



UNIVERSITÀ
DEGLI STUDI
DI PADOVA

Sede Amministrativa: Università degli Studi di Padova
Dipartimento di *Psicologia dello Sviluppo e dei Processi di Socializzazione*

SCUOLA DI DOTTORATO DI RICERCA IN: *Scienze Psicologiche*
INDIRIZZO: *Psicologia dello Sviluppo e dei Processi di Socializzazione*
CICLO: XXV°

Typical and Atypical Development of Numerical Representation

Direttore della Scuola : Ch.mo Prof. Clara Casco
Coordinatore d'indirizzo: Ch.mo Prof. Eloisa Valenza
Supervisore : Ch.mo Prof. Daniela Lucangeli

Dottorando : Francesco Sella

Abstract of the thesis

How numerical information is represented? Recent studies have highlighted the prominent role of preverbal core knowledge systems for representing numerical quantities: the Object Tracking System (OTS) and the Approximate Number System (ANS; or analogue magnitude system). The former is general mechanism which allows individuals to track the spatio-temporal characteristics of the objects and its capacity is limited (3-4 items). The latter is a quantitative mechanism which entails the representation of each numerosity as a distribution of activation on the mental number line. In the present work we investigated several aspects of these two systems along with numerical and non-numerical estimation ability in typical and atypical development.

In Study 1.1, we implemented an imitation task to investigate the spontaneous focusing on numerosity in 2 ½ year-old children. The results suggest that most of the children employed the analogue magnitude system when spontaneously encoding numerosity. The use of the analogue magnitude system may be related to both its low demanding of attentional resources and to the availability of other (non-numerical) quantitative cues which covariate with numerosity.

In Study 1.2, 2 ½ year-old children completed a categorization task in order to investigate their ability in estimating numerical sets. Children's estimations were independent from the visual characteristics of the stimuli (i.e. perimeter or density) within the OTS capacity. Conversely, the estimation of larger quantities (5-9 dots) was significantly affected by stimuli characteristics: in particular, the increase of perimeter with a constant density appears as the combination of visual characteristics which strongly increases the perceived numerosity.

In Study 2, Preschoolers, Grade 1 and Grade 3 pupils had to map continuous, discrete and symbolic quantities. The results indicated that different mechanisms are involved in the estimation of continuous quantities with respect to numerical (discrete and symbolic) quantities.

In Study 3, we devised a dual-task paradigm to investigate the relation between visual short term memory (VSTM) and subitizing. We found a striking correspondence between the number of elements retained in VSTM and the number of elements that can be subitized.

In Study 4.1, children with developmental dyscalculia (DD) in comorbidity with a profile of Non-Verbal syndrome (NVS) and typically developing (TD) children completed a numerical comparison task. We found a specific deficit in the comparison of numerical quantities in DD-NVS children with respect to TD. In particular, the OTS capacity seems to be reduced in the DD-NVS group as compared to TD.

In Study 4.2, children with developmental dyscalculia (DD) and typically developing (TD) children completed two number-line tasks. Children with DD displayed a less precise

estimation of symbolic quantities, thereby suggesting a specific deficit in the number representation with respect to TD children.

In Study 5, individuals with Down Syndrome (DS) and typically developing children matched for both mental (MA) and chronological age (CA) completed two numerical tasks in order to evaluate their ability to compare non-symbolic quantities (i.e. dots) and counting process. Kids with DS showed a specific deficit in comparing small quantities, within OTS capacity, with respect to both MA and CA matched kids. For the comparison of larger quantities, kids with DS displayed a performance similar to MA matched controls but lower as compared to CA matched controls. Finally, the counting ability appears similar between kids with DS and MA matched children.

Abstract della tesi

Come viene rappresentata l'informazione numerica? Recenti ricerche hanno evidenziato il ruolo fondamentale dei sistemi cognitive preverbal nella rappresentazione numerica: l'Object Tracking System (OTS) e l'Approximate Number System (ANS; o Analogue Magnitude System). Il primo è un meccanismo generale che permette di conservare in memoria le caratteristiche spazio-temporali degli stimoli e la sua capacità è limitata (3-4 elementi). Il secondo è un meccanismo quantitativo che rappresenta ogni numerosità come una distribuzione d'attivazione su teorica linea numerica mentale. Nella presente lavoro di tesi, presenteremo diversi studi volti ad indagare il funzionamento di questi meccanismi in interazione con processi di stima numerica e non-numerica in contesto di sviluppo tipico ed atipico.

Nello Studio 1.1, abbiamo utilizzato un compito di imitazione per indagare la capacità di concentrarsi spontaneamente sulla numerosità in bambini di 2 ½ anni. I risultati hanno evidenziato come la maggior parte dei bambini adotti un sistema analogico di quantità quando analizzano spontaneamente delle quantità numeriche. La selezione di questo meccanismo è probabilmente legata sia alla minor richiesta di risorse attentive, sia alla disponibilità di altri indizi quantitativi (non numerici) che covariano con la numerosità.

Nello Studio 1.2, bambini di 2 ½ anni hanno svolto un compito di categorizzazione per investigare la loro capacità di stimare la grandezza numerica di insiemi. Le stime dei bambini erano indipendenti dalle caratteristiche visive degli elementi dell'insieme (i.e. perimetro o densità) per le quantità dentro il range di OTS (1-4 elementi). Le stime di quantità più grandi (5-9 elementi) erano invece influenzate dalle caratteristiche visive degli stimoli: in particolare, l'aumento del perimetro con densità costante sembra essere la combinazione di caratteristiche visive degli stimoli che fa aumentare maggiormente la percezione di numerosità.

Nello Studio 2, bambini prescolari, di prima primaria e di terza primaria dovevano stimare quantità continue, discrete e simboliche. I risultati suggeriscono la presenza di differenti meccanismi coinvolti nella stima di quantità continue rispetto a quelle numeriche (discrete e simboliche).

Nello Studio 3, abbiamo utilizzato il paradigma del doppio compito per studiare la relazione tra memoria visiva a breve termine e subitizing. Dai risultati emerge una marcata corrispondenza tra il numero di elementi memorizzati ed il numero di elementi che possono essere velocemente enumerati attraverso il subitizing.

Nello Studio 4.1, bambini con diagnosi di Discalculia Evolutiva (DE) in comorbidità con sindrome non verbale (SNV) e bambini con sviluppo tipico hanno svolto un compito di confronto di quantità numeriche. Abbiamo riscontrato un deficit nella discriminazione di numerosità nel gruppo DE-SNV rispetto ai bambini a sviluppo tipico. In particolare, la capacità di OTS sembra essere ridotta nei bambini con DE-SNV rispetto ai bambini a sviluppo tipico.

Nello Studio 4.2, bambini con diagnosi di Discalculia Evolutiva (DE) e bambini con sviluppo tipico hanno completato due compiti di stima sulla linea numerica. I bambini con DE hanno mostrato minor precisione nella stima di quantità simboliche suggerendo una rappresentazione numerica deficitaria rispetto al gruppo con sviluppo tipico.

Nello Studio 5, ragazzi con sindrome di Down (SD) e bambini con sviluppo tipico pareggiati per età mentale (EM) ed età cronologica (EC) hanno svolto due compiti numerici per valutare le loro abilità di discriminazione numerica e di conteggio. I ragazzi con SD hanno mostrato un deficit nel discriminare piccole quantità, all'interno del range di OTS, rispetto ai bambini a sviluppo tipico pareggiati sia per EM che per EC. Nella comparazione di numerosità più grandi, i ragazzi con SD hanno ottenuto una performance simile ai bambini pareggiati per EM e minore rispetto ai ragazzi pareggiati per EC. Infine, l'abilità di conteggio appare simile tra i partecipanti con SD e i bambini pareggiati per EM.

Acknowledgments

The last three years have been the most instructive, stimulating, exciting, explorative, but also demanding, intensive, laborious, challenging of my entire life. I began the doctoral school with a strange mix of enthusiasm and a sort of freshman feeling. Nonetheless, it took a short time to realize that I had the valuable opportunity to work with brilliant persons whom I wish to express my gratefulness.

I owe sincere gratitude to my supervisor prof. Daniela Lucangeli who has constantly encouraged and guided me during my PhD study. Her knowledge, motivation and expertise aided me in the realization of the present thesis and other academic and professional goals. I would never forget all the help that I got from her: she has been both a scientific and a professional adviser for me.

I'm particularly in debt with prof. Marco Zorzi for his scientific support in the realization of several studies in this thesis. I feel fortunate for collaborating with him and taking advantage of his great experience and knowledge.

I consider an honor to have the opportunity to work with prof. Carlo Umiltà who has been my advisor for the first part of the doctoral school. He masterly led me working in different projects apart from the present thesis: I feel gratitude for this possibility.

I wish to thank Dr. Manuela Piazza and Dr. Marco Buiatti who supervised me during the period I spent at the INSERM Cognitive Neuroimaging Unit in Saclay (France): it was a valuable experience for my professional career.

I'm also in debt with prof. Cesare Cornoldi who was my supervisor for the master degree thesis and first suggested me to apply for the doctoral school at the University of Padova.

I feel gratitude for prof. Patrick Lemaire who guided me in a treasured collaboration: It was really a valuable experience working with him.

I am also very much grateful to prof. Patrizio Tressoldi for including me in a series of projects apart from this thesis.

I would like to thank my lab colleagues for their support: Dr. Matteo Lisi, Dr. Maria Grazia Ranzini, Dr. Maria Grazia Di Bono, Michele De Filippo De Grazia, Alberto Testolin, Dr. Veronica Montani, Dr. Ivilin Stoianov, Dr. Sara Caviola. I am especially indebted with senior lab members Dr. Ilaria Berteletti, Dr. Mario Bonato, Dr. Simone Cutini, who have been valuable scientific guides.

I wish to thank Dr. Anna Maria Re and Dr. Silvia Lanfranchi for their kind support in the realization of clinical studies in the thesis.

I am very much grateful to Luca Semenzato, Pietro Scatturin and all the technicians of the Department of Developmental Psychology and Socialization of the University of Padova for their technical support.

I wish to express my gratitude to all the participants who were involved in the presented studies. A special thanks goes to the children and their families: their kindness and availability made this thesis possible.

I would like to thank also all the undergraduates for their priceless help in the realization of the studies.

I wish to thank also my family for their for their love, support and encouragement throughout my entire life. In particular, I wish to thank my aunt Oriana who housed me during my study period in France.

Finally, my deepest thank goes to Nicoletta who has supported me in every single day of this demanding period. She definitely makes the difference in my life, so this thesis is dedicated to her.

Preface

In the present thesis, I included five different experimental studies related to several aspects of numerical cognition. After a brief general introduction and the explanation of the basic concepts, the Reader will find abstract, theoretical introduction, method, results and discussion sections separately for each study. I advise the Reader that the Study 1 and the Study 4 have been reported as preliminary results given that the data collection is still ongoing and some consistent methodological changing may be implemented. At the end, there is a main conclusion of the entire work.

Apart from the present work, I wish to mention two studies that were completed during the doctoral school but have not been reported in this thesis:

- Sella, F., Re, A. M., Lucangeli, D., Cornoldi, C., & Lemaire, P. (in press, 2012). Strategy Selection in ADHD Characteristics Children: A Study in Arithmetic. *Journal of attention disorders*. doi:10.1177/1087054712438766
- Sella, F., Bonato, M., Cutini, S., & Umiltà, C. (in press, 2012). Living on the edge: strategic and instructed slowing in the stop signal task. *Psychological research*. doi:10.1007/s00426-012-0419-y.

Summary

Introduction to the thesis	1
Basic concepts	3
Typical Development Studies	7
Study 1: <u>Numerical knowledge in 2-3 years-old children: two explorative studies.</u>	
Abstract	9
Study 1.1: <u>The Spontaneous Focusing on Numerosity entails an analogue magnitude system representation: a pilot study in 2-3 year-old children.</u>	
Abstract	10
Introduction	11
Method	13
Results	15
Discussion	17
Study 1.2: <u>Estimating numerosity via object-file and analogue magnitude: evidence from a categorization task in 2 ½ year-old children.</u>	
Abstract	21
Introduction	22
Method	23
Results	25
Discussion	27
Study 2: <u>Continuous, discrete and symbolic quantity estimation in preschool and school children.</u>	
Abstract	29
Introduction	30
Method	32
Results	34
Discussion	39
Study 3: <u>Visual Short Term Memory load disrupts subitizing limit.</u>	
Abstract	43
Introduction	44
Method	46
Results	48
Discussion	52
Atypical Development Studies	55
Study 4: <u>Numerical estimation in children with Developmental Dyscalculia: evidence from a delayed match to sample task and a number line estimation task.</u>	

Abstract Study 4.1 & Abstract Study 4.2	57
Introduction Study 4.1 & Abstract Study 4.2	59
Method Study 4.1	62
Results Study 4.1	64
Discussion Study 4.1	66
Method Study 4.2	67
Results Study 4.2	68
Discussion Study 4.2	71

Study 5: Subitizing, estimation and counting skills in Down syndrome: evidence from two delayed match-to-sample paradigms.

Abstract	73
Introduction	74
Method	77
Results	80
Discussion	85

Conclusion	89
------------	----

References	93
------------	----

Introduction to the thesis

We can define numerical cognition as a heterogeneous topic which is a convergence point for different scientific disciplines. Recently, several research branches have investigated numerical and quantitative competence from different perspectives. For instance, animal and comparative psychology has demonstrated that animals are able to represent and operate on numerical quantities thus suggesting the evolutionary role that the processing of numerical information has played in shaping the human cognitive architecture. Evidence from neuropsychology and cognitive neuroscience has provided important insights on the relation between numerosity processing and specific brain networks. Experimental psychology has provided important contributions to delineate the characteristics of numerical abilities in humans and how these competences can be the milestone for the acquisition of advanced mathematical concepts. On the other hand, developmental psychology has investigated the complex interaction between the progress of numerical competence and the acquisition of math knowledge both in typical and in atypical development condition (i.e., clinical developmental psychology). In this light, the intrinsic relation between the representation of numerosity and mathematical knowledge has created a fascinating scenario in which evolutionary and cultural features of cognition can be studied in interaction.

The present thesis has been planned on the basis of recent neuropsychological and psychological models of numerical cognition. The main aim is to provide a significant contribution to the contemporary theoretical framework making available new evidence to the scientific community. The thesis presents a series of different studies, each with specific hypotheses, displayed to respond to relevant experimental questions. We answered to these questions assuming different theoretical perspectives (e.g. developmental psychology, experimental, clinical developmental psychology). Indeed, each study includes a specific theoretical introduction, the pertaining method section, the results and the concerning discussion. Moreover, two main sections address the typical development studies and the atypical developing studies, respectively. As first experimental question (Study 1), we asked which are the quantification mechanisms and how these are employed by 2-3 year-old children. We aimed to provide new evidence regarding the ability to represent and process numerical information in young children. The main aim is to describe how basic systems for numerical representation are employed, spontaneously and under instruction, by pre-counter

children. Secondly, we expect that the implementation of new and adapted paradigms may promote future studies of numerical abilities in this age range. The Study 2 addressed children's ability to translate the magnitude of quantities, presented in different formats, into a spatial position. In particular, we aimed to evaluate similarities and differences in numerical and non-numerical estimation in order to obtain a better description of such abilities in preschool and primary school children, before and after entering into the formal education system. In Study 3, we employed a dual-task paradigm in order to highlight the intrinsic relation between visual short-term memory and the basic processing of small numerical quantities. In the atypical development sections, we investigated whether the functioning of the basic quantification processes are preserved, impaired or delayed in children with developmental dyscalculia (Study 4) and kids with Down syndrome (Study 5) as compared to matched typically developing children.

In summary, the scope of the thesis is to investigate the functioning of specific cognitive systems which are predisposed to process and interact with definite type of stimuli (e.g. numerosity and quantity). We aim to provide a better description of the developmental trend of numerical processes both considering the typical and atypical condition.

Basic concepts

In the eighties of the last century, a series of studies conducted on newborns revealed the ability to represent numerical quantities and suggested that humans have an innate number sense. After more than thirty years of research, the existence of two pre-verbal mechanisms that allow to proficiently represent numerosities is well established (Feigenson, Dehaene, & Spelke, 2004; Piazza, 2010): the Object-tracking System and the Approximate Number System (or analogue magnitude system). The former is a general cognitive mechanism which allows to track different objects in space and time and it is thought to be based on attentional mechanisms and memory capacity (Burr, Turi & Anobile, 2010; Piazza et al., 2011; Cutini & Bonato, 2012). The latter is a quantitative system which entails an approximate representation of numerosities on a mental number line (Dehaene, 1997; Feigenson, Dehaene, & Spelke, 2004; Meck & Church, 1983; Gallistel & Gelman, 1992). With development, the counting process represents the first connection between the pre-verbal systems and the culturally determined numerical (Arabic) system. Individuals can rely on an accurate serial counting procedure that permits to exactly identify the number of elements in a potentially infinite set by mapping the numerical magnitude into the Arabic system (Gelman & Gallistel, 1978).

The Object Tracking System. Human cognition is fundamentally based onto few basic core knowledge abilities that are enough flexible to allow the learning of new skills (Spelke & Kinzler, 2007). One of these abilities consists in object representation and it entails spatio-temporal principles of cohesion (objects move as bounded wholes), continuity (objects move on connected, unobstructed paths), and contact (objects do not interact at a distance). This system basically allows individuals to track objects through space and time. As demonstrated by several visual short term memory paradigms, the main characteristic of the OTS is its capacity limited to 3-4 elements. For instance, in the Multiple Object Tracking paradigm, it has been shown that individuals are able to track usually a maximum of four target objects moving in the display with other identical objects (for a review, Scholl, 2001). In the numerical domain, individuals are able to fast and correctly enumerate a small set of three-four elements even when the items are briefly presented or masked (Trick & Phylyshyn, 1994). This phenomenon, called subitizing, is a direct evidence of the OTS signature. Generally speaking, the OTS can subserve enumeration because it encompasses a one-to-one correspondence between the elements in the set and the object files memorized in the system.

For instance, when comparing one element versus two elements, individuals realize the mismatch between quantities through a one-to-one comparison between the object file stored for the first set and the object files stored for the second set. The developmental trajectory of the OTS reaches its peak early in development, indeed one year-old children display an OTS capacity similar to adults (Ross-Sheehy, Oakes, & Luck., 2003).

The Approximate Number System. The ANS (or analogue magnitude system) is a quantitative system which entails an approximate representation of numerosities on a mental number line. Two alternative models account for the ability in estimating quantities: the Logarithmic model (ANS; Feigenson, Dehaene, & Spelke, 2004; Piazza, 2010) and the Linear Model (Meck & Church, 1983; Gallistel & Gelman, 1992). The linear model (or accumulator model) argues that numerosities are linearly spaced but the access to the internal representation is noisy with scalar variability. Therefore, the noise in selecting a given numerical quantity becomes noisier with increasing numerosities. The Logarithmic model represents each numerosity as a distribution of activation on a logarithmically compressed number line (Dehaene, 1997; Dehaene, Piazza, Pinel, & Cohen, 2003). The progressive compression of the logarithmic scale causes an overlap of activations for adjacent representations as magnitude increases.

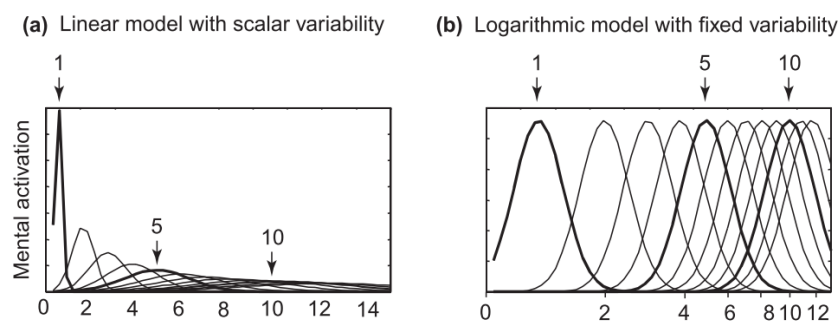


Figure 1. Linear model with scalar variability (*panel a*); Logarithmic model with fixed variability (*panel b*) (The figure is adapted from Feigenson, Dehaene, & Spelke, 2004)

As a consequence, two numerosities far apart are easier to discriminate compared to two adjacent ones because the amount of overlapping distribution of activation is smaller (i.e. distance effect). Moreover, because the compression increases with magnitude, the overlap between distributions of activations increases for a given numerical distance with increasing magnitude (i.e. size effect). Both models account for the distance effect, in which the speed and accuracy of judgment increases with increasing difference between the numerical values,

and the size effect, in which speed and accuracy decrease with increasing number magnitude. These two effects can be summarized in term of a ratio-dependent effect, where discrimination decreases when the ratio between two numbers or numerical quantities approaches one. This ability to notice the difference in numerosity between two sets, defined as number acuity, follows Weber's law and changes across development with a great improvement during the first years of life and a slight reduction in elder hood (Halberda & Feigenson, 2008; Piazza et al., 2010; Halberda, Ly, Wilmer, Naiman, & Germine, 2012). For instance, six months-old infants can notice the difference between 8 vs. 16 elements (1:2 ratio) but fail with the comparison 8 vs. 12 (ratio 2:3) (Xu & Spelke, 2000). Healthy adults reliably differentiate between sets with a 9:10 ratio despite a wide range of individual differences in the population (Halberda, Mazocco, & Feigenson, 2008; Halberda et al., 2012).

