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Nonparametric Estimation of ROC Surface Under Verification Bias: Supplementary Material

Khanh To Duc
Department of Statistical Sciences
University of Padua
Italy

Monica Chiogna
Department of Statistical Sciences
University of Padua
Italy

Gianfranco Adimari
Department of Statistical Sciences
University of Padua
Italy

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Department of Statistical Sciences
Via Cesare Battisti, 241
35121 Padova
Italy
tel: +39 049 8274168
fax: +39 049 8274170
http://www.stat.unipd.it

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Khanh To Duc
Department of Statistical Sciences
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S1 Asymptotic covariance matrix: technical details

The asymptotic covariance matrix of $\sqrt{n}(\widehat{\mathrm{TCF}}_{1,\mathrm{KNN}}, \widehat{\mathrm{TCF}}_{2,\mathrm{KNN}}, \widehat{\mathrm{TCF}}_{3,\mathrm{KNN}})^\top$, Ξ, is obtained by

$$\Xi = h'\Xi^* h'^\top, \tag{1}$$

where h' is the matrix of first-order derivatives of the function $h(\cdot)$ given in the proof of Theorem 3.2, main paper, i.e.,

$$h' = \begin{pmatrix} \frac{\beta_{11}}{\theta_1^2} & 0 & -\frac{1}{\theta_1} & 0 & 0 & 0 \\ 0 & -\frac{(\beta_{12}-\beta_{22})}{\theta_2^2} & 0 & \frac{1}{\theta_2} & -\frac{1}{\theta_2} & 0 \\ \frac{\beta_{23}}{(1-\theta_1-\theta_2)^2} & \frac{\beta_{23}}{(1-\theta_1-\theta_2)^2} & 0 & 0 & 0 & \frac{1}{(1-\theta_1-\theta_2)} \end{pmatrix}, \tag{2}$$

and Ξ^* is the asymptotic covariance matrix of $\sqrt{n}\big(\hat{\theta}_{1,\mathrm{KNN}}, \hat{\theta}_{2,\mathrm{KNN}}, \hat{\beta}_{11,\mathrm{KNN}}, \hat{\beta}_{12,\mathrm{KNN}}, \hat{\beta}_{22,\mathrm{KNN}}, \hat{\beta}_{23,\mathrm{KNN}}\big)^\top$. Recall that

$$\Xi = \begin{pmatrix} \xi_1^2 & \xi_{12} & \xi_{13} \\ \xi_{12} & \xi_2^2 & \xi_{23} \\ \xi_{13} & \xi_{23} & \xi_3^2 \end{pmatrix}.$$

The matrix Ξ^* is a 6×6 matrix such that its diagonal elements are the asymptotic variances of $\sqrt{n}\hat{\theta}_{k,\mathrm{KNN}}$ and $\sqrt{n}\hat{\beta}_{jk,\mathrm{KNN}}$. Let $\sigma_{12}^* = \mathrm{as}\mathbb{C}\mathrm{ov}(\sqrt{n}\hat{\theta}_{1,\mathrm{KNN}}, \sqrt{n}\hat{\theta}_{2,\mathrm{KNN}})$, $\sigma_{sjk} = \mathrm{as}\mathbb{C}\mathrm{ov}(\sqrt{n}\hat{\theta}_{s,\mathrm{KNN}}, \sqrt{n}\hat{\beta}_{jk,\mathrm{KNN}})$ and $\sigma_{jkls} = \mathrm{as}\mathbb{C}\mathrm{ov}(\sqrt{n}\hat{\beta}_{jk,\mathrm{KNN}}, \sqrt{n}\hat{\beta}_{ls,\mathrm{KNN}})$.

Recall that, in the main paper, $\sigma_{jk}^2 = \mathrm{as}\mathbb{V}\mathrm{ar}(\hat{\beta}_{jk,\mathrm{KNN}})$. We write

$$\Xi^* = \begin{pmatrix} \sigma_1^2 & \sigma_{12}^* & \sigma_{111} & \sigma_{112} & \sigma_{122} & \sigma_{123} \\ \sigma_{12}^* & \sigma_2^2 & \sigma_{211} & \sigma_{212} & \sigma_{222} & \sigma_{223} \\ \sigma_{111} & \sigma_{211} & \sigma_{11}^2 & \sigma_{1112} & \sigma_{1122} & \sigma_{1123} \\ \sigma_{112} & \sigma_{212} & \sigma_{1112} & \sigma_{12}^2 & \sigma_{1222} & \sigma_{1223} \\ \sigma_{122} & \sigma_{222} & \sigma_{1122} & \sigma_{1222} & \sigma_{22}^2 & \sigma_{2223} \\ \sigma_{123} & \sigma_{223} & \sigma_{1123} & \sigma_{1223} & \sigma_{2223} & \sigma_{23}^2 \end{pmatrix}.$$

Hence, from (1) and (2) ,

$$\begin{aligned} \xi_1^2 &= \mathrm{as}\mathbb{V}\mathrm{ar}\left(\sqrt{n}\widehat{\mathrm{TCF}}_{1,\mathrm{KNN}}(c_1)\right) \\ &= \frac{\beta_{11}^2}{\theta_1^4}\sigma_1^2 + \frac{\sigma_{11}^2}{\theta_1^2} - 2\frac{\beta_{11}}{\theta_1^3}\sigma_{111}, \\ \xi_2^2 &= \mathrm{as}\mathbb{V}\mathrm{ar}\left(\sqrt{n}\widehat{\mathrm{TCF}}_{2,\mathrm{KNN}}(c_1,c_2)\right) \\ &= \sigma_2^2\frac{(\beta_{12}-\beta_{22})^2}{\theta_2^4} + \frac{\sigma_{12}^2+\sigma_{22}^2-2\sigma_{1222}}{\theta_2^2} - 2\frac{\beta_{12}-\beta_{22}}{\theta_2^3}(\sigma_{212}-\sigma_{222}), \\ \xi_3^2 &= \mathrm{as}\mathbb{V}\mathrm{ar}\left(\sqrt{n}\widehat{\mathrm{TCF}}_{3,\mathrm{KNN}}(c_2)\right) \\ &= \beta_{23}^2\frac{\sigma_1^2+2\sigma_{12}^*+\sigma_2^2}{(1-\theta_1-\theta_2)^4} + \frac{\sigma_{23}^2}{(1-\theta_1-\theta_2)^2} + 2\beta_{23}\frac{\sigma_{123}+\sigma_{223}}{(1-\theta_1-\theta_2)^3}. \end{aligned} \quad (3)$$

Let $\lambda^2 = \mathrm{as}\mathbb{V}\mathrm{ar}(\sqrt{n}\hat{\beta}_{12,\mathrm{KNN}} - \sqrt{n}\hat{\beta}_{22,\mathrm{KNN}})$. Hence, $\sigma_{12}^2+\sigma_{22}^2-2\sigma_{1222} = \lambda^2$, and

$$\xi_2^2 = \sigma_2^2\frac{(\beta_{12}-\beta_{22})^2}{\theta_2^4} + \frac{\lambda^2}{\theta_2^2} - 2\frac{\beta_{12}-\beta_{22}}{\theta_2^3}(\sigma_{212}-\sigma_{222}).$$

Observe that $\hat{\theta}_{3,\mathrm{KNN}} = 1 - (\hat{\theta}_{1,\mathrm{KNN}} + \hat{\theta}_{2,\mathrm{KNN}})$. Thus,

$$\begin{aligned} \mathrm{as}\mathbb{V}\mathrm{ar}(\sqrt{n}\hat{\theta}_{3,\mathrm{KNN}}) &= \mathrm{as}\mathbb{V}\mathrm{ar}(\sqrt{n}\hat{\theta}_{1,\mathrm{KNN}} + \sqrt{n}\hat{\theta}_{2,\mathrm{KNN}}) \\ &= \mathrm{as}\mathbb{V}\mathrm{ar}(\sqrt{n}\hat{\theta}_{1,\mathrm{KNN}}) + \mathrm{as}\mathbb{V}\mathrm{ar}(\sqrt{n}\hat{\theta}_{2,\mathrm{KNN}}) \\ &\quad + 2\mathrm{as}\mathbb{C}\mathrm{ov}(\sqrt{n}\hat{\theta}_{1,\mathrm{KNN}}, \sqrt{n}\hat{\theta}_{2,\mathrm{KNN}}). \end{aligned}$$

This leads to the expression $\sigma_3^2 = \sigma_1^2 + 2\sigma_{12}^* + \sigma_2^2$. In addition,

$$\begin{aligned} \sigma_{123}+\sigma_{223} &= \mathrm{as}\mathbb{C}\mathrm{ov}(\sqrt{n}\hat{\theta}_{1,\mathrm{KNN}}, \sqrt{n}\hat{\beta}_{23,\mathrm{KNN}}) + \mathrm{as}\mathbb{C}\mathrm{ov}(\sqrt{n}\hat{\theta}_{2,\mathrm{KNN}}, \sqrt{n}\hat{\beta}_{23,\mathrm{KNN}}) \\ &= \mathrm{as}\mathbb{C}\mathrm{ov}(\sqrt{n}\hat{\theta}_{1,\mathrm{KNN}} + \sqrt{n}\hat{\theta}_{2,\mathrm{KNN}}, \sqrt{n}\hat{\beta}_{23,\mathrm{KNN}}) \\ &= -\mathrm{as}\mathbb{C}\mathrm{ov}(\sqrt{n} - (\sqrt{n}\hat{\theta}_{1,\mathrm{KNN}} + \sqrt{n}\hat{\theta}_{2,\mathrm{KNN}}), \sqrt{n}\hat{\beta}_{23,\mathrm{KNN}}) \\ &= -\sigma_{323}. \end{aligned}$$

Therefore, from (3), the asymptotic variance of $\sqrt{n}\widehat{\mathrm{TCF}}_{3,\mathrm{KNN}}(c_2)$ is

$$\xi_3^2 = \frac{\beta_{23}^2\sigma_3^2}{(1-\theta_1-\theta_2)^4} + \frac{\sigma_{23}^2}{(1-\theta_1-\theta_2)^2} - 2\frac{\beta_{23}\sigma_{323}}{(1-\theta_1-\theta_2)^3}.$$

Recall that $\sigma_k^2 = \theta_k(1-\theta_k) + \omega_k^2$ and $\sigma_{jk}^2 = \beta_{jk}(1-\beta_{jk}) + \omega_{jk}^2$, where ω_k^2 and ω_{jk}^2 are given in (3.4) and (3.5), main paper, respectively. To obtain σ_{kjk}, we observe that

$$\begin{aligned}
\beta_{jk} &= \Pr(T \geq c_j, D_k = 1) \\
&= \Pr(D_k = 1)\Pr(T \geq c_j | D_k = 1) \\
&= \Pr(D_k = 1)\left[1 - \Pr(T < c_j | D_k = 1)\right] \\
&= \Pr(D_k = 1) - \Pr(D_k = 1)\Pr(T < c_j | D_k = 1) \\
&= \Pr(D_k = 1) - \Pr(T < c_j, D_k = 1) \\
&= \theta_k - \gamma_{jk},
\end{aligned}$$

for $j = 1, 2$, $k = 1, 2, 3$ and $k \geq j$. Then, we consider

$$\hat{\gamma}_{jk,\mathrm{KNN}} = \frac{1}{n}\sum_{i=1}^{n} \mathrm{I}(T_i < c_j)\left[V_i D_{ki} + (1 - V_i)\hat{\rho}_{ki,K}\right].$$

The asymptotic variance of $\sqrt{n}\hat{\gamma}_{jk,\mathrm{KNN}}$, ζ_{jk}^2, is obtained as that of $\sqrt{n}\hat{\beta}_{jk,\mathrm{KNN}}$. In fact, we get $\zeta_{jk}^2 = \gamma_{jk}(1-\gamma_{jk}) + \eta_{jk}^2$, where

$$\begin{aligned}
\eta_{jk}^2 &= \frac{K+1}{K}\mathbb{E}\Big[\mathrm{I}(T < c_j)\rho_k(T,A)\{1 - \rho_k(T,A)\}\{1 - \pi(T,A)\}\Big] \\
&\quad + \mathbb{E}\Big[\mathrm{I}(T < c_j)\rho_k(T,A)\frac{\{1 - \rho_k(T,A)\}\{1 - \pi(T,A)\}^2}{\pi(T,A)}\Big].
\end{aligned}$$

It is straightforward to see that $\hat{\gamma}_{jk,\mathrm{KNN}} = \hat{\theta}_{k,\mathrm{KNN}} - \hat{\beta}_{jk,\mathrm{KNN}}$. Thus, we can compute the asymptotic covariances σ_{kjk} for $j = 1, 2$, $k = 1, 2, 3$ and $k \geq j$, using the fact that

$$\begin{aligned}
\mathrm{as}\mathbb{V}\mathrm{ar}(\sqrt{n}\hat{\gamma}_{jk,\mathrm{KNN}}) &= \mathrm{as}\mathbb{V}\mathrm{ar}(\sqrt{n}\hat{\theta}_{k,\mathrm{KNN}} - \sqrt{n}\hat{\beta}_{jk,\mathrm{KNN}}) \\
&= \mathrm{as}\mathbb{V}\mathrm{ar}(\sqrt{n}\hat{\theta}_{k,\mathrm{KNN}}) + \mathrm{as}\mathbb{V}\mathrm{ar}(\sqrt{n}\hat{\beta}_{jk,\mathrm{KNN}}) \\
&\quad - 2\mathrm{as}\mathbb{C}\mathrm{ov}(\sqrt{n}\hat{\theta}_{k,\mathrm{KNN}}, \sqrt{n}\hat{\beta}_{jk,\mathrm{KNN}}).
\end{aligned}$$

This leads to

$$\sigma_{kjk} = \frac{1}{2}\left(\sigma_k^2 + \sigma_{jk}^2 - \zeta_{jk}^2\right).$$

Hence,

$$\begin{aligned}
\sigma_{111} &= \frac{1}{2}\left(\sigma_1^2 + \sigma_{11}^2 - \zeta_{11}^2\right); \quad & \sigma_{212} &= \frac{1}{2}\left(\sigma_2^2 + \sigma_{12}^2 - \zeta_{12}^2\right); \\
\sigma_{222} &= \frac{1}{2}\left(\sigma_2^2 + \sigma_{22}^2 - \zeta_{22}^2\right); \quad & \sigma_{323} &= \frac{1}{2}\left(\sigma_3^2 + \sigma_{23}^2 - \zeta_{23}^2\right).
\end{aligned}$$

As for λ^2, note that

$$\hat{\beta}_{12,\mathrm{KNN}} - \hat{\beta}_{22,\mathrm{KNN}} = \frac{1}{n}\sum_{i=1}^{n} \mathrm{I}(c_1 \leq T_i < c_2)\left[V_i D_{2i} + (1 - V_i)\hat{\rho}_{2i,K}\right].$$

By analogy with other estimators, the asymptotic variance of

$$\sqrt{n}\left[(\hat{\beta}_{12,\mathrm{KNN}} - \hat{\beta}_{22,\mathrm{KNN}}) - (\beta_{12,\mathrm{KNN}} - \beta_{22,\mathrm{KNN}})\right]$$

has expression

$$\begin{aligned}\lambda^2 &= (\beta_{12}-\beta_{22})\left[1-(\beta_{12}-\beta_{22})\right] \\ &+ \left(1+\frac{1}{K}\right)\mathbb{E}\Big[\mathrm{I}(c_1 \le T_i < c_2)\rho_2(T,A)(1-\rho_2(T,A))(1-\pi(T,A))\Big] \\ &+ \mathbb{E}\Big[\mathrm{I}(c_1 \le T_i < c_2)\rho_2(T,A)\frac{(1-\rho_2(T,A))(1-\pi(T,A))^2}{\pi(T,A)}\Big] \\ &= (\beta_{12}-\beta_{22})\left[1-(\beta_{12}-\beta_{22})\right] + \omega_{12}^2 - \omega_{22}^2.\end{aligned}$$

Now, we focus on the elements ξ_{12}, ξ_{13} and ξ_{23} of the covariance matrix Ξ. We can write

$$\begin{aligned}\xi_{12} &= -\frac{1}{\theta_1\theta_2}(\sigma_{1112}-\sigma_{1122}) + \frac{\beta_{11}}{\theta_1^2\theta_2}(\sigma_{112}-\sigma_{122}) \\ &\quad - \frac{\beta_{12}-\beta_{22}}{\theta_2^2}\left(\frac{\beta_{11}}{\theta_1^2}\sigma_{12}^* - \frac{\sigma_{211}}{\theta_1}\right), \qquad (4) \\ \xi_{13} &= \frac{1}{1-\theta_1-\theta_2}\left(\frac{\beta_{11}}{\theta_1^2}\sigma_{123} - \frac{\sigma_{1123}}{\theta_1}\right) + \frac{\beta_{23}}{\theta_1(1-\theta_1-\theta_2)^2} \\ &\quad \times \left[\frac{\beta_{11}}{\theta_1}\left(\sigma_1^2+\sigma_{12}^*\right) - (\sigma_{111}+\sigma_{211})\right], \qquad (5)\end{aligned}$$

and

$$\begin{aligned}\xi_{23} &= \frac{1}{\theta_2(1-\theta_1-\theta_2)}\left[(\sigma_{1223}-\sigma_{2223}) - \frac{\beta_{12}-\beta_{22}}{\theta_2}\sigma_{223}\right] \\ &\quad + \frac{\beta_{23}}{\theta_2(1-\theta_1-\theta_2)^2}\Bigg[(\sigma_{112}-\sigma_{122}+\sigma_{212}-\sigma_{222}) \\ &\quad - \frac{\beta_{12}-\beta_{22}}{\theta_2}\left(\sigma_2^2+\sigma_{12}^*\right)\Bigg]. \qquad (6)\end{aligned}$$

Recall that (reference [12], main paper)

$$\begin{aligned}\hat{\theta}_{k,\mathrm{KNN}} - \theta_k &= \frac{1}{n}\sum_{i=1}^{n}\left[V_i D_{ki} + (1-V_i)\rho_{ki}\right] + \frac{1}{n}\sum_{i=1}^{n}(1-V_i)(\hat{\rho}_{ki,K}-\rho_{ki}) - \theta_k \\ &= \frac{1}{n}\sum_{i=1}^{n} V_i\left[D_{ki}-\rho_{ki}\right] + \frac{1}{n}\sum_{i=1}^{n}\left[\rho_{ki}-\theta_k\right] \\ &\quad + \frac{1}{n}\sum_{i=1}^{n}\left[\frac{1}{K}\sum_{l=1}^{K}\left(V_{i(l)}D_{ki(l)} - \rho_{ki(l)}\right)\right] + o_p\left(n^{-1/2}\right) \\ &= S_k + R_k + W_k + o_p\left(n^{-1/2}\right),\end{aligned}$$

and

$$
\begin{aligned}
\hat{\beta}_{jk,\mathrm{KNN}} - \beta_{jk} &= \frac{1}{n}\sum_{i=1}^{n} \mathrm{I}(T_i \geq c_j)\left[V_i D_{ki} + (1 - V_i)\rho_{ki}\right] \\
&\quad + \frac{1}{n}\sum_{i=1}^{n} \mathrm{I}(T_i \geq c_j)(1 - V_i)(\hat{\rho}_{ki,K} - \rho_{ki}) - \beta_{jk} \\
&= \frac{1}{n}\sum_{i=1}^{n} \mathrm{I}(T_i \geq c_j)V_i\left[D_{ki} - \rho_{ki}\right] + \frac{1}{n}\sum_{i=1}^{n}\left[\mathrm{I}(T_i \geq c_j)\rho_{ki} - \beta_{jk}\right] \\
&\quad + \frac{1}{n}\sum_{i=1}^{n} \mathrm{I}(T_i \geq c_j)(1 - V_i)\left[\frac{1}{K}\sum_{l=1}^{K}\left(V_{i(l)}D_{ki(l)} - \rho_{ki(l)}\right)\right] \\
&\quad + o_p\left(n^{-1/2}\right) \\
&= S_{jk} + R_{jk} + W_{jk} + o_p\left(n^{-1/2}\right).
\end{aligned}
$$

Then, we derive suitable expressions for some terms in (4)–(6). First, we consider the term, $\sigma_{1112} - \sigma_{1122}$. We have

$$
\begin{aligned}
\sigma_{1112} - \sigma_{1122} &= \mathrm{as}\mathbb{C}\mathrm{ov}(\sqrt{n}\hat{\beta}_{11,\mathrm{KNN}}, \sqrt{n}\hat{\beta}_{12,\mathrm{KNN}}) - \mathrm{as}\mathbb{C}\mathrm{ov}(\sqrt{n}\hat{\beta}_{11,\mathrm{KNN}}, \sqrt{n}\hat{\beta}_{22,\mathrm{KNN}}) \\
&= \mathrm{as}\mathbb{C}\mathrm{ov}(\sqrt{n}\hat{\beta}_{11,\mathrm{KNN}}, \sqrt{n}\hat{\beta}_{12,\mathrm{KNN}} - \sqrt{n}\hat{\beta}_{22,\mathrm{KNN}}) \\
&= \mathrm{as}\mathbb{C}\mathrm{ov}\Big(\sqrt{n}(S_{11} + R_{11} + W_{11}), \sqrt{n}(S_{12} - S_{22}) \\
&\quad + \sqrt{n}(R_{12} - R_{22}) + \sqrt{n}(W_{12} - W_{22})\Big) \\
&= \mathrm{as}\mathbb{C}\mathrm{ov}\left(\sqrt{n}S_{11}, \sqrt{n}(S_{12} - S_{22})\right) + \mathrm{as}\mathbb{C}\mathrm{ov}\left(\sqrt{n}S_{11}, \sqrt{n}(W_{12} - W_{22})\right) \\
&\quad + \mathrm{as}\mathbb{C}\mathrm{ov}\left(\sqrt{n}R_{11}, \sqrt{n}(R_{12} - R_{22})\right) + \mathrm{as}\mathbb{C}\mathrm{ov}\left(\sqrt{n}W_{11}, \sqrt{n}(S_{12} - S_{22})\right) \\
&\quad + \mathrm{as}\mathbb{C}\mathrm{ov}\left(\sqrt{n}W_{11}, \sqrt{n}(W_{12} - W_{22})\right).
\end{aligned}
$$

This result follows from the fact that terms of type R and S, as well as R and W, are uncorrelated (see Remark 1 below). By arguments similar to those used in reference [12], main paper, we also obtain

$$
\begin{aligned}
&\mathrm{as}\mathbb{C}\mathrm{ov}\left(\sqrt{n}S_{11}, \sqrt{n}(S_{12} - S_{22})\right) \\
&= \mathbb{E}\left\{\pi(T, A)\mathbb{C}\mathrm{ov}(\mathrm{I}(T \geq c_1)D_1, \mathrm{I}(c_1 \leq T < c_2)D_2 | T, A)\right\} \\
&= \mathbb{E}\left\{\pi(T, A)\mathrm{I}(c_1 \leq T < c_2)\mathbb{C}\mathrm{ov}(D_1, D_2 | T, A)\right\} \\
&= -\mathbb{E}\left\{\pi(T, A)\mathrm{I}(c_1 \leq T < c_2)\rho_1(T, A)\rho_2(T, A)\right\}.
\end{aligned}
$$

Similarly, we have that

$$
\begin{aligned}
&\text{as}\mathbb{C}\text{ov}\left(\sqrt{n}S_{11}, \sqrt{n}(W_{12}-W_{22})\right)\\
&\quad = \quad -\mathbb{E}\left\{[1-\pi(T,A)]\mathrm{I}(c_1 \le T < c_2)\rho_1(T,A)\rho_2(T,A)\right\},\\
&\text{as}\mathbb{C}\text{ov}\left(\sqrt{n}R_{11}, \sqrt{n}(R_{12}-R_{22})\right)\\
&\quad = \quad -\beta_{11}(\beta_{12}-\beta_{22}) + \mathbb{E}\left\{\mathrm{I}(c_1 \le T < c_2)\rho_1(T,A)\rho_2(T,A)\right\},\\
&\text{as}\mathbb{C}\text{ov}\left(\sqrt{n}W_{11}, \sqrt{n}(S_{12}-S_{22})\right)\\
&\quad = \quad -\mathbb{E}\left\{[1-\pi(T,A)]\mathrm{I}(c_1 \le T < c_2)\rho_1(T,A)\rho_2(T,A)\right\},\\
&\text{as}\mathbb{C}\text{ov}\left(\sqrt{n}W_{11}, \sqrt{n}(W_{12}-W_{22})\right)\\
&\quad = \quad -\frac{1}{K}\mathbb{E}\left\{[1-\pi(T,A)]\mathrm{I}(c_1 \le T < c_2)\rho_1(T,A)\rho_2(T,A)\right\}\\
&\qquad\quad -\mathbb{E}\left\{\frac{[1-\pi(T,A)]^2\mathrm{I}(c_1 \le T < c_2)\rho_1(T,A)\rho_2(T,A)}{\pi(T,A)}\right\}.
\end{aligned}
$$

This leads to

$$
\sigma_{1112} - \sigma_{1122} = -[\psi^2_{1212} + \beta_{11}(\beta_{12}-\beta_{22})], \tag{7}
$$

where

$$
\begin{aligned}
\psi^2_{1212} = &\left(1+\frac{1}{K}\right)\mathbb{E}\left\{[1-\pi(T,A)]\mathrm{I}(c_1 \le T < c_2)\rho_1(T,A)\rho_2(T,A)\right\}\\
&+\mathbb{E}\left\{[1-\pi(T,A)]^2\mathrm{I}(c_1 \le T < c_2)\frac{\rho_1(T,A)\rho_2(T,A)}{\pi(T,A)}\right\}.
\end{aligned}
$$

Secondly, we consider the term $\sigma_{112} - \sigma_{122}$. In this case, we have

$$
\begin{aligned}
\sigma_{112} - \sigma_{122} &= \text{as}\mathbb{C}\text{ov}(\sqrt{n}\hat{\theta}_{1,\mathrm{KNN}}, \sqrt{n}\hat{\beta}_{12,\mathrm{KNN}}) - \text{as}\mathbb{C}\text{ov}(\sqrt{n}\hat{\theta}_{1,\mathrm{KNN}}, \sqrt{n}\hat{\beta}_{22,\mathrm{KNN}})\\
&= \text{as}\mathbb{C}\text{ov}\left(\sqrt{n}\hat{\theta}_{1,\mathrm{KNN}}, \sqrt{n}(\hat{\beta}_{12,\mathrm{KNN}} - \hat{\beta}_{22,\mathrm{KNN}})\right)\\
&= \text{as}\mathbb{C}\text{ov}\Bigg(\sqrt{n}(S_1+R_1+W_1), \sqrt{n}(S_{12}-S_{22})\\
&\qquad + \sqrt{n}(R_{12}-R_{22}) + \sqrt{n}(W_{12}-W_{22})\Bigg)\\
&= \text{as}\mathbb{C}\text{ov}\left(\sqrt{n}S_1, \sqrt{n}(S_{12}-S_{22})\right) + \text{as}\mathbb{C}\text{ov}\left(\sqrt{n}S_1, \sqrt{n}(W_{12}-W_{22})\right)\\
&\quad + \text{as}\mathbb{C}\text{ov}\left(\sqrt{n}R_1, \sqrt{n}(R_{12}-R_{22})\right) + \text{as}\mathbb{C}\text{ov}\left(\sqrt{n}W_1, \sqrt{n}(S_{12}-S_{22})\right)\\
&\quad + \text{as}\mathbb{C}\text{ov}\left(\sqrt{n}W_1, \sqrt{n}(W_{12}-W_{22})\right).
\end{aligned}
$$

We obtain

$$\begin{aligned}
&\text{as}\mathbb{C}\text{ov}\left(\sqrt{n}S_1, \sqrt{n}(S_{12} - S_{22})\right) \\
&\quad = \quad -\mathbb{E}\left\{\pi(T,A)\mathrm{I}(c_1 \le T < c_2)\rho_1(T,A)\rho_2(T,A)\right\}, \\
&\text{as}\mathbb{C}\text{ov}\left(\sqrt{n}S_1, \sqrt{n}(W_{12} - W_{22})\right) \\
&\quad = \quad -\mathbb{E}\left\{[1-\pi(T,A)]\mathrm{I}(c_1 \le T < c_2)\rho_1(T,A)\rho_2(T,A)\right\}, \\
&\text{as}\mathbb{C}\text{ov}\left(\sqrt{n}R_1, \sqrt{n}(R_{12} - R_{22})\right) \\
&\quad = \quad -\theta_1(\beta_{12} - \beta_{22}) + \mathbb{E}\left\{\mathrm{I}(c_1 \le T < c_2)\rho_1(T,A)\rho_2(T,A)\right\}, \\
&\text{as}\mathbb{C}\text{ov}\left(\sqrt{n}W_1, \sqrt{n}(S_{12} - S_{22})\right) \\
&\quad = \quad -\mathbb{E}\left\{[1-\pi(T,A)]\mathrm{I}(c_1 \le T < c_2)\rho_1(T,A)\rho_2(T,A)\right\}, \\
&\text{as}\mathbb{C}\text{ov}\left(\sqrt{n}W_1, \sqrt{n}(W_{12} - W_{22})\right) \\
&\quad = \quad -\frac{1}{K}\mathbb{E}\Bigg\{[1-\pi(T,A)]\mathrm{I}(c_1 \le T < c_2)\rho_1(T,A)\rho_2(T,A)\Bigg\} \\
&\qquad\quad - \mathbb{E}\Bigg\{[1-\pi(T,A)]^2\mathrm{I}(c_1 \le T < c_2)\frac{\rho_1(T,A)\rho_2(T,A)}{\pi(T,A)}\Bigg\},
\end{aligned}$$

and then

$$\sigma_{112} - \sigma_{122} \quad = \quad -[\psi^2_{1212} + \theta_1(\beta_{12} - \beta_{22})]. \tag{8}$$

Similarly, it is straightforward to obtain

$$\sigma_{211} \quad = \quad -[\psi^2_{112} + \theta_2\beta_{11}] \tag{9}$$

and

$$\sigma_{123} \quad = \quad -[\psi^2_{213} + \theta_1\beta_{23}], \tag{10}$$

with

$$\begin{aligned}
\psi^2_{112} \quad = \quad & \left(1 + \frac{1}{K}\right)\mathbb{E}\left\{[1-\pi(T,A)]\mathrm{I}(T \ge c_1)\rho_1(T,A)\rho_2(T,A)\right\} \\
& + \mathbb{E}\left\{\frac{[1-\pi(T,A)]^2\mathrm{I}(T \ge c_1)\rho_1(T,A)\rho_2(T,A)}{\pi(T,A)}\right\}
\end{aligned}$$

and

$$\begin{aligned}
\psi^2_{213} \quad = \quad & \left(1 + \frac{1}{K}\right)\mathbb{E}\left\{[1-\pi(T,A)]\mathrm{I}(T \ge c_2)\rho_1(T,A)\rho_3(T,A)\right\} \\
& + \mathbb{E}\left\{\frac{[1-\pi(T,A)]^2\mathrm{I}(T \ge c_2)\rho_1(T,A)\rho_3(T,A)}{\pi(T,A)}\right\}.
\end{aligned}$$

The covariance between $\sqrt{n}\hat{\theta}_{1,\mathrm{KNN}}$ and $\sqrt{n}\hat{\theta}_{2,\mathrm{KNN}}$ is computed analogously, i.e.,

$$\sigma^*_{12} \quad = \quad -[\theta_1\theta_2 + \psi^2_{12}], \tag{11}$$

where

$$\psi_{12}^2 = \left(1+\frac{1}{K}\right)\mathbb{E}\left\{[1-\pi(T,A)]\rho_1(T,A)\rho_2(T,A)\right\} + \mathbb{E}\left\{\frac{[1-\pi(T,A)]^2\rho_1(T,A)\rho_2(T,A)}{\pi(T,A)}\right\}.$$

By using results (7), (8), (9) and (11) into (4), we obtain a suitable expression for the quantity $\mathrm{as}\mathbb{C}\mathrm{ov}\left(\sqrt{n}\widehat{\mathrm{TCF}}_{1,\mathrm{KNN}}(c_1), \sqrt{n}\widehat{\mathrm{TCF}}_{2,\mathrm{KNN}}(c_1,c_2)\right)$, which depends on easily estimable quantities.

