension of $T(o_1)$ contains an element more than what is accepted. According to (R), e has to be in the set $\bar{T}(o_1)$ because e is in the set $\bar{T}(o_1^s)$.

Observation

When we have a context o^s with an even number of equivalence classes, the following holds:

$$|\{[x]_{\sim_P} \in P(o^s)\}| = |\{[x]_{\sim_P} \in \overline{P}(o^s)\}|.$$

The addition of ± 1 in (EX) concerns contexts with an odd number of equivalence classes. If o^s is such a case, we can say there is one equivalence class that can be considered either part of the extension of P or of \overline{P} . The elements that are in that equivalence class are called *tolerant* elements. We can perhaps think that the tolerant elements are neither P nor \overline{P} .

3.3.9 Example of an application of the model

What the present model is allegedly able to do is to predict a correct use of vague gradable adjectives. Let us try to apply now the model to the pair of vague adjectives ‘tall’ and ‘short’. Let T stand for ‘tall’, \overline{T} for ‘short’.

Consider again the example given at the end of section 2.2.2. Take the comparison class of men and a context with three individuals, John, Bill and Marc. John is 190 cm tall, Bill is 188 cm, Marc 160 cm. Speakers that see the three men, do not know their precise heights. However, if they have to describe the height of John, Bill and Marc, they will naturally say that John and Bill are tall, Marc is short. Our model tries to explain why we have such a natural intuition.

Formally, we have the context $o = \{j, b, m\}$, with j, b, m standing for John, Bill, Marc, respectively. Different context structures give rise to different orderings between the elements in o ; for example:

1. $j \sim_T b \sim_T m$
2. $j \sim_T b >_T m$
3. $j >_T b >_T m$.

For each set of context structures, we also consider the ordering that it gives to the context o^s . This is obtained by filling o in with all the other individuals of the comparison class of men whose height is in-between John’s, Bill’s and Marc’s height according to the ordering given in each context structure. By (EX) the individuals who are between 190 and 175 cm tall are in the set $T(o^s)$, all the others in $\overline{T}(o^s)$.

Now, we do not consider the coarsest granular models, i.e. the context structures that give rise to only one equivalence class, namely 1.

Consider ordering 2: $j \sim_T b >_T m$. All the context structures that give this ordering give the following extensions for T and \overline{T} in o :

$$T(o) = \{j, b\}, \bar{T}(o) = \{m\}.$$

By (EX), j and b are in $T(o^s)$ and m in $\bar{T}(o^s)$. So, the context structures considered give the right prediction for $T(o)$ and $\bar{T}(o)$.

Consider now $j >_T b >_T m$. We can have some context structures that give the following extensions for T and \bar{T} in o :

$$T(o) = \{j, b\}, \bar{T}(o) = \{m\}.$$

But for the same ordering, we can have also other context structures that give the following extensions:

$$T(o) = \{j\}, \bar{T}(o) = \{b, m\}.$$

The latter kind of context structures will be ruled out by our rule (R). By (EX), we have that $T(o^s) = \{j, b\}$, $\bar{T}(o^s) = \{m\}$. So, Bill has to be considered tall in o as well. And that is exactly what we intuitively do: when we see John, Bill and Marc we say that John and Bill are tall, Marc short.

Analyzing our use of vague gradable adjectives in unproblematic cases, a model has been construed to accounts for it. It can then be applied to problematic cases, like in Sorites series or when we have borderline cases.

3.3.10 Context structures and semi-orders

In this section, a change for the model just sketched is proposed: instead of using a weak ordering relation, the suggestion is to make use of semi-orders.

An objection that might be raised against the proposal in the previous section concerns assumption (Q), according to which O_s contains all the subsets of s . By (Q) we get, as we have seen, that all the contexts of type o^s containing a soritical series are also ordered by some function P within each context structure. The ordering for the same context o^s differs on the basis of the context structure considered. This result might not be accepted if we want to preserve that in a soritical context no assignment of P or \bar{P} to any element of the context is admissible. According to our intuitions, it is very difficult and might seem unnatural to assign P or \bar{P} to individuals belonging to a soritical series in a definite and clear way, as we would be obliged to admit that two objects that taken by themselves are indistinguishable are actually one P , the other \bar{P} . We can now see that if we can get rid of the assumption (Q) we obtain that soritical contexts cannot be ordered by

function P . They can still be ordered, but the order we get is a semi-order, not a weak order. Let us look at this fact in more detail⁸.

If the relation $x >_P y$ is irreflexive and transitive, but not necessary almost connected, and the relation \sim_P as reflexive and symmetric, but not necessarily transitive, a set ordered according to the relation $>_P$ gives rise to a semi-order.

A *semi-order* is a structure $\langle I, R \rangle$ that is irreflexive (IR), semitransitive (STr) and satisfies the interval-order condition (IO) as follows:

$$\text{(IR)} \quad \forall x : \neg R(x, x)$$

$$\text{(STr)} \quad \forall x, y, z, v : (R(x, y) \wedge R(y, z)) \rightarrow (R(x, v) \vee R(v, z))$$

$$\text{(IO)} \quad \forall x, y, v, w : (R(x, y) \wedge R(v, w)) \rightarrow (R(x, w) \vee R(v, y)).$$

Together with the rejection of (Q), I assume that not all the subsets of s containing at least two elements are proper contexts. In general, as we have seen before, the contexts in which all the elements are indistinguishable with respect to P are not relevant: since there is no distinction to be detected, we do not get any information about the distinction between elements that are P and elements that are \bar{P} . This kind of contexts are not proper ones: they also contain a Sorites series. The proper contexts are only the ones which contain at least one difference pair.

If we assume that not all finite subsets of the domain are proper contexts and the subsets to rule out are the ones containing indistinguishable individuals with respect to some property, almost-connectedness of the comparative relation does not hold anymore. Recall the property of almost-connectedness:

$$\text{(AC)} \quad \forall x, y, z : R(x, y) \rightarrow (R(x, z) \vee R(z, y))$$

Loosing (AC), we also loose transitivity for the comparative relation $>_P$. However, we can add some closure conditions to get back transitivity.

The strategy here is to start with proper contexts containing only two elements. Then, put some constraints in order to build larger appropriate contexts.

Let us define now the set of proper contexts O_s^* . Consider the contexts containing only two elements for which the following holds:

$$\text{(U)} \quad \forall o \in O_s^* : \text{card}(o) = 2 \rightarrow \emptyset \neq P(o) \neq o.$$

⁸I will mainly refer to van Rooij [117].

By (U) we want to capture the idea that a context $\{x, y\}$ is a proper context iff its elements x, y are not indistinguishable with respect to P , i.e. they constitute a difference pair. That means, intuitively, that there is a gap between the P - and \overline{P} -elements in context $\{x, y\}$.

To get proper contexts with cardinality bigger than 2, given a comparison class s we close O_s^* under the following two conditions:

(OR1) $\forall o \in O_s^*$: if $\{x, y\} \in O_s^*$ then $o \cup \{x\} \in O_s^*$ or $o \cup \{y\} \in O_s^*$

(OR2) $\forall x, y, z, v \in s$: if $\{x, y\} \in O_s^*, \{y, z\} \in O_s^*, \{x, z\} \in O_s^*$, then $\{x, y, z, v\} \in O_s^*$.

These closure conditions are used to make it sure that (IR), (STr) and (IO) are met and to generate sets without Sorites series. (OR1) says that you can always add one element x from any proper context o' to another proper context o and get a new proper context. So, you can have contexts with more than two objects. Moreover, (OR1) guarantees that the comparative relation satisfies (IO). (OR2) guarantees that Semi Transitivity (STr) holds. Those two latter facts will be proved later on.

We can define the comparative and the similarity relations in the usual way:

Definition 14 $x >_P y$ iff_{def} $\{x, y\} \in O_s^*$ and $x \in P(\{x, y\})$ and $y \in \overline{P}(\{x, y\})$.

Definition 15 $x \sim y$ iff_{def} $x \not>_P y$ and $y \not>_P x$.

From those definitions and (U) also follows:

$x \sim_P y$ iff $\{x, y\} \notin O_s^*$.

We can now prove that we get semi-orders for all contexts $o \in O_s^*$.

Theorem 1 Any context structure $M = \langle \mathcal{D}, I_{CC}, P, \approx_{s_P} \rangle$, where P obeys axioms (NR), (DD), (UD) and (U), and where for all $s \in CC$ the set O_s^* is closed under (OR1), (OR2), gives rise to a semi-order $\langle I_{CC}, > \rangle$, if we define $x >_P y$ as $x \in P(\{x, y\})$ and $y \in \overline{P}(\{x, y\})$.

Proof The proof of theorem 1 consists of three parts⁹. We have to prove that the structure $\langle I_{CC}, > \rangle$ is 1. irreflexive, 2. semitransitive, 3. satisfies the interval-order condition, as the definition of semi-orders requires.

⁹I am adapting the proof provided by van Rooij in [117]

1. To prove: $\langle I_{CC}, > \rangle$ satisfies (IR).

It directly follows from the definition of the comparative relation, which is namely irreflexive.

2. To prove: $\langle I_{CC}, > \rangle$ satisfies (IO) ($\forall x, y, v, w : (x > y \wedge v > w) \rightarrow (x > w \vee v > y)$).

Assume $x > y$, $v > w$. So, $\{x, y\} \in O_s^*$ and $\{v, w\} \in O_s^*$. By (OR1) we have then to consider two cases: $\{v, w, x\} \in O_s^*$ and $\{v, w, y\} \in O_s^*$.

Consider the first case: $\{v, w, x\} \in O_s^*$. Since $v > w$ by assumption, by (UD) there is a difference pair in $\{v, w, x\}$. We have the following possibilities:

- $P(\{v, w, x\}) = \{v\}$, $\bar{P}(\{v, w, x\}) = \{w, x\}$; so, by (DD) $v > x$;
- $P(\{v, w, x\}) = \{v, w\}$, $\bar{P}(\{v, w, x\}) = \{x\}$; so, by (DD) $v > x$;
- $P(\{v, w, x\}) = \{v, x\}$, $\bar{P}(\{v, w, x\}) = \{w\}$; so, by (DD) $x > w$;
- $P(\{v, w, x\}) = \{x\}$, $\bar{P}(\{v, w, x\}) = \{v, w\}$; so, by (DD) $x > w$.

In the last two cases, $x > w$ holds. Consider the first two cases, where $v > x$ holds. By assumption, $x > y$ holds too. It is easy to verify that $\{v, x, y\} \in O_s^*$ and that there are difference pairs in $\{v, x, y\}$. We have the following possibilities:

- $P(\{v, x, y\}) = \{v\}$, $\bar{P}(\{v, x, y\}) = \{x, y\}$; so, by (DD) $v > y$;
- $P(\{v, x, y\}) = \{v, x\}$, $\bar{P}(\{v, x, y\}) = \{y\}$; so, by (DD) $v > y$.

So, in case $\{v, w, x\} \in O_s^*$ holds, then either $v > y$ or $x > w$ hold.

Consider now the second case: $\{v, w, y\} \in O_s^*$. Performing a similar argument as in the first case, it follows that either $v > y$ or $x > w$ hold.

So, both in the first and in the second case we have proven that the interval-order condition, i.e. $\forall x, y, v, w : (x > y \wedge v > w) \rightarrow (x > w \vee v > y)$, holds.

3. To prove: $\langle I_{CC}, > \rangle$ satisfies (STr) ($\forall x, y, z, v : (x > y \wedge y > z) \rightarrow (x > v \vee v > z)$).

Assume $x > y$ and $y > z$. So, from (IR) and (IO) follows $x > z$. It must then be the case that $\{x, y\} \in O_s$, $\{y, z\} \in O_s$ and $\{x, z\} \in O_s$. By (OR2) we have $\{x, y, z, v\} \in O_s$. Since $x > y$ and $y > z$, we have by (NR) and (UD) the following possibilities:

- $P(\{x, y, z, w\}) = \{x, y\}$, $\bar{P}(\{x, y, z, w\}) = \{z, v\}$; so, by (DD) $x > v$;
- $P(\{x, y, z, w\}) = \{x\}$, $\bar{P}(\{x, y, z, w\}) = \{y, z, v\}$; so, by (DD) $x > v$;
- $P(\{x, y, z, w\}) = \{x, y, v\}$, $\bar{P}(\{x, y, z, w\}) = \{z\}$; so, by (DD) $v > z$;
- $P(\{x, y, z, w\}) = \{x, v\}$, $\bar{P}(\{x, y, z, w\}) = \{y, z\}$; so, by (DD) $v > z$.

So, in all cases, either $x > v$ or $v > z$ holds. ■

Each comparison class s is then ordered in such a way that it originates a semi-order structure. In that case, though, we do not have any longer equivalence classes because the indistinguishability relation \sim_P turns out to be reflexive and symmetric, but not necessarily transitive. So, it is not an equivalence relation and therefore the model developed in the previous sections and based on equivalence classes does not apply. However, there is a way to move around the problem.

We know from Luce [72] that any semi-order can induce a weak order. Let us consider his contribution.

If R is an arbitrary relation on a set s , an indifference relation J can always be defined as follows: for $a, b \in s$: sJb iff neither aRb nor bRa . Then, the relation $(>, \sim)$ induced on s by a given relation R, J on s is defined as follows: $a > b$ if either:

- (i) aRb ,
- (ii) aJb and there exists $c \in S$ such that aJc and cRb , or
- (iii) aJb and there exists $d \in S$ such that aRd and dRb .

If neither $a > b$ nor $b > a$, then $a \sim b$.

Theorem 2 (R, J) is a semi-order if and only if R is transitive and $(>, \sim)$ is a weak order.

For the proof of Theorem 2, consider Luce [72], pp. 183-185.

Using Luce's theorem, we can claim that whenever we have a semi-order, we can define a relation $>$ (and consequently its symmetric counterpart \sim) that generates a weak order. Any semi-order obtained rejecting (Q) can be

mapped to one induced weak order. In such a way, we can then apply again the machinery described in the previous section that makes use of equivalence classes.

3.4 Results and remarks

In this final section I show how some interesting results can be proved in the model described in the previous section and, then, I state some general observations. The relevance of the results proved will be made clear in section 3.4.4.

3.4.1 Further results

In the model described by making use of weak orders and equivalence classes, we are able to prove the following interesting results¹⁰:

(A), (B), (C), (D), (E) hold for each context $o \in O_s$.

(A) $\forall x \in o$, if $|\{[y] \in o^s : [y] > [x]\}| <^c |\{[y] \in o^s : [y] < [x]\}|$, then $x \in P(o)$.

The intuitive reading of (A) is: for all the elements x in some context o such that in context o^s the number of equivalence classes $[y]$ that are ‘more P ’ than the equivalence class $[x]$ is smaller than the number of equivalence classes $[y]$ that are ‘less P ’ than $[x]$, then x is P . Put otherwise, again; for all $x \in o$, if the cardinality of the set of equivalence classes that are ‘more P ’ than $[x]$ is strictly lower than the cardinality of the set of equivalence classes that are ‘less P ’ than $[x]$, then x is in $P(o)$.

(B) $\forall x \in o$, if $|\{[y] \in o^s : [y] > [x]\}| >^c |\{[y] \in o^s : [y] < [x]\}|$, then $x \in \bar{P}(o)$.

The intuitive reading of (B) is: for all the elements $x \in o$ such that in context o^s the number of equivalence classes $[y]$ that are ‘more P ’ than the equivalence class $[x]$ is bigger than the number of equivalence classes $[y]$ that are ‘less P ’ than $[x]$, then x is \bar{P} . Put otherwise:

¹⁰For the sake of simplicity, read relations $>$, $<$, \leq or \geq as abbreviations of $>^*_P$ or $<^*_P$, \leq^*_P , \geq^*_P , respectively

for all $x \in o$, if the cardinality of the set of equivalence classes that are ‘more P ’ than $[x]$ is strictly greater than the cardinality of the set of equivalence classes that are ‘less P ’ than $[x]$, then x is in $\overline{P}(o)$.

- (C) $\forall x \in o$, if $|\{[y] \in o^s : [y] < [x]\}| = |\{[y] \in o^s : [y] > [x]\}|$, then $x \in P(o)$ or $x \in \overline{P}(o)$.

The intuitive reading of (C) is: for all the elements x in o such that in context o^s the number of equivalence classes $[y]$ that are ‘more P ’ than the equivalence class $[x]$ is equal to the number of equivalence classes $[y]$ that are ‘less P ’ than $[x]$, x is in the set given by $P(o)$ or in the set given by $\overline{P}(o)$, that is, x is P or \overline{P} . In other terms: for some x and some context o , if the cardinality of the set of equivalence classes that are ‘more P ’ than $[x]$ is equal to the cardinality of the set of equivalence classes that are ‘less P ’ than $[x]$, then x is in $P(o)$ or in $\overline{P}(o)$.

(C) allows for tolerant equivalence classes: whenever the number of equivalence classes in o^s is odd, the equivalence class which is equally distant from the greatest and the lowest equivalence class in the comparative relation can be considered as P or \overline{P} on the basis of speakers’ intentions and purposes. Whether to choose P or \overline{P} is not a syntactic nor a semantic matter, but only pragmatic.

- (D) $\forall x \in o$, if $x \in P(o)$, then either $|\{[y] \in o^s : [y] > [x]\}| <^c |\{[y] \in o^s : [y] < [x]\}|$ or $|\{[y] \in o^s : [y] > [x]\}| = |\{[y] \in o^s : [y] < [x]\}|$.

The intuitive reading of (D) is: for all the elements $x \in o$, if x is P in context o , then in context o^s the number of equivalence classes $[y]$ that are ‘more P ’ than the equivalence class $[x]$ is either smaller than or equal to the number of equivalence classes $[y]$ that are ‘less P ’ than $[x]$. Put otherwise, for any $x \in o$, if $x \in P(o)$, then the cardinality of the set of equivalence classes that are ‘more P ’ than $[x]$ is lower than or equal to the cardinality of the set of equivalence classes that are ‘less P ’ than $[x]$.

- (E) $\forall x \in o$, if $x \in \overline{P}(o)$ then either $|\{[y] \in o^s : [y] > [x]\}| >^c |\{[y] \in o^s : [y] < [x]\}|$ or $|\{[y] \in o^s : [y] > [x]\}| = |\{[y] \in o^s : [y] < [x]\}|$.

The intuitive reading of (E) is: for all the elements $x \in o$, if x is \overline{P} in context o , then in context o^s the number of equivalence classes that are ‘more P ’ than the equivalence class $[x]$ is either greater than or equal

to the number of equivalence classes that are ‘less P ’ than $[x]$. In other terms, for each $x \in o$, if $x \in \overline{P}(o)$, then the cardinality of the set of equivalence classes that are ‘more P ’ than $[x]$ is greater than or equal to the cardinality of the set of equivalence classes that are ‘less P ’ than $[x]$.

I only provide the proofs of (A) and (D). Since the proofs of (B) and (C) are similar to the one of (A), and the proof of (E) is similar to (D), I shall omit them.

3.4.2 Proof of (A)

(A) $\forall x \in o$, if $|\{[y] \in o^s : [y] > [x]\}| <^c |\{[y] \in o^s : [y] < [x]\}|$, then $x \in P(o)$.

Assume for some arbitrary $x \in o$: $|\{[y] \in o^s : [y] > [x]\}| <^c |\{[y] \in o^s : [y] < [x]\}|$. To prove: $x \in P(o)$.

Two cases are to be distinguished:

- $|\{[z] \in o^s\}|$, i.e. the number of equivalence classes in o^s , is even. So, $|\{[y] \in P(o^s)\}| = |\{[y] \in \overline{P}(o^s)\}|$, i.e. the number of equivalence classes in $P(o^s)$ is identical to the number of equivalence classes in $\overline{P}(o^s)$.
Assume: $x \notin P(o)$. So, $x \in \overline{P}(o)$, that means, the equivalence class $[x] \in \overline{P}(o)$. By (R), $[x] \in \overline{P}(o^s)$.

Given $[x] \in \overline{P}(o^s)$, by the definition of $>$ for equivalence classes the two following facts hold:

Fact 1 $\forall [y] \in o^s ([y] \in P(o^s) \rightarrow [y] > [x])$

Fact 2 $\forall [y] \in o^s ([y] < [x] \rightarrow [y] \in \overline{P}(o^s))$.

From Facts 1 and 2 two consequences follow.

Consequence of Fact 1: $|\{[y] \in P(o^s)\}| \leq^c |\{[y] \in o^s : [y] > [x]\}|$.

Consequence of Fact 2: $|\{[y] \in o^s : [y] < [x]\}| <^c |\{[y] \in \overline{P}(o^s)\}|$, because $[x] \in \overline{P}(o^s)$, so $|\{[y] \in o^s : [x] < [y]\}| <^c |\{[y] \in \overline{P}(o^s)\}|$ for, at least, one equivalence class, namely $[x]$.

From those two consequences of facts 1 and 2 and the assumption $|\{[y] \in o^s : [y] > [x]\}| <^c |\{[y] \in o^s : [y] < [x]\}|$, we obtain, by transitivity of $<^c$ and \leq^c : $|\{[y] \in P(o^s)\}| <^c |\{[y] \in \overline{P}(o^s)\}|$. That contradicts $|\{[y] \in P(o^s)\}| = |\{[y] \in \overline{P}(o^s)\}|$. Therefore: $x \in P(o)$.

■

- $|\{[y] \in o^s\}|$ is odd.

Two cases are possible:

- $|\{[y] \in P(o^s)\}| = |\{[y] \in \overline{P}(o^s)\}| - 1$. So, $|\{[y] \in P(o^s)\}| <^c |\{[y] \in \overline{P}(o^s)\}|$.

Assume $x \notin P(o)$. So, $x \in \overline{P}(o)$, that means, $[x] \in \overline{P}(o)$. By (R), $[x] \in \overline{P}(o^s)$.

Facts 1 and 2 and their consequences hold.

Consider the following case: take an equivalence class $[x]$ such that $\forall [y] \in \overline{P}(o^s) : [y] \leq [x]$. In such a case $[x]$ is a tolerant equivalence class, such that $|\{[y] \in o^s : [y] > [x]\}| = |\{[y] \in o^s : [y] < [x]\}|$. For such a case, consider the informal observations made for (C).

The interesting case to be considered is the following: assume that the equivalence class $[x]$ is such that: $\exists [y] \in \overline{P}(o^s) : [y] > [x]$.

We have, then: $|\{[y] \in P(o^s)\}| <^c |\{[y] \in o^s : [y] > [x]\}|$ and $|\{[y] \in o^s : [y] < [x]\}| <^c |\{[y] \in \overline{P}(o^s)\}|$ for more than one equivalence class. Since $|\{[y] \in P(o^s)\}| <^c |\{[y] \in \overline{P}(o^s)\}|$ for only one equivalence class, we have $|\{[y] \in o^s : [y] < [x]\}| <^c |\{[y] \in P(o^s)\}|$. By transitivity, then: $|\{[y] \in o^s : [y] < [x]\}| <^c |\{[y] \in o^s : [y] > [x]\}|$. We get a contradiction with our initial assumption: $|\{[y] \in o^s : [y] > [x]\}| <^c |\{[y] \in o^s : [y] < [x]\}|$. So, $x \in P(o^s)$. ■

- $|\{[y] \in P(o^s)\}| = |\{[y] \in \overline{P}(o^s)\}| + 1$. So, $|\{[y] \in P(o^s)\}| >^c |\{[y] \in \overline{P}(o^s)\}|$.

Assume $x \notin P(o)$. So, $x \in \overline{P}(o)$, that means, $[x] \in \overline{P}(o)$. By (R), $[x] \in \overline{P}(o^s)$.

Facts 1 and 2 hold. From $|\{[y] \in o^s : [x] < [y]\}| <^c |\{[y] \in \overline{P}(o^s)\}|$, $|\{[y] \in \overline{P}(o^s)\}| <^c |\{[y] \in P(o^s)\}|$ and $|\{[y] \in P(o^s)\}| \leq^c |\{[y] \in o^s : [y] > [x]\}|$ we get, by transitivity of $<^c$ and \leq^c : $|\{[y] \in o^s : [x] < [y]\}| <^c |\{[y] \in o^s : [y] > [x]\}|$. Again, that contradicts $|\{[y] \in o^s : [y] > [x]\}| <^c |\{[y] \in o^s : [y] < [x]\}|$. Then: $x \in P(o)$. ■

3.4.3 Proof of (D)

(D) $\forall x \in o$, if $x \in P(o)$, then either $|\{[y] \in o^s : [y] > [x]\}| <^c |\{[y] \in o^s : [y] < [x]\}|$ or $|\{[y] \in o^s : [y] > [x]\}| = |\{[y] \in o^s : [y] < [x]\}|$.

Take an arbitrary $x \in o$ and assume: $x \in P(o)$. So, $[x] \in P(o)$. By (R), $[x] \in P(o^s)$.

To prove: either $|\{[y] \in o^s : [y] > [x]\}| <^c |\{[y] \in o^s : [y] < [x]\}|$ holds or $|\{[y] \in o^s : [y] > [x]\}| = |\{[y] \in o^s : [y] < [x]\}|$ holds.

Two cases are to be distinguished:

- $|\{[z] \in o^s\}|$ is even.

So, we have: $|\{[y] \in P(o^s)\}| = |\{[y] \in \overline{P}(o^s)\}|$.

Given the assumption $[x] \in P(o^s)$, by the definition of $>$ as a comparative relation between equivalence classes the following facts and their consequences hold:

Fact 3 $\forall [y] \in o^s ([y] \in \overline{P}(o^s) \rightarrow [x] > [y])$

Fact 4 $\forall [y] \in o^s ([y] > [x] \rightarrow [y] \in P(o^s))$.

Consequence of Fact 3: $|\{[y] \in \overline{P}(o^s)\}| \leq^c |\{[y] \in o^s : [y] < [x]\}|$.

Consequence of Fact 4: $|\{[y] \in o^s : [y] > [x]\}| <^c |\{[y] \in P(o^s)\}|$ for at least one equivalence class, $[x]$, because $[x] \in P(o^s)$.

Given the consequence of Fact 4 $|\{[y] \in o^s : [y] > [x]\}| <^c |\{[y] \in P(o^s)\}|$, the assumption $|\{[y] \in P(o^s)\}| = |\{[y] \in \overline{P}(o^s)\}|$, and the consequence of Fact 3 $|\{[y] \in \overline{P}(o^s)\}| \leq^c |\{[y] \in o^s : [y] < [x]\}|$, we obtain $|\{[y] \in o^s : [y] > [x]\}| <^c |\{[y] \in o^s : [y] < [x]\}|$. ■

- $|\{[z] \in o^s\}|$ is odd.

Two cases are possible:

- $|\{[y] \in P(o^s)\}| = |\{[y] \in \overline{P}(o^s)\}| - 1$.

So, $|\{[y] \in P(o^s)\}| <^c |\{[y] \in \overline{P}(o^s)\}|$.

