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Heterogeneity of Agents in Diffusion of Innovation Modelling: Communication, Networks and Competition

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Dedicated to

My Beloved Father,

Late Md. Abu Taher

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Abstract

Diffusion of innovation is an attempt to study the behaviour of agents in the complex network structure, contagion of information and the related consequences. Pioneering approach by Bass [5] is further developed with numerous research works. Further considerations bring to the introduction of heterogeneity effect and marketing mix variables, that has been studied later. Empirical results show that the basic Bass model and its generalisations (GBM) can be studied as a modified form of the basic Logistic model. Recent work by Bemmaor [6] explains the diffusion dynamics as a mixture of probability distributions obtained from individual level heterogeneity. Therefore, the basis of this thesis is to formulate heterogeneous agent based diffusion modelling at the aggregate level, in order to predict the future behaviour of the diffusion path, based on its past behaviour.

This thesis extends the contribution of heterogeneous agents' behaviour and try to put some probabilistic assumption at the individual adoption attitude (adoption propensity) and obtains the diffusion dynamics as an aggregate level observed phenomenon (consequences). Special emphasis is given to the contribution of heterogeneous group of imitators towards the diffusion process. The existing Generalised Bass model (GBM) has been updated with additional information of the imitators' participation that results a mixture of GBM with Bemmaor heterogeneity effect, which is important to describe the social contagion of information.

To accommodate the effect of heterogeneity among the involved agents, both in innovators and imitators subgroup, this thesis suggests an extension of Bemmaor model, named modified Bemmaor model (MBM), which is further modified considering GBM with heterogeneity. Proposed models are applied to real data sets under technological diffusion of innovation perspectives. Obtained results highlight efficacy of the proposed models with additional heterogeneity parameters and strong agreements with the research outputs conducted with similar objectives. Proposed models are very useful in applied context and could be extended further for multiple innovation, simultaneous innovation or multiple regime of innovations diffusion contexts.

Sommario

La diffusione di innovazioni attiene allo studio del comportamento degli agenti in una rete complessa, al trasferimento di informazioni e alle conseguenze collegate. L'approccio pionieristico di Bass [5] è stato ampiamente sviluppato i letteratura attraverso numerosi lavori di ricerca. Ulteriori considerazioni hanno portato all'introduzione dell'effetto di eterogeneità e successivamente sono state studiate le variabili di *marketing mix*. I risultati empirici dimostrano che il modello di Bass e le sue generalizzazioni (GBM) possono essere studiate come una variante del modello Logistico di base. Un recente lavoro di Bemmaor [6] mostra che le dinamiche di diffusione possono essere spiegate come una mistura di distribuzioni di probabilità associate al livello individuale della componenta latente che esprime l'eterogeneitá. Pertanto, lo scopo di questa tesi è quello di formulare modelli statistici di diffusione a livello aggregato fondati sul comportamento di agenti eterogenei, al fine di prevedere l'andamento futuro della diffusione, in base al suo comportamento passato.

Questa tesi vuole estendere il contributo del comportamento degli agenti eterogenei e cerca di formulare alcune assunzioni probabilistiche sull'attitudine all'adozione individuale (propensione all'adozione), ottenendo così le dinamiche di diffusione come un fenomeno osservato a livello aggregato (conseguenze). Particolare enfasi viene attribuita al contributo del gruppo eterogeneo di imitatori verso le dinamiche di diffusione. L'attuale Modello Generalizzato di Bass (GBM) è stato aggiornato con informazioni aggiuntive sulla partecipazione dei imitatori, diventando un ibrido tra un GBM e un modello di Bemmaor con effetto di eterogeneità, importante nella descrizione del contagio sociale dell'informazione.

Per sistemare l'effetto di eterogeneitá tra gli agenti eterogenei coinvolti sia nei sottogruppi di innovatori che imitatori, questa tesi suggerisce un'estensione del modello di Bemmaor, denominato Modello Bemmaor modificato (MBM), che viene ulteriormente esteso considerando un GBM con eterogeneità. I modelli proposti sono applicati a due insiemi di dati reali nella prospettiva della diffusione di un'innovazione tecnologica. I risultati ottenuti sembrano evidenziare la validità dei modelli proposti con parametri di eterogeneità aggiuntivi e la coerenza con i risultati ottenuti in altri lavori di ricerca con obiettivi simili. I modelli presentati trovano applicazione in diversi contesti e possono essere estesi ulteriormente a prospettive della diffusione dell'innovazione multipla, simultanea o a regime multiplo.

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Chapter 1

Introduction

In recent decades, the pattern of diffusion of innovations for products or services has become an interesting matter of study for social scientists, mathematicians, marketing experts, statisticians, engineers and biologists. Researchers are drawn to the topic not only to examine trends and underlying factors in the diffusion process but also to forecast them. In this context, researchers try to understand the behaviour of the existing individuals (agents) in society and their attitudes toward newly introduced goods or services and explain the dynamics of their decisions with special mathematical models. The purchase action can be termed "adoption" within the marketing language that divides existing non-homogeneous agents into several mutually exclusive groups. Bass [5] considers two sub-populations of adopters, innovators and imitators, and develops an aggregate model for the adoption assuming homogeneity in the sub-population units. This paradigm of innovation diffusion modelling has been extended and examined in numerous studies and reviews (see, in particular, [3, 38, 45]).

The important issue is to model the social contagion of information and adoption of goods/services in a social system that may be characterized by specific regime changes. This adoption framework also depends on the structure of the social system, on its internal rules or external influences (policies, marketing strategies, etc.) that may vary in terms of time. Homophily and segregation patterns due to age, ethnicity, geography, profession, educational level etc. are common features in social structures that significantly influence the susceptibilities toward adopting a new product or new innovation. Staying in an era of innovative scientific developments, social structures also experience simultaneous innovations in competitive product markets and the diffusion dynamics of competing innovations emerges from the aggregation of consumers' (agents') decision. Rogers and Shoemaker [57] defines the diffusion of innovation as the process by which innovation spreads among the members of a social system. The innovation itself, adopters of the innovation, innovation channels, time and space modify the agents' behaviour through a process that determines the social system dynamics. The existing network structure among the agents in the social dynamics, the connections among products of the same family and their regime changes and the communication dynamics among the agents in the complex system should be carefully considered in innovation diffusion modelling.

The next Section makes a short overview of existing innovation diffusion modelling approaches that deal with the above topic and Section 1.2 discusses the main contribution of the thesis.

1.1 Overview

Various modelling approaches were followed by the researches to obtain the time pattern of a diffusion process. The simpler approach is to consider the diffusion process only as a direct function of time. The alternative approach is to consider the process as a function of the number of previous adopters over time through special differential equations which may include theoretical assumptions and therefore, easy to interpret parameters. Some other works extended the fundamental diffusion model to study the time and spatial aspects of diffusion process simultaneously. Some researchers also tried to model jointly the diffusion of competing products/brands. In this family of models we can find elements which are appropriate for different situations (according to the mutual relationships among the products which can be complementary, partially substitute or perfectly substitute).

Most of the developed models of diffusion of innovation are based upon the fundamental social network connection explained by Rogers [56]. Many products exhibit a cumulative S-shaped curve diffusion pattern. At the aggregate level, we try to use the adoption data of the earlier part of the life cycle to predict the future evolution. Existing social structure and communication among the individuals (agents) directly influences the expansion of the diffusion. Within the social network, individuals (agents) communicate their evaluation of an innovation to others and influence their adoption. Therefore, one of the vital direction of diffusion innovation research framework is devoted to obtain the conditional distribution of the tentative adopters to become adopters with respect to time.

The economic constraints may also have influential impact on diffusion. However, only a few contributions in the literature are devoted to the analysis of individual consuming intention. This happens because this approach requires individual data often impossible to obtain.

In the modern technological world, innovations of different products may be connected, even, within a single social network. Individuals have different options to choose a particular type of innovation. For example, if someone wants to buy a smartphone, he can choose any of the innovations of smartphones operating in the market. Even within the same brand, there exist different options based on specification and served activities. Other examples are notebooks and laptops, which replace the general consumption opportunity and take over the desktops computer technology. This replacement of innovations can be studied through a regime change of innovation within the social network structure. There are two structural features that affect the network effect. One is the rewiring probability that contributes to the diffusion of information about the innovation in a global manner and, the other, is the degree of connectivity, which determines the degree of clustered ties in a network, i.e., the number of neighbours. A greater degree of neighbourhood provides stronger social reinforcement for the adoption of an innovation [12] and complex contagions exhibit different diffusion patterns than simple ones [58].

Timing of adoption is another fundamental issue in innovation diffusion research. Adoptions of a new innovation is rarely instantaneous. Among the existing agents in society, some adopts early and others later, with an accelerating adoption process initially followed by a decreasing process when all interested individuals have adopted (see, for example, [23, 43, 44]). This adoption timing can be modelled through a probability distribution, and different innovations may be described by different adoption time distributions as well. For example, high tech products' adoption takes place at the early life cycle, and, as a result, the adoption timing can be modelled with an exponential-like distribution. Whereas, expensive durables, such as refrigerators, automobiles follow almost normal probability distribution, since, for these products a smaller degree of replacement is observed. Empirical studies by Rogers [56] show that the process of adoption over time can typically be illustrated as a normal distribution. Bemmaor and Lee [7] postulate that in a complete random network with heterogeneous agents, the individual adoption propensity can be described with a shifted Gompertz distribution set up. Individuals (agents) in a social network vary according to their propensity to adopt a new innovation. Also the network structure determines the communication dynamics that have influential effect on the diffusion dynamics.

This thesis focuses on the above issues about differences among adoption timing behaviour of the agents at the individual as well as at the aggregate level. The main matter of interest could be the identification of network structure and the adoption timing distribution that become appropriate for a dynamic market potential. In order to achieve this, we should however rely on detailed individual data. Therefore, the basis of this research is to formulate heterogeneous agent-based diffusion modelling at the aggregate level, in order to predict the future behaviour of the diffusion path. Following the research work by Bemmaor [6], this study considers the heterogeneity among the agents, and extends the adoption propensity behaviour with further propositions at the individual level as well as their consequences at the aggregate level.

1.2 Summary and main contribution of the thesis

The central goal of this thesis is to explore the diffusion dynamics when heterogeneity among the agents is under consideration. Distributional assumptions have been set up for individual adoption propensity that enable to extend existing aggregate level diffusion dynamics.

After an introductory chapter, Chapter 2 makes a review of the existing homogeneous and heterogeneous models. The Bass model and its extension, the generalised Bass model (GBM) are briefly discussed with their parameter estimation procedures. The chapter also has a short look up at the non-uniform influence models. After a short discussion on the limitation of Bass model, this chapter presents also the diffusion modelling approach that considers the individual heterogeneous adoption propensity. A brief focus on the modelling approach by Bemmaor, Bemmaor and Lee [6, 7] is given for further considerations. Some emphasis is given to the role of the asymmetry parameter, which acts essentially on the logistic component (imitators) of the Bass model and not on the monomolecular one (innovators).

Chapter 3 and Chapter 4 represent the main contribution of the thesis. In Chapter 3 an alternative approach of diffusion modelling is presented with reference to general exogenous interventions. Considering individual level heterogeneity in the imitator group, a new model is suggested to build a mixture of generalised Bass models using the Bemmaor approach. In the presence of completely heterogeneous agents in the social sub-populations (innovators and imitators), an innovation diffusion dynamics is presented in Chapter 4. We concentrate on the Bemmaor modelling approach and we try to capture the existing heterogeneity in both innovators and imitators groups. Since all the above models are nested, their parameters estimation procedure and the significance of considering a more complex model are also highlighted in this chapter.

The thesis discusses also the application of the existing models and proposed extensions to real data sets, under technological innovation diffusion modelling.

Chapter 5 is devoted to the discussion of the diffusion dynamics of South Asian natural gas production under technological innovation diffusion dynamics. The application of the existing Bass model and its extensions has been performed to see the identifiability strength to capture the real intervention consequences. A further application of extended diffusion models is discussed in Chapter 6. Using Algerian natural gas production data set, this chapter points out the comparative performance and reliability in the parameter estimates of the extended models and the forecasting strength under ARMAX set up.

An overall discussion of the obtained results with further research directions are given at the end in Chapter 7.

Chapter 2

Review of diffusion models

2.1 Introduction

Numerous modelling approaches have been followed by the researchers in last few decades to capture the shape of an innovation diffusion. In the existing social dynamics, individuals share their evaluations and make the diffusion speedy or delayed according to the evaluation performance. Social contagion of informations, which is termed simply as "word-of-mouth effect" plays a very important role in this regard. Consequences of economic and budgetary contexts and local interventions are also inevitable. Because of the influence of social contacts, social interactions, and interpersonal communication to the adoption of new behaviour [60], researchers tried initially to describe the evolutionary pattern of diffusion of innovations within the existing social structure. Further modifications and considerations have been adopted later, to capture the local interventions and heterogeneity of social structures as well. Recent research on diffusion innovations is much concentrated to model individual level adoption rate and obtain the aggregate diffusion pattern, which can be discussed as agent based modelling or bottom to the top modelling approach.

The mathematical modelling of diffusion of innovation has attracted strong interest to the researchers after the pioneering works by Fourt and Woodlock [20] (to model a simple penetration of grocery products), Mansfield [43] (to model the diffusion of a newly invented technology) and Bass [5], who introduced a differential equation growth model for consumer durable with a closed form solution. The reason motivating these traditional modelling is to make an empirical generalization and hence describe the spread of new products parsimoniously at the aggregate level. This modelling paradigm has produced a rich literature stream which has been reviewed by numerous authors (see, for instance [37, 38, 39, 42, 51, 45]) and most of the reported work has consisted of refinement and extensions of the Bass model [5] without alteration of its basic premise ([38, 6]). More recently, Kiesling et al. [36] review the wealth of literature of diffusion research streams and critically examine the strength and limitations of both aggregate level modelling and agent-based modelling of innovation diffusion.

In this chapter we make a short descriptive review of the Bass model and its extension, the generalised Bass model (GBM) and critically examine the related assumptions and parameter estimation procedure. The following sections present also the diffusion modelling approach that considers the individual heterogeneous adoption propensity. A brief focus has been given on the modelling approach by Bemmaor [6] and Bemmaor and Lee [7] to examine the role of the asymmetry parameter, which only acts on the logistic component of the Bass model.

2.2 Homogeneous models: Bass model and Its generalisation GBM

Bass model is build on the Roger's conceptual framework by developing a mathematical model that captures the non-linear structure of S-shaped diffusion dynamics [55].

Let us denote with $F(t)$ the cumulative distribution of adoptions over time and with $f(t) = F'(t)$ the corresponding density of the adoption process. Bass's [5] fundamental diffusion model is based on the assumption that the hazard or the probability of adoption of a new product or innovation at time t, given that it has not yet been adopted, $f(t)/[1 - F(t)]$, depends on a convex combination of three factors (following the conditional probability law): the conditional probability of adoption of innovators, 1, the corresponding conditional probability of imitators, $F(t)$, and the conditional probability of "neutrals," 0. In other words,

$$f(t)/[1 - F(t)] = p \cdot 1 + qF(t) + (1 - p - q) \cdot 0 = p + qF(t). \tag{2.1}$$

The innovation coefficient, $p > 0$, measures the propensity of potential adopters to become adopters, and the imitation coefficient, $q > 0$, measures the propensity of potential adopters to imitate previous adopters. A different interpretation of the meaning of parameters p and q is proposed by Van den Bulte [69] by considering p an external influence effect and q an internal influence effect both acting on an agent.

In a simplified form, the Bass model then may be described with the following normalised equation:

$$f(t) = (p + qF(t))\,(1 - F(t))\,. \tag{2.2}$$

Under the initial condition, $F(0) = 0$, the solution of Equation(2.2) by Bass [5] defines the following distribution function:

$$F(t) = \frac{\left(1 - e^{-(p+q)t}\right)}{\left(1 + \frac{q}{p}e^{-(p+q)t}\right)}\,, \qquad t \geq 0, \quad p, q > 0. \tag{2.3}$$

Let m be the number of potential adopters (or adoptions) in the market. Then, the total number of adoptions until time t is

$$Y(t) = mF(t) = m\frac{\left(1 - e^{-(p+q)t}\right)}{\left(1 + \frac{q}{p}e^{-(p+q)t}\right)}, \qquad t \geq 0. \tag{2.4}$$

If the information about the very early stages of the diffusion process is not available, this can be overcome by the modified model used by Guidolin and Mortarino [24]:

$$Y(t) = m\frac{1 - e^{-(p+q)(t+d)}}{1 + \frac{q}{p}e^{-(p+q)(t+d)}}, \qquad t + d \geq 0, \tag{2.5}$$

where d is an unknown translation parameter to be estimated such that $F(-d) = 0$.