Counting. Learning to count represents the first connection between the pre-verbal innate mechanisms of quantification and the culturally determined numerical system. In a seminal study, Gelman and Gallistel (1978) proposed the basic principles of counting: the one-to-one correspondence principle states that one and only one object must be associated with the corresponding word in the counting list; the stable-order principle states that the counting list must be recited in the correct and established order; the cardinality principles identifies the last word in the counting list as the numerosity (cardinality) of the entire set. Two other less fundamental principles have also been described: the abstraction principle states that every collection of objects can be counted, whether tangible or not; the order-irrelevance principle refers to the fact that the order in which the elements of a set are counted is irrelevant.

Different hypotheses have been proposed to explain the relation between the pre-verbal mechanisms of quantification and the acquisition of counting. The bootstrapping account (Carey, 2004) states that the progressive alignment of the counting list and the representation of small numbers, generated from OTS, allow children to solidly connect the number-words to the correspondent numerosity. In this light, children use the OTS to individuate the objects and create a mental model of the set using working memory. Therefore, they make a one-to-one correspondence between elements of the working-memory model and the long-term memory models (e.g., the counting list). Finally, children select the number of the list which perfectly matches the working-memory and the long term memory model (Le Corre & Carey, 2007). After a period for establishing the alignment of these

systems, children infer that the next number-word in the counting list corresponds to one element added in the set (i.e., successor function).

Another hypothesis states that children create a mapping between the counting list and the analogue magnitude representation of numerosities. Therefore, the connection between the two systems should be noisier as the numbers increase. At the beginning of learning to count, small numerosities are connected to the corresponding numbers in the list with small or no errors, whereas larger numerosities are connected with more errors. Apart from the approximate representation, pre-verbal infants are able to represent the concept of “greater than” and “less than” relation in analogical numerical quantities (Brannon, 2002). This ordering ability may be transposed from the analogue magnitude system to the counting list by analogy (Wynn, 1992). Thus, the numerosities which occupy the later position in the ordering of the numerical quantities are also those which appear later in the counting list.

The two proposed hypotheses might be both plausible. Indeed, children may use the OTS to track 3-4 elements in alignment with the counting list. Thereafter, children induce that the next verbal label in the counting list corresponds to the element added to the set which increases the analogue magnitude representation.

Typical development studies

JENNY: How old are you?

EDWARD: Eighteen.

JENNY: I'm eight. That means when I'm eighteen, you'll be 28. And when I'm 28, you'll only be 38.

EDWARD: You're pretty good at arithmetic.

JENNY: And when I'm 38, you'll be 48. And that's not much difference at all.

(From the movie "Big Fish", 2003)

Study 1

Magnitude knowledge in 2-3 years-old children: two explorative studies

Abstract

The aim of the present research is to implement two modified versions of already adopted numerical tasks in order to investigate some aspects of quantity estimation in 2 ½ year-old children. The data gathered using the new modified tasks are preliminary thus the character of the research is rather explorative. In Study 1.1, children completed a modified version of the Spontaneous Focusing on Numerosity task in order to highlight the underlying estimation mechanisms that children adopt when focusing on numerosity in a spontaneous way. In Study 1.2, we introduce a categorization task in which children categorize cardboards representing analogical numerical quantities (i.e. set of dots) as “few” or “many”. The aim of the task is to investigate the numerical representation and estimation competence in a numerical range within and beyond the Object Tracking System capacity.

Study 1.1

The Spontaneous Focusing on Numerosity (SFON) task entails an analogue magnitude system representation: a pilot study in 2-3 year-old children.*

Abstract

Spontaneous Focusing on Numerosity refers to children's predisposition to encode numerical aspects of the environment in the absence of any specific suggestion. The ability to concentrate on numerosity is a stable process that correlates with counting development and basic arithmetical skills even when verbal IQ, verbal comprehension and procedural abilities are controlled for. Therefore, the SFON can be considered a separate domain specific process that allows children to concentrate on numerical aspect of their environment and provides an advantageous predisposition for future mathematical achievement. In this light, it is worthwhile to understand which kind of enumeration process is used by children when focusing on numerosity already at an early age, even before counting abilities start to being mastered. Here we implemented a modified version of the SFON task to be used with pre-counter participants of 2 ½ years of age. In this task, children are required to imitate the experimenter's behavior by inserting some tokens in a puppet's mouth as if it was food. If a child concentrates on numerosity (i.e. a Focuser), she should replicate the experimenter's behavior by inserting the same number of elements, conversely, a non-focuser fails the imitation by inserting a different number of elements, maybe by focusing on the feeding action and giving to the puppet either a handful or all the tokens available. We hypothesized that Focusers may rely either on the object-file system or instead estimate the numerosity via the analogue magnitude system to individuate the number of pieces to feed the puppet. Results suggest that most of Focusers adopted an analogue magnitude estimation when spontaneously focusing on numerosity. The selection of an analogue magnitude system may be related to both lower attentional resources needed and to the saliency of other quantitative cues, such as time and total amount in size of tokens, which covariate with numerosity.

* In collaboration with Berteletti I., Lucangeli D., & Zorzi M.

Introduction

The basic idea of the present study is to highlight the enumeration mechanisms that children adopt when spontaneously focusing on numerosity (SFON; Hannula & Lehtinen, 2001, 2003, 2005; Hannula, Rasanen, & Lehtinen, 2007; Hannula, Lepola, & Lehtinen, 2010). The SFON refers to children's predisposition to encode numerical aspects of the environment in the absence of any specific suggestion. A child that focuses on numerosity is more prone to spontaneously consider the numerical characteristic of the objects: for instance, a basket with the yellow bananas is seen as a basket with *three* yellow bananas. Therefore, the number of elements, along with the color, is considered a salient aspect of the set. In a typical imitation task to assess the SFON, young children are introduced to an animal-like puppet that may be fed through its wide-open mouth (e.g. bird) and its favorite food (e.g. tokens). After a brief period of familiarization with the puppet, the experimenter inserts a small number of pieces of food (e.g., 1-2 elements at each trial) in the puppet's mouth and asks the child to replicate the same feeding. The experimenter voluntarily omits to introduce the task as a mathematical or numerical game in order to maximize the spontaneous aspect of the focusing. Some children focus on the number of elements that are inserted into the puppet whereas others simply overlook the numerical aspect of the task. Therefore, the focusers insert into the puppet's mouth the same number of elements already placed by the experimenter or at least demonstrate some quantification behavior (e.g. counting acts like whispering the numbers). Conversely, the non-focuser usually feed the puppet a handful or all the available pieces of food without showing any attention to the numerical aspect of the action. Notably, focusers and non-focusers have both a similar understanding of quantification concepts and a comparable level of cognitive skills needed to accomplish the task. Thus, spontaneous focusing can be considered as specific and separate process implemented by some children that consider numerosity as a relevant dimension of their environment. The ability to concentrate on numerosity is a stable process that correlates with counting development and basic arithmetical skills even when verbal IQ, verbal comprehension and procedural abilities are controlled for (Hannula & Lehtinen, 2005). Moreover, SFON ability assessed during kindergarten is a unique predictor of math achievement, but not of reading, at Grade 2 (Hannula, Rasanen & Lehtinen, 2010). To sum up, SFON ability may be considered a separate domain specific process that allows children to concentrate on the numerical aspect of their environment and provides an advantageous predisposition for future mathematical

achievement. Hannula and colleagues (2010) proposed three explanations of the association between SFON and the learning of arithmetical skills. First, SFON competence is a particular feature of a more general tendency of focusing on numerical and mathematical aspects of the environment. In this light, children who focus on numerosity should also turn their attention to other mathematical characteristics such as the meaning of the Arabic digits and arithmetical operations. The repeated concentration on numerical features should positively favor children's knowledge about numbers, quantity and math. Secondly, SFON predisposition may be a particular instance of more general motivational aspects of learning: children with an intrinsic motivation demonstrate goal oriented behaviors which makes them focus on task characteristics instead of looking for other type of cues (Lepola, Niemi, Kuikka & Hannula, 2005). Finally, SFON competences are interconnected with enumeration abilities and it is interlinked with arithmetical abilities. In particular, Hannula and colleagues (2007) found that the association between SFON and counting skills is mediated by the subitizing-based enumeration skill level thus demonstrating a strong association between SFON and the enumeration process. Nevertheless, this association between SFON and enumeration skills may be the byproduct of processes that children are adopting to accomplish the task. In this light, focusers are able to encode numerosity, retain the numerical information and then imitate the experimenter's behavior. A child who can focus on numerosity, without an efficient encoding of the number of elements that are inserted into the puppet, will fail in the imitation task. Then, it becomes particularly important to understand which kind of enumeration process children implement when performing the imitation task.

Here we propose that young pre-counting children may adopt two alternative non mutually-exclusive processes when encoding the number of elements inserted into the puppet: namely, individuation or estimation. Pupils may individuate the objects that are inserted in to the puppet using the Object Tracking System (OTS): such mechanism allows individuals to exactly track a limited number of objects (less than 3-4) in space and time (Feigenson, Dehaene & Spelke, 2004; Piazza, 2010). As already demonstrated in manual search experiments, 14 month-old children can track and search at most 3 elements when they are hidden into an opaque box. As soon as the number of elements increases to 4, children stop searching for the fourth element thus suggesting that the tracking capacity is completely loaded and no more space is available to track exceeding elements (Feigenson & Carey, 2003). In the SFON context, this lead to the hypothesis that focusers can keep track of at most

three elements that are inserted into the puppet. With more than three elements, children should fail to replicate the experimenter's behavior. On the other hand, children may estimate the number of elements fed to the puppet relying on the Approximate Number System (ANS; Feigenson, Dehaene, & Spelke, 2004; Piazza, 2010; Stoianov & Zorzi, 2012). Two models account for the ANS: the Logarithmic and the Linear model. In the logarithmic model of ANS, each numerosity is represented as a distribution of activation with a constant variability on a logarithmically spaced mental number line (Dehaene, 1997; Dehaene, Piazza, Pinel, & Cohen, 2003; Feigenson, Dehaene, & Spelke, 2004; Piazza, 2010). In the Linear model, the distributions of activation are linearly spaced and have scalar variability (Meck & Church, 1983; Gallistel & Gelman, 1992). Nonetheless, in both models, an increase in magnitude causes a progressive overlap of the distributions of activations making the estimation of larger numerosities less accurate as compared to smaller ones. In the SFON context, children who rely on an estimation process should be less accurate in reproducing the experimenter's behavior as the number of elements fed to the puppet increases. Nevertheless, the estimations should be centered on the correct number of elements with increasing variability as the number of elements becomes larger.

In the present study, we adapted the SFON imitation task to highlight the enumeration processes adopted by 2 ½ years-old children when focusing on numerosity. In our modified version the elements inserted into the puppet ranged from one to six and were proposed several times. Repeating the same numerosity several times should allow differentiating between the OTS and the ANS. We expect focusers' performance to be accurate up to three elements and drastically drops for larger numerosities if the individuation mechanism is adopted. On the other hand, we should observe a constant decline of focusers' performance from one to six if the estimation processes is employed.

Method

Participants. Forty-six pupils between 24 and 44 months (21 boys; $M_{\text{age-in-months}} = 30$, $SD = 4$) took part in the experimental session after parents gave their informed consent.

Procedure. Undergraduates in Educational Sciences from the University of Padova attended two lessons (one hour and half each) on how to properly administer the task and collected data as part of their academic internship. After the first lesson, undergraduates created their own materials and, in the second lesson, they completed a supervised simulation on how to

administer the task. Prepared materials were checked by the supervisors (first and second authors) to be appropriate according to the task characteristics. Non-adequate materials were discarded and undergraduates were requested to do them following specific corrections and checked again. Each undergraduate individually met one child, in a quiet room, for three sessions with an average time between sessions of 10 days (range: 2-24). Each child completed the task three times. The task was presented as a game, no time limit was given and items or questions could be repeated if necessary but neither feedback nor hints were given to the child. Children were free to stop the task either for an extra break or to terminate the testing session.

The SFON task. The SFON task was adapted from the original task by Hannula (2001). The experimenter introduced the child to an animal-like puppet called SFON that had to be fed with its favorite food. SFON is a homemade puppet with an open wide mouth allowing the introduction of pieces of food made of paperboard (small cubes with fixed color and dimension). The experimenter explained the game to the child as:

“Look here, this is my little friend SFON (showing SFON)! And this is its favorite food (pointing at the pieces of food)! Now look carefully what I do and when it’s your turn just do exactly as I did”. Therefore, the experimenter takes n pieces of food and put them in SFON’s mouth. The child is invited to do the same: “Now it’s your turn, do exactly what I did”.

If the child concentrates on numerosity, she will give to the puppet the same number of pieces of food as the experimenter gave. Conversely, the child may put a random number of elements (e.g. all the available pieces of food) indicating that she did not focus on numerosity. During the whole procedure, the experimenter never referred to the numerical aspect of the task and children are not told that they will do a game related to number or math. In each session, the experimenter gave to the puppet 1 to 6 elements. Each numerosity was repeated three times in each session for a total amount of 54 trials over the three sessions. The task always started by feeding the puppet with 2 pieces of food, while the numerosities for the following trials were randomly presented. There were 18 pieces of “food” available at the beginning of each trial; the pieces from the previous trial were restored in the starting position as soon as the child gave the response. Thus, when the experimenter gave one piece to the puppet, the child had 17 pieces available for answering and when 6 pieces of food were inserted into the puppet by the experimenter, the children had the remaining 12 pieces of food

available. Children’s responses were recorded on a scoring sheet for every single trial indicating the number of pieces given to the puppet. When the child gave more than 11 pieces, the answer was classified as “ALL” suggesting that she simply wanted to give all the pieces of food. Experimenters were not asked to record qualitative understanding of the task.

Results

We considered a child as Focuser if she replicated the experimenter’s behavior at least 8 times out of 9 when only one element was inserted into the puppet. Out of the 46 children, 20 were classified as Focusers and their performance were analyzed. One Focuser replicated the experimenter’s behavior perfectly, thereby suggesting the use of a counting strategy. Therefore, we decided to remove this participant from the following analyses. For the Focusers group, we calculated the percentage of correct responses for each numerosity, collapsing the three sessions (Figure 1).

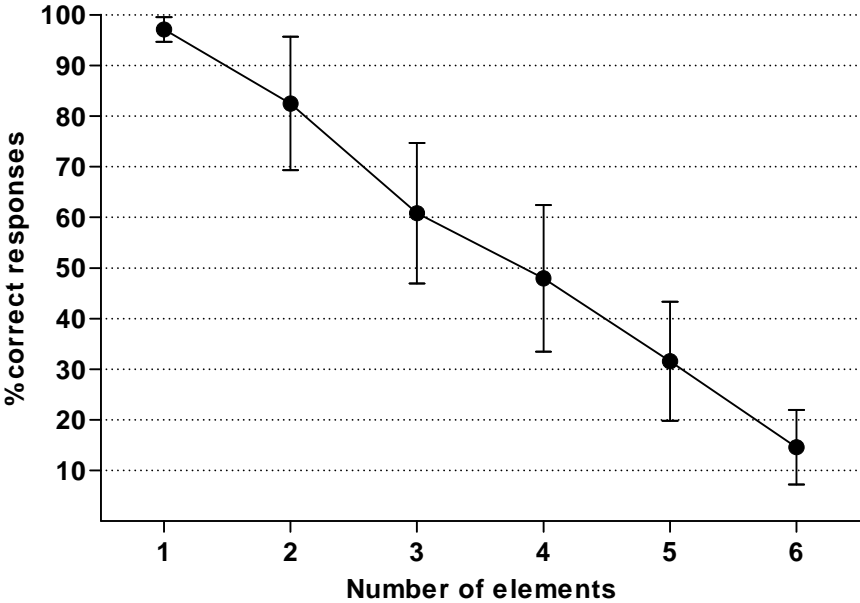


Figure 1. Focusers’ percentage of correct responses as a function of the number of elements insert into the puppet by the experimenter (error bars mean 95% CI).

We hypothesized that children used an individuation or an estimation system to encode the elements that were inserted into the puppet. If children adopted an individuation system we should observe a high accuracy for numerosities up to 3 and then a drop of accuracy for larger numerosities. On the other hand, if focusers adopted an estimation process, we should

observe a systematic decrease of accuracy from 1 to 6. To disentangle whether focusers adopted and individuation or approximate estimation of the number of elements, we calculated the individual slope of the regression analysis with accuracy as dependent variable and numerosities 1, 2 and 3 as predictors (Figure 2). If the focusers used an individuation process, the mean of the beta parameter should be equal to zero, conversely, it should be negative and different from zero if focusers adopted an estimation process. The mean of the beta parameter ($M = -.18, SD = .14$) was significantly different from zero, $t(18) = 5.58, p < 0.001$, thus suggesting that focusers adopted an estimation process to concentrate on the number of elements that were inserted into the puppet.

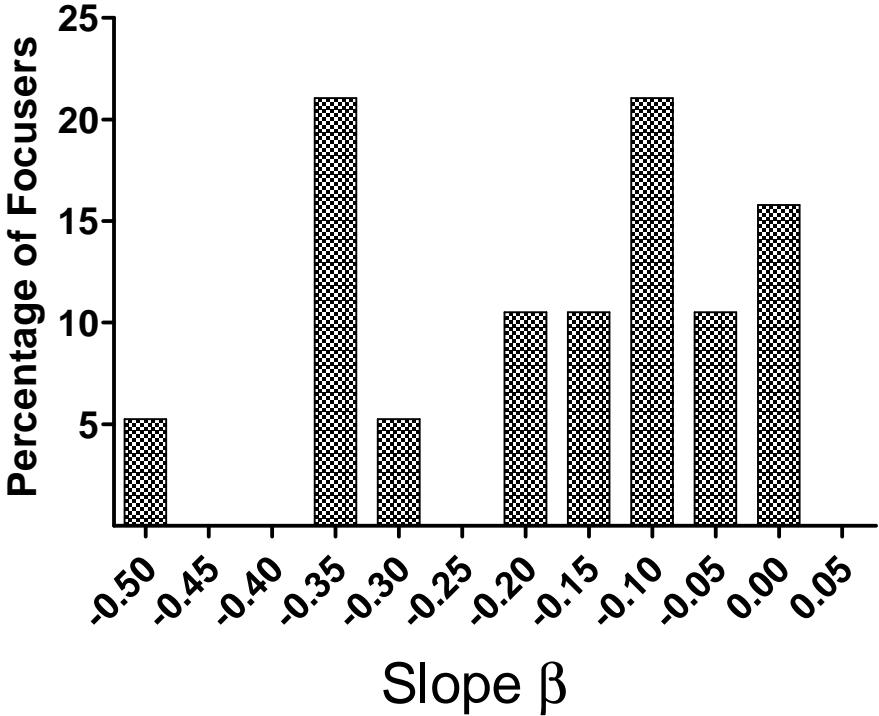


Figure 2. Histogram of percentage of Focusers' individual slope of the linear regression analysis on accuracy for one, two and three elements inserted into the puppet by the experimenter.

According to the ANS, each numerosity should be represented as a distribution of activation on a mental number line. To verify whether Focuser children rely on the ANS to represent the number of elements that were inserted into the puppet, we plotted for each number of elements (from 1 to 6) inserted into the puppet by the experimenter the frequency (in percentage) of the number of elements inserted into the puppet by the children (Figure 3).

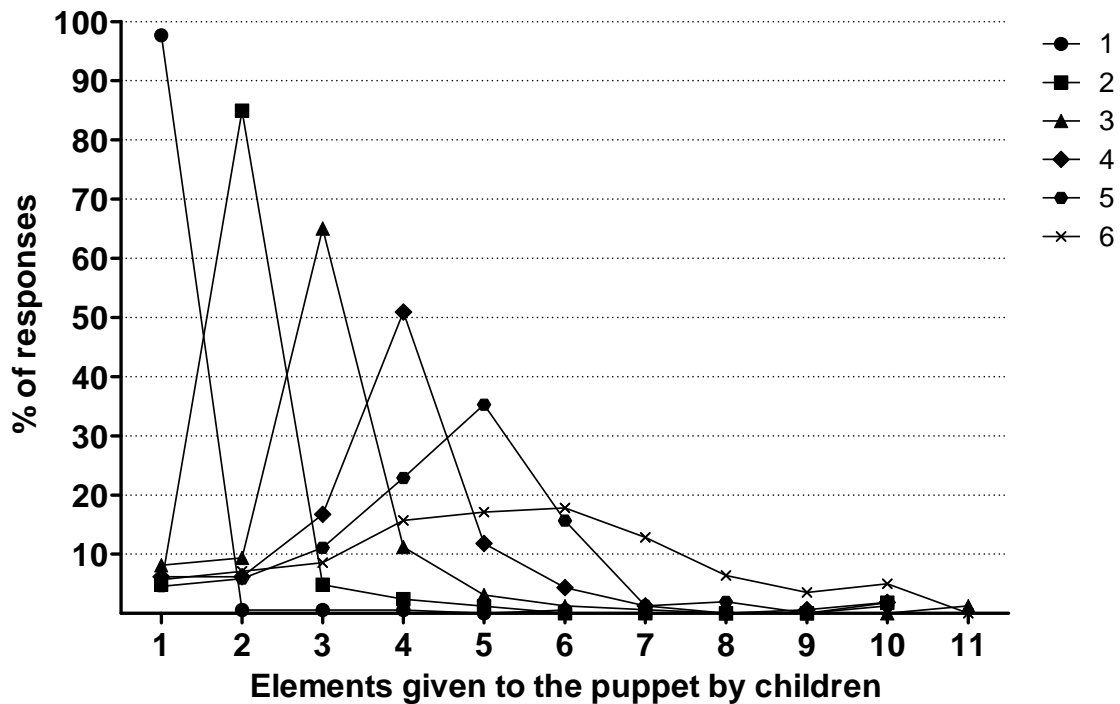


Figure 3. Distribution (in percentage) of the number of elements inserted into the puppet by the children for each number of elements (from 1 to 6) inserted into the puppet by the experimenter (see up-left legend) in the three experimental sessions. Responses with more than twelve elements were discarded from the graph.