Also in this case, Fact 3 and 4 and their consequences hold.

By transitivity of $<^c$ and \leq^c , from the consequence of Fact 4 $|\{[y] \in o^s : [y] > [x]\}| <^c |\{[y] \in P(o^s)\}|$, the assumption $|\{[y] \in P(o^s)\}| <^c |\{[y] \in \overline{P}(o^s)\}|$ and the consequence of Fact 3 $|\{[y] \in \overline{P}(o^s)\}| \leq^c |\{[y] \in o^s : [y] < [x]\}|$, we infer $|\{[y] \in o^s : [y] > [x]\}| <^c |\{[y] \in o^s : [y] < [x]\}|$. ■

- $|\{[y] \in P(o^s)\}| = |\{[y] \in \overline{P}(o^s)\}| + 1$.

So, $|\{[y] \in P(o^s)\}| >^c |\{[y] \in \overline{P}(o^s)\}|$.

Also in this case, Fact 3 and 4 and their consequences hold.

Consider the equivalence classes $[x]$ such that: $\exists [y] \in P(o^s) : [y] < [x]$.

Given the consequence of Fact 4 $|\{[y] \in o^s : [y] > [x]\}| <^c |\{[y] \in P(o^s)\}|$, the assumption $|\{[y] \in P(o^s)\}| <^c |\{[y] \in \overline{P}(o^s)\}|$,

and the consequence of Fact 3 $|\{[y] \in \overline{P}(o^s)\}| \leq^c |\{[y] \in o^s : [y] < [x]\}|$, by transitivity of the relation $<^c$ follows $|\{[y] \in o^s : [y] > [x]\}| <^c |\{[y] \in o^s : [y] < [x]\}|$. ■

Consider an equivalence class $[x]$ such that $\forall [y] \in P(o^s) : [y] \geq [x]$, that is, such that $[x]$ is less P than all the P -elements. $[x]$ is a tolerant equivalence class. So, $\forall [y] \in o^s ([y] < [x] \rightarrow [y] \in \overline{P}(o^s))$ holds. We also have: $|\{[y] \in o^s : [y] < [x]\}| = |\{[y] \in \overline{P}(o^s)\}|$ and $|\{[y] \in o^s : [y] > [x]\}| = |\{[y] \in P(o^s)\}| - 1$.

Now, $|\{[y] \in P(o^s)\}| - 1 = |\{[y] \in \overline{P}(o^s)\}|$ by assumption, so: $|\{[y] \in o^s : [y] > [x]\}| = |\{[y] \in \overline{P}(o^s)\}|$. Since $|\{[y] \in o^s : [y] < [x]\}| = |\{[y] \in \overline{P}(o^s)\}|$, we obtain $|\{[y] \in o^s : [y] > [x]\}| = |\{[y] \in o^s : [y] < [x]\}|$. ■

3.4.4 Theoretical remark

(A), (B) and (C) show that, given a certain element x in a context, if we know whether the number of equivalence classes that are more P than $[x]$ is equal to, greater than or smaller than the number of equivalence classes that are less P than $[x]$, then we can conclude whether x is in the extension of $P(o)$ or of $\overline{P}(o)$. The number of equivalence classes that are more P than $[x]$ compared with that are less P than $[x]$ tells us something about the position of x in the order given by the comparative relation $>_P$. Imagine such an order as a segment of a line. Equivalence classes are groups of objects lying on such a line. On the left end of the segment there is the equivalence class $[y]$ such that it is more P than all the other equivalence classes in the segment. That means that, if P is interpreted as ‘tall’, on the left end of the ordered segment there is the equivalence class containing the tallest men. On the right end, by contrast, there is the equivalence class $[z]$ such that it is less P than all the other equivalence classes in the segment. In our example, that is the equivalence class containing the shortest men. If the number of equivalence classes that are more P than a given $[x]$ is greater than the number of equivalence classes that are less P than $[x]$, then x is closer to the equivalence class of shortest men than to the equivalence class of tallest men. So, given the conditions described in the model, we can derive that x is in $\overline{P}(o)$. If the number of equivalence classes that are more P than a given $[x]$ is less than the number of equivalence classes that are less P than $[x]$, then x is closer to the equivalence class of tallest men than to the equivalence class of shortest men. Given the conditions described in the model, we can then derive that x is in $P(o)$.

(D) and (E) show that, given a certain element of a context x , if we know whether x is in $P(o)$ or in $\overline{P}(o)$, we know in which position it lies in the ordering, that is, we can infer whether the number of equivalence classes that are more P than $[x]$ is equal to, greater than or smaller than the number of equivalence classes that are less P than $[x]$. Let P be interpreted as ‘tall’. If x is tall, then x is closer to the equivalence class of the tallest men in the ordering, if x is short, then x is closer to the equivalence class of the shortest men.

In the description of the model the comparative relation has been defined over the function P . From a linguistic point of view, that operation corresponds to taking the positive form of adjectives as primitive and from it deriving the comparative form. What (A), (B), (C), (D), (E) show is that, if you prefer taking the comparative relation as primitive, instead of function P , you obtain the same results with respect to the model for polar adjectives.

There are some (in my opinion, strong) intuitions that might bring us to think that we always assign a polar adjective to an object after some sort of comparison between that object and some comparison class or context. Imagine there is only one object o^* in the universe: we cannot say if it is big or small, nor if tall or short, etc. We need to have at least another object to be able to properly attribute a property to o^* . When children learn how to use polar adjectives, they need to see a comparative set among the elements of which some objects are, for example, big, and others small.

By means of the primitive function P we arbitrarily say (or, we already know) what is big and what is small, and afterwards we build comparatives among the objects characterized as big and as small. Put otherwise, the function P itself “choose” which elements are P and which \overline{P} . However, it seems that when we use gradable adjectives in natural language, first we see how things are in the world and their relations and, then, we can distinguish big from small objects, and so on. An approach of vagueness and of the meaning of adjectives that starts from an analysis of comparatives might be closer to our intuitions. The results I proved in the previous section show that, if someone wants to assume such an approach, she can use the same model proposed.

3.5 Open problems

In this final section I intend to underline some problematic points and set some questions.

3.5.1 About infinity

The model developed and presented here is suitable for domains with a finite number of elements. But what happens to the domains with a countably infinite number of elements? Take the set of natural numbers \mathbb{N} and consider the predicates ‘small’ and ‘large’. The problem we have is that we cannot draw a boundary between small and large elements in the context $o *^{\mathbb{N}}$ when $|o *^{\mathbb{N}}| = |\mathbb{N}|$. But for all the other contexts $o \in O_{\mathbb{N}}$, which contain a finite number of elements, we can always fill the correspondent context $o^{\mathbb{N}}$ in and distinguish small from large elements. However, a question might arise: is the incapacity to treat infinite contexts a real defect of the model? Maybe, such incapacity is not a mere weakness. To ask whether a certain number is large does not make sense, if the comparison class considered is the whole set \mathbb{N} . In normal conversation, we can use the expression ‘large (or small) number’ when we have a context with a finite number of elements. The use of gradable adjectives with infinite contexts does not seem to be natural in ordinary language. With Parikh’s words (Parikh [82]):

Perhaps this is the explanation of why we use vague predicates in daily life without any serious problem and still avoid difficulties which a logician might run into.

3.5.2 About polarity

The model described in this chapter tries to explain why we are able to properly use vague polar adjectives. It does not account, though, for a *solution* to the problem of vagueness, nor does it explain why polar adjectives such as ‘tall’, ‘short’, ‘big’ and ‘small’ are vague. I made some assumptions about soritical series and used two mathematical structures to deal with soritical domains: weak and semi-orders. I also assumed

$$\text{(EX)} \quad |\{[y] \in P(o^s)\}| = |\{y \in \overline{P}(o^s)\}| \pm 1$$

that concerns the behavior of vague predicates on Sorites series. By (EX) I assumed that half of the elements of any soritical context o^s are P , half are not. An intuitive justification for such a strong assumption is based on the interpretation of P and \overline{P} as opposite poles at the ends of a segment. There is a tension between opposite poles. The objects that are closer to the positive pole P are more clearly P . As the distance from the positive pole increases, the objects are less clearly P , and the highest grade of tension (and uncertainty) is exactly in the middle of the series. After that point, the objects get closer to the negative pole and so they get more and more

clearly \overline{P} . The cut-off point between the extension of P and of \overline{P} then lies in the middle of the series. The ideal tension between the two poles as they were magnetic poles is the intuitive justification for assumption (EX); that is why I draw a line in the middle of the series to distinguish P -objects from \overline{P} -objects.

In fact, the vagueness problem consists exactly in where to draw the cut-off line. With my proposal I do not want to *solve* the problem of vagueness. I assume (EX) to explain why we agree on some unproblematic uses of polar adjectives. So, the model does not provide a way to draw the line to discriminate the extensions of P and \overline{P} . Analyzing unproblematic cases, it seemed to me that (EX) is useful to explain why natural language speakers choose the gradable adjectives to use in those cases.

A question which still remains unanswered is the following: do we refer to a possible soritical series to get the meaning of ‘tall’ and ‘short’ when we are using these adjectives in unproblematic contexts?

3.5.3 About the notion of granularity

In the Sorites series contiguous elements are indistinguishable with respect to an aspect (height, color, ...) from some point of view (usually, from an observational point of view). Using a more efficient way of measurement or adopting a higher-level standard of precision, a larger number of differences between the objects can be determined. The fact that we, as natural language speakers, are not able to discriminate between short quantities by ourselves is not due to some deficiencies or shortcomings of our cognitive system, but it shows that we have been

attuned to the aspects of our environment that are most likely to be relevant to our interests (Hobbs [42], p. 433).

Such an idea might bring about a change in the epistemic conception of vagueness. If we assume Hobbs’ view, vagueness is not seen as a mere defect of our epistemic capacities, that is, of our capacity to be acquainted with the world around us. On the contrary, it is positively considered, that is, as the result of human adaptation to the world. If we cannot distinguish some differences is also due to the fact that from a pragmatic point of view we do not usually *need* to do that.

Vagueness and granularity in formal ontology

The idea of connecting granularity to vagueness has been developed also in Bittner and Smith’s jointed papers, such as [7], [8] and [9]. Bittner and Smith

worked mainly on the problem of vagueness of proper names in formal ontology. They propose a formal framework connecting the idea of granularity with mereotopology, while what has been proposed in the present chapter is rather a semantic framework connecting granularity with algebra. Sets of fixed models (what I called *context structures*) are taken to correspond to granular levels, and granular partitions are meant to be equivalent classes from an algebraic point of view.

In Smith and Brogaard [105] it is pointed out that in their work the term ‘partition’ is not used to mean ‘equivalence class’. A granular partition is a grid of cells that gives an abstract classification of objects in reality (Smith and Brogaard [105], p. 6):

A granular partition is a way of dividing up the world, or some portion of the world, by means of cells.

While Bittner and Smith’s granular partitions are a way to divide things and get several categories of objects related one each other, the granular partitions proposed in the present research are ways to describe objects in the world taken one by one and considered in their similarity relations with the others. In other terms, while granular partitions as systems of cells can be used to give a conceptualization of the world itself and its constituents from an ontological point of view, granular partitions as equivalence relations can be used to describe single items from a semantic point of view.

3.5.4 Granular levels and precisifications

At first glance, fine-grained levels closely look like sharpenings in supervaluationist theories. Both constructions attempt to obtain more and more precision in the evaluation of vague predicates. There are, though, some differences between the two views.

- For the supervaluationists the truth-value of a sentence is given by a quantification over all the precisifications. That means, precisifications are functional to the semantic evaluation of sentences. By contrast, in the model presented here there is no quantification over context structures belonging to different granular levels. Each of them represents a way to look at the domain and they are functional to the establishment of the comparative relation.
- According to the supervaluationism, the valuation function of predicates changes from precisification to precisification. In the model presented here, the function representing predicates change within *each*

granular level. The variation across granular levels concerns primarily the comparative relations and the ordering that it gives rise to.

- Precisifications are a semantic device to capture the intuition that sentences concerning borderline cases are problematic to be evaluated. The intuition behind granular levels is different. Those are invoked to reflect the different ways we can look towards a domain with respect to standards of precision.

Moreover, granular levels are used as a device to represent epistemic aspects rather than semantic. The model has been developed to account for the use of gradable adjectives and epistemic considerations are brought into such an account, since they seem to influence the semantics of sentences containing gradable adjectives. The approach assumed in this research, then, is not entirely epistemic either.

3.5.5 A brief comparison with the degree-based approach

The model presents an account for gradable adjectives that is much more complex than the degree-based accounts. You have to take care of granular levels, comparison classes and contexts, while the degree-based theories need only a scale of degrees to explain how gradable adjectives are used. Someone could object that given such a difference in complexity, the degree-based approach must be preferred.

I would like to reply to such an objection with some observations. First of all, my goal is to respect our intuitions on the behavior of gradable adjectives, and our intuitions can be not always formalized in a simple way. In fact, I want to take into account many aspects that characterize gradable adjectives, namely vagueness, context sensitivity, granular sensitivity, and not just gradability. Of course, taking into account more aspects than the degree-based theory makes the account more complex. Secondly, while degree-based theories add degrees to the ontology, I do not add granular levels, comparison classes and contexts as independent elements to the ontology: they are just ways to look at or to group the elements in the domain and not elements added to the domain. What I add to the ontology are what I called *possible* objects. But I gave a justification of such an operation: I included those elements in the domain because they can build Sorites series, and since I want to explain how vague adjectives apply to Sorites series, I need to have the possibility to arrive at Sorites series in the domain. By contrast, degree-based theories add degrees as a tool to explain the behavior of adjectives, without justifying such an operation.

A problem about the formal representation of gradable adjectives remains open both for the degree-based approaches and my proposal. It concerns multidimensional adjectives like ‘clever’ and ‘beautiful’. How can we determine the truth value of a sentence like “Mary is cleverer than John”? We should consider all the different dimensions that compose the meaning of ‘clever’, or should we preferably take into account the speakers’ intentions? This question needs an analysis that would lead us a little too far from the purposes of this thesis.

Chapter 4

Vague Count Nouns and Sortal Concepts

4.1 Introduction

In the model for gradable adjectives given in the previous chapter an assumption has been made which needs to be discussed: comparison classes are assumed to be sets with precise boundaries. In this chapter I intend to take the discussion of that assumption as a starting point to handle further philosophical issues.

In the model comparison classes have been defined as subsets of a fixed domain and are the extensions of what have been called monadic predicates in the language \mathcal{L} . Monadic predicates A, B, C are taken to formally represent count nouns in the natural language, like ‘man’, ‘tree’, ‘child’, ‘mountain’. It has been assumed - for the sake of simplicity - that the comparison classes individuate sets of elements of the domain and that such sets have precise boundaries. This means that we can determine with precision what elements belong to each of them. However, if you consider count nouns in natural language, you can notice that some of them suffer from vagueness. If this is the case, is it then licit to formally represent vague count nouns with predicates that individuate sets with precise boundaries? More precisely, what I am going to discuss in this chapter is the assumption that comparison classes *have precise boundaries*.

Let us try to understand better the matter at issue. Consider a case of vague count noun, ‘mountain’, and the set of things to which ‘mountain’ applies, that is, the set of mountains. We can see that the three features that characterize vague terms - presented in Chapter 2 - pertain to ‘mountain’:

1. Borderline cases. Imagine you are in a mountainous region. Consider

a raised part of the Earth's surface which is a bit smaller than the mountains behind it, but at the same time higher than the hills in front of it. How can you classify that object? It is 'in-between' hills and mountains, so you have no evidence that makes you assert that it is a mountain, nor that it is a hill. So, it is not an easy task for you to also determine the truth value of the sentence "That is a mountain" (uttered indicating the raised part of the Earth's surface you are considering).

2. No sharp boundaries. Consider, again, a mountainous region. In the North of it there are the highest mountains, and going towards South, step by step, the mountains get smaller and smaller. In the most southern area of the region there are landforms that we intuitively call hills. Which is the boundary between mountains and hills? How high must be a raised part of the Earth's surface in order for it to be classified as a mountain? Moreover, the indecisiveness about some borderline cases, as described in 1, makes it difficult to draw a sharp line between the extension of the predicate relative to 'mountain' and its anti-extension. There is no agreement on the height a landform must have in order to be called a 'mountain'. If you check different dictionaries and encyclopedias, you will notice that there is no real common agreement on such a question.
3. Sorites paradox. Imagine ideally constructing a series of objects, such that each object corresponds to an elevation of the Earth's surface. The objects are ordered from the shortest to the tallest, and the difference between two successive objects in the series is no more than two meters. You start from the tallest one, and you say that it is clearly a mountain. Then, you consider the object that comes after it in the series. Since it is no more than two meters smaller than the first one, you say that it is a mountain, because two meters do not constitute a difference enough to determine a boundary between mountains and non-mountains. It is easy to verify that, going on in this way, also the smallest object will be said to be a mountain, and such a judgment is intuitively incorrect.

Count nouns such as 'man', 'tree', 'child', 'mountain' are represented in the model by monadic predicates that "cuts" the domain into subsets. Since some count nouns are vague, the assumption that such subsets have well determined boundaries is naive. In this chapter I wish to consider the problem of an adequate formal representation of vague count nouns to be integrated into the model for gradable adjectives. Moreover, I will take into account the problem of the vagueness of count nouns and distinguish it from the problem of vagueness of single terms, that is sometimes called 'ontic

vagueness'. I will try to show that the vagueness of count nouns is not to be treated as ontic vagueness, and then I will suggest a formal treatment of vague count nouns to be integrated in the model for gradable adjectives proposed in Chapter 3.

Count nouns are part of the set of the so-called “sortal terms” because they individuate sorts (kinds) of objects. Sortal terms are then associated with sortal concepts¹. For instance, the count noun ‘man’ individuates the sort of individuals that are men, and the concept **man** is then associated with the count noun ‘man’².

In this chapter I do not consider sortal terms (and concepts) associated with uncountable nouns (mass nouns) like ‘water’, ‘flour’, ‘grass’: my aim is to refine the assumption about the comparison classes denoted by count nouns in the model for gradable adjectives. Moreover, in that model only countable individuals are in the domain of quantification. For the sake of simplicity, then, I will consider only the sortal concepts related to count nouns. Both philosophical and linguistic considerations are taken into account. Some of the philosophical considerations about sortals concern the identity criteria associated to sortals. There will be room for discussion about the notion of identity criteria in the next chapter.

The structure of the present chapter is the following: in section 1 some linguistic considerations about sortal terms are offered; in section 2 some philosophical considerations on sortal concepts are sketched; in section 3 the philosophical thesis according to which ontological vagueness coincides with sortal vagueness is proven to be misleading and a formal account for vague sortal terms is presented. Such a formal account has been suggested by a research in conceptual modelling applied to medical issues: in the appendix (section 4) the details of this inspiring research are presented.

4.2 Linguistic considerations about sortal terms

The use of the term ‘sortal’ as it is used nowadays in philosophy of language was introduced by Locke (CFR Essay III, iii, 15):

[...] things are ranked under names into sorts or species only as they agree to certain abstract ideas, to which we have annexed

¹I sometimes use the term *sortal* as a substantive to indicate either a sortal term or a concept: the use will appear clear from the context.

²As a convention to distinguish the name from the concept I will continue writing names in inverted commas (e.g. ‘man’) and concepts in sans serif font (e.g. **man**).

those names, the essence of each *genus* or sort comes to be nothing but that abstract idea which the general, or *sortal* (if I may have leave so to call it so from *sort*, as I do *general* from *genus*), name stands for.

A sortal term is represented in logical form by a predicate³. Following Frege, a sortal predicate expresses a sense and, thus, it stands for a concept into which individuals may fall (See Wiggins [123], p. 9). In other words, count nouns are represented in systems of first-order logic by predicates that individuate sets of objects in the domain. Those sets are associated to concepts. To understand what concept a sortal predicate stands for means to grasp a rule that associates individuals with the predicate, that is, to understand what an entity must be to satisfy the predicate. According to this view, then, concepts are not abstractions: the concept **man** is not an abstract concept like **manhood**. It is a universal concept as much as entities falling into it can be spoken of or quantified over.

In this section I am concerned with the linguistic aspects concerning count nouns. I shall try to explain how count nouns (or, more generally, sortal terms) have been conceived in linguistic literature. Durrant [25], partially following Strawson, provides a tentative characterization of sortal terms. According to him, a sortal is a term

which furnishes us with a principle for distinguishing and counting particulars and which does so in its own right relying on no antecedent principle or method of so distinguishing and counting. Grammatically a sortal takes form of a common noun which: (i) takes the indefinite article in its own right; (ii) takes the plural form in its own right. We have as examples: ‘man’; ‘apple’; ‘house’; ‘dog’; ‘digit’. (Durrant [25], p. 1).

Sortal terms are primarily common nouns. Common nouns refer to classes of entities rather than to single entities: ‘tree’, ‘cat’, ‘child’ are common nouns and denote, respectively, the class of trees, the class of cats, and the class of children. Common nouns are distinguished from proper names: the latter refers to specific, single entities. ‘John’, ‘Everest’, ‘Italy’ are proper names, each of them referring to one single individual. Such a distinction between proper and common nouns is standard and reported by most (if not all) English grammar books.

³In section 4.1.1 we will see that there can be some exceptions to this (standard) assumption.

More specifically, following Durrant’s characterization, a sortal term takes the form of a common noun which:

(i) takes the indefinite article in its own right (e.g. “A man is running in the park”);

(ii) takes the plural form in its own right (e.g. “There are trees in the garden”).

Requirement (i) marks up the difference between adjectival and sortal predication. English grammar requires that the predication of sortal terms is given in the following way: “ x is a man”, “ x is an elm”, ... Compare adjectival predication: in this case, the indefinite article is not used. A sentence like “ x is a red” is not well-formed; its correct form is “ x is red”.

Lowe, following Dummett [23], makes a distinction between adjectival and sortal (general) terms. Dummett’s criteria for distinguishing between them are the following:

Criterion of application: both sortal and adjectival terms are associated with criteria of application, which are general principles determining to which individuals the considered terms correctly apply. A criterion of application for a term determines the extension of the term, for instance, the set of cats in the case of the sortal term ‘cat’ and the set of red things in the case of the adjectival term ‘red’.

Criterion of (numerical) identity: sortal terms are associated with criteria of application **and** criteria of numerical identity, which are meant to be principles determining the conditions under which one individual can be said to be either the same or distinct as another. A criterion of identity determines “whether or not, for instance, the cat that is now sitting on the mat is the same cat as the cat that was formerly sleeping on the sofa” (Lowe [67]). By contrast, there is no condition that any red thing must satisfy in order to be identical with another red thing: whether or not a certain red thing is identical with another depends on what sort of red things they are. The identity criterion determines whether, for instance, x is the same red apple than y , and not whether x is the same red than y .

Moreover, requirement (i) rules out common nouns derived from other linguistic expressions, e.g. from adjectives and verbs. For instance, ‘author’ is a common noun derived from the verb ‘to write’. ‘Author’ can take the indefinite article, but not in its own right. The reason is that we can refer to

an author only as a person (or man, woman, ...) who writes or has written. The fundamental sortal is ‘person’ (or ‘man’, ‘woman’, ...) and ‘author’ is derived from that fundamental sortal. The same can be said for verbal nouns such as ‘winner’ and ‘player’. Such expressions are ruled out by requirement (ii) too. If you ask someone to count the authors of a certain book, she will successfully complete the task only if she knows what counts for being an author; to do that, she must know that an author is a person (man, woman, ...) who writes or has written. The same for ‘player’: to count how many players are in a sports ground you count the persons who are playing there⁴. That means that the meaning of count nouns derived from adjectives, verbs and other parts of the discourse can be reduced to sortals related to the adjective, verb or anything else from which the noun derives.

Requirement (ii) rules out mass nouns, since those cannot take the plural form. However, (ii) is related to the issue of countability too. You can ask the following question:

How many books have you read?

By contrast, compare the behavior of mass nouns: Grammatically, a question like

* How many water are there?

is not well-formed, because ‘water’ cannot take a plural form nor can it be preceded by ‘many’ (rather, by ‘much’). Someone could object that you can count portions of water. But even in this case, there is no unique criterion to count portions of substances like water: to answer the question

How much water is in the bottle?

you can give different answers: “a litre”, “100 centilitres”, “1000 millilitres” (See Soavi et al. [106]). By contrast, the question

How many bottles are on the table?

⁴Someone could object to this treatment of ‘player’ taking into consideration the case of games played between humans and robots. In that case, counting how many players are in the sports ground does not coincide with counting how many persons are playing there, since robots are also among the players. In this case, one could say that ‘player’ relies on two sortals: ‘human being’ and ‘robot’.

has a unique answer, that is the number of particulars individuated as wholes falling under the concept **bottle**. According to Geach, countability is also what makes the conceptual difference between count and mass nouns (Geach [33], pp. 39-40):

we can speak of the same gold as being first a statue and then a great number of coins, but “How many golds?” does not make sense; thus “gold” is a substantival term, though we cannot use it for counting.

From requirement (ii) you can infer that count nouns can appear with determiners, like ‘some’, ‘all’ and ‘many’. This feature marks one of the differences between the category of common nouns and the category of proper nouns.

Excluding that mass terms are sortals because they are not countable is, however, a thorny issue. As mentioned above, I do not wish to discuss such a problem since my attention is focused only on count nouns. In general, sortals are often characterized as terms that allow the counting of the items with which they are associated. For instance, you can count how many persons are in a room, how many trees are in your garden, how many apples you eat in a week, etc. Nevertheless, as will be shown below, the fact that a term makes it possible to count the objects to which it applies is only a sufficient condition for it to be categorized as a sortal.

Requirement (i) and (ii) clarify which kind of terms has to be considered as correspondent to the monadic predicates in the model given in Chapter 3. Following (i) and (ii) we have count nouns such as ‘man’, ‘apple’ and ‘child’. The monadic predicates representing count nouns individuate sets of objects called ‘comparison classes’; for instance, we have the comparison class of men, of apples, of children. Comparison classes are subsets of a given domain, and can be divided further into subsets that I have called ‘contexts’. For instance, the set of authors of book Xyz can be considered a context: roughly speaking, you take the comparison class of persons, within it you select the individuals that wrote the book Xyz , and you get the context (set) to which the authors of book Xyz belong. The same for the (complex) count noun ‘basketball players’. You take the comparison class of persons and within it you select the individuals who are basketball players.

To be noticed: I considered ‘child’ as a sortal and not just as a term selecting a context. But both the set of children and the set of basketball players are subsets of the set of persons. Why is the former to be considered as a comparison class and the latter as a context? First of all, by the definition of context as subset of some comparison class, both of them are contexts. There

are, though, comparison classes that are subsets of other (more fundamental) comparison classes - as well as some sortal concepts, as we will see, are related to other sortals by the means of subordination. The comparison classes of children and adults are mutually exclusive, while they are part of the larger comparison class of persons. But there can be basketball players both within the comparison class of children and the comparison class of adults. Contexts of basketball players can be encountered in principle within (almost all) the comparison classes that are subsets of the general comparison class of persons, while comparison classes which are subsets of the comparison class of persons are mutually exclusive (compare, for instance, the comparison classes of children and adults)⁵.