Considering the time period as unity (year, quarter, month, week, day, etc.), the observed rate of diffusion, in other words, sales *S(t)*, in the time interval, $(t-1,\ t)$, can be described with the following regression model:

$$\begin{aligned} S(t) &= mf(t) + \epsilon(t) \\ &\approx m\left(F(t) - F(t-1)\right) + \epsilon(t) \\ &= m\left(\frac{1-e^{-(p+q)t}}{1+\frac{q}{p}e^{-(p+q)t}} - \frac{1-e^{-(p+q)(t-1)}}{1+\frac{q}{p}e^{-(p+q)(t-1)}}\right) + \epsilon(t). \end{aligned}$$

which emphasizes the non-cumulative version of the adoption process, which is useful when early adoption information is not available. The error term in the previous equation, $\epsilon(t)$, is assumed to be independently distributed with zero mean and variance σ^2. Often, these assumptions are weakened to consider stationary autocorrelated dynamics. The parameters p, q, and m may be estimated with a nonlinear least squares (NLS) procedure [67, 62].

A better approximation of $f(t) = F'(t)$, can also be obtained with the following representation:

$$f(t) \approx F(t+0.5) - F(t-0.5). \tag{2.6}$$

The Bass model is the first foremost formal way to separate innovators (leaders) and imitators (followers) in the innovation process. Innovators and imitators characterize a latent distinction, since the observed data merely report on the adoption by a susceptible agent without any other specification. This approach better explains Roger's [56] perspective based on a questionable normal distribution assumption.

A very important extension of the standard Bass model, developed by Bass et al. [4], is the Generalized Bass Model (GBM) that introduces the effect of a general intervention or control function, $x(t) = x(t;\theta)$, $\theta \in \Re^k$, depicting the possible effect of exogenous variables on the diffusion process. Thus, an extension of Equation (2.2) is given by,

$$f(t) = (1 - F(t))\,(p + qF(t))\,x(t), \tag{2.7}$$

and its solution, under $F(0) = 0$, gives an explicit expression for the total number of adopters until time t as follows:

$$Y(t) = mF(t) = m\frac{\left(1 - e^{-(p+q)\int_o^t x(\tau)d\tau}\right)}{\left(1 + \frac{q}{p}e^{-(p+q)\int_o^t x(\tau)d\tau}\right)}, \qquad 0 \le t < +\infty. \tag{2.8}$$

Bass et al. [4] called this function, $x(t)$, the "current marketing effort" that reflects the current effect of dynamic marketing variables on the conditional probability of adoption at time t. Notice that the closed-form solution Equation (2.8) is extremely general, because the control function, $x(t)$, may assume, under local integrability, any shape without special limitations. For $x(t) = 1$, the model reduces to the standard Bass model. For $x(t) > 1$, the adoption process is accelerated over time; otherwise, the process is delayed (see, in particular, [29]). Therefore, this intervention function may modify the time that elapses between adoption events within a general closed form solution, which is very powerful in applied contexts.

Researchers suggested many alternative structures to the intervention function, $x(t)$, to comply with existing population behaviour, modelled with shocks estimated through the observed dataset.

The standard Bass model fits very well to unimodal real data, and many other versions of the model, including the GBM, appeared later to explain different aspects of diffusion. A special application of the GBM has been made in the energy sector, for crude oil in particular ([28, 29, 27]), where the rationale for these applications is grounded on the related diffusion of technologies that are directly or indirectly energy-consuming.

The Bass model and the GBM have fixed market potential over the assumed life cycle. An important extension, the dynamic market potential, $m(t)$, is introduced in [25] and [30, 31, 32]. In particular, Guseo and Guidolin [30] obtain a

Riccati closed-form solution for general $m(t)$ and $x(t)$ functions. The results emphasize the different role of policies over time $x(t)$ and over scale $m(t)$, to describe the time modulation of a non-constant carrying capacity (market potential).

Despite recent developments, the standard Bass model and GBM still suffer conceptual limitations in application and forecasting. These models assume that the internal influence (word-of-mouth effect) remains uniform over the time frame of the diffusion process. Conversely, in practice, later adopters may not be as likely to discuss the product with non-adopters as early adopters, and may be less likely to exhibit the same enthusiasm in discussing the new product. In many occurrences, late adopters have different characteristics than early adopters and respond differently [56]. The diffusion model should allow for this difference. Furthermore, the Bass type models make very specific assumptions regarding the social interactions, stating that, the social structure consists of a fully connected network, and therefore, the influence of the adopters on non-adopters is a linear function of the number of adopters throughout the diffusion period [63]. Nevertheless, flexibility of the control function, $x(t)$, does not cover all heterogeneity aspects of the involved agents.

In the contemporary research paradigm, a number of non-uniform influence diffusion models have been introduced as extensions of Bass model, considering the coefficient of imitation as systematically varying over time (see [16], [35]). Easingwood et al. [17] suggests a model that can accommodate different diffusion patterns and therefore allows the diffusion curve to be symmetrical as well as asymmetrical, with a point of inflection that is allowed to change. Their model has the form,

$$f(t) = (p + qF(t))^{\delta} (1 - F(t)), \tag{2.9}$$

where the parameter, δ, is the non-uniform influence factor. For $\delta = 1$, the model reduces to the standard Bass model. Values of δ between 0 and 1 cause an acceleration in the diffusion speed, while for $\delta > 1$, a slower penetration rate is described. Therefore, the parameter δ can be interpreted as a measure aiming to the network structure of the agents in the diffusion dynamics. For a well connected and homogeneous group of imitators, the value of δ is smaller, and vice versa. Thus, the non-uniform influence diffusion models can accommodate different diffusion patterns, with time varying coefficient of imitation. Unfortunately, there exist no closed form solution for δ in Equation (2.9), and in practice, the inflection bound has to be obtained numerically. Thus, for these type of models, the maximum rate of adoption can be attained at any stage of the diffusion process.

2.3 Heterogeneous diffusion models: Bemmaor model

Quite recently, a limited number of diffusion models have been introduced that incorporates individual-level heterogeneity and/or heterogeneity in the diffusion penetration rate. Researchers have tried to develop a segmental diffusion model [55], or models that consider several distributional assumptions of the market penetration rate and adoption at the individual level (see, in particular, [6, 70, 33]).The principal matter of interest in this case is to obtain a parsimonious and flexible closed-form diffusion model that can accommodate symmetric and non-symmetric diffusion patterns with a point of inflection that can occur at any stage of the diffusion process [41].

The research paradigm on the diffusion of innovation in a social system by Bass [5], Mansfield [43], and the related generalizations addressed the market as an aggregate structure, with little attention to micro-level processes that characterize adoption decisions (see, in particular, [13, 38]). The main issue in this line is understanding and explaining the diffusion process across a population of adopting units. The existence of a heterogeneous population of adopters has been largely ignored in this perspective. In the individual-level perspective, the diffusion of innovation can be modelled either as individual adoption probability with the timing of adoption or through the derivation of adoption behaviour at the individual level in a decision-theoretic framework (see, for instance, [13, 59, 64]).

The model by Chatterjee and Eliashberg [13] considers a) heterogeneity in initial perceptions about the future performance of the innovation/product; b) consumers' preference structure, and c) perceived reliability of the informations' source. Chatterjee and Eliashberg postulate, in a bayesian framework, the initial assignments for these variables are dynamically updated on the basis of further information. This approach is an important step in modelling diffusion at the micro level, but its application is limited by its dependence on extensive perceptual data on adoption. Sinha and Chandrashekaran [64] use a hazard model approach that explicitly incorporates covariates in the adoption time specification, so the population is heterogeneous in adoption timing. The split hazard model framework models the adoption decision at the individual level, and describes and forecasts new product acceptance at the aggregate market level.

Bemmaor [6] suggests an alternative approach for explaining the changes in the parameter estimates of the Bass model that includes the underlying heterogeneity of the population. He assumes that diffusion can equivalently be explained by the variation in individual propensities to buy across consumers. Therefore, a shifted Gompertz density may explain the timing of the first purchase, and the individual propensity across consumers is assumed to follow a gamma distribution. The aggregate diffusion process results in a mixture of these two densities. Bemmaor and Lee [7] briefly analyzed the resulting misspecification in the Bass model discussed by Van den Bulte and Lilien [70] and found that the Bemmaor model had better forecasting capacity. Their results also suggest that the Gamma-shifted Gompertz model is a flexible model for analyzing the systematic changes in parameter estimates when specification error and ill-conditioning occur.

The Gamma-shifted Gompertz model postulates that the ratio q/p of the Bass model parameters varies with the scale parameter $1/\beta$ of the distribution of heterogeneity parameter η (related to the propensity to buy), and η is distributed according to a Gamma$(1/\beta, \alpha)$ law.

The individual-level model (model of first adoption timing) can be specified by a shifted Gompertz distribution with a scale parameter b and shift parameter η as follows:

$$F(t) = \left(1 - e^{-bt}\right) e^{-\eta e^{-bt}}, \quad t \geq 0, \quad \eta, b > 0, \tag{2.10}$$

or with the corresponding density function:

$$f(t|\eta, b) = be^{\left(-bt-\eta e^{-bt}\right)} \left[1 + \eta \left(1 - e^{-bt}\right)\right], \qquad t \geq 0, \quad \eta, b > 0. \tag{2.11}$$

For a fixed value of b, small values for η imply a low mean adoption time (i.e., a strong individual propensity to buy).

If the heterogeneity parameter η varies according to a Gamma distribution with shape parameter α, and scale parameter $1/\beta$, the aggregate-level diffusion model is described by the distribution function:

$$F(t) = \frac{(1 - e^{-bt})}{(1 + \beta e^{-bt})^{\alpha}} \quad , \quad \alpha, \beta > 0, \tag{2.12}$$

with density,

$$f(t) = be^{-bt} \left(1 + \beta e^{-bt}\right)^{-(\alpha+1)} \left[1 + \alpha\beta + \beta e^{-bt} (1 - \alpha)\right]. \tag{2.13}$$

If we re-parameterize Equation (2.12) with the equivalent Bass model parameters by letting $b = p+q$ and $\beta = q/p$, we obtain the following aggregate-level diffusion model:

$$F(t) = \frac{\left(1 - e^{-(p+q)t}\right)}{\left(1 + \frac{q}{p} e^{-(p+q)t}\right)^{\alpha}}, \quad t \geq 0, \quad \alpha, p, q > 0. \tag{2.14}$$

The parameter α can be interpreted as the measure of heterogeneity in the diffusion dynamics. Thus a larger value of α will indicate more heterogeneity (For high values of α the density becomes symmetric, and moves to the right. Vice versa for small values of α, the density tends to be concentrated around $t = 0$). For $\alpha = 1$, model (2.14) reduces to the standard Bass model. When $\alpha = 0$, the model reduces to an exponential model, and when $\alpha = \infty$, it converges to the shifted Gompertz model (see, for instance, [40]).

Therefore, as α approaches zero, the shape of the diffusion curve resembles an exponential diffusion curve, and for larger values of α, it approaches a logistic curve. The role of α becomes the vital driver in the diffusion process that can be used to explain the impact of contagion or lack thereof in the diffusion process which is very important to explain the social network structure. Thus, the Gamma- shifted Gompertz model encompasses several other models of diffusion, including the standard Bass model. Although, the analytical formulation of the Bemmaor model looks similar to the non-uniform model by Easingwood et al. [17], (see Equation (2.9)) this model captures an apparent non-influence over time, having advantage to demonstrate the importance of "extra-Bass" skew for descriptive fit, and also has a closed-form expression.

The Gamma-shifted Gompertz model by Bemmaor [6] is an important contribution in diffusion modelling that provides grounds for investigating jointly "the speed takeoff" [47], and "the diffusion speed after takeoff" [68] observed in the process through the heterogeneity parameter. The additional parameter included in this model become an indicator of heterogeneity and modifies the contagion process among the agents in the imitators subgroup of agents in the diffusion dynamics. But heterogeneity in the innovators subgroup could also be a feature of a diffusion process. For this reason, an extension of this approach could be very powerful.

2.4 Logistic, Bass and Bemmaor models: a generalised relationship

Diffusion models (homogeneous or heterogeneous) are developed to describe the growth or increments of adoption of innovations or a new products in the existing social (complex) contexts. The dynamic system consists of a large number of agents that are connected in terms of relational form through proper channel and works in a similar mechanism, as the spread of a viral agents in human population (as observed in [2, 56, 22, 5, 4]). Some opinion leaders quickly adopt innovation based on the actions of corporate communications; then the mechanism is activated in parallel by word of mouth, a very powerful tool that is critical to the success or the failure of almost all business initiatives.

Interestingly, logistic and Bass functions and their generalizations, determines relational self-sustaining evolutionary growth dynamics observed in the socio-economic networking contexts [26].

Consider a process where, m is the ultimate penetration obtained at the end (i.e., a carrying capacity). A logistic equation expresses the instantaneous change $\xi'(t)$ as a mathematical function of cumulative version of the existing process $\xi(t)$ and the activated residual $(m - \xi(t))$. The equation that governs the dynamics, may take the functional form:

$$\xi'(t) = \theta\xi(t)\{m - \xi(t)\}/m, \qquad \theta > 0, \quad t \in R. \tag{2.15}$$

The quantity θ controls the speed of the dynamics. With an initial positive condition, $\xi(0) = \xi_0 > 0$, the solution of the Equation (2.15) at the peak position $t_p = \frac{1}{\theta}log\frac{m-\xi_0}{\xi_0}$ may be found as:

$$\begin{aligned} \xi(t) &= \frac{m}{1 + e^{-\theta(t-t_p)}} \\ &= m\ L(t), \end{aligned} \tag{2.16}$$

where $L(t)$ represents the logistic distribution.

The above expression gives the instantaneous function:

$$\begin{aligned} \xi'(t) &= \frac{m\theta e^{-(t-t_p)}}{\{1 + e^{-\theta(t-t_p)}\}^2} \\ &= m\ l(t). \end{aligned} \tag{2.17}$$

Therefore, the time to pick t_p is connected with the initial condition $\xi_0 = m/(1 + e^{\theta t_p})$.

Analogously, the diffusion of innovation in a social system explained by Bass, can be outlined as the connection of the knowledge about an innovation by the innovators $(\xi(t)/m)$, and the proportional access to the residual dynamics due to the imitators, $(m - \xi(t))$:

$$\xi'(t) = (p + q\xi(t)/m)(m - \xi(t)) \qquad t \in [0,\ +\infty) \qquad p, q > 0. \tag{2.18}$$

Where the parameter, p, and q represents the dynamic contribution of innovators and imitators respectively. With the initial condition $\xi_0 = 0$, the solution of the cumulative Bass model is:

$$\xi(t) = m\ \frac{1 - e^{-(p+q)t}}{\{1 + \frac{q}{p} e^{-(p+q)t}\}} \qquad t \in [0,\ +\infty) \qquad p, q > 0. \tag{2.19}$$

The corresponding rate function is:

$$\xi'(t) = m\ \frac{(p+q)^2\ e^{-(p+q)t}}{p\{1 + \frac{q}{p} e^{-(p+q)t}\}^2} \qquad t \in [0,\ +\infty) \qquad p, q > 0. \tag{2.20}$$

A simple re-parametrisation of Equation (2.19) and Equation (2.20), letting $\theta = (p+q)$ and $t_p = (\ln \frac{q}{p})/(p+q)$ enables to compare the above to the logistic model as:

$$\begin{aligned} \xi(t) &= m\ \frac{1 - e^{-\theta t}}{1 + e^{-\theta(t-t_p)}} \qquad t \in [0,\ +\infty) \qquad p, q > 0 \\ &= m\ B(t), \end{aligned} \tag{2.21}$$

and

$$\begin{aligned} \xi'(t) &= m\ \theta\ \frac{\{e^{-\theta t} + e^{-\theta(t-t_p)}\}}{\{1 + e^{-\theta(t-t_p)}\}^2} \qquad t \in [0,\ +\infty) \qquad p, q > 0 \\ &= m\ b(t), \end{aligned} \tag{2.22}$$

where $B(t)$ and $b(t)$ are the Bass distribution function and Bass density function, respectively.