Discussion

Spontaneous focusing on numerosity is a domain specific process that allows children to grasp the numerical aspect of their environment and provide an advantageous predisposition for the future mathematical achievement. In particular, a correlation has been found between this predisposition and enumeration and arithmetical abilities (Hannula et al., 2005). This correlation may be the byproduct of processes that children use when focusing on numerosity. Focusers must adopt a mechanism to encode numerosity, keep in mind the numerical information and then imitate the experimenter's behavior. Despite the attentional or motivational aspect of the focusing, a child fails in imitating the behavior if she is not able to encode the number of elements that are inserted into the puppet. In this light, it becomes particularly important to understand which kind of enumeration process children implement when accomplishing the imitation task. We hypothesized that children may implement two non-exclusive systems, the OTS and the ANS. The former provides the exact representation of small quantities (3-4 elements) by means of tracking objects in space and time. In the SFON task, a focuser should replicate the experimenter's behavior only for small

numerosities because the capacity of the OTS is limited to three-four elements. In the ANS, instead, each numerosity is represented as a distribution of activation on a mental number line. Therefore, a focuser should be able to represent each numerosity with less accuracy as the number of elements inserted into the puppet increases.

We found that Focusers' accuracy in reproducing the experimenter's behavior immediately decreased as the number of elements inserted into the puppet increases. This result was evident in both global and individual analyses thus confirming the robustness of the evidence. We also highlighted that the responses of the focusers reliably reproduce the ANS signature with a distribution of responses centered on the numerosity inserted into the puppet with scalar variability. Therefore, it seems that 2 ½ years-old children rely on an approximate representation of numerosity when they accomplish the imitation task. Nonetheless, it is worth to notice that a small percentage of children seemed to obtain a performance compatible with OTS strategy. It might be claimed that both strategies can be implemented but children prefer to rely on the approximate representation of quantities also for small numerosities (Cantlon, Safford, & Brannon, 2010). We may interpret such a preference for an approximate representation as a consequence of the interaction between the enumeration processes and the task structure. First, the spontaneous focusing lacked the request of a specific goal and there were no feedbacks to shape children's performance. The OTS requires attentional resources (Burr, Turi, & Anobile, 2010) and it is conceivable that its use could be triggered only with a strong reinforcements or incentives. This feature is different from manual search paradigms in which children look for hidden items because the objects themselves represent an interesting reinforcement (Feigenson & Carey, 2003). Secondly, the tracking of the objects in time is a process that requires to neglect other quantitative information that may instead facilitate the use of the ANS. Indeed, there are several quantitative aspects that may guide children's performance. Children might rely their estimation on the total amount of food inserted into the puppet without computing the effective number of elements. In this light, the sum of the physical size of the pieces of food was the key aspect that guided children in reproducing the same "mass" of food. Similarly, children might have considered the total amount of time that the experimenter spent to insert the elements in the puppet as the crucial aspect of the task. Then, children imitate the feeding behavior for a similar time interval without considering the number of elements given to the puppet. The use of temporal or physical information mimics the approximate number system

in line with a more general tendency to process the magnitude (vanMarle & Wynn, 2006; Walsh, 2003). Future research should investigate the influence of physical and temporal cues controlling for the size of the pieces of food and for the total amount of time that the experimenter spends in introducing the elements into the puppet.

In summary, children who spontaneously focus on numerosity exhibited a pattern of responses that was more compatible with the implementation of the ANS as compared to the OTS. Therefore, the ANS seems the principal system to encode quantity at 2-3 years of age.

Study 1.2

Estimating numerosity via object-file and analogue magnitude: evidence from a categorization task in 2 ½ year-old children.*

Abstract

Several studies have demonstrated that infants can perceive differences within both small and large numerosities when confounding physical variables, such as surface area or size, are strictly controlled for. Despite these evidences, there is less knowledge about the internal magnitude representation of quantities, within and beyond the Object Tracking System capacity, in older children just before managing the counting principle. In the present study, we introduced a categorization task in which 2 ½ year-old children categorized cardboards representing analogical numerical quantities (i.e. set of dots from 1 to 9) by inserting them in a box with one dot (“few”) or nine dots (“many”). The aim of the task is to investigate the numerical representation and estimation competence in a numerical range inside and outside the OTS size signature. The probability of inserting a cardboard in the “many” box constantly increased when magnitude of the cardboard increased, thus children understood the aim of the task and were able to represent numerosities in a congruent way. It is worthwhile to notice that estimations within the OTS capacity were independent from the physical controls implemented (i.e. perimeter or density). Beyond the set size signature of OTS (i.e., more than 4 elements), the estimation mechanism related to ANS was significantly affected by the stimuli characteristics and physical parameters. The increase of perimeter with a constant density appeared as a physical variable which strongly covaried with the sense of numerosity. Conversely, the decrease of area with an increase of density induced a weaker sense of perceived numerosity.

* In collaboration with Berteletti I., Lucangeli D., & Zorzi M.

Introduction

Humans and other species are born with an innate ability to extract numerical information from the environment as a proficient tool shaped by the evolutionary process (Cantlon & Brannon, 2006). Several developmental studies have demonstrated that infants can discriminate between numerical quantities when presented in different formats, auditory and visual, and even when physical cues that usually positively covariate with numerosity such as area, perimeter and duration are strictly controlled for (Antell & Keating, 1983; Starkey & Cooper, 1980; Xu & Spelke, 2000; Lipton & Spelke, 2003; Wood & Spelke, 2005). Two systems have been proposed as foundational of numerical representation: the Object Tracking System and the Approximate Number System (Feigenson, Dehaene, & Spelke, 2004; Piazza, 2010). The former is a general cognitive system that tracks objects in space and time by creating a memory-file, which entails some characteristics for few items in the set (Kahneman, Treisman, & Gibbs, 1992; Tick & Pylyshyn, 1994). The system can subserve enumeration because it encompasses a one-to-one correspondence between the elements in the set and the number of the object files memorized in the system. When comparing one element versus two elements, infants realize the mismatch between quantities through a one-to-one comparison between the object file stored for the first set and the object files stored for the second set. The signature characteristic of the OTS is a capacity limited to three-four elements. For instance, in a manual search experiment, the pattern of searching elements hidden into an opaque box revealed that 12- to 14-month-old infants track up to three elements but failed with four elements signifying that the OTS capacity was completely full with 3 items and no more space is available to track another element (Feigenson & Carey, 2003). When larger quantities have to be compared, infants rely on the ANS, which represents each numerosity as a distribution of activation on the mental number line (Dehaene, 1997; Gelman & Gallistel, 1992). The distinctive feature of the ANS is the ratio dependent effect, which states that two numerosities are more difficult to discriminate as their ratio approaches to one. Indeed, two numerosities that are far apart are easier to discriminate because the overlap between their distributions of activations is minimal. For instance, 6-months-old infants can effectively note the difference between 8 and 16 dots (1:2 ratio) but fail with 8 and 12 (2:3 ratio) (Xu & Spelke, 2000). Besides the theoretical considerations, the distinction between the OTS and the ANS is well defined in adults whereas it is still under debate in early childhood (Revkin et al: Piazza, 2010; Feigenson, Carey & Spelke, 2002; Feigenson,

Carey & Hauser, 2002; see Cordes & Brannon, 2009). In particular, Feigenson, Carey and Spelke (2002) found that infants fail to notice the difference between small numerosities when continuous extents are controlled for. In this light, when the OTS creates an object file also confounding information such as the size of the item is stored into the memory. Therefore, infants operate also on these properties instead of considering only the numerical component of the item. This bias led infants to erroneous discriminations and confusions between representations of numerical quantities. For larger quantities, it is still debated whether individuals are able to extract numerosities or base their estimation on other physical variables. For instance, Clearfield and Mix (1999) found that infants dishabituated to the contour length of a set (or continuous extent) but they failed to dishabituate numerosity. In 3 ½ year-old children, the perceptual characteristics of contour length and density seem to facilitate a successful comparison between quantities (Rousselle, Palmers, & Noel, 2004). Other studies have reported that the combination of total convex hull, small item size, and then lower density are the more prominent visual variables in increasing the sense of magnitude (Gebuis & Reynvoet, 2012a, 2012b). Developmentally speaking, it is worthwhile to understand when children become able to ignore physical characteristics of the stimuli for enumeration purpose both in small and large range. Here we implemented an easy and intuitive task to investigate the estimation ability in 2 ½ year-old children. The numerical interval presented to the children is within and beyond the OTS size signature in order to highlight differences between the implementation of the two systems.

Method

Participants. 46 pupils (21 boys; $M_{\text{age-months}} = 30.37$, $SD = 4$; range: 24 - 47) completed the three sessions of the task after parents gave their informed consent.

Procedure. Undergraduates in Educational Sciences from the University of Padova attended two lessons (one hour and half each) on how to properly administer the task and collected data as part of their academic internship. After the first lesson, undergraduates prepared their own materials and, in the second lesson, they completed a supervised simulation on how to administer the task. Materials were checked by the supervisors (first and second authors) to be appropriate according to the task characteristics. Non-adequate materials were discarded and undergraduates were requested to do them following specific corrections and checked again. Each undergraduate individually met one child, in a quiet room, for three sessions with an

average time of 10 days (range: 2-24). Each child completed the categorization task three times. The task was presented as a game, no time limit was given and items or questions could be repeated if necessary but neither feedback nor hints were given to the child. Children were free to stop the task either for an extra break or to terminate the testing session.

The Classification task. In the Classification task, children were required to classify sets of dots deciding whether there were “few” or “many”. Two identical boxes were placed in front of the child: a card with one dot was attached on the box on the left side whereas a card with nine-dots was attached to the box on the right side (see Figure 1). The experimenter told the child: “We will play a game with these two boxes. Look, on this box (on the left side) there are a few dots. On the other one (on the right side) there are many dots. Now I show you how to play.” The experimenter took two cardboards with 1 and 9 dots respectively. Then, the experimenter said: “Here there are few dots (taking the cardboard with 1 dot) and I put it into this box (the one on the left hand side of the child). Here there are many dots (taking the cardboard with 9 dots) and I put it into this box (the one on the right hand side of the child). Now it is your turn! Look at this (a cardboard with 2 dots), is this few or many? Put it in the correct box!”. The children were given the cardboard and they were allowed to look and manipulate it as long as they wanted before putting it into one of the two boxes. If a child was not sure about the correct response, the experimenter repeated the instructions and kindly invited the child to provide a response. There were two types of sets, each with nine cardboards representing dots from 1 to 9. In one set, total enclosure of dots increased with numerosity whereas the density and the size of the dots were kept constant (except for one dot). In the other set, total enclosure of dots was constant, therefore the size of dots decreased and density increased with numerosity. In each session, a child had to sort 18 cards. For each trial, the response of the child was categorized as 0 when the cardboard was put into the “few” box whereas it was categorized as 1 when the cardboard was placed into the “many” box.

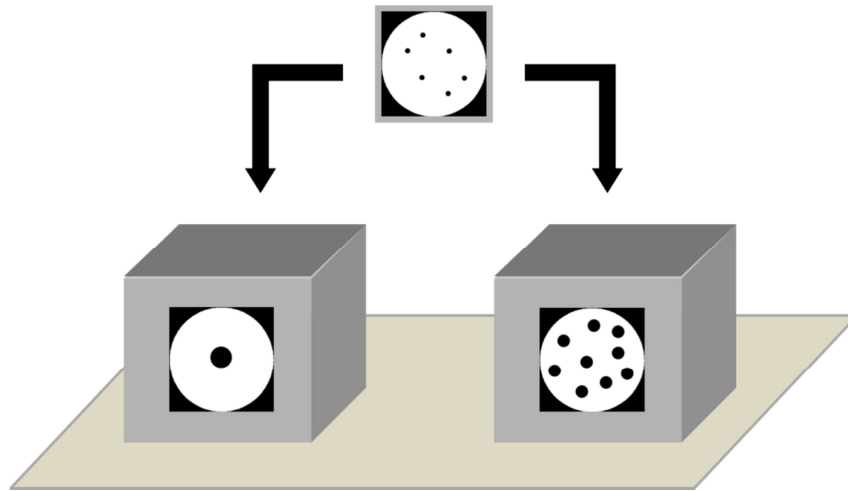


Figure 1. The box placed on the left side was named as “few” and a cardboard with one dot was attached in front of it. The box on the right side was named as “many” and a cardboard with nine dots was attached in front of it. The experimenter gave the cardboards with sets from one to nine dots to the children who had to put them in one of the two boxes.

Results

In the categorization task, children decided whether a number of dots is more similar to “few” or “many” by putting the given cardboard into the one-dot box or in the nine-dot box, respectively. We expect children to easily and correctly classify the extreme numerosities of the interval and, conversely, show much more indecision for numerosities in the middle of the interval. Indeed, the probability to insert a one dot cardboard in the “many” box should be minimum whereas the probability to put the nine dots cardboard in the “many” box should be closer to one. When the numerosities on the cardboards are in the middle of the interval as for 4-5-6 dots, the probability to put the cardboard in the “many” box should be around the chance level ($p = .5$) suggesting a greater indecision in the classification. First, we calculated the percentage of correct categorizations (cardboards with 1 to 4 dots in the “few” and cardboards with 6 to 9 dots in the “many”) for each participant. We selected only those participants who obtained a mean accuracy above chance level (50 % accuracy). Twenty-seven children (16 boys; $M_{\text{age-months}} = 31.4$, $SD = 5$; range: 24 - 47) were able to perform the task above chance level and were included in the analysis[†]. We analyzed the data using a Bayesian Graphical Model (Lee & Wagenmakers, 2010; Figure 3). We specified the parameter Theta as the prior rate with a beta distribution. The beta distribution is a continuous probability distribution appropriate to examine proportions of binary responses and it is

[†] The main findings of the study remain similar even including all the participants. Nevertheless, we preferred to include only participants with an above chance accuracy to a clearer picture of the results.

parameterized by two positive shape parameters, typically denoted as α and β . The α and β parameters can be interpreted as prior counters of the two possible events. For instance, if we flip a coin 4 times and we obtain two times head and two times tail, the α and β will be both equal to 2. Then, in the next flip we will specify the prior knowledge of the beta distribution as $\beta(2, 2)$. When there is no knowledge about an event, the parameters α and β are usually both set equal to 1. Such specification denotes a uniform prior distribution and assumes that the underlying process is binary. The R_i is the number of successful classifications in n number of events and it updates the value of Theta. We applied this model separately for each numerosity from 1 to 9 for the two conditions (i.e., perimeter and surface) considering a successful event when a child put a cardboard in the “many” box. The mean of the posterior distributions for each cardboard are presented in Figure 4: The probability of inserting a cardboard in the “many” box constantly increased with increasing numerosity.

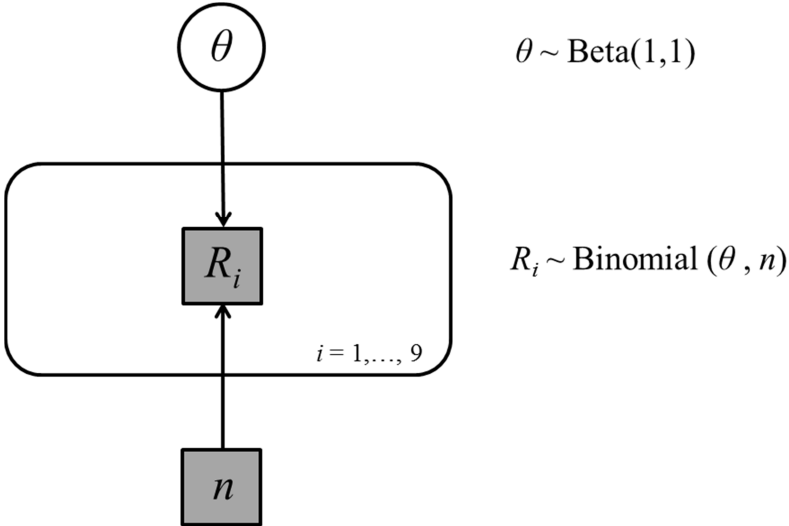


Figure 3. Bayesian Graphical Model for the estimation of Theta parameter underlying binary responses from a priori Beta distribution (Lee & Wagenmakers, 2010).

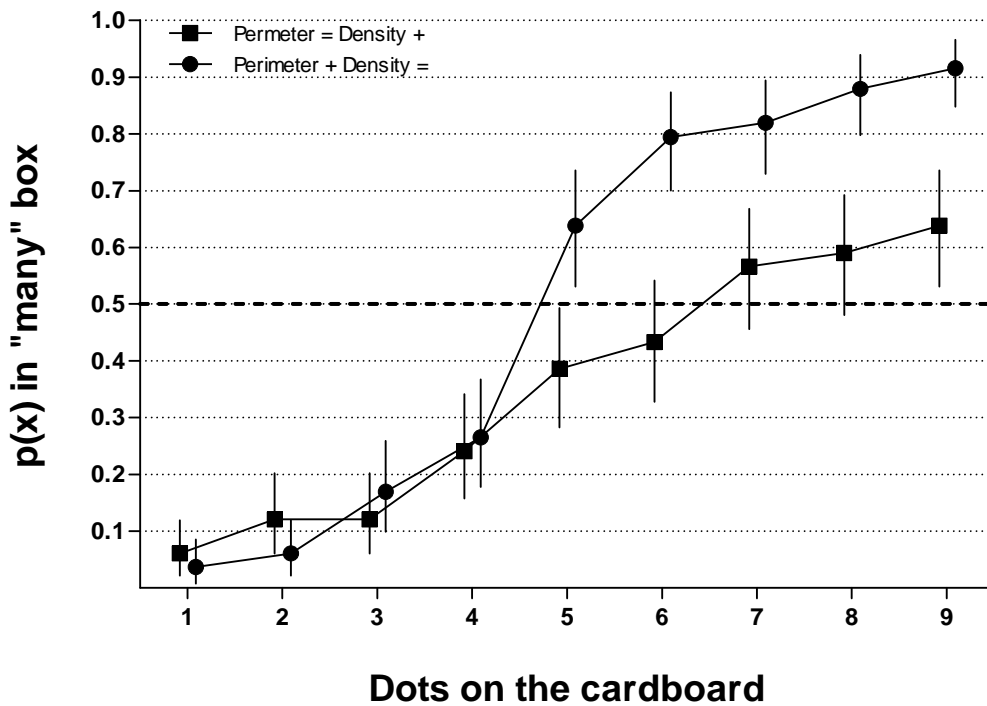


Figure 4. Average posterior distributions of the probability to insert a cardboard in the “many” box according to the specified Bayesian model (error bars indicate the 95% CI of the distribution). Results are divided for the physical controls implemented: increasing perimeter and constant density (circles) or increasing density and constant perimeter (squares).

Discussion

In the present study, children decided whether a numerosity represented on a cardboard was more similar to “few” or “many” by putting it into a one-dot or nine-dots box, respectively. We aimed to verify whether children were able to represent and categorized numerosities both within and beyond of the set size signature of the OTS. In particular, we investigated whether different physical controls in the stimuli may have a different impact of numerosity estimation for OTS and ANS. Feigenson and colleagues (2002) claimed that infants include in the object-files also physical cues, which are not suitable for enumeration purpose, thereby compromising their ability in detecting changes in numerosity. More than half 2 ½ year-old children of our sample were able to proficiently accomplish the task. Indeed, as expected, the smaller numerosities (i.e. range 1 - 4) had less probability to be inserted into the “many” box as compared to larger numerosities (i.e. range 6 - 9). The probability of inserting a cardboard in the “many” box constantly increased when numerosity of the cardboard increased, thus children understood the aim of the task and were able to represent numerosities in a congruent

way. It is worthwhile to notice that the estimations within the OTS set size were independent from the physical properties of the sets. We hypothesized two non-mutually exclusive scenarios. On the one hand, children created an object file for each dot in the set including both the stimuli numerosity and the stimulus physical characteristics. In a second step, the physical characteristics are ignored for enumeration and estimation purpose. On the other hand, being the aim of the task declared, children immediately ignored the physical characteristics of the stimuli and encoded only the one-to-one correspondence in order to accomplish the task. Beyond the set size signature of OTS (i.e., more than 4 elements), the estimation mechanism related to ANS was significantly affected by the stimuli characteristics and physical controls. The increase of perimeter with a constant density appeared as a physical variable which strongly covaried with the sense of numerosity (Clearfield & Mix, 1999, 2001; Mix, Huttenlocher, & Levine, 2002; Rousselle, Palmers, & Noel, 2004; Gebuis & Reinvot, 2012a, 2012b). The decrease of area with an increase of density induced a weaker sense of numerosity. A possible caveat might be the fact that the numerosity attached to “many” box was represented by a widespread array of nine dots. In this light, children might have based their estimation on the similarity between the spread out set with nine dots on the “many” box and the sets represented on the cardboards in which perimeter increased with numerosity. Nevertheless, the cardboard with increasing density and nine dots was inserted into “many” box with a probability above chance level despite the fact that the configuration of the stimuli occupied a small perimeter. Therefore, both occupied perimeter and density seem to increase the internal representation of numerosity as perceived by the children but with a different weight. The occupied perimeter seems to be stronger in augmenting internal magnitude representation as compared to density. Broadly speaking, the new paradigm as is prevents us to draw final conclusions. Indeed, children could base their estimations on different factors, namely, the similarity between images on the boxes and the images on the cardboards or the congruency with the verbal labels “few” and “many”. Nonetheless, the decisional process of assigning a numerosity to one of the two boxes was guided by mechanisms that are differentially influenced by physical cues of the stimuli. Up to four elements, we observed the effect of the OTS and its feature to completely discarding physical cues whereas beyond the set size signature of the OTS, the representation seems to be based on an ANS that is noticeably influenced by continuous visual cues.