4.2.1 Sortal logics

In logic, two classes of linguistic expressions, count nouns like ‘man’ and ‘tree’ and adjectives like ‘red’ and ‘fat’ are both represented by predicates. The linguistic distinction between count nouns and adjectives in natural language is not reflected in standard first-order logic.

The need of reflecting the difference between sortal and adjectival predication in formal systems has led to the development of the so-called sortal logics, whose aim is to treat sortal predicates differently from standard monadic predicates. A sortal logic is conceived as a formal system that emphasizes the difference between sortal and non-sortal predication, but this goal is far from easily achieved. Only few sortal logics have been presented so far: one of the most reliable (and existing) reviews of sortal logics is Pelletier [89]. It seems hard to provide a formal way to distinguish sortal from standard monadic predicates. I briefly consider here some issues about sortal logics because one could think that the model for gradable adjectives could be integrated into a sortal logic system. This means that sortal logic would be used to give a formal treatment to count nouns. However, it seems that the game would not be worth the candle: According to Pelletier, sortal logics are only apparently alternative to standard first-order logics (see especially Pelletier [89], pp. 126-127.).

The idea of providing sortal theories with sortal logics goes back to Strawson [109] and Geach [33]; some attempts to give formal systems of sortal logics have been pursued first by Smiley [104] and Wallace [122], followed then by Stevenson [108], Tennant [111], Gupta [38], Lowe [64] (those attempts have been followed by Freund [31] in recent years). Furthermore, according to Pelletier an analogy can be traced between the treatment of sortal predication

⁵The concept child will be discussed also in section 4.3.5.

in philosophy of logic and the treatment of generic predication in formal semantics. All those proponents of sortal logics tend to refute that quantifiers range over the elements of the domain. Such elements are instead treated as if they came pre-packaged as individuals of some sort. Formal systems of sortal logics are very closely related to (and, actually, translatable into) systems of restricted quantification theory. Consider the following classical formulae:

- $\forall x(Fx \rightarrow Gx)$;
- $\exists x(Fx \wedge Gx)$.

In a restricted quantification theory they are abbreviated by the following formulae:

- $(\forall x : Fx)(Gx)$;
- $(\exists x : Fx)(Gx)$.

In the latter formulae there are syntactic units that are called ‘quantifier phrases’: $(\forall x : Fx)$ (to read: every F) and $(\exists x : Fx)$ (to read: some F). But the formulae in unrestricted quantification theory have the same truth conditions as the unrestricted formulae. Moreover, in the two systems the same formulae are theorems and the same inferences valid, after the formulae in one system have been translated into the other system. So, systems of sortal logics that reside in restricted quantification theory differ from systems of classical first-order logic only in notation. It seems then that sortal logics fail to adequately represent the theory according to which individuals come as individuals of a certain sort: Such a difference cannot be just notational.

Pelletier’s remarks on sortal logics prevent me from integrating the model for gradable adjectives proposed in Chapter 3 within a system of sortal logic. I do not consider relevant for the purposes of the model to represent the elements of the domain that come pre-packaged as individuals of some sort. I rather consider relevant to underline the difference between sortal and adjectival predication by representing the former by the means of one-place predicates and the latter by the means of functions.

4.3 Philosophical considerations about sortal concepts

From a philosophical perspective, what is interesting in a research on count nouns is that their senses stand for concepts (in a Fregean view). An analysis

of sortal concepts is then the objective of many contributions in philosophical literature. First of all, let us try to understand what a sortal concept is.

There are different positions about what must be considered a (genuine) sortal concept and it is incredibly difficult to establish a theory for sortals, that is, a theory that defines, univocally characterizes sortals and provides an account for them⁶. However, it seems that philosophers agree on the idea that sortal concepts individuate sorts or kinds of entities, such that each of them answers the question “What is x ?”, for x an entity of the world.

Such a general function is not enough to characterize sortals as such. There are three further functions that are attributed to sortals and, in what follows, I wish to explain them and show some objections that can be raised against them (I refer here to Soavi et al. [106]). Those functions are the following:

Individuation A sortal is associated with criteria of individuation, that is, criteria that make it possible to single out individuals in a region of space.

Countability A sortal allows us to count how many entities fall under it.

Identification A sortal is associated with criteria of identity that allow us to re-identify the same entity across time.

Consider, first, the individuation function. To individuate an object means to isolate it from the region of space where it is and from the other objects that happen to be next to it. Performing the function of individuating objects, sortals are able to answer the question “What is it?”, referring to some object. The individuation process concerns the synchronic identity of objects, and not their diachronic identity. In other words, individuation does not take into account the qualitative transformations an object undergoes over time: for instance, a man can lose weight in a certain period of time, thus he has different properties at time t_0 , before losing weight, and at time t_n , after having lost some weight. The fact that we can say that at time t_0 he is the same man as at time t_n is due to the identification function of sortals.

Consider, now, the countability role of sortals. An aspect that seems to characterize sortal predicates is that we can always count how many elements are in their extensions. Each sortal comes with a criterion that allows us to count the entities falling under it. In other words, the following question

⁶Feldman [28] presents different accounts of sortalhood that have been given in the philosophical market and shows that they suggest non-equivalent criteria of sortalhood: they individuate different sets of sortal predicates (and concepts).

How many objects which are ϕ are there?

makes sense, with ϕ standing for a sortal predicate. To be able to give an answer to such a question we need to recognize which ϕ -objects are distinct and which are identical among them. To make the point clearer, consider the following questions (see Wright [129]):

- (i) How many chairs are there in this room?
- (ii) How many yellow things are on my desk?
- (iii) How many yellow books are on my desk?

To answer (i), we count how many things that are chairs are in the room. We need to know what makes something a chair and in which respect it differs from other objects. For this reason, as we will see, sortal terms come with identity criteria, i.e. criteria that make it possible for us to re-identify things we have already encountered. In the example of chairs, the identity criterion for chairs is supposed to allow us to determine, in a counting process, whether a chair that we have just counted is the same as the one that is in front of us now.

The answer to (ii) is not trivial. We can count the lemons and the yellow books that are on the desk, but also the yellow book covers and the lemon skins, and even more things, like parts of the lemon skins, etc. So, what should we count as a ‘yellow object’? There is no specific *kind* of object that exemplifies the predicate ‘yellow’. Moreover, there is no identity criteria that make it possible to identify and re-identify yellow things.

But if we relativize a question containing a non-sortal term (‘yellow’) to a question about sortals (‘book’) to which a non-sortal predicate is attributed (‘yellow object’), like in (iii), we can get rid of (most of the) ambiguity we had with questions like (ii). In this case, we know what to count, that is, we count the objects belonging to the sort ‘book’ and, among them, we select those which are yellow. Now, we are able to count how many yellow books are on my desk.

Countability has been thought of as a sufficient, but not necessary condition for a concept (term) to be a sortal. As we have already mentioned, classical examples of concepts that cannot be counted are mass nouns. Nevertheless, mass nouns seem to be able to answer the Aristotelian question: “What is it?”. If you accept them as sortals, you refute countability as a necessary condition for sortals.

Some people doubt that countability is even a sufficient condition for sortals. For instance, an expression like ‘man with a walking stick’ refers to a concept such that we can count individuals falling under it, but we exclude it from being a sortal. **Man with a walking stick** is a complex concept that relies on a more fundamental concept, that of **man**. However, the difference between simple and complex concepts does not rely purely on grammatical features. From a semantic point of view, a sortal term is simple when it is not semantically analyzable. In Lowe’s words (Lowe [64], pp. 30-31.),

its meaning is not a function of the meanings of certain other expressions, such as the meanings of its syntactic components (if it has any), or the meanings of the syntactic components of some syntactically complex sortal term which is synonymous with it. [...] a semantically simple sortal term is one which, superficial syntax notwithstanding, has no other semantic function than simply to designate a *distinct sort* of things or stuff..

Lowe’s example clarifies his claim. Consider the terms ‘ice’ and ‘frozen water’. They are synonymous, and even if the former is syntactically simple and the second complex, they both refer to the same kind of things: water. Their chemical composition is indeed the same as water (H_2O). Moreover, the meaning of ‘frozen water’ is given by the composition of the meanings of ‘water’ and ‘frozen’. By contrast, consider ‘heavy water’. The meaning of ‘heavy water’ is not given by the composition of the meanings of ‘water’ and ‘heavy’. ‘Heavy water’ refers to a specific substance (D_2O) which is chemically distinct from water and, thus, can be considered a different kind of substance in a metaphysically strong sense.

There is also another problem connected to the countability function. It is assumed to be always determinable how many objects fall under a sortal concept. But vague sortal concepts represent a counter-example: there is not always a definite way to count the objects falling under a sortal. Recall the mountain example. We are in a mountainous region, where there are mountains, hills and some landforms that are smaller than mountains but higher than hills. As we have seen, we have difficulty to categorize those landforms: are they to be considered as mountains or as hills? Consider the question:

How many mountains are there?

What kind of answer can be given, if we are uncertain about the categorization of some objects in the region? Nevertheless, instead of refusing the

countability function as one of the conditions for a concept to be a sortal, the vagueness problem related to countability can be faced together with the problem of vague sortal terms. In section 4.3.3 I will propose a way to treat the phenomenon of vagueness for sortal terms.

Consider now the identification function. Sortal concepts help to grasp the identity of the objects that fall under each of them. The understanding of any sortal concept requires the ability to recognize the differences between the objects falling under it and the objects not falling under it. Moreover, a sortal ϕ requires understanding what it means for an object a that exemplifies ϕ to be the same as, or distinct from, any object b that exemplifies the same ϕ . This does not happen with qualitative concepts, like **yellow**. Consider the sortal concept of **person**: to understand what it means for an individual to exemplify the concept **person** requires knowing whether that individual is the same as, or distinct from, another individual which also exemplifies **person**, that is, requires knowing what it means to meet the same person again. Being able to re-identify entities is essential also to counting entities: if you are counting how many persons are in a ground you need to know if the person you have in front of you is the same as the person you have encountered (and counted) five minutes ago. By contrast, to understand what it means for an object to be yellow does not require knowing if a yellow object is the same as another yellow object. Moreover, grasping the sortal concept of **person** requires to know whether an individual exemplifying **person** is distinct from the individuals belonging to other sorts, for instance, trees or cats. A person cannot be at the same time also a tree or a cat (at least in an essentialist view). By contrast, qualitative concepts like **yellow**, **flat** and the like are not exclusive: yellow objects can be cube-shaped or flat.

The identification function has to do both with synchronic and diachronic identity - while, as we have seen, individuation has only to do with synchronic identity. The possibility of re-identifying an object after a period of time depends on the knowledge of some essential features of the object. A principle of individuation is supposed to tell us what determines the identity of an object, that is, what determines which object it is. A criterion of identity is supposed to tell us what determines whether an object belonging to a sort is identical with another object belonging to the same sort. In the latter case, identity is conceived as a relation, whereas in the former case identity is conceived as individual essence, following the tradition of classical metaphysics (see Lowe [67], pp. 521-522).

A term ϕ expresses a sortal concept if and only if “is the same ϕ as” generates statements of genuine identity. To know what the world should look like in order for “ a is the same person as b ” to be true means to understand what being a person means. By contrast, to know how the world should

look like in order for “ a is the same yellow thing as b ” to be true you need first to know what sort of things a and b are and this is not part of your understanding of **yellow**. The relation “is the same ϕ as” generates statements of genuine identity iff “is the same ϕ as” is a congruence relation for any property of the objects that are compared. That is, the following hold:

1. From “ a has the property P ” and “ a is the same ϕ as b ” follows “ b has the property P ”.
2. From “ b has the property P ”, “ a is the same ϕ as b ” follows “ a has the property P ”.

However, there are some terms that are too general to be considered sortals. For instance, common nouns such as ‘thing’ and ‘object’ apply to all the elements of the domain. They do not provide any partition of the domain because the extension of each predicate coincides with the whole domain. Thus, they do not individuate any sort⁷.

Consider now the following tentative characterization of sortal concepts proposed by Wright (Wright [129], p. 2.):

Let us say that a concept is a sortal if to instantiate it is to exemplify a certain general kind of objects - not necessarily a natural kind - which the world contains.

The “general kind of object” mentioned by Wright in the passage above is mainly conceived as what Aristotle in the *Categories* called ‘secondary substance’. Just for completeness, let us briefly and roughly resume Aristotle’s doctrine of substance in his *Categories*. Substance is the first and most fundamental category; anything which is a substance is either an individual (e.g. a **particular**) or a sort (e.g. **universal**). Individual substances cannot be predicated of anything else and therefore are called **primary substances**. In the second case, sorts/kinds are predicable of individuals and are called **secondary substances**. Secondary substances differ from attributes like ‘yellow’, ‘fat’, ‘narrow’ as far as secondary substances characterize individuals as wholes. According to Aristotle, ‘yellow’, ‘fat’, ‘narrow’ and the like pick out features that could be said to be *in* individuals, but that do not essentially characterize them (Robinson [94]). There is an interdependence relation between primary and secondary substances: individuals can be only individuated/recognized as individuals of some sort, while sorts can be only conceived as sorts of individuals (Lowe [64], chap. 2).

⁷See also the considerations about the identity condition in Lombard [62], pp. 26-27.

According to the Aristotelian view, individuals or concrete particulars are themselves the constituents of the world: they make up the world and constitute the exemplification of universals. In other words, universals are kinds (secondary substances) under which individuals (primary substances) fall. Aristotle would agree with the idea that the being of a concrete individual is grounded in the attributes associated with it, but those attributes are not accidental properties, rather, universal properties that individuals exemplify. Universals or kinds mark what individuals are, they characterize individuals in their essence. In Loux's words (Loux [63], p. 119):

A concrete particular is such that were it not to exemplify its proper kind, it would not exist. The kind to which a concrete particular belongs, then, provides us with existence conditions for that particular.

4.3.1 On the debate about ontological realism

The view that sortals individuate sorts/kinds of individuals is usually associated with a realist position concerning the ontology of kinds and individuals in the world. I wish to say a few words on the positions held by supporters of ontological realism. In the debate on ontological realism it is possible to distinguish between strong and weak realism. Supporters of weak ontological realism argue that

(WOR) There is a world existing independently of all our mental states.

A strong ontological realism claims something more about the world; in particular it is combined with an epistemological thesis according to which we can know something about the nature of the world existing independently of all our mental states, beside its mere existence. In turn, it is possible to distinguish two main streams among the supporters of the strong ontological realism. Supporters belonging to the first stream commit to the following thesis:

(SOR1) The real world is unstructured, it is a sort of indistinct blob of matter.

Supporters belonging to the second stream argue the following thesis:

(SOR2) The world is not an indistinct blob of matter, but structured: there are distinct objects, properties etc.

Supporters of (SOR1) do not set the question about what the real elements of the world are, nor which are mere projections of our thoughts among the things we individuate. Since there is nothing to be individuated - the world is unstructured - if we individuate something, that is a mere projection of our thought. On the contrary, supporters of (SOR2) have to face the problem of selecting the objects, properties, tropes, events, facts really existing, i.e. existing independently of all our mental states, among all the objects, properties, etc. that we individuate. In other words, the problem for strong ontological realists supporting (SOR2) is that of selecting those objects that have ontological respectability. But what does it mean for an entity to have ontological respectability? One standard (Quinean) solution in analytic philosophy is to claim that only entities with clearly determined identity criteria are ontologically respectable, i.e. acceptable. But what does it mean for an identity criterion to be clearly determined? Which conditions must an identity criterion meet in order to be a good identity criterion? The clarification of the notion of identity criterion will be the main issue addressed in Chapter 5.

I will now consider the contributions of some philosophers that analyzed the notion of sortals as universals and that seem to support (SOR2): Strawson, Lowe, Lombard and Wiggins.

4.3.2 Strawson

According to Strawson's theory (Strawson [109]), some universals apply to or collect particulars. Among them, some are classified as characterizing universals and others as sortal universals. Such a conceptual distinction seems to correspond to the distinction between adjectival and sortal predication. Sortal universals are said to supply principles "for distinguishing and counting individual particulars" that they collect. On the contrary, characterizing universals are said also to supply principles of grouping particulars, but "only for particulars already distinguished, or distinguishable, in accordance with some antecedent principle or method" (Strawson [109], p. 168).

Consider an example. Take a single individual, for instance the individual named Fido. Fido is associated with a number of different sortal universals: it is a dog, an animal, a terrier. The sortals **dog**, **animal**, **terrier** are related one each other. **Animal** individuates the most general kind of objects. A sub-kind of **animal** is individuated by **dog**, that is, the set of dogs is a subset of the set of animals. In turn, **terrier** individuates a sub-kind of **dog**, i.e. the set of terriers is a subset of the set of dogs (and by logical means also of the set of animals). As a generalization, Strawson claims (p. 169):

the universals to which one and the same particular is sortally tied will have a characteristic relation to each other, which is sometimes described as that of a sub- or super-ordination.

In the case of characterizing universals, an individual can be tied to many characterizing universals. Take, for instance, the individual named Socrates. You can say that Socrates is wise, is short, is warm, talks, dies. The same individual collects different characterizing universals, sometimes at different times, and those characterizing universals may be not related to each other. This means that among the characterizing universals associated with Socrates there are not necessarily relations of sub- or super-ordination as in the case of sortal universals associated with him. Moreover, while - at least at first glance - sortal universals are essentially attributed to a particular at all times of its existence (e.g. Fido cannot cease to be a dog without ceasing to exist), characterizing universals can be differently associated with a particular at different times. For instance, Socrates can be said to be warm at a certain time and cold at a different time. That Socrates ceases to be cold does not mean that he ceases to be wise. The fact that different characterizing universals can be attributed at different times to the same individual is made possible by the continuing identity of the individual itself⁸.

4.3.3 Lowe

Lowe [64] presents an account for distinguishing individuals from kinds. According to him, there is an interdependence between the two notions of individual and sort. Let us start taking into account Lowe's definitions of individual and sort. In the definitions the symbol / signifies instantiation, so the expression X/Y is to be read: X instantiates (is an instance of/exemplifies) Y . X, Y are used here as meta-variables that vary on subjects and predicates (Lowe [64], pp. 38-39.).

Definition 16 *X is an individual if and only if X is an instance of something Y (other than itself) and X has no instances (other than itself). Formally:*

X is an individual iff_{def} $(\exists Y)((X/Y) \wedge (Y \neq X)) \wedge \neg(\exists Y)((Y/X) \wedge (Y \neq X))$.

⁸Leaving aside Strawson's theory, I would like to make a consideration that will be useful for the issues discussed in the next chapter. Given an individual, the continuing identity that makes it possible to attribute different characterizing universals to it at different times is related to the identity criterion associated with the sortal universal(s) to which the individual is associated.

Definition 17 *X is a sort if and only if there is something Y such that Y is an instance of X and Y is distinct from X. Formally:*

$$X \text{ is a sort iff}_{def} (\exists Y)((Y/X) \wedge (Y \neq X)).$$

According to those definitions, the individual named ‘Fido’ is distinct from the sort of dogs to which it belongs because Fido does not have any instances (except, maybe, itself), while there are different instances of the sort of dogs, that is, there are many different dogs.

A natural objection to Lowe’s definitions is of a modal nature. There is nothing that prevents us imagining encountering an individual which does not exemplify any sort: think of the case of a natural entity which does not belong to any kinds individuated so far by natural sciences, or the case of a sort that currently does not have any instantiation. For instance, consider the sort of dinosaurs. We know that they existed and that there are none anymore. So, by the definition for sorts, dinosaurs cannot be considered a kind of entity. However, we would like to consider them a sort, at least because they are not physically impossible entities. Lowe’s definitions, therefore, are not completely satisfactory. Lowe takes into account the possibility of employing modal expressions in his definitions, but he refutes it because it could bring about even more difficulties. For instance, if we revised the definition of individuals saying that *X* is an individual if and only if *X* cannot have instances (other than itself), we would obtain some counterintuitive results. A concept like **round square** cannot have instances in the sense that there cannot be any object falling under it. Therefore, according to the new definition, it is an individual. But we tend to exclude impossible objects to be genuine entities (see Lombard’s conditions in the following subsection). So, Lowe’s definitions are to be taken only as an attempt to express some intuitions on the distinction between individuals and sorts, and not as a complete or definitive characterization of them.

4.3.4 Lombard

Ontologists want to individuate basic, ultimate, or fundamental kinds of objects to which the other kinds of objects can be reduced. If basic kinds of objects can be individuated, then we must have a criterion to decide which sets of objects count as fundamental kinds. Lombard [62] takes into account this issue and I will summarize his position.

First of all, it must be noted that Lombard considers sorts or kinds as determined by sortal *properties*. Even if the notion of property recalls an intensional framework, speaking of sortal properties is not incoherent with

our extensional treatment of sortals. We can think of the kind/sort of persons as the set of individuals that have the property of ‘being persons’.

When we say which kind an individual belongs to we are saying something essential to the individual itself inasmuch as we say what it is. In this view, we would not accept, for instance, the class of bachelors as a (metaphysically relevant) kind of thing, because *being a bachelor* is an accidental property. Someone who has that property at time t_0 could lose it at time t_n without ceasing to exist. In contrast, the individuals to which such a property is attributed belong in their essence to the kind of men. So, according to Lombard, a sortal term ϕ denotes an essential kind if and only if:

1. it is possible that there are entities that are ϕ ;
2. it is necessarily true that, if some entity is ϕ , then it is necessary that, if that entity exists, it is ϕ ;
3. there is a criterion of identity for things which are ϕ ;
4. there is no sortal ψ such that (i) necessarily, anything that is also ϕ is ψ (but not vice versa), and (ii) there is a criterion of identity for the things that are ψ .

Consider how Lombard motivates the assumption of conditions 1-4.

Condition 1 is assumed to exclude impossible properties from sortal properties. Examples of impossible properties are ‘being a round square’, ‘being a furious green idea’, and the like. If there were sets of objects having those properties, they would be empty, because no object can satisfy any of the impossible properties. Moreover, impossible properties would then determine the same set (the empty set), so they do not seem to be proper *cookie-cutters*.

Condition 2 underlines the fact that an entity which exemplifies a sortal concept ϕ is ϕ whenever it exists: if such an entity ceased to exist, then it would also cease to be ϕ . Sortal properties are essential to any object possessing them.

Condition 3 is required to establish whether a property denotes a class of objects which is a kind. Lombard’s intention in giving condition 3 is to rule out some properties as properties that determine essential kinds, e.g. the property of being an abstract object, the property of being either a physical object or a number, and the like. Entities that possess properties like those just mentioned do not share an essence, because there is no criterion of identity for them. In Chapter 5 I will resume some conditions for identity criteria given by Lombard himself; they will also clarify condition 3.

Condition 4 is used to rule out metaphysically not interesting kinds, especially the kinds which are too small and reducible to broader kinds. For

instance, prime numbers are numbers that have a certain feature, so the kind of prime numbers rely on the basic kind of numbers (from a set-theoretical point of view, we would say that prime numbers are a subset of numbers). So, the kinds that are more interesting from the point of view of ontologists are the broadest (as required by condition 4) kinds whose members share an essence (as required by conditions 1 and 2) and a criterion of identity (as required by condition 3).

4.3.5 Wiggins

Wiggins [123] presents an account for sortal concepts in relation with a theory of individuation. Every object that exists and to which some predicates are applicable, is (or, ideally, could be) individuated by a sortal concept. Individuation is closely related to identification: if x is a K , and K answers to the question ‘What is x ?’, then K is associated to an identity criterion by means of which K -entities may be traced and re-identified across time. Wiggins tries to express the relation between individuation, identification and sortal concepts in the so-called Thesis of the Sortal Dependency of Individuation. According to it, given two entities a and b , a is identical to b if and only if there exist a sortal concept K such that (see Wiggins [123], chapter 2):

1. a and b fall under K ;
2. to say that x falls under K is to say what x is;
3. a is the same K as b iff the way in which x is K -related to Y is sufficient for whatever is true of x to be true of y and whatever is true of y to be true of x (logical requirements: congruence and equivalence);
4. K is a substance-concept only if it determines either a principle of activity (characteristic of living beings), a principle of functioning (characteristic of organs) or a principle of operation for members of its extension (characteristic of abstract entities).

What is the role of K ? According to Wiggins, K is a substance-concept that marks what things falling under K are, gives persistence conditions for them, and offers conditions for evaluating identity claims about them.

Two senses of sortal concepts can be distinguished: a strong one and a weak one. A sortal concept K in a strong sense is associated with an identity criterion which gives us not only a way to know whether two instances

of *K*-objects are the same object, but also the conditions under which *K*-objects are identical in the real world. On the contrary, a sortal concept in a weak sense is associated with an identity criterion having only an epistemic function: it tells us how we identify objects, but does not tell us what is for those objects to be identical.

Among the kinds of objects that are individuated by sortal concepts in a strong sense Wiggins detects words of natural kinds, like ‘horse’, ‘tree’, and the like. The instances of concepts of natural kinds are classified by virtue of their scientific resemblance, their specific constitution and mode of interaction with the environment. With Wiggins’ words, there are lawlike principles in nature that

determine directly or indirectly the characteristic development, the typical history, the limits of any possible development or history, and the characteristic mode of activity of anything that instantiates the kind. (Wiggins [123], p. 84).

If there is dispute about whether or not a certain entity is a member of a natural kind, one has to appeal to science and tries to find scientific facts that prove or disprove its belonging to a natural kind.

What about words of artefact kinds such as ‘clock’, ‘chair’ and the like? According to Wiggins, the concepts corresponding to such words are weak sortals: only identity criteria with an epistemic function are associated to them. First of all, ordinary artifacts are individuated by virtue of their function and not of a principle of natural activity. There is nothing nomological in the individuation of entities of artefact kinds. So, they fail to meet a metaphysical requirement: concepts of artefact kinds are undetermined inasmuch as “questions of artifact identity are matters of arbitrary decision” (Wiggins [123], p. 91). Moreover, concepts of artefact kinds fail to meet the logical requirements of equivalence and congruency because identity criteria associated with artefact kinds either fail to be transitive or lead to contradictions. So, according to Wiggins, those concepts are not sortal concepts in a strong sense: they fail to meet metaphysical and logical requirements. Nevertheless, we are able to individuate and identify objects like clocks and chairs for daily-life purposes: we can say that artefact kinds are associated only with weak sortal concepts.

I included in the previous sections the concept **child** among sortals. According to Wiggins’s characterization, though, a sortal concept constitutes the essence of the entities falling into it, that is, entities would cease to exist if they cease to fall under the sortal concept they pertain to. If so, then **child** is not a sortal concept because it does not apply to entities at every moment

of their existence. Children become adults after some years, and so they fall under a different concept, without ceasing to exist.