Comparing Equation (2.16) and Equation (2.21), it can be concluded that the cumulative Bass model can be determined by a monotonic transformation of the logistic model, when:

$$B(t) = L(t)\ \left(1 - e^{-\theta t}\right), \qquad t \in [0,\ +\infty). \tag{2.23}$$

Previous result is presented in Guseo [26].

Similarly, the generalisation of Bass model, GBM, with local intervention function $x(t)$, may be extended to the corresponding perturbed logistic model.

The equation for the logistic model with intervention $x(t)$ can be given by:

$$\xi'(t) = \frac{\theta}{m}\xi(t)\,(m - \xi(t))\;x(t) \qquad t \in R \qquad p, q > 0. \tag{2.24}$$

Under the initial condition, $\xi(0) = \xi_0 > 0$ with the peak point $t_p = \frac{1}{\theta}\ln(\frac{m-\xi_0}{\xi_o})$, the solution of Equation (2.24) may be obtained as:

$$\begin{aligned} \xi'(t) &= m\ f(t) \\ &= m\ \frac{\theta\ x(t)\ e^{-\theta\left\{\int_0^t x(\tau)d\tau - t_p\right\}}}{\left\{1 + e^{-\theta(\int_0^t x(\tau)d\tau - t_p)}\right\}^2} \qquad t \in [0,\ +\infty) \qquad p, q > 0. \end{aligned} \tag{2.25}$$

For $x(t) = 1$, we obtain the pure logistic density.

Now, let us consider a heterogeneous diffusion model with individual heterogeneous propensity. The cumulative version of the Bemmaor model is given by:

$$\xi(t) = \frac{\left(1 - e^{-(p+q)t}\right)}{\left(1 + \frac{q}{p}e^{-(p+q)t}\right)^{\alpha}}, \quad t \geq 0, \quad \alpha, p, q > 0. \tag{2.26}$$

The instantaneous version is:

$$\xi'(t) = \frac{(p+q)\ e^{-(p+q)t)}\ \left\{q(1-\alpha) + (p + \alpha\ q)\ e^{(p+q)t}\right\}}{\left\{1 + \frac{q}{p}\ e^{-(p+q)t}\right\}^{\alpha}\ \{q + p\ e^{(p+q)t}\}} \tag{2.27}$$

For $\alpha = 1$, and with the initial condition $\xi(0) = \xi_0 > 0$ with the peak point $t_p = \frac{1}{\theta}\ln(\frac{m-\xi_0}{\xi_o})$, the solution of Equation (2.27) can be obtained as:

$$\xi'(t) = m\ \theta\ \frac{\{e^{-\theta t} + e^{-\theta(t-t_p)}\}}{\{1 + e^{-\theta(t-t_p)}\}^2} \qquad t \in [0,\ +\infty) \qquad p, q > 0, \tag{2.28}$$

and

$$\xi(t) = m\ \frac{1 - e^{-\theta t}}{1 + e^{-\theta(t-t_p)}} \qquad t \in [0,\ +\infty) \qquad p, q > 0. \tag{2.29}$$

Which are exactly the same as in Equation (2.21) and Equation (2.22), the pure logistic density.

In the above circumstances, it can be concluded that, the Bass model is a modified logistic one under a monomolecular seeding action governed by $(1 - e^{-\theta t})$. In the existing social system, the diffusion model works as the accumulation of information spread, with basic foundation in the logistic model. Individual evaluations after adoption of a new innovation and the corresponding propensity to share this experience to neighbours in a social network generate a continuous update of the diffusion dynamics. Also local interventions (price devaluation, special offers, sudden scarcity) play an important role to speed up or slow down the diffusion dynamics and an adequate diffusion model should be able to capture this. More flexible diffusion models could be a useful tool to describe and highlights further features of the process under study.

2.5 Remarks

The paradigm of homogeneous innovation diffusion models by Bass, and their extensions provides a parsimonious and analytically tractable way to look the adoption as a whole and interpret its behaviour. Assuming that sufficient data points are available, those models can be fitted to the earlier portion of the adoption process of an innovation to obtain the parameter estimates. Various estimation methods, including ordinary least squares (OLS), maximum likelihood method (ML), nonlinear least squares (NLS), genetic algorithms (GA) and simulation based approaches are available for parameter estimation.

The modelling approach by Bemmaor is based upon the consumers' decision to adopt an innovation, or to reject it at the individual level, and infer the diffusion at the aggregate level. This paradigm of agent based modelling approach is more flexible as it consider the heterogeneous individual consumption within a random network structure. The model is easily identifiable, and the parameters can be estimated by nonlinear least squares method or simulated annealing. The advantage of Bemmaor model over the Bass model is that it is able to capture a wide variety of shapes of diffusion, and makes a basis for analysing the changes in parameter estimates of Bass model and its extension. Therefore, the model have potential to explain the nonlinear diffusion pattern observed in the real world, as the result of relatively simple local individual level interactions.

It is possible to describe both homogeneous and heterogeneous models as modified function obtained from the basic Logistic model. Bemmaor [6] shows that in the aggregate level, the diffusion dynamics can be explained as a mixture of distributions obtained from the individual level heterogeneity. Heterogeneity may be observed among the agents in both innovators and/or imitators subgroup. In this respect, parallel to the imitators, observed heterogeneity among the innovator subgroup can also play important role for explaining the diffusion structure. Therefore, in the following Chapters, we will try to make further modifications to the innovator and/ or imitators contribution to the existing models set up, and suggest some new models, that are more flexible and able to capture the local interventions to the systematic diffusion dynamics.

Chapter 3

Extensions of homogeneous models

3.1 Introduction

In Chapter 2 we discussed the existing homogeneous and heterogeneous models in brief with special emphasis on the Bass model, its generalisation GBM and the heterogeneous Bemmaor model. It has also been discussed that, the above models have their fundamental form as logistic model. The Bass model, characterize the diffusion of innovation as a contagious process initiated by the mass communication and propelled by word-of-mouth. The Generalised Bass model (GBM), introduced later by Bass et al. [4] is a further attempt to capture the local perturbation effect through an intervention function in general. These models provide an empirical generalization based on a differential equation formation and do not consider consumers' heterogeneity and the complex dynamics of social processes that shape the diffusion [36]. Bemmaor model [6] is an attempt to overcome those limitations and explicitly model the diffusion process considering consumers' heterogeneity, their social interactions and their decision making process based upon some probabilistic assumptions. A mixture of the above two modelling approaches would be a convenient way to describe the diffusion dynamics at the aggregate level.

In this chapter, we propose a mixture of Generalised Bass model with the heterogeneous modelling proposition by Bemmaor and discuss some important formulation of the model, providing also a parameter estimation procedure.

3.2 Extension of GBM mixture with Bemmaor model

3.2.1 Background

The diffusion models make an attempt to capture the cumulative nonlinear S-shaped diffusion structure with mathematical functions. The corresponding instantaneous shape takes a different structure, such as parabolic, exponential or symmetric depending on the type of innovation, adoption criterion, and the communication network among the existing agents in the social dynamics. These curves can be described very well by known mathematical equations. Changing the weights of the agents' subgroups in the mathematical models results in different trends and patterns of the diffusion dynamics, in order to capture the behaviour of available data. The realization of diffusion models indicate that most of the diffusion equations can be expressed as a composition of two parts [54, 34]. The first part, $G(1-F)$, is a function of the residual market, while the second one, $A(F)$ is a function of the number of subjects who already adopted the innovation. This second part models how the potential adopters get influenced by the level of diffusion that already reached. At any stage of the observed process, both components are constrained as $0 \leq A(F), G(1-F) \leq 1$. Most of the fundamental models, by Coleman [15] , Mansfield [44], Floyd [19], Fisher-Pry [18], Bass [5], Non-uniform influence (NUI) [17] etc., can be expressed with this formulation.

3.2.2 GBM mixture with Bemmaor model

Since many other diffusion patterns are nested within the Bemmaor modelling approach, our first idea is to extend the model incorporating related exogenous variables of the diffusion process within the logic of the GBM approach, in particular, through the intervention function $x(t)$.

Thus a mixture of homogeneous generalised Bass model, GBM with Bemmaor modelling with heterogeneous individual propensity (GBMBM) can be proposed, which has the distribution function:

$$F(t) = \frac{\left(1 - e^{-(p+q)\int_o^t x(\tau)d\tau}\right)}{\left(1 + \frac{q}{p}e^{-(p+q)\int_o^t x(\tau)d\tau}\right)^{\alpha}}, \qquad 0 \leq t < +\infty. \tag{3.1}$$

The instantaneous version of the proposed model will take various formulations, depending on the defined intervention function $x(t)$. Note that the additional parameter α has a behaviour similar to the one in the original Bemmaor model, representing a measure of heterogeneity. The larger is the value of α, the more heterogeneous are the agents in the imitators subgroup.

For example, let us consider a diffusion dynamics, experienced with an exponential shock, described by:

$$x(t) = 1 + c\ e^{b(t-a)}\ I_{[t \geq a]}. \tag{3.2}$$

In this expression, the exponential shock starts at time a, with intensity c, and persistence effect b. We may postulate a GBM Bemmaor mixed model (GBMBME1) to capture the random mixture of agents in the innovators and imitators group, by the following cumulative distribution function:

$$F(t) = \begin{cases} \frac{\left(1-e^{-(p+q)t}\right)}{\left(1+\frac{q}{p}e^{-(p+q)t}\right)^{\alpha}} & \text{if } t < a \\ \frac{\left(1-e^{-(p+q)[t+\frac{c}{b}e^{\{b(t-a)-1\}}]}\right)}{\left(1+\frac{q}{p}e^{-(p+q)[t+\frac{c}{b}e^{\{b(t-a)-1\}}]}\right)^{\alpha}} & \text{if } t \geq a. \end{cases} \tag{3.3}$$

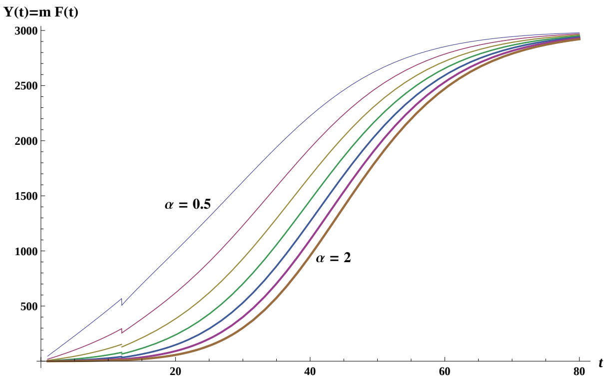
The instantaneous form of the function can be given by the equation:

$$f(t) = \begin{cases} \dfrac{(p+q)\ e^{-(p+q)t}\left\{(p+\alpha q)+q(1-\alpha)e^{-(p+q)t}\right\}}{p\ \left\{1+\frac{q}{p}\ e^{-(p+q)t}\right\}^{(1+\alpha)}\left\{q+p\ e^{(p+q)t}\right\}} & t < a \\ \\ \dfrac{(p+q)\ e^{-(p+q)u}\left\{1+c\ e^{b(t-a)-1}\right\}\left\{(p+\alpha q)+q(1-\alpha)e^{-(p+q)u}\right\}}{p\ \left\{1+\frac{q}{p}\ e^{-(p+q)u}\right\}^{(1+\alpha)}\left\{q+p\ e^{(p+q)u}\right\}} & t \geq a, \end{cases} \tag{3.4}$$

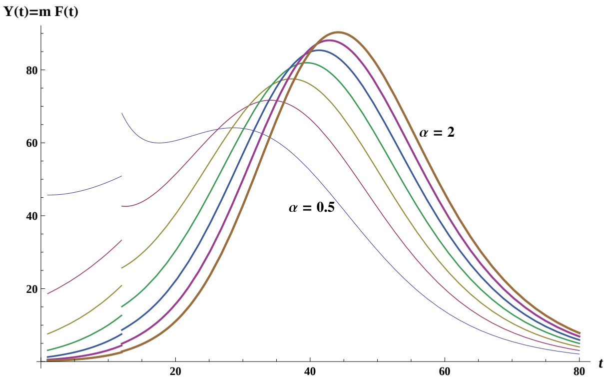
where $u = \left\{\frac{c}{b}\ e^{b(t-a)-1} + t\right\}$. Similar formulations are also possible for intervention functions with one rectangular shock, one rectangular and one exponential shock or two exponential shocks, etc.

In that situation, graph if a GBMBME1 in Figure 3.1 shows various shapes of the diffusion dynamics for different values of the parameter α. From the graphical presentation of cumulative adoption data, it is evident that, the larger is the value of α, the slower is the diffusion rate over time. The generalised Bass model, GBM with one exponential shock obtained when $\alpha = 1$ represented by the third line from the left, as in the Figure 3.1, is characterized by a graph showing a symmetric shaped normal curve for the instantaneous adoption data. Departures from this line when $\alpha < 1$, represent the existence of strong homogeneity. In that case, an exponential type model would perform better. For the reverse case (when $\alpha > 1$, below the third line in the graph), the diffusion dynamics is run by comparatively more heterogeneous agents and the graph of the instantaneous adoptions points to an asymmetrical shape of the curve. For a larger value of the α parameter, the diffusion rate is slower and the instantaneous adoption curve seems to have a longer tail to the left (as shown in the lowest line in the graph) indicating the existence of more heterogeneous agents. Bemmaor and Lee [7] observed that the value of $\alpha < 1$ is represents a more right-skewed distribution than Bass and the value of $\alpha > 1$, represents a more left-skew distribution than Bass.

The following figures show the possible effect of the introduced heterogeneity parameter α both for the instantaneous and cumulative version of the fitted model (for the case $m = 3000$, $p = 0.0023$, $q = 0.0987$, $a = 12$, $b = -0.3$, and $c = 1$).



(a) Cumulative adoption



(b) Instantaneous adoption

Figure 3.1: Effect of heterogeneity parameter α for $m = 3000$, $p = 0.0023$, $q = 0.0987$, $a = 12$, $b = -0.3$, and $c = 1$ in a GBMBME1 model.

It is very interesting to observe the shape and scale departures of the diffusion dynamics due to the changing value of the heterogeneity parameter. For the changing value of α, the curvature of the diffusion dynamics changes, also modification is observed in the instantaneous adoption peak. Homogeneous agents speed up the diffusion and the heterogeneous agents cause slow diffusion in the cumulative adoption. The most important feature is that, the heterogeneity introduced by Bemmaor [6] mostly influences the initial part of the diffusion dynamics for low level of α. That is, the introduced parameter α captures the adoption behaviour among the agents in the imitator subgroup only. Therefore, the proposed mathematical formulation of the diffusion dynamics may be extended, to identify the existing heterogeneity in both innovators and imitators subgroups.

3.3 Remarks

A simple but effective extension of the generalised Bass model, GBM, after introducing the heterogeneity among the innovator subgroup has been proposed in this chapter. This extension is important, in order to clarify the initial behaviour of the diffusion dynamics, characterized by the related innovators subgroup. The model parameters can be estimated through nonlinear least squares (NLS) method using the Levenberg-Marquardt algorithm [62]. Methodology are also available to obtain the accuracy of the proposed model. At the same time justification for the inclusion of an additional parameter can also be possible to verify. A brief discussion on the above is given in the following chapter (see, Section 4.3).

The proposed extension has two-fold advantages. In one hand, it has the capability to identify the level of heterogeneity among the agents, and on the other hand, a closed form equation is available for the model without further complexity, specially, for the generalised Bass model with various shocks. If the local perturbations in the diffusion dynamics correspond to a larger number of shocks and variations, the intervention function, $x(t)$, requires to include more parameters to identify them. In that case, it is not so easy to define the model and obtain the estimates of parameters, considering heterogeneity among the agents. Also heterogeneity among the innovators group should not be ignored in any perspective. That will be the considerable matter in the following chapters.