Study 2

Continuous, discrete and symbolic quantity estimation in preschool and school children.*

Abstract

It has previously been shown that children's numerical estimations, in the number to position task, shift from an intuitive (logarithmic) to a formal (linear) and more accurate representation with age and practice. The shift in representation concerns the symbolic digits and less is known about other types of quantity estimation. In the present study, Preschoolers, Grade 1 and Grade 3 pupils had to map continuous, discrete and symbolic quantities onto a visual line. The same numerosities were used for the discrete and the symbolic conditions, whereas the continuous condition was matched to the discrete condition in terms of cumulative surface area. Crucially, children could base their estimations in the discrete condition either on cumulative area or (approximate) visual numerosity. Preschoolers and older children showed a linear mapping for continuous quantities, whereas a developmental shift from a logarithmic to a linear representation was observed for both discrete and symbolic quantities. Analyses of individual children's estimates and response variability indicated that different mechanisms are involved in the estimation of continuous vs. numerical (discrete and symbolic) quantities. The finding that discrete quantities were processed as numerosities rather than as continuous quantities confirms the saliency of numerosity with respect to other non-numerical visual cue.

* In collaboration with Berteletti I., Lucangeli D., & Zorzi M.

Introduction

A growing number of studies have recently investigated numerical estimation abilities thus demonstrating how humans and other animal species can represent and operate on numerical quantities (Cantlon & Brannon, 2006; Cantlon & Brannon, 2007; Agrillo, Piffer, & Bisazza, 2010). However, humans are the only able to represent numerical quantities in an exact way by means of numerical symbols. An open question is how non-symbolic types of estimation develop and whether distinct estimation mechanisms operate depending on the type of quantity to estimate.

Numerate children and adults are able to linearly map numbers (i.e. Arabic digits) to the corresponding numerical internal magnitude (Zorzi & Butterworth, 1999). This exact representation has shown to emerge with numerical expertise and education (Berteletti, Lucangeli, Piazza, Dehaene & Zorzi, 2010; Siegler & Opfer, 2003; Siegler & Booth, 2004). In a seminal study, Siegler & Opfer (2003) have shown, using the number to position task (NP-task), that children shift from an intuitive to an exact representation. Children were required to place Arabic numbers (i.e. 25), onto a black horizontal bounded line (i.e. a line going from 0 to 100). This task entails a translation of the numerical value 25 into a spatial position on the physical line. Performances of younger children are characterized by an overestimation of small numbers and an underestimation of larger numbers displaying a logarithmic positioning. According to the Approximate Number System (ANS; Feigenson, Dehaene, & Spelke, 2004; Piazza, 2010) model, each numerosity is a distribution of activation on a logarithmically compressed number line (Dehaene, 1997; Dehaene, Piazza, Pinel, & Cohen, 2003). The progressive compression of the logarithmic scale causes an overlap of activations for adjacent representations as magnitude increases. Thus, in the NP task, the distance between small numbers is greater compared to larger numbers, suggesting that children use the logarithmic and more intuitive representation to accomplish the task. With increasing age, children shift from this progressively compressed representation to a formal and linear representation thus accurately placing numbers in correspondence of the correct position.

An open question is to understand whether the representation upon which the estimation of a numerical quantity is based depends on the format of the elements to be estimated. Indeed, if children rely on the ANS also for estimating non symbolic quantities, we should observe the same logarithmic signature as the one observed when Arabic digits are positioned on the lines at least when the formal representation (linear) is not yet reached. This

data would also confirm that the logarithmic positioning observed in the symbolic tasks is not an artifact or the consequence of poor knowledge of the elements to position. A second concern is whether the estimation of non-symbolic numerical quantities also changes with development. In particular, different types of estimation might be related to distinct mechanisms which could have different developmental trajectories.

In this study we directly compare performance of children, from preschool to third grade, on three positioning tasks that differ for the format of the items to be mapped onto the line: symbolic (i.e. the classical NP-task), non-symbolic discrete and non-symbolic continuous. In the two latter versions, participants are required to position non-symbolic quantities, either sets of squares or a certain continuous amount, onto lines that are bounded either by an empty square – corresponding to zero – or by a full square (i.e. completely black or completely filled with one-hundred squares) – corresponding to the maximum possible quantity. The discrete and the continuous conditions differ substantially since the former may be processed either as a numerical quantity or as a continuous quantity depending on whether the estimation focuses on the total area occupied by the squares or the number of squares. In order to directly compare performances across tasks and ages, the quantities were exactly the same across the three conditions.

In the continuous condition, the quantity (cumulative area) must be mapped onto another continuous quantity (length of the segment). Thus, we predicted that the estimates would be fairly linear even for young children because the transformation takes place within the visuo-spatial domain. In the symbolic condition, we expected to observe the widely replicated developmental shift from logarithmic to linear mapping as a function of age (Berteletti et al., 2010; Siegler & Opfer, 2003; Siegler & Booth, 2004; Booth & Siegler, 2006). The discrete condition could yield either a linear or a logarithmic mapping depending on how the discrete quantities are processed. If children use the continuous visual cues (i.e., cumulative surface area) as input to the estimation process, the type of mapping should mirror the mapping observed in the continuous condition. In contrast, if children automatically encode numerosity (Cantlon, Safford & Brannon, 2010; Cordes & Brannon, 2008, 2009; Stoianov & Zorzi, 2012), we should observe a logarithmic signature in the estimates of the youngest children. If that is the case, improvement across age groups due to increasing reliance on a formal and linear representation should be present for both the symbolic and the discrete conditions. Indeed, studies with adults in which the quantity to estimate was

underpinned by discrete stimuli have shown the ANS signature thus indicating that adults prefer to rely on numerical information instead of continuous properties (Pica, Lemer, Izard & Dehaene, 2004; Dehaene, Izard, Spelke & Pica, 2008). Moreover, the comparison of different estimation types (e.g. symbolic, non-symbolic discrete and non-symbolic continuous) at different time points will allow delineating developmental trajectories for each underlying estimation mechanisms.

Method

Participants. Two hundred and three children from preschool to grade 3 were recruited from middle socioeconomic schools located in northern Italy. There were 40 preschoolers (17 boys; $Age_{range} = 5:6$), 68 from Grade 1 (30 boys; $Age_{range} = 6:7$) and 95 from Grade 3 (44 boys; $Age_{range} = 7:8$).

Procedure. Undergraduates in Educational Sciences from the University of Padova attended an hour and half course on how to properly administer the tasks and collected the data as part of their course internship. Children were met individually, in a quiet room, and completed the three paper-pencil estimation tasks. Experimental tasks and the quantities to be mapped were randomly administered. They were presented as games, no time limit was given and items or questions could be repeated if necessary but neither feedback nor hints were given to the child. Children were free to stop at any time.

Tasks. The estimation tasks are adaptations from the Number-to-Position task (NP-task) of Siegler and Opfer (2003). For all three conditions, a 20 cm black line was presented in the center of a half A4 landscape white sheet (see Figure 1). In the Symbolic condition, the left-end was labeled 0 and the right-end was labeled 100. Children were required to estimate the position of ten numbers (i.e. 2, 3, 4, 6, 18, 25, 42, 67, 71, 86; Siegler & Opfer, 2003) making a pen mark on the line. For each trial, the number to be positioned was presented inside a box in the upper left corner of the sheet. For the Continuous condition, an empty box (2 x 2 cm) was placed just below the left-end of the line, whereas a full black box was placed just below the right-end. Children were told that the black box was a box full of liquid (e.g. juice) while the other one was empty and the horizontal line meant the level of fullness. The quantity to be positioned was represented by a partially filled box (i.e. 2, 3, 4, 6, 18, 25, 42, 67, 71, 86 percentage of fullness) placed in the upper left corner. For the Discrete condition, the same empty box was placed just below the left-end whereas a box filled with one hundred small

black squares (0.2 x 0.2 cm) was placed just below the right-end of the line. The quantity to be positioned was represented by a box filled with a variable amount of randomly spread small squares (i.e. 2, 3, 4, 6, 18, 25, 42, 67, 71, 86 squares). Children were told that the squares were chocolate pieces and the line went from an empty box to a full box of chocolate pieces. Children were not allowed to count the squares.

Instructions were similar for the three estimation tasks except for specific changes for each type of stimuli:

Symbolic condition instructions:

“We will now play a game with number lines. In this page there is a line that goes from 0 to 100. In the upper left box there is a number that I want you to place on the line making a mark using your pencil”. While pointing to the relevant elements on the sheet, the experimenter went on with the question: *“If 0 is here and 100 is here, where would you place 25?”*

Discrete/Continuous condition instructions:

“We will now play a game. In this page there is a line that goes from an empty box of chocolate/juice a full box of chocolate/juice. In the upper left box there is a quantity of chocolates/juice that I want you to place on the line making a mark using your pencil”. While pointing to the relevant elements on the sheet, the experimenter went on with the question: *“If the empty box is here and the full box is here, where would you place this quantity of chocolates/juice?”*

To verify whether children had understood the question and were aware of the interval size, they were asked to place 0 (empty box) and 100 (full box) on the line. Only on these two practice trials the experimenter gave feedback for wrong responses by saying: *“This line goes from 0 (empty box) to 100 (full box), if I want to place 0/100 (empty box/full box), this (making the mark) is the right place”.* After the two examples, the task started and no other feedbacks were given.

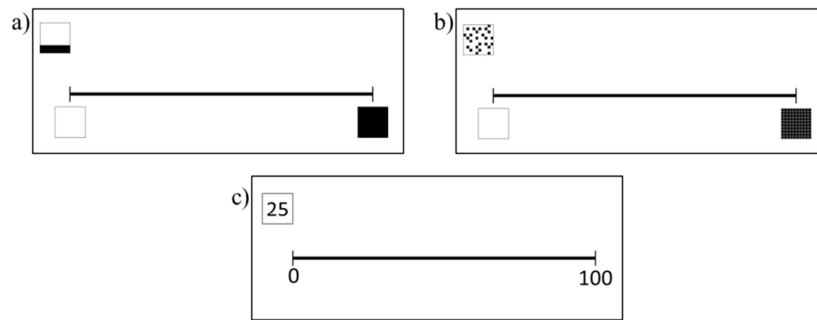


Figure 1. An example of three trials with (a) Continuous, (b) Discrete and (c) Symbolic representation of the same quantity (i.e. 25).

Results

Group analysis. Analyses were conducted following the procedure of Siegler and colleagues (Siegler & Booth, 2004; Siegler & Opfer, 2003) and post-hoc comparisons were always corrected with the Bonferroni formula. In case of inhomogeneous variances in the t-test with a violation of Levene's test, we corrected the degrees of freedom using the Welch-Satterthwaite correction. Estimation accuracy was assessed using the Percentage of Absolute Error of estimation (PAE) for each participant and condition. This was calculated as follows: $PAE = |\text{Estimate} - \text{Target Number or Quantity}|$. A mixed ANOVA was calculated with Grade as between-subject factor (Preschool, Grade 1 and Grade 3) and Estimation Task as within-subject factor (Continuous, Symbolic and Discrete). Mean PAEs, from preschool to Grade 3, in the Continuous condition were 19%, 14% and 11%, in the Discrete condition were 21%, 20% and 13%, and in the Symbolic condition were 24%, 18%, and 9% (see Figure 2).

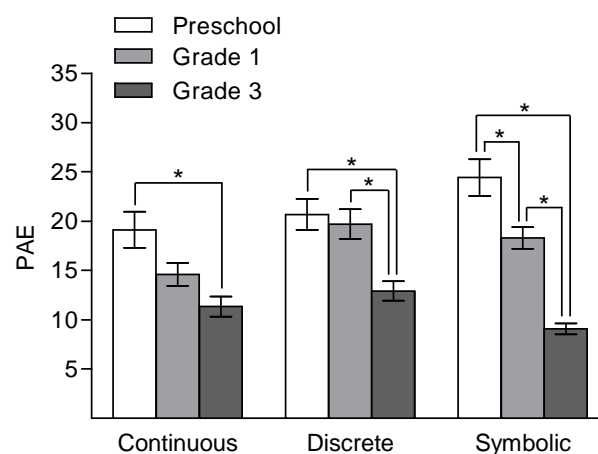


Figure 2. Percentage of absolute error in the three age groups is shown for each Estimation Task. Bars represent mean standard error. * $p < .05$ (Bonferroni corrected).

The main effect of Estimation Task ($F_{(2, 200)} = 6.84, p = .001$) and the main effect of Grade ($F_{(2, 200)} = 29.95, p < .001$) were significant. Since the interaction was also significant ($F_{(4, 200)} = 6.55, p < .001$), we performed separate one-way ANOVAs for each condition with Grade as between-subject factor. Group was significant for the three separate ANOVAs showing an increase in estimation precision with Grade (Continuous: $F_{(2, 200)} = 8.36, p < .001$; Discrete: $F_{(2, 200)} = 11.27, p < .001$; Symbolic: $F_{(2, 200)} = 56.45, p < .001$). For the Continuous condition, post-hoc comparisons revealed only a significant difference between Grade 3 pupils and Preschool children ($t_{(64.83)} = 3.71, p < .001$); in the Discrete condition, Grade 3 pupils were more precise compared to Grade 1 and Preschool pupils with $t_{(119.98)} = 3.74, p < .001$ and $t_{(133)} = 4.24, p < .001$, respectively; finally, in the Symbolic condition Grade 3 pupils outperformed Grade 1 pupils ($t_{(97.03)} = 7.49, p < .001$) while both, Grade 3 and Grade 1 pupils outperformed Preschool children ($t_{(45.45)} = 7.96, p < .001$ and $t_{(106)} = 3.01, p = .003$, respectively).

In order to understand the pattern of estimates for each condition, we fitted the linear and the logarithmic functions on group medians first (Siegler & Opfer, 2003). Group median estimates and the corresponding best linear or logarithmic fit are reported in Figure 3.

The difference between linear and logarithmic models was tested with paired-sample *t*-test on absolute distances between children's median estimate for each number and the predicted values according to the linear and the logarithmic model. If the *t*-test indicated a significant difference between the two distances, the best fitting model was attributed to the group. In the Symbolic condition, the logarithmic model had the highest R^2 for both Preschool and Grade 1 and significantly differed from the linear model ($t_{(9)} = -3.92, p = .004, R^2 \text{ lin} = 78\%$ vs. $R^2 \text{ log} = 98\%$ and $t_{(9)} = 2.77, p = .022, R^2 \text{ lin} = 88\%$ vs. $R^2 \text{ log} = 99\%$, respectively). For Grade 3 children, the linear fit was significantly better ($t_{(9)} = -2.35, p = .043, R^2 \text{ lin} = 98\%$ vs. $R^2 \text{ log} = 90\%$). In the Discrete condition, for Preschool and Grade 1, the difference between the two models did not reach significance, indicating an intermediate stage of performances (Preschool: $t_{(9)} = -1.64, p = .135, R^2 \text{ lin} = 97\%$ vs. $R^2 \text{ log} = 91\%$; Grade 1: $t_{(9)} = 1.61, p = .142, R^2 \text{ lin} = 93\%$ vs. $R^2 \text{ log} = 98\%$). For Grade 3 children however, the linear model showed the best fit ($t_{(9)} = -2.52, p = .033, R^2 \text{ lin} = 98\%$ vs. $R^2 \text{ log} = 92\%$). Finally, in the Continuous condition, the linear model had the highest R^2 and was significantly different from the logarithmic model for all groups (Preschool: $t_{(9)} = -3.5, p = .007, R^2 \text{ lin} = 96\%$ vs. $R^2 \text{ log} = 72\%$; Grade 1: $t_{(9)} = -4.23, p = .002, R^2 \text{ lin} = 98\%$ vs. $R^2 \text{ log} = 75\%$; Grade 3: $t_{(9)} = -4.22, p = .002, R^2 \text{ lin} = 98\%$ vs. $R^2 \text{ log} = 75\%$).

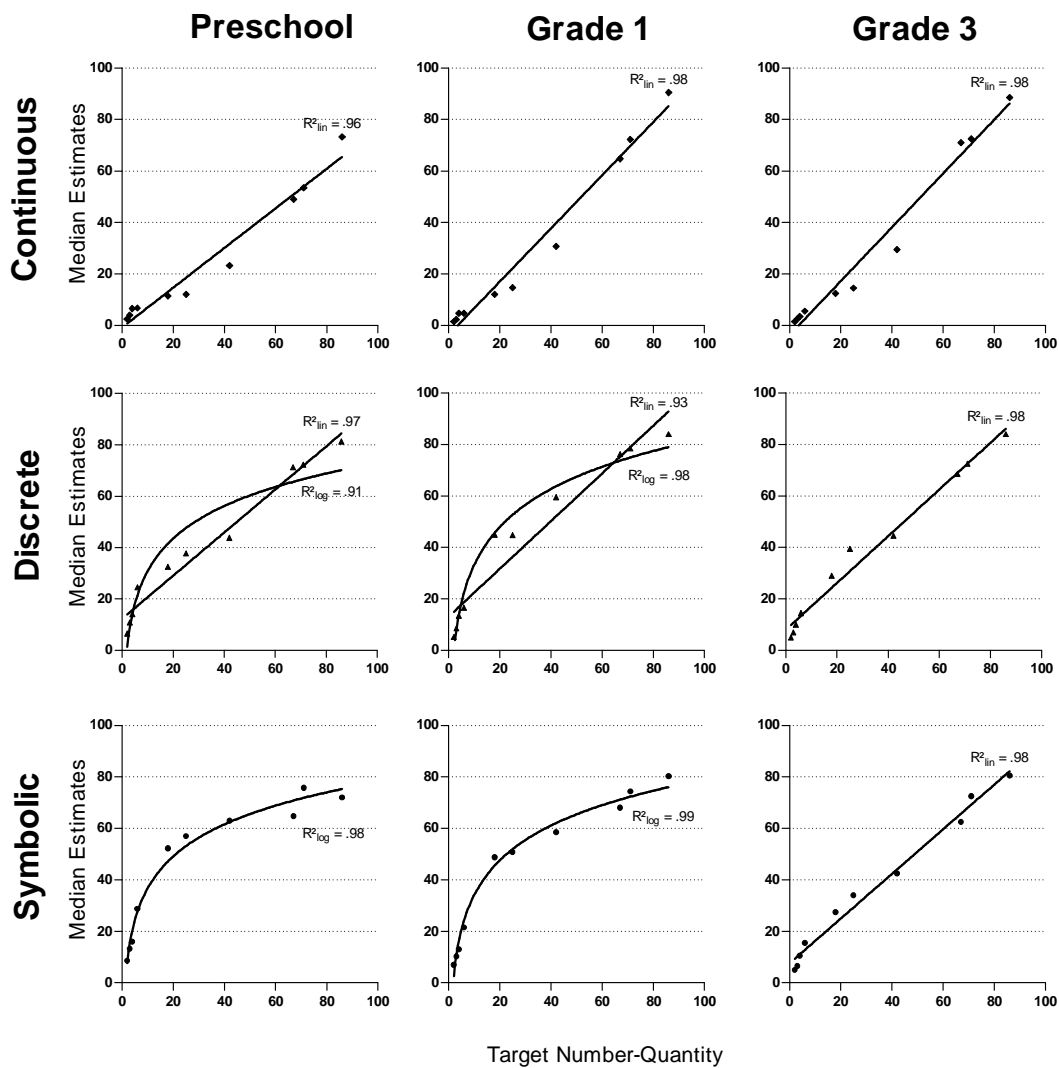


Figure 3. Children estimates and best fitting models as a function of age group for Continuous, Discrete and Symbolic type of estimation.