Wiggins call *phased-sortals* the concepts which apply only to a phase of the existence of entities, like *child* and *caterpillar*. He does not exclude them to be sortals: they answer the question “What is x ?”. They have only some specific features. A phased-sortal denotes part of the life history of an entity, which, in its essence, is denoted by another sortal. So, for instance, ‘child’ is a phased-sortal which applies to a phase of life of the entities fallen under the sortal *human being*. And the same for *caterpillar*: it denotes a kind of entities that are involved in a process of transformation. So, *caterpillar* does not apply to an entity for the whole of its existence as living being. When philosophers think of fundamental kinds of entities, they think of some necessary conditions associated to them such that, whenever they apply to something, they apply “in a present-tensed manner to the thing through the whole of its existence” (Robinson [94]).

Phased-sortals are good candidates for being vague terms. The count noun ‘child’ constitutes a common example in the literature about vagueness: there are some cases where its use is problematic. Consider the three features that characterize vagueness: (i) fuzzy boundaries, (ii) borderline cases and (iii) Sorites paradox.

(i) There is no clear boundary that determines when a child becomes an adolescent. It does not seem reasonable to establish a precise time or a (physical, psychological, ...) fact that makes possible to state whether a person is (still) a child or is (already) an adolescent.

(ii) There can be borderline cases of *child*: some persons (probably all, at a certain time of their existence) present some physical and psychological features that are typical of the childhood and other physical and psychological features typical of the teen years.

(iii) It is not difficult to build a Sorites series that shows the application of ‘child’ to be paradoxical. Consider a series of individuals whose age is between seven and seventeen years old. Those individuals are ordered from the youngest to the oldest in such a way that each individual is older than the previous one in the series for only one day. For the sake of simplicity, consider only the time issue: the difference between a child and an adolescent concerns the amount of time they have already lived. The example can be revised to get a more complicated (but more realistic) picture considering further aspects concerning the physical and psychological aspects of the individuals considered. However, consider the series of the individuals ordered by their age as depicted above. The first individual is exactly seven years old and you will certainly say that she is a child. Since the second individual in the series is only one day older than him, you will naturally think that one

day does not make a substantial difference between the first and the second individual. Thus, the second individual is also said to be a child. You continue applying such reasoning to each individual and its successor in the series and in the end you get that the eldest individual, who is seventeen, is still a child (counterintuitive conclusion). So, the term ‘child’ gives rise to a Sorites paradox.

In the model for gradable adjectives I accepted monadic predicates to represent count nouns like ‘child’. For the purposes of the model the distinction between count nouns associated to fundamental sortals and count nouns associated to phased-sortals is not really relevant. The choice can be made according to the ontology you prefer. You can consider, for instance, the set given by ‘child’ as a genuine comparison class or as a context, that is, as a subset of a more fundamental sortal, like ‘human being’. In the latter case, you might want to assume that only count nouns associated with fundamental kinds are the interpretations of monadic predicates. The ontology behind this choice is that only fundamental kinds are genuine *cookie-cutters* of the world. On this view, ‘child’ denotes the subclass of the set denoted by ‘human being’ and is treated as a context in the model. However, that you consider the set of children as a comparison class or as a context, no substantial change in the model of gradable adjectives is produced.

4.4 Sortal vagueness vs. ontic vagueness

In this section I wish to analyze the relation between the phenomenon of vagueness of sortal concepts and the so-called phenomenon of (alleged) ontic vagueness, and to propose a formal treatment of vague count nouns to be integrated in the model for vague adjectives in Chapter 3.

As mentioned in Chapter 2, some philosophers claim that the source of vagueness is not merely linguistic or cognitive, but ontological or, in other terms, that vagueness is a phenomenon in the world, not only in natural languages. If this were the case, then you could claim that there are, for instance, vague objects or vague properties (it depends on the ontology you assume).

In this section I wish (i) to clarify the notion of vague objects as used in the philosophical debate, (ii) to defend the thesis that the problem of sortal vagueness is distinct from the problem of ontic vagueness, (iii) to consider the problem of vague count nouns and suggest a formal treatment for it.

The confusion between the two sorts of vagueness (sortal and ontic) is probably due to the fact that some sortal terms are vague in two ways: as much as it is not clear whether some amounts of matter are part of each

instance and as much as the boundaries of their extensions are not sharp. To clarify such a claim, consider the following example concerning the sortal term ‘mountain’ given by Quine [92] (p. 126):

... take the general term ‘mountain’: it is vague on the score of how much terrain to reckon into each of the indisputable mountains, and it is vague on the score of what lesser eminences to count as mountains at all.

The first sense for which ‘mountain’ is vague concerns the boundaries of each instance of **mountain**, i.e., concerns each individual fallen into the set of mountains. The problem of vagueness here does not regard the extension of the corresponding concept **mountain**, but each individual falling into it, taken by itself. The difficulty relies on the determination of what is and what is not part of a certain mountain or, in other words, it relies on the individuation of an object, a mountain, as a whole. In formal ontology you can encounter the expression “principle of unity” to refer to the individuation function of sortals. Unity is distinguished from identity in as much as the former is related to the problem of distinguishing the parts of an entity from the space region where the entity is, and to decide exactly which parts constitute the entity, while the latter is related to the problem of whether two items (considered at different times) are the same object. With Guarino and Welty’s words (Guarino and Welty [37]):

asking “Is that my dog?” would be a problem of identity, whereas asking “Is the collar part of my dog?” would be a problem of unity.

Guarino and Welty claim that each sortal carries a principle of unity when what it exemplifies is a whole. The principle of unity is what allows us to determine what is part and what is not part of an individual which is conceived as a whole.

However, the vagueness of the sortal term ‘mountain’ is properly given by the second sense individuated by Quine. We are not sure how many mountains to count: there are some entities which are higher than hills but smaller than mountains and we do not have any evidence to count them as mountains or hills. The problem with this kind of objects regards the extension of **mountain**, as seen in the introduction of the present chapter.

The first sense in which ‘mountain’ is vague is related to the alleged problem of ontic vagueness, i.e. of vague objects. The second sense is to be considered as the problem of vague sortal terms (and concepts). To understand the difference between the two sources of vagueness, let us make a short journey through the debate on vague objects.

4.4.1 Vague objects

Philosophers refer to the notion of vague objects either to support the thesis that there are vague objects in the world or to refute the same thesis and support, instead, the thesis that there are no vague objects and that rather proper names (or definite descriptions) are vague.

How has the notion of (alleged) vague objects been characterized in the philosophical debate? Tye [113] proposes the following definition:

Definition 18 *An object o is vague iff (i) o has borderline spatio-temporal parts and (ii) there is no determinate fact of the matter about whether there are objects that are neither parts, borderline parts, nor non-parts of o .*

Given Tye's definition, the most common cases of (alleged) vague objects are the following:

- objects that have indefinite boundaries, like, for instance, clouds;
- singular objects corresponding to geographical singular terms, like *Everest* and *Sahara*.

There are some analogies and some differences among the objects of the two cases. In the first case we have that for some sort of objects, like, for instance, the sort of clouds, each object belonging to that sort has indeterminate boundaries: there are some portions of matter that do not clearly belong to the objects considered. What is at issue here is not the vagueness of the sortal term 'cloud'. Let me try to clarify. Consider a cloud c . It is not clear how many bits of water vapor c consists of, that is, there are some areas of water vapor that do not clearly belong to c . They can be considered borderline cases of the parts constituting c . You can also get a Sorites paradox along a series of areas of water vapor constituting a cloud. This seems to be possible for all the clouds, that is, for all the singular instantiations of the word 'cloud'. It seems that clouds somewhat *essentially* lack boundaries. This phenomenon is different from the phenomenon of vagueness of the concept cloud. As the borderline cases of the sort of clouds are objects that are not clearly clouds, and not bits of water vapor that do not clearly belong to some cloud. The problem is finding the exact extension of the sort, that is, determining which objects are definitely clouds and which are not. Consider a clear case of cloud: the unique cloud in the sky I can see from the window of my room now. Taking into account the item which the description refers to, you can observe that it has indefinite boundaries, there are borderline cases (areas of water vapor that are not clearly part of the cloud), and a Sorites paradox can be generated. In short, there are all the

features that mark vagueness. Now, what is to be considered vague? The definite description ‘the unique cloud in the sky I can see from the window of my room now’ or the object denoted by the description? Giving an answer to such a question, and therefore take a position about ontic vagueness, is out of the scope of the present research. I want to consider only the use of vague linguistic terms.

Compare now the case of clouds with the case of Everest and Sahara. Consider the singular term ‘Everest’. It refers to a singular mountain and it is not a borderline case of the concept **mountain**. From a linguistic point of view, ‘Everest’ is a proper name, while in the case of clouds you refer to a singular entity usually by pointing at a cloud in the sky or using a definite description. Such a difference is not a relevant difference, as in both cases we have to deal with linguistic expressions that refer to singular objects.

Unlike mountains, clouds seem to have an evanescent nature because of the material of which they are constituted (water vapor). Moreover, clouds can be differentiated by what is around them by means of the material of which clouds are constituted. Clouds are constituted mainly of water vapor, while the air around them is not. But what about Everest? It is constituted of soil, rock, and so on, the same materials of which the valleys or the other mountains around it are also constituted. The materials constituting Everest cannot help in demarcating it from its surroundings. You can determine some of its boundaries (the peak of Everest partially demarcates its boundaries), but when you are at the foot of the mountain, you do not have any material evidence to determine where its boundaries lie.

Just like the case of clouds, you can build a Sorites paradox for ‘Everest’. Suppose you are on the top of Mount Everest. You can clearly state that the ground on which you are is (part of) Mount Everest. Now, you start descending. After the first step, you would agree that you are still on Everest. And the same also after the second, third, n th step. When you arrive at the town Katmandu, you would be forced from the argument that you are still on Mount Everest, but you would reasonably say that you are not any longer on Mount Everest. If so, at which step did you exactly stop descending the mountain?

The phenomenon of vagueness related to Everest seems to show that the term ‘Everest’ does not refer to a precise volume of matter well demarcated from its surroundings (See Varzi [119]). Just like the case of clouds, this fact can be thought of as a failure of reference or conceptualization of the singular term ‘Everest’ or as evidence for the vagueness of the object named ‘Everest’. The two interpretations can be thought of as two readings of vagueness: *de dicto* and *de re*, as suggested by Varzi [119]. According to the reading of vagueness *de re*, the singular term ‘Everest’ is vague because its referent,

Mount Everest, is vague insofar as there is no fact of the matter about which chunks of matter are part of Everest and which are not. According to the reading of vagueness *de dicto*, by contrast, the singular term ‘Everest’ is vague because it vaguely designates an object, i.e. the referent of ‘Everest’ is not well fixed. When we decided to call a certain mountain ‘Everest’, we did not specify exactly which are the boundaries of the mountain.

Since I do not wish to go into the debate about the existence of vague objects, I will instead speak of vague singular terms or definite descriptions, leaving aside the ontological questions concerning the phenomena of vagueness described in this section. Nevertheless, vagueness has been defined in Chapter 2 as a problem which is primarily linguistic. So, I would concentrate on the problem of vagueness regarding linguistic expressions. Whether or not there are other kinds of vagueness, like ontological or cognitive, is out of the scope of the present research.

Vague objects and indeterminate identities

Not all philosophers agree with Tye’s definition of vague objects. According to some of them, a vague object is (also) an object whose identity is indeterminate. For some scholars, the alleged phenomenon of ontic vagueness is related to the issue of indeterminacy of identity. In the philosophical literature, such an issue arises from the so-called puzzles of identity. Among those, the most well-known is the puzzle of Theseus’ ship, introduced by Plutarch in his *Life of Theseus*. Consider the following formulation: Theseus possesses a wooden ship, *ship*₁. On day 1, he replaces one piece of it with a new piece, similar to the old one in all its respects. On day 2, Theseus replaces a plank with a new plank, and so on every day, until the ship gets completely rebuilt. Call the new ship *ship*₂. Is the resulting ship with new parts identical to the original one? In his *De Corpore* Hobbes adds a further issue: suppose the old parts of *ship*₁ are reassembled to create another ship, *ship*₃, which is exactly alike the original. Which one of *ship*₂ and *ship*₃ is (identical to) the original ship? We have two ships at the end of the process of dismantling and reassembling, so at the most only one of them can be considered identical to *ship*₁.

We can be lead by different criteria of identity to state either that *ship*₁ is identical to *ship*₂ or that *ship*₁ is identical to *ship*₃. Consider the following criterion: ship *x* is identical to ship *y* if and only if *x* and *y* are alike with respect to the spatio-temporal continuity. Following such a criterion, we claim that the process of parts replacement preserves identity. So, *ship*₁ = *ship*₂. Consider now another identity criterion: ship *x* is identical to ship *y* if and only if *x* and *y* are composed of the same parts. In this case, the process

of dismantling and reassembling preserves identity. So, $ship_1 = ship_3$. The two identity criteria are incompatible, because they give conflicting results: by the first criterion $ship_2$ is the same ship as the original, by the second criterion $ship_3$ is identical to the original ship.

For some philosophers identity puzzles show that ships and other ordinary objects are vague because their identities are indeterminate. But are cases of indeterminate identity to be considered as cases of vague objects? Philosophers who support the thesis that indeterminacy of identity implies ontic vagueness are, for instance, Evans [26] - even if he refutes the thesis that there are objects with indeterminate identities - and van Inwagen [116] - who is instead a supporter of ontic vagueness. By contrast, philosophers who claim that the problem of indeterminate identities must be distinguished from the problem of vague objects are, for instance, Parsons [84], Sainsbury [98], and Tye [113]. The second group of authors maintain the view that either there is genuine indeterminacy in the world or vague objects exist. I consider here the famous proof provided by Evans to demonstrate that there cannot be indeterminate identities, and therefore no vague object either, and the response of Tye, who shows that ontic vagueness is not to be thought of as related to indeterminate identities.

Consider first the well-known one-page article by G. Evans that appeared in 1978. I do not intend to discuss the cogency of his famous proof. Instead, I want to take into account the way he pretends to connect indefinite (with respect to their truth value) identity statements and worldly vagueness. Evans explains the theory of worldly vagueness as built on two ideas:

(i) the world might be vague. Accordingly, vagueness affects any true description of the world. This means that we use vague terms to describe states of affairs because they are themselves vague;

(ii) statements that are affected by vagueness may not have a definite truth value. (Some) identity statements are among them.

According to Evans, from (i) and (ii) follows

(iii) “the world might contain certain objects about which it is a *fact* that they have fuzzy boundaries” (Evans [26]).

After having expressed the thesis in these terms, Evans wants to argue against it, he raises the question whether (iii) is coherent and goes on with his famous proof. He shows that assuming the indefiniteness of $a = b$ you conclude that it is not the case that $a = a$. Contradiction. So, the assumption

is incoherent.

Now, it is not clear how and why the thesis, that there are objects with fuzzy boundaries, follows from (i) and (ii). It seems that (iii) is just a reinforcement of (i). What is the rule of (ii) in Evans's reasoning? (ii) says that vagueness makes linguistic statements containing vague terms indefinite with respect to their truth value. Among such statements, there are also identity statements. Now, what is the connection between vague or indefinite identity statements and ontic vagueness? Assume, as Evans does, that ontic vagueness refers to vagueness of objects in the world, and not of properties. Suppose that truth-value indefiniteness of statements is due to the vagueness of some singular terms contained in the statements. Some theorists can say that such a vagueness is due to the language, others, namely who accept (i), think that it is due to the world, i.e. to objects that have fuzzy boundaries (since one of the features of vagueness are fuzzy boundaries). Now, consider the latter thesis. It is what (iii) expresses. But it is not necessary to appeal to identity statements to claim something like (iii). Consider the former thesis, according to which vagueness of singular terms is linguistic. Again, when we have an indefinite identity statement, why should we think that the vagueness of singular terms contained in it is due to entities in the world? I cannot see any evidence of the connection between the indeterminate identity statements and vague objects. The burden of Evans' formal proof seems to address the problem of indeterminate identity and not the problem of worldly vagueness. That is also Tye's objection to Evans' proof.

Consider Tye's reply. Tye emphasizes that Evans' argument is a good proof for denying the thesis that a vague identity statement is not indefinite in truth-value, but is not a proof that rejects the thesis that there are vague objects. To understand Tye's argument, recall first Tye's definition of a vague object (**Definition 18**):

An object o is vague iff (i) o has borderline spatio-temporal parts and (ii) there is no determinate fact of the matter about whether there are objects that are neither parts, borderline parts, nor non-parts of o .

Tye refutes the view according to which vague objects are objects whose identity is indeterminate. His argument goes as follows⁹. That an object, either a or b (or both) is vague does not determine that an identity statement as $a = b$ has an indefinite truth-value. Consider a vague proper name: 'Everest'. Suppose that ' m ' is an amount of chunks of matter that lacks some chunks that are indefinite constituents of Everest. Now, the identity

⁹Tye [113], p. 556

statement ‘ $m = \text{Everest}$ ’ is vague because (at least) ‘Everest’ is vague. But such a statement is not indefinite with respect to its truth-value. It is indeed false. Consider a chunk of matter t that is an indefinite constituent of Everest but it is definitely not a constituent of m . Applying Leibniz’s Law and resembling Evans’ reasoning in his famous proof (see Evans [26]), we can say that m does not have the property of having t as an indefinite constituent, while Everest possesses such a property. Everest and m are different, hence ‘ $m = \text{Everest}$ ’ is false.

As Paganini [81] also shows, Tye’s argument does leave open the possibility of vague objects existing, while discarding the hypothesis that there is indeterminate identity. The two problems, therefore, are to be kept apart.

4.4.2 Vagueness of singular terms vs. sortal vagueness

It seems clear that the phenomena of vagueness concerning singular terms or definite descriptions are not the same as the phenomena of vagueness concerning sortal terms. For a singular term to be vague does not mean that it is a borderline case of a vague sortal term. Everest is clearly a mountain and the phenomenon of vagueness of the concept ‘mountain’ does not have anything to do with the vagueness of the term ‘Everest’. As it has been shown, borderline cases, fuzzy boundaries and Sorites paradox affect both ‘Everest’ and ‘mountain’, but in a different way. For instance, borderline cases for the application of ‘Everest’ are chunks of matter of which we doubt whether or not they are part of the mountain called ‘Everest’, while borderline cases for the application of ‘mountain’ are landforms of which we doubt whether or not they are mountains. The problems presented by the general term ‘mountain’ regard the extension of the concept *mountain*, that is, the application of the concept and its extensions: there are cases of landforms such that we do not know if they are to be classified as mountains or, for instance, as hills. The use of singular terms like ‘Everest’ sets questions about the boundaries of the singular object that is the referent of ‘Everest’.

Hence, I disagree with Varzi [119], when he claims:

If we wish, we can add that it is ultimately the vagueness of the relevant sortal concept (the concept *mountain*, in this case) that is responsible for the way in which the referent of ‘Everest’ is vaguely fixed.

One could claim, though, that the name of an object is vague not only when the object does not have demarcated boundaries, but also when it is doubtful which sortal predicate applies to the object (see, for instance, Romerales [95]). In this case, we would say that the name of a borderline

case of mountain is vague because the referent cannot be individuated in a precise manner: we cannot decide whether a certain landform is a mountain or a hill. What is the difference between conceiving the term *Everest* as vague (or Everest as a vague object) and the name of a borderline case of mountain as vague? A borderline case of mountain suffers for the same kind of vagueness as Everest: they both lack sharp boundaries. In this sense, the names referring to the two landforms are vague. Thus, we can say that the name of a borderline case of mountain is vague because the referent is vaguely fixed just like the case of Everest. But I refute the thesis that such a name is vague because its referent is a borderline case of a sortal term. Consider the following example: imagine seeing an object in front of you which is pretty similar to a chair but slightly larger than normal and with only one arm. We are not sure whether we can apply the concept **chair** to that object or, in other words, we are not able to individuate it by means of a sortal (or general) term. Can this case be considered a case of alleged ontic vagueness or vagueness of singular terms/definite descriptions? My answer is no. First of all, we can baptize that object and, thus, confer a proper name upon it, but that object does not apparently have fuzzy boundaries: we can single it out from the surroundings. So, in this case we cannot say that the referent of the name is vaguely fixed. What is problematic is that we are not able to say what kind of object it is. The problem is then conceptual, epistemic or linguistic. The object can be considered as a borderline case of **chair** but it does not present by itself any fuzzy boundaries or other features related to vagueness of a single item. The fact that a borderline case of **mountain** can be considered as a case of vague object (or its proper name/definite description as a vague linguistic expression) does not mean that all the borderline cases of sortal concepts are vague objects (or the names/descriptions with which we refer to them are vague).

As anticipated, the confusion between sortal vagueness and ontic vagueness relies on the fact that for some sortals, like ‘mountain’, the following two cases both occur: the boundaries of their extensions are not sharp and neither of their instances have sharp boundaries. But only the first case is sortal vagueness: what is vague is the sortal concept. The second case is related to the vagueness of the expressions by the means of which we refer to each instantiation of the sortal concept. But the two cases of vagueness must be kept distinct, in order to treat them in a proper way.

4.4.3 A formal treatment of vague sortal terms

The problem of vagueness of sortal terms is often generally treated as a problem of vagueness of predicates. This means that vague general terms

are treated at the same rate as vague adjectives and vague verbs, since these classes of linguistic expressions are formally represented by predicates. But as pointed out in the previous sections, there is a distinction between sortal and adjectival predication. Even if among both sortal terms and adjectives there are vague terms, a model that aims to formalize these two classes of linguistic expressions should take into account the difference between their linguistic features. In the model for gradable adjectives presented in Chapter 3 a choice had already been made: adjectives are represented by functions, while count nouns by one-place predicates.

The question I aim to answer in this section is: if you agree that some count nouns are vague and you want to extend the model for gradable adjectives including an account for vague count nouns, how can you treat the monadic predicates that stand for them? What do their extensions look like? The problem is to understand which individuals are to be considered as members of the sets given by (vague) monadic predicates.

Instead of developing a separate model for vague count nouns (sortal terms), I consider an account for vague terms (and concepts) that has been suggested by Schlobach et al. [101] and try to integrate it in the model for gradable adjectives. Such an account has been developed for terms which refer to vague concepts of diseases. I try to extend it to a general treatment for vague sortal terms. I present Schlobach et al.'s account in its details in the appendix to this chapter.

In the model for gradable adjectives I have taken count nouns to be represented by monadic predicates which individuate sets of elements in the domain: the set of children, the set of trees, and so on. As we have discussed, the problem is to individuate the boundaries of the sets denoted by vague sortal terms, because they seem to lack sharp boundaries. If so, then how can we represent them in set-theoretical terms? Such a question has been considered by Sainsbury [97]. According to him, non vague concepts (not only sortals) have sharp boundaries, while vague concepts do not have sharp boundaries and that means that they do not have any boundaries at all. He wants to maintain the view that even if vague concepts do not set boundaries, they can nevertheless classify and categorize things. He argues for the thesis that a set-theoretic description of vague concepts will never be adequate.

By contrast, the idea I am going to support here is to provide the sets that represent vague concepts with two sharp boundaries, as according to Schlobach et al.'s proposal: given a set corresponding to a vague concept, if any real boundary of it exists, it lies in-between the two boundaries that are provided. So, instead to refute providing a set-theoretical description of vague concepts, what I suggest is to give a set-theoretical **approximation** of vague concepts.

The tools for conceptual modelling used by Schlobach et al. are Description Logics (from now on, DL) and their extensions. DLs have the goal to represent concepts and they focus on sufficient and necessary conditions for set membership. They help to recognize instances of certain sets. The extension of DLs proposed by Schlobach et al. is Rough DL. That logic is able to distinguish a sufficient and a necessary condition for being members of a set. As Guarino and Welty [36] emphasize, membership conditions are not to be confused with identity conditions (or identity criteria):

This is a common confusion that is important to keep clear: membership conditions determine when an entity is an instance of a class, i.e. they can be used to answer the question, “Is that a dog?” but not, “Is that my dog?”

Identity criteria are used to determine whether or not two instances of a concept are the same object. Membership conditions, by contrast, are used to determine which objects of the domain fall under a concept.

How can we apply Schlobach et al.’s framework to vague sortals in the model for gradable adjectives? For each sortal (linguistically: count noun) forming a comparison class, we can define two approximations: a lower and an upper one. This means that in the model we define two approximations for each comparison class correspondent to a vague sortal. Let A be a monadic predicate standing for a vague count noun, say ‘mountain’. The problem is to determine the boundaries of the comparison class c_A individuated by A in the domain. Instead of exactly determining the boundaries of c_A , we determine two approximations: $\overline{c_A}$, which is the upper approximation and individuates all the objects of the domain that are possible mountains, and $\underline{c_A}$, which is the lower approximation and individuates all the objects of the domain that are definitely mountains. Let c_i be a comparison class correspondent to a vague sortal. The following two definitions of upper and lower approximation for c_i can be given:

Definition 19 $\overline{c_i} =_{def} \{x_1 : \exists x_2 : (x_1 \sim x_2) \wedge (x_2 \in c_i)\}$.

Definition 20 $\underline{c_i} =_{def} \{x_1 : \forall x_2 : (x_1 \sim x_2) \rightarrow (x_2 \in c_i)\}$.

We can either consider the similarity relation \sim as an equivalence relation or as a relation which is reflexive and symmetric but not necessarily transitive. In the first case the approximations $\overline{c_i}$ and $\underline{c_i}$ can be weakly or linearly ordered and in the second case they can be semi-ordered.

It must be noted that some elements belonging to a given upper approximation $\overline{c_1}$ may belong to another approximation, say $\overline{c_2}$. More precisely,

the elements of the complement $\overline{c_1} \setminus \underline{c_1}$ may belong to the complement $\overline{c_2} \setminus \underline{c_2}$. Such elements are the borderline cases of predicates A_1 and A_2 that give rise, respectively, to c_1 and c_2 . Let A_1 stand for ‘mountain’ and A_2 for ‘hill’. The landforms for which we are not able to determine whether they are mountains or hills are borderline cases both of ‘mountain’ and ‘hill’. From a model-theoretical perspective, that means that the elements of the domain which correspond to such borderline cases belong to the set of possible mountains and to the set of possible hills, and do not belong to the set of definite mountains, nor to the set of definite hills. That means, they belong to the complements $\overline{c_1} \setminus \underline{c_1}$ and $\overline{c_2} \setminus \underline{c_2}$. We can then define the set of borderline cases for a comparison class c_i as follows¹⁰:

Definition 21 $B(c_i) =_{def} \overline{c_i} \setminus \underline{c_i}$

The definitions given above can then be added to the description of the formal model given in Chapter 3. In this way, we obtain a model that is able to differentiate between count nouns (represented by monadic predicates) and gradable adjectives (represented by functions); moreover, the model describes the semantics of gradable adjective as functions whose domains are the sets denoted by monadic predicates. For such sets, called comparison classes, two approximations are defined. Given a comparison class, if it has always sharp boundaries, then the two approximations coincides. In case the comparison class corresponds to a vague count noun, the two approximations are distinct.