Clearly, a similar approach can be used to get suitable expressions for ξ_{13} and ξ_{23} too. In particular, the estimable version of ξ_{13} can be obtained by using suitable expressions for σ_{123}, σ_{1123} and $\sigma_{111}+\sigma_{211}$. The quantity σ_{123} is already computed in (10), and the formula for σ_{1123} can be obtained as

$$\sigma_{1123} = -[\psi_{213}^2 + \beta_{11}\beta_{23}].$$

To compute $\sigma_{111}+\sigma_{211}$, we notice that

$$\begin{aligned}\mathrm{as}\mathbb{C}\mathrm{ov}\left(\sqrt{n}\hat{\theta}_{3,\mathrm{KNN}}, \sqrt{n}\hat{\beta}_{11,\mathrm{KNN}}\right) &= \mathrm{as}\mathbb{C}\mathrm{ov}\left(\sqrt{n} - \sqrt{n}(\hat{\theta}_{1,\mathrm{KNN}} + \hat{\theta}_{1,\mathrm{KNN}}), \sqrt{n}\hat{\beta}_{11,\mathrm{KNN}}\right)\\ &= -\mathrm{as}\mathbb{C}\mathrm{ov}\left(\sqrt{n}(\hat{\theta}_{1,\mathrm{KNN}} + \hat{\theta}_{1,\mathrm{KNN}}), \sqrt{n}\hat{\beta}_{11,\mathrm{KNN}}\right).\end{aligned}$$

It leads to $\sigma_{111}+\sigma_{211} = -\sigma_{311}$. Similarly to (9), we have that

$$\sigma_{311} = -[\psi_{113}^2 + \theta_3\beta_{11}],$$

where

$$\psi_{113}^2 = \left(1+\frac{1}{K}\right)\mathbb{E}\left\{[1-\pi(T,A)]\mathrm{I}(T\geq c_1)\rho_1(T,A)\rho_3(T,A)\right\} + \mathbb{E}\left\{\frac{[1-\pi(T,A)]^2\mathrm{I}(T\geq c_1)\rho_1(T,A)\rho_3(T,A)}{\pi(T,A)}\right\}.$$

For the last term ξ_{23}, we need to make some other calculations. First, the quantity $\sigma_{1223}-\sigma_{2223}$ is obtained as $\sigma_{1112}-\sigma_{1122}$. We have

$$\sigma_{1223} - \sigma_{2223} = -\beta_{23}(\beta_{12}-\beta_{22}),$$

because $\mathrm{I}(c_1 \leq T < c_2)\mathrm{I}(T\geq c_2) = 0$. Second, the term σ_{223} is obtained as

$$\sigma_{223} = -[\psi_{223}^2 + \theta_2\beta_{23}],$$

where

$$\psi_{223}^2 = \left(1+\frac{1}{K}\right)\mathbb{E}\left\{[1-\pi(T,A)]\mathrm{I}(T\geq c_2)\rho_2(T,A)\rho_3(T,A)\right\} + \mathbb{E}\left\{\frac{[1-\pi(T,A)]^2\mathrm{I}(T\geq c_2)\rho_2(T,A)\rho_3(T,A)}{\pi(T,A)}\right\}.$$

Moreover, it is straightforward to show that

$$-(\sigma_{312}-\sigma_{322}) = \sigma_{112}-\sigma_{122}+\sigma_{212}-\sigma_{222},$$

and that

$$\sigma_{312}-\sigma_{322} \quad = \quad -[\psi^2_{1223}+\theta_3(\beta_{12}-\beta_{22})],$$

with

$$\begin{aligned}\psi^2_{1223} \quad = \quad & \left(1+\frac{1}{K}\right)\mathbb{E}\Big\{[1-\pi(T,A)]\mathrm{I}(c_1\leq T<c_2)\rho_2(T,A)\rho_3(T,A)\Big\} \\ & +\mathbb{E}\Big\{[1-\pi(T,A)]^2\mathrm{I}(c_1\leq T<c_2)\frac{\rho_2(T,A)\rho_3(T,A)}{\pi(T,A)}\Big\}.\end{aligned}$$

Remark 1. Here we show that pairs R_{jk} and $S_{j'k'}$ and R_{jk} and $W_{j'k'}$ (with $j\in\{1,2\}$, $k\in\{1,2,3\}$ and $k\geq j$, $j'\in\{1,2\}$, $k'\in\{1,2,3\}$ and $k'\geq j'$) are uncorrelated. Recall that

$$\begin{aligned}R_{jk} \quad &= \quad \frac{1}{n}\sum_{i=1}^{n}\left[\mathrm{I}(T_i\geq c_j)\rho_{ki}-\beta_{jk}\right], \\ S_{jk} \quad &= \quad \frac{1}{n}\sum_{i=1}^{n}\mathrm{I}(T_i\geq c_j)V_i(D_{ki}-\rho_{ki}), \\ W_{jk} \quad &= \quad \frac{1}{n}\sum_{i=1}^{n}\mathrm{I}(T_i\geq c_j)(1-V_i)\left[\frac{1}{K}\sum_{l=1}^{K}\left(V_{i(l)}D_{ki(l)}-\rho_{ki(l)}\right)\right].\end{aligned}$$

Since, $\mathbb{E}(R_{jk})=\mathbb{E}(S_{jk})=\mathbb{E}(W_{jk})=0$, we consider

$$\begin{aligned}&\mathbb{E}(R_{jk}S_{j'k'}) \\ &\quad = \quad \mathbb{E}\left(\left\{\frac{1}{n}\sum_{i=1}^{n}\left[\mathrm{I}(T_i\geq c_j)\rho_{ki}-\beta_{jk}\right]\right\}\left\{\frac{1}{n}\sum_{i=1}^{n}\mathrm{I}(T_i\geq c_{j'})V_i(D_{k'i}-\rho_{k'i})\right\}\right) \\ &\quad = \quad \frac{1}{n^2}\sum_{i=1}^{n}\sum_{m=1}^{n}\mathbb{E}\left\{\left[\mathrm{I}(T_i\geq c_j)\rho_{ki}-\beta_{jk}\right]\mathrm{I}(T_m\geq c_{j'})V_m(D_{k'm}-\rho_{k'm})\right\} \\ &\quad = \quad \frac{1}{n^2}\sum_{i=1}^{n}\sum_{m=1}^{n}\mathbb{E}(U_{im}).\end{aligned}$$

When $i=m$, we have

$$\begin{aligned}&\mathbb{E}(U_{ii}) \\ &\quad = \quad \mathbb{E}\left\{\left[\mathrm{I}(T_i\geq c_j)\rho_{ki}-\beta_{jk}\right]\mathrm{I}(T_i\geq c_{j'})\mathbb{E}\left\{V_i(D_{k'i}-\rho_{k'i})|T_i,A_i\right\}\right\} \\ &\quad = \quad \mathbb{E}\left\{\left[\mathrm{I}(T_i\geq c_j)\rho_{ki}-\beta_{jk}\right]\mathrm{I}(T_i\geq c_{j'})\mathbb{E}(V_i|T_i,A_i)\left[\mathbb{E}(D_{k'i}|T_i,A_i)-\rho_{k'i}\right]\right\} \\ &\quad = \quad \mathbb{E}\left\{\left[\mathrm{I}(T_i\geq c_j)\rho_{ki}-\beta_{jk}\right]\mathrm{I}(T_i\geq c_{j'})\mathbb{E}(V_i|T_i,A_i)\left[\Pr(D_{k'i}=1|T_i,A_i)-\rho_{k'i}\right]\right\} \\ &\quad = \quad 0.\end{aligned}$$

Similarly, we can show that the conditional expectation of U_{im} given T_i, A_i, T_m, A_m equals to 0 for $i \neq m$ by using the independence of units i and m. Therefore, we conclude that $\mathbb{E}(R_{jk} S_{j'k'}) = 0$. On the other hand,

$$\begin{aligned}
&\mathbb{E}(R_{jk} W_{j'k'}) \\
&= \mathbb{E}\Bigg(\left\{ \frac{1}{n} \sum_{i=1}^{n} [\mathrm{I}(T_i \geq c_j)\rho_{ki} - \beta_{jk}] \right\} \\
&\quad \times \left\{ \frac{1}{n} \sum_{i=1}^{n} \mathrm{I}(T_i \geq c_{j'})(1 - V_i) \left[\frac{1}{K} \sum_{l=1}^{K} \left(V_{i(l)} D_{k'i(l)} - \rho_{k'i(l)} \right) \right] \right\} \Bigg) \\
&= \frac{1}{n^2} \sum_{i=1}^{n} \sum_{m=1}^{n} \mathbb{E} \Bigg\{ [\mathrm{I}(T_i \geq c_j)\rho_{ki} - \beta_{jk}]\, \mathrm{I}(T_m \geq c_{j'})(1 - V_m) \\
&\quad \times \left[\frac{1}{K} \sum_{l=1}^{K} \left(V_{m(l)} D_{k'm(l)} - \rho_{k'm(l)} \right) \right] \Bigg\} \\
&= \frac{1}{n^2} \sum_{i=1}^{n} \sum_{m=1}^{n} \mathbb{E} \left(\mathbb{E}(Z_{im} | T_i, A_i, T_m, A_m) \right).
\end{aligned}$$

If $i = m$, we have

$$\begin{aligned}
&\mathbb{E}(Z_{ii} | T_i, A_i) \\
&= [\mathrm{I}(T_i \geq c_j)\rho_{ki} - \beta_{jk}]\, \mathrm{I}(T_i \geq c_{j'}) \mathbb{E}(1 - V_i | T_i, A_i) \mathbb{E} \left[\frac{1}{K} \sum_{l=1}^{K} \left(V_{i(l)} D_{k'i(l)} - \rho_{k'i(l)} \right) \right] \\
&= [\mathrm{I}(T_i \geq c_j)\rho_{ki} - \beta_{jk}]\, \mathrm{I}(T_i \geq c_{j'}) \mathbb{E}(1 - V_i | T_i, A_i) \\
&\quad \times \frac{1}{K} \sum_{l=1}^{K} \mathbb{E} \left[\mathbb{E} \left(V_{i(l)} D_{k'i(l)} - \rho_{k'i(l)} \middle| T_{i(l)}, A_{i(l)} \right) \right] \\
&= [\mathrm{I}(T_i \geq c_j)\rho_{ki} - \beta_{jk}]\, \mathrm{I}(T_i \geq c_{j'}) \mathbb{E}(1 - V_i | T_i, A_i) \\
&\quad \times \frac{1}{K} \sum_{l=1}^{K} \mathbb{E} \left[\mathbb{E} \left(D_{k'i(l)} - \rho_{k'i(l)} \middle| T_{i(l)}, A_{i(l)} \right) \right],
\end{aligned}$$

where the first equality comes from the fact that the variables that refer to the neighbourhood of unit i do not depend on $(T_i, A_i, V_i, D_{1i}, D_{2i}, D_{3i})$. Easily follows that $\mathbb{E}(Z_{ii} | T_i, A_i) = 0$. In cases of $i \neq m$, we can obtain that $\mathbb{E}(Z_{im} | T_i, A_i, T_m, A_m) = 0$ by similar arguments. Hence, $\mathbb{E}(R_{jk} W_{j'k'}) = 0$.

S2 Simulation study: correctly specified parametric models; unidimensional covariate A

Tables (1 – 6) refer to Section 5.1 of the main paper and report simulation results when the sample size is set equal to 500 and 1000. The number of replications in each simulation experiment is 5000.

S3 Simulation study: correctly specified parametric models; multidimensional covariate A

In this section, we present simulation result based on the introduction of a multidimensional vector A of observed covariates. In particular, we consider $A = (A_1, A_2, A_3)^\top$. The data are generated in a similar way as in Section 5.1 of the main paper. More precisely, the disease indicator $\mathcal{D}$ is a trinomial random vector (D_1, D_2, D_3), such that D_k is a Bernoulli random variable with success probability θ_k, $k = 1, 2, 3$. We set $\theta_1 = 0.4, \theta_2 = 0.35$ and $\theta_3 = 0.25$. The continuous test results T and covariates A_1, A_2, A_3 are generated by the following conditional models

$$T, A_1, A_2, A_3|D_k \sim \mathcal{N}_4(\mu_k, \Sigma), \qquad k = 1, 2, 3,$$

where $\mu_k = (2k, k, 1.5k, 0.5k)^\top$ and

$$\Sigma = \begin{pmatrix} 1.75 & 0.1 & -0.2 & 0.5 \\ 0.10 & 2.5 & 0.5 & -0.3 \\ -0.20 & 0.5 & 1.0 & 0.7 \\ 0.50 & -0.3 & 0.7 & 1.2 \end{pmatrix}.$$

The verification status V is generated by the model

$$\text{logit}\{\Pr(V = 1|T, A_1, A_2, A_3)\} = -0.7 - 0.35T + 0.2A_1 + 0.8A_2 - 0.6A_3.$$

This choice corresponds to a verification rate of about 0.51. We consider six pairs of cut points (c_1, c_2), i.e., $(2, 4), (2, 5), (2, 7), (4, 5), (4, 7)$ and $(5, 7)$.

Since the conditional distribution of T given D_k is a normal distribution, the true values of TCF's are

$$\begin{aligned} \text{TCF}_1(c_1) &= \Phi\left(\frac{c_1 - 2}{\sigma_{T|D}}\right), \\ \text{TCF}_2(c_1, c_2) &= \Phi\left(\frac{c_2 - 4}{\sigma_{T|D}}\right) - \Phi\left(\frac{c_1 - 4}{\sigma_{T|D}}\right), \\ \text{TCF}_3(c_2) &= 1 - \Phi\left(\frac{c_2 - 6}{\sigma_{T|D}}\right). \end{aligned}$$

For the (partially) parametric estimators FI, MSI, IPW and SPE, disease probabilities and verification probabilities are estimated by using correctly specified models. For the KNN estimators, the Mahalanobis distance is employed. In addition, we use $\bar{K} = 2$ for the estimation of standard deviations. The number of replications in each simulation experiment is 5000. The sample size is 500.

As expected, results given in Table 7, show a certain loss of efficiency of KNN estimators, compared to parametric competitors.

S4 Simulation study: misspecified models with moderate sample sizes

Tables 8 – 9 refer to Section 5.2 of the main paper and show simulation results for KNN estimators in case of misspecified models, with sample sizes 250 and 500. The number of replications in each simulation experiment is 5000.

Overall, results show that KNN estimators behave pretty satisfactorily, having a relatively small bias if 1NN is used. Nevertheless, they have a variance larger than the one shown at larger sample sizes. Moreover, a moderate sample size seems has an evident impact on variance estimators, which, as expected, behave poorly when compared with the behavior shown at larger sample sizes (see results in Table 4, main paper).

S5 Simulation study on boundary effect

Here, we present the results of a simulation study carried out to evaluate the so-called boundary effect on KNN estimators $\hat{\rho}_{ki,K}$. In the study, the true disease status $\mathcal{D} = (D_1, D_2, D_3)$ is a trinomial random vector, with D_k Bernoulli random variable and $\theta_1 = 0.1, \theta_2 = 0.5$ and $\theta_3 = 0.4$. The continuous test T and the covariate A are generated conditionally to D_k, for $k = 1, 2, 3$, and are (conditionally) independent. At each replication, based on generated data, we estimate the probabilities

$$\rho_k(t, a) = \Pr\{D_k = 1 | T = t, A = a\} = \frac{\Pr(T = t | D_k = 1) \Pr(A = a | D_k = 1)\theta_k}{\sum_{k=1}^{3} \Pr(T = t | D_k = 1) \Pr(A = a | D_k = 1)\theta_k},$$

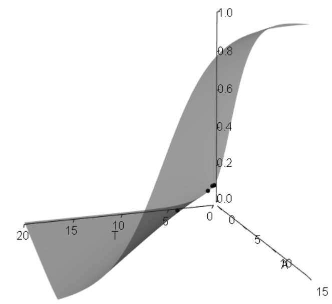
at points (t, a) that are internal to the support of (T, A) and points that are close to its boundary. We use KNN estimator based on Euclidean distance, with K varying in $\{1, 3, 6, 10\}$. The number of replications for each experiment is 5000.

In a first case (Scenario I), we generate T from models $T|D_k = 1 \sim \text{Exp}(1/k)$ and A from $A|D_k = 1 \sim \text{Exp}(k/2)$. We consider four points (t, a), i.e., $(0.05, 0.05)$, $(0.5, 0.5)$, $(1, 0.5)$ and $(5, 2)$, and use two sample sizes: 50 and 250. In a second case (Scenario II), we generate T from models $T|D_k = 1 \sim \text{Gamma}\,(2, 1.75/2k)$ and A from $A|D_k = 1 \sim \text{Gamma}\,(2, 2.5/k)$. We consider four points (t, a), i.e., $(0.05, 0.75)$, $(1, 1.5)$, $(0.75, 2.5)$ and $(2.5, 2.5)$, and use sample sizes 50, 250 and 1000. In both cases, we calculate the probabilities on the squares of side 0.1, centered at the chosen points (t, a), from the marginal law of (T, A), i.e., the quantities

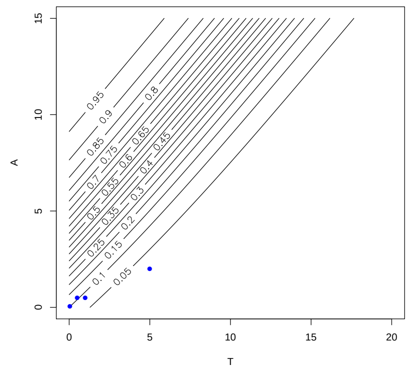
$$p = \Pr(t - 0.05 \leq T \leq t - 0.05, a - 0.05 \leq A \leq a + 0.05)$$

Such probabilities provide an indication of how dense might data be around the points. In both scenarios, the first point is close to the boundary, the second and third points belong to the interior of the domain but share with the first point a comparable value for p. Finally, the last point, belongs to the interior of the domain but shows a remarkably lower value for p. Plots of the functions $\rho_k(t, a)$ and values to be estimated are shown in Figure 1 and 2.

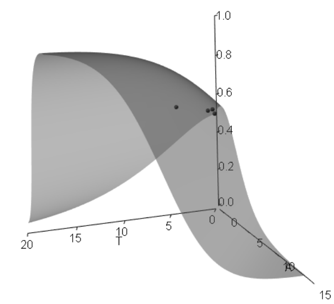
Overall, results in Tables 10–14 seem to show that the bias, when present, is driven more by sparsity of data issues than by boundary effects and that KNN estimators have their poorest performances on largest values of K, regardless of the position of points in the domain. As expected, such bias decreases for increasing sample sizes.



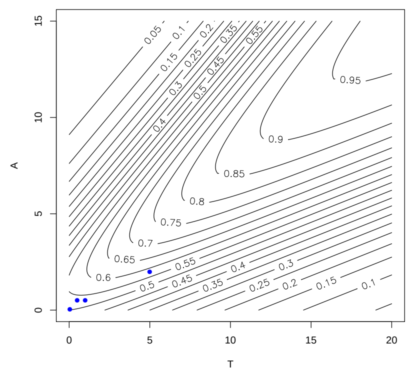
(a) Surface of $\rho_1(t, a)$



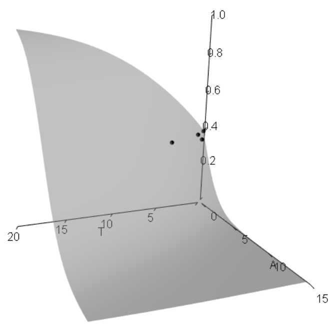
(b) Contour lines of $\rho_1(t, a)$



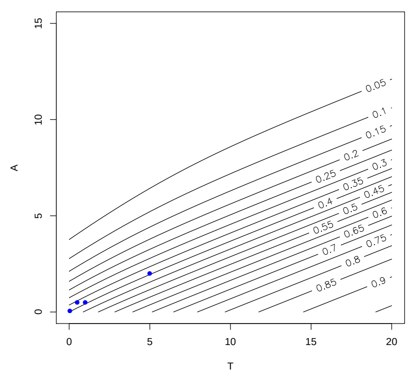
(c) Surface of $\rho_2(t, a)$



(d) Contour lines of $\rho_2(t, a)$

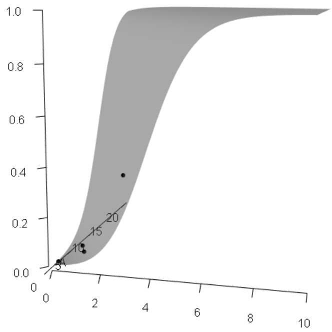


(e) Surface of $\rho_3(t, a)$

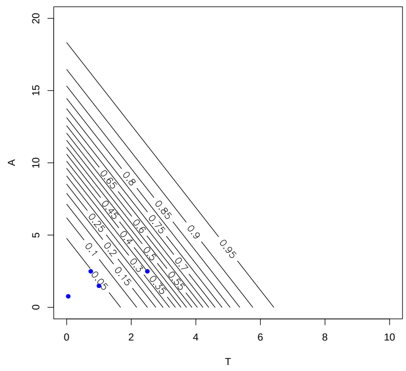


(f) Contour lines of $\rho_3(t, a)$

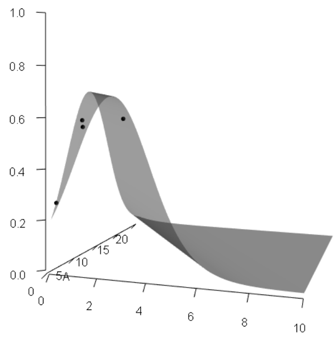
Figure 1 – *Scenario I: surfaces and contour lines of $\rho_k(t, a)$. Points represent the values to be estimated at the chosen coordinates, i.e., (0.05, 0.05), (0.5, 0.5), (1, 0.5), (5,2).*



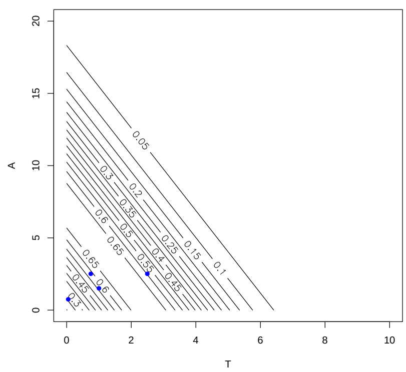
(a) Surface of $\rho_1(t,a)$



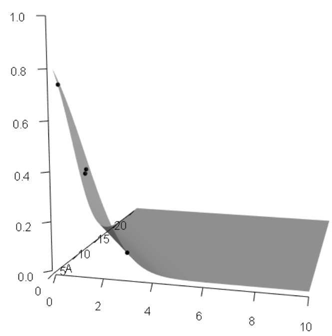
(b) Contour lines of $\rho_1(t,a)$



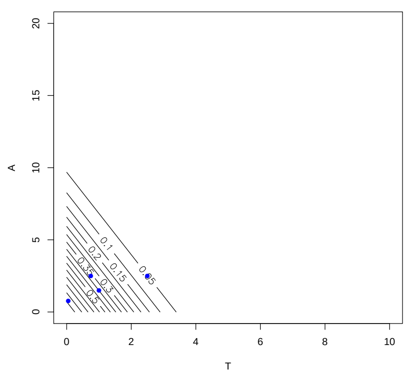
(c) Surface of $\rho_2(t,a)$



(d) Contour lines of $\rho_2(t,a)$



(e) Surface of $\rho_3(t,a)$



(f) Contour lines of $\rho_3(t,a)$

Figure 2 – *Scenario II: surfaces and contour lines of $\rho_k(t,a)$. Points represent the values to be estimated at the chosen coordinates, i.e., (0.05, 0.75), (1, 1.5), (0.75, 2.5), (2.5,2.5).*

Table 1 – *Monte Carlo means, Monte Carlo standard deviations and estimated standard deviations of the estimators for true class fractions in case of sample size equals to* 500 *and the first value of* Σ *is considered. "True" denotes the true parameter value.*

	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
				cut points = $(2,4)$					
True	0.5000	0.4347	0.9347						
FI	0.5007	0.4353	0.9344	0.0373	0.0335	0.0193	0.0309	0.0278	0.0349
MSI	0.5008	0.4353	0.9343	0.0384	0.0372	0.0231	0.0328	0.0335	0.0375
IPW	0.5017	0.4352	0.9341	0.0517	0.0506	0.0270	0.0495	0.0503	0.0287
SPE	0.5010	0.4354	0.9342	0.0406	0.0450	0.0264	0.0399	0.0453	0.0258
1NN	0.5000	0.4343	0.9333	0.0419	0.0464	0.0285	0.0393	0.0443	0.0269
3NN	0.4991	0.4343	0.9329	0.0401	0.0430	0.0266	0.0386	0.0431	0.0262
				cut points = $(2,5)$					
True	0.5000	0.7099	0.7752						
FI	0.5007	0.7109	0.7754	0.0373	0.0329	0.0375	0.0309	0.0279	0.0408
MSI	0.5008	0.7104	0.7750	0.0384	0.0366	0.0396	0.0328	0.0330	0.0441
IPW	0.5017	0.7104	0.7746	0.0517	0.0497	0.0468	0.0495	0.0482	0.0503
SPE	0.5010	0.7105	0.7749	0.0406	0.0441	0.0441	0.0399	0.0430	0.0436
1NN	0.5000	0.7077	0.7732	0.0419	0.0450	0.0466	0.0393	0.0417	0.0442
3NN	0.4991	0.7066	0.7722	0.0401	0.0415	0.0437	0.0386	0.0406	0.0431
				cut points = $(2,7)$					
True	0.5000	0.9230	0.2248						
FI	0.5007	0.9229	0.2240	0.0373	0.0169	0.0368	0.0309	0.0214	0.0302
MSI	0.5008	0.9231	0.2242	0.0384	0.0201	0.0375	0.0328	0.0247	0.0313
IPW	0.5017	0.9232	0.2235	0.0517	0.0268	0.0537	0.0495	0.0263	0.0527
SPE	0.5010	0.9233	0.2242	0.0406	0.0257	0.0408	0.0399	0.0253	0.0408
1NN	0.5000	0.9214	0.2235	0.0419	0.0261	0.0407	0.0393	0.0256	0.0401
3NN	0.4991	0.9199	0.2225	0.0401	0.0240	0.0392	0.0386	0.0248	0.0396
				cut points = $(4,5)$					
True	0.9347	0.2752	0.7752						
FI	0.9349	0.2756	0.7754	0.0174	0.0285	0.0375	0.0126	0.0235	0.0408
MSI	0.9348	0.2751	0.7750	0.0192	0.0324	0.0396	0.0157	0.0285	0.0441
IPW	0.9349	0.2752	0.7746	0.0298	0.0474	0.0468	0.0285	0.0469	0.0503
SPE	0.9350	0.2751	0.7749	0.0269	0.0403	0.0441	0.0261	0.0402	0.0436
1NN	0.9332	0.2734	0.7732	0.0269	0.0398	0.0466	0.0242	0.0391	0.0442
3NN	0.9317	0.2723	0.7722	0.0242	0.0371	0.0437	0.0235	0.0381	0.0431
				cut points = $(4,7)$					
True	0.9347	0.4883	0.2248						
FI	0.9349	0.4876	0.2240	0.0174	0.0372	0.0368	0.0126	0.0310	0.0302
MSI	0.9348	0.4878	0.2242	0.0192	0.0390	0.0375	0.0157	0.0349	0.0313
IPW	0.9349	0.4880	0.2235	0.0298	0.0513	0.0537	0.0285	0.0512	0.0527
SPE	0.9350	0.4879	0.2242	0.0269	0.0456	0.0408	0.0261	0.0459	0.0408
1NN	0.9332	0.4871	0.2235	0.0269	0.0465	0.0407	0.0242	0.0447	0.0401
3NN	0.9317	0.4855	0.2225	0.0242	0.0434	0.0392	0.0235	0.0436	0.0396
				cut points = $(5,7)$					
True	0.9883	0.2132	0.2248						
FI	0.9881	0.2121	0.2240	0.0051	0.0306	0.0368	0.0037	0.0234	0.0302
MSI	0.9883	0.2126	0.2242	0.0068	0.0329	0.0375	0.0059	0.0274	0.0313
IPW	0.9886	0.2128	0.2235	0.0141	0.0459	0.0537	0.0132	0.0447	0.0527
SPE	0.9887	0.2128	0.2242	0.0133	0.0400	0.0408	0.0126	0.0393	0.0408
1NN	0.9875	0.2136	0.2235	0.0129	0.0400	0.0407	0.0120	0.0376	0.0401
3NN	0.9867	0.2133	0.2225	0.0113	0.0369	0.0392	0.0118	0.0365	0.0396