Individual analysis. As for group analysis, a paired t -test on residuals was computed for each child on their linear and logarithmic regression data and they were classified as Linear, Intermediate, Logarithmic or with No Representation. If the difference between the two fits was significant and both models (or at least one) were significant, the child was assigned to the representation of the model with less absolute residuals. If the t -test on absolute residuals did not reach significance and the two models were significant, the highest R^2 determined the type of representation displayed by the child (Berteletti, Lucangeli & Zorzi, 2012). Indeed, when the data is almost, but not perfectly, linear, the logarithmic model also fits very well the data yielding a null difference in the t -test on residuals. Finally, whenever both models were

not significant, the child was considered unable to perform the task properly and classified as not having an appropriate representation (Berteletti et al., 2010). In Table 1 are shown the percentages of children with each type of representation for each task.

Overall, at group level, the Symbolic task shows the previously described developmental pattern with a progressive shift from logarithmic to linear positioning. For the Continuous task, kids as young as 5-y.o. are already able to properly match two continuous quantities. However, for the Discrete condition, children seemed to prefer using a numerical strategy rather than a continuous strategy. Indeed, both the linear and logarithmic models fit the group medians for Preschool and Grade 1 children. Moreover, at single subject level, we observe that for both the Symbolic and the Discrete tasks a large percentage of children are classified as positioning items following a logarithmic distribution. Only a very small number of kids were doing so for the Continuous task (i.e. approximately 2% in the 3 age groups). Finally, we selected only those children who performed a linear mapping in the Continuous condition and we asked whether their mapping remained linear in the Discrete condition. Of 168 children in this sample, 7% were classified as No representation, 42% as Logarithmic, 51% as Linear. Such a trend confirms that in the Discrete condition children adopt a numerical and logarithmic process in positioning quantity instead of basing their estimations on the visual continuous cue.

Condition	Type of representation			Total
	None	Logarithmic	Linear	
Continuous				
Preschool ($n = 40$)	27.5	12.5	60	100%
Grade 1 ($n = 68$)	10.3	4.4	85.3	100%
Grade 3 ($n = 95$)	4.2	5.3	90.5	100%
Discrete				
Preschool ($n = 40$)	22.5	42.5	35	100%
Grade 1 ($n = 68$)	11.8	51.5	36.8	100%
Grade 3 ($n = 95$)	1.1	38.9	60	100%
Symbolic				
Preschool ($n = 40$)	22.5	70	7.5	100%
Grade 1 ($n = 68$)	5.9	79.4	14.7	100%
Grade 3 ($n = 95$)	1.1	26.3	72.6	100%

Table 1. Cell values represent percentages of children.

Item-based variability analysis. Our hypothesis predicts different processes for the Continuous task compared to the other two tasks involving numerical processing, this implies that distinct processes could account for the same linear outcome. To further investigate the different nature of the processes, we asked whether the response variability differed between tasks.

First, for each condition, we removed from the analysis those children who had no representation (categorized as “None”) to avoid confounds due to extreme and random responses. Thus, for the Continuous condition 29 from Preschool, 61 from Grade 1 and 91 from Grade 3 children were kept into the analysis. For the Discrete condition there were 31 from Preschool, 60 from Grade 1 and 94 from Grade 3 children. For the Symbolic condition there were 31 from Preschool, 64 from Grade 1 and 94 from Grade 3 children. We analyzed the standard deviation for each item in a mixed ANOVA with Grade as between-subjects factor (Preschool, Grade 1 and Grade 3) and Estimation Task as within-subjects factor (Continuous, Symbolic and Discrete). Mean standard deviations of responses from Preschool to Grade 3 in the Continuous condition were 16, 17 and 17; in the Discrete condition they were 21, 19 and 16; and in the Symbolic condition they were 16, 14, and 9, respectively (see Figure 4). The main effect of Estimation Task was significant ($F_{(2, 27)} = 20.89, p < .001$), whereas the main effect of Grade was not ($F_{(2, 27)} = 1.96, p = .161$). Since the interaction was significant ($F_{(2, 27)} = 3.57, p = .012$), we performed separate one-way ANOVAs for each condition with Grade as a between-subject factor. For the Continuous condition the effect of Grade was not significant ($F < 1$) indicating that years of schooling do not influence the ability to estimate a continuous quantity. Instead, for Discrete and Symbolic conditions the reduction of response variability with increasing Grade was significant, indicating an increasing precision in estimating items’ positions (Discrete: $F(2, 27) = 5.28, p = .012$; Symbolic: $F(2, 27) = 4.26, p = .025$). In both conditions, post-hoc comparisons revealed a significant difference between Preschool and Grade 3 pupils in the Discrete condition (Discrete: $t(15.09) = 3.18, p < 0.05$) whereas in the Symbolic condition the difference was marginally significant ($t(18) = 2.84, p < 0.10$). Importantly, the variability for the Symbolic conditions in Grade 3 is smaller than the variability at the same time period for the Continuous condition.

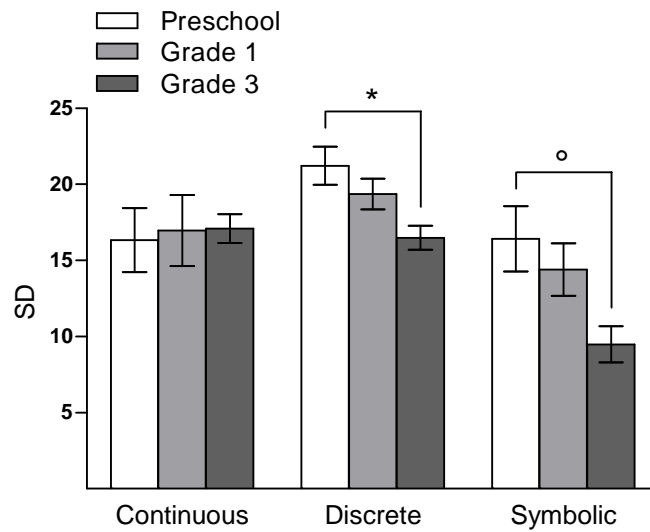


Figure 4. Item-based mean Standard Deviations in each condition (Continuous, Discrete and Symbolic) and school grade (Preschool, Grade 1 and Grade 3). Bars represent mean standard error. ° $p < .10$, * $p < .05$ (Bonferroni corrected)

Discussion

In the present study, we directly compared performance of children, from preschool to third grade, on three positioning tasks in which the quantities to be placed were continuous, discrete and symbolic. To directly compare the Continuous and the Discrete conditions, the quantities to estimate occupied the same amount of surface area but were different in their presentation. Crucially, in the Discrete condition children could base their estimations on the total amount of occupied area or, alternatively, encode the numerosity of the items. We expected to obtain similar patterns of responses between the Continuous and Discrete condition if children performed area-based judgments to provide estimates. Conversely, we expected a similar trend between the Discrete and the Symbolic condition if children performed numerosity-based judgments because the two tasks would rely on the same numerical representation. Consequently, we anticipated a shift from a logarithmic to a linear representation as previously shown with the symbolic number to position task (Siegler & Opfer, 2003). The latter evidence would also confirm children's preference to encode discrete quantities using the number of elements although other physical cues are available (Cordes & Brannon, 2009). Finally, if separate estimation mechanisms operate for different types of quantities, we should observe distinct developmental trajectories.

For the continuous task, a linear representation is already acquired at preschool and it is displayed across all ages groups. Median estimates were better fit by the linear model than

the logarithmic model for all groups, and at individual level only 2% of children were categorized as logarithmic in the Continuous condition. The accuracy in positioning the items slightly improved during development, because Grade 3 children outperformed only Preschoolers. In the Continuous condition, children made a simple transformation within the same visuo-spatial domain, thereby yielding an unbiased performance already at Preschool[†].

In the Discrete condition, both logarithmic and linear fits were good models for preschoolers and Grade 1 children, whereas the linear was the best fitting model for Grade 3 pupils. At the individual level, a large percentage of children were classified as positioning items following a logarithmic distribution. Thus, estimating discrete quantities shifts from a logarithmic to a linear representation as for the symbolic NP-task. It might be argued that younger children base their estimations on the contour length of the collection of the squares more than their numerosity or occupied area (Clearfield & Mix, 1999). However, the overestimation persists also for small numerosities that occupy a small perimeter and could be easily subitized (less than 4). Children overestimated small quantities despite the evidence that the area occupied by the squares was a small proportion of the whole full box. Thus, as previously shown by Cordes and Brannon (2009), the numerical cue seems to be more salient than other physical and spatial cues.

The Symbolic estimation replicated the shift from a logarithmic to a linear representation as already observed in previous studies (Berteletti et al., 2010; Siegler & Opfer, 2003).

The Discrete and the Symbolic condition appear more similar with a shift from a logarithmic to a linear representation compared to the Continuous condition. The latter appears substantially linear – if anything, inspecting the data in Figure 3 suggests a tendency to underestimate small quantities, which is a pattern opposite to that observed for the Discrete and Symbolic conditions. The item-based analysis of variability of responses also confirmed the similarity between the Discrete and Symbolic condition in contrast with the Continuous condition. From kindergarten to Grade 3, a reduction of the variability of responses makes children more similar in their estimations for both the Discrete and Symbolic condition. In particular, for the Grade 3 pupils, there was a strong reduction of variability in the Symbolic

[†] We note that experience might play a significant role given the fact that Italian preschoolers are well acquainted with these kinds of quantities through playing with rods of different lengths and shapes of varying size.

condition that could be ascribed to their greater familiarity with Arabic numbers and the 0-100 interval. Conversely, the variability of responses remained stable in the Continuous condition across age: indeed, Preschoolers, Grade 1 and Grade 3 children showed the same amount of variability of responses.

Continuous estimation is more accurate already in the early stages and follows a separate developmental trajectory compared to the other two types of estimation. Indeed, from kindergarten to Grade 3, the continuous estimation seems to be less influenced by schooling and maturation as compared to the discrete and symbolic estimation. Conversely, the discrete estimation appears to improve largely with maturation and schooling thus mimicking a developmental trajectory similar to the symbolic one.

In conclusion, the Continuous estimation is consistent with a linear mapping already at preschool whereas in the Discrete and Symbolic estimations we observed a shift from a logarithmic to a linear representation. The patterns of estimates in the latter conditions indicate a mapping between numerical and spatial domains with the typical ANS signature. Children's representations and response variability indicate that different mechanisms operate for the different estimation processes opposing the continuous one to the discrete and symbolic ones, as recently suggested in a quantity comparison task (Odic, Libertus, Feigenson, & Halberda, 2012). The similarity between the discrete and the symbolic estimation also suggest that children prefer to encode discrete quantities as numerosities despite the availability of physical cues thus confirming the salience of the numerosity as compared to the other physical cues (Cordes & Brannon, 2009). Taken together, the whole study seems to confirm the presence of a visual number sense which emerges as domain specific factor among other physical dimensions (Burr & Ross, 2008; Stoianov & Zorzi, 2012).

Study 3

Visual Short Term Memory load disrupts subitizing limit.*

Abstract

Early visual processing is characterized by the parallel elaboration of a massive amount of information, nonetheless the number of objects that can be simultaneously tracked and memorized is surprisingly limited, as observed both in visual short-term memory (VSTM) and non-symbolic visual enumeration tasks. Here we devised a dual-task paradigm approach to address the nature and the selectivity of the link between VSTM and subitizing capacities. Verbal memory load was used as control condition and a stringent method to evaluate the individual subitizing range was employed. Our results demonstrate that VSTM load, but not verbal load, modulated performance in the subitizing range without affecting the estimation ability. This finding provides converging evidence regarding the presence of two distinct mechanisms specifically associated to subitizing and estimation. Importantly, we found a striking correspondence between the number of elements retained in VSTM and the decrement in the number of elements that can be subitized. In particular, the trade-off between VSTM load and enumeration accuracy at the subitizing limit strongly suggests that VSTM and subitizing share the same cognitive resources.

* In collaboration with Cutini S., & Zorzi M.

Introduction

Early visual processing is characterized by the parallel elaboration of a massive amount of information, nonetheless the number of objects that can be simultaneously tracked and memorized is surprisingly limited, as observed both in visual short-term memory and visual enumeration tasks. Visual short-term memory (VSTM) refers to the ability to retain visual information for a limited period of time. One of the most commonly adopted paradigms for assessing VSTM is the change detection task: a memory array, consisting in a variable number of items (e.g., colored squares), is briefly presented and then, after a short retention interval, a test array is displayed. Participants are asked to detect whether any item included in the memory array is different from those presented in the test array. Critically, a successful change detection is possible only by comparing the items present in the visual field (i.e., test array) and those retained in VSTM (i.e., memory array): this can occur only if the items of the memory array, that are no longer in view, have been stored into VSTM. The typical capacity limit of VSTM is around three-four items (Luck & Vogel, 1997).

In visual enumeration tasks, participants are required to judge the numerosity of a set of items; in this domain, the subitizing (i.e. in an extremely rapid, precise and confident judgment of items numerosity) is one of the most intriguing phenomena (Kaufman, Lord, & Volkman, 1949). Subitizing can be usually observed only up to a few objects, usually in the number of four (Trick & Pylyshyn, 1994): performance below this limit is apparently effortless, with typical reaction time (RT) increases of around 50 ms per item.

One recent theoretical contribution (Piazza, 2010) has explicitly suggested the presence of an intimate link between VSTM and subitizing capacities. According to this view, VSTM and subitizing should be heavily related to a pre-verbal system, the Object Tracking System (OTS) (Trick & Pylyshyn, 1994), where objects are represented as distinct individuals that can be simultaneously tracked across different dimensions. The constitutive mechanism of this system is individuation, that allows to separate one item from the others, so that items are perceived as specific entities, each one with a definite identity and location (Mazza & Caramazza, 2011; Melcher & Piazza, 2011; Piazza, Fumarola, Chinello, & Melcher, 2011). Notably, the relation between VSTM and visual enumeration task seems to be circumscribed to the enumeration of quantities that fall within the limits of subitizing. When the subitizing limit is exceeded and there is no time to count the items, the OTS gives way to another pre-verbal system for numerical quantification: the Approximate Number System (ANS)

(Feigenson, Dehaene, & Spelke, 2004; Piazza, 2010; Stoianov & Zorzi, 2012). The ANS is devoted to approximate representation of numerosity and its performance is qualitatively different from that resulting from the OTS processing. There seems to be substantial evidence that subitizing and estimation are related to two different mechanisms, although there are some controversial results (Beran, 2007; Brannon & Terrace, 1998; Cordes, Gelman, Gallistel, & Whalen, 2001).

It is worth noting that the investigation of capacity limits might represent a fruitful approach to assess the features of specific cognitive architectures; despite an intuitive relationship between VSTM capacity and subitizing, and the naïve observation that in humans the strikingly similar behavioral limits of subitizing and VSTM (Cutini & Bonato, 2012), only one seminal study to date (Piazza et al., 2011) aimed at highlighting the relation between the processing of non-symbolic magnitudes within the subitizing range and VSTM. The authors adopted a dual-task paradigm to see whether these similar capacity limits are subserved by the same cognitive resources. In each trial, participants performed two tasks: an enumeration task and a change detection VSTM task. Participants were first presented with a memory array of either two or four colored circles (low vs. high VSTM load), briefly replaced by a counting set (1-8 items). This set was then masked and the participants were asked to report its numerosity (primary task), then they were presented with a test array (same number of colored circles of the memory set) and performed a same-different judgment with respect to the memory array (secondary task). Interestingly, the amount of VSTM load selectively impaired performance in the enumeration task, by reducing the individual subitizing range, but had no significant effect on the estimation of large quantities; furthermore, the interference between the two tasks exhibited a predictable pattern, consistently with the presence of a core component whose resources are shared between VSTM and visual enumeration of small numerosities (i.e., subitizing). Although it is undeniable that the aforementioned study made a significant breakthrough in understanding the relation between VSTM and subitizing, it is worth noting that there are some critical points that might moderate the impact of the results and thus need to be further investigated. In the paradigm used by Piazza et al. (2011), 90% of trials were composed by 1, 2, 4, 6 or 8 stimuli, and the other 10% of trials were composed by 3, 5, 7 or 9. This inhomogeneous distribution of the trials might pose a problem with regard to the calculation of the subitizing range, potentially biasing the results. For instance, such disparity might facilitate the task, thereby artefactually

increasing the subitizing limit, given that the participants might tend to respond with even numbers (except 1) when unsure about the correct numerosity.

Moreover, the control condition adopted by the authors included only a visual enumeration single-task with no concurrent load at all: this limitation allows to draw only weak inferences about the selectivity of the relation between subitizing and VSTM. For instance, it is conceivable to think that subitizing might be disrupted by a concurrent non-visual load, or simply by a dual-task condition. Based on this observation, it cannot be excluded that subitizing abilities might be simply affected by higher unspecific task demands, which are not necessarily confined to the VSTM domain.

Here we devised a dual-task paradigm approach, combined with an appropriate control condition in order to maximize the amount of information derivable from the experiment, and to precisely address the nature and the selectivity of the link between VSTM and subitizing capacities. In addition to the inclusion of a stringent control condition, we used the same number of trials for all the numerosities and to implement a stringent method to evaluate the individual subitizing range. These choices allowed us to take full advantage of the amount of information obtainable with the present experiment.

Method

Participants. Twelve participants (5 males; $M_{\text{age}} = 23.6$, $SD = 3$) took part in the experimental session after providing their informed consent. They had normal or corrected to normal vision and they did not report any past history of neurological disease or auditory deficit.

Stimuli. Participants accomplished two different memory tasks in a dual-task condition. In the Verbal Memory (VM) task, four disyllabic pseudo-words (Sartori, Job, Tressoldi, 1995) were presented through earphones for four seconds (1 second each). The words were randomly selected from a pool of 16 words. In the low-load condition only one word was played and repeated four times whereas, in the high-load condition, four different words were played. After words presentation, a cloud of random dimension dots (numerosity from 1 to 9) was shown for 200 ms and then immediately masked by a dots shaped figure for 100 ms. Participants reported the number of dots they saw by pressing the corresponding key on a numeric key-pad of a QWERTY keyboard. At the end of the trial, participants heard a target word and had to decide whether it was present or not in the previous memory set by pressing the key “Y” or “N”, respectively. There were 216 randomly presented trials, half in

the low-load condition and half in the high-load condition. In each load condition, there were 12 trials for each numerosity, from 1 to 9.

In the VSTM task, a 2 x 2 memory array (approximately 100 (l) x 132 (h) pixels) with four figures was presented for 500 ms. The figures were randomly selected among a pool of six black mushroom shapes in which the cap was the only varied element. In the low-load condition, the four figures were identical whereas, in the high-load condition, the four figures were all different. After an inter-trial interval of 1000 ms, a cloud of dots (numerosity from 1 to 9) was shown for 200 ms and then immediately masked by a dots shaped figure for 100 ms. Then, participants reported the number of dots they saw by pressing the corresponding key on a numeric key-pad of a QWERTY keyboard. At the end of the trial, a memory target figure appeared in the center of the screen and participants decided whether the target has been showed or not in the previous memory array by pressing the key “Y” or “N”, respectively (see Figure 1). There were 216 randomly presented trials, half in the low-load condition and half in the high-load condition. Inside each load condition, there were 12 trials for each numerosity, from 1 to 9.

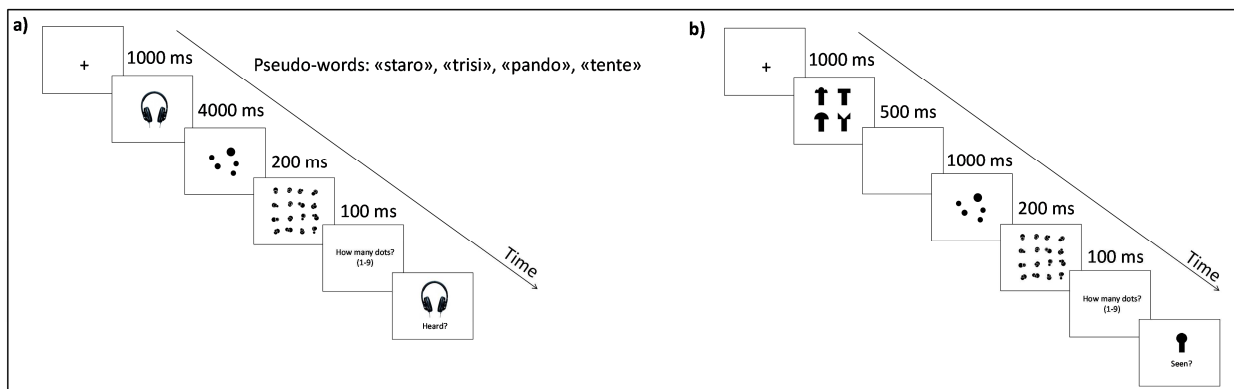


Figure 1. An example of high-load condition in the VM task (panel a). Four different disyllabic pseudo-words were presented through earphones for four seconds (1 second each). After words presentation, a cloud of dots was shown for 200 ms and then immediately masked by a dots shaped figure for 100 ms. Participants reported how many dots they saw by pressing the corresponding key on a numeric key-pad. At the end of the trial, participants heard a target word and had to decide whether it was present or not in the previous memory set. An example of high-load condition in the VSTM task (panel b). A 2 x 2 memory array with four different figures was presented for 500 ms. After an inter-trial of 1000 ms, a cloud of dots was shown for 200 ms and then immediately masked by a dots shaped figure for 100 ms. Participants reported how many dots they saw by pressing the corresponding key on a numeric key-pad. At the end of the trial, a memory target figure appeared in the center of the

screen and participants decided whether the target was present or not in the previous memory array.