Let M stand for a vague count noun. The comparison class correspondent to M is c_M . We define two approximations: $\overline{c_M}$ and $\underline{c_M}$. The former represents the set of possible M -objects and the latter the set of definitely M -objects. The boundary of the real kind is not known to us and lies in-between the boundaries of the two approximations.

4.5 Appendix: a DL account for vague medical concepts

In this section I present the formal account that inspired the treatment of vague sortal concepts in the previous section. Such an account has been developed by Schlobach et al. [101] and applied in a medical domain. They consider a specific case of vague concepts: the concept of **septic patient**. I will first present their study case, then the DL-account they offer and, finally, some problematic issues in their treatment.

¹⁰An objection to this point is that assuming such an approach we get a set of borderline cases which is well determined: borderline cases of borderline cases are then excluded.

4.5.1 A study case of vague sortals: diseases

In this section I wish to consider a typical domain where concepts cannot be easily characterized in a crisp manner, the medical domain. Consider for example the case of **sepsis**. The concept of this particular disease is not crispy specified and it is not easy to give it a formal definition because its underlying pathology is unclear. The problem that arises is how to clearly determine the set of septic patients. There are cases of patients that are clearly septic, as well as cases of patients that are clearly non-septic. However, it is difficult to trace the boundary between the septic and non-septic patients because there are some patients that present symptoms similar to clearly septic patients, but at the same time present also some features that make them similar to non-septic patients. Those patients cannot be said to be clearly septic, nor clearly non-septic.

Moreover, in the medical literature we cannot find a precise definition of sepsis but at the most a *consensus definition*, such as the definition given by Bone in 1992 (Bone [10]). According to this definition, septic patients are patients that have a confirmed infection with at least two out of four Systemic Inflammatory Response Syndrome (SIRS) criteria:

- temperature $> 38^{\circ}\text{C}$ OR temperature $< 36^{\circ}\text{C}$
- respiratory rate > 20 breaths/min OR $\text{PaCO}_2 < 32$ mmHg
- heart rate > 90 beats/minute
- leucocyte count $< 4,000$ mm^3 OR $> 12,000$ mm^3

and organ dysfunction, hypoperfusion, or hypotension. Bone criteria can be thought of as criteria of application of the **septic patient** concept. They are, in fact, used to determine to which patients the property of being septic applies.

Schlobach et al. in [101] find an interesting and effective way to give a crisp specification of approximations of concepts with uncertain boundaries. The advantage of their approach is that they do not introduce uncertainty in the model, but they model vague knowledge in a crisp manner.

4.5.2 Sepsis: its characterization in Rough DL

In this section I resume Schlobach et al.'s formal treatment of vague medical concepts. They characterize the concept of **septic patient** using Rough Description Logic (from now on, Rough DL).

Rough DL is an extension of classical Description Logic (DL) adding two operators that are called the lower and upper approximations. DL constitutes a family of knowledge representation formalisms¹¹. Given a domain, they formally represent the knowledge of it by defining concepts and specifying the properties of the objects of the domain. The specification of the properties represents what is called ‘domain description’ or ‘world description’. The main task for DLs is the classification or categorization of concepts and individuals. The language of DLs is first-order logic and can be easily extended: in Rough DL two operators are added to classical description logics. Let us consider Rough DL more carefully.

The basic idea of Rough DL is traced back from Rough Set Theory (Pawlak [86]): given a concept C with undetermined boundaries, two concepts that approximate it can be defined. The two concepts represent, respectively, a sub- and a super-concept of C . The subconcept corresponds to the lower approximation of C and the superconcept corresponds to the upper approximation of C . The former denotes a set which contains the elements that are **definitely** elements of the concept C , the latter denotes a set which contains the elements that are **possibly** elements of C .

Let Septic be the set of septic patients that cannot be formally defined.

The lower approximation of Septic is defined as the set containing all and only the patients such that all the patients which are indiscernible from them are septic. Formally, let $\underline{\text{Septic}}$ be such an approximation and be defined as follows:

Definition 22 $\underline{\text{Septic}} =_{def} \{pat_1 : \forall pat_2 : (pat_1 \sim pat_2) \rightarrow (pat_2 \in \text{Septic})\}$.

The lower approximation of Septic is the union of all the equivalence classes that are subsets of it.

Call the upper approximation $\overline{\text{Septic}}$, let $pat_1, pat_2, \dots, pat_n$ range over patients, and define $\overline{\text{Septic}}$ as follows:

Definition 23 $\overline{\text{Septic}} =_{def} \{pat_1 : \exists pat_2 : (pat_1 \sim pat_2) \wedge (pat_2 \in \text{Septic})\}$.

It is easy to verify that the upper approximation is the union of all the equivalent classes which have a non-empty intersection with the set Septic .

$\underline{\text{Septic}}$ is the set of definitely septic patients; $\overline{\text{Septic}}$ is, instead, the set of possibly septic patients. What is the relation between $\underline{\text{Septic}}$, $\overline{\text{Septic}}$ and Septic ? It is easy to verify that being a member of $\overline{\text{Septic}}$ constitutes a sufficient condition for being a member of Septic , while being a member of $\underline{\text{Septic}}$ constitutes a necessary condition for being a member of Septic .

¹¹For an introduction to DLs, see Baader and Nutt [2].

Schlobach *et al.* provide a picture that represents the general idea: see Figure 1 below.

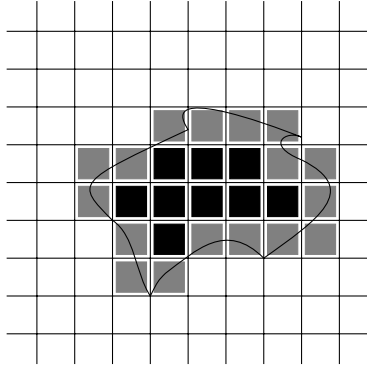


Figure 1

The domain is divided into many cells that represent groups of elements that are indiscernible (with respect to the fulfillment of the consensus criteria). **Septic** is formally difficult to be defined because it does not individuate a set of cells well determined. In the picture **Septic** is represented by the black curved line. There are some cells (namely, the grey ones) that have some elements that belong to **Septic**, and some other elements that do not belong to **Septic**. Septic is a set that contains all and only the cells of elements that fall completely under **Septic**: in the picture it is the set formed by the black cells. In other words, there is no cell in Septic such that it contains some elements that are not in **Septic**. On the contrary, $\overline{\text{Septic}}$ contains all the cells in Septic plus the cells that have some elements belonging to **Septic** and some other elements not belonging to **Septic**: it contains both the black and the grey cells. In the picture, the white cells belong to the set of definitely non-septic patients.

The relation between the set **Septic** and its approximations is represented as a DL axiom concerning subsumption relations among concepts:

$$\mathbf{Definitely_Septic} \sqsubseteq \mathbf{Septic} \sqsubseteq \mathbf{Possibly_Septic}.$$

In short, what the authors of [101] do is: given a set of patients of some disease for which no formal definition can be given, define two approximations to such a set, an upper and a lower approximation. The upper approximation defines the set of possibly ill patients, while the lower approximation defines the set of definitely ill patients.

4.5.3 Problematic issues

Consider the set of possibly septic patients, $\overline{\text{Septic}}$. It differs from the set of definitely septic patients Septic for some relevant aspects. Clearly, in $\overline{\text{Septic}}$

there is a group of cells containing some objects that belong to **Septic** and some other that do not belong to it. Such a group is illustrated by the grey cells in the picture given above. Those cells are elements of the complement $\overline{\text{Septic}} \setminus \text{Septic}$. Call such a set **BSeptic** (standing for *borderline septic patients*, since it is undetermined whether or not its elements are septic). Formally:

Definition 24 $B\text{Septic} = \overline{\text{Septic}} \setminus \text{Septic}$

Or, alternatively:

Definition 25 $B\text{Septic} = \{pat_i : (pat_i \in \overline{\text{Septic}}) \wedge (pat_i \notin \text{Septic})\}$

Schlobach et al. make two assumptions on the set **BSeptic**:

1. In each cell of **BSeptic** there are some elements that belong to **Septic** and some other that do not belong to **Septic**. The authors assume that there is a boundary of **Septic** and it falls in **BSeptic**. Formally:

$$\exists x(x \in B\text{Septic} \wedge x \in \text{Septic}) \wedge \exists y(y \in B\text{Septic} \wedge y \notin \text{Septic})$$

2. **BSeptic** is composed by a group of cells (depicted by squares in the figure) that, as all the other cells (squares) in the picture, “denote a set of domain elements, which cannot further be discerned by any available criteria” (Schlobach et al. [101] p. 557).

Assumption 2 is about the relation of indiscernibility that is represented by a similarity relation \sim . Given two individuals x and y , $x \sim y$ if and only if they are indiscernible with respect to the properties given by Bone’s consensus definition. Such a definition, indeed, presents a list of symptoms that can be thought of as (measurable) properties. Take an individual x . You measure x ’s temperature, respiratory rate, heart rate, leucocyte count, and so on. Consider, now, an individual y different from x . You measure the same properties with respect to y . If the results of x ’s and y ’s measurements are the same, then you can state that they are indiscernible: $x \sim y$. x and y belong then to the same cell (according to assumption 2). Consider the symptoms of the Bone criteria: temperature (P_1), respiratory rate (P_2), heart rate (P_3), leucocyte count (P_4), organ dysfunction(P_5), hypoperfusion (P_6), hypotension (P_7). Call Q the set of properties $\{P_1, \dots, P_6\}$. The elements that are indiscernible with respect to all $P_i \in Q$ are contained in a specific cell. This fact suggests that the cells can be thought of as equivalence classes formed by the similarity relation \sim which is an equivalence relation: it is reflexive, symmetric and transitive.

The above considerations can be applied to the cells (equivalence classes) in **BSeptic**: the individuals of each of them are indiscernible with respect to the symptoms of the Bone criteria. Thus, they are supposed to have the same disease, if any, or the same form of disease. But according to assumption 1, in each cell (equivalence class) of **BSeptic** there are some elements that are septic and some that are not. There is a boundary that separates the former from the latter, but we do not know where it lies. However, if some of the elements are septic and some others are not, it seems problematic that the cells in **BSeptic** are to be conceived as genuine equivalence classes. We take them “as if” they were equivalence classes. By contrast, consider the set of definitely septic patients and the set of clearly non-septic patients. The cells of those two sets seem to be genuine equivalence classes. The elements of each cell are indiscernible with respect to Q and all of them either belong or do not belong to the set **Septic**. Given any two patients that are indiscernible with respect to the Bone criteria, they turn out to be both either ill or not ill. It does not happen that they are indiscernible and at the same time one is ill and another is not. By contrast, it can happen for the elements of the cells in **BSeptic** to be indiscernible with respect to Q , but by assumption 1 some of them are septic and some are not.

Thus, there are some conceptual difficulties in conceiving the cells of **BSeptic** as genuine equivalence classes. To overcome those difficulties we have two paths: either refute assumption 1 or refute assumption 2. If we choose to refute 1, we refute that there are borderline cases. It will turn out then that the vagueness problem vanishes, and Schlobach et al’s attempt to treat vague concepts too. Let us maintain the fact that there are borderline cases and that we do not know whether some patients are septic or not. What happens, then, if we refute assumption 2? We refute that the cells correspond to genuine equivalence classes, that is, we refute that the relation of indiscernibility of symptoms is an equivalence relation. Something must happen in the cells of **BSeptic**. One solution is to think that individuals do actually differ with respect to some of the properties of Q , but we cannot decide which property is: our measurement does not give us an answer. Another way to see the problem is to think of the relation \sim as not transitive. In this way the problem seems to be closely related to the non-transitivity issues of vague adjectives encountered in Chapter 3. Given three patients x, y, z , if x is indiscernible from y and y from z with respect to Q , it does not follow that x is indiscernible from z too. Somewhere there is something which marks the difference between septic patients and non-septic patients, but we do not know where.

The authors of [101] are aware of the fact that the use of equivalence classes can be objected and indicate two directions one could take (Schlobach

et al. [101], p. 562.):

To model vague concepts, one might also study approximation operators based on tolerance relations (reflexive and symmetric). Also one could think of sets of equivalence classes according to different similarity relations.

The first direction is to define approximation operators based on a relation which is not necessarily transitive; the second is less clear: the idea seems to be to think of equivalence classes given by a different similarity relation. However, if you do not accept, for all the reasons provided, that the similarity relation is transitive, then the first direction seems to be the one to take. Septic and Septic are sets defined on the relation \sim , which is considered to be an equivalence relation. If we refute that \sim is a transitive relation, we can still maintain the definitions of the two approximation sets, but we do not get any longer equivalence classes. The alternative way to treat the sets Septic and Septic is to conceive them as semi-ordered sets, according to the definitions given in Chapter 3. As already mentioned, a semi-order is characterized by a similarity relation which is reflexive and symmetric but not (necessarily) transitive.

Chapter 5

Towards a Logical Adequacy of Identity Criteria

5.1 Introduction

In the previous chapter it has been claimed that sortals are associated with identity criteria. This chapter is devoted to analyzing the notion of identity criteria, focusing on the requirements that their logical form demands. In particular, some cases of identity criteria that fail to meet the formal requirement of transitivity will be presented and a solution to confer logical adequacy to non-transitive identity criteria will be considered.

In a loose and philosophically popular view, derived from Quine, identity criteria are required for ontological respectability: only entities with clearly determined identity criteria are ontologically acceptable. It seems that to answer questions such as “What exists?” and “What kind of objects are there in the world?” we need to understand and explain how the entities in the world are epistemically accessible to us. Behind this thesis lies the concept that we cannot claim that certain objects exist without us being able to explain how we can know something of them (see Cozzo [17]).

The credit for introducing the notion of an identity criterion (from now on, IC) is usually attributed to Frege. In his *Foundations of Arithmetic* Frege introduces the idea of IC in a context where he wonders how we can grasp or formulate the concept of numbers (see Frege [29], §62):

If we are to use the symbol a to signify an object, we must have a criterion for deciding in all cases whether b is the same as a , even if it is not always in our power to apply this criterion.

Frege wants to prove that natural numbers are objects, even though they are neither physical objects, nor mental constructions. Some philosophers

then argued that Frege intended to introduce the notion of IC only for abstract objects, and not for concrete ones¹. However, the Fregean notion of IC has been largely used by Wittgenstein in his *Philosophische Untersuchungen* to deal with questions about empirical objects.

Even if it is not completely clear whether or not Frege thought of ICs as related only to abstract entities, his considerations about ICs seem to adapt both for concrete and abstract objects. He suggests that an IC has the function of providing a general way of answering the following question, with a and b objects in a given domain:

Fregean Question: How can we know whether a is identical to b ?,

Consider two famous examples of ICs provided by Frege [29]:

- IC for directions: if a and b are lines, then the direction of line a is identical to the direction of line b if and only if a is parallel to b ;
- IC for sets: if a and b are sets, then they are the same set if and only if they have the same members (axiom of extensionality).

In the philosophical literature, the Fregean question has been reformulated in the following ways:

Ontological Question (OQ): If a and b are Ks, what is it for the object a to be identical to b ?

Epistemic Question (EQ): If a and b are Ks, how can we know that a is the same as b ?

Semantic Question (SQ): If a and b are Ks, when do ‘ a ’ and ‘ b ’ refer to the same object?

The difference between an answer to (EQ) and an answer to (OQ) is not purely formal. When answering (EQ), we think of conditions associated with a procedure for deciding the identity questions concerning objects of some kind K. In answering (OQ), we think of conditions which are meant to provide an ontological analysis of the identity between objects of kind K. Finally, an answer to (SQ) concerns sameness and difference of reference of simple or complex names. (OQ), (EQ) and (SQ) set identity questions that deal both

¹But Dummett, for instance, supports the thesis that Frege had something more general in mind, that is, Frege’s thesis on ICs applies both to concrete and to abstract objects. See Dummett [23], p. 73 ff., 545 ff.

with the synchronic identity problem and the diachronic identity problem. Identity can hold between two objects at a certain time (synchronic identity) or across time (diachronic identity). In the first case, you can reformulate (OQ), (EQ) and (SQ) with regard to the identity of objects taken at a specific time. For instance, you can formulate an ontological, an epistemic or a semantic question concerning the identity at time t of a statue and the clay constituting it: Are the statue j and the piece of clay the same object (OQ)? How can we know whether they are the same object (EQ)? Do ‘statue j ’ and ‘the piece of clay constituting statue j ’ refer to the same object (SQ)²? The same can be done for the case of diachronic identity. Consider a picture of yourself when you were three years old. Is the child at the time the picture was taken the same person as yourself now (OQ)? How can we know that you are the same person as the child in the picture (EQ)? Alternatively, suppose that you find a picture of one of your classmates at college. At the time the picture was taken, her name was Mary Brown. After marriage, she changed her surname into Smith. Is Mary Brown the same individual as Mary Smith (SQ)?

It must be noted that ICs do not have the function of saying what the relation of identity consists of. ICs cannot give a definition nor explain the identity relation without presupposing identity itself (Carrara and Giaretta [15], p. 430). Identity is a reflexive, symmetrical and transitive relation between any object and itself: the relation of identity in its intended meaning is not problematic (Lewis [59], pp. 192-193, Perry, [90], p. 254). Take the examples of ICs given for sets: if a and b are sets, then they are the same set iff they have the same members. A *sameness* relation is both in the left- and right-hand side of the biconditional *iff*. If the IC had the function of saying something about the sameness or identity relation, it could not have a sameness relation on both sides of the biconditional, since you cannot explain a concept using the same concept.

If then ICs do not have the role of saying something about the identity relation, what do ICs do in answering (OQ), (EQ) and (SQ)? What exactly is the role of ICs in answering (OQ), (EQ) and (SQ)? The following section is devoted to answering those questions.

²In this case, (SQ) asks whether the referent of a proper name is identical to the referent of a definite description: both proper names and definite descriptions are taken to be singular terms.

5.2 The identification role of ICs

In Chapter 4 we saw that ICs have an identification function with respect to sortal concepts. The specification of the identification role of ICs depends on the different notions of IC that have been in the marketplace of the philosophical debate. In this section I will first show some roles that can be misleadingly attributed to ICs, and then the peculiar role that they seem to play.

First of all, it is important to make clear that ICs are not definitions, neither of identity, as we have seen, nor of identity restricted to a sort of objects, nor of the sortal terms which they are associated with (See Lowe [71], [64], Williamson [125]). The IC for directions, for instance, does not define what a direction is, but gives the conditions under which two lines can be said to have the same direction. ICs do not have the function of determining the meaning of sortal terms; that is given, instead, by membership conditions. In the Appendix of Chapter 4 we saw the case of the concept **septic patient**. In that case there is a consensus definition about the symptoms individuals must present in order to be considered septic patients. The consensus definition does not provide a condition, the satisfaction of which by a pair of individuals x and y is necessary and sufficient for identifying x and y , but only the membership conditions for the set denoted by **septic patient**.

In the previous chapter we have seen that an entity is individuated by the means of the sortal concept which that entity exemplifies. ICs do not have, though, the function of deciding whether a certain entity belongs to some sort K . Consider Frege's discussion about ICs for natural numbers. He proposes the following IC for natural numbers: the number of the F s is identical to the number of the G s iff there is a one-to-one correspondence between the F s and the G s.

Such a criterion leaves open the question of how we individuate a number among the entities in some domain. We must already know what K -objects look like and what is it for an x to be a K in order to formulate and employ an IC for K -objects. Put otherwise, "an acceptable criterion of identity for K can only flow from a prior philosophical theory of K " (Horsten [43])³.

Sometimes ICs are thrown into the definition of sortals: a concept is a sortal iff an IC is associated to it. It seems, then, that ICs are conceived as conditions for determining whether a certain concept is a sortal. If it is - at

³The criterion above does not tell us, for instance, whether number five is identical to Julius Caesar. This kind of question is known as the 'Caesar problem' and it is largely discussed within the field of Philosophy of Mathematics. Is Julius Caesar a number? The IC does not give any answer to such a question. It seems that an IC for K -objects cannot decide whether or not a given object is a K .

least in principle - possible to associate an IC to a concept, then that concept is a sortal. Why? Because ICs offer the conditions for identifying (and re-identifying) the instances falling under a (sortal) concept. If some concept has instances such that it is not possible to have a criterion for identifying and re-identifying them across time, then the concept is not considered to be a sortal. Moreover, ontologists claim that entities without clearly determined ICs are not to be considered part of the furniture of the world: they are not real constituents of the world. The thesis that ICs are what mark concepts as sortals and the things falling under such sortals as real constituents of the world constitutes a strong metaphysical position. In this section I will try to illustrate some arguments supporting this position.

I will start by considering the following question: What is the relation between ICs and the sortal concept which they are associated with? To face this question it is worthwhile considering a contribution by Brand [12]. In his paper, Brand proposes a classification of the interpretations of ICs. Such a classification depends on the different functions attributed to ICs.

According to Brand's analysis, there are three different roles that have been attributed to ICs in the philosophical literature:

1. An IC can be considered as a criterion for judging how many objects of a certain kind there are or, also, for (re-)identifying a single object of some kind. ICs are considered to have only an epistemic function; one of providing identifying conditions. But ICs do not provide them if they do not say anything, for instance, about the features of the kind which the objects belong to. Consider, for instance, the following possible candidate of an IC for persons:

' x and y are the same person iff they have the same parents and siblings.'

Parents and siblings also are persons: on the right side of the biconditional the entities which are mentioned are of the same kind as the entities for which the IC is meant to be provided. The IC does not say anything about the essence of persons. Consider also the following IC for persons:

' x and y are the same person iff their bodies occupy the same spatio-temporal regions.'

Some object to this IC by also claiming that you need to specify mental states (memories, beliefs, etc.) in order to provide logically sufficient conditions for personal identity. Nevertheless, the spatio-temporal coincidence of bodies can be considered as a plausible epistemic criterion for counting persons or judging whether the person I see now is the

same person than the one I saw before, and can be used, for instance, in legal situations.

According to Brand, then, the ICs given above have a pragmatic and epistemic role: they probably are enough to answer the epistemic question (EQ), but they do not say anything about the ontological aspects of personal identity.

2. The second interpretation of the role of ICs is provided by Quine: criteria of identity are necessary to single out the members of a domain (“No entity without identity”). Only if the elements of the domain are singled out and individuated, can you quantify over the entities of the domain.

According to this interpretation, an IC for persons is supposed to provide “the most general individuating conditions” (Brand [12], p. 330), for instance:

‘ x and y are the same person iff they have the same properties.’

Such a formulation corresponds to the Leibniz Law (taken as the conjunction of the Law of Indiscernibility of Identicals and the Law of Identity of Indiscernibles). The Leibniz Law is sufficient if the goal is to find some condition to make possible the quantification over the domain of person. But the Leibniz Law does not provide the uniquely sufficient properties for the identity of persons, since the conditions it gives for the identity of persons are the same as the conditions for the identity for, say, chairs. And so, how can we count and distinguish things from one to another kind? The Leibniz Law is not sufficiently informative if the task of an IC is also to indicate some essential properties of persons, i.e. the properties that characterize persons and nothing else. Moreover, suppose we have only the Leibniz Law at our disposal as the IC for persons. How can we determine whether a is the same person as b by virtue of the Leibniz Law? We should examine all the properties of a and all the properties of b but, as we know, that is an infinite process. Moreover, Cozzo [17] shows that for some property P , it is impossible to know if a certain individual a has P without knowing if a is identical to b . Let a and b both be rivers. If I indicate as a the main river that flows in Vienna, how can I establish whether a has the property of flowing through a certain area in Budapest without having been acquainted with the knowledge that a is the same river as the one I am indicating from a certain bridge in that area in Budapest (Cozzo [17])? If we are looking for both the necessary and the sufficient properties for identity of objects of some sort K , this second interpretation

of the notion of ICs does not help us.

3. In the third place, you can talk of an IC as a criterion that specifies the nature of some kind of objects by the means of a universal statement. That means, an IC for some kind of objects “must specify non-trivial essential properties of objects of that kind” (Brand [12], p. 330). Consider an example of an IC for sets:

‘ x is the same set as y iff x has the same members as y .’

Now, ‘to have the same members as’ is a relation, that is, a dyadic property. In a more general way, we can express the same IC by means of the following logical form:

$$\exists P(\forall x\forall y((x \in \phi \wedge y \in \phi) \rightarrow \Box(x = y \leftrightarrow Pxy))).$$

The relational property is represented by P . P is a property of pairs of objects of kind ϕ . It is not a property essential to *each* object of ϕ . It is neither a property such that, whenever an object x possesses it, x belongs to a sort ϕ . Call instead a property with the aforementioned features P' : x belongs to a sort ϕ iff x has P' . Is there a relation between P and P' ? Let ϕ be the sort of persons. Consider:

P : x has the same memories as y ,

P' : x has memories.

P' is not implied by P . Think of the trivial case when a and b have no memories: a and b have the same memories (P is satisfied), without a and b having memories (P' is not satisfied).

P' seems to tell us something about the nature of items belonging to kind ϕ (persons). But, according to the third interpretation, an IC for persons specifies the nature of persons not by making sure that some instances of the sort of persons have P' , but by detecting a relational property that is a sufficient condition for stating the identity of pairs of items belonging to the set of persons.

To make the point clear, consider another example. Let P'' be the following property: ‘to have at least a mental state’ (assume here a general conception of mental states; mental states are, for instance, memories, beliefs, hopes, desires, etc.). A property such as P'' could be conceived as a membership condition for the set of persons: an individual is a person iff it has a mental state. Put otherwise, an object belongs to the set of persons

iff it possesses P'' . By contrast, consider the following relation P''' : ‘to have the same mental states as’. P''' is a dyadic property that is related to P'' as much as P is related to P' in the example given by Brand. The dyadic properties do not tell us the nature of persons, nor can we infer from them properties that are essential to persons. With Brand’s words (Brand [12] p. 330),

identity conditions specify the nature of sorts of objects, but in the roundabout way of specifying a relational property which is sufficient for identity of pairs of these objects.

Brand actually emphasizes the ontological function of ICs over the epistemic one (see his critical remarks on the first interpretation of the function of ICs). I believe that the epistemic role of ICs cannot be considered apart from their ontological role. Of course, it is not sufficient to claim that ICs play an epistemic identification role. ICs must say something about the world too, not only about our way of recognizing objects. The identification role of ICs can be further specified in connection with their ontological, epistemic and semantic functions. That is the goal of the following section.