Table 2 – *Monte Carlo means, Monte Carlo standard deviations and estimated standard deviations of the estimators for true class fractions in case of sample size equals to* 1000 *and the first value of* Σ *is considered. "True" denotes the true parameter value.*

	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
				cut points = $(2, 4)$					
True	0.5000	0.4347	0.9347						
FI	0.5007	0.4349	0.9345	0.0265	0.0238	0.0134	0.0218	0.0195	0.0245
MSI	0.5008	0.4351	0.9343	0.0270	0.0269	0.0159	0.0232	0.0236	0.0265
IPW	0.5013	0.4349	0.9342	0.0356	0.0361	0.0185	0.0354	0.0358	0.0201
SPE	0.5008	0.4351	0.9342	0.0284	0.0322	0.0182	0.0283	0.0321	0.0183
1NN	0.5002	0.4349	0.9340	0.0295	0.0336	0.0194	0.0278	0.0313	0.0189
3NN	0.4997	0.4346	0.9336	0.0283	0.0318	0.0183	0.0273	0.0305	0.0183
				cut points = $(2, 5)$					
True	0.5000	0.7099	0.7752						
FI	0.5007	0.7104	0.7750	0.0265	0.0234	0.0263	0.0218	0.0196	0.0288
MSI	0.5008	0.7104	0.7749	0.0270	0.0257	0.0279	0.0232	0.0233	0.0311
IPW	0.5013	0.7104	0.7749	0.0356	0.0348	0.0328	0.0354	0.0345	0.0355
SPE	0.5008	0.7105	0.7750	0.0284	0.0307	0.0312	0.0283	0.0304	0.0310
1NN	0.5002	0.7093	0.7741	0.0295	0.0319	0.0331	0.0278	0.0295	0.0312
3NN	0.4997	0.7083	0.7733	0.0283	0.0297	0.0312	0.0273	0.0287	0.0304
				cut points = $(2, 7)$					
True	0.5000	0.9230	0.2248						
FI	0.5007	0.9231	0.2244	0.0265	0.0118	0.0265	0.0218	0.0150	0.0213
MSI	0.5008	0.9233	0.2245	0.0270	0.0141	0.0268	0.0232	0.0174	0.0221
IPW	0.5013	0.9234	0.2238	0.0356	0.0187	0.0377	0.0354	0.0187	0.0377
SPE	0.5008	0.9234	0.2246	0.0284	0.0180	0.0289	0.0283	0.0180	0.0288
1NN	0.5002	0.9223	0.2239	0.0295	0.0187	0.0292	0.0278	0.0179	0.0283
3NN	0.4997	0.9213	0.2234	0.0283	0.0173	0.0283	0.0273	0.0174	0.0280
				cut points = $(4, 5)$					
True	0.9347	0.2752	0.7752						
FI	0.9351	0.2755	0.7750	0.0124	0.0199	0.0263	0.0089	0.0166	0.0288
MSI	0.9349	0.2753	0.7749	0.0137	0.0230	0.0279	0.0111	0.0201	0.0311
IPW	0.9348	0.2754	0.7749	0.0214	0.0335	0.0328	0.0207	0.0335	0.0355
SPE	0.9349	0.2755	0.7750	0.0194	0.0288	0.0312	0.0188	0.0285	0.0310
1NN	0.9337	0.2745	0.7741	0.0198	0.0286	0.0331	0.0172	0.0277	0.0312
3NN	0.9329	0.2737	0.7733	0.0179	0.0268	0.0312	0.0166	0.0270	0.0304
				cut points = $(4, 7)$					
True	0.9347	0.4883	0.2248						
FI	0.9351	0.4882	0.2244	0.0124	0.0263	0.0265	0.0089	0.0218	0.0213
MSI	0.9349	0.4882	0.2245	0.0137	0.0281	0.0268	0.0111	0.0246	0.0221
IPW	0.9348	0.4885	0.2238	0.0214	0.0370	0.0377	0.0207	0.0365	0.0377
SPE	0.9349	0.4883	0.2246	0.0194	0.0329	0.0289	0.0188	0.0325	0.0288
1NN	0.9337	0.4874	0.2239	0.0198	0.0342	0.0292	0.0172	0.0316	0.0283
3NN	0.9329	0.4867	0.2234	0.0179	0.0322	0.0283	0.0166	0.0308	0.0280
				cut points = $(5, 7)$					
True	0.9883	0.2132	0.2248						
FI	0.9883	0.2127	0.2244	0.0036	0.0218	0.0265	0.0026	0.0164	0.0213
MSI	0.9883	0.2129	0.2245	0.0048	0.0233	0.0268	0.0042	0.0193	0.0221
IPW	0.9883	0.2130	0.2238	0.0101	0.0322	0.0377	0.0098	0.0320	0.0377
SPE	0.9883	0.2129	0.2246	0.0096	0.0279	0.0289	0.0094	0.0277	0.0288
1NN	0.9876	0.2130	0.2239	0.0096	0.0289	0.0292	0.0085	0.0266	0.0283
3NN	0.9871	0.2130	0.2234	0.0084	0.0268	0.0283	0.0083	0.0259	0.0280

Table 3 – *Monte Carlo means, Monte Carlo standard deviations and estimated standard deviations of the estimators for true class fractions in case of sample size equals to* 500 *and the second value of* Σ *is considered. "True" denotes the true parameter value.*

	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
				cut points = $(2, 4)$					
True	0.5000	0.3970	0.8970						
FI	0.5003	0.3974	0.8965	0.0363	0.0297	0.0252	0.0303	0.0246	0.0326
MSI	0.5002	0.3976	0.8966	0.0373	0.0354	0.0287	0.0325	0.0318	0.0362
IPW	0.5005	0.3975	0.8969	0.0469	0.0481	0.0369	0.0461	0.0484	0.0376
SPE	0.5003	0.3976	0.8968	0.0404	0.0439	0.0358	0.0398	0.0439	0.0354
1NN	0.4987	0.3958	0.8975	0.0419	0.0466	0.0379	0.0396	0.0438	0.0347
3NN	0.4975	0.3947	0.8971	0.0400	0.0431	0.0349	0.0388	0.0425	0.0335
				cut points = $(2, 5)$					
True	0.5000	0.6335	0.7365						
FI	0.5003	0.6339	0.7362	0.0363	0.0305	0.0401	0.0303	0.0265	0.0388
MSI	0.5002	0.6337	0.7360	0.0373	0.0360	0.0425	0.0325	0.0331	0.0431
IPW	0.5005	0.6333	0.7368	0.0469	0.0474	0.0505	0.0461	0.0472	0.0530
SPE	0.5003	0.6338	0.7364	0.0404	0.0439	0.0475	0.0398	0.0435	0.0480
1NN	0.4987	0.6318	0.7376	0.0419	0.0466	0.0510	0.0396	0.0436	0.0477
3NN	0.4975	0.6306	0.7372	0.0400	0.0433	0.0472	0.0388	0.0423	0.0464
				cut points = $(2, 7)$					
True	0.5000	0.8682	0.2635						
FI	0.5003	0.8678	0.2623	0.0363	0.0216	0.0390	0.0303	0.0205	0.0334
MSI	0.5002	0.8678	0.2623	0.0373	0.0259	0.0407	0.0325	0.0257	0.0358
IPW	0.5005	0.8680	0.2628	0.0469	0.0349	0.0487	0.0461	0.0347	0.0487
SPE	0.5003	0.8682	0.2625	0.0404	0.0331	0.0425	0.0398	0.0329	0.0420
1NN	0.4987	0.8675	0.2634	0.0419	0.0352	0.0441	0.0396	0.0325	0.0425
3NN	0.4975	0.8667	0.2633	0.0400	0.0324	0.0427	0.0388	0.0313	0.0420
				cut points = $(4, 5)$					
True	0.8970	0.2365	0.7365						
FI	0.8974	0.2365	0.7362	0.0206	0.0254	0.0401	0.0169	0.0212	0.0388
MSI	0.8973	0.2361	0.7360	0.0231	0.0307	0.0425	0.0207	0.0274	0.0431
IPW	0.8975	0.2358	0.7368	0.0270	0.0416	0.0505	0.0264	0.0413	0.0530
SPE	0.8973	0.2362	0.7364	0.0262	0.0375	0.0475	0.0256	0.0371	0.0480
1NN	0.8964	0.2360	0.7376	0.0279	0.0388	0.0510	0.0263	0.0372	0.0477
3NN	0.8954	0.2359	0.7372	0.0264	0.0362	0.0472	0.0255	0.0362	0.0464
				cut points = $(4, 7)$					
True	0.8970	0.4711	0.2635						
FI	0.8974	0.4704	0.2623	0.0206	0.0357	0.0390	0.0169	0.0289	0.0334
MSI	0.8973	0.4703	0.2623	0.0231	0.0395	0.0407	0.0207	0.0347	0.0358
IPW	0.8975	0.4704	0.2628	0.0270	0.0489	0.0487	0.0264	0.0483	0.0487
SPE	0.8973	0.4706	0.2625	0.0262	0.0454	0.0425	0.0256	0.0446	0.0420
1NN	0.8964	0.4717	0.2634	0.0279	0.0480	0.0441	0.0263	0.0446	0.0425
3NN	0.8954	0.4720	0.2633	0.0264	0.0448	0.0427	0.0255	0.0433	0.0420
				cut points = $(5, 7)$					
True	0.9711	0.2347	0.2635						
FI	0.9710	0.2340	0.2623	0.0088	0.0282	0.0390	0.0073	0.0235	0.0334
MSI	0.9710	0.2342	0.2623	0.0116	0.0327	0.0407	0.0111	0.0292	0.0358
IPW	0.9711	0.2346	0.2628	0.0142	0.0403	0.0487	0.0143	0.0399	0.0487
SPE	0.9711	0.2344	0.2625	0.0141	0.0379	0.0425	0.0141	0.0372	0.0420
1NN	0.9707	0.2357	0.2634	0.0150	0.0400	0.0441	0.0148	0.0376	0.0425
3NN	0.9701	0.2360	0.2633	0.0142	0.0373	0.0427	0.0144	0.0366	0.0420

Table 4 – *Monte Carlo means, Monte Carlo standard deviations and estimated standard deviations of the estimators for true class fractions in case of sample size equals to* 1000 *and the second value of* Σ *is considered. "True" denotes the true parameter value.*

	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
				cut points = $(2,4)$					
True	0.5000	0.3970	0.8970						
FI	0.4997	0.3968	0.8971	0.0248	0.0206	0.0180	0.0214	0.0173	0.0230
MSI	0.4997	0.3967	0.8970	0.0256	0.0247	0.0208	0.0230	0.0225	0.0256
IPW	0.4998	0.3966	0.8971	0.0324	0.0340	0.0266	0.0327	0.0344	0.0266
SPE	0.4998	0.3968	0.8972	0.0278	0.0309	0.0259	0.0281	0.0312	0.0252
1NN	0.4989	0.3958	0.8974	0.0291	0.0328	0.0279	0.0280	0.0310	0.0246
3NN	0.4981	0.3950	0.8974	0.0279	0.0307	0.0258	0.0274	0.0301	0.0238
				cut points = $(2,5)$					
True	0.5000	0.6335	0.7365						
FI	0.4997	0.6336	0.7369	0.0248	0.0210	0.0286	0.0214	0.0187	0.0274
MSI	0.4997	0.6333	0.7366	0.0256	0.0246	0.0305	0.0230	0.0234	0.0305
IPW	0.4998	0.6331	0.7362	0.0324	0.0333	0.0368	0.0327	0.0335	0.0377
SPE	0.4998	0.6332	0.7366	0.0278	0.0304	0.0344	0.0281	0.0309	0.0341
1NN	0.4989	0.6322	0.7372	0.0291	0.0325	0.0369	0.0280	0.0308	0.0338
3NN	0.4981	0.6312	0.7372	0.0279	0.0303	0.0345	0.0274	0.0299	0.0328
				cut points = $(2,7)$					
True	0.5000	0.8682	0.2635						
FI	0.4997	0.8683	0.2635	0.0248	0.0152	0.0277	0.0214	0.0143	0.0236
MSI	0.4997	0.8683	0.2636	0.0256	0.0179	0.0288	0.0230	0.0181	0.0254
IPW	0.4998	0.8684	0.2635	0.0324	0.0243	0.0342	0.0327	0.0246	0.0345
SPE	0.4998	0.8683	0.2637	0.0278	0.0230	0.0300	0.0281	0.0234	0.0298
1NN	0.4989	0.8684	0.2643	0.0291	0.0247	0.0313	0.0280	0.0229	0.0300
3NN	0.4981	0.8678	0.2643	0.0279	0.0228	0.0304	0.0274	0.0221	0.0297
				cut points = $(4,5)$					
True	0.8970	0.2365	0.7365						
FI	0.8970	0.2368	0.7369	0.0144	0.0182	0.0286	0.0119	0.0149	0.0274
MSI	0.8970	0.2365	0.7366	0.0161	0.0218	0.0305	0.0146	0.0194	0.0305
IPW	0.8969	0.2365	0.7362	0.0190	0.0297	0.0368	0.0188	0.0293	0.0377
SPE	0.8970	0.2364	0.7366	0.0185	0.0265	0.0344	0.0182	0.0263	0.0341
1NN	0.8963	0.2364	0.7372	0.0198	0.0276	0.0369	0.0185	0.0263	0.0338
3NN	0.8958	0.2362	0.7372	0.0186	0.0260	0.0345	0.0179	0.0256	0.0328
				cut points = $(4,7)$					
True	0.8970	0.4711	0.2635						
FI	0.8970	0.4715	0.2635	0.0144	0.0249	0.0277	0.0119	0.0204	0.0236
MSI	0.8970	0.4715	0.2636	0.0161	0.0277	0.0288	0.0146	0.0245	0.0254
IPW	0.8969	0.4718	0.2635	0.0190	0.0344	0.0342	0.0188	0.0343	0.0345
SPE	0.8970	0.4715	0.2637	0.0185	0.0318	0.0300	0.0182	0.0316	0.0298
1NN	0.8963	0.4726	0.2643	0.0198	0.0342	0.0313	0.0185	0.0316	0.0300
3NN	0.8958	0.4728	0.2643	0.0186	0.0320	0.0304	0.0179	0.0307	0.0297
				cut points = $(5,7)$					
True	0.9711	0.2347	0.2635						
FI	0.9710	0.2347	0.2635	0.0061	0.0195	0.0277	0.0051	0.0166	0.0236
MSI	0.9711	0.2350	0.2636	0.0083	0.0225	0.0288	0.0078	0.0206	0.0254
IPW	0.9711	0.2353	0.2635	0.0103	0.0283	0.0342	0.0101	0.0283	0.0345
SPE	0.9712	0.2352	0.2637	0.0102	0.0262	0.0300	0.0100	0.0264	0.0298
1NN	0.9709	0.2362	0.2643	0.0111	0.0281	0.0313	0.0104	0.0266	0.0300
3NN	0.9707	0.2366	0.2643	0.0103	0.0262	0.0304	0.0100	0.0259	0.0297

Table 5 – *Monte Carlo means, Monte Carlo standard deviations and estimated standard deviations of the estimators for true class fractions in case of sample size equals to* 500 *and the third value of* Σ *is considered. "True" denotes the true parameter value.*

	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
				cut points = $(2, 4)$					
True	0.5000	0.3031	0.8031						
FI	0.5000	0.3032	0.8033	0.0357	0.0239	0.0345	0.0294	0.0199	0.0329
MSI	0.5000	0.3028	0.8031	0.0373	0.0319	0.0387	0.0326	0.0289	0.0384
IPW	0.5001	0.3024	0.8033	0.0448	0.0445	0.0464	0.0438	0.0440	0.0454
SPE	0.5000	0.3027	0.8033	0.0408	0.0412	0.0450	0.0400	0.0406	0.0438
1NN	0.4994	0.3024	0.8036	0.0428	0.0441	0.0489	0.0403	0.0412	0.0442
3NN	0.4990	0.3020	0.8033	0.0409	0.0412	0.0454	0.0394	0.0399	0.0427
				cut points = $(2, 5)$					
True	0.5000	0.4682	0.6651						
FI	0.5000	0.4686	0.6654	0.0357	0.0272	0.0426	0.0294	0.0226	0.0377
MSI	0.5000	0.4681	0.6649	0.0373	0.0352	0.0459	0.0326	0.0321	0.0432
IPW	0.5001	0.4677	0.6651	0.0448	0.0479	0.0529	0.0438	0.0475	0.0528
SPE	0.5000	0.4681	0.6650	0.0408	0.0443	0.0507	0.0400	0.0441	0.0501
1NN	0.4994	0.4677	0.6653	0.0428	0.0476	0.0545	0.0403	0.0447	0.0509
3NN	0.4990	0.4672	0.6650	0.0409	0.0447	0.0512	0.0394	0.0434	0.0494
				cut points = $(2, 7)$					
True	0.5000	0.7027	0.3349						
FI	0.5000	0.7025	0.3353	0.0357	0.0266	0.0410	0.0294	0.0222	0.0351
MSI	0.5000	0.7022	0.3352	0.0373	0.0334	0.0441	0.0326	0.0306	0.0398
IPW	0.5001	0.7022	0.3350	0.0448	0.0439	0.0509	0.0438	0.0438	0.0507
SPE	0.5000	0.7022	0.3354	0.0408	0.0414	0.0469	0.0400	0.0413	0.0466
1NN	0.4994	0.7018	0.3353	0.0428	0.0448	0.0495	0.0403	0.0418	0.0477
3NN	0.4990	0.7018	0.3353	0.0409	0.0419	0.0473	0.0394	0.0404	0.0466
				cut points = $(4, 5)$					
True	0.8031	0.1651	0.6651						
FI	0.8032	0.1654	0.6654	0.0276	0.0196	0.0426	0.0228	0.0161	0.0377
MSI	0.8033	0.1653	0.6649	0.0300	0.0259	0.0459	0.0270	0.0235	0.0432
IPW	0.8032	0.1653	0.6651	0.0340	0.0354	0.0529	0.0336	0.0349	0.0528
SPE	0.8033	0.1654	0.6650	0.0331	0.0322	0.0507	0.0325	0.0322	0.0501
1NN	0.8027	0.1653	0.6653	0.0357	0.0342	0.0545	0.0334	0.0328	0.0509
3NN	0.8024	0.1652	0.6650	0.0335	0.0319	0.0512	0.0324	0.0319	0.0494
				cut points = $(4, 7)$					
True	0.8031	0.3996	0.3349						
FI	0.8032	0.3993	0.3353	0.0276	0.0297	0.0410	0.0228	0.0243	0.0351
MSI	0.8033	0.3994	0.3352	0.0300	0.0367	0.0441	0.0270	0.0327	0.0398
IPW	0.8032	0.3998	0.3350	0.0340	0.0465	0.0509	0.0336	0.0456	0.0507
SPE	0.8033	0.3995	0.3354	0.0331	0.0436	0.0469	0.0325	0.0428	0.0466
1NN	0.8027	0.3994	0.3353	0.0357	0.0469	0.0495	0.0334	0.0436	0.0477
3NN	0.8024	0.3997	0.3353	0.0335	0.0438	0.0473	0.0324	0.0423	0.0466
				cut points = $(5, 7)$					
True	0.8996	0.2345	0.3349						
FI	0.8997	0.2339	0.3353	0.0192	0.0248	0.0410	0.0158	0.0204	0.0351
MSI	0.8996	0.2341	0.3352	0.0222	0.0314	0.0441	0.0203	0.0281	0.0398
IPW	0.8994	0.2345	0.3350	0.0255	0.0396	0.0509	0.0252	0.0389	0.0507
SPE	0.8995	0.2340	0.3354	0.0250	0.0373	0.0469	0.0247	0.0365	0.0466
1NN	0.8990	0.2341	0.3353	0.0271	0.0397	0.0495	0.0256	0.0374	0.0477
3NN	0.8988	0.2345	0.3353	0.0254	0.0374	0.0473	0.0247	0.0363	0.0466

Table 6 – *Monte Carlo means, Monte Carlo standard deviations and estimated standard deviations of the estimators for true class fractions in case of sample size equals to* 1000 *and the third value of* Σ *is considered. "True" denotes the true parameter value.*

	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
				cut points = $(2, 4)$					
True	0.5000	0.3031	0.8031						
FI	0.5004	0.3030	0.8033	0.0243	0.0172	0.0239	0.0208	0.0140	0.0232
MSI	0.5002	0.3028	0.8028	0.0254	0.0230	0.0265	0.0230	0.0204	0.0271
IPW	0.5002	0.3027	0.8027	0.0307	0.0315	0.0316	0.0310	0.0311	0.0322
SPE	0.5002	0.3028	0.8027	0.0278	0.0293	0.0308	0.0283	0.0288	0.0311
1NN	0.4999	0.3023	0.8027	0.0295	0.0313	0.0341	0.0285	0.0291	0.0313
3NN	0.4994	0.3023	0.8030	0.0281	0.0295	0.0316	0.0278	0.0283	0.0302
				cut points = $(2, 5)$					
True	0.5000	0.4682	0.6651						
FI	0.5004	0.4684	0.6651	0.0243	0.0192	0.0298	0.0208	0.0159	0.0265
MSI	0.5002	0.4682	0.6650	0.0254	0.0254	0.0322	0.0230	0.0227	0.0305
IPW	0.5002	0.4681	0.6650	0.0307	0.0341	0.0370	0.0310	0.0336	0.0373
SPE	0.5002	0.4682	0.6650	0.0278	0.0320	0.0355	0.0283	0.0312	0.0355
1NN	0.4999	0.4677	0.6650	0.0295	0.0344	0.0384	0.0285	0.0316	0.0360
3NN	0.4994	0.4676	0.6653	0.0281	0.0324	0.0363	0.0278	0.0307	0.0349
				cut points = $(2, 7)$					
True	0.5000	0.7027	0.3349						
FI	0.5004	0.7031	0.3345	0.0243	0.0187	0.0281	0.0208	0.0156	0.0247
MSI	0.5002	0.7030	0.3348	0.0254	0.0238	0.0305	0.0230	0.0216	0.0281
IPW	0.5002	0.7031	0.3348	0.0307	0.0312	0.0353	0.0310	0.0310	0.0358
SPE	0.5002	0.7030	0.3350	0.0278	0.0297	0.0324	0.0283	0.0292	0.0329
1NN	0.4999	0.7029	0.3351	0.0295	0.0323	0.0340	0.0285	0.0295	0.0336
3NN	0.4994	0.7027	0.3351	0.0281	0.0302	0.0328	0.0278	0.0286	0.0328
				cut points = $(4, 5)$					
True	0.8031	0.1651	0.6651						
FI	0.8035	0.1654	0.6651	0.0193	0.0137	0.0298	0.0161	0.0114	0.0265
MSI	0.8033	0.1654	0.6650	0.0208	0.0183	0.0322	0.0191	0.0167	0.0305
IPW	0.8032	0.1654	0.6650	0.0235	0.0247	0.0370	0.0238	0.0247	0.0373
SPE	0.8032	0.1654	0.6650	0.0228	0.0227	0.0355	0.0230	0.0228	0.0355
1NN	0.8029	0.1653	0.6650	0.0245	0.0240	0.0384	0.0236	0.0232	0.0360
3NN	0.8025	0.1652	0.6653	0.0233	0.0226	0.0363	0.0229	0.0225	0.0349
				cut points = $(4, 7)$					
True	0.8031	0.3996	0.3349						
FI	0.8035	0.4000	0.3345	0.0193	0.0209	0.0281	0.0161	0.0171	0.0247
MSI	0.8033	0.4002	0.3348	0.0208	0.0255	0.0305	0.0191	0.0231	0.0281
IPW	0.8032	0.4004	0.3348	0.0235	0.0322	0.0353	0.0238	0.0323	0.0358
SPE	0.8032	0.4003	0.3350	0.0228	0.0302	0.0324	0.0230	0.0303	0.0329
1NN	0.8029	0.4006	0.3351	0.0245	0.0323	0.0340	0.0236	0.0308	0.0336
3NN	0.8025	0.4003	0.3351	0.0233	0.0304	0.0328	0.0229	0.0299	0.0328
				cut points = $(5, 7)$					
True	0.8996	0.2345	0.3349						
FI	0.8999	0.2346	0.3345	0.0132	0.0176	0.0281	0.0112	0.0144	0.0247
MSI	0.8999	0.2348	0.3348	0.0152	0.0218	0.0305	0.0143	0.0199	0.0281
IPW	0.8999	0.2350	0.3348	0.0176	0.0272	0.0353	0.0177	0.0275	0.0358
SPE	0.8999	0.2349	0.3350	0.0172	0.0257	0.0324	0.0174	0.0259	0.0329
1NN	0.8997	0.2352	0.3351	0.0186	0.0274	0.0340	0.0181	0.0264	0.0336
3NN	0.8994	0.2351	0.3351	0.0175	0.0258	0.0328	0.0175	0.0257	0.0328

Table 7 – *Monte Carlo means, Monte Carlo standard deviations and estimated standard deviations of the estimators for true class fractions in case of sample size equals to* 500*. Dimension of covariate A is 3. KNN estimators are based in the Mahalanobis distance."True" denotes the true parameter value.*

	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
					cut points $= (2, 4)$				
True	0.5000	0.4347	0.9347						
FI	0.5001	0.4361	0.9344	0.0369	0.0383	0.0222	0.0310	0.0333	0.0507
MSI	0.5000	0.4358	0.9344	0.0370	0.0387	0.0227	0.0310	0.0335	0.0508
IPW	0.5017	0.4370	0.9340	0.0625	0.0566	0.0247	0.0600	0.0557	0.0297
SPE	0.4999	0.4355	0.9344	0.0372	0.0404	0.0230	0.0369	0.0403	0.0226
1NN	0.5009	0.4401	0.9304	0.0388	0.0418	0.0239	0.0393	0.0435	0.0262
3NN	0.5006	0.4420	0.9278	0.0384	0.0404	0.0232	0.0390	0.0425	0.0259
					cut points $= (2, 5)$				
True	0.5000	0.7099	0.7752						
FI	0.5001	0.7106	0.7756	0.0369	0.0358	0.0383	0.0310	0.0374	0.0504
MSI	0.5000	0.7101	0.7754	0.0370	0.0362	0.0385	0.0310	0.0376	0.0506
IPW	0.5017	0.7113	0.7739	0.0625	0.0627	0.0453	0.0600	0.0597	0.0528
SPE	0.4999	0.7096	0.7751	0.0372	0.0380	0.0392	0.0369	0.0375	0.0384
1NN	0.5009	0.7085	0.7671	0.0388	0.0401	0.0404	0.0393	0.0417	0.0421
3NN	0.5006	0.7060	0.7620	0.0384	0.0380	0.0394	0.0390	0.0406	0.0417
					cut points $= (2, 7)$				
True	0.5000	0.9230	0.2248						
FI	0.5001	0.9226	0.2246	0.0369	0.0200	0.0377	0.0310	0.0409	0.0275
MSI	0.5000	0.9225	0.2247	0.0370	0.0204	0.0377	0.0310	0.0410	0.0276
IPW	0.5017	0.9230	0.2216	0.0625	0.0318	0.0630	0.0600	0.0300	0.0622
SPE	0.4999	0.9226	0.2250	0.0372	0.0217	0.0388	0.0369	0.0220	0.0385
1NN	0.5009	0.9121	0.2155	0.0388	0.0233	0.0395	0.0393	0.0274	0.0402
3NN	0.5006	0.9054	0.2106	0.0384	0.0220	0.0383	0.0390	0.0270	0.0395
					cut points $= (4, 5)$				
True	0.9347	0.2752	0.7752						
FI	0.9347	0.2745	0.7756	0.0193	0.0327	0.0383	0.0096	0.0262	0.0504
MSI	0.9347	0.2743	0.7754	0.0194	0.0332	0.0385	0.0097	0.0264	0.0506
IPW	0.9373	0.2742	0.7739	0.0518	0.0550	0.0453	0.0460	0.0542	0.0528
SPE	0.9348	0.2742	0.7751	0.0223	0.0355	0.0392	0.0226	0.0364	0.0384
1NN	0.9299	0.2685	0.7671	0.0236	0.0347	0.0404	0.0266	0.0384	0.0421
3NN	0.9261	0.2640	0.7620	0.0214	0.0326	0.0394	0.0263	0.0375	0.0417
					cut points $= (4, 7)$				
True	0.9347	0.4883	0.2248						
FI	0.9347	0.4866	0.2246	0.0193	0.0380	0.0377	0.0096	0.0341	0.0275
MSI	0.9347	0.4867	0.2247	0.0194	0.0383	0.0377	0.0097	0.0343	0.0276
IPW	0.9373	0.4860	0.2216	0.0518	0.0589	0.0630	0.0460	0.0581	0.0622
SPE	0.9348	0.4871	0.2250	0.0223	0.0401	0.0388	0.0226	0.0407	0.0385
1NN	0.9299	0.4720	0.2155	0.0236	0.0416	0.0395	0.0266	0.0441	0.0402
3NN	0.9261	0.4633	0.2106	0.0214	0.0399	0.0383	0.0263	0.0430	0.0395
					cut points $= (5, 7)$				
True	0.9883	0.2132	0.2248						
FI	0.9879	0.2121	0.2246	0.0080	0.0317	0.0377	0.0036	0.0216	0.0275
MSI	0.9880	0.2124	0.2247	0.0080	0.0319	0.0377	0.0037	0.0218	0.0276
IPW	0.9893	0.2117	0.2216	0.0280	0.0599	0.0630	0.0236	0.0575	0.0622
SPE	0.9882	0.2130	0.2250	0.0112	0.0338	0.0388	0.0115	0.0340	0.0385
1NN	0.9831	0.2035	0.2155	0.0120	0.0348	0.0395	0.0162	0.0369	0.0402
3NN	0.9802	0.1994	0.2106	0.0105	0.0322	0.0383	0.0162	0.0358	0.0395

Table 8 – *Monte Carlo means, Monte Carlo standard deviations and estimated standard deviations of KNN estimators of the true class fractions when both models for $\rho_k(t,a)$ and $\pi(t,a)$ are misspecified and sample size equals to 250. "True" denotes the true parameter value.*