Procedure. Participants sat in a quiet room approximately 60 centimeters far from a 17-inch screen (1024 x 768 pixels). The instructions to accomplish the tasks were written on the screen and also verbally presented by the experimenter. Participants were explicitly instructed to not trade-off memory task for enumeration task or vice versa. The order of the tasks was counterbalanced across participants and the experimental session lasted for approximately one hour depending on single participant's ability. Participants were allowed to take a rest between the two tasks.

Results

Results are divided into three sections: a) the visual and verbal memory tasks; b) the dots enumeration task; c) VSTM load and subitizing. As first step, we removed from analysis the responses for 8 and 9 dots given that the percentage of correct responses in dots enumeration tended to unnaturally increase in these conditions. This anchoring effect was probably due to the fact that participants realized that the maximum amount of displayed dots was nine (numeric keypad limit) thus responding 8 or 9 for large numerosities

a) Visual and Verbal memory task. In this section, we controlled whether participants properly completed the words and pictures memory tasks. Indeed, some participants might prefer to discard the VSTM and the VM task to obtain a better performance in the dots enumeration task. This was not the case because participants yielded a percentage of correct responses higher than the chance level (i.e. 50%) in the VSTM task, both in low-load condition ($M = 91\%$, $SD = 6.1$), $t(11) = 23.45$, $p < 0.001$, and in the high-load condition ($M = 70\%$, $SD = 9.2$), $t(11) = 7.14$, $p < 0.001$. Moreover, participants showed a worse performance in the high-load condition as compared to the low-load condition, $t(11) = 7.8$, $p < 0.001$. In the VM task, accuracy was higher than chance level both in low-load condition ($M = 95\%$, $SD = 3.3$), $t(11) = 47.09$, $p < 0.001$, and in the high-load condition ($M = 96\%$, $SD = 3.3$), $t(11) = 48.55$, $p < 0.001$. Furthermore, there was no significant difference in percentage of correct responses between the low-load and the high-load condition in the VM task, $t < 1$. We also checked whether participants traded-off their accuracy in the memory tasks with the increasing number of dots in the enumeration task. For the VSTM task and the VM task, we collapsed the percentages of correct responses in the low-load and high-load condition. The Spearman's rank correlation analysis between percentages of correct responses

and number of dots was not significant for all participants except one. Only one participant decreased percentage of correct memory responses with the increase of number of dots in the VM task, $r = -0.79$, $p = 0.034$. However, the mean accuracy of this participant in the VM task was 99% suggesting that the trade-off did not affect the memory performance.

b) Dots enumeration. We analyzed percentage of correct responses in dots enumeration in a 2 (Load: low-load, high-load) x 7 (dots) repeated measures ANOVA, separately for VM task and VSTM task.

In the VM task, the main effect of Dots was significant, $F(1, 11) = 36.02$, $p < 0.001$, $\eta^2_p = .766$, whereas the main effect of Load and the interaction Load x Dots were not significant ($F < 1$). Participants consistently reduced their percentage of correct responses with the increase of the number of dots. This trend was not influenced by the verbal memory load, suggesting the independence between dots estimation and verbal memory.

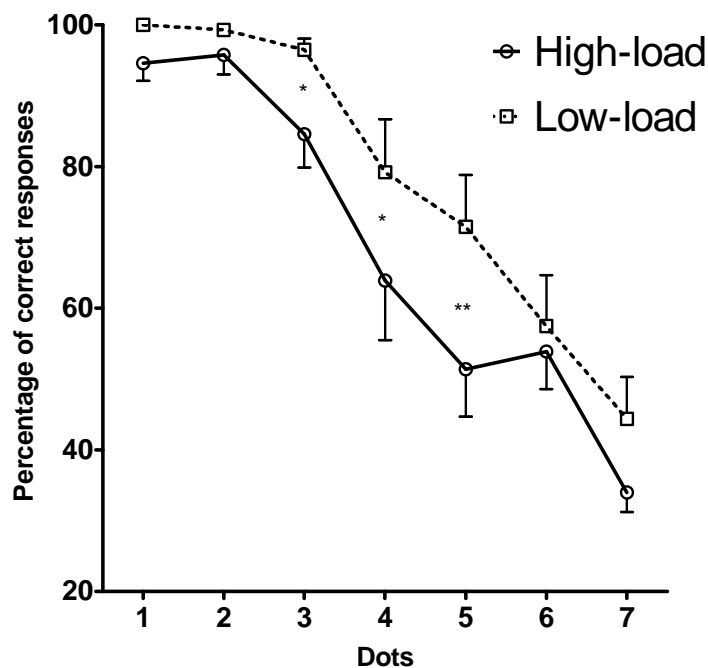


Figure 2. The percentage of correct responses (y-axis) as a function of the number of dots (x-axis) in the VSTM task (error bars mean SEM). The straight line means the high-load condition whereas the dashed line means the low-load condition. Participants showed a reduced accuracy for 3, 4, and 5 dots in the high-load condition as compared to the low-load condition. ** $p < 0.01$.

In the VSTM task, the main effect of Load was significant, $F(1, 11) = 12.98$, $p = 0.004$, $\eta^2_p = .541$, as well as the main effect of Dots, $F(6, 11) = 32.67$, $p < 0.001$, $\eta^2_p = .748$.

Crucially, the interaction Load x Dots was also significant, $F(6, 11) = 2.79$, $p = 0.018$, $\eta^2_p = .202$; thus, we performed a series of t -tests (one-tailed with Bonferroni correction for multiple comparisons) comparing accuracy in the low-load vs. high-load conditions for each numerosity. We found a significant difference for 3 dots, $t(11) = 3.03$, $p < 0.05$, 4 dots, $t(11) = 3.08$, $p < 0.05$, and 5 dots, $t(11) = 4.05$, $p < 0.05$. In the high-load condition the percentages of correct responses in the enumeration task was reduced as compared to low-load condition. This result directly suggests that VSTM load has a detrimental effect on sets that have a numerosity around the subitizing limit (Figure 2).

c) VSTM load and subitizing. To highlight the effect of VSTM load on subitizing range, we fitted a sigmoidal dose-response curve to the enumeration accuracy data as a function of the number of dots for each participant separately in VM task (accuracy in enumeration of the low-load and high-load condition in the VM task have been averaged because of the lack of interaction between load and dots) and in the VSTM task for the low-load condition and the high-load condition. We then calculated the second derivative point for each sigmoidal curve thus obtaining a precise measure of the first flex-point in the s-shaped curve. The first flex-point of the curve specifically underlines the moment in which subitizing gives the way to approximate estimation (Revkin, Piazza, Izard, Cohen, & Dehaene, 2008). The resulting flex-points were expressed in x-axis values so we rounded down to the nearest unit to have a subitizing limit measured in number of dots unit for each participant. Therefore, we retrieved the percentage of correct responses in dots enumeration task for each participant at his/her own subitizing limit in the VM task, in the low-load condition and in the high-load condition in the VSTM task. Finally, we analyzed accuracy of responses at the individual subitizing limit in a repeated measure ANOVA with Task (Verbal, low-load VSTM, high-load VSTM) as within subject factor. The main effect of Task was significant, $F(2, 11) = 8.16$, $p = 0.002$, $\eta^2_p = .426$, suggesting a different effect of the memory load on the enumeration accuracy at the subitizing limit as precisely assessed for each participant (Fig. 3). The post-hoc t -test comparisons (one-tailed) revealed a significant difference between VM task and low-load condition in VSTM task, $t(11) = 2.00$, $p = 0.035$, similarly the difference in accuracy was significant between low-load condition and high-load condition in VSTM task, $t(11) = 2.27$, $p = 0.022$. Consequently, the difference in dots enumeration accuracy was significant between VM task and high-load condition in VSTM task, $t(11) = 3.64$, $p = 0.002$. To ensure that participants' decrease of accuracy in dots enumeration at subitizing limit was

directly connected to memory load, we assessed Cowan’s K (Cowan, 2000; $K = (\text{hit rate} + \text{correct rejection rate} - 1) \times N$ ($N = \text{memory set size}$)) in order to obtain a reliable measure of the number of elements correctly stored in the VSTM. We assessed Cowan’s K for each participant separately in the VM task, in the low-load condition and the high-load condition in the VSTM task. In the VM, we considered that the K was equal to zero given the absence of visual stimuli to be memorized. We analyzed K values in a repeated measure ANOVA with Task (VM, low-load VSTM, high-load VSTM) as within subject factor. The main effect of Task was significant, $F(2, 11) = 50.72, p < 0.001, \eta^2_p = .822$. In a post-hoc comparison, we found that the Cowan’s K was significantly lower in low-load VSTM ($M = 0.82, SD = 0.11$) as compared to high-load VSTM condition ($M = 1.65, SD = .7$), $t(11) = 4.26, p = 0.01$ (Figure 3). This result confirmed the fact that participants properly accomplished the two memory tasks: the number of elements correctly memorized by participants were approximately one in the low-load condition and almost two in the high-load condition. Furthermore, we found a significant positive correlation between Cowan’s K and the individual flex-point of subitizing in the high-load VSTM condition, $r = .57, p = 0.05$. To sum up, the individual VSTM capacity positively covaried with the individual subitizing capacity.

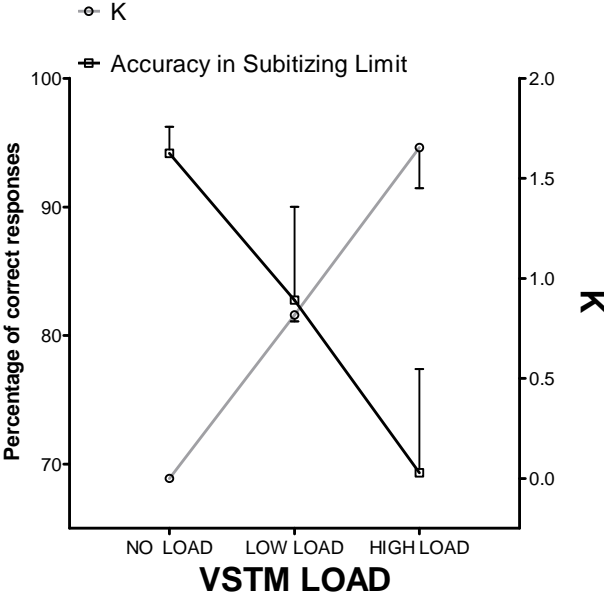


Figure 3. Black line: accuracy of visual enumeration calculated on the individual subitizing limit under VM load (i.e., NO LOAD), and the correspondent accuracy for low and high VSTM load. Gray line: Cowan’s K value under VM load (NO LOAD set to 0), low and high VSTM load. Values indicate mean and standard deviation.

Discussion

The interplay between enumeration and VSTM is rapidly becoming a crucial topic in cognitive science, and the strikingly similar limits of VSTM and subitizing strongly suggest that their underlying mechanisms might share some critical components. The present investigation aimed at taking advantage of these limits to unveil the genuine nature of the relation between VSTM and subitizing. Here we provided a series of relevant findings that might help to better understand the characteristics of the visuo-spatial mechanisms underlying VSTM and subitizing.

Our results demonstrate that the amount of VSTM load influenced the performance in the subitizing range without affecting the estimation ability. Visual enumeration performance was significantly modulated by the amount of VSTM load only for numerosities 3, 4 and 5, indicating that the strongest modulation of the VSTM for numerosities around the subitizing limit. Thus, we found compelling evidence that subitizing and estimation are likely to be related to two different mechanisms, rather than being two extremes belonging to the same continuum (Hyde, 2011). This result is in line with a number of previous observations (e.g. Piazza et al., 2011), like those from a forced choice enumeration task (Revkin et al., 2008), or those obtained by an investigation of attention and visual enumeration (Burr et al., 2010). Even one study (Vetter, Butterworth, & Bahrami, 2008) that provided results in apparent contradiction with Burr et al. (2010), in fact revealed that, although the attentional load affected both subitizing and estimation, the impact on the subitizing range was clearly more pronounced. In addition to the behavioral evidence, neuroimaging studies revealed the presence of a different neural signatures for subitizing and estimation (Ester, Drew, Klee, Vogel, & Awh, 2012; Hyde & Spelke, 2009; Vetter, Butterworth, & Bahrami, 2011). Thus, the present results provide converging evidence regarding the presence of two distinct mechanisms specifically associated to subitizing and estimation.

More importantly, our results provide compelling evidence with regard to the existence of a specific and selective link between VSTM and subitizing. Instead of using reaction times (which are more appropriate for separating subitizing from counting), we calculated the individual subitizing limit with a stringent procedure. Crucially, we found a marked correspondence between the number of elements retained in VSTM and the decrement in the number of elements that can be subitized. In particular, the trade-off between

VSTM load and enumeration accuracy at the subitizing limit might be regarded as a strong evidence that VSTM and subitizing share the same cognitive resources.

A recent study found a link between multiple object tracking (MOT) and subitizing (Chesney & Haladjian, 2011). The authors adopted a dual-task paradigm with an enumeration task (0-9 elements) and a tracking task (with 0, 2 or 4 elements to be tracked). Although the authors argued that the number of items that participants could subitize decreased by one for each tracked item, their results are not clear-cut as those provided here: for instance, they only found a marginally significant interaction between enumeration condition and tracking condition. Another recent study (Feng, Pratt, & Spence, 2012) employed a dual task paradigm with change detection and visual enumeration. Their results diverged from the present ones, failing to show a clear influence of VSTM load on the subitizing ability possibly because the to-be-enumerated stimuli were not masked and participants were allowed to count the elements.

As noted in the introduction, only one study (Piazza et al., 2011) aimed at addressing the same issues of the present work by using a similar approach. Notably, the added value provided by the present study resides in the relevant amount of information that confirms and enriches the findings of the previous investigation. Indeed, adding a verbal memory condition with two different amounts of load allowed us to rule out the hypothesis that the simple addition of a concurring task might produce an impairment of subitizing abilities. Our results show that the interaction between memory and subitizing is limited to VSTM. Indeed, verbal memory load did not affect performance, given that the amount of verbal memory load was far from influencing enumeration abilities, and the subitizing capacity was affected by the amount of load of the concurrent task only when the nature of the load was visual. This finding is confirmed by the fact that visual enumeration performance under verbal memory load was very similar to the usually observed performance with no concurrent load (e.g., Revkin, Piazza, Izard, Cohen, & Dehaene, 2008). Furthermore, the adoption of a homogeneous distribution of the trials excluded a potential bias in determining the subitizing range, providing more strength to the present results.

In the future research, the relation between VSTM and subitizing should be investigated by other paradigms to corroborate the present results, and it could be very informative to spotlight a possible covariation between the two abilities during typical and atypical development.

Atypical development studies.

*"Let me see: four times five is twelve, and four times six is thirteen, and four times seven is --
oh dear! I shall never get to twenty at that rate!"*

(Alice's Adventures in Wonderland, 1865)

Study 4

Numerical estimation in children with Developmental Dyscalculia: evidence from a delayed match to sample task and a number line estimation task.

Abstract Study 4.1

Developmental dyscalculia is a learning disability characterized by an evident deficit in math achievement scores despite an adequate general intelligence and preserved perceptual abilities. It has been claimed that DD might be related to a specific deficit of analogical numerosity representation as a signature of an impaired number sense. Such a deficit seems to regard the representation of both small and larger quantities, stemming from a reduced Object Tracking System capacity and a weaker acuity of the Approximate Number System, respectively. In particular, recent studies have proposed that children with DD may adopt a serial counting procedure when enumerating small numerosities thus suggesting a reduced subitizing range capacity. Nevertheless, these results could be attributed both to a weak numerical representation or in turn to a deficit in translating from analogical to symbolic representations and vice versa. In Study 4.1, children with DD in comorbidity with a profile of Non-Verbal syndrome (NVS) and typically developing (TD) children completed a delayed match-to-sample task in order to verify the accuracy in the comparison of analogical quantities (i.e. set of dots) within and beyond the OTS capacity (i.e. 1-4 and 5-9, respectively). We found a specific reduction of OTS functioning in NVS-DD children with respect to TD children as suggested by a decreased accuracy in comparison of small quantities (e.g. 3 vs. 4 dots). Also the comparison of larger numerosities seems to be less precise in NVS-DD children as compared to matched TD children. The evidence of the present study confirms and extends the results of previous research by showing a specific reduction of OTS capacity in the absence of any involvement of the access to symbolic representation of numerosity.

Abstract Study 4.2

Several studies have shown that typically developing children shift from a logarithmic to a linear representation in mapping symbolic digits to a spatial position on a line. The initial pattern of overestimation of small numbers and the underestimation of larger numbers is

compensated by means of maturation and education. Children with mathematical disability seem to show less accuracy in placing numbers on the line and their representation tends to be more logarithmic than linear. Here we evaluate to what extent this hypothesis holds for a sample of Italian children who have received a formal diagnosis of developmental Dyscalculia (DD). Ten children with DD ($M_{age-months} = 121$, $SD = 23$) and ten typically developing (TD) children ($M_{age-months} = 111$, $SD = 23$), matched for age and gender, completed two number to position tasks (intervals: 0-100, 0-1000). For the interval 0-100, children with DD obtained a representation in an intermediate stage between logarithmic and linear mapping whereas the TD reached a linear representation. For the interval 0-1000, children with DD exhibited a logarithmic mapping whereas TD children had a linear representation. This results highlights the specific deficit of basic numerical processing in DD.

Introduction to Study 4.1 and Study 4.2

Successful math achievement can be considered as the by-product of several cognitive, educational and motivational factors which can differently interact across lifetime. In this light, various reasons could be responsible for a weak math attainment in those children who obtain a performance at the lower bound of standardized mathematical tests. Beyond the educational and the motivational aspects, children with math difficulties may present relatively different cognitive profiles thus composing a rather heterogeneous group (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). For instance, a child with a specific deficit of phonological working memory might present a severe problem in learning basic arithmetic skills which are fundamental to solve calculations (Lemaire, Abdi, & Fayol, 1996). Conversely, another child could display similar calculation problems which are instead related to poor attentional control in the execution of the calculation procedures (Hitch, 1978). Therefore, the identification of the impaired cognitive subcomponents of math achievement can explain the source of difference between typically developing and math disabled children. One crucial aspect is to investigate whether children with math disability can represent and estimate numerical quantities as well as typically developing children.

Two mechanisms have been individuated as fundamental for quantification process: the Object Tracking System (OTS) and the Approximate Number System (ANS). The OTS is a general mechanism which tracks the spatio-temporal characteristics of the stimuli by creating a memory-file for each element. The main signature of the OTS is a capacity limited to 3-4 elements. Indeed, in the numerical context, when small sets (less than 3-4 items) have to be enumerated, individuals can quickly and exactly individuate the number of the elements with a minimum effort resulting in a behavioral effect called subitizing (Xu, Spelke & Goddard, 2005; Trick & Pylyshyn, 1994; Mandler & Shebo, 1982). Recent results have highlighted that children with developmental dyscalculia display a less efficient subitizing and tend to adopt serial counting to determine the numerosity of small sets (Schleifer & Landerl, 2010; Moeller, Neuburger, Kaufmann, Landerl & Nuerk, 2009; Landerl, Bevan & Butterworth, 2004). Nevertheless, such studies also involved symbolic quantity processing thereby preventing to disentangle whether the deficit observed in developmental dyscalculia might be link to a pure OTS impairment or related to the access of the symbolic quantity from the non-symbolic format (for this account, Rouselle & Noel, 2007).

For larger quantities (beyond the subitizing range 1-4), according to the Approximate Number System, each numerosity is represented as a distribution of activation with constant variability on a logarithmically compressed number line (Dehaene, 1997; Dehaene, Piazza, Pinel, & Cohen, 2003; Feigenson, Dehaene, & Spelke, 2004; Piazza, 2010; for another account see, Gallistel & Gelman, 1992). Two numerosities that are far apart are easier to discriminate as compared to numerosities that are close to each other (i.e., distance effect). As consequence of the logarithmic compression, the overlapping between distributions increases as the numerosity increases. Then, it is easier to discriminate between small quantities because they are more spaced apart as compared to larger quantities (i.e., size effect): for instance, on the mental number line the distance between 4 and 5 is larger than between 8 and 9. The distance and the size effects can be summarized in the ratio dependent effect: a discrimination between two numerosities becomes more difficult as their ratio approaches one. Recent studies have highlighted that a finer ability in discriminating between analogical quantities (e.g. dots) is positively correlated with math achievement as measured with standardized tests (Halberda, Mazocco, & Feigenson, 2008; Lourenco, Bonny, Fernandes, & Rao, 2012). Moreover, children with developmental dyscalculia (DD) appear less able to discriminate between quantities as a signature of a weak ANS acuity. Piazza and colleagues (2010) demonstrated that ten year-old children with DD had a reduced accuracy in comparing numerosities, displayed by a number acuity comparable to that of five year-old typically developing children (Piazza, Facoetti, Trussardi, Berteletti, Conte, Lucangeli, Dehaene, & Zorzi, 2010). Conversely, Rouselle and Noel (2007) found that children with math disability (MD) had a deficit in comparing symbolic numerosities (i.e. Arabic digits) whereas the comparison of the non-symbolic quantities was preserved thereby suggesting a deficit in accessing number magnitude from symbols more than an impairment in processing numerosity.