5.2.1 Ontological, epistemic and semantic functions of ICs

In this section I wish to analyze the relation between the ontological, epistemic and semantic functions of ICs. To do that, I will consider a contribution by Savellos [100] where he develops Brand’s notion of ICs. Savellos’ remarks can further clarify the notion of the IC. He starts his analysis quoting the following two passages:

The function of identity conditions is to specify conceptually significant properties of ontological types, not to define ‘identity’ or to introduce relativistic identity relations. True identity conditions indicate the nature of persons, or the nature of physical objects, or the nature of events, or so on. (Brand [13], p. 62.)

A criterion of identity, by giving conditions under which ϕ s are identical, captures and articulates the essence of what it is to be a ϕ . (Lombard [62], p. 47.)

Following the two quotations, we can state that ICs must somehow encode the characteristic nature of the objects belonging to the kinds which they are

associated with. That idea rules out Leibniz Law as a suitable IC because it does not specify an identity condition for a specific kind of objects. As we have seen in the previous chapter, ICs associated with sortals have the function of determining, in a counting process, whether a K -object that you counted as one some minutes ago is the same K -object as the one that is in front of you now. What must be emphasized is that what we want to determine is not just whether some object is identical to another, but whether one object is the same K -object as another. So, the identification role of ICs is related to a specification of the features of K -objects, and such a specification can be given from an ontological and epistemic (as well as semantic, I would add) point of view.

According to Savellos [100], in encoding the characteristic features of entities belonging to kinds K , ICs perform an ontological and an epistemic function. As far as the ontological function is concerned, to specify ICs for a sort K of objects is to specify what makes K -objects different from one another. For instance, in the case of the IC for sets, what makes sets differ one from another are the number of their elements. ICs for K -objects reveal some conceptually relevant features associated with the nature of the sort K , but do not completely specify such a nature. In the case of sets, the IC shows that sets are conceived as aggregates of elements (allowing for the case that they have no element) but, for instance, it does not reveal that sets are abstract entities - and that is an important feature of sets as well. What ICs do is to capture the nature of K -objects inasmuch as ICs are able to distinguish K -objects from K' -objects and each K -object from the other K -objects.

As far as the epistemic function of ICs is concerned, it must be emphasized that ICs do not tell us which individuals belong to some sort K . As we saw in Chapter 4, membership conditions for the set denoted by some K do not coincide with identity conditions for K . Moreover, ICs do not give us rules for individuating the boundaries of individuals: that is instead a task for unity principles (section 4.3). According to Savellos, the epistemic function of ICs is to specify the necessary and sufficient conditions for the truth of the statement that a K -object named a is identical to a K -object named b .

I wish to make some remarks on Savellos' claim. What he considers the epistemic function of ICs corresponds to what I called the semantic function of ICs. The epistemic function of ICs is instead that of providing identifying principles, i.e. conditions that are necessary or sufficient for us to determine whether two items are identical. It might happen that for some sorts of objects it is difficult to find ICs with a clear ontological import. Sometimes we can express only an approximation of them, that means we can express conditions that are only either necessary or sufficient for us to distinguish

and identify things of a certain sort K , without being able to say anything about the nature of K . In section 5.4 we will consider some examples of these kinds of ICs that have only an epistemic function; for instance, the following IC for phenomenal colors is one of them:

‘Two phenomenal colors are the same iff they are perceptually indistinguishable’.

Such a criterion has a pragmatic function: it helps us to distinguish colors, but it does not say anything about the nature of colors. Moreover, it appeals to our perception which is fallible; a strong metaphysical IC for colors must be grounded on something less fallible than our perception.

Something more needs to be said about the epistemic function of ICs. As Cozzo [17] suggests, to be able to use the notion of identity we need some rules or guidelines that provide applicable conditions to correctly assert identity statements. The epistemic function of ICs can be expressed in this way: they let us know on the basis of which evidences (or arguments) the assertion of identity statements is justified. In other words, ICs tell us how it is possible to know whether a certain a is identical to some b .

Nevertheless, there might be cases of ICs which determine truth conditions for identity statements, but we are not able to apply them, that is, they do not play an epistemic role. This is what Frege seems to suggest in the passage quoted above (Frege [30], p. 73):

If we are to use the symbol a to signify an object, we must have a criterion for deciding in all cases whether b is the same as a , even if it is not always in our power to apply this criterion.

What does Frege intend to say by adding “even if it is not always in our power to apply this criterion”? It does not seem that Frege had in mind the somewhat practical impossibility of applying ICs; rather, he seems to underline the fact that for some identity statements on K -objects the IC associated to K determines the truth or falsity of those statements, but it might happen that the IC is inapplicable for us, i.e. we do not grasp its truth conditions⁴. Even if the IC is at our disposal and by itself it would be enough to decide about the truth conditions of some identity statements, we might still not be able to decide about them. For example, assume that the identity condition for some K -objects is represented by an algorithm.

⁴Linnebo [60] observes that within a Fregean account, to understand an identity statement means to know its sense, that is, its truth condition. But this knowledge could remain un verbalized and might not be shared by everyone (Linnebo [60], p. 205).

Such an algorithm is able to decide the truth value of identity statements containing names of K -objects, but that algorithm could remain unknown to us, or could be so complex that we are not able to apply it (see Williamson [125]).

Let us accept the idea that we are not able to apply ICs in some cases. Is that in contrast with the idea that ICs have the epistemic function of recognizing objects? According to Cozzo [17], the two ideas, that can both be attributed to Frege, are not incoherent, but show that the (Fregean) notion of ICs is two-fold. On one side, criteria associated to sortals are necessary to get conditions for recognizing objects and to make possible the assertion of identity statements; for this reason, Leibniz Law is not a suitable IC, even if it determines perfect truth conditions. On the other side, since the (Fregean) sense of a statement is given by its truth conditions, an IC for K should fix necessary and sufficient conditions to make true an identity statement with singular terms referring to K -objects. In some cases, though, ICs determining truth conditions for identity statements could turn out to be inapplicable.

Let us consider the connection between the ontological, epistemic and semantic functions of ICs in more detail. For some philosophers ICs cannot have all those three functions at the same time. According to them, ICs are either ontological, epistemic or semantic principles (and sometimes intermediate positions are possible). I shall resume here those different positions following Horsten [43].

Lowe [70], [69], [64] and De Clercq [20] claim that ICs are mainly semantic principles, connected with metaphysical but not with epistemic issues. De Clercq claims, for instance (De Clercq [20], p. 23):

the question to which identity criteria seek to provide an answer is ‘When do two names refer to the same object?’. Or, if this sounds too much like an issue concerning the semantics of names: ‘When is the object referred to by one name the same as the object referred to by another name?’.

De Clercq accepts the standard thesis that ICs are associated with sortal concepts K or, better said, an IC associated with a concept K is concerned with the objects falling under K . For this reason, ICs are semantic-metaphysical principles. He excludes, though, ICs having an epistemic component. According to De Clercq, to search for an IC for K -objects is not to search for a “reliable *epistemic procedure*” (De Clercq [20], p. 25). ICs offer necessary semantic-metaphysical conditions for the identity of K -objects, but the procedures of getting to know such conditions are contingent and

variable from one possible world to another. For instance, suppose that the IC for persons is given by DNA traces. Such a criterion gives us necessary metaphysical conditions to determine the identity of persons and therefore necessary semantic conditions for the truth value of identity statements involving names for persons. Suppose there is a world where people are cloned. In such a world, epistemic procedures of getting to know the DNA traces of persons are not completely reliable.

Lowe [70], [69], [64] asserts that ICs are not to be conceived as epistemic or heuristic principles through which it is possible to determine whether identity statements are true or false, exactly because they are not always applicable, as Frege himself says. According to Lowe, ICs are then semantic or “metaphysical-cum-semantic” (Lowe [70], p. 62-63) principles: grasping such semantic principles is essential in order to understand sortal terms. As Cozzo [17] underlines, it does not seem to be right, though, to claim that the Fregean conception of ICs excludes attributing an epistemic function or role to ICs.

In contrast, Strawson [110] supports the thesis that ICs have a basic epistemic component. ICs inasmuch as they are criteria are something to be applied. To say what an IC is should tell us how it is that we can apply it. Strawson does not accept cases of not clearly determined or inapplicable ICs. Examples of cases of well-formed ICs are the IC for directions and the IC for natural numbers (i.e. examples of ICs for abstract objects). Only inapplicable ICs can be found for ordinary substantial individuals such as dogs, persons, trees. In short, his position can be summarized as follows (Strawson [110], p. 22):

you cannot talk sense about a thing unless you know, at least in principle, how it might be identified.

Williamson [125], followed some years later by Lowe [71], [66], argues for the thesis that ICs are metaphysical principles: they are associated with a concept K and their function is to tell what identity *consists of* for instances of K . One can support the thesis that ICs have an ontological function and not a semantic one because if ICs were semantic principles one could expect them to contain semantic notions (‘reference’, ‘truth’, ...) but they actually do not (see Horsten [43]). They specify conditions under which the *referents* of singular terms are identical whenever they belong to the same sort K .

However, even if ICs are primarily metaphysical principles, they do have some semantic implications: they can be used to determine sameness of reference of individual expressions. I would tend to agree that ICs are basically meant to be as metaphysical (ontological) principles, but I would not deny that they have a semantic function as well. Moreover, I would not exclude

attributing an epistemic function to them either. In expressing what the identity of directions consists of, the IC for directions suggests how we can know whether two lines have the same direction. And moreover, ICs help us to recognize the identity of objects of a certain kind. Sometimes, though, there can be situations where we do not know exactly the necessary and sufficient conditions under which it is possible to determine what it is for an object a to be identical to b . Some philosophers, for instance, claim that we do not have necessary and sufficient conditions for identity statements concerning objects belonging to natural (Strawson) or artefact (Wiggins) kinds. However, also for this kind of objects, we can find some criteria that can be used to identify objects, even if they do not rely on strong ontological issues. In sections 5.4 and 5.5 my focus will be on ICs that are not precise nor clearly determined, and therefore they do not say exactly what the identity of K -objects consists of, but they are nevertheless used in our everyday life for simple identifying purposes.

5.3 Formal formulations of ICs

After having discussed the role and the functions of ICs, it is worthwhile considering what their logical form looks like. The reason is that there are some requirements that ICs must satisfy to provide acceptable identity conditions, and part of those requirements are formal.

There is more than one way to express the logical form of ICs. When an IC is taken to be what answers the question “When is an entity x of kind K identical to an entity y of kind K ?”, then the following logical form seems to be the appropriate one for it:

$$\forall x \forall y ((x \in K \wedge y \in K) \rightarrow (x = y \leftrightarrow \Phi(x, y))). \quad (\text{IC1})$$

Φ is called the *identity condition* and consists of a relation R . IC1 can then also take the following form:

$$\forall x \forall y ((x \in K \wedge y \in K) \rightarrow (x = y \leftrightarrow R(x, y))). \quad (\text{IC1}^*)$$

The formula can be read as follows: two objects x and y of a given domain are identical iff some relation R holds between x and y . IC1* expresses the logical form of, for instance, the IC for sets: if a and b are sets, then they

are identical (they are the same set) iff they have the same elements. In this case, K represents the sort of sets, x and y vary in the elements of K , and R represents the relation ‘have the same number as’.

IC1 does not seem, though, to be the appropriate logical form of the IC for directions. In this case, in fact, we have objects belonging to the sort of lines; however, the IC is not outlined for the identity of lines, but for directions of lines. More generally, the question “When is an entity x of kind K identical to an entity y of kind K ?” is replaced by “When is the f of a thing x equal to the f of a thing y ?” (see Horsten [43]). The logical form of an IC that answers such a question can be the following:

$$\forall x \forall y ((x \in K \wedge y \in K) \rightarrow (f(x) = f(y) \leftrightarrow \Phi(x, y))). \quad (\text{IC2})$$

Also in this case, we can rewrite the formulation in the following way:

$$\forall x \forall y ((x \in K \wedge y \in K) \rightarrow (f(x) = f(y) \leftrightarrow R(x, y))), \quad (\text{IC2}^*)$$

where R is a relation holding between objects belonging to some kind K . Here we have some elements x and y of a given domain D which belong to some kind K , and f is a function whose domain is K itself and the range is a set of elements which constitute a different set, $f(K)$. In this case, the IC is outlined for elements belonging to $f(K)$, but the relation constituting the identity condition holds between elements of K . Consider again the IC for directions: objects x and y belong to the sort K of lines and $f(x)$ denotes the direction of x (and analogously for $f(y)$). The identity condition for directions is given by the relation of parallelism that does not hold between directions $(f(x), f(y))$ but between lines (x, y) . So, in this case we have two kinds of entities that are employed in the IC: directions and lines. For this reason the ICs of form IC2 are called *two-level criteria*, while the ICs of form IC1 are called *one-level criteria*. It must be noted that IC1* is not to be taken as a logical form different from IC1. IC1* is just a specification of IC1. The same consideration applies to IC2* and IC2.

There has been a discussion between Lowe and Williamson about which of the two types of criteria is the most fundamental one. In a very rough way, the positions of the two philosophers can be summarized in this way: according to Williamson [125], [126], the two types of criteria are not reducible one to the other; according to Lowe [69], [65], the one-level criteria are the most fundamental type and the two-level criteria can be reduced to one-level ones. Such a debate does not seem to have lead to any evident result. Whether

or not the two types of ICs can be reduced one to another is not a relevant issue for the purposes of this research work. What is important is to keep in mind the distinction between the two types.

From now on I will focus on the logical forms of ICs IC1* and IC2*. They present some common features. One of them is that the formulations both have a relation R on the right-hand side of a biconditional, while on the left-hand side they both have an identity relation. As we have seen, R constitutes the condition Φ under which two items (x and y in IC1*, $f(x)$ and $f(y)$ in IC2*) are said to be identical.

It is worthwhile making some observations about the properties of the relation R . Namely, to represent the identity condition Φ there can be more than one candidate relation R . Which is the best one? How can we determine the relation R that represents the identity condition? It is plausible to think that there are some requirements that R must meet. In the following section I wish to list these requirements or, at least, some of them. I do not think that a complete list of requirements for R has been yet individuated. The requirements I am going to discuss are very general and most of them connected to the logical form of ICs; future work is needed to complete the list.

5.3.1 Requirements for R

In this section, some constraints for the relation R are listed and discussed. For the sake of simplicity, I consider here the formulation of one-level criteria of form IC1*: when I mention, for instance, kind K I mean the kind of objects for which the identity condition is sought. The remarks given below can be, in any case, adapted for ICs of form IC2*. The relation R is what the identity condition consists of or, put otherwise, given an identity statement $a = b$ R is a relation that holds between a and b , is other than identity and analyzes what it is for the referents of a and b to be identical (See Linnebo [60], p. 206). How should R look to be a good candidate for being the identity condition of objects of some kind K ? To answer this question, I take into account three contributions: Carrara and Giaretta [14], Brand [12] and Lombard [62].

Non-vacuousness The identity condition cannot have parts that are vacuously satisfiable. Consider the following example (see Lombard [62], p. 32-33). Let PO be the set of physical objects, S the set of sets, $R(x, y)$ the identity condition for PO and $R'(x, y)$ the identity condition for S :

$$\forall x \forall y ((x \in PO \vee x \in S) \wedge (y \in PO \vee y \in S)) \rightarrow (x = y \leftrightarrow (R(x, y) \vee R'(x, y))).$$

The condition given above for the identity of x and y is not associated with a kind of entities in a metaphysically interesting sense, since the members of the alleged kind do not share an essence. The identity condition must specify a relation that holds between elements of a certain kind, such that all of them are alike with respect to the properties associated to such a kind. Put otherwise, the identity condition supplies a property of properties. Such a property is called by Lombard *determinable* since it determines a class of properties, called *determinates*, having that property. An example of a determinable is ‘being a spatio-temporal property’, which can be considered a good candidate for an IC for objects: if o and o' are physical objects, they are identical iff they are alike with respect to all the properties that are spatio-temporal properties. A criterion of identity for K -objects, to be acceptable, cannot provide a determinable such that it makes non-vacuous sense to attribute to each K -object determinates falling under the determinable.

Informativeness R should contribute to specify the nature of the kind K of objects for which R acts as the identity condition. The identity condition does not completely characterize the nature of instances of K . According to Frege, to decide about identity questions concerning a K we need the concept of K , that is not provided by the ICs. Nevertheless, an IC specifies some non-trivial essential properties of objects of kind K . That means, the form of the relation cannot be tautological, for instance, it cannot have the following form:

$$R(x, y) \vee \neg R(x, y).$$

Partial exclusivity An identity condition for a kind K of objects cannot be so general that it can be applied to other kinds of objects. The example provided by Lombard is the following:

‘If x and y are both non-physical objects, x and y are identical iff they have the same individual essence’.

Now, the properties falling under the ‘large’ property ‘having an individual essence’ do not apply only to non-physical objects and can be part of the identity conditions for many kinds of objects. Lombard’s suggestion is to request a *partial* exclusivity (Lombard [62], p. 37):

for each kind there must be *at least one* determinable specified by the identity condition, such that it does not make

(non-vacuous) sense to say of any object not of that sort that it has or lacks a determinate falling under that determinable.

Minimality The identity condition for K -objects is required to specify the smallest number of determinables such that the determinates falling under them turn out to be necessary and sufficient to ensure identity between two objects of kind K . The determinables specified in the identity condition cannot be superfluous.

Non-circularity The identity condition for K -objects cannot make use of the concept of K itself, otherwise it is circular. There has been a long debate about the circularity of the IC for events proposed by Davidson (see Davidson [19]):

‘If x and y are events, $x = y$ iff x and y have the same causes and effects’.

Since some causes and effects are events, the identity condition for events involves identity between events: in fact, to determine whether two events are the same we are required to determine, first, the identity of events taken as their causes or effects.

Non-tautologicity R cannot be a property that every two objects of kind K share. Formally:

$$R \subset K \times K. \tag{C1}$$

C1 says that the relation R is a *proper* subset of the set $K \times K$, that is, there is some pair of objects that are K such that the objects of the pair are not in the extension of R .

K -Maximality R must be maximal with respect to K , i.e. it must be the widest dyadic property that makes an identity condition true. A dyadic property G is wider than a property G' iff for any x and y , if $G'(x, y)$ is the case, then $G(x, y)$, but not vice versa. In other words, the ordered pairs of G' are a subset of the set of ordered pairs of G . Formally, for all the relations R' that are possible candidates for the identity condition Φ :

$$R' \subseteq R. \tag{C2}$$

Uniqueness R is unique with respect to K . That means, if there are R_1, R_2, \dots, R_n , such that each R_i satisfies the identity condition for K -objects, and every R_k is independent of each R_j , that is, every R_k is neither narrower nor wider than each R_j , then at most one of R_1, R_2, \dots, R_n provides a correct IC for K -objects.

Equivalence R must be an equivalence relation. Why? The argument goes as follows: on the left side of the biconditional in IC1* (as well as IC2*) there is an identity relation that is an equivalence relation. Consequently, the right side of the conditional is supposed to present an equivalence relation too. R must then be reflexive, symmetric and transitive.

5.4 Logical adequacy of ICs

An IC for objects of some kind K is acceptable if the relation R satisfies the requirements listed in the previous section. In the present and in the following section I will focus on a specific formal requirement for R , that of being an equivalence relation. Williamson [124], [125] and De Clercq and Horsten [21] consider the problem of intuitively plausible ICs that are not logically adequate because the relation R fails to be transitive. Their point is to show that we do not need to refuse such ICs: we can “adjust” the problem from a logical point of view. In this section I will present De Clercq and Horsten’s work in detail and in the following section I will refine their formal framework.

De Clercq and Horsten [21] take into account Williamson’s formulation of second-level ICs in [124] and [125]. The logical requirements that they discuss are independent of the debate about the possibility to reduce second-level criteria to first-level ones: they apply to both the two types of criteria. So, consider again IC2*:

$$\forall x \forall y ((x \in K \wedge y \in K) \rightarrow (f(x) = f(y) \leftrightarrow R(x, y))) \quad (\text{IC2}^*)$$

In the debate about ICs the relations considered as candidates for R often fail to be transitive. Consider some examples offered by Williamson:

- Let x, y, z, \dots range over color samples and f be the function that maps color samples to perceived colors. A plausible candidate for R might be the relation of indistinguishability. It is easy to verify, though, that such an R is not necessarily transitive: it might happen that x is indistinguishable from y and y from z , but x and z can be perceived different in color.

- Let x, y, z, \dots be person-stages and $f(x), f(y), f(z), \dots$ the persons that correspond to each stage. Consider a condition of identity for persons the following: $f(x) = f(y)$ iff x and y remember the same previous life-stages. But a middle-aged person can remember when she was young, and when she becomes old she can remember when she was middle-aged, but it may happen that she cannot remember when she was young.
- If $f(x)$ is a physical magnitude, to determine $f(x) = f(y)$ you measure x and y . If x and y differed by little, the measurement operation could give the identity of the physical magnitudes as a result. If R were defined on the basis of the measurement operations, it would turn out to be not transitive, since the sum of many little differences is not itself little.

Even if the examples just mentioned look like plausible as identity conditions, they do not meet the logical requirement that IC2* demands as they are non-transitive. In what follows I will consider especially the first and the third examples, that is, the examples concerning perceived (or phenomenal) colors and the measurement of physical magnitudes. I do not wish to consider the example concerning IC for persons because the relation ‘have the same memories of the previous life-stages as’ suggested by Williamson is not a satisfactory condition for personal identity. Suppose that two different persons, J and L get in a coma after an accident at two distinct moments of time, and that they awake from the coma exactly at the same moment of time t . Suppose that at time t they have both lost all the memories of their previous life. At t , then, they trivially have the same memories, namely no one. So, by Williamson’s identity condition, we should conclude they are the same person. But we would never claim that J and L are the same person. To provide the sortal *person* with a satisfactory identity condition is a complicated matter. I prefer not to go into it but keep the focus on examples of identity conditions that appear to be intuitively plausible.

Consider in particular the example of perceived colors, i.e. colors as they appear to us. The IC can be considered in two ways: as a two-level or as a one-level criterion. In the first case, given a set of color samples, the relation R that is supposed to represent the identity condition holds between color samples and the relation is ‘being indistinguishable in color from’. In the second case, you can rewrite the example as follows: the set on which R holds is the set of perceived (phenomenal) colors and the relation R is ‘being indistinguishable/indiscriminable from’.

It is well known that the relation ‘being indistinguishable in color from’

(or, if you prefer, ‘being indistinguishable/indiscriminable from’) is not transitive. However, people use such a relation when they have to judge about colors in normal cases. They usually do not have at their disposal an instrument (e.g. a Munsell Chart) with which they can compare colors in a precise way. They rely only on their perception, even if it is fallible.

In normal cases, relying on oneself’s perception is enough to correctly assert identity statements about colors. Therefore, Williamson and De Clercq and Horsten do not want to exclude ICs commonly used by people, even if they are not transitive. They want to save intuitions on ICs. So, the problem they have to face is the following: how can we stay as close as possible to the intuitive candidates for identity conditions and at the same time get logically adequate ICs?

Williamson’s suggestion is to find a relation which approximates a non-transitive relation R . The relation which is the best approximation to R must be reflexive, symmetric and transitive and it will be used to replace R . To find such a relation, Williamson gives up the requirement for the identity condition to be both necessary and sufficient.

Take a relation R to be the best candidate for the identity condition Φ with respect to some kind of objects $f(x)$ s, and let R be reflexive and symmetric, but not transitive. Then, define some equivalent relations R', R'', \dots which are approximations to R . To determine whether R' is the best approximation to R , Williamson proposes two constraints that R' must meet:

Weak constraint : no candidate relation R'' should approximate R better than R' .

Strong constraint : R' should approximate R better than any other candidate R'' .

Williamson proposes two ways to find an adequate equivalence relation R' to substitute a non-transitive R :

Approach from above This approach seeks the smallest equivalence relation R^+ such that $R \subseteq R^+$. That means, some $f(x)$ and $f(y)$ that are not identical under R turn out to be identical under R^+ or, equivalently, R^+ is a super-relation of R . The equivalence classes given by R^+ are numerically more than the equivalence classes given by R . R^+ always exists and is unique. The IC of this form:

$$\forall x \forall y ((x \in K \wedge y \in K) \rightarrow (f(x) = f(y) \leftrightarrow R^+(x, y))) \quad (\text{IC}^+)$$

provides a sufficient, but not necessary, condition for the identity of $f(x)$ s.

Approach from below This approach seeks the largest equivalence relation R^- such that $R^- \subseteq R$. That means, R^- is a sub-relation of R since not all the ordered pairs in R are ordered pairs in R^- . R^- always exists on the assumption of the Axiom of Choice but it is not unique. To decide which relation can be preferable over others, some constraints can be put. One of it is what Williamson calls *Minimality Constraint* (see section 5.4.4). According to it, the relation R^- to be preferred is the one with the minimum number of equivalence classes. An IC of this form:

$$\forall x \forall y ((x \in K \wedge y \in K) \rightarrow (f(x) = f(y) \leftrightarrow R^-(x, y))) \quad (\text{IC}^-)$$

provides a necessary, but not sufficient, condition for the identity of $f(x)$ s.

There are cases where a proposed identity condition is necessary for some kind of entities. For instance, the condition of being perceptually indistinguishable is a plausible, necessary identity condition for colors. On the contrary, there are other kinds of entities for which a good IC is sufficient: certain forms of mental continuity can be considered as a sufficient condition for personal identity. But this is not so obviously sufficient. There are not always good reasons to consider a condition as obviously necessary or sufficient for the identity of some kinds of entities:

As long as there is no compelling reason to regard a condition either as obviously necessary or as obviously sufficient (as will often be the case) it seems more reasonable to keep all options open instead of retreating immediately to either of the two fall-back positions considered by Williamson. (De Clercq and Horsten [21], p. 373.)

De Clercq and Horsten go for a third option: giving up both the necessity and the sufficiency of the identity condition.

De Clercq and Horsten propose an approach to find approximating relations that is alternative to Williamson's ones and is called *overlapping approach*: the equivalence relation that is sought partially overlaps R , instead of being a sub- or a super-relation with respect to R . The relation defined in the overlapping approach is called R^\pm .

The advantages of the overlapping approach are the following: (i) it can be used for cases where the most plausible identity condition is neither sufficient nor necessary and (ii) it can generate closer approximations than Williamson's approach.

De Clercq and Horsten's proposal is based on the assumption that R is not indeterminate: any two objects either stand in the relation R or they do not. De Clercq and Horsten show that their approach is able to deal with the same problems with which Williamson's approach deals.

5.4.1 Overlapping approach: foundational issues

To make clear the goal of their proposal, the authors address some foundational issues.