Chapter 4

Extensions of heterogeneous models

4.1 Introduction

In the previous Chapters, we have discussed the fundamental diffusion models and their extensions. We also underlined that the described models have their fundamental form as Logistic model. Therefore, in Chapter 3, an attempt has been made to propose extensions to explain the diffusion dynamics in a social system with both homogeneous and heterogeneous agents. The proposals, directly expressed through a closed form expression, are able to capture the modification in the diffusion dynamics due to the imitators subgroup. A further improvement in the model's flexibility could be obtained as soon as we consider that, heterogeneity in the agents' behaviour can occur within the imitators subgroup and in the innovators subgroup.

In this chapter, we consider the heterogeneity for both the innovators and imitators subgroups and try to capture the heterogeneity in the diffusion dynamics with a proposal of some extensions in the existing models.

4.2 Extension of Bemmaor model

4.2.1 Background

It has been observed in the previous Chapter that, the exponential formulation in the denominator of standard Bemmaor model is emphasized the heterogeneity among the imitators subgroup. The graphical representation in Figure 3.1 shows the effect on the shape and peak of the diffusion curve for various values of the parameter α. The innovators may show acceptance/reluctance attitude towards an innovation and their consumption propensity may affect the diffusion path, making it slower or faster. Therefore, it is also important to allow for a heterogeneity component among the agents in this subgroup.

4.2.2 Modified Bemmaor model

Let us consider the cumulative version of the Bemmaor model with the Bass parametrization. The diffusion equation is given by:

$$F(t) = \frac{\left(1 - e^{-(p+q)t}\right)}{\left(1 + \frac{q}{p} e^{-(p+q)t}\right)^{\alpha}}, \quad t \geq 0, \quad \alpha, p, q > 0. \tag{4.1}$$

From the above equation, it should be noticed that, the Bemmaor model can be considered as a composition of two parts, important for the real life interpretation purpose. The numerator that explains, mainly, the influence of innovators, and the denominator explains the prevalent contribution of imitators on the ultimate penetration. We point out that the ratio in Equation (4.1) is a product between two special distribution functions in a common range giving rise to a new distribution. In particular, the numerator is a monomolecular distribution while the reciprocal of denominator is a simple power through α of a logistic distribution.

Therefore, in the Bemmaor model, heterogeneity within the imitators is considered through a power in the denominator. To allow for heterogeneity also in the innovators subgroup, we can introduce a simple modification, based on a non-negative exponent for the innovators component. This intuitive idea gives rise to the following model (modified Bemmaor model, MBM) :

$$F(t) = \frac{\left(1 - e^{-(p+q)t}\right)^{\delta}}{\left(1 + \frac{q}{p} e^{-(p+q)t}\right)^{\alpha}}, \quad t \geq 0, \quad \alpha, \delta, p, q > 0. \tag{4.2}$$

The rate function can easily obtained as the equation:

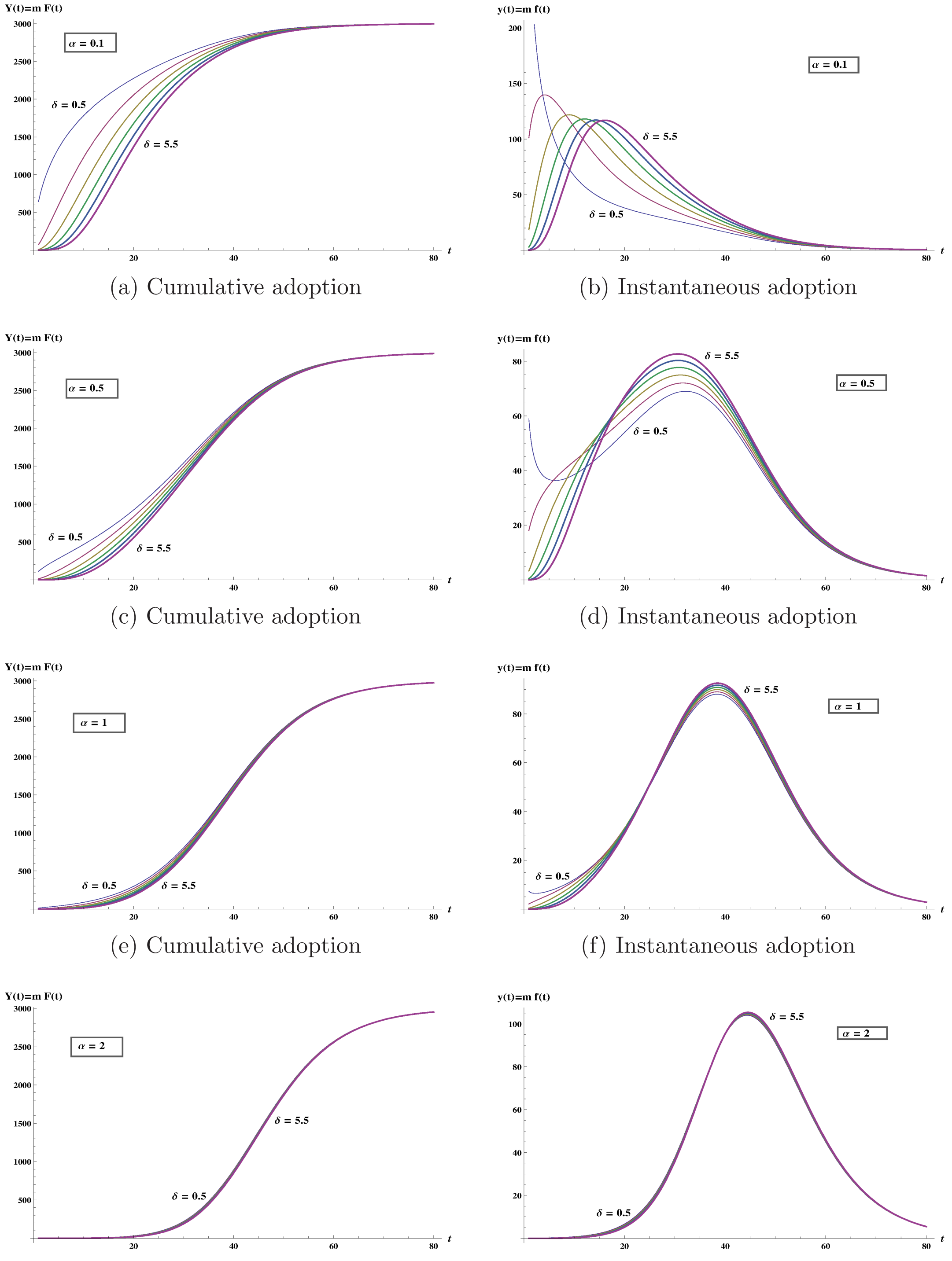
$$f(t) = \frac{(p+q)e^{-(p+q)t}\left[(p\delta + q\alpha) + q(\delta - \alpha)e^{-(p+q)t}\right]}{p\left(1 + \frac{q}{p} e^{-(p+q)t}\right)^{\alpha+1} \left(1 - e^{-(p+q)t}\right)^{-(\delta-1)}} \quad t \geq 0, \quad \alpha, \delta, p, q > 0. \tag{4.3}$$

Notice that, the numerator $\left(1 - e^{-(p+q)}\right)$ defines a special monomolecular distribution function over $t \in [0, +\infty)$, and from the probability calculus, any positive power of a distribution function is a distribution. This extension may be used in shifting a Gompertz with propensity η controlled by a gamma distribution that mimics Bemmaor and Lee's work [7].

The new parameter δ will speed up/suppress the initial starting behaviour of the adoption process and modify the curve peakedness. This extension is important for describing the quick/delayed entrance of the innovators, which could be explained as an effect of social contagion among the agents in this subgroup. For a fixed α, when $\delta = 1$, the modified model reduces to the standard Bemmaor model; a value for δ greater than 1 will delay the innovators' contagion process, whereas $\delta < 1$ will speed up the diffusion at the very beginning. In other words, the parameter δ may be considered as a measure of innovators' propensity to delay participation in the adoption process. Propensity to participate is high for small values of δ.

For $\alpha = \delta = 1$, the proposed modified model equals the standard Bass model. A more realistic situation, in which the degree of heterogeneity is high, would be characterized by values for α and δ significantly greater than from 1.

The following figures show the possible effect of the new innovators' heterogeneity parameter δ for different values of the imitators' heterogeneity parameter α. Figure 4.1 shows both the instantaneous and cumulative version of the diffusion dynamics with coefficients $m = 3000$, $p = 0.0013$, and $q = 0.1155$.



(a) Cumulative adoption

(b) Instantaneous adoption

(c) Cumulative adoption

(d) Instantaneous adoption

(e) Cumulative adoption

(f) Instantaneous adoption

(g) Cumulative adoption

(h) Instantaneous adoption

Figure 4.1: Effect of heterogeneity parameters α, and δ for $m = 3000$, $p = 0.0013$, and $q = 0.1155$ in a MBM

With a fixed α, the innovators contagion varies with the value of δ. As written before, the value of introduced parameter δ can be considered as a measure of heterogeneity in the innovators initialization. A higher value for δ corresponds to the case of a higher degree of heterogeneity among innovators in their subgroup. This behaviour delays the diffusion dynamics. The effect of the parameter δ is comparatively higher for lower values of α. In case of the existence of homogeneous agents in the imitators group (when $\alpha < 1$), the changes of δ are more effective. For $\alpha > 1$, the relative effect for δ is less appreciable.

The parameters α and δ can be interpreted in terms of the degree of connection in the communication networks among the agents in the respective subgroups in the existing social structure. If the early adopters are well connected and show homogeneous consumption intensity, the diffusion speeds up very rapidly, and touches the peak of curve at very early life cycle. On the other hand, a group of less connected early adopters, with more homogeneous imitators consumption propensity ($\delta > 1, \alpha < 1$) may results a speedy and asymmetric diffusion structure that contracts the life cycle of an innovation. A much stable and asymmetric diffusion pattern could be observed for the existence of a well connected and homogeneous agents. Vice versa, for high values of α and δ, heterogeneity in both subgroups defines a slow and almost symmetrical adoption process over time.

A further development of the above ideas may follow the consideration of bimodal or multimodal consumption propensities for both the early adopters and the late majority group. In the following subsection, we try to implement the proposed extensions in order to capture the local perturbations in the diffusion dynamics. The description starts with the illustration of the generalised Bass model, GBM.

4.2.3 GBM and Modified Bemmaor Mixed model

In Chapter 3 an attempt is proposed to extend the generalised Bass model with the introduction of a Bemmaor heterogeneous modelling approach. Let us recall the Generalised Bass Bemmaor Mixture model (GBMBM), which has the equation:

$$F(t) = \frac{\left(1 - e^{-(p+q)\int_o^t x(\tau)d\tau}\right)}{\left(1 + \frac{q}{p}e^{-(p+q)\int_o^t x(\tau)d\tau}\right)^{\alpha}} \quad , \qquad 0 \leq t < +\infty. \tag{4.4}$$

Function $x(t)$ represents the local intervention characterised by the effect of dynamic marketing and networking efforts on the conditional probability of adoption at time t, which may take different formulations based on the observed diffusion dynamics. The parameter α represents the heterogeneity of the imitators' subgroup.

Therefore, an extension of the above model, considering heterogeneity in both early adopters (innovators), and late majority (imitators) (GBM mixture with modified Bemmaor model, GBMBMM) may be proposed with the following cumulative distribution function:

$$F(t) = \frac{\left(1 - e^{-(p+q)\int_o^t x(\tau)d\tau}\right)^{\delta}}{\left(1 + \frac{q}{p}e^{-(p+q)\int_o^t x(\tau)d\tau}\right)^{\alpha}} \quad , \qquad 0 \leq t < +\infty. \tag{4.5}$$

The model will take different instantaneous versions depending on the characterization of defined intervention function $x(t)$.

For example, a modified Bemmaor model with an exponential shock has the following cumulative distribution function (GBMBMME1):

$$F(t) = \begin{cases} \frac{\left(1-e^{-(p+q)t}\right)^{\delta}}{\left(1+\frac{q}{p}e^{-(p+q)t}\right)^{\alpha}} & \text{if } t < a \\ \frac{\left(1-e^{-(p+q)[t+\frac{c}{b}e^{\{b(t-a)-1\}}]}\right)^{\delta}}{\left(1+\frac{q}{p}e^{-(p+q)[t+\frac{c}{b}e^{\{b(t-a)-1\}}]}\right)^{\alpha}} & \text{if } t \geq a. \end{cases} \tag{4.6}$$

The instantaneous version of the model can be expressed with the equation:

$$f(t) = \begin{cases} \frac{(p+q)e^{-(p+q)t}\left[(p\delta+q\alpha)+q(\delta-\alpha)e^{-(p+q)t}\right]}{p\left(1+\frac{q}{p}e^{-(p+q)t}\right)^{\alpha+1}\left(1-e^{-(p+q)t}\right)^{-(\delta-1)}} & \text{if } t < a \\ \frac{(p+q)e^{-(p+q)u}\left(1+ce^{b(t-a)-1}\right)\left[(p\delta+q\alpha)+q(\delta-\alpha)e^{-(p+q)u}\right]}{p\left(1+\frac{q}{p}e^{-(p+q)u}\right)^{\alpha+1}\left(1-e^{-(p+q)u}\right)^{-(\delta-1)}} & \text{if } t \geq a, \end{cases} \tag{4.7}$$

where, $u = \frac{c}{b}e^{b(t-a)-1} + t$.

The following graphs show the possible effect of introduced innovators' heterogeneity parameter δ for various value of imitators' heterogeneity parameter α, both on the instantaneous and cumulative versions of the diffusion dynamics with one exponential shock. The model coefficients i.e, $m = 3000$, $p = 0.00068$, $q = 0.1155$, $a = 12.75$, $b = -0.255$, and $c = 0.64$ have been considered for the example.

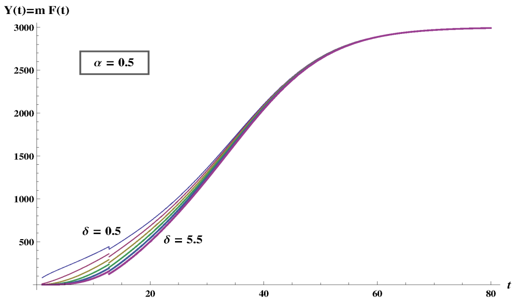
It is evident from Figure 4.2 that the heterogeneity is effectively influencing the diffusion dynamics at the early stage of adoption. The existence of more heterogeneity is observed among the agents in different subgroup of populations, the slower the diffusion pattern and resulted a longer life cycle of an innovation. The reverse situation may observed for the opposite case. The shock intensity and its persistence are, in general, uncorrelated to the heterogeneity measured by α and δ. However, we observe that a fixed intensity c for a shock starting at a specific time point a, has a different effect according to the $f(t)$ value reached by the process at that time point. Since that $f(t)$ value is influenced by α and δ, this explains why the same shock appears as different for different (α, δ) values.

The modified models proposed in this chapter are very simple but important in terms of explaining the contagion effect with specific parametrization of innovators' and imitators' penetration in the diffusion process. The empirical validation of these postulates is discussed in Chapter 6 with a real dataset considering an example of the diffusion dynamics of a technological innovation.