Beyond the approximate representation, numerate individuals are able to represent numerical quantities in an exact way by means of numerical symbols. Indeed, numerate children and adults are able to linearly map numbers (i.e. Arabic digits) to the corresponding numerical internal magnitude (Zorzi & Butterworth, 1999; Verguts, Fias, & Stevenson, 2005). In a seminal study, Siegler & Opfer (2003) have shown, using the number to position task (NP-task), that children shift from an intuitive to an exact representation from 2nd to 6th grade. Participants were required to place Arabic numbers (i.e. 25), onto a black horizontal bounded

line (i.e. a line going from 0 to 100). This task entails a translation of the numerical value 25 into a spatial position on the physical line. Performance of younger children is characterized by an overestimation of small numbers and an underestimation of larger numbers, yielding a logarithmic pattern. According to the ANS, the distance between small numbers is greater as compared to larger numbers, suggesting that children use the logarithmic and more intuitive representation to accomplish the task. With increasing age, children shift from this compressed representation to a formal and linear representation thus accurately placing numbers in correspondence of the correct position. This shift, from a logarithmic to a linear representation, is influenced by the context, namely, the scale of the line interval. Preschoolers show a linear representation for small intervals such as 1-10, whereas their representation is still logarithmic for a larger scale such as 0-100 (Berteletti et al., 2010). During the first two years of elementary school, the linear representation is progressively acquired for the 0-100 interval (Siegler & Booth, 2004) whereas the linearity is mastered around 4th grade for the 0-1000 interval (Booth & Siegler, 2006) and around 6th grade for the 0-10000 interval (Thompson & Opfer, 2010). Interestingly, at a same time point, a child may be able to position numbers linearly on a smaller scale but revert to an informal representation to perform the task on a larger interval although being perfectly able to name and recite the entire sequence of the larger interval (Berteletti, Lucangeli & Zorzi, 2012). Thus, children's logarithmic representation is not merely an artifact of the task itself or poor knowledge of the items presented but it entails a specific representation of numerosity. Finally, other studies have shown that performance in the NP-task correlates with other estimation tasks (Booth & Siegler, 2006), memory for small versus large numbers (Thompson & Siegler, 2010) and future mathematical achievement (Booth & Siegler, 2008).

Geary, Hoard, Nugent and Byrd-Craven (2008), using standardized mathematical achievement tests, classified 1st and 2nd grade children into mathematical learning disability (below the 11th percentile), low math achievement (between 11th and 25th percentile), and typical achievement groups. In the number line task with the interval 0-100, Grade 1 pupils with math disability displayed a logarithmic representation as compared to the other groups, which showed a linear mapping. Only at Grade 2, children with math disability displayed a representation in an intermediate stage between the logarithmic and the linear mapping but they still lacked a complete linear representation. In a subsequent study, Landerl, Fussenberg, Moll and Willburger (2009) analysed the performance in the number to position task of

typically developing, dyscalculic, dyslexic, and dyslexic-dyscalculic children. Children categorized as dyslexic had a score below 1 standard deviation (SD) in a reading fluency test and an adequate score in the arithmetic test. Conversely, dyscalculics had a score below 1 SD in the arithmetic test but had an adequate score in the reading test. Children with performance below 1 SD in both the reading and the arithmetic test were categorized as dyslexic-dyscalculics. In the 0-1000 interval, dyscalculic children had an almost logarithmic representation whereas the dyslexic-dyscalculics displayed a worse performance with a clear logarithmic fit.

In summary, children with developmental dyscalculia seem to display a deficit in basic numerical processing that need further investigations. On one hand, there is a need to verify whether the OTS deficit is purely related to the memory file creation rather than to the access to the symbolic quantity. Similarly, there is need to gather more evidence regarding the ANS acuity deficit in children with developmental dyscalculia (Study 4.1). On the other hand, it appears that the number to position task might play a role in highlight the internal representation of numbers but also be a potential diagnostic tool (Study 4.2).

Method Study 4.1^{*†}

Participants. Twenty-eight children from the middle socioeconomic status from northern Italy took part in the study. There were 14 children with Non-Verbal syndrome in comorbidity with developmental dyscalculia (NVS-DD) and 14 typically developing (TD) children matched for chronological age and verbal IQ using the Vocabulary and Digit Span subtests of WISC III (Wechsler, 1991). Participants' characteristics are presented in Table 1. Children with NVS-DD met the criteria for the diagnosis of developmental dyscalculia (DD): general IQ above 85; performance below 2 SD on math standardized tests; normal or corrected to normal vision; no other neurological disorders; normal education; the absence of comorbidity with attention deficit Hyperactivity disorder. Moreover, they presented the classical pattern of the NVS with a discrepancy between the preserved verbal abilities and the compromised spatial abilities (Rourke, 1989).

* The data presented in this study have been already presented in a study by Trussardi A. (PhD dissertation thesis, 2008). Here the data have been reanalyzed in light of a different research hypothesis. The author wish to thank Dr. Trussardi and her collaborators for the permission to reanalyzing the dataset.

† In collaboration with Zorzi M.

	NVS-DD (<i>n</i> = 14)	TD (<i>n</i> = 14)	<i>t</i> (26)
	<i>M</i> (<i>SD</i>)	<i>M</i> (<i>SD</i>)	
Age (months)	126 (16)	118(15)	-1.24
Vocabulary	9.8(2.9)	11.6(1.9)	2.01
Digit Span	7.3(3.1)	9.1(3.7)	1.33
Block Design	5.6(3.1)	12.8(2.3)	6.89**
Object Assembly	5.3(2)	11.6(3.5)	5.86**

** $p < 0.001$

Task. Participants completed a delayed dots-to-dots match-to-sample task (Figure 1).

Dots-to-dots Match to sample

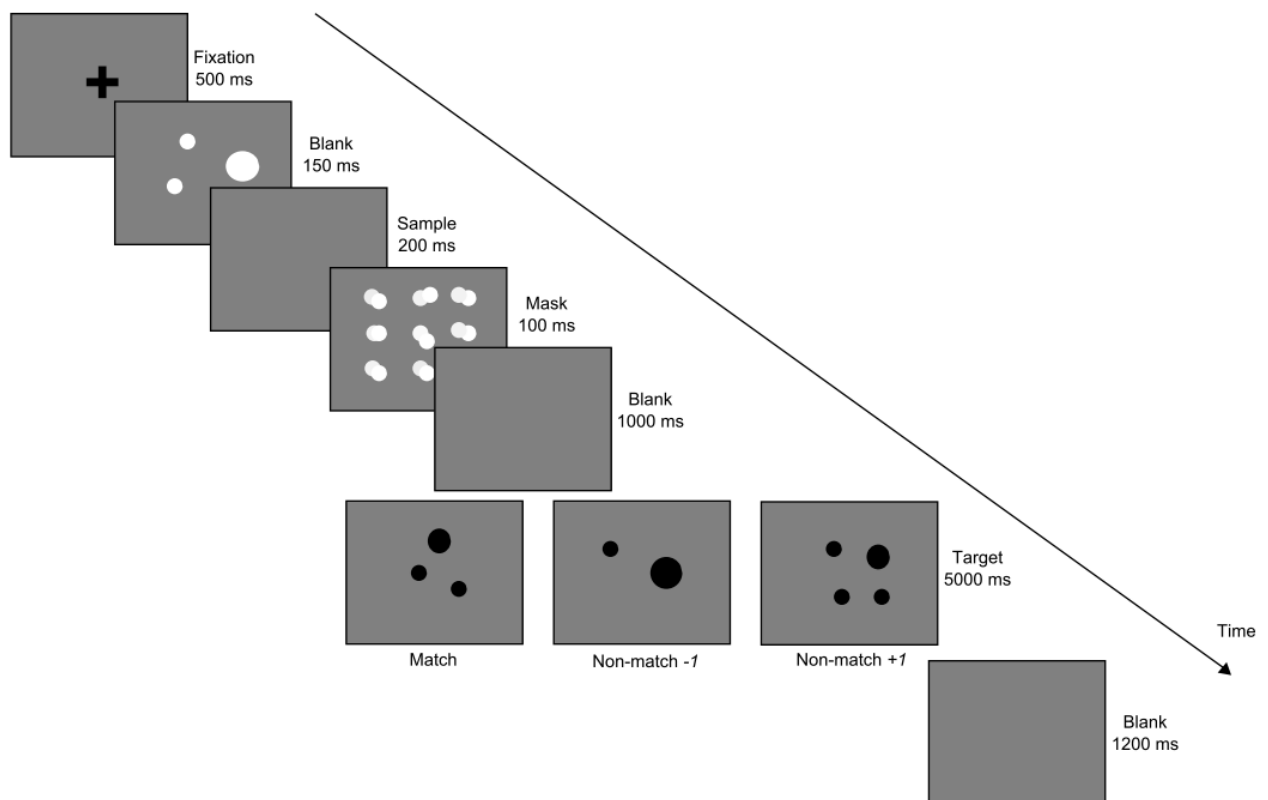


Figure 1. In the dots-to-dots match-to-sample, participants decided whether the numerosity in the target set was the same (match condition) or different (non-match condition) as compared to the sample set.

Each trial began with a fixation cross in the middle of the screen for 500 ms immediately followed by a blank screen for 150 ms. Thereafter, a sample set of dots was shown in the middle of the screen for 200 ms and immediately replaced by a mask for 100 ms. The size and the area of dots was randomly manipulated in order to prevent participants to base their

matching on physical size (e.g. area and perimeter) instead of extracting numerical information from the set. Then, after 1000 ms of black screen, a target set appeared and participants reported whether the target set had the same or a different numerosity with respect to the sample set by pressing the left or the right button of the keypad, respectively. The time allowed to provide a response was 5000 ms, otherwise the program skipped to the next trial and the response was categorized as missing. The target set had the same numerosity of the sample set (match condition) in half of the trials whereas in the other half the target numerosity was minus one or plus one dot with respect to the sample set (non-match condition). When the sample set numerosity was one dot or nine dots, the target in the non-match condition was two dots or eight dots, respectively. There were 12 trials for each numerosity from 1 to 9 in the sample set, thus resulting in a total of 108 trials.

Results Study 4.1

The missing responses and responses under 200 ms were removed from the accuracy and reaction times analysis. We analysed the mean percentages of correct responses in a 2 (Group [NVS-DD, TD]) x 8 (Numerosity [1, 2, 3, 4, 5, 6, 7, 8]) mixed ANOVA. The main effect of Numerosity, $F(4.48, 26) = 49.21, p < 0.001, \eta_p^2 = .654$, and the main effect of Group, $F(1, 26) = 9.4, p = 0.005, \eta_p^2 = .265$, were both statistically significant. The interaction Numerosity x Group approached significance ($F(5.67, 26) = 1.88, p = 0.093, \eta_p^2 = .067$). In the post-hoc comparisons, we used the Mann-Whitney U test with significance level set according to the Bonferroni formula. The NVS-DD group showed a lower performance as compared to TD group for numerosity three, $Z = 3.18, p = 0.004$, and the numerosity 5, $Z = 2.67, p = 0.032$. In our analysis, we used the number of elements in the sample set as factor, thus the comparisons entailed numerosities in the target set that could be either minus one or plus one element with respect to the sample set. For instance, 3 dots in the sample set were in turn compared against 2, 3, and 4 elements in the target set, whereas 5 dots in the sample set were compared against 4, 5, and 6 elements in the target set. This coding procedure prevent us to define those numerosity comparisons whose ratio is more difficult to discriminate for NVS-DD children as compared to TD children. To address this potential caveat in the interpretation of the results, we calculated the mean percentage of correct responses based on the ratio of the sets. As an example, the ratio 2vs3 included the trials in which the numerosity of sample set was 2 and the numerosity of target was 3 or vice versa, and the trials in which the numerosity of the

sample and the target set corresponded to 2. We then analysed the mean percentage of correct responses in the ratios 2vs3, 3vs4, 4vs5, and 5vs6 in a series of Mann-Whitney U tests with Bonferroni correction. The NVS-DD group had a lower performance as compared to TD group only for the ratio 3vs4, $Z = 2.92$, $p = 0.016$.

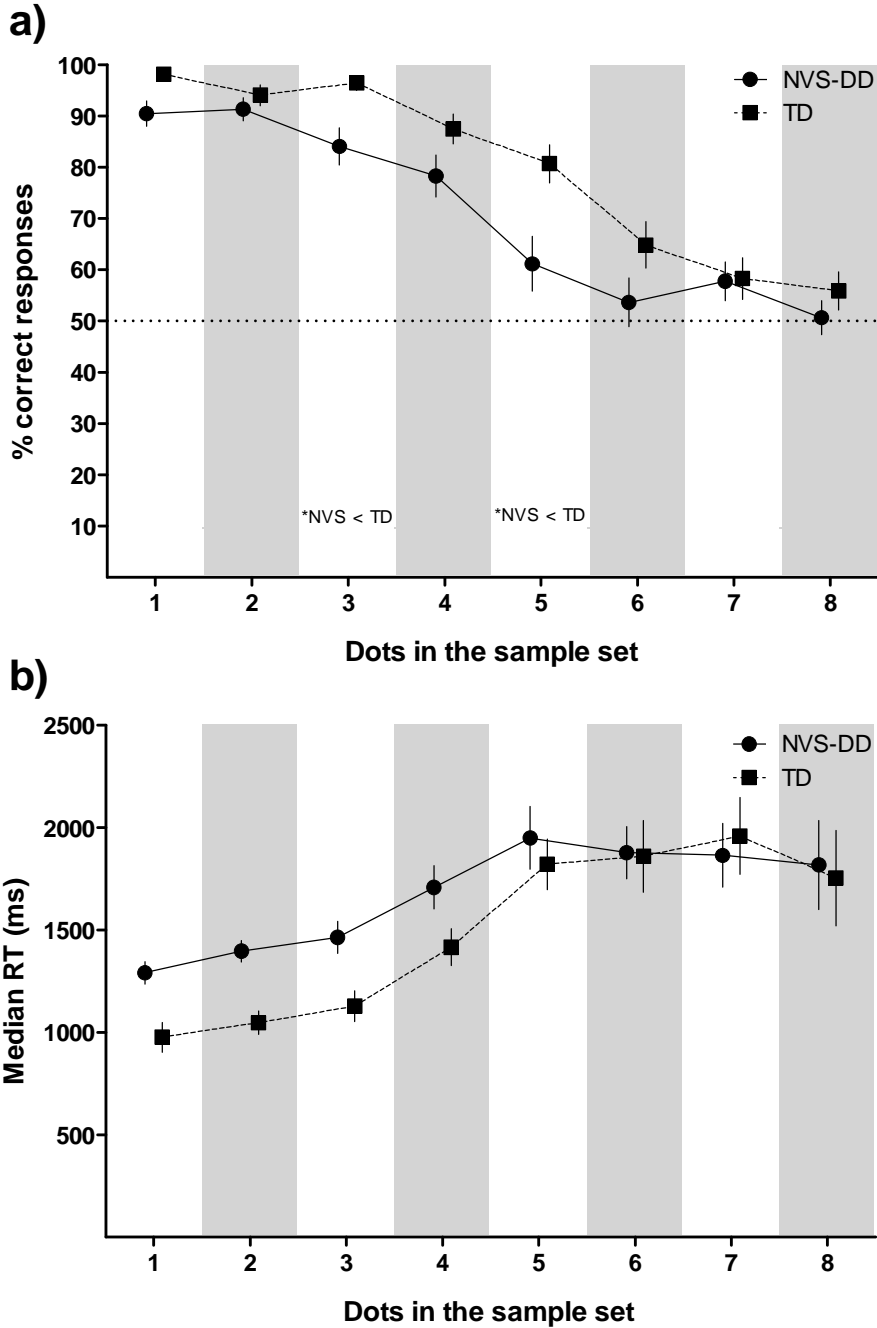


Figure 2. Mean percentage of correct responses (*panel a*) and median of reaction times (*panel b*) as a function of the number of dots in the sample set (error bars mean 95% CI, dashed line means the chance level; * $p < 0.05$).

We analysed the median reaction times in a 2 (Group [NVS-DD, TD]) x 8 (Numerosity [1, 2, 3, 4, 5, 6, 7, 8]) mixed ANOVA. The main effect of Numerosity was significant, $F(2.33, 26) = 19.19, p < 0.001, \eta_p^2 = .425$, suggesting a different speed depending on the number of dots to match. The main effect of the Group and the interaction Numerosity x Group failed to reach significance ($F(1, 26) = 1.79, p = 0.193, \eta_p^2 = .064$; $F(2.33, 26) = 1.31, p = 0.279, \eta_p^2 = .048$, respectively).

Discussion Study 4.1

The comparison of numerical quantities can rely onto two different mechanisms, the OTS and the ANS. In the match-to-sample task, when the sets are composed of few elements (less than 3-4), individuals create an memory object-file for each element in the first and the second set. The one-to-one correspondence between the elements store in memory allows participants to accurately determine whether the numerosities in the two sets match or mismatch. When in the sample there are more than the 3-4 elements, individuals can rely on the ANS which entails a less precise discrimination as a function of the ratio of the numerosities that are compared. Previous studies have identified in DD children a reduced subitizing limit, so their speed in enumerating small numerosities seems to be compromised (Moeller et al., 2009; Schleifer et al., 2009). Nevertheless, these studies adopted paradigms which entail also the access to the symbolic representation of numbers which has been claimed to be responsible for DD more than a deficit in numerosity processing per se (Rouselle & Noel, 2007). In the present study, we explicitly avoid any involvement of access to a symbolic representation of numerosity by asking participants to simply indicate whether the numerosities match or mismatch. Children with NVS-DD displayed a reduced OTS capacity with respect to the TD group as suggested by the decreased accuracy in the comparison of 3 vs. 4 elements. Also the accuracy in the ANS range seems to be compromised with a general lower accuracy for larger numerosity discrimination. The present results confirm and extend those of the previous studies. NVS-DD had a reduced OTS capacity (2-3 elements), which cannot be ascribed to a possible deficit in the connection between analogical and symbolic representations. We speculate that the reduced OTS capacity might play a detrimental role in counting procedure acquisition in preschool (Carey, 2001). The compromised learning to count could have cause a negative snowball effect on the subsequent acquisition of Arabic meaning, basic calculus and arithmetical facts learning. Moreover, the counting skills have been identified as one of

the most prominent predictor of math achievement at the end of the first year of primary school (Passolunghi, Vercelloni, & Schadee, 2007). Also the representation of larger numerical quantities appears to be weaker in children with NVS-DD as already demonstrated with DD children (Piazza et al., 2010). Nevertheless, our sample of children with DD in comorbidity with NVS prevents us from disentangling the contribution that each clinical condition may separately provide to the present results. Future studies may compare individuals with NVS, DD and NVS-DD in order to obtain a finer description of OTS and ANS functioning in each cognitive profile.

Method Study 4.2[‡]

Participants. Ten children with DD (2 boys; $M_{age-months} = 121$, $SD = 23$) were recruited from the Regional Center for Research in Learning Disabilities of Padova. They all received a formal diagnosis of DD by an expert clinician with a specific specialization in the diagnosis and treatment of learning disabilities. Children with DD obtained a global scores 2 SD below the mean in a standardized math test, had a normal IQ (above 85), had neither sensorial deficits nor comorbidity with Attention Deficit Hyperactivity Disorder. Four children of the DD sample also satisfied the criteria for the diagnosis of Dyslexia. The typically developing group was composed of ten children (2 boys; $M_{age-months} = 111$, $SD = 23$) recruited from the north Italian middle-socioeconomic schools. The prevalent teacher reported that TD children were not characterized by any specific difficulties or disabilities for math and reading as well as not considered as inattentive or hyperactive children.

Procedure. Children were met individually, in a quiet room, and completed the two computerized version of the number to position task (Siegler & Opfer, 2003): They were presented as games, no time limit was given and items or questions could be repeated if necessary but neither feedback nor hints were given to the child. Children were free to stop at any time.

Task. The Number-to-Position task (NP task) was a computer adaptation of Siegler and Opfer's (2003). An approximately 17 cm black line was presented in the centre of the screen with a mild yellow background. In the 0-100 interval, the left end was labelled by 0 and the right end was labelled by 100. Children were required to estimate the position on the line of ten numbers (2, 3, 4, 6, 18, 25, 42, 67, 71, 86; Siegler & Opfer, 2003) by clicking on the line

[‡] In collaboration with Berteletti I., Lucangeli D., & Zorzi M.

using the mouse. The movements of the cursor were constrained to the line to facilitate the answer and avoid the collection of unreliable responses. For each trial, the number to be positioned was presented in the upper left corner of the screen. In the interval 0-1000, the left end was labelled by 0 and the right end was labelled by 1000 and there were twenty-two numbers to be placed (2, 5, 18, 34, 56, 78, 100, 122, 147, 150, 163, 179, 246, 366, 486, 606, 722, 725, 738, 754, 818, 938; Siegler & Opfer, 2003). The order of the tasks was sequential: children first completed the 0-100 interval task and then the 0-1000 interval task. At the beginning of the experiment, children were asked to place the number 0, 100 and 50 in the interval 0-100 and 0, 1000 and 500 in the interval 0-1000. This procedure was implemented to ensure that children understood the task and also to make them practice with the mouse response. Moreover, when a response was provided a small red circle appeared in the clicking point in order to provide a visual feedback for the placement of the estimation. After, this practice phase, the other numbers were presented randomly.