- What is the aim of searching for a technical substitute? Recall the problem at issue: given a kind K of objects, we have a relation that is a good intuitive candidate as an IC for K -objects, but for logical reasons we cannot accept it. So, another relation suitable for replacing the candidate is sought. The candidate for such a replacement can be regarded as a sketch of our common sense IC or, in alternative, as the common sense criterion itself. Depending on which of those two options you choose, the substitute that is sought can be considered in two different ways: if you assume the first interpretation of the candidate relation, the search for the substitute turns out to be an attempt to formulate the common sense IC in a more complete way, while if you assume the second interpretation, you want to improve the IC. De Clercq and Horsten remain neutral with respect to those interpretation. The choice between the alternatives depends on the IC at issue from time to time and on how much improvement we think our common sense needs.
- Should we search for a definable (meaningful) substitute? That means, should we be able to define the relation in terms of concepts that we already possess? It seems to be plausible to require identity conditions to be meaningful. Nevertheless, the approximation process can lead to a condition that can be grasped only by enumerating the elements of its extension and is not intuitively captured by concepts in our possession. Moreover, there is nothing that can make sure that our concepts are sufficient to define any best approximation to the candidate relation.
- The fact that ICs need improvement reflects a more general linguistic phenomenon, that is, reflects imperfection of ordinary language concepts. In spite of their imprecision, the concepts associated to ordinary language are useful in our communication and submitted to rules. Imprecision in identity conditions does not prevent us from applying ICs in ordinary circumstances. Even if ICs are not always fully adequate (for

instance, they can lack some logical requirements), they are sufficiently adequate for our purposes.

5.4.2 Overlapping approximations

In the main part of the article, De Clercq and Horsten provide an equivalence relation R^\pm that closely approximates R and achieves that task better than R^+ or R^- . For the sake of clarity, consider an example.

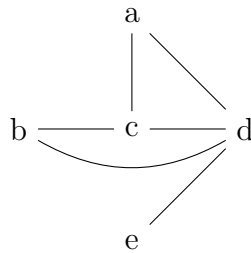
Given a function f , let the domain of objects for f be the following:

$$\mathcal{D} = \{a, b, c, d, e\}.$$

Let R be a candidate relation for the identity condition for $f(x)$ s, reflexive and symmetric. Assume that for all individuals x in \mathcal{D} , $R(x, x)$ holds. If R holds between two elements x and y , write the pair as follows: \overline{xy} . Let R on \mathcal{D} be the following⁵:

$$R = \{\langle a, c \rangle, \langle a, d \rangle, \langle b, c \rangle, \langle b, d \rangle, \langle c, d \rangle, \langle d, e \rangle\}.$$

R is not an equivalence relation. In fact, it fails to be transitive. For instance, R holds between a and d and between d and e , but it does not hold between a and e , as we would instead expect from an equivalence relation. A relation can be displayed by the means of a graph whose nodes represent the objects of the domain and edges connecting the nodes represent the relation holding between them. R is represented by the following graph:

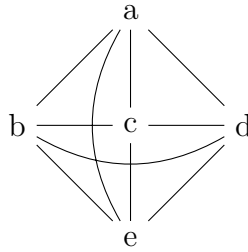


Consider now what R^+ looks like in this case. It is unique and it is the smallest equivalence relation that is a superset of R , that is:

$$R^+ = \{\langle a, b \rangle, \langle a, c \rangle, \langle a, d \rangle, \langle a, e \rangle, \langle b, c \rangle, \langle b, d \rangle, \langle b, e \rangle, \langle c, d \rangle, \langle c, e \rangle, \langle d, e \rangle\}.$$

⁵For the sake of simplicity, I omit writing pairs of form $\langle x, x \rangle$, given that $R(x, x)$ holds for every element x of the domain.

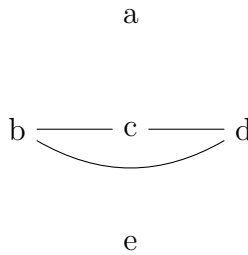
It is represented by the following graph:



On the contrary, R^- is not unique. For instance, one of the largest equivalence relations included in R is the following:

$$R^- = \{\langle b, c \rangle, \langle b, d \rangle, \langle c, d \rangle\}.$$

The correspondent graph is the following:



To determine whether R^+ or R^- is the best approximation to R , first you measure the degree of unfaithfulness of R^+ and R^- with respect to R . Such a degree is the number of revisions you must make to get R^+ or R^- from R . A revision is any adding or removing of an ordered pair to or from R . In the example considered above, R^+ is obtained by adding four ordered pairs to R and R^- by removing three ordered pairs. The degree of unfaithfulness of R^+ is 4 and the degree of R^- is 3. Thus, R^- is closer to R than R^+ . That means that with R^- , you stay closer to your intuitive identity condition R , because R^- modifies R less than R^+ .

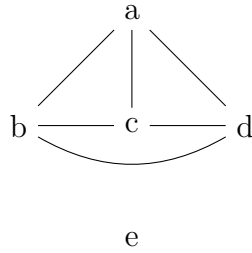
Consider now the following equivalence relation:

$$R^\pm = \{\langle a, b \rangle, \langle a, c \rangle, \langle a, d \rangle, \langle b, c \rangle, \langle b, d \rangle, \langle c, d \rangle\}.$$

With respect to R , R^\pm adds one ordered pair and removes another one. So the degree of unfaithfulness of R^\pm is 2; that is, less than both R^+ and R^- . It is, then, the best approximation to R . An overlapping relation can

be closer to R than the relations obtained with the approaches from below and from above.

R^\pm is an overlapping relation with respect to R and is a kind of hybrid relation between R^+ and R^- , since it both adds and removes one ordered pair. The following is its corresponding graph:



An overlapping relation can be closer to R than the relations obtained with the approach from below and from above in virtue of the fact that it is allowed both for adding and removing pairs to or from R . In other words, if you allow a relation to be neither necessary nor sufficient for being an identity condition, you are able to get closer approximations to a given, non-transitive R .

5.4.3 Quantitative vs. qualitative approach

In this section a quantitative and a qualitative standard of closeness are defined and compared. Williamson criticizes a quantitative standard based on “counting pairs” judging it as unpractical. By contrast, De Clercq and Horsten criticize the qualitative approach proposed by Williamson, define a quantitative approach and emphasize its positive aspects.

Consider first the quantitative approach to closeness suggested by De Clercq and Horsten. The basic ideas have been already employed above when the measurement process of degrees of unfaithfulness has been described.

Consider the graphs given above. The number of edges corresponds to the number of ordered pairs given by the relation R .

Definition 26 For any given finite R and any equivalence relation E on the same domain as R : $\mu_R^+(E)$ is the set of edges belonging to E but not to R and $\mu_R^-(E)$ the set of edges belonging to R but not to E .

Definition 27 Let $\mu_R(E)$ be the set of mistakes of E and be defined as follows:

$$\mu_R^+(E) \cup \mu_R^-(E).$$

Informally speaking, $\mu_R(E)$ represents the number of mistakes of E with respect to R insofar as it is the number of modifications of R , including both the additions and the removals of edges.

Definition 28 *Given a relation R , a relation E is the quantitatively best approximation to R iff the size of $\mu_R(E)$ is minimal.*

We can also read definition 28 as follows: given some potential approximations to R , E_1, E_2, \dots, E_n , the quantitatively best approximation E_k is such that the size of $\mu_R(E_k)$ is minimal.

In order to find the best equivalence approximation to a given R , Williamson does not require to count the number of revisions of R . He supports a qualitative approach that is defined as follows:

Definition 29 *Given a reflexive and symmetric relation R , E is a qualitatively best approximation to R iff E is an equivalence relation on the same domain of R and there is no equivalence relation E' such that $\mu_R(E') \subset \mu_R(E)$.*

Roughly speaking, a best approximation E to R is a relation such that any further refinement of it would produce a too large modification of R . The best approximation in the quantitative sense is always the best approximation in the qualitative sense, but not viceversa. For instance, the relation R^- as stated in the example above is the best approximation to the given R only according to the qualitative approach. Compare $\mu_R(R^-)$ with $\mu_R(R^\pm)$:

$$\mu_R(R^-) = \{\overline{ad}, \overline{de}, \overline{ac}\}$$

$$\mu_R(R^\pm) = \{\overline{ab}, \overline{de}\}$$

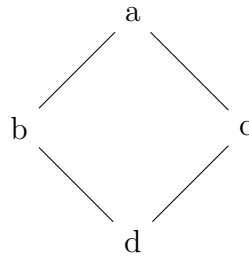
$\mu_R(R^\pm)$ is not a subset of $\mu_R(R^-)$, so R^- is considered by Williamson a qualitative best approximation, but according to the quantitative approach R^\pm is the best approximation as $\mu_R(R^\pm)$ contains less edges than $\mu_R(R^-)$.

5.4.4 Possible refinements

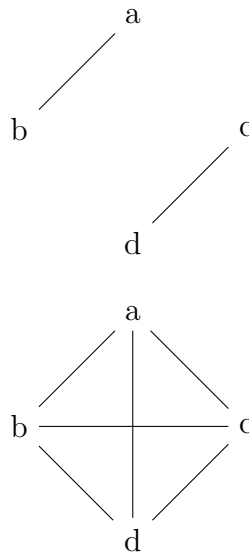
The overlapping approach is more general than the approach form above and from below and leads to the closest approximations to a given relation R in virtue of the fact that identity conditions are allowed to be neither necessary nor sufficient. Nevertheless, best approximations are not unique. How to choose among them?

In case we get too many best approximations, some conditions can be put in order to choose one among them. Williamson suggests using the so-called *Minimality Constraint*, according to which the relation with the smallest number of equivalence classes is preferable to be chosen.

The quantitative approach does not always satisfy the Minimality Constraint. Given a candidate R , it can happen that more than one relation has the minimal size of $\mu_R(E)$. But this minimal number does not also ensure that there is a minimal number of ordered pairs. Consider, for instance, the reflexive and symmetric relation R^* represented by the following graph ($R^*(x, x)$ is assumed to hold for all x):



With respect to R^* two equivalence relations that are equivalently best approximations can be defined. They are represented by the following graphs:



In fact, the size of $\mu_{R^*}(E)$ is 2 for both the two relations. The difference between them is that they have a different number of ordered pairs and according to the Minimality Constraint, the one with the smallest number must be preferred.

Even if we could find further constraints and obtain, at the end, only one best approximation, we might still wonder whether that approximation is a correct IC.

According to Williamson, there are two views from which consider the problem of the non-uniqueness of the identity condition:

Ignorance view : There is exactly one best approximation, but because of our epistemic shortcomings we may never be able to determine which one it is.

Supervaluation view : there is no fact of the matter that makes an equivalence relation to be the best approximation. The identity condition consists of a (unique) partial equivalence relation R_S whose extension $\mathcal{E}(R_S)$ and anti-extension $\mathcal{A}(R_S)$ are defined as follows (let $BA(R)$ be the set of best approximations of the relation R):

- $\langle a, b \rangle \in \mathcal{E}(R_S) \Leftrightarrow$ for all $E \in BA(R) : \langle a, b \rangle \in E$;
- $\langle a, b \rangle \in \mathcal{A}(R_S) \Leftrightarrow$ for all $E \in BA(R) : \langle a, b \rangle \notin E$.

$\mathcal{E}(R_S)$ together with $\mathcal{A}(R_S)$ does not exhaust the domain of R and R_S turns out to be indeterminate.

De Clercq and Horsten propose a third view, that, in my opinion, can be called *degree view*. Given a domain \mathcal{D} and a relation R , you assign weights to the ordered pairs of elements of \mathcal{D} . This means that a number is assigned to each ordered pair; such a number represents the degree of certainty at which R holds between the two elements of the pair. The weight assigned to an ordered pair is a number of the scale of real values between 1 and 0. If the weight assigned to the pair $\langle a, b \rangle$ is 1, then a and b are R -related with absolute certainty. If the weight is 0, a and b are not R -related with absolute certainty. Thank to the weights, we can also calculate the degree of unfaithfulness with real numbers. Consider an example. Let E be an equivalence relation that approximates R . If the weight assigned to the pair $\langle d, e \rangle$ is 0.65 in R and, according to E , d and e are R -related, then the unfaithfulness of E is increased with 0.35.

After all, though, the authors seem to prefer a pragmatic solution to the problem of non-uniqueness. Such a problem is considered as it reflected the indeterminacy of our identity concepts and the preference for the best equivalence relation depends on the purposes of the linguistic agents.

5.5 Approximating identity conditions. A granular proposal

In this section I will present two problematic aspects of De Clercq and Horsten’s approach. I will then focus mainly on one of them and suggest how to improve their formal framework with respect to it. More precisely, I wish to embody De Clercq and Horsten’s proposal in an enlarged framework that takes into account contexts and levels of granularity, adapting the formal model for gradable adjectives presented in Chapter 3.

In my proposal I will take into account and revise the example about phenomenal colors given by Williamson. The case of colors is a well-known example of failure of transitivity and it has been discussed also in other places in the philosophical literature. I think that some observations by Hardin [40] on this issue are remarkable.

Hardin observes that many philosophers endorse a view according to which the following principle (that I will call **NT** from now on) holds:

There exist triples of phenomenal colors x , y and z , such that x is indiscriminable from y and y is indiscriminable from z , but x is discriminable from z .

By ‘indiscriminability between colors’ Hardin means ‘perceptual indistinguishability’. By **NT** the relation of perceptual indistinguishability fails to be transitive; therefore, the IC for colors based on the relation of indistinguishability is incoherent: on the left side of the biconditional there is a necessarily equivalent relation (identity), on the right side a not necessarily equivalent relation. This problem seems to affect any semantic account of color terms relying on everyday uses of color predicates. Hardin argues that phenomenal colors are themselves indeterminate, that is, there is no sharp color-discrimination threshold; since the truth of **NT** is based on the assumption that there is a discrimination threshold, refusing this assumption implies refusing **NT** too.

In non problematic cases, i.e. when we have to make judgments on very different colors, we report our observations using coarse-grained predicates because we do not need to express shade differences. When we have to deal with borderline cases of colors, we tend to be more precise in using color predicates. More fine-grained color predicates are used in color science and technology, but in everyday life people do not use them; that is not just because there are limits of hue discriminability, but because of “something like the limit of useful naming of phenomenal hues for the purposes of communicating between people” (Hardin [40], p. 221). Put otherwise, the number of

possible color discriminations is much higher than the number of color terms normally used. Why? First of all, there is a variability in discrimination between observers; second, people observe colors under normal conditions such as changing light, contrast, shadows, and not under standard conditions, and normal conditions make color comparisons problematic; third, it is more difficult to compare a color with a mental standard (like the standard of ‘red’ that one could have seen in the Munsell Chart) than with another color perceived at the same time.

So, color perception is influenced by many factors and the use of color predicates is somewhat sloppy. Hardin suggests that to answer a question like “What are the boundaries of red?” we must first

specify, explicitly or tacitly, a context and a level of precision and [...] realize the margin of error or indeterminacy which that context and level carry with them. (Hardin [40], p. 230.)

In the following analysis, I wish to show that De Clercq and Horsten’s framework can be improved if you consider the use of ICs in a context and in a level of precision. Moreover, I agree with Hardin’s belief that the nature of our purposes imposes limits on the precision of our utterances: a too large set of color predicates would make our judgments more precise, but would also hinder a profitable communication among agents.

The IC for phenomenal colors is an example of an IC that has mostly an epistemic function: we do not know precisely whether two items belonging to the kind of colors are identical. We rely only on our perception which is fallible. So, we express an IC for colors in a logically inadequate way. Williamson and De Clercq and Horsten believe that there are logically adequate ICs and try to capture them by approximating our intuitively good, but logically inadequate ICs.

5.5.1 Problematic issues

It is worthwhile considering two problematic aspects in De Clercq and Horsten’s proposal.

The first problem concerns what they take to be the candidate relation for identity condition R . They claim that the reflexive and symmetrical R taken as common-sense identity condition is *obvious*. The authors do not characterize, though, what they mean with *obviousness*. The first question is: what does an obvious candidate relation look like? From which point of view do you consider such a relation obvious? From a cognitive, epistemic, ontological point of view? Suppose that the notion of obvious candidate has

been characterized, and thus the notion of obviousness has been made precise, and consider the following case: given some K -objects, we have more than one obvious candidate IC for K s. Which one should we choose? De Clercq and Horsten would probably answer: the most obvious one. But suppose there are two equally obvious criteria of identity for K -objects, better than (= more obvious than) all the other ones. How to choose among them? The two authors as well as Williamson discuss some ways to choose among alternative equivalence approximations to R , but they do not deal with the problem of how to choose the initial relation R . Since choosing R is a fundamental step for all their technical apparatus, it would be worthy to make some considerations on what such an R is supposed to look like. With respect to this issue, some requirements on R has already been given in section 5.3.1.

Consider a second problematic issue. De Clercq and Horsten assume that the candidate R for the identity condition is discrete in this sense: given two objects x and y , either it is clear that R holds between x and y or it is clear that it does not hold. This assumption seems to be plausible and acceptable. Consider, for instance, Williamson's example concerning perceived colors: unlike vague predicates or relations, given two color samples either you are able to distinguish them, or you are not. It would not make sense to say that two color samples are perceived neither as the same, nor as different. In fact, what happens when you see two color samples? If you notice a difference among them you will claim that they are different colors. If you do not notice any difference, you will claim that they are the same color. However, can we imagine some case where the relation R for perceived colors holds between two objects x and y in a given situation, while in another situation R does not hold between the same objects x and y ? If the identity condition is taken to be neither necessary nor sufficient, nothing prevent us to think of this kind of cases. Consider the following situations:

- a** You see two monochromatic spots, A and B, and you do not detect any difference with respect to their colour. Following Williamson, you claim that they have the same colour, because they are perceptually indistinguishable. Now, suppose you add two further monochromatic spots, C and D, such that they are perceptually distinguishable. However, A is indistinguishable from C and B from D. In such a scenario, you can accept to revise your previous judgement and say that A and B are distinct.
- b** You see two colour samples A and B from a distant point of view such that you are not able to distinguish A-colour from B-colour. You say that A and B have the same colour. Now, you get closer to them and detect a

difference between them. So, you revise your previous judgement and say that A and B are distinct.

- c** You see two monochromatic spots again, A and B. You perceive them as equally, say, orange. Nevertheless a friend of yours, who is a painter, tells you that she perceives them actually different: B is more yellowish than A. According to her color perception, which is more refined than yours, there are more differences among color samples than you detect.

What can we say about **a-c**? The objects which identity statements are about are objects which we have a sensible experience of. Someone could think that, in such cases, the truth value of identity statements differs from context to context. Similar situations may happen also for abstract entities, like, for instance, concepts, but that is not of my concern here. Even if identity is maintained to be an absolute relation in each context, when we have to make an identity statement on the basis of an identity condition that represents a non necessarily transitive relation, it may happen that two objects that are judged as identical in a context can be judged as distinct in a different context, for instance when the context contains a larger number of objects. I think that the context-dependency issue is relevant for all the cases of identity conditions where transitivity fails. Nevertheless, not only contextualist issues play a role here. Let us analyze examples **a**, **b** and **c** in a deeper way.

5.5.2 Contexts and levels of granularity

Example **a** shows that our perception of colors depends on the range of colors we see at some moment. Better said, comparing a color sample with one or more color samples makes our judgments about colors differ. Thus, R can vary across contexts. For instance, consider a domain $\mathcal{D} = \{a, b, c, d, e\}$ and a context o , that is a subset of \mathcal{D} : $o = \{a, b\}$. Suppose $R = \{\overline{ab}\}$ in context o . Consider now an enlarged context, o' containing a and b plus two other elements, c and d : $o' = \{a, b, c, d\}$. In o' you may have the following R -pairs: $\overline{ac}, \overline{ad}$, but not \overline{ab} . Informally, two objects, a and b , that are equally indistinguishable in a context, and therefore judged as identical, can be judged to be different in another context.

Examples **b** and **c** present a different issue than example **a**. The context is fixed in **b** and **c**: we have always the same two color samples in front of us, no color sample is added or removed. Given the same context, R varies along different granular levels of observation. Suppose that from a distant and coarse point of view, you make an identity statement about some objects

x and y in a context o via the relation R : for instance, $x = y$. From a more precise, fine-grained point of view, you can make a different identity statement about the same objects x and y in the same context o via R : for instance, $x \neq y$. That means that you can look at the elements of a context under different standards of precision, which I call granular levels. The finer the level is, the more differences between the individuals can be detected⁶.

In the following paragraph I try to formalize the notions of contexts and granular levels on a model-theoretic fashion and integrate them with De Clercq and Horsten's formal treatment of approximating relations.

5.5.3 Granular models

Let \mathcal{L} be a formal language through which we can represent English expressions. \mathcal{L} consists of:

- individual constant symbols: \bar{a}, \bar{b}, \dots (there is a constant symbol for each element of the domain);
- individual variables: x_0, x_1, x_2, \dots (countably many);
- two-places predicate symbols P_1, P_2, \dots (countably many);
- usual logical connectives with identity, quantifiers.

The language is very simple. It gives us exactly what we need for a formal characterization of identity conditions.

The set of terms consists of individual constant and individual variable symbols.

Formulas are defined as follows:

1. If t_1, t_2 are terms, then $P_1(t_1, t_2), P_2(t_1, t_2), \dots$ are formulas;
2. If t_1, t_2 are terms, then $t_1 = t_2$ is a formula;
3. If ϕ, ψ are formulas, then $\phi \square \psi$ is a formula, where \square is one of the usual logical connectives;
4. If ϕ is a formula, then $\neg \phi$ is a formula;
5. If ϕ is a formula, then $\forall x_i \phi, \exists x_i \phi$ are formulas.

⁶The point of view of the painter too can be seen as a fine-grained observational level.

Let me give now an interpretation to \mathcal{L} . Let \mathcal{D}_K be a fixed non empty domain of objects of kind K^7 . A context o is defined as a subset of the domain \mathcal{D}_K . So, the set of all contexts O in \mathcal{D}_K is the powerset of \mathcal{D}_K :

Definition 30 $O = \wp(\mathcal{D}_K)$.

Let $\mathcal{M} = \langle \mathcal{D}_K, R \rangle$ be a fixed model or granular structure. \mathcal{M} is a structure consisting of the sorted-domain \mathcal{D}_K , and a binary relation R (a two-places predicate). \mathcal{D}_K is the domain of K -objects. Assume that R is reflexive and symmetric, but not (necessarily) transitive. Moreover, R is a primitive relation. R pairs the elements that are indistinguishable according to the identity condition it represents. For instance, in the case of color samples R gives rise to a set of ordered pairs, each of them consisting of elements that are indistinguishable with respect to their (perceived) color.

R varies across contexts. Given a granular structure \mathcal{M} , if the relation R fails to be transitive with respect to some (if not all) contexts $o \subseteq O$, then the formal framework given by De Clercq and Horsten is applied. This means that in each context o an equivalence overlapping relation R^\pm can be defined for a non-transitive R^8 . If a relation R is transitive in a context o , then in that case R^\pm coincides with the given R .

R does not vary only across contexts, but also across granular levels. Granular structures belong to different granular levels. Given the same context $o \subseteq O$, different granular structures can give different sets of ordered pairs generated by R . For instance, it can happen that according to the most coarse-grained granular structure the relation R holds among all the elements of the context considered. No difference is detected among them (with respect to some property), so all of them are considered indistinguishable. On the contrary, according to more fine-grained granular structures R holds between a less number of elements of o .

We can partially order the granular structures from the coarsest to the finest with respect to any context $o \in O$. I define a partial order for granular structures⁹. First, we need to define the relation \geq^c between cardinality of sets:

Definition 31 *Given an $o \in O$, for all the pairs \overline{xy} in M and \overline{xy} in M' , $|\{\overline{xy}^{M'}\}| \leq^c |\{\overline{xy}^M\}|$ iff the number of $\overline{xy}^{M'}$ is less than or equal to the number of \overline{xy}^M in o .*

⁷I assume a sorted-domain for the sake of simplicity. In alternative, I could have assumed that the domain is partitioned into comparison classes and that contexts are subsets of comparison classes, like the model for gradable adjectives.

⁸If you prefer to maintain one of Williamson's approaches, instead of R^\pm you can define R^+ or R^- .

⁹The idea is the same as in Chapter 3.

Now we can define the relation \leq^* (*finer than*) between granular structures with respect to some context $o \in O$:

Definition 32 *Given a context $o \in O$, M' is finer than M iff the number of $\overline{xy}^{M'}$ is less than the number of \overline{xy}^M , that is:*

$$M' \leq^* M \text{ iff } |\{\overline{xy}^{M'}\}| \leq |\{\overline{xy}^M\}|.$$

What we get, then, is a series of granular structures linearly ordered by the relation \leq^* .

5.5.4 Example

To make the point clear and see how the model works, consider the following example. Let $o = \{a, b, c, d, e\}$ be a given context. Consider two granular structures, $\mathcal{M}_1 = \langle \mathcal{D}_K, R \rangle$ and $\mathcal{M}_2 = \langle \mathcal{D}_K, R \rangle$. According to \mathcal{M}_1 , we have: $R = \{\overline{ab}, \overline{bc}, \overline{de}\}$. It is not transitive (a is indistinguishable from b and b from c , but a is not indistinguishable from c). The best overlapping approximation is the following: $R^\pm = \{\overline{ab}, \overline{bc}, \overline{ac}, \overline{de}\}$. The pair \overline{ac} has been added. The degree of unfaithfulness of R^\pm is 1. According to \mathcal{M}_2 , we have: $R = \{\overline{ab}, \overline{bc}, \overline{cd}, \overline{de}, \overline{ce}\}$. In this case R it is not transitive either. The best overlapping approximation removes the pairs \overline{ab} and \overline{bc} : $R^\pm = \{\overline{cd}, \overline{de}, \overline{ce}\}$.

Given the context o , from the definitions given above follows that $\mathcal{M}_1 \leq^* \mathcal{M}_2$: \mathcal{M}_1 is finer than \mathcal{M}_2 because its relation R gives a less number of pairs than the relation R in \mathcal{M}_2 .

5.5.5 Objections and replies

Some objections can be raised against the proposed formal characterization of ICs, as well as some problems in the account are to be underlined. I try to outline here some objections and problems, and sketch a reply to them.

- It has been claimed that ICs are associated with sortal concepts, that is, with concepts that answer the question “What is x ?”. The examples of non-transitive ICs considered are associated to kinds of objects like colors and physical magnitudes. It is not clear, though, whether colors or physical magnitudes are to be considered sortal concepts. For instance, the adjective ‘red’ does not correspond to a sortal concept: we do not individuate an object x saying “ x is a red”.

This first objection seems to attack the notion of IC itself or, better said, the thesis that ICs are necessarily associated with sortal concepts.

In my thesis I accepted the standard thesis according to which only concepts associated with ICs are sortals. Being associated with an IC is a necessary condition for concepts to be sortals, but not a sufficient one. The possibility for some concepts to be associated with ICs without being sortals is not excluded.