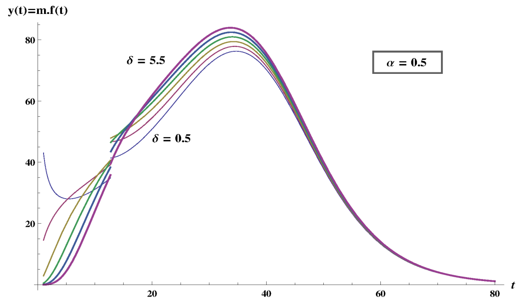
4.3 Model parameter estimates and inference

4.3.1 Parameter estimation and validation of forecasts

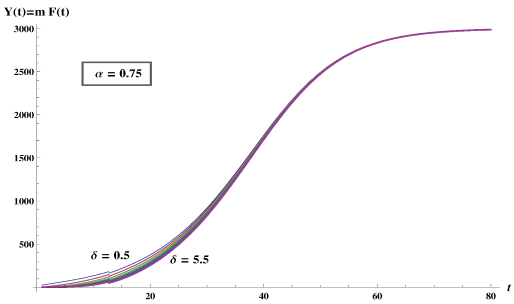
Like other nonlinear model estimation, time series data for the innovation diffusion dynamics can be specified in a nonlinear regression framework. The traditional Bass model, the GBM in Equations (2.3, 2.8) and heterogeneity models in Equations (2.14), (3.1), (4.2), and (4.5) can be specified in a nonlinear regressive



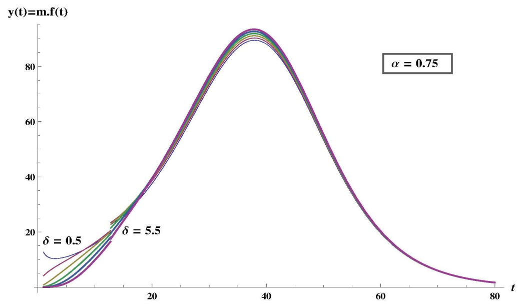
(a) Cumulative adoption



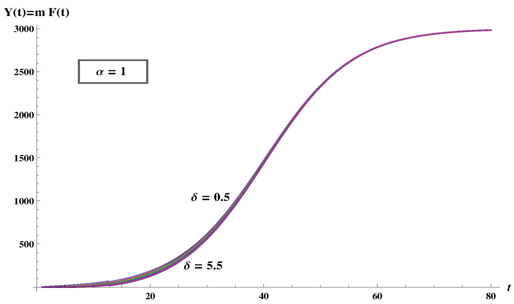
(b) Instantaneous adoption



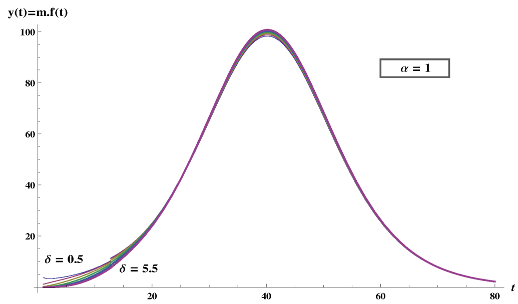
(c) Cumulative adoption



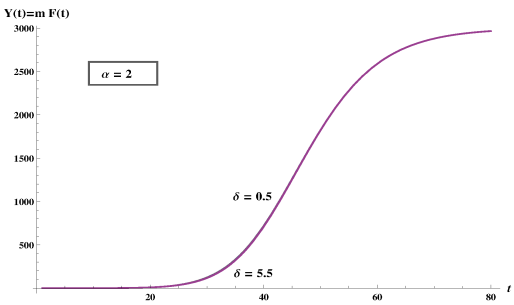
(d) Instantaneous adoption



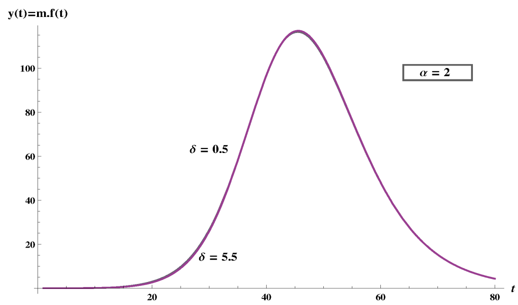
(e) Cumulative adoption



(f) Instantaneous adoption



(g) Cumulative adoption



(h) Instantaneous adoption

Figure 4.2: Effect of heterogeneity parameters α and δ for $m = 3000$, $p = 0.00068$, and $q = 0.1155$ under an exponential shock characterized by parameters $a = 12.75$, $b = -0.255$, and $c = 0.64$ in a GBMBMME1

equation as follows:

$$Y(t) = g(\beta, t) + \epsilon(t), \tag{4.8}$$

where $Y(t)$ represents the cumulative observed data, $g(\beta, t)$ is the cumulative deterministic component of the model specified through the cumulative mean process $m \cdot F(t)$ of adoption over time, β is the parameter vector, and $\epsilon(t)$ is a white noise process. The model parameters can be estimated using the nonlinear least squares (NLS) method following the Levenberg-Marquardt algorithm [62]. Moreover, for a sequence of models with increasing complexity, the nonlinear least squares (NLS) estimates of the common parameters of a simpler model can be used as a starting point for the parameters' iterations of a more complex model. This is a good practice, useful for avoiding convergence problems in the search algorithm, which is very sensitive to initial tentative values. The NLS estimates for the nonlinear regression parameters are based on a nonparametric methodology that does not depend on distributional aspects of residuals if the mean trajectory of the estimated regression model is essentially correct.

Aiming to obtain a more accurate forecast, the predicted models can be combined with an autoregressive integrated moving average with exogenous inputs (ARMAX), or Box-Jenkins forecasting approach. Thus, the estimated function $g(\hat{\beta}, t)$ can be used in an ARMAX model, as an input variable, to obtain a convenient expression of the residual structure in $\epsilon(t)$ that may be characterized by auto-dependence effects very far from a standard white noise assumption.

The combination of ARMAX with nonlinear diffusion models is presented through previous two-stage procedure in Guseo and Dalla Valle [28] and Guseo et al. [29]. A similar approach may be found in Christodoulos et al. [14]. The basic idea is to estimate conditionally the Box-Jenkins parameters of the following model:

$$\Phi(B)\left(y(t) - g(\hat{\beta}, t)\right) = \theta(B) a_t, \tag{4.9}$$

where, a_t is a white noise process. Properties of the resulting composite predictor $\tilde{Y}_t$ may be studied with the usual tools, for instance, $\rho^2_{Y(t),\tilde{Y}_t} = R^2$ or similar index, such as, Mean Square Error (MSE), Mean Absolute Error (MAE) and

Mean Absolute Percentage Error (MAPE) with respective expressions:

$$MSE = \frac{1}{T}\sum_{t=1}^{T}\left(\tilde{Y}_t - Y_t\right)^2, \tag{4.10}$$

$$MAE = \frac{1}{T}\sum_{t=1}^{T}|\tilde{Y}_t - Y_t|, \tag{4.11}$$

$$MAPE = \frac{1}{T}\sum_{t=1}^{T}\frac{|\tilde{Y}_t - Y_t|}{|Y_t|}, \tag{4.12}$$

where $\tilde{Y}_t$ and Y_t are the predicted and actual value at time t respectively and T is the number of predictions.

Previous type of analysis is usually performed in order to study the residual term $\epsilon(t)$ for short term prediction under the nonlinear evolutionary behaviour of $g(\beta,\ t)$.

4.3.2 Validation of additional parameter in the model

Following Guseo et al. [29], the significance of the gain from a simpler model (M_1) to a more complex model (M_2) can be evaluated in two steps. As the first step, the squared multiple partial correlation coefficient is computed with the following:

$$\tilde{R}^2 = \left(R^2_{M_2} - R^2_{M_1}\right) / \left(1 - R^2_{M_1}\right). \tag{4.13}$$

If N denotes the number of observations used to fit the models, and λ is the number of parameters considered for model M_2, the significance of the κ parameters in M_2 not included in model M_1 can be evaluated with a special form of *F-statistics* defined as follows:

$$F = \left[\tilde{R}^2\left(N-\lambda\right)\right] / \left[\left(1-\tilde{R}^2\right)\kappa\right]. \tag{4.14}$$

If $\epsilon(t)$ is also i.i.d. normal, then $F \sim F_{\kappa,(N-\lambda)}$. More generally, considering the common threshold 4 for the *F-ratio* in Equation (4.14) as an approximate robust criterion to compare model (M_1) nested in model (M_2), the comparative performance can be evaluated (see, in particular, [29]).

Alternatively, under residuals' normality assumption for $\epsilon(t)$, the BIC criterion (see, for instance, [61, 53]) can be used to identify the best model:

$$BIC = N \cdot \ln(RSS/N) + k \cdot \ln(N), \tag{4.15}$$

where RSS represents the residual sum of squares for the fitted model and k is the number of parameters. A lower value for BIC is expected for the best fitted model.

4.4 Remarks

Considering heterogeneity among the agents in both innovators and imitators subgroup, an extension of the existing Bemmaor model is proposed in this chapter. Bemmaor [6] postulates that the agents/individuals in the existing system are heterogeneous with respect to their consumption propensity which is distributed as Gamma distribution. In this proposal, we follow the same criterion allowing for heterogeneity. The advantage of such an extension is that it can identify the heterogeneity among the agents in both innovators and imitators subgroups.

The considered additional parameter can be interpretable with reference to the communication and network structure among the agents in the diffusion dynamics. The model also has the strength to identify the local interventions with adequate GBM specifications. The extended models can perform well with better forecasting accuracy when at least a part of perturbation is included in the observed time series.

In real situations, the agents in a diffusion process may show distinct behaviour with respect to the consumption behaviour, communication network and contagion process. Sometimes, those process are not compatible to describe with an unimodal distribution. Even the local intervention characterisation of shocks may need to include a higher number of parameters to describe the process. Therefore, this modelling approach requires, obviously, further development to face identification problems with insufficient data, or explaining a diffusion dynamics which is at initial stage.

Chapter 5

Application of existing models to South Asian natural gas production data

5.1 Introduction

In the previous Chapters, a number of existing diffusion models and their proposed extensions are discussed in brief. In order to describe their efficacy in practice, an application of the existing homogeneous models and their extensions to a real data set will be discussed in this chapter. We consider the production of natural gas in the technological diffusion framework and try to figure out the real consequences of the social contagion effect and the local intervention strategies that is captured by the respective diffusion modelling approach.

In this Chapter, we study natural gas production in four selected south Asian states, i.e., Bangladesh, Myanmar, India and Pakistan. These neighbouring states have a history of natural gas production starting in the early 1970s and later growing with a complicated administrative system, legislative constraints, architectural and technological limitations and political crisis. Like other technological diffusion processes, natural gas production in these regions is affected not only by endogenous mechanisms (geographical structure of the ground level or scientific discovery) but also by some exogenous mechanisms (government decisions, the availability of technological assistance and equipment, or investment support). A comparison among these countries would be useful in describing the regional energy scenario, in highlighting and explaining historical growth patterns and in providing insights on the future of natural gas consumption.

5.2 Background

During the last decades, forecasting and estimates of the natural gas reserves appeared in different studies considering various methodological aspects and using available data at global, national, and/or regional levels. With recent technological advancements and use of modern equipment, extraction and distribution system for natural gas have been greatly upgraded to meet the increasing consumption demand. New forecasting models appeared with respect to the conventional Hubbert approach. Econometric approaches, statistical and mathematical modelling, engineering frameworks, software simulations, and, spatio-temporal contexts defined the main directions of research. A state-of-the-art survey and a synthesis of published research in the field of forecasting natural gas consumption can be found in Soldo [65]. Brandt [9] presented a review of mathematical models of future oil supplies and concluded that future developments in oil depletion modelling lie in simulation models that combine both physical and economic aspects of oil production. Rao and Kishore [54] argued that the theory of diffusion modelling allows analysis of diffusion processes and studied the growth patterns of different technologies, considering the underlying diffusion factors. They have also presented a brief review of technological diffusion models with reference to renewable energy technologies.

Although the consumers are the main driver in the success of a newly introduced technology and its diffusion process, the response from the existing social structure, which is very much involved with the existing support mechanisms, cannot be ignored. To this end, diffusion of the adoption of natural gas production in the selected states could be suitably modelled with the traditional Bass model (BM) (Equation (2.4)) and its generalization (GBM) (Equation (2.7)) for evaluating both endogenous and exogenous mechanisms. The motivation for this choice is that any production process in an energy context (and elsewhere) is strongly associated with and determined by the diffusion of the specific technologies that depend on those resources. This approach follows Rogers' concept [56] of complex social systems for the inclusion of innovative and imitative behaviour of the adopters of a particular technology. At the same time, an assessment of the strength of the incentive policies passed by the local government of a given country could also be investigated.

5.3 The data

This study uses annual natural gas production (in billion cubic meters, BCM) in four neighbouring states in South Asia (Bangladesh, India, Myanmar and Pakistan) by British Petroleum [11] for the period 1971 to 2011. Given that the selected countries share a similar geographical structure and cultural heritage, these states are experiencing an almost equivalent pattern of socio-economic and demographic structures with a recent rapid increasing trend of economic development.

As shown in Figure5.1, the trend of natural gas production is almost equivalent for two pairs of neighbouring countries, Bangladesh-Myanmar and India-Pakistan. The major gas extraction in the region is still done by state-owned companies. Since Bangladesh and Myanmar are on the same geographical plates, they have experienced an almost identical historical natural gas extraction. Bangladesh uses its extracted gas for domestic consumption needs, and, for Myanmar, natural gas represents an important contribution to annual exports. Both countries have undertaken initiatives to open the extraction of natural gas to foreign companies in the early 2000s, which accelerated the production and extraction of their natural resources.

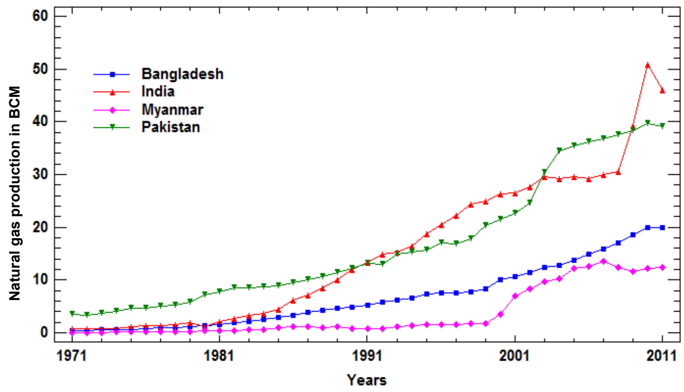


Figure 5.1: Natural gas production in selected South Asian countries

India and Pakistan are the nuclear-power holders in this sub-continent, two rising economic powers that are nevertheless experiencing a crisis with respect to sustainable energy policies for their large domestic consumption. Pakistan's natural gas production history began in the early 1960s with a rapidly increasing rate of extraction for domestic consumption needs. Conversely, India's natural gas extraction history started a little later but rapidly accelerated as a part of the energy mix. This acceleration was due to technological developments with sophisticated extraction equipment and the recent discovery of new gas fields in different geographic locations in the aforementioned countries. Now, however, the production is showing a decreasing trend that requires explanation and judgement for strategic purposes.

First, we consider the standard Bass model (BM) (Equation (2.3)). Like other studies on the statistical implementation of the Bass model, we use a non-linear regression approach for the parameter estimates. We then consider a GBM (Equation (2.8)) with specific intervention functions $x(t)$ that may be able to capture the shocks observed in the dataset. The adequacy of the fitness of proposed models can be evaluated through the determination index R^2, the *BIC* (see, Equation (4.15))([61]; [53]) and/or *F-tests* based on the adjusted partial correlation coefficient [29] (see Equation 4.14).

5.4 Results and discussion

The selected four South Asian countries have different histories of natural gas production, varying with the number of gas fields discovered and the estimated reserves. Of course, these countries have also passed through different strategic policies for gas extraction with respect to time, technological developments and extraction support by the foreign communities. We try to model the observed trends of annual gas production and cluster them into two pairs, employing convenient models. In this way, we are able to understand the intervention aspects that either sped up the extraction process or slowed it down, eventually highlighting different strategic policies.

5.4.1 Bangladesh and Myanmar

We start the analysis with the Bangladesh dataset. The standard Bass model (BM) presents a determination index R^2 value of 0.999488 with a prediction for the first observed value of $z_{1970} = 0.64$; this clearly depicts the start of natural gas extraction in 1970. As the observed data series experienced shocks at different time points, we may enhance our prediction by introducing a GBM. Table 5.1 presents the results obtained for GBM with one and two exponential shocks. The observed values of the *F-statistic* and the *BIC* indicate that a GBM with two exponential shocks (GBM2) exhibits a better fit to the observed data.