	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
				cut points = $(-1.0, -0.5)$					
True	0.1812	0.1070	0.9817						
1NN	0.1787	0.0983	0.9726	0.0438	0.0551	0.0838	0.0421	0.0499	0.0364
3NN	0.1736	0.0891	0.9731	0.0421	0.0411	0.0639	0.0404	0.0438	0.0389
				cut points = $(-1.0, 0.7)$					
True	0.1812	0.8609	0.4469						
1NN	0.1787	0.8241	0.4056	0.0438	0.0976	0.2236	0.0421	0.1019	0.2072
3NN	0.1736	0.7917	0.3985	0.0421	0.0662	0.1573	0.0404	0.0910	0.2230
				cut points = $(-1.0, 1.3)$					
True	0.1812	0.9732	0.1171						
1NN	0.1787	0.9576	0.0993	0.0438	0.0347	0.0873	0.0421	0.0559	0.1278
3NN	0.1736	0.9488	0.0949	0.0421	0.0243	0.0610	0.0404	0.0513	0.1408
				cut points = $(-0.5, 0.7)$					
True	0.4796	0.7539	0.4469						
1NN	0.4747	0.7258	0.4056	0.0720	0.0980	0.2236	0.0636	0.1060	0.2072
3NN	0.4632	0.7026	0.3985	0.0692	0.0662	0.1573	0.0607	0.0949	0.2230
				cut points = $(-0.5, 1.3)$					
True	0.4796	0.8661	0.1171						
1NN	0.4747	0.8592	0.0993	0.0720	0.0627	0.0873	0.0636	0.0748	0.1278
3NN	0.4632	0.8597	0.0949	0.0692	0.0467	0.0610	0.0607	0.0673	0.1408
				cut points = $(0.7, 1.3)$					
True	0.9836	0.1122	0.1171						
1NN	0.9788	0.1334	0.0993	0.0292	0.0769	0.0873	0.0258	0.0907	0.1278
3NN	0.9709	0.1570	0.0949	0.0299	0.0533	0.0610	0.0257	0.0817	0.1408

Table 9 – *Monte Carlo means, Monte Carlo standard deviations and estimated standard deviations of KNN estimators of the true class fractions when both models for $\rho_k(t,a)$ and $\pi(t,a)$ are misspecified and sample size equals to 500. "True" denotes the true parameter value.*

	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
				cut points $= (-1.0, -0.5)$					
True	0.1812	0.1070	0.9817						
1NN	0.1797	0.1000	0.9802	0.0312	0.0398	0.0398	0.0297	0.0354	0.0253
3NN	0.1771	0.0935	0.9791	0.0301	0.0317	0.0327	0.0290	0.0322	0.0273
				cut points $= (-1.0, 0.7)$					
True	0.1812	0.8609	0.4469						
1NN	0.1797	0.8323	0.4278	0.0312	0.0788	0.1681	0.0297	0.0723	0.1433
3NN	0.1771	0.8084	0.4205	0.0301	0.0596	0.1269	0.0290	0.0671	0.1519
				cut points $= (-1.0, 1.3)$					
True	0.1812	0.9732	0.1171						
1NN	0.1797	0.9621	0.1086	0.0312	0.0270	0.0718	0.0297	0.0400	0.0908
3NN	0.1771	0.9546	0.1038	0.0301	0.0198	0.0486	0.0290	0.0378	0.0982
				cut points $= (-0.5, 0.7)$					
True	0.4796	0.7539	0.4469						
1NN	0.4765	0.7323	0.4278	0.0503	0.0772	0.1681	0.0444	0.0750	0.1433
3NN	0.4708	0.7149	0.4205	0.0481	0.0575	0.1269	0.0431	0.0696	0.1519
				cut points $= (-0.5, 1.3)$					
True	0.4796	0.8661	0.1171						
1NN	0.4765	0.8621	0.1086	0.0503	0.0449	0.0718	0.0444	0.0531	0.0908
3NN	0.4708	0.8611	0.1038	0.0481	0.0349	0.0486	0.0431	0.0494	0.0982
				cut points $= (0.7, 1.3)$					
True	0.9836	0.1122	0.1171						
1NN	0.9805	0.1298	0.1086	0.0215	0.0620	0.0718	0.0168	0.0645	0.0908
3NN	0.9770	0.1462	0.1038	0.0206	0.0468	0.0486	0.0167	0.0601	0.0982

Table 10 – *Monte Carlo means of KNN estimates of* $\rho_k(t,a)$*, at various points* (t,a)*.* T *and* A *are generated as described in Scenario I (Exponential distributions). Sample size is 50.* p *indicates* $\Pr(t-0.05 \leq T \leq t-0.05, a-0.05 \leq A \leq a+0.05)$.

(t,a)	p		$\rho_1(t,a)$	$\rho_2(t,a)$	$\rho_3(t,a)$
		True	0.101	0.503	0.396
		1NN	0.099	0.503	0.399
(0.05, 0.05)	4.611e-03	3NN	0.101	0.507	0.391
		6NN	0.103	0.518	0.379
		10NN	0.104	0.519	0.376
		True	0.107	0.533	0.361
		1NN	0.109	0.531	0.360
(0.5, 0.5)	2.218e-03	3NN	0.107	0.528	0.365
		6NN	0.105	0.527	0.367
		10NN	0.106	0.525	0.369
		True	0.082	0.529	0.389
		1NN	0.087	0.524	0.389
(1, 0.5)	1.741e-03	3NN	0.085	0.514	0.401
		6NN	0.086	0.520	0.394
		10NN	0.085	0.519	0.396
		True	0.026	0.581	0.393
		1NN	0.024	0.538	0.438
(5, 2)	4.785e-05	3NN	0.022	0.510	0.469
		6NN	0.022	0.482	0.497
		10NN	0.024	0.475	0.501

Table 11 – *Monte Carlo means of KNN estimates of* $\rho_k(t,a)$*, at various points* (t,a)*.* T *and* A *are generated as described in Scenario I (Exponential distributions). Sample size is 250.* p *indicates* $\Pr(t-0.05 \leq T \leq t-0.05, a-0.05 \leq A \leq a+0.05)$.

(t,a)	p		$\rho_1(t,a)$	$\rho_2(t,a)$	$\rho_3(t,a)$
		True	0.101	0.503	0.396
		1NN	0.108	0.503	0.389
(0.05, 0.05)	4.611e-03	3NN	0.105	0.512	0.383
		6NN	0.104	0.505	0.391
		10NN	0.102	0.510	0.387
		True	0.107	0.533	0.361
		1NN	0.104	0.517	0.379
(0.5, 0.5)	2.218e-03	3NN	0.108	0.528	0.364
		6NN	0.108	0.529	0.363
		10NN	0.107	0.529	0.364
		True	0.082	0.529	0.389
		1NN	0.078	0.539	0.383
(1, 0.5)	1.741e-03	3NN	0.080	0.531	0.389
		6NN	0.080	0.529	0.391
		10NN	0.081	0.527	0.392
		True	0.026	0.581	0.393
		1NN	0.023	0.572	0.405
(5, 2)	4.785e-05	3NN	0.024	0.560	0.416
		6NN	0.024	0.544	0.432
		10NN	0.023	0.526	0.451

Table 12 – *Monte Carlo means for KNN estimates for* $\rho_k(t,a)$ *at various points* (t,a)*.* T *and* A *are generated as described in Scenario II (Gamma distributions). Sample size is 50.* p *indicates* $\Pr(t-0.05 \le T \le t-0.05, a-0.05 \le A \le a+0.05)$.

(t,a)	p		$\rho_1(t,a)$	$\rho_2(t,a)$	$\rho_3(t,a)$
(0.05, 0.75)	1.124e-03	TRUE	0.005	0.260	0.736
		1NN	0.006	0.277	0.717
		3NN	0.007	0.304	0.689
		6NN	0.008	0.322	0.670
		10NN	0.010	0.343	0.648
(1, 1.5)	1.368e-03	TRUE	0.040	0.562	0.398
		1NN	0.034	0.556	0.411
		3NN	0.037	0.546	0.418
		6NN	0.035	0.534	0.432
		10NN	0.034	0.517	0.450
(0.75, 2.5)	1.308e-03	TRUE	0.047	0.584	0.369
		1NN	0.047	0.569	0.384
		3NN	0.045	0.564	0.391
		6NN	0.045	0.555	0.400
		10NN	0.044	0.545	0.411
(2.5, 2.5)	7.831e-05	TRUE	0.353	0.596	0.051
		1NN	0.264	0.626	0.110
		3NN	0.220	0.637	0.142
		6NN	0.175	0.638	0.187
		10NN	0.142	0.624	0.234

Table 13 – *Monte Carlo means of KNN estimates of* $\rho_k(t,a)$*, at various points* (t,a)*.* T *and* A *are generated as described in Scenario II (Gamma distributions). Sample size is 250.* p *indicates* $\Pr(t-0.05 \le T \le t-0.05, a-0.05 \le A \le a+0.05)$.

(t,a)	p		$\rho_1(t,a)$	$\rho_2(t,a)$	$\rho_3(t,a)$
(0.05, 0.75)	1.124e-03	True	0.005	0.260	0.736
		1NN	0.004	0.287	0.709
		3NN	0.004	0.285	0.711
		6NN	0.005	0.289	0.706
		10NN	0.005	0.294	0.700
(1, 1.5)	1.368e-03	True	0.040	0.562	0.398
		1NN	0.040	0.564	0.396
		3NN	0.038	0.560	0.403
		6NN	0.038	0.555	0.407
		10NN	0.037	0.551	0.412
(0.75, 2.5)	1.308e-03	True	0.047	0.584	0.369
		1NN	0.045	0.585	0.370
		3NN	0.046	0.578	0.376
		6NN	0.046	0.575	0.379
		10NN	0.045	0.568	0.386
(2.5, 2.5)	7.831e-05	True	0.353	0.596	0.051
		1NN	0.331	0.607	0.061
		3NN	0.304	0.622	0.074
		6NN	0.276	0.633	0.090
		10NN	0.248	0.641	0.111

Table 14 – *Monte Carlo means of KNN estimates of $\rho_k(t,a)$, at various points (t,a). T and A are generated as described in Scenario II (Gamma distributions). Sample size is 1000. p indicates* $\Pr(t-0.05 \leq T \leq t-0.05, a-0.05 \leq A \leq a+0.05)$.

(t,a)	p		$\rho_1(t,a)$	$\rho_2(t,a)$	$\rho_3(t,a)$
(0.05, 0.75)	1.124e-03	True	0.005	0.260	0.736
		1NN	0.006	0.262	0.733
		3NN	0.005	0.266	0.729
		6NN	0.005	0.273	0.723
		10NN	0.005	0.276	0.718
(1, 1.5)	1.368e-03	True	0.040	0.562	0.398
		1NN	0.040	0.556	0.404
		3NN	0.040	0.561	0.399
		6NN	0.039	0.562	0.399
		10NN	0.039	0.559	0.402
(0.75, 2.5)	1.308e-03	True	0.047	0.584	0.369
		1NN	0.047	0.591	0.362
		3NN	0.045	0.581	0.374
		6NN	0.046	0.579	0.375
		10NN	0.046	0.580	0.374
(2.5, 2.5)	7.831e-05	True	0.353	0.596	0.051
		1NN	0.343	0.605	0.052
		3NN	0.338	0.605	0.057
		6NN	0.327	0.610	0.063
		10NN	0.314	0.616	0.069

S6 Preliminary simulation study: choice of K and the distance measure

In this study, we compare the behavior of the KNN estimators for several choices of the distance measure (Euclidean, Manhattan, Canberra and Mahalanobis) and the size of the neighborhood ($K = 1, 3, 5, 10, 20$). Data are generated as in Section 5.1 of the main paper.

Table 15 – *Monte Carlo means, Monte Carlo standard deviations and estimated standard deviations of the estimators for true class fractions corresponding to the case of the first value of* Σ *and sample size* = 250. *"True" denotes the true parameter value.*

	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
				cut points = $(2, 4)$					
True	0.5000	0.4347	0.9347						
				Euclidean					
1NN	0.4989	0.4334	0.9331	0.0592	0.0665	0.0387	0.0555	0.0626	0.0382
3NN	0.4975	0.4325	0.9322	0.0567	0.0617	0.0364	0.0545	0.0608	0.0372
5NN	0.4965	0.4320	0.9315	0.0559	0.0600	0.0360	0.0543	0.0604	0.0372
10NN	0.4943	0.4306	0.9297	0.0551	0.0580	0.0357	0.0542	0.0600	0.0376
20NN	0.4902	0.4278	0.9258	0.0542	0.0557	0.0358	0.0541	0.0595	0.0384
				Manhattan					
1NN	0.4993	0.4337	0.9332	0.0595	0.0667	0.0387	0.0555	0.0626	0.0382
3NN	0.4981	0.4329	0.9323	0.0569	0.0619	0.0366	0.0546	0.0608	0.0372
5NN	0.4972	0.4325	0.9315	0.0560	0.0603	0.0361	0.0544	0.0604	0.0372
10NN	0.4952	0.4312	0.9297	0.0553	0.0583	0.0358	0.0542	0.0600	0.0375
20NN	0.4912	0.4287	0.9260	0.0545	0.0562	0.0359	0.0542	0.0595	0.0384
				Canberra					
1NN	0.4858	0.4336	0.9279	0.0585	0.0636	0.0399	0.0560	0.0632	0.0421
3NN	0.4797	0.4325	0.9240	0.0550	0.0583	0.0384	0.0548	0.0614	0.0414
5NN	0.4773	0.4316	0.9213	0.0538	0.0568	0.0382	0.0546	0.0609	0.0416
10NN	0.4812	0.4324	0.9166	0.0542	0.0554	0.0376	0.0551	0.0602	0.0420
20NN	0.4882	0.4343	0.9092	0.0550	0.0542	0.0375	0.0560	0.0593	0.0428
				Mahalanobis					
1NN	0.4997	0.4343	0.9325	0.0596	0.0665	0.0388	0.0559	0.0628	0.0391
3NN	0.4986	0.4341	0.9308	0.0570	0.0614	0.0367	0.0550	0.0610	0.0381
5NN	0.4978	0.4336	0.9296	0.0562	0.0594	0.0361	0.0548	0.0606	0.0382
10NN	0.4955	0.4325	0.9262	0.0553	0.0572	0.0359	0.0548	0.0601	0.0388
20NN	0.4910	0.4299	0.9196	0.0542	0.0546	0.0360	0.0548	0.0595	0.0401
				cut points = $(2, 5)$					
True	0.5000	0.7099	0.7752						
				Euclidean					
1NN	0.4989	0.7068	0.7738	0.0592	0.0627	0.0652	0.0555	0.0591	0.0625
3NN	0.4975	0.7038	0.7714	0.0567	0.0576	0.0615	0.0545	0.0574	0.0610
5NN	0.4965	0.7016	0.7698	0.0559	0.0558	0.0607	0.0543	0.0571	0.0609
10NN	0.4943	0.6967	0.7662	0.0551	0.0535	0.0599	0.0542	0.0568	0.0612
20NN	0.4902	0.6881	0.7595	0.0542	0.0535	0.0594	0.0541	0.0567	0.0612
				Manhattan					
1NN	0.4993	0.7071	0.7737	0.0595	0.0631	0.0653	0.0555	0.0591	0.0625
3NN	0.4981	0.7044	0.7714	0.0569	0.0578	0.0618	0.0546	0.0573	0.0610
5NN	0.4972	0.7024	0.7697	0.0560	0.0561	0.0609	0.0544	0.0570	0.0609
10NN	0.4952	0.6976	0.7660	0.0553	0.0539	0.0601	0.0542	0.0567	0.0612

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	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
20NN	0.4912	0.6894	0.7591	0.0545	0.0539	0.0596	0.0542	0.0566	0.0612
				Canberra					
1NN	0.4858	0.6987	0.7649	0.0585	0.0591	0.0648	0.0560	0.0605	0.0648
3NN	0.4797	0.6930	0.7592	0.0550	0.0537	0.0616	0.0548	0.0587	0.0635
5NN	0.4773	0.6891	0.7550	0.0538	0.0522	0.0611	0.0546	0.0583	0.0634
10NN	0.4812	0.6861	0.7470	0.0542	0.0503	0.0602	0.0551	0.0577	0.0633
20NN	0.4882	0.6834	0.7352	0.0550	0.0503	0.0593	0.0560	0.0568	0.0633
				Mahalanobis					
1NN	0.4997	0.7068	0.7724	0.0596	0.0628	0.0651	0.0559	0.0595	0.0632
3NN	0.4986	0.7035	0.7686	0.0570	0.0575	0.0616	0.0550	0.0577	0.0617
5NN	0.4978	0.7007	0.7658	0.0562	0.0554	0.0605	0.0548	0.0573	0.0617
10NN	0.4955	0.6948	0.7598	0.0553	0.0529	0.0597	0.0548	0.0570	0.0621
20NN	0.4910	0.6846	0.7490	0.0542	0.0529	0.0589	0.0548	0.0569	0.0621
				cut points = $(2, 7)$					
True	0.5000	0.9230	0.2248						
				Euclidean					
1NN	0.4989	0.9201	0.2233	0.0592	0.0372	0.0577	0.0555	0.0366	0.0570
3NN	0.4975	0.9177	0.2216	0.0567	0.0340	0.0558	0.0545	0.0355	0.0563
5NN	0.4965	0.9157	0.2205	0.0559	0.0330	0.0550	0.0543	0.0355	0.0561
10NN	0.4943	0.9112	0.2184	0.0551	0.0318	0.0542	0.0542	0.0358	0.0560
20NN	0.4902	0.9031	0.2145	0.0542	0.0318	0.0531	0.0541	0.0366	0.0559
				Manhattan					
1NN	0.4993	0.9201	0.2230	0.0595	0.0376	0.0579	0.0555	0.0366	0.0571
3NN	0.4981	0.9176	0.2210	0.0569	0.0341	0.0557	0.0546	0.0356	0.0563
5NN	0.4972	0.9156	0.2196	0.0560	0.0330	0.0550	0.0544	0.0356	0.0561
10NN	0.4952	0.9111	0.2171	0.0553	0.0319	0.0542	0.0542	0.0358	0.0559
20NN	0.4912	0.9030	0.2125	0.0545	0.0319	0.0532	0.0542	0.0366	0.0558
				Canberra					
1NN	0.4858	0.9101	0.2173	0.0585	0.0374	0.0570	0.0560	0.0407	0.0575
3NN	0.4797	0.9032	0.2142	0.0550	0.0336	0.0548	0.0548	0.0398	0.0566
5NN	0.4773	0.8988	0.2122	0.0538	0.0327	0.0539	0.0546	0.0399	0.0563
10NN	0.4812	0.8939	0.2076	0.0542	0.0320	0.0525	0.0551	0.0399	0.0555
20NN	0.4882	0.8882	0.2011	0.0550	0.0320	0.0509	0.0560	0.0399	0.0544
				Mahalanobis					
1NN	0.4997	0.9184	0.2216	0.0596	0.0375	0.0577	0.0559	0.0375	0.0572
3NN	0.4986	0.9151	0.2191	0.0570	0.0340	0.0554	0.0550	0.0365	0.0563
5NN	0.4978	0.9122	0.2172	0.0562	0.0329	0.0545	0.0548	0.0365	0.0561
10NN	0.4955	0.9060	0.2138	0.0553	0.0316	0.0534	0.0548	0.0370	0.0558
20NN	0.4910	0.8957	0.2081	0.0542	0.0316	0.0520	0.0548	0.0378	0.0555
				cut points = $(4, 5)$					
True	0.9347	0.2752	0.7752						
				Euclidean					
1NN	0.9322	0.2734	0.7738	0.0374	0.0572	0.0652	0.0342	0.0553	0.0625
3NN	0.9303	0.2712	0.7714	0.0328	0.0526	0.0615	0.0332	0.0538	0.0610
5NN	0.9288	0.2696	0.7698	0.0315	0.0512	0.0607	0.0332	0.0534	0.0609
10NN	0.9255	0.2662	0.7662	0.0301	0.0489	0.0599	0.0335	0.0529	0.0612
20NN	0.9196	0.2603	0.7595	0.0291	0.0467	0.0594	0.0342	0.0522	0.0612
				Manhattan					
1NN	0.9323	0.2734	0.7737	0.0372	0.0575	0.0653	0.0341	0.0553	0.0625
3NN	0.9308	0.2715	0.7714	0.0328	0.0529	0.0618	0.0331	0.0538	0.0610
5NN	0.9293	0.2700	0.7697	0.0315	0.0514	0.0609	0.0331	0.0534	0.0609
10NN	0.9260	0.2665	0.7660	0.0303	0.0494	0.0601	0.0334	0.0528	0.0612
20NN	0.9199	0.2606	0.7591	0.0292	0.0472	0.0596	0.0341	0.0522	0.0612
				Canberra					

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	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
1NN	0.9101	0.2651	0.7649	0.0380	0.0542	0.0648	0.0388	0.0550	0.0648
3NN	0.9023	0.2605	0.7592	0.0331	0.0498	0.0616	0.0381	0.0533	0.0635
5NN	0.8998	0.2576	0.7550	0.0319	0.0484	0.0611	0.0382	0.0528	0.0634
10NN	0.9025	0.2537	0.7470	0.0305	0.0469	0.0602	0.0382	0.0520	0.0633
20NN	0.9069	0.2491	0.7352	0.0292	0.0453	0.0593	0.0385	0.0509	0.0633
					Mahalanobis				
1NN	0.9322	0.2725	0.7724	0.0373	0.0569	0.0651	0.0345	0.0554	0.0632
3NN	0.9299	0.2695	0.7686	0.0330	0.0521	0.0616	0.0336	0.0538	0.0617
5NN	0.9282	0.2670	0.7658	0.0318	0.0504	0.0605	0.0337	0.0533	0.0617
10NN	0.9240	0.2622	0.7598	0.0304	0.0480	0.0597	0.0341	0.0526	0.0621
20NN	0.9165	0.2547	0.7490	0.0295	0.0455	0.0589	0.0351	0.0518	0.0621
					cut points = $(4, 7)$				
True	0.9347	0.4883	0.2248						
					Euclidean				
1NN	0.9322	0.4867	0.2233	0.0374	0.0680	0.0577	0.0342	0.0633	0.0570
3NN	0.9303	0.4852	0.2216	0.0328	0.0630	0.0558	0.0332	0.0615	0.0563
5NN	0.9288	0.4837	0.2205	0.0315	0.0615	0.0550	0.0332	0.0611	0.0561
10NN	0.9255	0.4807	0.2184	0.0301	0.0597	0.0542	0.0335	0.0606	0.0560
20NN	0.9196	0.4753	0.2145	0.0291	0.0577	0.0531	0.0342	0.0602	0.0559
					Manhattan				
1NN	0.9323	0.4864	0.2230	0.0372	0.0679	0.0579	0.0341	0.0632	0.0571
3NN	0.9308	0.4848	0.2210	0.0328	0.0630	0.0557	0.0331	0.0614	0.0563
5NN	0.9293	0.4831	0.2196	0.0315	0.0616	0.0550	0.0331	0.0610	0.0561
10NN	0.9260	0.4799	0.2171	0.0303	0.0599	0.0542	0.0334	0.0606	0.0559
20NN	0.9199	0.4743	0.2125	0.0292	0.0579	0.0532	0.0341	0.0601	0.0558
					Canberra				
1NN	0.9101	0.4764	0.2173	0.0380	0.0651	0.0570	0.0388	0.0638	0.0575
3NN	0.9023	0.4707	0.2142	0.0331	0.0602	0.0548	0.0381	0.0619	0.0566
5NN	0.8998	0.4673	0.2122	0.0319	0.0586	0.0539	0.0382	0.0614	0.0563
10NN	0.9025	0.4615	0.2076	0.0305	0.0571	0.0525	0.0382	0.0606	0.0555
20NN	0.9069	0.4539	0.2011	0.0292	0.0555	0.0509	0.0385	0.0596	0.0544
					Mahalanobis				
1NN	0.9322	0.4841	0.2216	0.0373	0.0678	0.0577	0.0345	0.0635	0.0572
3NN	0.9299	0.4811	0.2191	0.0330	0.0628	0.0554	0.0336	0.0616	0.0563
5NN	0.9282	0.4785	0.2172	0.0318	0.0609	0.0545	0.0337	0.0612	0.0561
10NN	0.9240	0.4735	0.2138	0.0304	0.0590	0.0534	0.0341	0.0606	0.0558
20NN	0.9165	0.4658	0.2081	0.0295	0.0566	0.0520	0.0351	0.0601	0.0555
					cut points = $(5, 7)$				
True	0.9883	0.2132	0.2248						
					Euclidean				
1NN	0.9868	0.2133	0.2233	0.0177	0.0567	0.0577	0.0172	0.0532	0.0570
3NN	0.9860	0.2139	0.2216	0.0151	0.0519	0.0558	0.0168	0.0516	0.0563
5NN	0.9851	0.2141	0.2205	0.0142	0.0502	0.0550	0.0170	0.0512	0.0561
10NN	0.9833	0.2145	0.2184	0.0135	0.0479	0.0542	0.0174	0.0508	0.0560
20NN	0.9800	0.2150	0.2145	0.0131	0.0453	0.0531	0.0183	0.0505	0.0559
					Manhattan				
1NN	0.9869	0.2130	0.2230	0.0177	0.0568	0.0579	0.0172	0.0531	0.0571
3NN	0.9860	0.2133	0.2210	0.0151	0.0519	0.0557	0.0168	0.0515	0.0563
5NN	0.9851	0.2131	0.2196	0.0142	0.0502	0.0550	0.0170	0.0511	0.0561
10NN	0.9832	0.2134	0.2171	0.0136	0.0480	0.0542	0.0175	0.0507	0.0559
20NN	0.9795	0.2136	0.2125	0.0132	0.0453	0.0532	0.0185	0.0504	0.0558
					Canberra				
1NN	0.9717	0.2113	0.2173	0.0212	0.0525	0.0570	0.0247	0.0531	0.0575
3NN	0.9670	0.2103	0.2142	0.0179	0.0479	0.0548	0.0246	0.0513	0.0566

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	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
5NN	0.9655	0.2097	0.2122	0.0171	0.0464	0.0539	0.0248	0.0509	0.0563
10NN	0.9668	0.2078	0.2076	0.0155	0.0445	0.0525	0.0248	0.0501	0.0555
20NN	0.9680	0.2047	0.2011	0.0141	0.0426	0.0509	0.0250	0.0491	0.0544
					Mahalanobis				
1NN	0.9865	0.2116	0.2216	0.0180	0.0563	0.0577	0.0178	0.0532	0.0572
3NN	0.9853	0.2116	0.2191	0.0155	0.0515	0.0554	0.0175	0.0515	0.0563
5NN	0.9842	0.2115	0.2172	0.0147	0.0495	0.0545	0.0178	0.0511	0.0561
10NN	0.9817	0.2112	0.2138	0.0140	0.0470	0.0534	0.0184	0.0506	0.0558
20NN	0.9770	0.2111	0.2081	0.0138	0.0442	0.0520	0.0197	0.0502	0.0555

Table 16 – *Monte Carlo means, Monte Carlo standard deviations and estimated standard deviations of the estimators for true class fractions corresponding to the case of the second value of Σ and sample size $= 250$. "True" denotes the true parameter value.*

	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
					cut points $= (2, 4)$				
True	0.5000	0.3970	0.8970						
					Euclidean				
1NN	0.4982	0.3953	0.8976	0.0587	0.0642	0.0537	0.0561	0.0618	0.0487
3NN	0.4960	0.3933	0.8970	0.0556	0.0595	0.0494	0.0548	0.0600	0.0472
5NN	0.4941	0.3917	0.8966	0.0548	0.0578	0.0484	0.0545	0.0596	0.0471
10NN	0.4904	0.3885	0.8955	0.0540	0.0561	0.0473	0.0542	0.0591	0.0473
20NN	0.4841	0.3834	0.8926	0.0533	0.0539	0.0467	0.0540	0.0587	0.0479
					Manhattan				
1NN	0.4984	0.3954	0.8976	0.0587	0.0643	0.0536	0.0560	0.0618	0.0486
3NN	0.4962	0.3934	0.8970	0.0556	0.0596	0.0494	0.0548	0.0599	0.0471
5NN	0.4943	0.3918	0.8967	0.0549	0.0580	0.0484	0.0545	0.0595	0.0470
10NN	0.4907	0.3887	0.8955	0.0540	0.0563	0.0474	0.0542	0.0591	0.0472
20NN	0.4844	0.3837	0.8927	0.0533	0.0542	0.0469	0.0540	0.0587	0.0479
					Canberra				
1NN	0.4894	0.3901	0.8943	0.0581	0.0611	0.0536	0.0566	0.0620	0.0508
3NN	0.4828	0.3859	0.8912	0.0544	0.0560	0.0504	0.0553	0.0599	0.0494
5NN	0.4793	0.3834	0.8889	0.0534	0.0545	0.0494	0.0550	0.0594	0.0494
10NN	0.4768	0.3811	0.8853	0.0530	0.0532	0.0482	0.0549	0.0590	0.0498
20NN	0.4737	0.3795	0.8770	0.0533	0.0521	0.0469	0.0549	0.0585	0.0508
					Mahalanobis				
1NN	0.4979	0.3950	0.8943	0.0587	0.0635	0.0543	0.0566	0.0620	0.0505
3NN	0.4954	0.3928	0.8921	0.0557	0.0581	0.0497	0.0554	0.0600	0.0490
5NN	0.4936	0.3912	0.8898	0.0550	0.0565	0.0487	0.0552	0.0595	0.0490
10NN	0.4895	0.3879	0.8852	0.0541	0.0540	0.0474	0.0551	0.0590	0.0495
20NN	0.4828	0.3829	0.8768	0.0530	0.0511	0.0468	0.0551	0.0585	0.0505
					cut points $= (2, 5)$				
True	0.5000	0.6335	0.7365						
					Euclidean				
1NN	0.4982	0.6304	0.7400	0.0587	0.0645	0.0721	0.0561	0.0615	0.0672
3NN	0.4960	0.6283	0.7396	0.0556	0.0600	0.0670	0.0548	0.0597	0.0654
5NN	0.4941	0.6260	0.7392	0.0548	0.0587	0.0661	0.0545	0.0594	0.0652
10NN	0.4904	0.6212	0.7378	0.0540	0.0569	0.0649	0.0542	0.0591	0.0653
20NN	0.4841	0.6133	0.7345	0.0533	0.0569	0.0641	0.0540	0.0590	0.0653