Moreover, what happens if we consider ‘red’ as a substantive standing for the color red, e.g. “Red suits you”? In this case, ‘red’ can be considered as a sortal noun and, therefore, it would be easy to accommodate the problem via a revision of the formulation of the IC. We can formulate a one-level IC for colors as follows: given two perceived colors x and y , x is identical to y iff x is indistinguishable from y .

- A second objection runs as follows: what changes from context to context or from granular level to granular level is the extension of the relation. But we are dealing also with epistemic issues. In a certain context and granular level we make an identity judgment according to a certain relation R . When the context or the granular level changes we make a sort of revision of our previous identity judgment. So, if we want to be faithful to our intuitions, an intensional treatment would be more appropriate in order to account also for the epistemic issues.

I decided to provide an extensional model following Williamson’s and De Clercq and Horsten’s approaches. However, this second objection is very important. An intensional treatment of ICs would be interesting to be provided especially if you consider not only the ontological function, but also the epistemic one. If the goal is to model how we know and use ICs, we should think of an intensional formal framework. That is a possible further development of the account.

- The proposed model for accounting for ICs is not suitable for an infinite domain. The domain of objects must be finite. The applicability of the model is then reduced to some specific cases, while it should be generalized.

As already mentioned, the model for approximating ICs has been developed to face logical problems arising from the intuitive use of ICs for everyday problems (color comparisons and the like). De Clercq and Horsten too are aware of the problem that their approach is applicable only to finite domains. However, they attempt to accommodate the problem and suggest reducing infinite graphs to finitary graphs. In a nutshell, it is worthwhile considering infinite graphs because we deal with relations that are potentially infinite, for instance the relations underlying the Sorites paradox. However, the transitivity failure of some

relations is at concern here. Such a problem is shown by finite graphs, so there is nothing bad to represent the problem and the solution only using finite graphs. Moreover, as already mentioned in the previous chapters, it is rare that people in ordinary life make inferences with a great (even infinite) number of steps. The fact that infinite relations are not easily handled is a logical problem. Nevertheless, since we are dealing with ICs as they are commonly used by people and not by logicians, the infinity issue does not play a relevant role in the treatment of ICs.

- Consider the following problem: If ICs have the function of answering questions (EQ), (OQ), and (SQ), which of those questions is answered by an intuitive IC that contains a non-transitive relation R ? Moreover, does an IC with an approximated relation like R^\pm answer the same question or a different one?

It seems plausible to claim that an IC containing a non-transitive relation R answers (EQ). Consider the IC for phenomenal colors: as we have seen, we do not know precisely whether two perceived colors are identical. We rely only on our perception, which is fallible. Therefore, the IC for colors we express is not logically adequate. This means that this IC cannot establish the identity of colors in reality, since it relies only on our fallible perception. However, it is sufficient for our pragmatic or epistemic purposes of color comparison.

Which question does an IC containing an approximated relation such as R^\pm answer? The relation R^\pm is logically adequate; therefore, thank to it we can determine whether or not two items are actually identical in reality. So, it is plausible to think that an IC containing an approximating relation answers (OQ).

5.5.6 Summing up

In this section I have tried to improve De Clercq and Horsten's formal framework by considering the relation R in relation with contexts and granular levels. The suggestion is, briefly, the following: before determining the closest approximation to R , you have to fix a context and a granular level of observation (a granular structure), because R can vary across contexts and granular levels. If, in the context and according to the granular level you fixed, R fails to be transitive, you can build the closest approximation to R for that context and that level.

Chapter 6

Conclusion

Throughout the dissertation I have tried to show that the relation of indistinguishability, which we normally take to be an equivalence relation, fails to be transitive in some cases. I analyzed two of those cases: vague terms and identity criteria.

It has been observed that natural language speakers use vague terms and identity criteria to make judgments on reality in an effective way; in other words, they can communicate information using vague terms and identity criteria, even if those present shortcomings from a logical point of view. In a nutshell, the leading question of my research has been the following: how is it possible to communicate or to make meaningful judgments about reality using logically inadequate tools? I have tried to answer this question by providing two models: the first accounts for vague count nouns and gradable adjectives and the second for identity criteria. More precisely, the aims of the research have been (i) to show how natural language speakers use adjectives and identity criteria to make judgments about the objects of their experience and (ii) to try to solve some logical problems that arise from such a use. I have suggested using two notions to account for the use of gradable adjectives and identity criteria: context dependence and granularity. However, the models still present some open problems and further refinements are desirable.

As far as the model for gradable adjectives is concerned, I have proposed considering it as an alternative to the degree-based approach. However, the latter is able to account for several aspects of gradable adjectives. Is my model able to account for all the same aspects? For instance, degree-based theories are also able to effectively deal with measure phrases when those are added to adjectives. Consider, for instance, the following sentences:

- (1) John is 1.80 m tall.

(2) John is 30 cm taller than Mary.

Measure phrases can be associated with positive gradable adjectives (e.g. they can be associated with ‘tall’, but not with ‘short’) and with comparatives, as in (2). Measure phrases denote or quantify over degrees that can be mapped onto a countable, finite set of units. Sentences containing measure phrases associated with adjectives find a natural interpretation within a degree-based theory, which assumes scales of degrees to account for gradable adjectives.

It is not clear if and how measure phrases can be modeled in the granular and contextualist approach as I have suggested. A further refinement of the model consists in verifying whether or not it is possible to interpret sentences containing measure phrases within the model itself. If it is possible, then the model can be improved; if it is not possible, then the reasons for this weakness must be clarified.

As far as the model for identity criteria is concerned, it could be considered as a first step towards a formal treatment of identity criteria. In Chapter 5 I have considered only the transitivity requirement for the identity condition (given by the relation R) and I came up with a model that attempts to accommodate identity conditions that fail to be transitive. However, it would also be worthwhile taking into account other requirements for the identity condition, such as those given by Lombard and Brand: non-vacuousness, informativeness, partial exclusivity, minimality, non-circularity, non-tautologicity, K -maximality and uniqueness (section 5.3.1). If we want to provide a general, formal treatment for identity criteria, first of all we should complete the list of the requirements, if possible. Secondly, we should try to formalize those requirements. The objective is to get a model that tells us if a given relation R is a suitable identity condition. However, to find the best identity condition is not an entirely abstract process. The requirements given are formal and do not tell us anything about the content of such a condition. Our knowledge, intuitions or experience about the domain of discourse suggest the content of the identity condition for a certain kind of objects; then, the formal requirements can be used to rule out the inadequate identity conditions and find - possibly - the *form* of the best condition, i.e. the form of the criterion that tells us how things are in the world. Finding the *content* of the identity condition, however, lies out of the scope of a logic framework.

Bibliography

- [1] ARMOUR-GARB, B. Diagnosing Dialetheism. In *The Law of Non-Contradiction: New Philosophical Essays*, G. Priest, J. Beall, and B. Armour-Garb, Eds. Oxford: Clarendon Press, 2004, pp. 113–25.
- [2] BAADER, F., AND NUTT, W. Basic Description Logics. In *The Description Logic Handbook*, F. Baader, D. Calvanese, D. McGuinness, D. Nardi, and P. Patel-Schneider, Ed. Cambridge: Cambridge University Press, 2003, pp. 47–100.
- [3] BARKER, C. The Dynamics of Vagueness. *Linguistics and Philosophy* 25, 1 (2002), 1–36.
- [4] BARKER, C. Vagueness. In *Encyclopedia of language and linguistics (2nd ed.)*. Amsterdam: Elsevier, 2006.
- [5] BEALL, J., AND COLYVAN, M. Looking for Contradictions. *Australasian Journal of Philosophy* 79, 4 (2001), 564–569.
- [6] BENNETT, B. A theory of vague adjectives grounded in relevant observables. In *Proceedings of the Tenth International Conference on Principles of Knowledge Representation and Reasoning (2006)*, J. M. P. Doherty and C.A. Welty, Eds., Menlo Park: AAAI Press, pp. 36–45.
- [7] BITTNER, T., AND SMITH, B. Granular Partitions and Vagueness. In *Proceedings of Formal Ontology in Information Systems Conference (FOIS 2001)*, C. Welty and B. Smith, Eds. New York: ACM Press New York, 2001, pp. 309–320.
- [8] BITTNER, T., AND SMITH, B. A Theory of Granular Partitions. In *Foundations of Geographic Information Science*, M. Duckham, M. Goodchild, and M. Worboys, Eds. London: CRC Press, 2002, pp. 117–151.
- [9] BITTNER, T., AND SMITH, B. Vague Reference and Approximating Judgments. *Spatial Cognition and Computation* 3 (2003), 137–156.

- [10] BONE, R. Definitions for Sepsis and Organ Failure and Guidelines for the Use of Innovative Therapies in Sepsis. *Critical Care Medicine* 20, 6 (1992), 864–874.
- [11] BOSCH, P. Vagueness is Context-Dependence. A Solution to the Sorites Paradox. In *Approaching Vagueness*, T. Ballmer and M. Pinkal, Eds. Amsterdam: Elsevier, 1983, pp. 189–210.
- [12] BRAND, M. Identity Conditions for Events. *American Philosophical Quarterly* 14 (1977), 329–337.
- [13] BRAND, M. *Intending and Acting: Toward a Naturalized Action Theory*. Cambridge (Mass.): MIT Press, 1980.
- [14] CARRARA, M., AND GIARETTA, P. Identity Criteria and Sortal Concepts. *Proceedings of Formal Ontology in Information Systems Conference (FOIS 2001)* (2001), 234–243.
- [15] CARRARA, M. AND GIARETTA, P. On Identity Criteria. In *Arguments and Analysis. Papers contributed to the Sections of the 4th International Congress of the Society for Analytical Philosophy (Bielefeld 2000)* (2002), C. Nimtz, Ed., Paderborn: Mentis Verlag, pp. 426–435.
- [16] COCCHIARELLA, N. Sortals, Natural Kinds and Re-Identification. *Logique et Analyse* 20, 80 (1977), 439–474.
- [17] COZZO, C. Identità: logica e ontologia. *Almanacchi nuovi* (1996), 33–79.
- [18] CRESSWELL, M. The Semantics of Degree. In *Montague Grammar*, B. Partee, Ed. New York: Academic Press, 1976, pp. 261–292.
- [19] DAVIDSON, D. *Essays on actions and events*. Oxford: Oxford University Press, 1980.
- [20] DE CLERCQ, R. A criterion of diachronic identity based on Lockes Principle. *Metaphysica*, 6 (2005).
- [21] DE CLERCQ, R., AND HORSTEN, L. Closer. *Synthese* 146, 3 (2005), 371–393.
- [22] DEUTSCH, H. Relative Identity. In *The Stanford Encyclopedia of Philosophy*, E. N. Zalta, Ed.

- [23] DUMMETT, M. *Frege: Philosophy of Language*. London: Duckworth, 1973.
- [24] DUMMETT, M. Wang's Paradox. *Synthese* 30, 3 (1975), 301–324.
- [25] DURRANT, M. *Sortals and the Subject-Predicate Distinction*. Aldershot: Ashgate Publishing Lim., 2001. Edited by Stephen Horton.
- [26] EVANS, G. Can there Be Vague Objects? *Analysis* 38, 4 (1978), 208.
- [27] FARA, D. Shifting Sands: An Interest-Relative Theory of Vagueness. *Philosophical Topics* 28, 1 (2000), 45–81.
- [28] FELDMAN, F. Sortal Predicates. *Nous* 7, 3 (1973), 268–282.
- [29] FREGE, G. *Die Grundlagen der Arithmetik: eine logisch-mathematische Untersuchung über den Begriff der Zahl*. Breslau: Kōbner, 1884.
- [30] FREGE, G. *Die Grundlagen der Arithmetik (English translation: The Foundations of Arithmetic)*. Oxford: Basil Blackwell, 1953.
- [31] FREUND, M. A Complete and Consistent Formal System for Sortals. *Studia Logica* 65, 3 (2000), 367–381.
- [32] GARRETT, B. Vagueness and Identity. *Analysis*, 48 (1988), 130–134.
- [33] GEACH, P. *Reference and Generality. An Examination of Some Medieval and Modern Theories (Emended Edition)*. Ithaca: Cornell University Press, 1968.
- [34] GIARETTA, P. Concetti e stipulazioni arbitrarie. In *La filosofia di Gottlob Frege*, N. Vassallo, Ed. Milano: Franco Angeli, 2003, pp. 113–127.
- [35] GRAFF, D., AND WILLIAMSON, T. *Vagueness*. Dartmouth: Ashgate, 2002.
- [36] GUARINO, N., AND WELTY, C. A Formal Ontology for Properties. *Lecture Notes in Computer Science* (2000), 97–112.
- [37] GUARINO, N., AND WELTY, C. Identity, Unity, and Individuality: Towards a Formal Toolkit for Ontological Analysis. In *Proceedings of ECAI-2000* (2000), W. Horn, Ed., Amsterdam: IOS Press.

- [38] GUPTA, K. *The Logic of Common Nouns: An Investigation in Quantified Modal Logic*. New Haven; London: Yale University Press, 1980.
- [39] HALPERN, J. Intransitivity and Vagueness. *The Review of Symbolic Logic* 1, 04 (2009), 530–547.
- [40] HARDIN, C. Phenomenal Colors and Sorites. *Nous* 22, 2 (1988), 213–234.
- [41] HIRSCH, E. *The Concept of Identity*. Oxford: Oxford University Press, 1982.
- [42] HOBBS, J. Granularity. In *Proceedings of the Ninth International Joint Conference on Artificial Intelligence*, A. Joshi, Ed. Los Angeles: Morgan Kaufmann, 1985, pp. 432–435.
- [43] HORSTEN, L. Impredicative Identity Criteria. *Philosophy and Phenomenological Research* (2009). forthcoming.
- [44] KAMP, H. Two Theories of Adjectives. In *Formal Semantics of Natural Language*, E. Keenan, Ed. Cambridge: Cambridge University Press, 1975, pp. 123–155.
- [45] KEEFE, R. *Theories of Vagueness*. Cambridge: Cambridge University Press, 2000.
- [46] KEEFE, R., AND SMITH, P. Introduction: Theories of Vagueness. In *Vagueness: A Reader*, R. Keefe and P. Smith, Eds. Cambridge (Mass.): MIT Press, 1997, pp. 1–57.
- [47] KEEFE, R., AND SMITH, P. *Vagueness: A Reader*. Cambridge(Mass.): MIT Press, 1997.
- [48] KENNEDY, C. Modes of Comparison. *Proceedings of CLS 43*. forthcoming.
- [49] KENNEDY, C. Comparison and Polar Opposition. In *Proceedings of SALT*, vol. 7. Ithaca: CLC Publications, 1997, pp. 240–257.
- [50] KENNEDY, C. Gradable Adjectives Denote Measure Functions, Not Partial Functions. *Studies in the Linguistic Sciences* 29, 1 (1999), 65–80.

- [51] KENNEDY, C. On the Monotonicity of Polar Adjectives. In *Perspectives on Negation and Polarity Items*, V. S.-V. J. Hoeksema, H. Rullmann and T. van der Wouden, Eds. Amsterdam: John Benjamins Publishing Co., 2001, pp. 201–221.
- [52] KENNEDY, C. Polar Opposition and the Ontology of Degrees. *Linguistics and Philosophy* 24, 1 (2001), 33–70.
- [53] KENNEDY, C. Vagueness and Grammar: the Semantics of Relative and Absolute Gradable Adjectives. *Linguistics and Philosophy* 30, 1 (2007), 1–45.
- [54] KLEIN, E. The Semantics of Positive and Comparative Adjectives. *Linguistics and Philosophy* 4 (1980), 1–45.
- [55] KLEIN, E. Comparatives. In *Semantik: Ein internationales Handbuch der zeitgenössischen Forschung*, A. von Stechow and D. Wunderlich, Eds. Berlin: Walter de Gruyter, 1991, pp. 673–691.
- [56] KRAUSE, D., AND FRENCH, S. Quantum sortal predicates. *Synthese* 154, 3 (2007), 417–430.
- [57] KYBURG, A., AND MORREAU, M. Fitting Words: Vague Language in Context. *Linguistics and Philosophy* 23, 6 (2000), 577–597.
- [58] LEHRER, K., AND WAGNER, C. Intransitive indifference: The semi-order problem. *Synthese* 65, 2 (1985), 249–256.
- [59] LEWIS, D. *On the Plurality of Worlds*. Oxford: Basil Blackwell, 1986.
- [60] LINNEBO, O. To Be Is to Be an *F*. *Dialectica*, 59(2) (2005), 201–222.
- [61] LÖBNER, S. *Wahr neben Falsch: Duale Operatoren als die Quantoren natürlicher Sprache*. Tübingen: Niemeyer, 1990.
- [62] LOMBARD, L. *Events: A Metaphysical Study*. London: Routledge & Kegan Paul Books, 1986.
- [63] LOUX, M. *Metaphysics. A Contemporary Introduction*. London: Routledge, 1998.
- [64] LOWE, E. *Kinds of Being: A Study of Individuation, Identity, and the Logic of Sortal Terms*. Oxford: Basil Blackwell, 1989.
- [65] LOWE, E. One-level Versus Two-Level Identity Criteria. *Analysis* 51, 4 (1991), 192–194.

- [66] LOWE, E. *The Possibility of Metaphysics*. New York: Oxford University Press, 1998.
- [67] LOWE, E. Sortals and the individuation of objects. *Mind and Language* 22, 5 (2007), 514–533.
- [68] LOWE, E. *More Kinds of Being: A Further Study of Individuation, Identity, and the Logic of Sortal Terms*. Oxford: Wiley-Blackwell, 2009.
- [69] LOWE, E. J. What is a Criterion of Identity? *The Philosophical Quarterly* 39, 154 (1989), 1–21.
- [70] LOWE, J. Substance, Identity and Time. *Proceedings of the Aristotelian Society - Supplementary Volumes*, 62 (1988), 61–78.
- [71] LOWE, E.J. Objects and Criteria of Identity. In *A Companion to the Philosophy of Language*, B. Hale and C. Wright, Ed. Oxford: Blackwell, 1997, pp. 613–633.
- [72] LUCE, R. Semiorders and a Theory of Utility Discrimination. *Econometrica* 24, 2 (1956), 178–191.
- [73] MANI, I. A Theory of Granularity and Its Application to Problems of Polysemy and Underspecification of Meaning. In *Proceedings of the Sixth International Conference on Principles of Knowledge Representation and Reasoning* (1998), L. K. S. A. G. Cohn and S. C. Shapiro, Eds., Morgan Kaufmann, pp. 245–257.
- [74] MCGEE, V. Kilimanjaro. *Canadian Journal of Philosophy Supplementary Volume* 23 (1998), 141–163.
- [75] MERRICKS, T. There are no criteria of identity over time. *Nous*, 32 (1998), 106–124.
- [76] MILLER, K. Vagueness, Persistence and Indeterminate Identity. *Erkenntnis* 64, 2 (2006), 223–230.
- [77] MOLTMANN, F. Comparatives without Degrees: A Trope-Based Analysis. unpublished.
- [78] MOLTMANN, F. Properties and Kinds of Tropes: New Linguistic Facts and Old Philosophical Insights. *Mind* 113, 449 (2004), 1–43.

- [79] NOONAN, H. Identity. In *The Stanford Encyclopedia of Philosophy*, E. N. Zalta, Ed. Fall 2008.
- [80] OLIN, D. *Paradox*. Chesham: Acumen, 2003.
- [81] PAGANINI, E. *La vaghezza*. Roma: Carocci, 2008.
- [82] PARIKH, R. The Problem of Vague Predicates. In *Language, Logic and Method*, R.S. Cohen and M.W. Wartofsky, Ed. Dordrecht: Reidel, 1983, pp. 241–261.
- [83] PARIKH, R. Vagueness and Utility: The Semantics of Common Nouns. *Linguistics and Philosophy* 17, 6 (1994), 521–535.
- [84] PARSONS, T. *Indeterminate Identity: Metaphysics and Semantics*. Oxford: Oxford University Press, 2000.
- [85] PARSONS, T., AND WOODRUFF, P. Worldly Indeterminacy of Identity. *Proceedings of the Aristotelian Society* 95 (1995), 171–191.
- [86] PAWLAK, Z. Rough Sets. *International Journal of Computer and Information Sciences* 11 (1982), 341356.
- [87] PEELEN, L., KLEIN, M., SCHLOBACH, S., DE KEIZER, N., AND PECK, N. Analyzing Differences in Operational Disease Definitions Using Ontological Modeling. In *Proceedings of the 11th Conference on Artificial Intelligence in Medicine (AIME 07)* (2007), R. Bellazzi, A. Abu-Hanna, and J. Hunter, Eds., Springer, pp. 297–302.
- [88] PELLETIER, F. Mass terms. In *Handbook of Metaphysics and Ontology*, H. Burkhardt and B. Smith, Eds. Munich: Philosophia Verlag, 1991, pp. 495–499.
- [89] PELLETIER, F. Review of EJ Lowes *Kinds of being: a study of individuation, identity and the logic of sortal terms*. *History and Philosophy of Logic* 13, 1 (1992), 125–128.
- [90] PERRY, J. *Identity, Personal Identity, and the Self*. Indianapolis, Hackett Publishing, 2002.
- [91] PRINZ, J. Vagueness, Language, and Ontology. *Electronic Journal of Analytical Philosophy* 6 (1998). <http://ejap.louisiana.edu/EJAP/1998/prinz98.html>.
- [92] QUINE, W. *Word and Object*. Cambridge (Mass.): MIT Press, 1960.

- [93] RAFFMAN, D. Vagueness Without Paradox. *Philosophical Review* 103, 1 (1994), 41–74.
- [94] ROBINSON, H. Substance. In *The Stanford Encyclopedia of Philosophy*, E. N. Zalta, Ed., fall 2008 ed. 2008.
- [95] ROMERALES, E. Amounts of Vagueness, Degrees of Truth. *Sorites* 11 (1999), 41–65.
- [96] RUSSELL, B. Vagueness. *Australasian Journal of Philosophy* 1, 2 (1923), 84–92.
- [97] SAINSBURY, R. Concepts without boundaries. In *Vagueness: A Reader*, R. Keefe and P. Smith, Eds. Cambridge (Mass.): MIT Press.
- [98] SAINSBURY, R. What is a Vague Object? *Analysis*, 49(2) (1989), 99–103.
- [99] SAINSBURY, R. Is There Higher-Order Vagueness? *Philosophical Quarterly* 41, 163 (1991), 167–182.
- [100] SAVELLOS, E. E. Criteria of Identity and the Individuation of Natural-Kind Events. *Philosophy and Phenomenological Research* 52, 4 (1992), 807–831.
- [101] SCHLOBACH, S., KLEIN, M., AND PEELEN, L. Description Logics with Approximate Definitions. Precise Modeling of Vague Concepts. In *Proceedings of IJCAI-07* (2007), M. M. Veloso, Ed., Breckenridge (CO): AAAI Press, pp. 557–562.
- [102] SEUREN, P. The Comparative. In *Generative Grammar in Europe*, F. Kiefer and N. Ruwet, Eds. Dordrecht: Riedel, 1973, pp. 528–564.
- [103] SEUREN, P. The Comparative Revisited. *Journal of Semantics* 3, 1-2 (1984), 109–141.
- [104] SMILEY, T. Syllogism and Quantification. *The Journal of Symbolic Logic* 27, 1 (1962), 58–72.
- [105] SMITH, B., AND BROGAARD, B. Quantum Mereotopology. *Annals of Mathematics and Artificial Intelligence* 36, 1 (2002), 153–175.
- [106] SOAVI, M., CARRARA, M., AND GIARETTA, P. È un uccello, è un aereo, è Superman! Visione e identificazione sortale. *Rivista di Estetica* 39, 3 (2008), 265–280.

- [107] SORENSEN, R. *Vagueness and Contradiction*. Oxford: Oxford University Press, 2001.
- [108] STEVENSON, L. A Formal Theory of Sortal Quantification. *Notre Dame Journal of Formal Logic* 16, 2 (1975), 185–207.
- [109] STRAWSON, P. *Individuals: An Essay in Descriptive Metaphysics*. London: Methuen, 1959.
- [110] STRAWSON, P. Entity and Identity. In *Contemporary British Philosophy, Fourth Series*, H. Lewis, Ed. London: George Allen & Unwin, 1976. repr. in Strawson, P.F., *Entity and Identity and Other Essays*, Oxford: Clarendon Press, 1997, pp. 21–51.
- [111] TENNANT, N. Continuity and Identity. *Journal of Philosophical Logic* 6, 1 (1977), 223–231.
- [112] THOMASSON, A. *Ordinary Objects*. Oxford: Oxford University Press, 2007.
- [113] TYE, M. Vague Objects. *Mind* 99 (1990), 535–557.
- [114] UNGER, P. There Are No Ordinary Things. *Synthese* 41, 2 (1979), 117–154.
- [115] VAN BENTHEM, J. Later Than Late. On the Logical Origin of the Temporal Order. *Pacific Philosophical Quarterly* 63 (1982), 193–203.
- [116] VAN INWAGEN, P. How to Reason about Vague Objects. *Philosophical Topics* 16, 1 (1988), 255–284.
- [117] VAN ROOIJ, R. Semi-Orders and Satisficing Behaviour. to appear.
- [118] VAN ROOIJ, R. Vagueness and Pragmatics. In *The Vagueness Handbook*, G. Ronzitti, Ed. Springer. forthcoming.
- [119] VARZI, A. Vague Names for Sharp Objects. In *Proceedings of the Workshop on Semantic Approximation, Granularity, and Vagueness* (2000), L. Obrst and I. Mani, Eds., Breckenridge (CO): AAAI Press, pp. 73–78.
- [120] VELTMAN, F., AND MUSKENS, R. Lecture Notes on Logical Analyse. unpublished.
- [121] VON STECHOW, A. My Reaction to Cresswell’s, Hellan’s, Hoeksema’s, and Seuren’s Comments. *Journal of Semantics* 3 (1984), 183–199.

- [122] WALLACE, J. Sortal Predicates and Quantification. *The Journal of Philosophy* 62, 1 (1965), 8–13.
- [123] WIGGINS, D. *Sameness and Substance Renewed*. Cambridge: Cambridge University Press, 2001.
- [124] WILLIAMSON, T. Criteria of Identity and the Axiom of Choice. *The Journal of Philosophy* 83, 7 (1986), 380–394.
- [125] WILLIAMSON, T. *Identity and Discrimination*. Oxford: Basil Blackwell, 1990.
- [126] WILLIAMSON, T. Fregean Directions. *Analysis* 51, 4 (1991), 194–195.
- [127] WILLIAMSON, T. *Vagueness*. London-New York: Routledge, 1994.
- [128] WOODRUFF, P., AND PARSONS, T. Indeterminacy of Identity of Objects and Sets. *Nous (Supplement: Philosophical Perspectives, 11, Mind, Causation and World)*, 31 (1997), 321–348.
- [129] WRIGHT, C. *Frege's Conception of Numbers as Objects*. Aberdeen: Aberdeen University Press, 1983.