Table 5.1: Bangladesh: model selection

Model	R^2	df	$\tilde{R}^2_{Vs.BM}$	$F_{Vs.BM}$	$\tilde{R}^2_{Vs.GBM1}$	$F_{Vs.GBM1}$	RSS	BIC
BM	0.999488	38	–	–	–	–	129.33	58.2413
GBM1	0.999952	35	0.90625	112.778	–	–	11.22	33.501
GBM2	0.999986	32	0.972656	189.712	0.70833	25.904	2.94	30.787

Table 5.2 represents the parameter estimates for the GBM with two exponential shocks (GBM2). The respective asymptotic confidence intervals denote stability

Table 5.2: Bangladesh: parameter estimates for GBM with two exponential shocks

Model parameters		Estimates	Asymptotic standard errors	Asymptotic 95% confidence intervals	
				Lower	Upper
General penetration parameters	m	1191.53	269.049	643.491	1739.56
	p	0.000394248	0.0000663284	0.000259141	0.000529355
	q	0.0978616	0.0120549	0.0733064	0.122417
First observed exponential shock parameters	a_1	11.3492	0.485556	10.3691	12.3382
	b_1	-0.123598	0.0649745	-0.255947	0.00875072
	c_1	0.570926	0.0818882	0.404125	0.737727
Second observed exponential shock parameters	a_2	25.2846	0.367149	24.5367	26.0324
	b_2	-0.108772	0.0768354	-0.26528	0.0477373
	c_2	-0.164139	0.0254792	-0.216039	-0.11224

in the estimates for the general penetration parameters that indicate a strong imitative behaviour in natural gas production (and, therefore, consumption) in Bangladesh. The parameter m gives an estimate for the cumulative production until 2030, which indicates that, by 2011, Bangladesh had exhausted almost 24% of its available gas resources. As indicated by the prediction shown in Figure 5.2, the production is still following an increasing trend, and peak production will be observed between 2021 and 2026.

The process experienced two mean-reverting (since $b_1, b_2 < 0$) exponential shocks around $1982 \approx (1971 + a_1)$ and $1996 \approx (1971 + a_2)$ that accelerated production process. The historical consequences of these shocks are important. New gas field discoveries and new strategic and administrative policies implemented by the government or the controlling authorities over the course of time introduced an intervention in natural gas extraction. The first observed shock may have been caused by the new gas field discovery at Beani Bazar in 1981 by Bangladesh Petroleum Exploration and Production Company Limited (BAPEX), which was the first gas field discovered after independence was achieved in 1971.

From 1995 to 2001, the Ministry of Power, Energy and Mineral resources, Bangladesh, welcomed international oil companies to collaborate with BAPEX in the mining of oil and other natural resources. This approach created a new era of discovery of natural resources and resulted in one off-shore gas field discovery (Sangu in 1996 by Cairns Energy) and two on-shore gas fields (Bibiyana in 1997

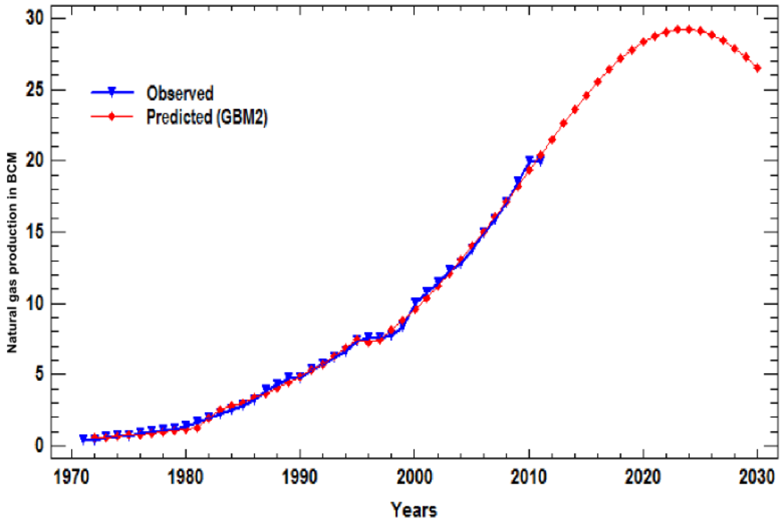


Figure 5.2: Bangladesh: observed gas production and estimated predictions

and Moulavibazar in 1998 by Oxidental/Unicol) that accelerated production. At present, 23 gas fields have been discovered, and 16 have been set to production.

Although Myanmar is one of the oldest oil producing countries in the world, its natural gas production history is quite recent, but most of its annual revenue now comes from the natural gas exports. The production process has experienced frequent ups and downs. We proceed with the standard Bass model, and, later, considering the shocks observed in the graphs of the data series, we try to fit the GBM models with one and two exponential shocks, respectively. Let us examine the results in Table 5.3. It is evident that, although a little development is observed in the value of the determination index R^2 for the GBM with two exponential shocks (GBM2), GBM1 and GBM2 are potentially equivalent (the second shock is not significant), and the second shock intensity c_2 may be zero. The major variation occurs due to a large shock around the year $2000 \approx (1971 + a_1)$ that minimised local perturbation effects. The parameter estimates in the GBM model with one exponential shock (GBM1) and the results are presented in Table 5.4.

Table 5.3: Myanmar: model selection

Model	R^2	df	$\tilde{R}^2_{Vs.BM}$	$F_{Vs.BM}$	$\tilde{R}^2_{Vs.GBM1}$	$F_{Vs.GBM1}$	RSS	BIC
BM	0.994438	38	–	–	–	–	360.627	100.2858
GBM1	0.999800	35	0.964042	312.786	–	–	11.9735	-28.1845
GBM2	0.999819	32	0.967458	158.5574	0.095	1.11971	9.90302	-24.8278

Table 5.4: Myanmar: parameter estimates for GBM with one exponential shock

Model parameters		Estimates	Asymptotic standard errors	Asymptotic 95% confidence intervals	
				Lower	Upper
General penetration parameters	m	226.262	24.3843	176.759	276.765
	p	0.00064712	0.00007451	0.00049586	0.00079837
	q	0.100935	0.003134	0.0945723	0.107297
Observed exponential shock parameters	a_1	30.0146	0.191302	29.6263	30.403
	b_1	-0.043977	0.0338031	-0.112601	0.024647
	c_1	1.71503	0.159097	1.39204	2.03801

As in Bangladesh, the natural gas production of Myanmar is showing stable imitative behaviour and already experienced peak production in 2007 (as shown in Figure 5.3). The stable parameter m gives an estimate for the cumulative production until 2030 and indicates that, by 2011, Myanmar had exhausted 65% of its available gas resources, and, in the near future, the process will show a sudden decrease in natural gas production if new discoveries are not made. The diffusion process experienced a large positive shock ($c_1 > 1$) around 2000 that has already run its course ($b_1 < 0$). In the early 1990s, the Myanmar ruling military authority SLORC (State Law and Order Restoration Council) opened up the opportunity for foreign companies to search for oil and gas. Consequently, two major offshore gas fields, Yadana and Yetagun, were discovered in 1998, with approximate reserves of 150 BCM and 48 BCM, respectively. Production started in 2000 and intensified after 2004 when Myanmar authorities accelerated the opening of gas exploration and new gas fields were discovered along the Arakan coastline. In the meantime, Myanmar came to an agreement with Thailand, China and India for energy security concerns and started importing natural gas. At present, 27 companies from 13 different countries are active in Myanmar's natural gas and oil industry.

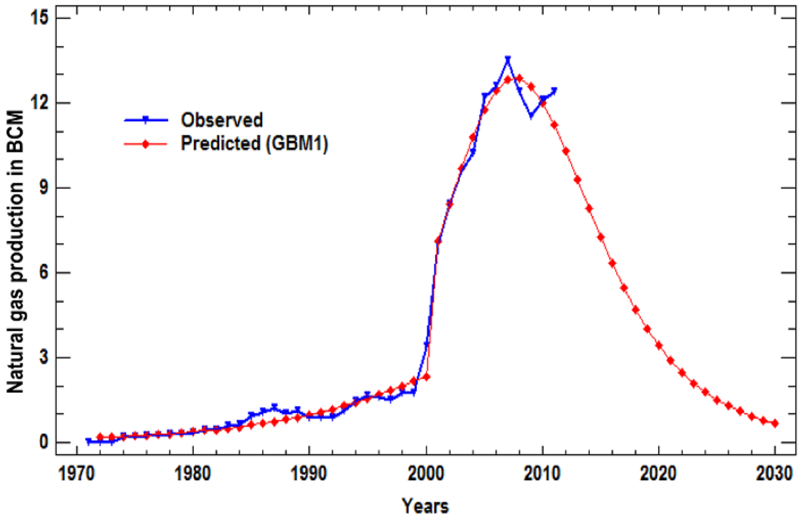


Figure 5.3: Myanmar: observed gas production and estimated predictions

5.4.2 India and Pakistan

India, the seventh-largest country in the world and the fifth-highest energy consumer because of its fast-growing economy and population growth, has suffered from a significant energy crisis in recent times. Natural gas production in India is showing a rapid increasing trend, but it is still unable to meet half of the demand, so authorities are looking for more imports and unconventional sources like shale gas [21]. The production structure according to the observed data, shows standard imitative behaviour with a sudden shock in 2009. The model fitness results shown in Table 5 indicate that a GBM with two exponential shocks (GBM2) performs better for data description and prediction purposes.

Table 5.5: India: model selection

Model	R^2	df	$\tilde{R}^2_{Vs.BM}$	$F_{Vs.BM}$	$\tilde{R}^2_{Vs.GBM1}$	$F_{Vs.GBM1}$	RSS	BIC
BM	0.998532	38	–	–	–	–	2068.82	171.9083
GBM1	0.999395	35	0.59039	16.816	–	–	779.974	5.4204
GBM2	0.999991	32	0.99391	870.419	0.98512	706.182	10.059	-24.1871

Table 5.6: India: parameter estimates for GBM with two exponential shocks

Model parameters		Estimates	Asymptotic standard errors	Asymptotic 95% confidence intervals	
				Lower	Upper
General penetration parameters	m	903.351	21.7108	859.127	947.574
	p	0.000708234	0.000026245	0.000654775	0.000761693
	q	0.118954	0.00765065	0.10337	0.134538
First observed exponential shock parameters	a_1	14.6661	0.261476	14.1335	15.1987
	b_1	-0.0947156	0.0188191	-0.133049	-0.0563822
	c_1	0.716346	0.0765113	0.560498	0.872195
Second observed exponential shock parameters	a_2	38.542	0.0530763	38.4339	38.6501
	b_2	-0.0430395	0.0656805	-0.176827	0.0907476
	c_2	0.97833	0.0916385	0.791668	1.16499

Table 5.6 presents the parameter estimates of the selected model, and the stable general penetration parameter estimates indicate that the diffusion process is characterized by strong imitative behaviour. The production of natural gas shows a standard diffusion process with a sudden jump in 2008, and, by 2011, almost 70% of the gas URR had been extracted.

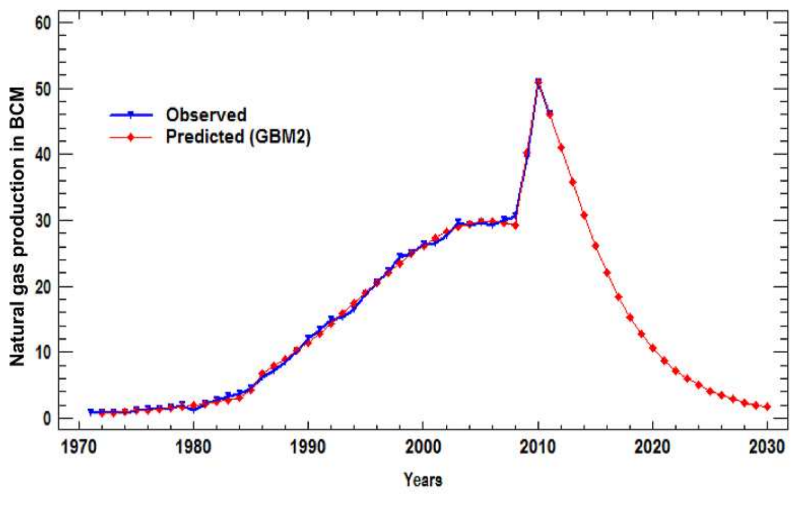


Figure 5.4: India: observed gas production and estimated predictions

One important and significant mean-reverting shock can be observed around 1985 ≈ (1971 + a_1) just after the state-owned Gas Authority of India Limited

(GAIL) was structured to promote gas use and develop the midstream and downstream gas structure in 1984. GAIL held a monopoly in the effective transmission and distribution of natural gas until 2006. Some other public-private partnership companies, such as Reliance, were also approved later to join the production line to meet high domestic consumption. In addition, probably due to the government policy to convert petroleum-driven vehicles to Compressed Natural Gas (CNG) engines, production process experienced another large shock around the middle of 2009 $\approx$ $(1971 + a_2)$. Although India started importing natural gas with pipelines and agreements with Myanmar, Qatar, Turkmenistan, Pakistan and Israel in 1994, in the future, the situation may become worse than expected (as shown in Figure 5.4), and India will require a great deal of imports to meet the internal consumption demand.

In the South Asian context, Pakistan has a long history of natural gas production that began in the early 1960s. Unfortunately, the current dataset does not cover the complete history, and, for comparison, we consider the production history from 1971. This modified data set must be described by the translated Bass model that has the capability to identify the starting point for the diffusion process as well.

We use the translated Bass model (BMT) (Equation (2.5)) for Pakistan natural gas production data. Table 5.7 displays the results for different model choices in the translated version for a suitable description of the diffusion process from the available historical data. The F-statistic of the partial correlation coefficient does not support a strong preference between GBM1T and GBM2T (they are equivalent), so we proceed with a GBM with one exponential shock (GBM1T) under the translated parametric set-up.

Table 5.7: Pakistan: model selection

Model	R^2	df	$\tilde{R}^2_{Vs.BM}$	$F_{Vs.BM}$	$\tilde{R}^2_{Vs.GBM1}$	$F_{Vs.GBM1}$	RSS	BIC
BMT	0.999635	37	–	–	–	–	528.907	119.7012
GBM1T	0.999977	34	0.936986	168.521	–	–	31.2548	25.7426
GBM2T	0.999982	31	0.950684	99.600	0.21739	2.870	21.2745	10.2371

Table 5.8: Pakistan: parameter estimates for GBM with one exponential shock in the translated state

Model parameters		Estimates	Asymptotic standard errors	Asymptotic 95% confidence intervals	
				Lower	Upper
General penetration parameters	m	2498.9	504.554	1473.52	3524.28
	p	0.0014985	0.0002703	0.00094926	0.0020479
	q	0.0666232	0.00205941	0.0624379	0.0708084
	d	-0.576818	0.134038	-0.849217	-0.304419
Observed exponential shock parameters	a_1	32.1567	0.171246	31.8087	32.5048
	b_1	-0.140606	0.0486995	-0.239575	-0.0416361
	c_1	0.418448	0.027464	0.362634	0.474261

The parameter estimates and diffusion prediction results proposed in Table 5.8 and Figure 5.5 show a very unstable and slow diffusion process for natural gas production in Pakistan. Although primary energy consumption has grown up almost by 80% in the last 15 years,natural gas production and consumption are showing a very low increasing trend due to low gas prices and other disincentives. The estimate of parameter d indicates that, although the production started a few years before our selected starting point, the past history does not affect the diffusion process in practice, and the estimate of m suggests that almost 27% of the natural gas reserves had been extracted by 2011. Peak production will be observed between 2020 and 2025.

Figure 5.5 shows that Pakistan's natural gas production process experienced two different shocks. The large shock observed around 2003 $\approx (1971 + a1)$ was probably due to the merger of the the two operating private sector gas companies KGC (Karachi Gas Company) and IGC (Indus Gas Company) with the Sui Northern Gas Pipelines Limited and the sanctioning of a large number of household connections. At the same time, the government mandated Compressed Natural Gas (CNG)conversion of vehicle engines, increasing consumption, and this resulted in a sudden jump in production. The Pakistan Ministry of Petroleum and Natural Resources annual report [46] states that natural gas comprises 48% of the total energy mixture, almost 28% of which is used for power generation purposes. To meet the additional consumption, Pakistan has signed gas pipeline agreements with Turkmenistan, Qatar and Iran.