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	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
					Manhattan				
1NN	0.4984	0.6308	0.7400	0.0587	0.0648	0.0720	0.0560	0.0615	0.0672
3NN	0.4962	0.6284	0.7394	0.0556	0.0601	0.0672	0.0548	0.0597	0.0653
5NN	0.4943	0.6262	0.7392	0.0549	0.0587	0.0661	0.0545	0.0593	0.0651
10NN	0.4907	0.6216	0.7378	0.0540	0.0571	0.0650	0.0542	0.0591	0.0653
20NN	0.4844	0.6137	0.7345	0.0533	0.0571	0.0643	0.0540	0.0589	0.0653
					Canberra				
1NN	0.4894	0.6226	0.7386	0.0581	0.0630	0.0715	0.0566	0.0623	0.0682
3NN	0.4828	0.6164	0.7357	0.0544	0.0573	0.0671	0.0553	0.0604	0.0663
5NN	0.4793	0.6123	0.7331	0.0534	0.0557	0.0660	0.0550	0.0600	0.0661
10NN	0.4768	0.6083	0.7283	0.0530	0.0542	0.0647	0.0549	0.0597	0.0662
20NN	0.4737	0.6037	0.7188	0.0533	0.0542	0.0632	0.0549	0.0593	0.0662
					Mahalanobis				
1NN	0.4979	0.6286	0.7358	0.0587	0.0639	0.0723	0.0566	0.0619	0.0682
3NN	0.4954	0.6249	0.7330	0.0557	0.0590	0.0670	0.0554	0.0600	0.0662
5NN	0.4936	0.6220	0.7305	0.0550	0.0574	0.0658	0.0552	0.0596	0.0660
10NN	0.4895	0.6159	0.7252	0.0541	0.0551	0.0643	0.0551	0.0592	0.0662
20NN	0.4828	0.6065	0.7158	0.0530	0.0551	0.0631	0.0551	0.0590	0.0662
					cut points = $(2, 7)$				
True	0.5000	0.8682	0.2635						
					Euclidean				
1NN	0.4982	0.8672	0.2672	0.0587	0.0495	0.0629	0.0561	0.0458	0.0609
3NN	0.4960	0.8657	0.2671	0.0556	0.0452	0.0610	0.0548	0.0442	0.0601
5NN	0.4941	0.8642	0.2671	0.0548	0.0442	0.0605	0.0545	0.0440	0.0601
10NN	0.4904	0.8608	0.2667	0.0540	0.0430	0.0602	0.0542	0.0441	0.0602
20NN	0.4841	0.8539	0.2655	0.0533	0.0430	0.0598	0.0540	0.0444	0.0604
					Manhattan				
1NN	0.4984	0.8674	0.2672	0.0587	0.0495	0.0628	0.0560	0.0457	0.0609
3NN	0.4962	0.8658	0.2670	0.0556	0.0453	0.0610	0.0548	0.0442	0.0601
5NN	0.4943	0.8642	0.2669	0.0549	0.0443	0.0606	0.0545	0.0440	0.0601
10NN	0.4907	0.8609	0.2664	0.0540	0.0430	0.0602	0.0542	0.0440	0.0602
20NN	0.4844	0.8541	0.2650	0.0533	0.0430	0.0598	0.0540	0.0444	0.0604
					Canberra				
1NN	0.4894	0.8610	0.2684	0.0581	0.0493	0.0629	0.0566	0.0480	0.0617
3NN	0.4828	0.8546	0.2673	0.0544	0.0444	0.0610	0.0553	0.0466	0.0607
5NN	0.4793	0.8507	0.2663	0.0534	0.0429	0.0605	0.0550	0.0465	0.0605
10NN	0.4768	0.8468	0.2641	0.0530	0.0414	0.0597	0.0549	0.0465	0.0603
20NN	0.4737	0.8416	0.2595	0.0533	0.0414	0.0588	0.0549	0.0466	0.0600
					Mahalanobis				
1NN	0.4979	0.8634	0.2651	0.0587	0.0493	0.0622	0.0566	0.0469	0.0609
3NN	0.4954	0.8598	0.2635	0.0557	0.0447	0.0602	0.0554	0.0454	0.0599
5NN	0.4936	0.8571	0.2625	0.0550	0.0435	0.0597	0.0552	0.0453	0.0596
10NN	0.4895	0.8510	0.2603	0.0541	0.0416	0.0587	0.0551	0.0454	0.0595
20NN	0.4828	0.8416	0.2563	0.0530	0.0416	0.0577	0.0551	0.0458	0.0593
					cut points = $(4, 5)$				
True	0.8970	0.2365	0.7365						
					Euclidean				
1NN	0.8958	0.2352	0.7400	0.0388	0.0540	0.0721	0.0373	0.0524	0.0672
3NN	0.8946	0.2350	0.7396	0.0362	0.0502	0.0670	0.0361	0.0510	0.0654
5NN	0.8933	0.2343	0.7392	0.0355	0.0490	0.0661	0.0360	0.0507	0.0652
10NN	0.8905	0.2328	0.7378	0.0348	0.0474	0.0649	0.0360	0.0503	0.0653
20NN	0.8857	0.2299	0.7345	0.0343	0.0455	0.0641	0.0364	0.0499	0.0653
					Manhattan				
1NN	0.8960	0.2354	0.7400	0.0387	0.0541	0.0720	0.0373	0.0524	0.0672

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	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
3NN	0.8946	0.2350	0.7394	0.0361	0.0503	0.0672	0.0361	0.0510	0.0653
5NN	0.8933	0.2344	0.7392	0.0356	0.0491	0.0661	0.0360	0.0507	0.0651
10NN	0.8906	0.2329	0.7378	0.0348	0.0476	0.0650	0.0360	0.0503	0.0653
20NN	0.8857	0.2300	0.7345	0.0343	0.0458	0.0643	0.0364	0.0498	0.0653
				Canberra					
1NN	0.8881	0.2325	0.7386	0.0384	0.0518	0.0715	0.0390	0.0522	0.0682
3NN	0.8843	0.2305	0.7357	0.0359	0.0481	0.0671	0.0380	0.0507	0.0663
5NN	0.8821	0.2289	0.7331	0.0353	0.0469	0.0660	0.0380	0.0502	0.0661
10NN	0.8800	0.2271	0.7283	0.0346	0.0455	0.0647	0.0380	0.0497	0.0662
20NN	0.8761	0.2242	0.7188	0.0345	0.0439	0.0632	0.0385	0.0492	0.0662
				Mahalanobis					
1NN	0.8955	0.2335	0.7358	0.0387	0.0531	0.0723	0.0376	0.0523	0.0682
3NN	0.8939	0.2322	0.7330	0.0361	0.0490	0.0670	0.0365	0.0508	0.0662
5NN	0.8922	0.2308	0.7305	0.0356	0.0476	0.0658	0.0365	0.0504	0.0660
10NN	0.8887	0.2279	0.7252	0.0349	0.0456	0.0643	0.0368	0.0498	0.0662
20NN	0.8824	0.2236	0.7158	0.0344	0.0434	0.0631	0.0374	0.0492	0.0662
				cut points = $(4, 7)$					
True	0.8970	0.4711	0.2635						
				Euclidean					
1NN	0.8958	0.4719	0.2672	0.0388	0.0666	0.0629	0.0373	0.0630	0.0609
3NN	0.8946	0.4724	0.2671	0.0362	0.0627	0.0610	0.0361	0.0611	0.0601
5NN	0.8933	0.4725	0.2671	0.0355	0.0612	0.0605	0.0360	0.0607	0.0601
10NN	0.8905	0.4723	0.2667	0.0348	0.0600	0.0602	0.0360	0.0604	0.0602
20NN	0.8857	0.4705	0.2655	0.0343	0.0584	0.0598	0.0364	0.0600	0.0604
				Manhattan					
1NN	0.8960	0.4720	0.2672	0.0387	0.0667	0.0628	0.0373	0.0629	0.0609
3NN	0.8946	0.4724	0.2670	0.0361	0.0627	0.0610	0.0361	0.0611	0.0601
5NN	0.8933	0.4724	0.2669	0.0356	0.0614	0.0606	0.0360	0.0607	0.0601
10NN	0.8906	0.4722	0.2664	0.0348	0.0599	0.0602	0.0360	0.0604	0.0602
20NN	0.8857	0.4705	0.2650	0.0343	0.0584	0.0598	0.0364	0.0600	0.0604
				Canberra					
1NN	0.8881	0.4708	0.2684	0.0384	0.0651	0.0629	0.0390	0.0632	0.0617
3NN	0.8843	0.4687	0.2673	0.0359	0.0611	0.0610	0.0380	0.0613	0.0607
5NN	0.8821	0.4673	0.2663	0.0353	0.0599	0.0605	0.0380	0.0608	0.0605
10NN	0.8800	0.4657	0.2641	0.0346	0.0581	0.0597	0.0380	0.0603	0.0603
20NN	0.8761	0.4620	0.2595	0.0345	0.0561	0.0588	0.0385	0.0599	0.0600
				Mahalanobis					
1NN	0.8955	0.4684	0.2651	0.0387	0.0664	0.0622	0.0376	0.0631	0.0609
3NN	0.8939	0.4670	0.2635	0.0361	0.0618	0.0602	0.0365	0.0611	0.0599
5NN	0.8922	0.4659	0.2625	0.0356	0.0604	0.0597	0.0365	0.0607	0.0596
10NN	0.8887	0.4631	0.2603	0.0349	0.0584	0.0587	0.0368	0.0601	0.0595
20NN	0.8824	0.4587	0.2563	0.0344	0.0563	0.0577	0.0374	0.0597	0.0593
				cut points = $(5, 7)$					
True	0.9711	0.2347	0.2635						
				Euclidean					
1NN	0.9701	0.2368	0.2672	0.0217	0.0549	0.0629	0.0213	0.0533	0.0609
3NN	0.9695	0.2375	0.2671	0.0200	0.0519	0.0610	0.0206	0.0517	0.0601
5NN	0.9689	0.2382	0.2671	0.0197	0.0507	0.0605	0.0205	0.0515	0.0601
10NN	0.9675	0.2395	0.2667	0.0194	0.0492	0.0602	0.0207	0.0512	0.0602
20NN	0.9648	0.2406	0.2655	0.0192	0.0478	0.0598	0.0212	0.0510	0.0604
				Manhattan					
1NN	0.9701	0.2366	0.2672	0.0217	0.0551	0.0628	0.0213	0.0532	0.0609
3NN	0.9695	0.2374	0.2670	0.0200	0.0519	0.0610	0.0206	0.0517	0.0601
5NN	0.9688	0.2380	0.2669	0.0197	0.0506	0.0606	0.0206	0.0514	0.0601

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	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
10NN	0.9674	0.2393	0.2664	0.0194	0.0493	0.0602	0.0207	0.0512	0.0602
20NN	0.9646	0.2405	0.2650	0.0192	0.0479	0.0598	0.0213	0.0510	0.0604
					Canberra				
1NN	0.9664	0.2384	0.2684	0.0220	0.0538	0.0629	0.0232	0.0534	0.0617
3NN	0.9645	0.2382	0.2673	0.0203	0.0502	0.0610	0.0227	0.0518	0.0607
5NN	0.9632	0.2384	0.2663	0.0199	0.0491	0.0605	0.0228	0.0514	0.0605
10NN	0.9614	0.2385	0.2641	0.0195	0.0475	0.0597	0.0230	0.0510	0.0603
20NN	0.9585	0.2379	0.2595	0.0195	0.0461	0.0588	0.0237	0.0506	0.0600
					Mahalanobis				
1NN	0.9699	0.2349	0.2651	0.0217	0.0547	0.0622	0.0217	0.0531	0.0609
3NN	0.9689	0.2349	0.2635	0.0200	0.0508	0.0602	0.0211	0.0515	0.0599
5NN	0.9680	0.2350	0.2625	0.0198	0.0496	0.0597	0.0211	0.0512	0.0596
10NN	0.9661	0.2352	0.2603	0.0196	0.0478	0.0587	0.0215	0.0508	0.0595
20NN	0.9622	0.2351	0.2563	0.0194	0.0460	0.0577	0.0223	0.0504	0.0593

Table 17 – *Monte Carlo means, Monte Carlo standard deviations and estimated standard deviations of the estimators for true class fractions corresponding to the case of the third value of* Σ *and sample size* = 250. *"True" denotes the true parameter value.*

	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
					cut points = $(2, 4)$				
True	0.5000	0.3031	0.8031						
					Euclidean				
1NN	0.4997	0.3021	0.8047	0.0592	0.0602	0.0682	0.0571	0.0584	0.0621
3NN	0.4984	0.3018	0.8043	0.0561	0.0565	0.0632	0.0556	0.0566	0.0601
5NN	0.4975	0.3016	0.8039	0.0554	0.0552	0.0621	0.0553	0.0562	0.0598
10NN	0.4956	0.3006	0.8029	0.0549	0.0539	0.0611	0.0550	0.0558	0.0597
20NN	0.4915	0.2986	0.8005	0.0543	0.0523	0.0605	0.0549	0.0555	0.0600
					Manhattan				
1NN	0.4997	0.3023	0.8049	0.0594	0.0602	0.0681	0.0571	0.0584	0.0621
3NN	0.4984	0.3019	0.8041	0.0562	0.0565	0.0634	0.0556	0.0566	0.0601
5NN	0.4977	0.3016	0.8039	0.0554	0.0553	0.0620	0.0553	0.0562	0.0598
10NN	0.4956	0.3006	0.8030	0.0548	0.0540	0.0611	0.0550	0.0558	0.0597
20NN	0.4913	0.2985	0.8004	0.0543	0.0523	0.0606	0.0549	0.0555	0.0600
					Canberra				
1NN	0.4963	0.3011	0.8039	0.0585	0.0588	0.0680	0.0571	0.0584	0.0627
3NN	0.4940	0.3001	0.8018	0.0553	0.0544	0.0633	0.0557	0.0565	0.0607
5NN	0.4932	0.2998	0.8008	0.0546	0.0533	0.0618	0.0554	0.0561	0.0604
10NN	0.4911	0.2990	0.7980	0.0542	0.0520	0.0606	0.0552	0.0557	0.0604
20NN	0.4863	0.2973	0.7910	0.0543	0.0505	0.0597	0.0553	0.0553	0.0607
					Mahalanobis				
1NN	0.4982	0.3014	0.8017	0.0591	0.0592	0.0675	0.0574	0.0585	0.0631
3NN	0.4962	0.3010	0.7993	0.0561	0.0546	0.0630	0.0560	0.0565	0.0610
5NN	0.4947	0.3003	0.7971	0.0552	0.0530	0.0617	0.0558	0.0561	0.0607
10NN	0.4912	0.2987	0.7930	0.0542	0.0508	0.0602	0.0556	0.0556	0.0608
20NN	0.4855	0.2963	0.7853	0.0531	0.0482	0.0591	0.0557	0.0552	0.0612
					cut points = $(2, 5)$				
True	0.5000	0.4682	0.6651						
					Euclidean				
1NN	0.4997	0.4676	0.6668	0.0592	0.0661	0.0780	0.0571	0.0634	0.0717

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	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
3NN	0.4984	0.4670	0.6666	0.0561	0.0619	0.0729	0.0556	0.0614	0.0695
5NN	0.4975	0.4665	0.6664	0.0554	0.0606	0.0717	0.0553	0.0610	0.0692
10NN	0.4956	0.4652	0.6654	0.0549	0.0593	0.0706	0.0550	0.0606	0.0691
20NN	0.4915	0.4623	0.6630	0.0543	0.0593	0.0698	0.0549	0.0604	0.0691
				Manhattan					
1NN	0.4997	0.4678	0.6671	0.0594	0.0663	0.0779	0.0571	0.0634	0.0717
3NN	0.4984	0.4671	0.6665	0.0562	0.0618	0.0730	0.0556	0.0614	0.0695
5NN	0.4977	0.4667	0.6664	0.0554	0.0606	0.0716	0.0553	0.0610	0.0692
10NN	0.4956	0.4652	0.6654	0.0548	0.0593	0.0706	0.0550	0.0606	0.0691
20NN	0.4913	0.4623	0.6629	0.0543	0.0593	0.0699	0.0549	0.0604	0.0691
				Canberra					
1NN	0.4963	0.4659	0.6666	0.0585	0.0650	0.0779	0.0571	0.0635	0.0721
3NN	0.4940	0.4648	0.6649	0.0553	0.0600	0.0728	0.0557	0.0615	0.0699
5NN	0.4932	0.4641	0.6640	0.0546	0.0588	0.0712	0.0554	0.0610	0.0695
10NN	0.4911	0.4626	0.6611	0.0542	0.0575	0.0699	0.0552	0.0606	0.0694
20NN	0.4863	0.4596	0.6542	0.0543	0.0575	0.0688	0.0553	0.0602	0.0694
				Mahalanobis					
1NN	0.4982	0.4661	0.6637	0.0591	0.0651	0.0767	0.0574	0.0636	0.0723
3NN	0.4962	0.4648	0.6613	0.0561	0.0603	0.0722	0.0560	0.0614	0.0700
5NN	0.4947	0.4637	0.6592	0.0552	0.0588	0.0709	0.0558	0.0610	0.0696
10NN	0.4912	0.4611	0.6550	0.0542	0.0567	0.0691	0.0556	0.0605	0.0695
20NN	0.4855	0.4571	0.6474	0.0531	0.0567	0.0678	0.0557	0.0602	0.0695
				cut points = $(2, 7)$					
True	0.5000	0.7027	0.3349						
				Euclidean					
1NN	0.4997	0.7024	0.3366	0.0592	0.0633	0.0712	0.0571	0.0592	0.0675
3NN	0.4984	0.7016	0.3362	0.0561	0.0590	0.0680	0.0556	0.0572	0.0660
5NN	0.4975	0.7011	0.3361	0.0554	0.0578	0.0674	0.0553	0.0568	0.0657
10NN	0.4956	0.6995	0.3353	0.0549	0.0565	0.0666	0.0550	0.0565	0.0656
20NN	0.4915	0.6959	0.3332	0.0543	0.0565	0.0659	0.0549	0.0564	0.0656
				Manhattan					
1NN	0.4997	0.7024	0.3367	0.0594	0.0632	0.0713	0.0571	0.0592	0.0675
3NN	0.4984	0.7018	0.3363	0.0562	0.0591	0.0681	0.0556	0.0572	0.0660
5NN	0.4977	0.7012	0.3360	0.0554	0.0578	0.0673	0.0553	0.0568	0.0658
10NN	0.4956	0.6995	0.3351	0.0548	0.0565	0.0666	0.0550	0.0565	0.0657
20NN	0.4913	0.6957	0.3330	0.0543	0.0565	0.0658	0.0549	0.0564	0.0657
				Canberra					
1NN	0.4963	0.7011	0.3369	0.0585	0.0625	0.0713	0.0571	0.0595	0.0677
3NN	0.4940	0.6995	0.3359	0.0553	0.0573	0.0678	0.0557	0.0575	0.0662
5NN	0.4932	0.6986	0.3351	0.0546	0.0560	0.0669	0.0554	0.0571	0.0659
10NN	0.4911	0.6963	0.3331	0.0542	0.0544	0.0659	0.0552	0.0568	0.0656
20NN	0.4863	0.6916	0.3286	0.0543	0.0544	0.0648	0.0553	0.0566	0.0652
				Mahalanobis					
1NN	0.4982	0.6996	0.3341	0.0591	0.0622	0.0703	0.0574	0.0596	0.0675
3NN	0.4962	0.6975	0.3323	0.0561	0.0577	0.0671	0.0560	0.0575	0.0658
5NN	0.4947	0.6961	0.3310	0.0552	0.0561	0.0662	0.0558	0.0571	0.0655
10NN	0.4912	0.6925	0.3282	0.0542	0.0541	0.0648	0.0556	0.0568	0.0651
20NN	0.4855	0.6870	0.3232	0.0531	0.0541	0.0633	0.0557	0.0567	0.0648
				cut points = $(4, 5)$					
True	0.8031	0.1651	0.6651						
				Euclidean					
1NN	0.8032	0.1655	0.6668	0.0487	0.0481	0.0780	0.0472	0.0466	0.0717
3NN	0.8020	0.1651	0.6666	0.0460	0.0450	0.0729	0.0457	0.0451	0.0695
5NN	0.8011	0.1649	0.6664	0.0453	0.0439	0.0717	0.0455	0.0448	0.0692

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	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
10NN	0.7996	0.1646	0.6654	0.0446	0.0427	0.0706	0.0453	0.0446	0.0691
20NN	0.7965	0.1637	0.6630	0.0441	0.0412	0.0698	0.0454	0.0443	0.0691
					Manhattan				
1NN	0.8031	0.1655	0.6671	0.0489	0.0480	0.0779	0.0472	0.0466	0.0717
3NN	0.8019	0.1652	0.6665	0.0460	0.0450	0.0730	0.0457	0.0451	0.0695
5NN	0.8012	0.1651	0.6664	0.0453	0.0440	0.0716	0.0454	0.0448	0.0692
10NN	0.7995	0.1646	0.6654	0.0446	0.0427	0.0706	0.0453	0.0446	0.0691
20NN	0.7963	0.1637	0.6629	0.0441	0.0413	0.0699	0.0454	0.0443	0.0691
					Canberra				
1NN	0.8008	0.1648	0.6666	0.0489	0.0469	0.0779	0.0475	0.0465	0.0721
3NN	0.7990	0.1647	0.6649	0.0455	0.0434	0.0728	0.0460	0.0451	0.0699
5NN	0.7980	0.1644	0.6640	0.0448	0.0425	0.0712	0.0458	0.0448	0.0695
10NN	0.7958	0.1636	0.6611	0.0442	0.0411	0.0699	0.0457	0.0444	0.0694
20NN	0.7916	0.1623	0.6542	0.0440	0.0397	0.0688	0.0461	0.0440	0.0694
					Mahalanobis				
1NN	0.8019	0.1646	0.6637	0.0488	0.0466	0.0767	0.0476	0.0465	0.0723
3NN	0.7999	0.1638	0.6613	0.0460	0.0431	0.0722	0.0462	0.0450	0.0700
5NN	0.7984	0.1634	0.6592	0.0452	0.0418	0.0709	0.0461	0.0446	0.0696
10NN	0.7954	0.1624	0.6550	0.0443	0.0400	0.0691	0.0461	0.0442	0.0695
20NN	0.7901	0.1608	0.6474	0.0436	0.0383	0.0678	0.0464	0.0439	0.0695
					cut points = $(4, 7)$				
True	0.8031	0.3996	0.3349						
					Euclidean				
1NN	0.8032	0.4003	0.3366	0.0487	0.0660	0.0712	0.0472	0.0619	0.0675
3NN	0.8020	0.3998	0.3362	0.0460	0.0617	0.0680	0.0457	0.0600	0.0660
5NN	0.8011	0.3995	0.3361	0.0453	0.0604	0.0674	0.0455	0.0596	0.0657
10NN	0.7996	0.3989	0.3353	0.0446	0.0594	0.0666	0.0453	0.0592	0.0656
20NN	0.7965	0.3973	0.3332	0.0441	0.0581	0.0659	0.0454	0.0589	0.0656
					Manhattan				
1NN	0.8031	0.4001	0.3367	0.0489	0.0658	0.0713	0.0472	0.0619	0.0675
3NN	0.8019	0.3999	0.3363	0.0460	0.0617	0.0681	0.0457	0.0600	0.0660
5NN	0.8012	0.3996	0.3360	0.0453	0.0606	0.0673	0.0454	0.0596	0.0658
10NN	0.7995	0.3989	0.3351	0.0446	0.0593	0.0666	0.0453	0.0592	0.0657
20NN	0.7963	0.3972	0.3330	0.0441	0.0581	0.0658	0.0454	0.0589	0.0657
					Canberra				
1NN	0.8008	0.3999	0.3369	0.0489	0.0649	0.0713	0.0475	0.0620	0.0677
3NN	0.7990	0.3995	0.3359	0.0455	0.0606	0.0678	0.0460	0.0600	0.0662
5NN	0.7980	0.3988	0.3351	0.0448	0.0595	0.0669	0.0458	0.0596	0.0659
10NN	0.7958	0.3973	0.3331	0.0442	0.0579	0.0659	0.0457	0.0591	0.0656
20NN	0.7916	0.3943	0.3286	0.0440	0.0563	0.0648	0.0461	0.0587	0.0652
					Mahalanobis				
1NN	0.8019	0.3982	0.3341	0.0488	0.0646	0.0703	0.0476	0.0620	0.0675
3NN	0.7999	0.3966	0.3323	0.0460	0.0603	0.0671	0.0462	0.0599	0.0658
5NN	0.7984	0.3957	0.3310	0.0452	0.0589	0.0662	0.0461	0.0594	0.0655
10NN	0.7954	0.3937	0.3282	0.0443	0.0571	0.0648	0.0461	0.0590	0.0651
20NN	0.7901	0.3907	0.3232	0.0436	0.0549	0.0633	0.0464	0.0586	0.0648
					cut points = $(5, 7)$				
True	0.8996	0.2345	0.3349						
					Euclidean				
1NN	0.9000	0.2348	0.3366	0.0373	0.0556	0.0712	0.0361	0.0531	0.0675
3NN	0.8992	0.2346	0.3362	0.0349	0.0520	0.0680	0.0349	0.0515	0.0660
5NN	0.8987	0.2346	0.3361	0.0345	0.0510	0.0674	0.0347	0.0511	0.0657
10NN	0.8974	0.2343	0.3353	0.0340	0.0499	0.0666	0.0346	0.0508	0.0656
20NN	0.8952	0.2335	0.3332	0.0336	0.0485	0.0659	0.0348	0.0506	0.0656

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	TCF$_1$	TCF$_2$	TCF$_3$	MC.sd$_1$	MC.sd$_2$	MC.sd$_3$	asy.sd$_1$	asy.sd$_2$	asy.sd$_3$
					Manhattan				
1NN	0.9000	0.2346	0.3367	0.0372	0.0556	0.0713	0.0361	0.0530	0.0675
3NN	0.8992	0.2347	0.3363	0.0350	0.0520	0.0681	0.0349	0.0515	0.0660
5NN	0.8986	0.2346	0.3360	0.0345	0.0510	0.0673	0.0347	0.0511	0.0658
10NN	0.8974	0.2343	0.3351	0.0340	0.0498	0.0666	0.0346	0.0508	0.0657
20NN	0.8949	0.2335	0.3330	0.0336	0.0484	0.0658	0.0348	0.0506	0.0657
					Canberra				
1NN	0.8988	0.2351	0.3369	0.0372	0.0548	0.0713	0.0363	0.0531	0.0677
3NN	0.8973	0.2348	0.3359	0.0346	0.0509	0.0678	0.0352	0.0515	0.0662
5NN	0.8966	0.2345	0.3351	0.0341	0.0500	0.0669	0.0351	0.0511	0.0659
10NN	0.8948	0.2337	0.3331	0.0336	0.0485	0.0659	0.0351	0.0507	0.0656
20NN	0.8915	0.2320	0.3286	0.0334	0.0471	0.0648	0.0355	0.0503	0.0652
					Mahalanobis				
1NN	0.8991	0.2335	0.3341	0.0373	0.0543	0.0703	0.0365	0.0531	0.0675
3NN	0.8978	0.2328	0.3323	0.0350	0.0504	0.0671	0.0354	0.0513	0.0658
5NN	0.8966	0.2323	0.3310	0.0345	0.0493	0.0662	0.0353	0.0509	0.0655
10NN	0.8942	0.2314	0.3282	0.0337	0.0477	0.0648	0.0355	0.0505	0.0651
20NN	0.8901	0.2299	0.3232	0.0332	0.0458	0.0633	0.0359	0.0502	0.0648

Table 18 – *Monte Carlo means, Monte Carlo standard deviations and estimated standard deviations of the estimators for true class fractions corresponding to the case of the first value of* Σ *and sample size* $= 500$. *"True" denotes the true parameter value.*

	TCF$_1$	TCF$_2$	TCF$_3$	MC.sd$_1$	MC.sd$_2$	MC.sd$_3$	asy.sd$_1$	asy.sd$_2$	asy.sd$_3$
					cut points $= (2, 4)$				
True	0.5000	0.4347	0.9347						
					Euclidean				
1NN	0.5000	0.4343	0.9333	0.0419	0.0464	0.0285	0.0393	0.0443	0.0269
3NN	0.4991	0.4343	0.9329	0.0401	0.0430	0.0266	0.0386	0.0431	0.0262
5NN	0.4984	0.4339	0.9324	0.0396	0.0419	0.0262	0.0385	0.0429	0.0261
10NN	0.4971	0.4332	0.9315	0.0391	0.0407	0.0258	0.0384	0.0426	0.0262
20NN	0.4948	0.4317	0.9296	0.0386	0.0394	0.0256	0.0383	0.0424	0.0265
					Manhattan				
1NN	0.5004	0.4346	0.9335	0.0421	0.0464	0.0283	0.0393	0.0442	0.0269
3NN	0.4995	0.4345	0.9329	0.0401	0.0431	0.0266	0.0386	0.0431	0.0261
5NN	0.4989	0.4343	0.9324	0.0396	0.0421	0.0262	0.0385	0.0428	0.0261
10NN	0.4978	0.4336	0.9315	0.0392	0.0409	0.0259	0.0384	0.0426	0.0261
20NN	0.4958	0.4323	0.9296	0.0388	0.0397	0.0258	0.0383	0.0424	0.0264
					Canberra				
1NN	0.4903	0.4340	0.9296	0.0421	0.0444	0.0292	0.0395	0.0445	0.0289
3NN	0.4849	0.4340	0.9270	0.0395	0.0407	0.0274	0.0387	0.0434	0.0284
5NN	0.4816	0.4335	0.9250	0.0386	0.0393	0.0271	0.0385	0.0432	0.0285
10NN	0.4770	0.4322	0.9212	0.0375	0.0380	0.0272	0.0384	0.0429	0.0288
20NN	0.4812	0.4329	0.9165	0.0378	0.0374	0.0270	0.0388	0.0424	0.0291
					Mahalanobis				
1NN	0.5007	0.4350	0.9331	0.0421	0.0464	0.0284	0.0394	0.0444	0.0272
3NN	0.4999	0.4353	0.9321	0.0402	0.0429	0.0266	0.0388	0.0432	0.0265
5NN	0.4995	0.4354	0.9314	0.0397	0.0418	0.0262	0.0387	0.0430	0.0265
10NN	0.4983	0.4349	0.9296	0.0393	0.0403	0.0259	0.0386	0.0427	0.0266
20NN	0.4961	0.4337	0.9263	0.0388	0.0389	0.0257	0.0387	0.0424	0.0271