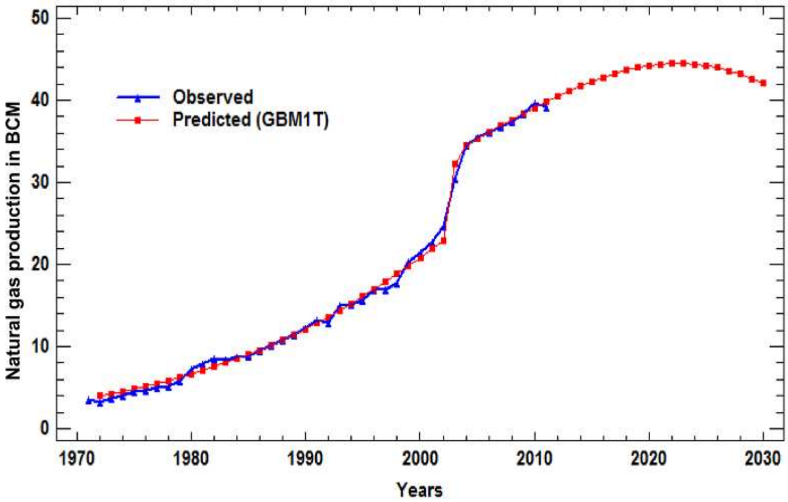


Figure 5.5: Pakistan: observed gas production and estimated predictions

5.5 Remarks

The modelling of a complex system that is interrupted by various exogenous and endogenous factors is neither simple nor easy. However, so far, scholars have tried to overcome the difficulties and model the process so that most influential events can be identified in the observed trend. The research output of the present study has dual importance. On one hand, it explains the existing trend in natural gas production in the selected countries based on the historical data, and, on the other hand, it proves the strength and efficacy of the existing diffusion models to capture the important consequences influencing the natural gas extraction trend with statistical validity.

The results indicating the existing trend and forecast for the South Asian energy sector, with a special emphasis on natural gas production research output, provides a picture of an area where the countries show different complementary patterns. According to the historical data of natural gas production, it is evident that, with respect to energy deficiency, India and Pakistan are on one side, while Myanmar (a natural gas exporter) and Bangladesh (a prospective reserve holder) are on the opposite side. Except for Myanmar, in all the countries that extract natural gas for internal consumption, a majority is used for power generation for domestic and industrial use.

Although Pakistan and India are the nuclear-power holders in this region, they are still late in providing natural gas for the energy mix. Due to recent off-shore discoveries of new gas fields and after the economic sea border settlement, Bangladesh and Myanmar have bright prospects to increase their natural gas production level.

India and Pakistan are now looking for possible options to import gas to meet their domestic consumption needs. A joint energy pact among these four states can give rise to the creation of a sustainable and stable energy zone in this area. Behind the existing complementaries because of the flip side of massive fossil fuel consumption, the negative impact of global warming should not be overlooked.

This application of diffusion modelling has a two-fold significance. In one way, it shows the efficacy of the available diffusion models to interpret the real life situations which is interrupted with strategic policy interventions. On the other hand, it obtains an estimate of future reserves of URR in a very conventional way without considering the monetary interventions in a homogeneous state of consumption.

Chapter 6

Application to Algerian natural gas production data

6.1 Introduction

In technology diffusion, rates of diffusion are context specific; they depend on socio-economic acceptance, technological advancements and institutional factors that facilitate or hinder diffusion and drive the inter-linked process as a complex phenomenon [49]. Non-renewable energy productions over time follow life cycle patterns that may be interpreted as diffusion of innovation processes, or related approaches. For specific modelling see, for instance, [1, 8, 9, 71].

Following, in particular, the world oil extraction dynamics analyzed in [27], we observe that, in a historical natural gas production series, the extraction dynamics can be considered as a function of local and international demand driven by related natural gas-consuming technologies, direct and indirect extraction costs, energy return on energy investment, strategic opportunities and environmental constraints. This complex system can be modelled under the assumption of deterministic or semi-deterministic regulatory interventions. Therefore, this system is suitable for examining natural gas reserves through the characterization of an evolutionary production pattern of an ultimately recoverable resource (URR) under a finite life cycle hypothesis. In this perspective, the production process can be considered as a result of all concurring forces, and may be easily interpreted in a diffusion of innovation frame.

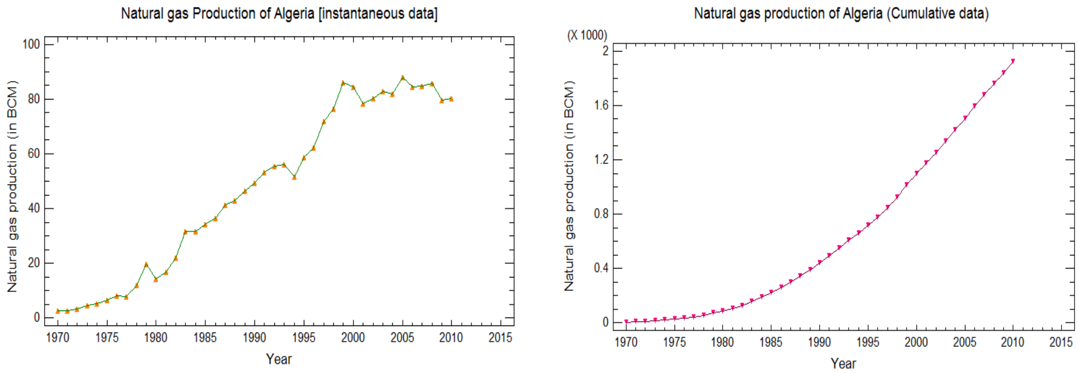
In Chapter 5, we discussed the application of usual diffusion models to natural gas production data and appreciating to what extent diffusion models are capable to capture the local perturbation effects with a valid estimate of the URR. The aim of the present Chapter is to apply the existing diffusion of innovation models and their extensions proposed in Chapter 3 and 4, in order to make a valid comparison of the models' parameter estimates. The following Sections are devoted to analyzing Algerian natural gas production within the logic of a diffusion of innovation process. Some interesting results are provided for the evolving dynamics, peak time and reserves estimation.

6.2 The data

Algeria owns the eighth-largest natural gas reserve, with 159 trillion cubic feet (about 4500 billion cubic meters) of proven natural gas, according to *Oil and Gas Journal*. Results from the BP Statistical Review of World Energy 2010 [10] indicate Algeria holds 2.4% of the total world gas reserves. The reserve-to-production ratio is 55.3 years, but this type of index is often questioned, because it does not consider the nonlinear extraction dynamics. The country is the fourth-largest exporter of natural gas to Europe. Algeria's natural gas sector expanded rapidly on the heels of increased production.

Recent successes were aided by international partnerships and technological advances, and the country is, at the same time, looking forward to solidifying its standing as a regional transit hub for natural gas. Sonatrach dominates the country's natural gas production and wholesale distribution; however, foreign investments in the sector are continuously increasing. Foreign producers such as PCI, BP, Statoil, Total, BHP-Billiton, Eni and Repsol have entered into partnership agreements with Sonatrach since the early 1970s.

The present study uses Algerian natural gas production data (in billion cubic meters, BCM) obtained from [10] for the period from 1970 to 2010. As seen in



(a) Instantaneous data

(b) Cumulative data

Figure 6.1: Natural gas production in Algeria

Figure 6.1, starting from the 1970s, the scenario of Algerian gas production follows an increasing trend until 2005 with some ups and downs at different sections but shows a slow decreasing trend afterwards. The rising trend is due to the increasing demand from the three top consumers (i.e., Italy, Spain and France), but the unexpected slow declining trend after 2005 is a matter of discussion.

6.3 Results and discussion

The history of Algerian natural gas production is strongly associated with extraction technology, transportation, construction of pipelines, new agreements with consumers and internal demand, etc. Therefore, selected historical production data may be interpreted within a diffusion of innovation framework, which may also include sub-models for observed exogenous interventions that may modify the diffusion trajectories. Under the finite life-cycle assumption, the current reserve can indirectly be obtained as a simple difference between the estimated URR, through the historical production and the actual cumulative production that considers nonlinear extraction dynamics. Statistical analysis is based upon nonlinear methodologies and more flexible autoregressive structures of residuals are also considered.

To describe cumulative annual Algerian natural gas production, here we consider the models described in the previous Chapters in Equations (2.3), (2.8), (2.14), and (3.1) with different specifications. Starting with the standard Bass model, a GBM with an exponential shock is then fitted. Then to consider heterogeneity, the standard BM and the GBMBM are examined. Further generalizations proposed in Equations (4.2) and (4.5) the BMM and the GBMBMM, are also considered to identify the best fitted model and validate the parameters' estimates.

Table 6.1: Parameter estimates and respective asymptotic 95% confidence intervals.

Model Parameters		Bass	BM	BMM	GBM	GBMBM	GBMBMM
General penetration parameters	m	2687 (2565.83, 2807.57)	4948 (4060.67, 5835.57)	3029 (2652.85, 3405.08)	2993 (2863.17, 3123.55)	3152 (2710.72, 3594.18)	2833 (2448.23, 3218.45)
	p	0.0018 (0.0017, 0.0019)	0.0379 (0.0353, 0.0405)	0.0013 (-0.00006, 0.00262)	0.0012 (0.00110, 0.00130)	0.0023 (-0.00163, 0.00617)	0.00068 (-0.0009, 0.0023)
	q	0.124 (0.1186, 0.1286)	0.0096 (-0.0011, 0.0203)	0.1155 (0.0847, 0.1463)	0.1118 (0.1068, 0.1170)	0.09876 (0.0627, 0.1348)	0.1298 (0.0806 0.1789)
Exponential shock parameters	a				11.42 (10.577, 12.254)	11.56 (10.4665, 12.6491)	12.75 (11.6686, 13.8393)
	b				-0.264 (-0.3620, -0.1472)	-0.3005 (-0.5091, -0.0919)	-0.255 (-0.5547, 0.04408)
	c				1.158 (0.8570, 1.4599)	1.015 (0.5992, 1.4300)	0.6384 (0.3262, 0.9505)
Propensity parameters (asymmetries)	α		22.17 (2.4064, 41.9398)	0.763 (0.5388, 0.9864)		1.2092 (0.5095, 1.9090)	0.756 (0.3174, 1.1954)
	δ			3.233 (2.3156, 4.1512)			2.218 (1.279, 3.156)
R^2		0.999407	0.999877	0.999907	0.999924	0.999925	0.999946
Model Standard Error		14.9322	6.81086	5.9028	5.33395	5.31204	4.50681

As shown in Table 6.1, the results from the Algerian gas production dataset prove the efficacy of the newly introduced modifications of the existing Bemmaor model for the parameter estimates and the goodness of fit of the model. Compared to the standard Bass model or the BM, the modified Bemmaor models

(BMM, GBMBM and GBMBMM) reach better R^2 values. Parameter estimates for m, the carrying capacity, provide an indication of the limiting behaviour of the cumulative production process and represent a current estimate of the URR. All these models except the BM suggest that natural gas production has crossed the middle of the life cycle and that the maximum production level has already been achieved.

The standard Bass model predicts a moderate amount for the net natural gas reserve and shows a very limited contribution by innovators. Since 2011, Algeria has produced 1921 BCM of natural gas, and according to the Bass model, only 28% of the total reserve remains for the future. The R^2, however, indicates the requirements for further modification of the fitted model.

The GBM with one exponential shock shows a better fit to the data. This model shows a mean-reverting positive shock around 1981/1982 when the Algerian state-dominated oil and gas company commissioned the Sonatrach Skikda LNG plant and refinery (GL-1K complex) and the government signed a 20-year agreement with France.

The BM and the GBMBM improve the model goodness of fit in terms of the R^2 and the estimated standard error. A large value of the additional heterogeneity parameter in the BM indicates the existing heterogeneity in the annual gas production and, therefore, describes the possibility for explaining the observed process in a shifted Gompertz set up. The BM indicates that 61% of the total natural gas remains. The GBMBM, on the other hand, indicates the suitability of the usual GBM with an estimate for the heterogeneity parameter not significantly different from 1 and an observed positive exponential shock in gas production around 1982 that was absorbed in time.

The BMM and the GBMBMM include an additional parameter for innovators' heterogeneous propensity. Both models fit well with this additional parameter in terms of R^2 compared to all other considered models. The models have somewhat similar values for the imitators' propensity level. The large innovators' propensity coefficients for the BMMs indicate that heterogeneity exists among initial productions, and a very accelerated trend with a late start is observed at the early stage of the diffusion process. The predicted reserves level for GBMBMM is intermediate in the predictions' range (higher than predicted by the Bass model, smaller than predicted by the BM). Also the results indicate that the maximum level of production was already reached and that 67.8% of the Algerian natural gas URR had been extracted by 2011. The process also shows a positive mean-reverting exponential shock around 1983, when Algeria signed another gas export agreement with Italy and the first BTUs of gas were delivered through the TRANSMED pipeline.

Finally, based upon the comparative parameter estimates, model standard errors and R^2 values help to select the best fitted model among the postulated models. In all respects, the GBMBMM provides a more accurate description of the natural gas production. To obtain an improved short-term prediction for the regressive approach of the postulated models, an ARMAX model, based upon one regressor or more lagged regressors depending upon the predictive values of the first regressive step, was implemented. The forecasts for the different models are given in Figures 6.2 – 6.7. The results for model estimation and forecast performance are described in Table 2.

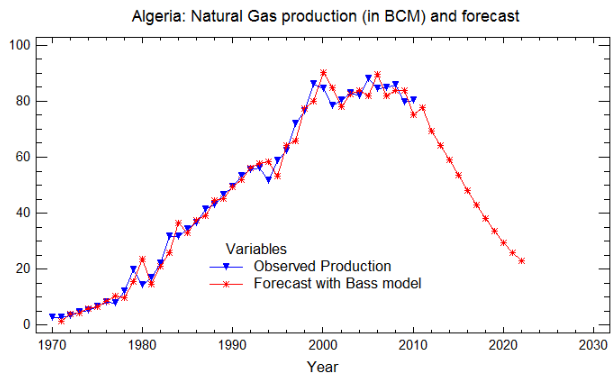


Figure 6.2: Algerian natural gas forecast with the Bass model and ARMA(2,2).

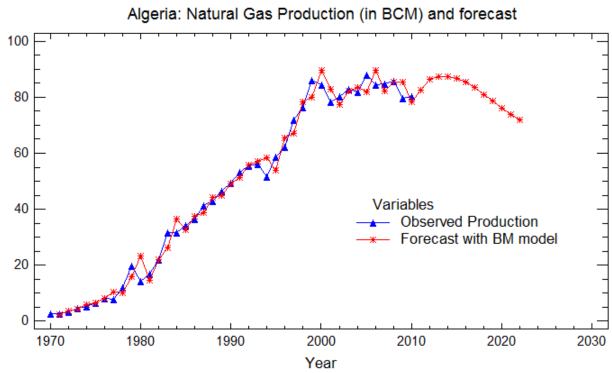


Figure 6.3: Algerian natural gas forecast with the Bemmaor (BM) model and ARMA(2,2).

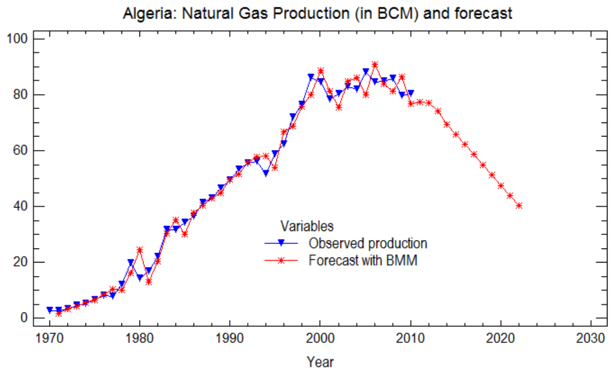


Figure 6.4: Algerian natural gas forecast with the modified Bemmaor (BMM) model and ARMA(1,4).

Figures 6.2–6.4 show the graphs for the observed Algerian natural gas production and the forecast with the standard Bass model, BM and BMM, respectively, with a suitable ARMAX sharpening for better short-term prediction.

The results show that Algerian natural gas production crossed the maximum production level in 2000 according to the Bass model and BM predictions, whereas the BMM indicated that maximum production was reached in 2006. The BM shows a recovery trend and forecasts another peak production level between 2012 and 2016. Therefore, the predicted life cycle increases. Other models do not support this prediction, and a decreasing trend is observed after maximum production in 2000 with some stationarity in the process.