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	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
				cut points = $(2, 5)$					
True	0.5000	0.7099	0.7752						
				Euclidean					
1NN	0.5000	0.7077	0.7732	0.0419	0.0450	0.0466	0.0393	0.0417	0.0442
3NN	0.4991	0.7066	0.7722	0.0401	0.0415	0.0437	0.0386	0.0406	0.0431
5NN	0.4984	0.7053	0.7712	0.0396	0.0404	0.0430	0.0385	0.0403	0.0429
10NN	0.4971	0.7025	0.7693	0.0391	0.0391	0.0423	0.0384	0.0402	0.0430
20NN	0.4948	0.6975	0.7656	0.0386	0.0391	0.0417	0.0383	0.0401	0.0430
				Manhattan					
1NN	0.5004	0.7080	0.7733	0.0421	0.0450	0.0464	0.0393	0.0417	0.0442
3NN	0.4995	0.7070	0.7722	0.0401	0.0416	0.0439	0.0386	0.0405	0.0431
5NN	0.4989	0.7058	0.7712	0.0396	0.0405	0.0431	0.0385	0.0403	0.0429
10NN	0.4978	0.7032	0.7692	0.0392	0.0393	0.0424	0.0384	0.0401	0.0430
20NN	0.4958	0.6984	0.7654	0.0388	0.0393	0.0419	0.0383	0.0400	0.0430
				Canberra					
1NN	0.4903	0.7008	0.7663	0.0421	0.0427	0.0460	0.0395	0.0424	0.0454
3NN	0.4849	0.6973	0.7624	0.0395	0.0391	0.0435	0.0387	0.0413	0.0445
5NN	0.4816	0.6945	0.7598	0.0386	0.0380	0.0429	0.0385	0.0411	0.0444
10NN	0.4770	0.6893	0.7546	0.0375	0.0367	0.0424	0.0384	0.0409	0.0445
20NN	0.4812	0.6865	0.7467	0.0378	0.0367	0.0419	0.0388	0.0405	0.0445
				Mahalanobis					
1NN	0.5007	0.7078	0.7725	0.0421	0.0452	0.0465	0.0394	0.0419	0.0445
3NN	0.4999	0.7065	0.7704	0.0402	0.0415	0.0435	0.0388	0.0407	0.0434
5NN	0.4995	0.7052	0.7690	0.0397	0.0404	0.0429	0.0387	0.0405	0.0433
10NN	0.4983	0.7018	0.7655	0.0393	0.0389	0.0421	0.0386	0.0403	0.0434
20NN	0.4961	0.6957	0.7595	0.0388	0.0389	0.0415	0.0387	0.0402	0.0434
				cut points = $(2, 7)$					
True	0.5000	0.9230	0.2248						
				Euclidean					
1NN	0.5000	0.9214	0.2235	0.0419	0.0261	0.0407	0.0393	0.0256	0.0401
3NN	0.4991	0.9199	0.2225	0.0401	0.0240	0.0392	0.0386	0.0248	0.0396
5NN	0.4984	0.9186	0.2218	0.0396	0.0235	0.0388	0.0385	0.0248	0.0395
10NN	0.4971	0.9160	0.2205	0.0391	0.0229	0.0384	0.0384	0.0248	0.0394
20NN	0.4948	0.9115	0.2183	0.0386	0.0229	0.0379	0.0383	0.0251	0.0394
				Manhattan					
1NN	0.5004	0.9214	0.2232	0.0421	0.0261	0.0408	0.0393	0.0256	0.0401
3NN	0.4995	0.9199	0.2222	0.0401	0.0240	0.0394	0.0386	0.0248	0.0396
5NN	0.4989	0.9185	0.2213	0.0396	0.0236	0.0388	0.0385	0.0248	0.0395
10NN	0.4978	0.9159	0.2197	0.0392	0.0229	0.0384	0.0384	0.0248	0.0394
20NN	0.4958	0.9114	0.2171	0.0388	0.0229	0.0380	0.0383	0.0251	0.0393
				Canberra					
1NN	0.4903	0.9138	0.2185	0.0421	0.0258	0.0404	0.0395	0.0276	0.0404
3NN	0.4849	0.9090	0.2164	0.0395	0.0237	0.0388	0.0387	0.0271	0.0399
5NN	0.4816	0.9053	0.2150	0.0386	0.0231	0.0383	0.0385	0.0271	0.0398
10NN	0.4770	0.8989	0.2124	0.0375	0.0225	0.0377	0.0384	0.0275	0.0395
20NN	0.4812	0.8939	0.2077	0.0378	0.0225	0.0368	0.0388	0.0276	0.0390
				Mahalanobis					
1NN	0.5007	0.9205	0.2225	0.0421	0.0262	0.0407	0.0394	0.0260	0.0402
3NN	0.4999	0.9182	0.2208	0.0402	0.0241	0.0392	0.0388	0.0253	0.0397
5NN	0.4995	0.9165	0.2197	0.0397	0.0235	0.0386	0.0387	0.0253	0.0395
10NN	0.4983	0.9128	0.2174	0.0393	0.0228	0.0380	0.0386	0.0254	0.0394
20NN	0.4961	0.9065	0.2139	0.0388	0.0228	0.0374	0.0387	0.0258	0.0392
				cut points = $(4, 5)$					
True	0.9347	0.2752	0.7752						

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	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
					Euclidean				
1NN	0.9332	0.2734	0.7732	0.0269	0.0398	0.0466	0.0242	0.0391	0.0442
3NN	0.9317	0.2723	0.7722	0.0242	0.0371	0.0437	0.0235	0.0381	0.0431
5NN	0.9308	0.2713	0.7712	0.0233	0.0360	0.0430	0.0234	0.0379	0.0429
10NN	0.9287	0.2693	0.7693	0.0225	0.0350	0.0423	0.0235	0.0376	0.0430
20NN	0.9254	0.2658	0.7656	0.0215	0.0337	0.0417	0.0237	0.0373	0.0430
					Manhattan				
1NN	0.9333	0.2733	0.7733	0.0269	0.0399	0.0464	0.0241	0.0391	0.0442
3NN	0.9321	0.2725	0.7722	0.0242	0.0373	0.0439	0.0234	0.0381	0.0431
5NN	0.9311	0.2715	0.7712	0.0234	0.0363	0.0431	0.0233	0.0379	0.0429
10NN	0.9292	0.2696	0.7692	0.0225	0.0352	0.0424	0.0234	0.0376	0.0430
20NN	0.9260	0.2661	0.7654	0.0216	0.0340	0.0419	0.0236	0.0373	0.0430
					Canberra				
1NN	0.9163	0.2668	0.7663	0.0273	0.0375	0.0460	0.0272	0.0389	0.0454
3NN	0.9085	0.2633	0.7624	0.0241	0.0348	0.0435	0.0267	0.0379	0.0445
5NN	0.9040	0.2610	0.7598	0.0232	0.0339	0.0429	0.0268	0.0376	0.0444
10NN	0.8988	0.2571	0.7546	0.0223	0.0328	0.0424	0.0271	0.0372	0.0445
20NN	0.9022	0.2535	0.7467	0.0214	0.0320	0.0419	0.0272	0.0367	0.0445
					Mahalanobis				
1NN	0.9333	0.2728	0.7725	0.0268	0.0396	0.0465	0.0244	0.0391	0.0445
3NN	0.9317	0.2712	0.7704	0.0243	0.0367	0.0435	0.0237	0.0381	0.0434
5NN	0.9305	0.2698	0.7690	0.0234	0.0357	0.0429	0.0237	0.0379	0.0433
10NN	0.9281	0.2669	0.7655	0.0226	0.0345	0.0421	0.0238	0.0375	0.0434
20NN	0.9239	0.2620	0.7595	0.0218	0.0331	0.0415	0.0242	0.0371	0.0434
					cut points $= (4, 7)$				
True	0.9347	0.4883	0.2248						
					Euclidean				
1NN	0.9332	0.4871	0.2235	0.0269	0.0465	0.0407	0.0242	0.0447	0.0401
3NN	0.9317	0.4855	0.2225	0.0242	0.0434	0.0392	0.0235	0.0436	0.0396
5NN	0.9308	0.4847	0.2218	0.0233	0.0424	0.0388	0.0234	0.0433	0.0395
10NN	0.9287	0.4829	0.2205	0.0225	0.0414	0.0384	0.0235	0.0431	0.0394
20NN	0.9254	0.4798	0.2183	0.0215	0.0403	0.0379	0.0237	0.0428	0.0394
					Manhattan				
1NN	0.9333	0.4867	0.2232	0.0269	0.0465	0.0408	0.0241	0.0447	0.0401
3NN	0.9321	0.4854	0.2222	0.0242	0.0434	0.0394	0.0234	0.0435	0.0396
5NN	0.9311	0.4843	0.2213	0.0234	0.0425	0.0388	0.0233	0.0433	0.0395
10NN	0.9292	0.4823	0.2197	0.0225	0.0414	0.0384	0.0234	0.0430	0.0394
20NN	0.9260	0.4791	0.2171	0.0216	0.0404	0.0380	0.0236	0.0428	0.0393
					Canberra				
1NN	0.9163	0.4798	0.2185	0.0273	0.0447	0.0404	0.0272	0.0450	0.0404
3NN	0.9085	0.4750	0.2164	0.0241	0.0413	0.0388	0.0267	0.0438	0.0399
5NN	0.9040	0.4718	0.2150	0.0232	0.0401	0.0383	0.0268	0.0435	0.0398
10NN	0.8988	0.4667	0.2124	0.0223	0.0390	0.0377	0.0271	0.0433	0.0395
20NN	0.9022	0.4609	0.2077	0.0214	0.0381	0.0368	0.0272	0.0428	0.0390
					Mahalanobis				
1NN	0.9333	0.4855	0.2225	0.0268	0.0464	0.0407	0.0244	0.0449	0.0402
3NN	0.9317	0.4829	0.2208	0.0243	0.0433	0.0392	0.0237	0.0437	0.0397
5NN	0.9305	0.4812	0.2197	0.0234	0.0423	0.0386	0.0237	0.0434	0.0395
10NN	0.9281	0.4779	0.2174	0.0226	0.0410	0.0380	0.0238	0.0431	0.0394
20NN	0.9239	0.4728	0.2139	0.0218	0.0398	0.0374	0.0242	0.0428	0.0392
					cut points $= (5, 7)$				
True	0.9883	0.2132	0.2248						
					Euclidean				
1NN	0.9875	0.2136	0.2235	0.0129	0.0400	0.0407	0.0120	0.0376	0.0401

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	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
3NN	0.9867	0.2133	0.2225	0.0113	0.0369	0.0392	0.0118	0.0365	0.0396
5NN	0.9861	0.2133	0.2218	0.0107	0.0359	0.0388	0.0118	0.0363	0.0395
10NN	0.9850	0.2135	0.2205	0.0102	0.0345	0.0384	0.0120	0.0361	0.0394
20NN	0.9833	0.2140	0.2183	0.0098	0.0332	0.0379	0.0123	0.0359	0.0394
					Manhattan				
1NN	0.9876	0.2134	0.2232	0.0128	0.0401	0.0408	0.0119	0.0376	0.0401
3NN	0.9868	0.2129	0.2222	0.0113	0.0368	0.0394	0.0117	0.0365	0.0396
5NN	0.9862	0.2128	0.2213	0.0108	0.0359	0.0388	0.0117	0.0363	0.0395
10NN	0.9851	0.2128	0.2197	0.0103	0.0346	0.0384	0.0119	0.0360	0.0394
20NN	0.9832	0.2130	0.2171	0.0098	0.0333	0.0380	0.0122	0.0359	0.0393
					Canberra				
1NN	0.9758	0.2130	0.2185	0.0147	0.0374	0.0404	0.0170	0.0376	0.0404
3NN	0.9708	0.2117	0.2164	0.0129	0.0344	0.0388	0.0170	0.0365	0.0399
5NN	0.9680	0.2108	0.2150	0.0125	0.0334	0.0383	0.0172	0.0362	0.0398
10NN	0.9650	0.2096	0.2124	0.0121	0.0322	0.0377	0.0175	0.0359	0.0395
20NN	0.9667	0.2074	0.2077	0.0109	0.0311	0.0368	0.0176	0.0355	0.0390
					Mahalanobis				
1NN	0.9874	0.2126	0.2225	0.0129	0.0401	0.0407	0.0123	0.0377	0.0402
3NN	0.9864	0.2117	0.2208	0.0115	0.0367	0.0392	0.0121	0.0365	0.0397
5NN	0.9857	0.2114	0.2197	0.0110	0.0356	0.0386	0.0122	0.0363	0.0395
10NN	0.9841	0.2110	0.2174	0.0106	0.0341	0.0380	0.0125	0.0360	0.0394
20NN	0.9817	0.2108	0.2139	0.0102	0.0326	0.0374	0.0129	0.0358	0.0392

Table 19 – *Monte Carlo means, Monte Carlo standard deviations and estimated standard deviations of the estimators for true class fractions corresponding to the case of the second value of Σ and sample size = 500. "True" denotes the true parameter value.*

	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
					cut points = $(2, 4)$				
True	0.5000	0.3970	0.8970						
					Euclidean				
1NN	0.4987	0.3958	0.8975	0.0419	0.0466	0.0379	0.0396	0.0438	0.0347
3NN	0.4975	0.3947	0.8971	0.0400	0.0431	0.0349	0.0388	0.0425	0.0335
5NN	0.4965	0.3937	0.8968	0.0395	0.0423	0.0342	0.0386	0.0423	0.0334
10NN	0.4945	0.3918	0.8962	0.0391	0.0412	0.0335	0.0384	0.0420	0.0334
20NN	0.4909	0.3886	0.8951	0.0386	0.0400	0.0328	0.0383	0.0418	0.0336
					Manhattan				
1NN	0.4989	0.3956	0.8974	0.0420	0.0467	0.0379	0.0396	0.0438	0.0347
3NN	0.4975	0.3947	0.8971	0.0401	0.0431	0.0350	0.0388	0.0425	0.0335
5NN	0.4966	0.3938	0.8968	0.0394	0.0423	0.0342	0.0386	0.0423	0.0334
10NN	0.4947	0.3919	0.8963	0.0391	0.0413	0.0335	0.0384	0.0420	0.0334
20NN	0.4912	0.3889	0.8951	0.0387	0.0402	0.0329	0.0383	0.0418	0.0336
					Canberra				
1NN	0.4929	0.3922	0.8948	0.0416	0.0451	0.0379	0.0398	0.0438	0.0357
3NN	0.4884	0.3895	0.8931	0.0394	0.0414	0.0351	0.0389	0.0425	0.0346
5NN	0.4850	0.3873	0.8915	0.0387	0.0401	0.0346	0.0387	0.0422	0.0346
10NN	0.4797	0.3835	0.8884	0.0380	0.0386	0.0342	0.0386	0.0418	0.0347
20NN	0.4773	0.3812	0.8848	0.0378	0.0378	0.0335	0.0385	0.0416	0.0351
					Mahalanobis				
1NN	0.4988	0.3960	0.8953	0.0419	0.0459	0.0383	0.0398	0.0439	0.0356

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	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
3NN	0.4974	0.3947	0.8937	0.0400	0.0426	0.0353	0.0391	0.0426	0.0345
5NN	0.4965	0.3937	0.8923	0.0396	0.0416	0.0344	0.0389	0.0423	0.0344
10NN	0.4941	0.3915	0.8895	0.0391	0.0402	0.0336	0.0388	0.0420	0.0345
20NN	0.4902	0.3881	0.8848	0.0386	0.0386	0.0329	0.0388	0.0417	0.0349
				cut points = (2, 5)					
True	0.5000	0.6335	0.7365						
				Euclidean					
1NN	0.4987	0.6318	0.7376	0.0419	0.0466	0.0510	0.0396	0.0436	0.0477
3NN	0.4975	0.6306	0.7372	0.0400	0.0433	0.0472	0.0388	0.0423	0.0464
5NN	0.4965	0.6294	0.7371	0.0395	0.0426	0.0464	0.0386	0.0420	0.0462
10NN	0.4945	0.6268	0.7366	0.0391	0.0416	0.0456	0.0384	0.0419	0.0461
20NN	0.4909	0.6222	0.7355	0.0386	0.0416	0.0448	0.0383	0.0418	0.0461
				Manhattan					
1NN	0.4989	0.6316	0.7374	0.0420	0.0465	0.0509	0.0396	0.0436	0.0477
3NN	0.4975	0.6305	0.7372	0.0401	0.0434	0.0473	0.0388	0.0423	0.0464
5NN	0.4966	0.6295	0.7370	0.0394	0.0425	0.0464	0.0386	0.0420	0.0462
10NN	0.4947	0.6269	0.7365	0.0391	0.0416	0.0456	0.0384	0.0418	0.0461
20NN	0.4912	0.6225	0.7354	0.0387	0.0416	0.0449	0.0383	0.0417	0.0461
				Canberra					
1NN	0.4929	0.6267	0.7367	0.0416	0.0453	0.0504	0.0398	0.0439	0.0481
3NN	0.4884	0.6224	0.7354	0.0394	0.0419	0.0468	0.0389	0.0426	0.0468
5NN	0.4850	0.6190	0.7341	0.0387	0.0407	0.0462	0.0387	0.0423	0.0466
10NN	0.4797	0.6132	0.7309	0.0380	0.0394	0.0456	0.0386	0.0421	0.0466
20NN	0.4773	0.6091	0.7262	0.0378	0.0394	0.0447	0.0385	0.0419	0.0466
				Mahalanobis					
1NN	0.4988	0.6310	0.7349	0.0419	0.0462	0.0509	0.0398	0.0438	0.0482
3NN	0.4974	0.6289	0.7330	0.0400	0.0429	0.0473	0.0391	0.0424	0.0468
5NN	0.4965	0.6270	0.7315	0.0396	0.0420	0.0464	0.0389	0.0422	0.0466
10NN	0.4941	0.6231	0.7281	0.0391	0.0407	0.0453	0.0388	0.0419	0.0466
20NN	0.4902	0.6170	0.7229	0.0386	0.0407	0.0443	0.0388	0.0418	0.0466
				cut points = (2, 7)					
True	0.5000	0.8682	0.2635						
				Euclidean					
1NN	0.4987	0.8675	0.2634	0.0419	0.0352	0.0441	0.0396	0.0325	0.0425
3NN	0.4975	0.8667	0.2633	0.0400	0.0324	0.0427	0.0388	0.0313	0.0420
5NN	0.4965	0.8661	0.2634	0.0395	0.0316	0.0424	0.0386	0.0311	0.0419
10NN	0.4945	0.8644	0.2634	0.0391	0.0307	0.0423	0.0384	0.0311	0.0420
20NN	0.4909	0.8610	0.2632	0.0386	0.0307	0.0422	0.0383	0.0311	0.0421
				Manhattan					
1NN	0.4989	0.8674	0.2633	0.0420	0.0352	0.0441	0.0396	0.0324	0.0425
3NN	0.4975	0.8667	0.2632	0.0401	0.0324	0.0427	0.0388	0.0313	0.0420
5NN	0.4966	0.8661	0.2632	0.0394	0.0315	0.0425	0.0386	0.0311	0.0419
10NN	0.4947	0.8644	0.2632	0.0391	0.0307	0.0423	0.0384	0.0310	0.0420
20NN	0.4912	0.8611	0.2629	0.0387	0.0307	0.0422	0.0383	0.0311	0.0421
				Canberra					
1NN	0.4929	0.8637	0.2644	0.0416	0.0346	0.0441	0.0398	0.0333	0.0429
3NN	0.4884	0.8600	0.2643	0.0394	0.0315	0.0429	0.0389	0.0323	0.0424
5NN	0.4850	0.8567	0.2639	0.0387	0.0307	0.0426	0.0387	0.0322	0.0423
10NN	0.4797	0.8510	0.2628	0.0380	0.0297	0.0423	0.0386	0.0323	0.0423
20NN	0.4773	0.8471	0.2606	0.0378	0.0297	0.0418	0.0385	0.0323	0.0422
				Mahalanobis					
1NN	0.4988	0.8654	0.2619	0.0419	0.0351	0.0439	0.0398	0.0330	0.0425
3NN	0.4974	0.8631	0.2610	0.0400	0.0320	0.0425	0.0391	0.0319	0.0419
5NN	0.4965	0.8613	0.2603	0.0396	0.0313	0.0421	0.0389	0.0318	0.0417

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	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
10NN	0.4941	0.8575	0.2589	0.0391	0.0302	0.0416	0.0388	0.0318	0.0416
20NN	0.4902	0.8515	0.2567	0.0386	0.0302	0.0411	0.0388	0.0319	0.0415
				cut points = $(4, 5)$					
True	0.8970	0.2365	0.7365						
				Euclidean					
1NN	0.8964	0.2360	0.7376	0.0279	0.0388	0.0510	0.0263	0.0372	0.0477
3NN	0.8954	0.2359	0.7372	0.0264	0.0362	0.0472	0.0255	0.0362	0.0464
5NN	0.8947	0.2357	0.7371	0.0259	0.0353	0.0464	0.0253	0.0360	0.0462
10NN	0.8932	0.2350	0.7366	0.0254	0.0343	0.0456	0.0253	0.0358	0.0461
20NN	0.8905	0.2335	0.7355	0.0250	0.0332	0.0448	0.0254	0.0356	0.0461
				Manhattan					
1NN	0.8965	0.2360	0.7374	0.0279	0.0388	0.0509	0.0263	0.0371	0.0477
3NN	0.8955	0.2359	0.7372	0.0265	0.0362	0.0473	0.0254	0.0362	0.0464
5NN	0.8947	0.2357	0.7370	0.0259	0.0354	0.0464	0.0253	0.0360	0.0462
10NN	0.8932	0.2349	0.7365	0.0254	0.0344	0.0456	0.0253	0.0358	0.0461
20NN	0.8906	0.2336	0.7354	0.0251	0.0334	0.0449	0.0254	0.0356	0.0461
				Canberra					
1NN	0.8906	0.2345	0.7367	0.0279	0.0374	0.0504	0.0273	0.0371	0.0481
3NN	0.8872	0.2329	0.7354	0.0261	0.0348	0.0468	0.0266	0.0360	0.0468
5NN	0.8850	0.2317	0.7341	0.0256	0.0339	0.0462	0.0265	0.0358	0.0466
10NN	0.8819	0.2297	0.7309	0.0251	0.0329	0.0456	0.0266	0.0354	0.0466
20NN	0.8798	0.2279	0.7262	0.0249	0.0320	0.0447	0.0267	0.0352	0.0466
				Mahalanobis					
1NN	0.8962	0.2350	0.7349	0.0281	0.0383	0.0509	0.0264	0.0372	0.0482
3NN	0.8951	0.2342	0.7330	0.0263	0.0354	0.0473	0.0256	0.0361	0.0468
5NN	0.8942	0.2333	0.7315	0.0258	0.0346	0.0464	0.0256	0.0359	0.0466
10NN	0.8923	0.2316	0.7281	0.0253	0.0333	0.0453	0.0256	0.0356	0.0466
20NN	0.8888	0.2289	0.7229	0.0250	0.0319	0.0443	0.0258	0.0352	0.0466
				cut points = $(4, 7)$					
True	0.8970	0.4711	0.2635						
				Euclidean					
1NN	0.8964	0.4717	0.2634	0.0279	0.0480	0.0441	0.0263	0.0446	0.0425
3NN	0.8954	0.4720	0.2633	0.0264	0.0448	0.0427	0.0255	0.0433	0.0420
5NN	0.8947	0.4723	0.2634	0.0259	0.0439	0.0424	0.0253	0.0431	0.0419
10NN	0.8932	0.4726	0.2634	0.0254	0.0430	0.0423	0.0253	0.0429	0.0420
20NN	0.8905	0.4723	0.2632	0.0250	0.0421	0.0422	0.0254	0.0427	0.0421
				Manhattan					
1NN	0.8965	0.4718	0.2633	0.0279	0.0480	0.0441	0.0263	0.0446	0.0425
3NN	0.8955	0.4720	0.2632	0.0265	0.0448	0.0427	0.0254	0.0433	0.0420
5NN	0.8947	0.4723	0.2632	0.0259	0.0440	0.0425	0.0253	0.0431	0.0419
10NN	0.8932	0.4725	0.2632	0.0254	0.0430	0.0423	0.0253	0.0428	0.0420
20NN	0.8906	0.4722	0.2629	0.0251	0.0421	0.0422	0.0254	0.0427	0.0421
				Canberra					
1NN	0.8906	0.4716	0.2644	0.0279	0.0470	0.0441	0.0273	0.0447	0.0429
3NN	0.8872	0.4705	0.2643	0.0261	0.0439	0.0429	0.0266	0.0434	0.0424
5NN	0.8850	0.4694	0.2639	0.0256	0.0430	0.0426	0.0265	0.0431	0.0423
10NN	0.8819	0.4675	0.2628	0.0251	0.0421	0.0423	0.0266	0.0428	0.0423
20NN	0.8798	0.4659	0.2606	0.0249	0.0409	0.0418	0.0267	0.0425	0.0422
				Mahalanobis					
1NN	0.8962	0.4694	0.2619	0.0281	0.0478	0.0439	0.0264	0.0447	0.0425
3NN	0.8951	0.4684	0.2610	0.0263	0.0446	0.0425	0.0256	0.0434	0.0419
5NN	0.8942	0.4676	0.2603	0.0258	0.0437	0.0421	0.0256	0.0431	0.0417
10NN	0.8923	0.4660	0.2589	0.0253	0.0424	0.0416	0.0256	0.0428	0.0416
20NN	0.8888	0.4634	0.2567	0.0250	0.0411	0.0411	0.0258	0.0425	0.0415

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	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
				cut points = $(5, 7)$					
True	0.9711	0.2347	0.2635						
				Euclidean					
1NN	0.9707	0.2357	0.2634	0.0150	0.0400	0.0441	0.0148	0.0376	0.0425
3NN	0.9701	0.2360	0.2633	0.0142	0.0373	0.0427	0.0144	0.0366	0.0420
5NN	0.9697	0.2367	0.2634	0.0139	0.0366	0.0424	0.0143	0.0364	0.0419
10NN	0.9690	0.2376	0.2634	0.0137	0.0358	0.0423	0.0143	0.0362	0.0420
20NN	0.9676	0.2388	0.2632	0.0136	0.0350	0.0422	0.0145	0.0361	0.0421
				Manhattan					
1NN	0.9707	0.2358	0.2633	0.0150	0.0399	0.0441	0.0148	0.0376	0.0425
3NN	0.9701	0.2361	0.2632	0.0142	0.0374	0.0427	0.0144	0.0366	0.0420
5NN	0.9697	0.2366	0.2632	0.0140	0.0367	0.0425	0.0143	0.0364	0.0419
10NN	0.9690	0.2376	0.2632	0.0137	0.0358	0.0423	0.0143	0.0362	0.0420
20NN	0.9675	0.2386	0.2629	0.0136	0.0350	0.0422	0.0145	0.0361	0.0421
				Canberra					
1NN	0.9677	0.2371	0.2644	0.0154	0.0388	0.0441	0.0160	0.0377	0.0429
3NN	0.9661	0.2376	0.2643	0.0144	0.0365	0.0429	0.0156	0.0366	0.0424
5NN	0.9650	0.2376	0.2639	0.0141	0.0357	0.0426	0.0157	0.0364	0.0423
10NN	0.9634	0.2378	0.2628	0.0139	0.0347	0.0423	0.0158	0.0362	0.0423
20NN	0.9617	0.2380	0.2606	0.0137	0.0338	0.0418	0.0160	0.0360	0.0422
				Mahalanobis					
1NN	0.9705	0.2344	0.2619	0.0150	0.0396	0.0439	0.0150	0.0376	0.0425
3NN	0.9699	0.2342	0.2610	0.0142	0.0371	0.0425	0.0145	0.0365	0.0419
5NN	0.9694	0.2343	0.2603	0.0139	0.0362	0.0421	0.0145	0.0362	0.0417
10NN	0.9683	0.2345	0.2589	0.0138	0.0351	0.0416	0.0146	0.0360	0.0416
20NN	0.9663	0.2345	0.2567	0.0137	0.0340	0.0411	0.0150	0.0358	0.0415

Table 20 – *Monte Carlo means, Monte Carlo standard deviations and estimated standard deviations of the estimators for true class fractions corresponding to the case of the third value of Σ and sample size = 500. "True" denotes the true parameter value.*

	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
				cut points = $(2, 4)$					
True	0.5000	0.3031	0.8031						
				Euclidean					
1NN	0.4994	0.3024	0.8036	0.0428	0.0441	0.0489	0.0403	0.0412	0.0442
3NN	0.4990	0.3020	0.8033	0.0409	0.0412	0.0454	0.0394	0.0399	0.0427
5NN	0.4984	0.3017	0.8032	0.0403	0.0402	0.0447	0.0391	0.0397	0.0425
10NN	0.4971	0.3010	0.8028	0.0398	0.0392	0.0438	0.0390	0.0395	0.0423
20NN	0.4951	0.3000	0.8019	0.0394	0.0384	0.0433	0.0388	0.0393	0.0424
				Manhattan					
1NN	0.4993	0.3022	0.8035	0.0429	0.0441	0.0490	0.0403	0.0412	0.0442
3NN	0.4990	0.3021	0.8034	0.0409	0.0412	0.0455	0.0393	0.0399	0.0427
5NN	0.4984	0.3017	0.8032	0.0404	0.0402	0.0446	0.0391	0.0397	0.0424
10NN	0.4972	0.3010	0.8028	0.0398	0.0392	0.0438	0.0389	0.0394	0.0423
20NN	0.4951	0.3000	0.8019	0.0394	0.0384	0.0433	0.0388	0.0393	0.0424
				Canberra					
1NN	0.4971	0.3011	0.8028	0.0424	0.0431	0.0491	0.0404	0.0412	0.0445
3NN	0.4956	0.3006	0.8019	0.0404	0.0397	0.0456	0.0394	0.0399	0.0430
5NN	0.4945	0.3001	0.8012	0.0398	0.0387	0.0448	0.0391	0.0396	0.0428

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	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
10NN	0.4928	0.2994	0.7997	0.0392	0.0377	0.0437	0.0390	0.0394	0.0427
20NN	0.4908	0.2985	0.7969	0.0390	0.0370	0.0430	0.0389	0.0392	0.0428
					Mahalanobis				
1NN	0.4985	0.3020	0.8017	0.0429	0.0433	0.0487	0.0405	0.0412	0.0447
3NN	0.4974	0.3012	0.8002	0.0407	0.0402	0.0454	0.0396	0.0399	0.0431
5NN	0.4964	0.3006	0.7990	0.0401	0.0392	0.0446	0.0394	0.0396	0.0429
10NN	0.4944	0.2996	0.7965	0.0395	0.0377	0.0436	0.0393	0.0393	0.0428
20NN	0.4909	0.2980	0.7919	0.0389	0.0361	0.0427	0.0392	0.0391	0.0429
					cut points = $(2,5)$				
True	0.5000	0.4682	0.6651						
					Euclidean				
1NN	0.4994	0.4677	0.6653	0.0428	0.0476	0.0545	0.0403	0.0447	0.0509
3NN	0.4990	0.4672	0.6650	0.0409	0.0447	0.0512	0.0394	0.0434	0.0494
5NN	0.4984	0.4668	0.6648	0.0403	0.0437	0.0505	0.0391	0.0431	0.0491
10NN	0.4971	0.4659	0.6645	0.0398	0.0428	0.0498	0.0390	0.0429	0.0490
20NN	0.4951	0.4645	0.6638	0.0394	0.0428	0.0493	0.0388	0.0427	0.0490
					Manhattan				
1NN	0.4993	0.4676	0.6652	0.0429	0.0477	0.0545	0.0403	0.0447	0.0509
3NN	0.4990	0.4673	0.6650	0.0409	0.0447	0.0513	0.0393	0.0434	0.0494
5NN	0.4984	0.4668	0.6648	0.0404	0.0438	0.0504	0.0391	0.0431	0.0491
10NN	0.4972	0.4658	0.6644	0.0398	0.0429	0.0498	0.0389	0.0429	0.0490
20NN	0.4951	0.4644	0.6636	0.0394	0.0429	0.0493	0.0388	0.0427	0.0490
					Canberra				
1NN	0.4971	0.4664	0.6651	0.0424	0.0470	0.0545	0.0404	0.0447	0.0511
3NN	0.4956	0.4654	0.6642	0.0404	0.0436	0.0512	0.0394	0.0434	0.0495
5NN	0.4945	0.4647	0.6636	0.0398	0.0426	0.0504	0.0391	0.0431	0.0493
10NN	0.4928	0.4636	0.6622	0.0392	0.0415	0.0496	0.0390	0.0428	0.0491
20NN	0.4908	0.4620	0.6593	0.0390	0.0415	0.0489	0.0389	0.0427	0.0491
					Mahalanobis				
1NN	0.4985	0.4668	0.6634	0.0429	0.0472	0.0544	0.0405	0.0448	0.0512
3NN	0.4974	0.4657	0.6619	0.0407	0.0439	0.0510	0.0396	0.0434	0.0496
5NN	0.4964	0.4649	0.6606	0.0401	0.0429	0.0501	0.0394	0.0431	0.0493
10NN	0.4944	0.4631	0.6580	0.0395	0.0416	0.0492	0.0393	0.0428	0.0491
20NN	0.4909	0.4604	0.6534	0.0389	0.0416	0.0484	0.0392	0.0426	0.0491
					cut points = $(2,7)$				
True	0.5000	0.7027	0.3349						
					Euclidean				
1NN	0.4994	0.7018	0.3353	0.0428	0.0448	0.0495	0.0403	0.0418	0.0477
3NN	0.4990	0.7018	0.3353	0.0409	0.0419	0.0473	0.0394	0.0404	0.0466
5NN	0.4984	0.7014	0.3352	0.0403	0.0410	0.0469	0.0391	0.0401	0.0464
10NN	0.4971	0.7004	0.3349	0.0398	0.0403	0.0466	0.0390	0.0399	0.0463
20NN	0.4951	0.6988	0.3342	0.0394	0.0403	0.0463	0.0388	0.0399	0.0464
					Manhattan				
1NN	0.4993	0.7019	0.3354	0.0429	0.0449	0.0494	0.0403	0.0418	0.0477
3NN	0.4990	0.7018	0.3353	0.0409	0.0420	0.0474	0.0393	0.0404	0.0466
5NN	0.4984	0.7013	0.3352	0.0404	0.0411	0.0470	0.0391	0.0401	0.0464
10NN	0.4972	0.7004	0.3348	0.0398	0.0403	0.0466	0.0389	0.0400	0.0463
20NN	0.4951	0.6987	0.3340	0.0394	0.0403	0.0463	0.0388	0.0399	0.0464
					Canberra				
1NN	0.4971	0.7011	0.3357	0.0424	0.0444	0.0495	0.0404	0.0418	0.0478
3NN	0.4956	0.7002	0.3354	0.0404	0.0412	0.0474	0.0394	0.0405	0.0467
5NN	0.4945	0.6994	0.3350	0.0398	0.0403	0.0468	0.0391	0.0402	0.0466
10NN	0.4928	0.6981	0.3339	0.0392	0.0392	0.0463	0.0390	0.0401	0.0464
20NN	0.4908	0.6958	0.3320	0.0390	0.0392	0.0458	0.0389	0.0399	0.0463

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	TCF$_1$	TCF$_2$	TCF$_3$	MC.sd$_1$	MC.sd$_2$	MC.sd$_3$	asy.sd$_1$	asy.sd$_2$	asy.sd$_3$
				Mahalanobis					
1NN	0.4985	0.7005	0.3343	0.0429	0.0447	0.0490	0.0405	0.0419	0.0477
3NN	0.4974	0.6991	0.3332	0.0407	0.0413	0.0468	0.0396	0.0406	0.0466
5NN	0.4964	0.6980	0.3322	0.0401	0.0403	0.0464	0.0394	0.0403	0.0463
10NN	0.4944	0.6956	0.3303	0.0395	0.0391	0.0457	0.0393	0.0401	0.0461
20NN	0.4909	0.6920	0.3272	0.0389	0.0391	0.0451	0.0392	0.0400	0.0459
				cut points = $(4, 5)$					
True	0.8031	0.1651	0.6651						
				Euclidean					
1NN	0.8027	0.1653	0.6653	0.0357	0.0342	0.0545	0.0334	0.0328	0.0509
3NN	0.8024	0.1652	0.6650	0.0335	0.0319	0.0512	0.0324	0.0319	0.0494
5NN	0.8020	0.1652	0.6648	0.0329	0.0313	0.0505	0.0322	0.0317	0.0491
10NN	0.8010	0.1648	0.6645	0.0325	0.0304	0.0498	0.0320	0.0315	0.0490
20NN	0.7995	0.1645	0.6638	0.0322	0.0295	0.0493	0.0320	0.0314	0.0490
				Manhattan					
1NN	0.8028	0.1654	0.6652	0.0355	0.0342	0.0545	0.0334	0.0328	0.0509
3NN	0.8023	0.1652	0.6650	0.0335	0.0320	0.0513	0.0323	0.0319	0.0494
5NN	0.8019	0.1651	0.6648	0.0330	0.0313	0.0504	0.0321	0.0317	0.0491
10NN	0.8010	0.1648	0.6644	0.0325	0.0305	0.0498	0.0320	0.0315	0.0490
20NN	0.7994	0.1644	0.6636	0.0322	0.0296	0.0493	0.0320	0.0314	0.0490
				Canberra					
1NN	0.8014	0.1652	0.6651	0.0351	0.0332	0.0545	0.0335	0.0328	0.0511
3NN	0.8002	0.1648	0.6642	0.0332	0.0310	0.0512	0.0325	0.0318	0.0495
5NN	0.7994	0.1647	0.6636	0.0326	0.0302	0.0504	0.0323	0.0316	0.0493
10NN	0.7980	0.1642	0.6622	0.0321	0.0293	0.0496	0.0322	0.0314	0.0491
20NN	0.7957	0.1635	0.6593	0.0318	0.0285	0.0489	0.0323	0.0312	0.0491
				Mahalanobis					
1NN	0.8019	0.1647	0.6634	0.0356	0.0334	0.0544	0.0336	0.0328	0.0512
3NN	0.8012	0.1645	0.6619	0.0333	0.0309	0.0510	0.0326	0.0318	0.0496
5NN	0.8004	0.1642	0.6606	0.0328	0.0300	0.0501	0.0324	0.0316	0.0493
10NN	0.7986	0.1635	0.6580	0.0324	0.0289	0.0492	0.0324	0.0313	0.0491
20NN	0.7955	0.1624	0.6534	0.0319	0.0277	0.0484	0.0325	0.0311	0.0491
				cut points = $(4, 7)$					
True	0.8031	0.3996	0.3349						
				Euclidean					
1NN	0.8027	0.3994	0.3353	0.0357	0.0469	0.0495	0.0334	0.0436	0.0477
3NN	0.8024	0.3997	0.3353	0.0335	0.0438	0.0473	0.0324	0.0423	0.0466
5NN	0.8020	0.3997	0.3352	0.0329	0.0432	0.0469	0.0322	0.0421	0.0464
10NN	0.8010	0.3994	0.3349	0.0325	0.0424	0.0466	0.0320	0.0418	0.0463
20NN	0.7995	0.3988	0.3342	0.0322	0.0416	0.0463	0.0320	0.0417	0.0464
				Manhattan					
1NN	0.8028	0.3997	0.3354	0.0355	0.0468	0.0494	0.0334	0.0436	0.0477
3NN	0.8023	0.3997	0.3353	0.0335	0.0440	0.0474	0.0323	0.0423	0.0466
5NN	0.8019	0.3996	0.3352	0.0330	0.0433	0.0470	0.0321	0.0420	0.0464
10NN	0.8010	0.3994	0.3348	0.0325	0.0425	0.0466	0.0320	0.0418	0.0463
20NN	0.7994	0.3988	0.3340	0.0322	0.0417	0.0463	0.0320	0.0417	0.0464
				Canberra					
1NN	0.8014	0.3999	0.3357	0.0351	0.0462	0.0495	0.0335	0.0436	0.0478
3NN	0.8002	0.3996	0.3354	0.0332	0.0433	0.0474	0.0325	0.0423	0.0467
5NN	0.7994	0.3994	0.3350	0.0326	0.0426	0.0468	0.0323	0.0420	0.0466
10NN	0.7980	0.3987	0.3339	0.0321	0.0416	0.0463	0.0322	0.0418	0.0464
20NN	0.7957	0.3973	0.3320	0.0318	0.0407	0.0458	0.0323	0.0416	0.0463
				Mahalanobis					
1NN	0.8019	0.3985	0.3343	0.0356	0.0461	0.0490	0.0336	0.0436	0.0477

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	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
3NN	0.8012	0.3979	0.3332	0.0333	0.0431	0.0468	0.0326	0.0423	0.0466
5NN	0.8004	0.3973	0.3322	0.0328	0.0423	0.0464	0.0324	0.0420	0.0463
10NN	0.7986	0.3960	0.3303	0.0324	0.0412	0.0457	0.0324	0.0417	0.0461
20NN	0.7955	0.3940	0.3272	0.0319	0.0400	0.0451	0.0325	0.0415	0.0459
				cut points = $(5, 7)$					
True	0.8996	0.2345	0.3349						
				Euclidean					
1NN	0.8990	0.2341	0.3353	0.0271	0.0397	0.0495	0.0256	0.0374	0.0477
3NN	0.8988	0.2345	0.3353	0.0254	0.0374	0.0473	0.0247	0.0363	0.0466
5NN	0.8985	0.2345	0.3352	0.0249	0.0368	0.0469	0.0246	0.0361	0.0464
10NN	0.8979	0.2345	0.3349	0.0246	0.0361	0.0466	0.0245	0.0359	0.0463
20NN	0.8968	0.2343	0.3342	0.0244	0.0353	0.0463	0.0245	0.0358	0.0464
				Manhattan					
1NN	0.8990	0.2343	0.3354	0.0270	0.0397	0.0494	0.0256	0.0374	0.0477
3NN	0.8988	0.2345	0.3353	0.0254	0.0375	0.0474	0.0247	0.0363	0.0466
5NN	0.8985	0.2345	0.3352	0.0250	0.0368	0.0470	0.0246	0.0361	0.0464
10NN	0.8979	0.2346	0.3348	0.0246	0.0361	0.0466	0.0245	0.0359	0.0463
20NN	0.8967	0.2343	0.3340	0.0244	0.0354	0.0463	0.0245	0.0358	0.0464
				Canberra					
1NN	0.8983	0.2347	0.3357	0.0269	0.0394	0.0495	0.0258	0.0374	0.0478
3NN	0.8975	0.2348	0.3354	0.0251	0.0370	0.0474	0.0249	0.0363	0.0467
5NN	0.8969	0.2347	0.3350	0.0247	0.0363	0.0468	0.0248	0.0361	0.0466
10NN	0.8959	0.2345	0.3339	0.0243	0.0354	0.0463	0.0247	0.0359	0.0464
20NN	0.8941	0.2338	0.3320	0.0241	0.0345	0.0458	0.0248	0.0357	0.0463
				Mahalanobis					
1NN	0.8985	0.2337	0.3343	0.0271	0.0391	0.0490	0.0258	0.0373	0.0477
3NN	0.8979	0.2334	0.3332	0.0252	0.0366	0.0468	0.0250	0.0362	0.0466
5NN	0.8973	0.2331	0.3322	0.0248	0.0359	0.0464	0.0248	0.0360	0.0463
10NN	0.8960	0.2325	0.3303	0.0244	0.0349	0.0457	0.0248	0.0357	0.0461
20NN	0.8937	0.2316	0.3272	0.0242	0.0338	0.0451	0.0250	0.0355	0.0459

Table 21 – *Monte Carlo means, Monte Carlo standard deviations and estimated standard deviations of the estimators for true class fractions corresponding to the case of the first value of Σ and sample size = 1000. "True" denotes the true parameter value.*

	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
				cut points = $(2, 4)$					
True	0.5000	0.4347	0.9347						
				Euclidean					
1NN	0.5002	0.4349	0.9340	0.0295	0.0336	0.0194	0.0278	0.0313	0.0189
3NN	0.4997	0.4346	0.9336	0.0283	0.0318	0.0183	0.0273	0.0305	0.0183
5NN	0.4993	0.4344	0.9333	0.0280	0.0313	0.0180	0.0272	0.0304	0.0183
10NN	0.4985	0.4341	0.9328	0.0276	0.0304	0.0178	0.0272	0.0302	0.0182
20NN	0.4973	0.4333	0.9318	0.0273	0.0296	0.0176	0.0271	0.0301	0.0183
				Manhattan					
1NN	0.5002	0.4349	0.9341	0.0295	0.0336	0.0193	0.0278	0.0313	0.0188
3NN	0.5000	0.4348	0.9336	0.0283	0.0318	0.0183	0.0273	0.0305	0.0183
5NN	0.4997	0.4346	0.9334	0.0280	0.0313	0.0181	0.0272	0.0304	0.0182
10NN	0.4990	0.4344	0.9328	0.0277	0.0305	0.0178	0.0272	0.0302	0.0182
20NN	0.4979	0.4337	0.9319	0.0274	0.0298	0.0176	0.0271	0.0301	0.0183

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	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
				Canberra					
1NN	0.4935	0.4343	0.9313	0.0297	0.0324	0.0197	0.0279	0.0315	0.0198
3NN	0.4895	0.4347	0.9296	0.0283	0.0303	0.0186	0.0274	0.0307	0.0194
5NN	0.4867	0.4347	0.9283	0.0278	0.0295	0.0184	0.0272	0.0305	0.0195
10NN	0.4818	0.4341	0.9257	0.0270	0.0285	0.0183	0.0271	0.0304	0.0197
20NN	0.4765	0.4326	0.9217	0.0263	0.0277	0.0184	0.0270	0.0303	0.0200
				Mahalanobis					
1NN	0.5006	0.4354	0.9338	0.0295	0.0337	0.0194	0.0279	0.0314	0.0190
3NN	0.5003	0.4355	0.9332	0.0284	0.0318	0.0183	0.0274	0.0306	0.0185
5NN	0.5001	0.4354	0.9328	0.0281	0.0310	0.0180	0.0274	0.0304	0.0184
10NN	0.4996	0.4354	0.9319	0.0277	0.0302	0.0178	0.0273	0.0303	0.0184
20NN	0.4985	0.4350	0.9301	0.0274	0.0293	0.0176	0.0273	0.0301	0.0186
				cut points = $(2, 5)$					
True	0.5000	0.7099	0.7752						
				Euclidean					
1NN	0.5002	0.7093	0.7741	0.0295	0.0319	0.0331	0.0278	0.0295	0.0312
3NN	0.4997	0.7083	0.7733	0.0283	0.0297	0.0312	0.0273	0.0287	0.0304
5NN	0.4993	0.7075	0.7728	0.0280	0.0292	0.0306	0.0272	0.0285	0.0303
10NN	0.4985	0.7057	0.7715	0.0276	0.0282	0.0301	0.0272	0.0284	0.0302
20NN	0.4973	0.7028	0.7694	0.0273	0.0282	0.0298	0.0271	0.0283	0.0302
				Manhattan					
1NN	0.5002	0.7094	0.7741	0.0295	0.0318	0.0331	0.0278	0.0295	0.0312
3NN	0.5000	0.7086	0.7733	0.0283	0.0298	0.0312	0.0273	0.0287	0.0304
5NN	0.4997	0.7078	0.7728	0.0280	0.0291	0.0307	0.0272	0.0285	0.0303
10NN	0.4990	0.7062	0.7715	0.0277	0.0283	0.0302	0.0272	0.0284	0.0302
20NN	0.4979	0.7034	0.7694	0.0274	0.0283	0.0299	0.0271	0.0283	0.0302
				Canberra					
1NN	0.4935	0.7037	0.7685	0.0297	0.0303	0.0328	0.0279	0.0298	0.0318
3NN	0.4895	0.7013	0.7659	0.0283	0.0281	0.0311	0.0274	0.0290	0.0312
5NN	0.4867	0.6993	0.7639	0.0278	0.0274	0.0305	0.0272	0.0289	0.0311
10NN	0.4818	0.6953	0.7603	0.0270	0.0264	0.0302	0.0271	0.0288	0.0312
20NN	0.4765	0.6897	0.7551	0.0263	0.0264	0.0299	0.0270	0.0288	0.0312
				Mahalanobis					
1NN	0.5006	0.7096	0.7737	0.0295	0.0321	0.0330	0.0279	0.0296	0.0313
3NN	0.5003	0.7085	0.7725	0.0284	0.0297	0.0311	0.0274	0.0288	0.0306
5NN	0.5001	0.7076	0.7715	0.0281	0.0290	0.0306	0.0274	0.0286	0.0304
10NN	0.4996	0.7057	0.7695	0.0277	0.0282	0.0301	0.0273	0.0285	0.0304
20NN	0.4985	0.7021	0.7659	0.0274	0.0282	0.0297	0.0273	0.0284	0.0304
				cut points = $(2, 7)$					
True	0.5000	0.9230	0.2248						
				Euclidean					
1NN	0.5002	0.9223	0.2239	0.0295	0.0187	0.0292	0.0278	0.0179	0.0283
3NN	0.4997	0.9213	0.2234	0.0283	0.0173	0.0283	0.0273	0.0174	0.0280
5NN	0.4993	0.9206	0.2229	0.0280	0.0169	0.0281	0.0272	0.0173	0.0279
10NN	0.4985	0.9190	0.2220	0.0276	0.0164	0.0279	0.0272	0.0173	0.0279
20NN	0.4973	0.9163	0.2208	0.0273	0.0164	0.0276	0.0271	0.0174	0.0278
				Manhattan					
1NN	0.5002	0.9222	0.2238	0.0295	0.0187	0.0292	0.0278	0.0179	0.0283
3NN	0.5000	0.9213	0.2232	0.0283	0.0173	0.0284	0.0273	0.0174	0.0280
5NN	0.4997	0.9206	0.2226	0.0280	0.0169	0.0281	0.0272	0.0173	0.0279
10NN	0.4990	0.9190	0.2217	0.0277	0.0164	0.0279	0.0272	0.0173	0.0278
20NN	0.4979	0.9163	0.2201	0.0274	0.0164	0.0276	0.0271	0.0174	0.0278
				Canberra					
1NN	0.4935	0.9168	0.2200	0.0297	0.0183	0.0287	0.0279	0.0189	0.0284

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	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
3NN	0.4895	0.9136	0.2184	0.0283	0.0170	0.0279	0.0274	0.0185	0.0282
5NN	0.4867	0.9111	0.2173	0.0278	0.0166	0.0277	0.0272	0.0185	0.0281
10NN	0.4818	0.9061	0.2155	0.0270	0.0160	0.0274	0.0271	0.0187	0.0281
20NN	0.4765	0.8991	0.2128	0.0263	0.0160	0.0270	0.0270	0.0191	0.0280
					Mahalanobis				
1NN	0.5006	0.9218	0.2233	0.0295	0.0187	0.0291	0.0279	0.0181	0.0283
3NN	0.5003	0.9204	0.2223	0.0284	0.0173	0.0283	0.0274	0.0176	0.0280
5NN	0.5001	0.9193	0.2216	0.0281	0.0169	0.0280	0.0274	0.0175	0.0279
10NN	0.4996	0.9170	0.2201	0.0277	0.0164	0.0278	0.0273	0.0176	0.0278
20NN	0.4985	0.9132	0.2178	0.0274	0.0164	0.0274	0.0273	0.0177	0.0278
					cut points = $(4, 5)$				
True	0.9347	0.2752	0.7752						
					Euclidean				
1NN	0.9337	0.2745	0.7741	0.0198	0.0286	0.0331	0.0172	0.0277	0.0312
3NN	0.9329	0.2737	0.7733	0.0179	0.0268	0.0312	0.0166	0.0270	0.0304
5NN	0.9323	0.2731	0.7728	0.0173	0.0263	0.0306	0.0166	0.0269	0.0303
10NN	0.9310	0.2717	0.7715	0.0166	0.0256	0.0301	0.0166	0.0267	0.0302
20NN	0.9289	0.2695	0.7694	0.0159	0.0248	0.0298	0.0166	0.0266	0.0302
					Manhattan				
1NN	0.9339	0.2745	0.7741	0.0197	0.0287	0.0331	0.0171	0.0277	0.0312
3NN	0.9332	0.2738	0.7733	0.0179	0.0269	0.0312	0.0166	0.0270	0.0304
5NN	0.9326	0.2732	0.7728	0.0173	0.0264	0.0307	0.0165	0.0269	0.0303
10NN	0.9314	0.2718	0.7715	0.0166	0.0257	0.0302	0.0165	0.0267	0.0302
20NN	0.9294	0.2697	0.7694	0.0159	0.0249	0.0299	0.0166	0.0266	0.0302
					Canberra				
1NN	0.9216	0.2694	0.7685	0.0205	0.0271	0.0328	0.0190	0.0276	0.0318
3NN	0.9149	0.2666	0.7659	0.0179	0.0252	0.0311	0.0187	0.0269	0.0312
5NN	0.9106	0.2646	0.7639	0.0171	0.0246	0.0305	0.0187	0.0267	0.0311
10NN	0.9040	0.2612	0.7603	0.0164	0.0239	0.0302	0.0189	0.0265	0.0312
20NN	0.8979	0.2571	0.7551	0.0158	0.0232	0.0299	0.0191	0.0263	0.0312
					Mahalanobis				
1NN	0.9338	0.2741	0.7737	0.0199	0.0284	0.0330	0.0173	0.0277	0.0313
3NN	0.9329	0.2731	0.7725	0.0180	0.0266	0.0311	0.0167	0.0270	0.0306
5NN	0.9323	0.2722	0.7715	0.0173	0.0259	0.0306	0.0167	0.0269	0.0304
10NN	0.9308	0.2702	0.7695	0.0167	0.0253	0.0301	0.0167	0.0267	0.0304
20NN	0.9284	0.2671	0.7659	0.0160	0.0244	0.0297	0.0168	0.0265	0.0304
					cut points = $(4, 7)$				
True	0.9347	0.4883	0.2248						
					Euclidean				
1NN	0.9337	0.4874	0.2239	0.0198	0.0342	0.0292	0.0172	0.0316	0.0283
3NN	0.9329	0.4867	0.2234	0.0179	0.0322	0.0283	0.0166	0.0308	0.0280
5NN	0.9323	0.4861	0.2229	0.0173	0.0316	0.0281	0.0166	0.0307	0.0279
10NN	0.9310	0.4849	0.2220	0.0166	0.0308	0.0279	0.0166	0.0305	0.0279
20NN	0.9289	0.4830	0.2208	0.0159	0.0300	0.0276	0.0166	0.0304	0.0278
					Manhattan				
1NN	0.9339	0.4873	0.2238	0.0197	0.0341	0.0292	0.0171	0.0316	0.0283
3NN	0.9332	0.4865	0.2232	0.0179	0.0322	0.0284	0.0166	0.0308	0.0280
5NN	0.9326	0.4859	0.2226	0.0173	0.0316	0.0281	0.0165	0.0307	0.0279
10NN	0.9314	0.4846	0.2217	0.0166	0.0309	0.0279	0.0165	0.0305	0.0278
20NN	0.9294	0.4825	0.2201	0.0159	0.0301	0.0276	0.0166	0.0304	0.0278
					Canberra				
1NN	0.9216	0.4824	0.2200	0.0205	0.0326	0.0287	0.0190	0.0318	0.0284
3NN	0.9149	0.4789	0.2184	0.0179	0.0306	0.0279	0.0187	0.0310	0.0282
5NN	0.9106	0.4764	0.2173	0.0171	0.0299	0.0277	0.0187	0.0308	0.0281

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	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
10NN	0.9040	0.4720	0.2155	0.0164	0.0291	0.0274	0.0189	0.0307	0.0281
20NN	0.8979	0.4665	0.2128	0.0158	0.0283	0.0270	0.0191	0.0305	0.0280
					Mahalanobis				
1NN	0.9338	0.4863	0.2233	0.0199	0.0341	0.0291	0.0173	0.0317	0.0283
3NN	0.9329	0.4849	0.2223	0.0180	0.0321	0.0283	0.0167	0.0309	0.0280
5NN	0.9323	0.4839	0.2216	0.0173	0.0314	0.0280	0.0167	0.0307	0.0279
10NN	0.9308	0.4816	0.2201	0.0167	0.0306	0.0278	0.0167	0.0305	0.0278
20NN	0.9284	0.4781	0.2178	0.0160	0.0298	0.0274	0.0168	0.0304	0.0278
					cut points $= (5, 7)$				
True	0.9883	0.2132	0.2248						
					Euclidean				
1NN	0.9876	0.2130	0.2239	0.0096	0.0289	0.0292	0.0085	0.0266	0.0283
3NN	0.9871	0.2130	0.2234	0.0084	0.0268	0.0283	0.0083	0.0259	0.0280
5NN	0.9869	0.2131	0.2229	0.0080	0.0263	0.0281	0.0083	0.0257	0.0279
10NN	0.9862	0.2132	0.2220	0.0075	0.0253	0.0279	0.0083	0.0256	0.0279
20NN	0.9851	0.2135	0.2208	0.0072	0.0244	0.0276	0.0084	0.0255	0.0278
					Manhattan				
1NN	0.9876	0.2128	0.2238	0.0096	0.0289	0.0292	0.0085	0.0266	0.0283
3NN	0.9873	0.2128	0.2232	0.0084	0.0269	0.0284	0.0082	0.0259	0.0280
5NN	0.9870	0.2127	0.2226	0.0080	0.0262	0.0281	0.0082	0.0257	0.0279
10NN	0.9863	0.2128	0.2217	0.0075	0.0253	0.0279	0.0083	0.0256	0.0278
20NN	0.9852	0.2128	0.2201	0.0072	0.0245	0.0276	0.0084	0.0255	0.0278
					Canberra				
1NN	0.9791	0.2131	0.2200	0.0108	0.0269	0.0287	0.0116	0.0266	0.0284
3NN	0.9748	0.2123	0.2184	0.0093	0.0248	0.0279	0.0116	0.0259	0.0282
5NN	0.9721	0.2118	0.2173	0.0089	0.0242	0.0277	0.0118	0.0257	0.0281
10NN	0.9680	0.2108	0.2155	0.0087	0.0233	0.0274	0.0121	0.0256	0.0281
20NN	0.9645	0.2094	0.2128	0.0084	0.0226	0.0270	0.0124	0.0254	0.0280
					Mahalanobis				
1NN	0.9875	0.2122	0.2233	0.0097	0.0288	0.0291	0.0086	0.0267	0.0283
3NN	0.9870	0.2119	0.2223	0.0085	0.0267	0.0283	0.0084	0.0259	0.0280
5NN	0.9866	0.2117	0.2216	0.0081	0.0260	0.0280	0.0084	0.0257	0.0279
10NN	0.9858	0.2113	0.2201	0.0077	0.0251	0.0278	0.0085	0.0256	0.0278
20NN	0.9843	0.2110	0.2178	0.0074	0.0242	0.0274	0.0087	0.0254	0.0278

Table 22 – *Monte Carlo means, Monte Carlo standard deviations and estimated standard deviations of the estimators for true class fractions corresponding to the case of the second value of Σ and sample size $= 1000$. "True" denotes the true parameter value.*

	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
					cut points $= (2, 4)$				
True	0.5000	0.3970	0.8970						
					Euclidean				
1NN	0.4989	0.3958	0.8974	0.0291	0.0328	0.0279	0.0280	0.0310	0.0246
3NN	0.4981	0.3950	0.8974	0.0279	0.0307	0.0258	0.0274	0.0301	0.0238
5NN	0.4975	0.3943	0.8973	0.0275	0.0301	0.0252	0.0273	0.0300	0.0236
10NN	0.4962	0.3931	0.8972	0.0271	0.0294	0.0246	0.0272	0.0298	0.0236
20NN	0.4940	0.3910	0.8966	0.0268	0.0288	0.0242	0.0271	0.0297	0.0236
					Manhattan				
1NN	0.4990	0.3959	0.8975	0.0291	0.0327	0.0279	0.0279	0.0310	0.0246