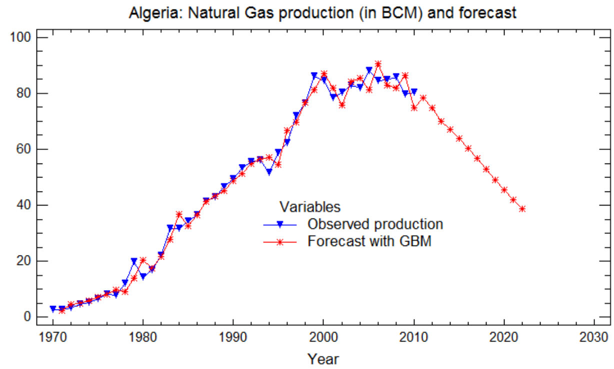


Figure 6.5: Algerian natural gas forecast with the generalized Bass (GBM) model and ARMA(2,3).

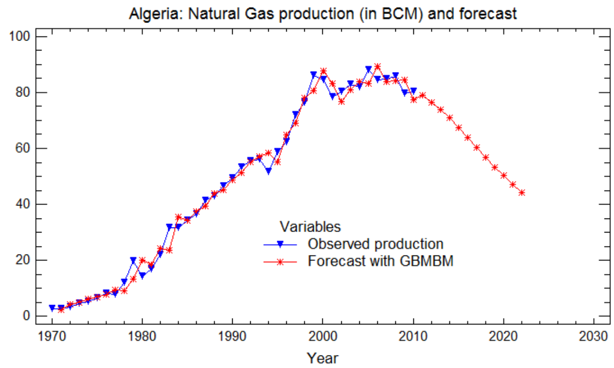


Figure 6.6: Algerian natural gas forecast with the generalized Bass & Bemmaor (GBMBM) model and ARMA(2,1).

Model forecasts for the Algerian gas production data with the GBM with one exponential shock, described by the GBM, GBMBM and GBMBMM, are shown in Figures 6.5 –6.7, respectively.

Results show that the maximum production level was achieved in 2006. This is different from the year predicted by the standard Bass model or the BM, which cannot consider the exponential shock in the data. A rapidly decreasing production process was also predicted by the GBMBMM followed by the respective prediction with the GBM and the GBMBM set up with ARMAX corrections for short-term prediction.

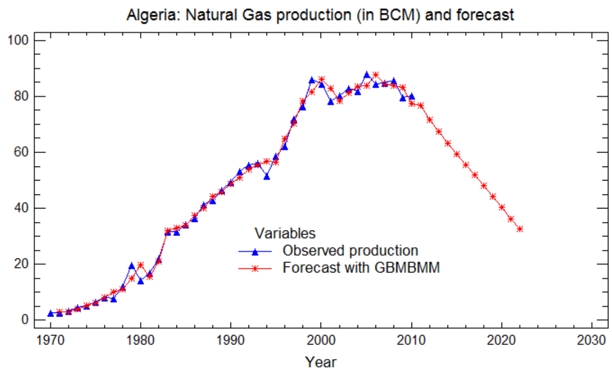


Figure 6.7: Algerian natural gas forecast with the generalized Bass & modified Bemmaor (GBMBMM) model and ARMA(2,1).

Results in Table 6.2 indicate that, the forecast with ARMA(2,1) set-up, when regressed with the predicted estimates the root-mean squared error (RMSE) and the mean-absolute prediction error (MAPE) attain minimum values for the GBMBMM. When compared with the standard Bass model, the significance for including additional parameters is confirmed by the *F-test* for all other postulated models.

Similar results are also found for the parameters when compared with the BM. For the GBMBMM, when compared with the standard Bass model, BM and GBM, respectively, the squared partial correlation coefficients, $\tilde{R}^2$, equal 0.9089 ($F = 65.85$), 0.5610 ($F = 14.483$) and 0.2895 ($F = 6.723$). At the same time, the BIC value is minimum for this model. Therefore, strong evidence for the significance of an additional parameter in the GBMBMM and its forecast capacity is established for this dataset.

6.4 Remarks

Diffusion of innovation modelling faces new challenges in incorporating the influencing variables within a parsimonious model that helps explaining the changes

Table 6.2: Model performance for estimation and forecast.

Models	RMSE	MAPE	No. of parameters	R^2	$\tilde{R}^2$ w.r.to Bass (F)	$\tilde{R}^2$ w.r.to BM (F)	$\tilde{R}^2$ w.r.to GBM (F)	RSS	BIC
Bass + ARMA(2,2)	2.9690	2.1389	3	0.999407	NA	NA	NA	8472.89	229.71
BM +ARMA(2,2)	2.63093	2.13023	4	0.99877	0.7626 (141.40)	NA	NA	1716.35	167.96
BMM +ARMA(1,4)	2.78532	2.77534	5	0.999907	0.8432 (96.80)	0.2439 (11.612)	NA	1254.33	158.82
GBM +ARMA(2,3)	2.37584	3.29014	6	0.999924	0.8718 (79.34)	0.3821 (10.822)	NA	995.79	153.07
GBMBM +ARMA(2,1)	2.51599	2.57759	7	0.999925	0.8735 (58.69)	0.3902 (6.558)	NA	959.40	155.26
GBMBMM +ARMA(2,1)	1.72866	1.77848	8	0.999946	0.9089 (65.85)	0.5610 (14.483)	0.2895 (6.723)	670.28	144.28

of the evolutionary shape of the curve in time. The main aim of this Chapter was to investigate and compare the existing diffusion models and their proposed extensions. In order to take into account different patterns in individual propensity of the agents, various parametrizations have been considered to identify the heterogeneity level so that the complete life cycle of an innovation can be studied more efficiently.

The application of the proposed modified models, in parallel with other existing models of innovation diffusion, gives a fruitful comparison in terms of efficacy and the estimates' stability. Results obtained for the Algerian gas production data perfectly match those of recent studies, which support the decreasing trend identified by the model forecasts. Overall production of natural gas decreased by 3% in 2011 compared to the previous year, as British Petroleum [11] reports.

We emphasize that the different and partially nested models in Table 6.1 have an equivalent URR estimate of about 3000 billion cubic feet. These results differ from the reported natural gas reserves, which are only a component of URR, by Oil and Gas Journal. The proposed reserve estimate of about 4500 billion cubic meters does not match our estimates based on the production data, which are, in our opinion, more reliable.

Finally, initial curvature of the diffusion dynamics is important to identify a good diffusion model, to see the future trend and identify the local interventions. If the model includes many parameters with limited data, identification problems arises which are not easy to solve. In some cases, the application of this procedure is complicated in the first step when initial values for the parameters have to be decided. However, the approach applied in this Chapter proved to be useful, although further developments to resolve the above computational limitations may be required.

Chapter 7

Discussions and further research directions

7.1 Introduction

Research studies on diffusion of innovations provide an attempt to explain the mechanism by which new ideas, products, new trends or inventions spread in the society. New innovations take time to diffuse and a basic puzzle in diffusion of innovations research is to investigate the reasons why there is often a long interim between an innovation's first appearance and the time of substantial adoption. In fact, diffusion is often affected by the heterogeneity among the potential adopters in society. Individuals (agents) share their evaluations about a newly introduced innovation with their neighbourhood and this behaviour speeds up or delays the diffusion process. Also the socio-economic factors/levels of the agents influences the acceptance or rejection of an innovation. Therefore, it is important to consider both the heterogeneity among the agents in a diffusion process and the existing communication network structure under the hood of social interaction.

In the introductory part of this thesis we discussed the fundamental diffusion models and also some diffusion models with heterogeneity. The Bass diffusion model describes how a new innovation is adopted through the interaction between the early adopters and potential adopters. Its generalisation incorporates the local interventions i.e., the controlling process that extends or contracts the life cycle of an innovation. In the paradigm of heterogeneous diffusion models, the Bemmaor model provides a convenient way for explaining the heterogeneous individual adoption propensity with probabilistic distributional assumptions. The idea can be further developed by incorporating the existing marketing efforts and local interventions in the wider range. Therefore, the thesis combines under Bemmaor heterogeneity concept, the Generalised Bass model mixture with Bemmaor effect (GBMBM) (as proposed in Chapter 3).

Since the Bass model has been developed with the assumption of the existence of two groups of agents with a different behaviour (innovators and imitators), it may be useful to consider heterogeneity in both subgroups. In particular, it can be postulated that the individuals/agents in a society are heterogeneous with respect to their adoption propensity, no matter whether they belong to the innovators or imitators subgroups. An extension of the Bemmaor model considering this hypothesis has been proposed in Chapter 4, further extended with the introduction of local interventions in the diffusion dynamics. In order to validate the efficacy of the proposed extensions, in Chapter 5, the existing models are applied to evaluate the reserves of South Asian natural gas production and in Chapter 6 to the Algerian natural gas production.

7.2 Discussion on findings

Innovation is embedded in a complex system dynamics. Spielman [66] defines an innovation as anything new successfully introduced into an economic or social process, which is not just trying something new but successfully integrating a new idea or product into a process that includes technical, economic, and social components. The most important feature of innovation is the creative use of different types of knowledge in response to social or economic needs and opportunities [50]. It is also important to analyze the nature and structure of interactions among agents which are linked to one another within social networks. The adoption of an innovation by an agent in the social structure may have a positive or negative impact on other agents that are unpredictable or unintended [48]. Individuals may realize the effectiveness, benefits and cost of an innovation and convey the information to their neighbourhood. Therefore, individual decision making process and agents network are important elements to consider in the diffusion of innovations research.

The main focus of this thesis is to make an attempt to clarify the effect of heterogeneity among the agents in the existing diffusion dynamics. This approach may explain in part, the nature and structure of communication among members of a social system. Individual adoption and heterogeneous consumption propensity have been taken under considerations, that are supposed to follow a probability distribution with suitable heterogeneity parameters. The further requirement here met also to be able to formulate the diffusion dynamics in a closed form at the aggregate level. Local interventions are also incorporated to observe the consequences of the strategic interruptions in the diffusion dynamics.

Almost all the cumulative diffusion dynamics can be described through an S-shaped curve. It has been discussed in the introductory part of this thesis that almost all the considered models have their origin in the logistic diffusion model. Also a simple modification of the agents in the existing diffusion models can capture further changes in the diffusion dynamics caused by either the local perturbation or, an interaction pattern in the existing social network dynamics.

In Chapter 4, an attempt was performed to modify the existing Bemmaor model introducing heterogeneity in both innovator and imitators subgroups. Therefore, the proposed extension with parameters α and δ can be used to describe the heterogeneity and communication among the agents in the diffusion dynamics. It has been observed that, the parameter δ, representing the innovators' heterogeneity, has a strong effect in a comparative homogeneous state (for lower values of α, e.g. $\alpha < 1$). In particular, in a well connected social network, diffusion dynamics is much influenced by the early adoptions of innovators and therefore we observe a speedy life cycle. On the other hand, in a complete random network structure, agents stay in a scattered manner and their random information transmission delays the diffusion dynamics in the society. In a neutral state, when both $\alpha = \delta = 1$, the diffusion dynamics can be described by a symmetric diffusion curve, which is the standard Bass model shape.

All the postulated models have closed form expressions, easily identifiable and parameter estimation can be done with the existing non-parametric estimation methods. Because of the fitting to diffusion dynamics is done with cumulative data points, we should not rely on the observed value of the determination indices that in these situations, always give very hight values. Alternatively, we may compare the performance of the fitted model with a standard benchmark, e.g., the simpler Bass model. In order to do this, we can use an F-test based on partial determination indices proposed by Guseo and Dalla Valle [28] or the *BIC* criterion for obtaining the best fitted model. Model forecasting can be obtained with an ARMAX set up with the estimated model as the regressor that avoids normality assumptions for the residuals.

This thesis describes the application of the existing and the postulated models in order to observe the evolutionary trend of natural gas production in two different regions. Although, the technological diffusions are context specific, the response from the existing social structure that is involved with supporting mechanism should not be ignored. In a historical natural gas production series, the extraction dynamics can be considered as a function of local and international demand, driven by related natural gas-consuming technologies, direct and indirect extraction costs, energy return on energy investment, strategic opportunities and environmental constraints. The complex system of gas extraction can be modelled under the assumption of deterministic or semi-deterministic regulatory interventions and, therefore, historical cumulative data series can be analysed through usual diffusion models considering a finite life cycle of this ultimately recoverable resources (URR).

The results obtained in Chapter 5 describe the capability of the existing models in identifying the local perturbations and their consequences in south Asian natural gas production. Considering a homogeneous state of natural gas consumption, the estimated models and respective forecasts represent a complementary situation of natural gas reserves in the studied area. Identified local intervention times and their consequences have perfect match the production interruptions by strategic policy implications. Also a useful future reserves estimate was obtained under the considerations of nonlinear extraction dynamics.

Results from the application of the diffusion models to the Algerian historical natural gas production reveals that, introducing heterogeneity into the both innovators and imitators subgroups, the complete life cycle of the diffusion process can be studied more efficiently with minimum prediction errors. The proposed models give a better reserves estimate, explaining the local interventions significance in a more intuitive way that mimics the forecastings revealed by the recent studies by Oil and Gas journals (for instance, see British Petroleum [11].

7.3 Further research directions

This thesis describes the diffusion dynamics at the aggregate level which is postulated to be affected by the heterogeneity at the individual level. Considering the existence of heterogeneity in both innovators and imitators subgroups, the thesis proposed some extensions of the existing models that motivates further research directions.

First, the considered heterogeneous propensity of agents has been assumed to follow a distribution at the individual level and this leads to a unimodal behaviour. But repeated consumptions and information interactions may allow for heterogeneity to show a bimodal or multimodal curvature. One possibility is then to use either a univariate bimodal/multimodal distribution, or to consider a distribution with multiple parameters so that it can capture the observed heterogeneity.

Second, the postulated models consider a single innovation perspective which could be further developed for multiple innovations, simultaneous innovations or multiple regime of innovations with heterogeneity of agents at different levels. New formulations of models could also be possible with re-parametrization of the proposed α and δ parameters to explain the heterogeneity in terms of propensity levels and communication networks.

Pegoretti et al. [52] briefly discuss how the structure of social networks affects innovation diffusion and competition under different information regimes. They show that a single innovation in a completely random network with perfect information diffuses faster, and with imperfect information, the fastest pace is reached in small-world networks. Also the competition structure among alternative similar innovations should be considered for multiple diffusion studies. Modelling approaches discussed in this thesis can also be extended to this dimension.

A stream of further research can be done in terms of agent based modelling, since, adoption choices made by the heterogeneous agents reflect both their own evaluations and the social influences passed through others. Therefore, diffusion of an innovation can be studied as a combined dynamics resulted from the above. Modelling approaches may cover the independent decisions with social influence and /or, mixed decision with partial social influence. Further modifications can also be introduced by considering the impact of network topological properties. Results obtained by Namatame et al. [48] highlight that fastest diffusion is possible when agents are locally connected, where as a slowest and unstable diffusion process is observed for a fully connected network and in case of a scale free network, the diffusion pattern takes the middle of the two extreme cases, which is almost the similar phenomenon described by the parameters α and δ proposed in this thesis.

The Bayesian approach of inference is another less cultivated area in diffusion research. There exists possibilities to use the communication characteristics among the agents and existing network structure as prior information of the process that be updated conditioning to the adoption behaviour of agents.

7.4 Final remarks

Following the research stream by Bemmaor [6], the thesis forwarded a theoretical model to consider the heterogeneous behaviour of imitators in the generalised Bass model set-up. The results actually show that, the presence of homogeneous agents results in an early and speedy diffusion, whereas, heterogeneous agents delay the life cycle of an innovation and generate a slower rate of diffusion dynamics. The extension of the Bemmaor model [6] validates the impact of the early or late participation of the innovators to the diffusion process that is also affected by the imitators adoption behaviour.

The findings of the thesis seem to be interesting in two perspectives. First of all, the model developed here puts together both the Generalised Bass model and the Bemmaor model. In particular, it combines two main classes of models: homogeneous and heterogeneous modelling paradigms. Second, it extends the Bemmaor approach of modelling innovations diffusion with considerations of innovators heterogeneous propensity.

The models have straightforward application to the technological diffusion. Although, this thesis applied the models to the historical natural gas consumption, where extraction dynamics are considered as a function of local and international demands and the historical production series at least crossed the initial trend of the process, models can also be applied to describe the diffusion process of essentials and durable products with similar set up.

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