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	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
3NN	0.4981	0.3950	0.8974	0.0279	0.0307	0.0258	0.0274	0.0301	0.0238
5NN	0.4975	0.3944	0.8973	0.0276	0.0302	0.0252	0.0273	0.0300	0.0237
10NN	0.4963	0.3932	0.8971	0.0272	0.0294	0.0247	0.0272	0.0298	0.0236
20NN	0.4942	0.3912	0.8967	0.0268	0.0288	0.0242	0.0271	0.0297	0.0236
					Canberra				
1NN	0.4951	0.3932	0.8958	0.0291	0.0319	0.0278	0.0280	0.0310	0.0251
3NN	0.4921	0.3914	0.8948	0.0276	0.0297	0.0258	0.0274	0.0301	0.0243
5NN	0.4899	0.3900	0.8939	0.0271	0.0290	0.0254	0.0273	0.0299	0.0242
10NN	0.4853	0.3868	0.8919	0.0267	0.0279	0.0250	0.0272	0.0297	0.0243
20NN	0.4792	0.3825	0.8885	0.0261	0.0270	0.0249	0.0271	0.0295	0.0245
					Mahalanobis				
1NN	0.4988	0.3958	0.8963	0.0292	0.0325	0.0280	0.0281	0.0311	0.0251
3NN	0.4980	0.3949	0.8952	0.0280	0.0304	0.0259	0.0275	0.0302	0.0242
5NN	0.4974	0.3942	0.8945	0.0276	0.0298	0.0254	0.0274	0.0300	0.0241
10NN	0.4960	0.3929	0.8929	0.0272	0.0289	0.0248	0.0274	0.0298	0.0241
20NN	0.4937	0.3906	0.8900	0.0268	0.0280	0.0243	0.0274	0.0296	0.0243
					cut points = $(2, 5)$				
True	0.5000	0.6335	0.7365						
					Euclidean				
1NN	0.4989	0.6322	0.7372	0.0291	0.0325	0.0369	0.0280	0.0308	0.0338
3NN	0.4981	0.6312	0.7372	0.0279	0.0303	0.0345	0.0274	0.0299	0.0328
5NN	0.4975	0.6305	0.7372	0.0275	0.0298	0.0338	0.0273	0.0298	0.0327
10NN	0.4962	0.6289	0.7371	0.0271	0.0291	0.0332	0.0272	0.0296	0.0326
20NN	0.4940	0.6261	0.7367	0.0268	0.0291	0.0328	0.0271	0.0296	0.0326
					Manhattan				
1NN	0.4990	0.6323	0.7372	0.0291	0.0325	0.0369	0.0279	0.0308	0.0338
3NN	0.4981	0.6312	0.7372	0.0279	0.0304	0.0345	0.0274	0.0299	0.0329
5NN	0.4975	0.6305	0.7371	0.0276	0.0299	0.0338	0.0273	0.0298	0.0327
10NN	0.4963	0.6289	0.7370	0.0272	0.0291	0.0332	0.0272	0.0296	0.0326
20NN	0.4942	0.6263	0.7367	0.0268	0.0291	0.0328	0.0271	0.0296	0.0326
					Canberra				
1NN	0.4951	0.6285	0.7369	0.0291	0.0318	0.0365	0.0280	0.0310	0.0340
3NN	0.4921	0.6257	0.7364	0.0276	0.0295	0.0343	0.0274	0.0301	0.0330
5NN	0.4899	0.6236	0.7358	0.0271	0.0288	0.0338	0.0273	0.0299	0.0329
10NN	0.4853	0.6188	0.7341	0.0267	0.0278	0.0332	0.0272	0.0298	0.0329
20NN	0.4792	0.6124	0.7309	0.0261	0.0278	0.0329	0.0271	0.0297	0.0329
					Mahalanobis				
1NN	0.4988	0.6315	0.7357	0.0292	0.0323	0.0366	0.0281	0.0309	0.0341
3NN	0.4980	0.6300	0.7345	0.0280	0.0302	0.0343	0.0275	0.0300	0.0331
5NN	0.4974	0.6288	0.7336	0.0276	0.0295	0.0338	0.0274	0.0298	0.0329
10NN	0.4960	0.6264	0.7317	0.0272	0.0287	0.0331	0.0274	0.0297	0.0328
20NN	0.4937	0.6226	0.7285	0.0268	0.0287	0.0325	0.0274	0.0296	0.0328
					cut points = $(2, 7)$				
True	0.5000	0.8682	0.2635						
					Euclidean				
1NN	0.4989	0.8684	0.2643	0.0291	0.0247	0.0313	0.0280	0.0229	0.0300
3NN	0.4981	0.8678	0.2643	0.0279	0.0228	0.0304	0.0274	0.0221	0.0297
5NN	0.4975	0.8674	0.2644	0.0275	0.0223	0.0301	0.0273	0.0220	0.0296
10NN	0.4962	0.8665	0.2645	0.0271	0.0217	0.0299	0.0272	0.0219	0.0296
20NN	0.4940	0.8648	0.2645	0.0268	0.0217	0.0299	0.0271	0.0219	0.0297
					Manhattan				
1NN	0.4990	0.8684	0.2642	0.0291	0.0247	0.0312	0.0279	0.0229	0.0300
3NN	0.4981	0.8678	0.2643	0.0279	0.0228	0.0304	0.0274	0.0221	0.0297
5NN	0.4975	0.8675	0.2643	0.0276	0.0223	0.0301	0.0273	0.0220	0.0296

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	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
10NN	0.4963	0.8665	0.2644	0.0272	0.0217	0.0299	0.0272	0.0219	0.0296
20NN	0.4942	0.8648	0.2643	0.0268	0.0217	0.0299	0.0271	0.0219	0.0297
				Canberra					
1NN	0.4951	0.8660	0.2653	0.0291	0.0246	0.0312	0.0280	0.0233	0.0303
3NN	0.4921	0.8639	0.2655	0.0276	0.0224	0.0303	0.0274	0.0225	0.0300
5NN	0.4899	0.8620	0.2655	0.0271	0.0218	0.0302	0.0273	0.0224	0.0299
10NN	0.4853	0.8574	0.2650	0.0267	0.0211	0.0300	0.0272	0.0225	0.0299
20NN	0.4792	0.8510	0.2639	0.0261	0.0211	0.0298	0.0271	0.0226	0.0299
				Mahalanobis					
1NN	0.4988	0.8668	0.2634	0.0292	0.0248	0.0311	0.0281	0.0232	0.0300
3NN	0.4980	0.8653	0.2628	0.0280	0.0227	0.0302	0.0275	0.0225	0.0296
5NN	0.4974	0.8641	0.2624	0.0276	0.0222	0.0299	0.0274	0.0223	0.0295
10NN	0.4960	0.8618	0.2616	0.0272	0.0216	0.0297	0.0274	0.0223	0.0295
20NN	0.4937	0.8580	0.2602	0.0268	0.0216	0.0294	0.0274	0.0223	0.0294
				cut points = $(4, 5)$					
True	0.8970	0.2365	0.7365						
				Euclidean					
1NN	0.8963	0.2364	0.7372	0.0198	0.0276	0.0369	0.0185	0.0263	0.0338
3NN	0.8958	0.2362	0.7372	0.0186	0.0260	0.0345	0.0179	0.0256	0.0328
5NN	0.8954	0.2361	0.7372	0.0183	0.0254	0.0338	0.0178	0.0255	0.0327
10NN	0.8945	0.2358	0.7371	0.0179	0.0248	0.0332	0.0178	0.0254	0.0326
20NN	0.8930	0.2351	0.7367	0.0177	0.0242	0.0328	0.0178	0.0253	0.0326
				Manhattan					
1NN	0.8963	0.2364	0.7372	0.0199	0.0277	0.0369	0.0185	0.0263	0.0338
3NN	0.8958	0.2362	0.7372	0.0187	0.0260	0.0345	0.0179	0.0256	0.0329
5NN	0.8954	0.2361	0.7371	0.0183	0.0254	0.0338	0.0178	0.0255	0.0327
10NN	0.8945	0.2358	0.7370	0.0179	0.0248	0.0332	0.0178	0.0254	0.0326
20NN	0.8930	0.2351	0.7367	0.0177	0.0243	0.0328	0.0178	0.0253	0.0326
				Canberra					
1NN	0.8923	0.2354	0.7369	0.0196	0.0268	0.0365	0.0191	0.0262	0.0340
3NN	0.8897	0.2344	0.7364	0.0182	0.0248	0.0343	0.0186	0.0255	0.0330
5NN	0.8880	0.2336	0.7358	0.0179	0.0243	0.0338	0.0185	0.0254	0.0329
10NN	0.8850	0.2320	0.7341	0.0176	0.0237	0.0332	0.0186	0.0252	0.0329
20NN	0.8815	0.2299	0.7309	0.0174	0.0231	0.0329	0.0187	0.0250	0.0329
				Mahalanobis					
1NN	0.8962	0.2357	0.7357	0.0197	0.0274	0.0366	0.0186	0.0263	0.0341
3NN	0.8957	0.2352	0.7345	0.0185	0.0256	0.0343	0.0180	0.0256	0.0331
5NN	0.8952	0.2346	0.7336	0.0182	0.0250	0.0338	0.0179	0.0254	0.0329
10NN	0.8941	0.2335	0.7317	0.0179	0.0243	0.0331	0.0179	0.0253	0.0328
20NN	0.8921	0.2319	0.7285	0.0176	0.0235	0.0325	0.0180	0.0251	0.0328
				cut points = $(4, 7)$					
True	0.8970	0.4711	0.2635						
				Euclidean					
1NN	0.8963	0.4726	0.2643	0.0198	0.0342	0.0313	0.0185	0.0316	0.0300
3NN	0.8958	0.4728	0.2643	0.0186	0.0320	0.0304	0.0179	0.0307	0.0297
5NN	0.8954	0.4731	0.2644	0.0183	0.0314	0.0301	0.0178	0.0305	0.0296
10NN	0.8945	0.4734	0.2645	0.0179	0.0307	0.0299	0.0178	0.0304	0.0296
20NN	0.8930	0.4737	0.2645	0.0177	0.0302	0.0299	0.0178	0.0303	0.0297
				Manhattan					
1NN	0.8963	0.4725	0.2642	0.0199	0.0340	0.0312	0.0185	0.0316	0.0300
3NN	0.8958	0.4728	0.2643	0.0187	0.0319	0.0304	0.0179	0.0307	0.0297
5NN	0.8954	0.4730	0.2643	0.0183	0.0313	0.0301	0.0178	0.0305	0.0296
10NN	0.8945	0.4734	0.2644	0.0179	0.0308	0.0299	0.0178	0.0304	0.0296
20NN	0.8930	0.4736	0.2643	0.0177	0.0302	0.0299	0.0178	0.0303	0.0297

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	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
					Canberra				
1NN	0.8923	0.4728	0.2653	0.0196	0.0332	0.0312	0.0191	0.0316	0.0303
3NN	0.8897	0.4725	0.2655	0.0182	0.0311	0.0303	0.0186	0.0307	0.0300
5NN	0.8880	0.4720	0.2655	0.0179	0.0306	0.0302	0.0185	0.0305	0.0299
10NN	0.8850	0.4706	0.2650	0.0176	0.0300	0.0300	0.0186	0.0303	0.0299
20NN	0.8815	0.4685	0.2639	0.0174	0.0294	0.0298	0.0187	0.0302	0.0299
					Mahalanobis				
1NN	0.8962	0.4710	0.2634	0.0197	0.0338	0.0311	0.0186	0.0316	0.0300
3NN	0.8957	0.4704	0.2628	0.0185	0.0316	0.0302	0.0180	0.0307	0.0296
5NN	0.8952	0.4699	0.2624	0.0182	0.0311	0.0299	0.0179	0.0305	0.0295
10NN	0.8941	0.4690	0.2616	0.0179	0.0304	0.0297	0.0179	0.0303	0.0295
20NN	0.8921	0.4674	0.2602	0.0176	0.0297	0.0294	0.0180	0.0302	0.0294
					cut points = $(5,7)$				
True	0.9711	0.2347	0.2635						
					Euclidean				
1NN	0.9709	0.2362	0.2643	0.0111	0.0281	0.0313	0.0104	0.0266	0.0300
3NN	0.9707	0.2366	0.2643	0.0103	0.0262	0.0304	0.0100	0.0259	0.0297
5NN	0.9704	0.2369	0.2644	0.0101	0.0257	0.0301	0.0100	0.0258	0.0296
10NN	0.9700	0.2376	0.2645	0.0099	0.0252	0.0299	0.0100	0.0257	0.0296
20NN	0.9692	0.2386	0.2645	0.0098	0.0247	0.0299	0.0100	0.0256	0.0297
					Manhattan				
1NN	0.9709	0.2361	0.2642	0.0111	0.0280	0.0312	0.0104	0.0267	0.0300
3NN	0.9707	0.2366	0.2643	0.0103	0.0262	0.0304	0.0100	0.0259	0.0297
5NN	0.9705	0.2370	0.2643	0.0101	0.0257	0.0301	0.0100	0.0258	0.0296
10NN	0.9700	0.2376	0.2644	0.0099	0.0252	0.0299	0.0100	0.0257	0.0296
20NN	0.9692	0.2385	0.2643	0.0098	0.0247	0.0299	0.0100	0.0256	0.0297
					Canberra				
1NN	0.9689	0.2375	0.2653	0.0111	0.0273	0.0312	0.0110	0.0267	0.0303
3NN	0.9677	0.2381	0.2655	0.0102	0.0255	0.0303	0.0108	0.0260	0.0300
5NN	0.9668	0.2384	0.2655	0.0101	0.0250	0.0302	0.0108	0.0258	0.0299
10NN	0.9653	0.2386	0.2650	0.0099	0.0244	0.0300	0.0108	0.0257	0.0299
20NN	0.9635	0.2386	0.2639	0.0098	0.0239	0.0298	0.0110	0.0255	0.0299
					Mahalanobis				
1NN	0.9709	0.2353	0.2634	0.0110	0.0277	0.0311	0.0104	0.0266	0.0300
3NN	0.9705	0.2353	0.2628	0.0103	0.0259	0.0302	0.0101	0.0259	0.0296
5NN	0.9702	0.2353	0.2624	0.0101	0.0253	0.0299	0.0101	0.0257	0.0295
10NN	0.9697	0.2354	0.2616	0.0099	0.0248	0.0297	0.0101	0.0256	0.0295
20NN	0.9685	0.2354	0.2602	0.0098	0.0242	0.0294	0.0102	0.0255	0.0294

Table 23 – *Monte Carlo means, Monte Carlo standard deviations and estimated standard deviations of the estimators for true class fractions corresponding to the case of the third value of* Σ *and sample size* = 1000. *"True" denotes the true parameter value.*

	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
					cut points = $(2,4)$				
True	0.5000	0.3031	0.8031						
					Euclidean				
1NN	0.4999	0.3023	0.8027	0.0295	0.0313	0.0341	0.0285	0.0291	0.0313
3NN	0.4994	0.3023	0.8030	0.0281	0.0295	0.0316	0.0278	0.0283	0.0302
5NN	0.4991	0.3021	0.8030	0.0277	0.0290	0.0309	0.0277	0.0281	0.0300

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	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
10NN	0.4986	0.3018	0.8029	0.0274	0.0284	0.0303	0.0276	0.0279	0.0299
20NN	0.4975	0.3012	0.8026	0.0271	0.0280	0.0299	0.0275	0.0278	0.0299
				Manhattan					
1NN	0.4999	0.3023	0.8028	0.0295	0.0313	0.0341	0.0285	0.0291	0.0313
3NN	0.4995	0.3024	0.8029	0.0281	0.0295	0.0315	0.0278	0.0283	0.0302
5NN	0.4991	0.3021	0.8030	0.0277	0.0290	0.0309	0.0277	0.0281	0.0300
10NN	0.4986	0.3018	0.8029	0.0274	0.0285	0.0303	0.0276	0.0279	0.0299
20NN	0.4975	0.3012	0.8027	0.0271	0.0280	0.0299	0.0275	0.0278	0.0299
				Canberra					
1NN	0.4983	0.3018	0.8027	0.0293	0.0310	0.0339	0.0285	0.0291	0.0314
3NN	0.4970	0.3013	0.8023	0.0279	0.0288	0.0314	0.0278	0.0282	0.0304
5NN	0.4963	0.3010	0.8021	0.0273	0.0282	0.0308	0.0277	0.0281	0.0302
10NN	0.4949	0.3003	0.8013	0.0269	0.0274	0.0302	0.0276	0.0279	0.0301
20NN	0.4930	0.2995	0.7996	0.0266	0.0269	0.0298	0.0275	0.0278	0.0301
				Mahalanobis					
1NN	0.4992	0.3022	0.8019	0.0295	0.0311	0.0341	0.0286	0.0291	0.0315
3NN	0.4986	0.3017	0.8010	0.0279	0.0290	0.0315	0.0279	0.0282	0.0305
5NN	0.4980	0.3014	0.8003	0.0277	0.0284	0.0308	0.0278	0.0280	0.0303
10NN	0.4968	0.3008	0.7990	0.0273	0.0276	0.0301	0.0277	0.0279	0.0302
20NN	0.4947	0.2998	0.7965	0.0269	0.0269	0.0297	0.0277	0.0277	0.0302
				cut points $= (2, 5)$					
True	0.5000	0.4682	0.6651						
				Euclidean					
1NN	0.4999	0.4677	0.6650	0.0295	0.0344	0.0384	0.0285	0.0316	0.0360
3NN	0.4994	0.4676	0.6653	0.0281	0.0324	0.0363	0.0278	0.0307	0.0349
5NN	0.4991	0.4674	0.6653	0.0277	0.0319	0.0356	0.0277	0.0305	0.0346
10NN	0.4986	0.4670	0.6652	0.0274	0.0313	0.0350	0.0276	0.0303	0.0345
20NN	0.4975	0.4662	0.6649	0.0271	0.0313	0.0346	0.0275	0.0302	0.0345
				Manhattan					
1NN	0.4999	0.4677	0.6651	0.0295	0.0344	0.0384	0.0285	0.0316	0.0360
3NN	0.4995	0.4676	0.6652	0.0281	0.0324	0.0362	0.0278	0.0307	0.0349
5NN	0.4991	0.4674	0.6652	0.0277	0.0319	0.0357	0.0277	0.0305	0.0347
10NN	0.4986	0.4670	0.6652	0.0274	0.0314	0.0350	0.0276	0.0303	0.0345
20NN	0.4975	0.4662	0.6649	0.0271	0.0314	0.0346	0.0275	0.0302	0.0345
				Canberra					
1NN	0.4983	0.4670	0.6656	0.0293	0.0340	0.0383	0.0285	0.0316	0.0360
3NN	0.4970	0.4663	0.6652	0.0279	0.0317	0.0360	0.0278	0.0307	0.0349
5NN	0.4963	0.4659	0.6650	0.0273	0.0312	0.0355	0.0277	0.0305	0.0347
10NN	0.4949	0.4650	0.6643	0.0269	0.0305	0.0348	0.0276	0.0303	0.0346
20NN	0.4930	0.4638	0.6627	0.0266	0.0305	0.0344	0.0275	0.0302	0.0346
				Mahalanobis					
1NN	0.4992	0.4672	0.6642	0.0295	0.0341	0.0384	0.0286	0.0316	0.0361
3NN	0.4986	0.4666	0.6632	0.0279	0.0320	0.0361	0.0279	0.0307	0.0350
5NN	0.4980	0.4660	0.6625	0.0277	0.0314	0.0354	0.0278	0.0305	0.0348
10NN	0.4968	0.4650	0.6611	0.0273	0.0306	0.0348	0.0277	0.0303	0.0346
20NN	0.4947	0.4634	0.6585	0.0269	0.0306	0.0343	0.0277	0.0302	0.0346
				cut points $= (2, 7)$					
True	0.5000	0.7027	0.3349						
				Euclidean					
1NN	0.4999	0.7029	0.3351	0.0295	0.0323	0.0340	0.0285	0.0295	0.0336
3NN	0.4994	0.7027	0.3351	0.0281	0.0302	0.0328	0.0278	0.0286	0.0328
5NN	0.4991	0.7025	0.3351	0.0277	0.0296	0.0325	0.0277	0.0284	0.0327
10NN	0.4986	0.7021	0.3350	0.0274	0.0291	0.0323	0.0276	0.0282	0.0326
20NN	0.4975	0.7014	0.3348	0.0271	0.0291	0.0321	0.0275	0.0282	0.0326

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	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
					Manhattan				
1NN	0.4999	0.7028	0.3351	0.0295	0.0322	0.0340	0.0285	0.0295	0.0336
3NN	0.4995	0.7027	0.3351	0.0281	0.0302	0.0328	0.0278	0.0286	0.0328
5NN	0.4991	0.7025	0.3351	0.0277	0.0297	0.0325	0.0277	0.0284	0.0327
10NN	0.4986	0.7022	0.3350	0.0274	0.0291	0.0323	0.0276	0.0282	0.0326
20NN	0.4975	0.7013	0.3347	0.0271	0.0291	0.0321	0.0275	0.0282	0.0326
					Canberra				
1NN	0.4983	0.7023	0.3355	0.0293	0.0318	0.0339	0.0285	0.0296	0.0336
3NN	0.4970	0.7015	0.3354	0.0279	0.0297	0.0328	0.0278	0.0286	0.0329
5NN	0.4963	0.7012	0.3352	0.0273	0.0292	0.0325	0.0277	0.0284	0.0328
10NN	0.4949	0.7002	0.3348	0.0269	0.0285	0.0322	0.0276	0.0283	0.0327
20NN	0.4930	0.6989	0.3338	0.0266	0.0285	0.0319	0.0275	0.0282	0.0327
					Mahalanobis				
1NN	0.4992	0.7019	0.3344	0.0295	0.0321	0.0341	0.0286	0.0296	0.0336
3NN	0.4986	0.7010	0.3336	0.0279	0.0298	0.0326	0.0279	0.0287	0.0328
5NN	0.4980	0.7003	0.3332	0.0277	0.0292	0.0322	0.0278	0.0285	0.0327
10NN	0.4968	0.6988	0.3321	0.0273	0.0286	0.0319	0.0277	0.0283	0.0325
20NN	0.4947	0.6966	0.3303	0.0269	0.0286	0.0315	0.0277	0.0282	0.0324
					cut points = $(4, 5)$				
True	0.8031	0.1651	0.6651						
					Euclidean				
1NN	0.8029	0.1653	0.6650	0.0245	0.0240	0.0384	0.0236	0.0232	0.0360
3NN	0.8025	0.1652	0.6653	0.0233	0.0226	0.0363	0.0229	0.0225	0.0349
5NN	0.8023	0.1653	0.6653	0.0229	0.0222	0.0356	0.0227	0.0224	0.0346
10NN	0.8019	0.1652	0.6652	0.0226	0.0218	0.0350	0.0226	0.0223	0.0345
20NN	0.8011	0.1650	0.6649	0.0224	0.0213	0.0346	0.0226	0.0222	0.0345
					Manhattan				
1NN	0.8029	0.1653	0.6651	0.0245	0.0240	0.0384	0.0236	0.0232	0.0360
3NN	0.8025	0.1652	0.6652	0.0233	0.0226	0.0362	0.0229	0.0225	0.0349
5NN	0.8023	0.1652	0.6652	0.0229	0.0222	0.0357	0.0227	0.0224	0.0347
10NN	0.8019	0.1652	0.6652	0.0226	0.0218	0.0350	0.0226	0.0223	0.0345
20NN	0.8011	0.1650	0.6649	0.0224	0.0214	0.0346	0.0226	0.0222	0.0345
					Canberra				
1NN	0.8018	0.1652	0.6656	0.0243	0.0235	0.0383	0.0237	0.0232	0.0360
3NN	0.8010	0.1650	0.6652	0.0230	0.0220	0.0360	0.0230	0.0225	0.0349
5NN	0.8005	0.1649	0.6650	0.0227	0.0217	0.0355	0.0228	0.0224	0.0347
10NN	0.7995	0.1647	0.6643	0.0224	0.0212	0.0348	0.0227	0.0222	0.0346
20NN	0.7980	0.1644	0.6627	0.0221	0.0206	0.0344	0.0227	0.0222	0.0346
					Mahalanobis				
1NN	0.8024	0.1650	0.6642	0.0246	0.0236	0.0384	0.0237	0.0232	0.0361
3NN	0.8020	0.1649	0.6632	0.0233	0.0221	0.0361	0.0230	0.0225	0.0350
5NN	0.8015	0.1646	0.6625	0.0229	0.0216	0.0354	0.0229	0.0223	0.0348
10NN	0.8005	0.1643	0.6611	0.0226	0.0209	0.0348	0.0228	0.0222	0.0346
20NN	0.7987	0.1636	0.6585	0.0223	0.0203	0.0343	0.0228	0.0221	0.0346
					cut points = $(4, 7)$				
True	0.8031	0.3996	0.3349						
					Euclidean				
1NN	0.8029	0.4006	0.3351	0.0245	0.0323	0.0340	0.0236	0.0308	0.0336
3NN	0.8025	0.4003	0.3351	0.0233	0.0304	0.0328	0.0229	0.0299	0.0328
5NN	0.8023	0.4004	0.3351	0.0229	0.0300	0.0325	0.0227	0.0298	0.0327
10NN	0.8019	0.4003	0.3350	0.0226	0.0295	0.0323	0.0226	0.0296	0.0326
20NN	0.8011	0.4001	0.3348	0.0224	0.0292	0.0321	0.0226	0.0295	0.0326
					Manhattan				
1NN	0.8029	0.4005	0.3351	0.0245	0.0322	0.0340	0.0236	0.0308	0.0336

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	TCF_1	TCF_2	TCF_3	$MC.sd_1$	$MC.sd_2$	$MC.sd_3$	$asy.sd_1$	$asy.sd_2$	$asy.sd_3$
3NN	0.8025	0.4003	0.3351	0.0233	0.0303	0.0328	0.0229	0.0299	0.0328
5NN	0.8023	0.4003	0.3351	0.0229	0.0300	0.0325	0.0227	0.0298	0.0327
10NN	0.8019	0.4003	0.3350	0.0226	0.0296	0.0323	0.0226	0.0296	0.0326
20NN	0.8011	0.4001	0.3347	0.0224	0.0293	0.0321	0.0226	0.0295	0.0326
				Canberra					
1NN	0.8018	0.4006	0.3355	0.0243	0.0321	0.0339	0.0237	0.0309	0.0336
3NN	0.8010	0.4003	0.3354	0.0230	0.0302	0.0328	0.0230	0.0299	0.0329
5NN	0.8005	0.4001	0.3352	0.0227	0.0297	0.0325	0.0228	0.0298	0.0328
10NN	0.7995	0.3999	0.3348	0.0224	0.0292	0.0322	0.0227	0.0296	0.0327
20NN	0.7980	0.3994	0.3338	0.0221	0.0287	0.0319	0.0227	0.0295	0.0327
				Mahalanobis					
1NN	0.8024	0.3997	0.3344	0.0246	0.0321	0.0341	0.0237	0.0309	0.0336
3NN	0.8020	0.3993	0.3336	0.0233	0.0301	0.0326	0.0230	0.0299	0.0328
5NN	0.8015	0.3989	0.3332	0.0229	0.0296	0.0322	0.0229	0.0297	0.0327
10NN	0.8005	0.3980	0.3321	0.0226	0.0291	0.0319	0.0228	0.0296	0.0325
20NN	0.7987	0.3968	0.3303	0.0223	0.0285	0.0315	0.0228	0.0294	0.0324
				cut points = $(5, 7)$					
True	0.8996	0.2345	0.3349						
				Euclidean					
1NN	0.8997	0.2352	0.3351	0.0186	0.0274	0.0340	0.0181	0.0264	0.0336
3NN	0.8994	0.2351	0.3351	0.0175	0.0258	0.0328	0.0175	0.0257	0.0328
5NN	0.8992	0.2351	0.3351	0.0172	0.0254	0.0325	0.0173	0.0255	0.0327
10NN	0.8989	0.2351	0.3350	0.0170	0.0250	0.0323	0.0173	0.0254	0.0326
20NN	0.8983	0.2351	0.3348	0.0168	0.0247	0.0321	0.0172	0.0253	0.0326
				Manhattan					
1NN	0.8996	0.2352	0.3351	0.0186	0.0274	0.0340	0.0181	0.0264	0.0336
3NN	0.8994	0.2351	0.3351	0.0175	0.0259	0.0328	0.0175	0.0257	0.0328
5NN	0.8992	0.2351	0.3351	0.0172	0.0254	0.0325	0.0173	0.0255	0.0327
10NN	0.8989	0.2351	0.3350	0.0170	0.0250	0.0323	0.0173	0.0254	0.0326
20NN	0.8982	0.2351	0.3347	0.0168	0.0247	0.0321	0.0172	0.0253	0.0326
				Canberra					
1NN	0.8992	0.2354	0.3355	0.0184	0.0272	0.0339	0.0181	0.0265	0.0336
3NN	0.8985	0.2352	0.3354	0.0173	0.0256	0.0328	0.0175	0.0257	0.0329
5NN	0.8981	0.2352	0.3352	0.0171	0.0251	0.0325	0.0174	0.0255	0.0328
10NN	0.8974	0.2352	0.3348	0.0168	0.0247	0.0322	0.0174	0.0254	0.0327
20NN	0.8963	0.2351	0.3338	0.0167	0.0243	0.0319	0.0174	0.0253	0.0327
				Mahalanobis					
1NN	0.8994	0.2347	0.3344	0.0187	0.0271	0.0341	0.0181	0.0264	0.0336
3NN	0.8990	0.2344	0.3336	0.0175	0.0255	0.0326	0.0175	0.0257	0.0328
5NN	0.8986	0.2342	0.3332	0.0172	0.0251	0.0322	0.0175	0.0255	0.0327
10NN	0.8978	0.2338	0.3321	0.0170	0.0245	0.0319	0.0174	0.0253	0.0325
20NN	0.8964	0.2332	0.3303	0.0168	0.0240	0.0315	0.0175	0.0252	0.0324

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