Results Study 4.2

Analyses were conducted according to the method recommended by Siegler and colleagues (Siegler & Booth, 2004; Siegler & Opfer, 2003) and post-hoc comparisons were always corrected with the Bonferroni formula. Estimation accuracy was assessed using the Percentage of Absolute Error of estimation (PAE, corrected for statistical testing) for each participant for each condition. This was calculated as follows:

$$PAE = 2 \times \text{Arcsin} \left(\sqrt{\frac{|\text{Estimate} - \text{Target Number or Quantity}|}{100}} \right)$$

A mixed ANOVA was calculated with Group as between-subject factor (TD, DD) and Interval as within-subject factor (0-100, 0-1000). Mean PAEs in the 0-100 interval were 7% for TD children and 9% for children with DD. In the 0-1000, interval the mean PAEs were 12% for TD children and 26% for children with DD (see Figure 2). The main effect of Interval, $F(1, 18) = 55.12, p < .001, \eta_p^2 = .754$, and the main effect of the Group, $F(1, 18) = 10.8, p = .004, \eta_p^2 = .375$, were both significant. Because the interaction was also significant, $F(1, 18) = 8.95, p = .008, \eta_p^2 = .332$, we performed separate *t*-tests to compare groups' performance in each interval. In the interval 0-100, the DD group obtained a performance similar to controls, $t(18) = 2.2, p > 0.05$, whereas in the interval 0-1000 the difference was significant, $t(18) = 3.57, p < 0.05$. Children with DD showed less accuracy in placing number

on the larger interval as compared to matched TD controls. We also analysed the median of reaction times for the two groups in the two tasks separately. In the interval 0-100, children with DD showed a similar response time (Median = 4.5s, SD = 2) as compared to TD controls (Median = 5.6s, SD = 2.2), (Kolmogorov $Z = .894$, $p > 0.05$). Also in the interval 0-1000, there was no difference between the two groups (Kolmogorov $Z = .671$, $p > 0.05$): median of reaction times were 4.9s (SD = 2.2) for the control group and 4.8 (SD = 2.7) for the DD group.

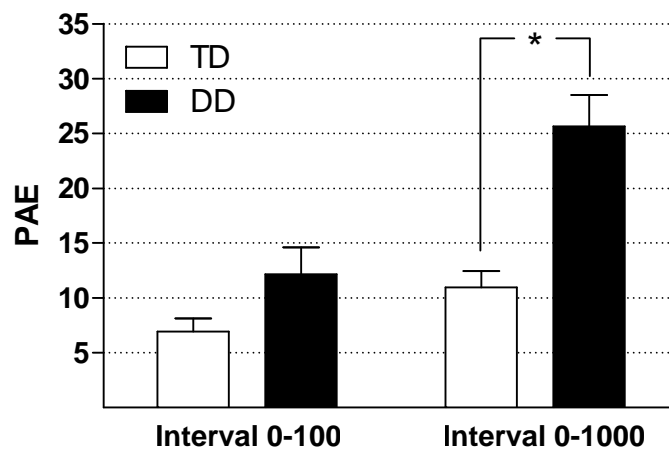


Figure 2. Percentage of absolute error in DD and TD children for the two number lines. Children with DD showed less accuracy in placing numbers in the 0-1000 interval as compared to the TD group.

Representation analysis. In order to understand the pattern of estimates, we fitted the linear and the logarithmic functions on group medians and subsequently individually for each child (Siegler & Opfer, 2003).

Group analysis. Group median estimates and best fits are reported in Figure 3. We tested the difference between linear and logarithmic models with paired-sample t -test on absolute distances between children's median estimate for each number and the predicted values according to the linear and the logarithmic model. If the t -test indicated a significant difference between the two distances, the best fitting model was attributed to the group. In the interval 0-100, the linear model had the highest R^2 and was significantly different from the logarithmic model for the TD group ($t(9) = 3.82$, $p = .004$, R^2 lin = 99%, $p < .001$ vs. R^2 log = 88%, $p < .001$) but not for the DD group ($t(9) < 1$, R^2 lin = 97%, $p < .001$ vs. R^2 log = 92%, $p < .001$). In the interval 0-1000, the linear model had the highest R^2 and was significantly

different from the logarithmic model for the TD group ($t(21) = 7.19, p < .001, R^2_{lin} = 97\%, p < .001$ vs. $R^2_{log} = 73\%, p < .001$) whereas for the DD group the logarithmic model had the highest R^2 and was significantly different from the linear model ($t(21) = 3.32, p < .003, R^2_{lin} = 66\%, p < .001$ vs. $R^2_{log} = 96\%, p < .001$).

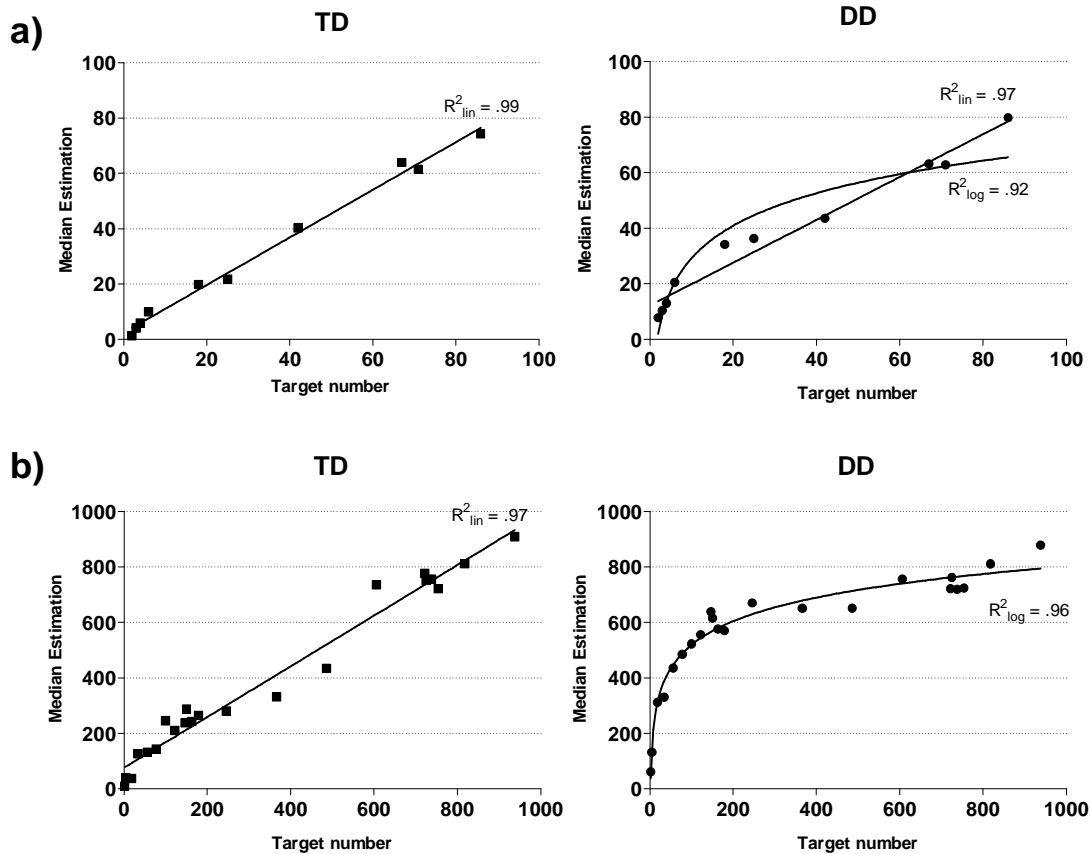


Figure 3. Children estimates and best fitting models divided for the DD and TD group in the 0-100 interval (*panel a*) and for the 0-1000 interval (*panel b*). The TD group obtained a linear representation in both intervals whereas the DD group showed an intermediate stage, between logarithmic and linear representation, in the 0-100 interval (*panel a*, right) and an evident logarithmic representation in the 0-1000 interval (*panel b*, right).

Individual analysis. We run linear and logarithmic regression analyses also on individual data. Paired t -test on residuals was computed for each child and accordingly they were classified as Linear, Intermediate, Logarithmic or No Representation. That is, if the difference between the two fits was significant and both models (or at least one) were significant, the highest R^2 determined what type of representation was displayed by the child. If the t -test on absolute residuals did not reach significance and the two models were both significant, the child was

considered to have an intermediate representation between logarithmic and linear. Indeed, when the data is almost, but not perfectly, linear, the logarithmic model also fits very well the data yielding a null difference in the *t*-test on residuals. Finally, whenever both models were not significant, the child was considered unable to perform the task properly and classified as not having a representation (Table 2).

Table 2.

Line interval	Type of representation			
	None	Logarithmic	Intermediate	Linear
<i>Interval 0-100</i>				
TD (<i>N</i> = 10)	0	0	4	6
DD (<i>N</i> = 10)	0	0	7	3
<i>Interval 0-1000</i>				
TD (<i>N</i> = 10)	0	0	3	7
DD (<i>N</i> = 10)	0	5	3	2

Note. Cell values represent number (per row) of children.

The individual analysis also confirmed the results already highlighted in the group analysis. In the interval 0-100, all children were in an intermediate or linear stage of mapping whereas no one were categorized as having a logarithmic representation. It is worth to notice that most of the TD children were clearly classified as linear and most of the DD were still in an intermediate stage. In the interval 0-1000, the individual analysis confirmed that half of the DD group still relied on a logarithmic representation when placing numbers onto the line. Finally, no child, in both intervals, was categorized as “no representation”, thus suggesting that all participants properly accomplished the task.

Discussion Study 4.2

Several studies have demonstrated that children shift from a logarithmic to linear mapping in the number to position task (Berteletti et al., 2010; Siegler & Opfer, 2003). The logarithmic representation is considered a direct evidence that children assigned more space to small numerosities than to larger numerosities, with a logarithmically compression that is a signature of the ANS (Berteletti et al., 2010; Dehaene, 1997; for different accounts, see Barth & Palladino, 2011; Moeller, Pixner, Kaufmann, & Nuerk, 2009). With education, children learn to linearly translate numbers into the correct spatial position onto the bounded line. Such

fine mapping correlates both with other numerical cognition tasks and also, and more importantly, with math achievement as measured by standardized tests (Booth & Siegler, 2006; Booth & Siegler, 2008). Moreover, children with math disability and DD seem to rely on an intuitive logarithmic representation instead of a formal linear representation (Geary et al., 2008; Landerl et al., 2009). In the present study, we tested the mapping between numbers and spatial position onto the line in a selected sample of 10 year-old children with formal diagnosis of DD as compared to a group of TD children matched for age. In line with previous studies, children with DD mainly relied on a less accurate logarithmic representation as compared to TD controls. These results are confirmed both at group and individual levels, indeed, at least half of the DD group children showed a logarithmic and less accurate representation. According to the ANS, children with DD represent small numbers as more spaced apart as compared to larger magnitudes which are logarithmic compressed. Therefore, the development of numerical representations seems to be delayed as compared to the TD children. Indeed, a logarithmic representation for the interval 0-1000 can be observed in second grade children (7-8 year-old) whereas the fourth-grade (9-10 year-old) children, as observed in our TD sample, display a linear and accurate mapping (Opfer & Siegler, 2007). Generally speaking, the present study highlights the specific deficit of in basic numerical processing in DD: indeed, Children with DD seem to display a delayed representation of numbers with respect to TD children.

Study 5

Subitizing, estimation and counting skills in Down syndrome: evidence from two delayed match-to-sample paradigms.*

Abstract

Individuals with Down syndrome (DS) exhibit various math difficulties which can be ascribed both to global intelligence level and/or to their atypical cognitive profile. In this light, it is worthwhile to understand whether math underachievement in DS can be attributed to deficits in basic enumeration and quantification processes. In the present study, individuals with DS and typically developing (TD) children matched for both mental and chronological age completed two delayed match-to-sample tasks in order to evaluate the functioning of subitizing, estimation and counting process. Kids with DS showed a specific deficit in subitizing as compared to both mental and chronological age matched TD kids. The estimation ability, instead, was similar to mental age matched controls but lower as compared to chronological age matched controls. The automaticity of counting routine appears to be weaker in kids with DS whereas the understanding of cardinality seems to be preserved in DS. The results provide new highlights regarding the source of difference in math achievement between DS and TD individuals.

* In collaboration with Lanfranchi S. & Zorzi M.

Introduction

Down syndrome (DS) is due to abnormalities on chromosome 21 and it is the most common cause of intellectual disability (Kittler et al., 2008). The cognitive profile of this syndrome is characterized by a relative weakness in verbal abilities, while visuospatial skills seem to be relatively preserved (Dykens, Hodapp, & Finucane, 2000).

It is well known that children and adults with Down syndrome (DS) exhibit several mathematical difficulties as compared to typically developing (TD) individuals (Brigstocke, Hulme & Nye, 2008). Children with DS obtain lower scores in a wide range of tests assessing basic math knowledge, arithmetic abilities and counting skills (Carr, 1988; Buckley & Sacks, 1987; Gelman, & Cohen, 1988; Porter, 1999). These mathematical deficits can be attributed to the general intelligence level or to the atypical cognitive profile of DS. In this light, it is worthwhile to understand whether math underachievement in DS can be ascribed to the low level of cognitive functioning or to specific deficits in basic enumeration and quantification processes. Such an investigation may be useful to provide new highlights regarding the source of difference in math achievement between DS and TD individuals.

There are basically three methods to determine the numerosity of a set: subitizing, estimation and serial counting. When small sets (less than 3-4 items) have to be enumerated, individuals can quickly and exactly perceive the number of elements with a minimum effort resulting in a behavioral effect called subitizing (from the Latin, *subitus* means immediate). Subitizing is possible by means of a general domain system that tracks objects in space and time, the Object Tracking System (OTS; Xu, Spelke & Goddard, 2005; Trick & Pylyshyn, 1994; Mandler & Shebo, 1982). Despite the fact that OTS is primarily a non-numerical mechanism, the individuation of distinct objects along with the one-to-one correspondence are considered essential in learning to count (Gallistel & Gelman, 1992; for this account, Carey, 2001). Moreover, the connection between OTS capacity and numerical abilities is also suggested by the fact that children with developmental dyscalculia have a less efficient subitizing and tend to adopt serial counting to determine the numerosity of small sets (Schleifer & Landerl, 2010; Moeller, Neuburger, Kaufmann, Landerl & Nuerk, 2009; Landerl, Bevan & Butterworth, 2004).

When the number of elements increases and serial counting is precluded, the numerosity of a set can be determined by means of the estimation process which relies on the Approximate Number System (ANS). Two alternative models account for the ANS: the

Logarithmic Model (Dehaene, 1997; Dehaene, Piazza, Pinel, & Cohen, 2003; Feigenson, Dehaene, & Spelke, 2004; Piazza, 2010) and the Linear Model (Meck & Church, 1983; Gallistel & Gelman, 1992). The former represents each numerosity as a distribution of activation on a logarithmically compressed number line whereas the Linear Model entails linearly spaced distributions of activation with scalar variability. Despite the theoretical differences, both models account for ratio-dependent effect which states that correct discrimination decreases when the ratio between numerosities approaches to one. This ability to notice the difference in numerosity between two sets, defined as number acuity, varies during development with a greater improvement during the first years of life and a slight decrease in elder hood (Halberda & Feigenson, 2008; Halberda, Ly, Wilmer, Naiman, & Germine, 2012). For instance, six months-old infants can notice the difference between 8 vs. 16 elements (1:2 ratio) but fail with the comparison 8 vs. 12 (ratio 2:3) (Xu & Spelke, 2000). Healthy adults reliably differentiate between sets with a 9:10 ratio despite a wide range of individual differences in the population (Halberda, Mazocco, & Feigenson, 2008; Halberda et al., 2012). The relevant point is that number acuity has been found to correlate with mathematical achievement and also to be weakened in children with developmental dyscalculia (Halberda, Mazocco, & Feigenson, 2008; Lourenco, Bonny, Fernandes, & Rao, 2012; Piazza et al., 2010).

When the number of elements in a set is larger than 3-4 elements and there is time at disposal, individuals can rely on accurate serial counting procedure that permits to exactly identify the number of elements in a potential infinite set by mapping the numerical magnitude into the Arabic system. There is still debate whether counting is an innate mechanism or a consequence of a repeated imitation of other's behavior (Gelman & Gallistel, 1978; Fuson, 1988; Briars & Siegler, 1984). Nevertheless, there is large agreement on the three basic principles of counting: the one-to-one correspondence principle claims that one and only one object must be associated with the corresponding word in the counting list; the stable-order principle states that the counting list must be recited in the correct and established order; the cardinality principles identified the last word in the counting list as the numerosity (cardinality) of the entire set (Gelman & Gallistel, 1978).

Enumeration skills have been relatively investigated in DS, even though these abilities might be the source of the differences between DS and typical development in math achievement. Paterson et al. (2006) investigated the OTS and the ANS in children with DS as

compared to children with Williams syndrome and typically developing individuals matched for mental age and chronological age. In a preferential looking paradigm (experiment 1), children with DS lacked a significant preferential looking for a novel card representing three elements as compared to the habituation card with two elements. The authors concluded that children with DS were not able to identify and to create an object file for each of the elements on the cards, thereby suggesting a deficit in OTS. In a dots comparison task (experiment 2), young adults with DS demonstrated fast responses in comparing sets of dots with large numerical distance with respect to sets with small distance, indexing a robust distance effect. Therefore, individuals with DS seem to correctly represent numerosities as a distribution of activation on the mental number line. In this regard, Camos (2009) recently found that six year-old children with DS had a performance comparable to typically developing pupils in a dots comparison task. Children were able to discriminate between 16 and 8 dots but failed to distinguish between 12 and 8, thus suggesting the ratio-dependent effect as a signature of a typical ANS. Nevertheless, the adopted ratios might be insufficient to highlight differences between typical and atypical development groups. Summarizing, the ANS appears to be preserved in individuals with DS whereas the OTS seems to be less efficient, at least in young children.

The main point regarding counting skills is whether individuals with DS have a superficial or a deep understanding of counting (for a review, Abdelahmeed, 2007). On one hand, some studies suggest that individuals with DS use counting as a mere routine lacking the understanding of cardinality principle. In fact, Gelman and Cohen (1988) maintained that children with DS learn to count by rote and lack the knowledge of the cardinality principle. Porter (1999) also reported that kids with DS can count by rote but are less efficient to detect counting errors performed by other individuals. On the other hand, other studies support the idea that individuals with DS properly understand the cardinality principle as well as the counting procedure. For example, Caycho, Gunn, and Siegel (1991) found a similar understanding of counting principles in children with DS and children matched for receptive vocabulary. Similarly, Bashash, Outhred and Bochner (2003) examined the performance of a sample of kids with DS ranging from 7 to 18 years-old: the entire sample was able to apply the three fundamental principles of counting in several counting tasks. Finally, Nye, Fluck and Buckley (2001) reported a pattern of results in which children with DS demonstrated a conceptual understanding of cardinality, although they made more errors in the counting

procedure. It clearly appears that the picture of the counting ability in the DS is still controversial and remains to be fully understood.

The aim of the present study was to explore enumeration abilities in kids with DS in comparison to typically developing groups matched for both mental and chronological age. We employed two delayed match-to-sample tasks in order to evaluate the functioning of subitizing, estimation and counting processes. In both tasks, children had to decide whether the numerosity presented in a sample set is equal or different from the numerosity displayed in a target set.

In the dots-to-dots match-to-sample task, we assessed the subitizing and estimation functioning by asking participants to match small numerosities, within the subitizing range, and large numerosities, beyond the subitizing range. Given that we precluded the serial counting, participants were forced to use the OTS and the ANS to estimate the number of elements in the sample set. Our aim was to verify whether DS kids have a deficit in OTS and also to highlight possible differences in ANS acuity by asking participants to compare sets whose numerosities entail several ratios. In the digit-to-dots match-to-sample, we ask whether kids with DS master the cardinality principle by asking to compare the sample numerosity, this time conveyed by an Arabic digit, to the numerosity of the target set of dots. In the target set, we expected participants to individuate small numerosities through subitizing, whereas enumeration of larger quantities could rely on serial counting.

Method

Participants. Sixty-three participants from the middle socioeconomic status from northern Italy took part to the study. There were 21 children with DS (9 males; $M_{age} = 14;2$, $SD = 4;0$), 21 typically developing children (9 males; $M_{age} = 5;6$, $SD = 0;7$) matched for mental-age (MA), and 21 typically developing kids (9 males; $M_{age} = 14;2$, $SD = 4;0$) matched for chronological age (CA). For the matching purpose a measure of receptive vocabulary, the Peabody Picture Vocabulary Scale (Dunn & Dunn 1997) was used. Moreover, in order to have also a measure of fluid intelligence the Raven's Colored Matrices (Raven, Raven, & Court, 1992) were administered to DS and MA groups. Participants' characteristics are presented in Table 1. In order to have a fine matching between groups (Bonato, Sella, Berteletti, & Umiltà, 2012), participants with DS and MA controls also completed the a standardized battery (BIN – Batteria Intelligenza Numerica; Molin, Poli, & Lucangeli, 2007) to assess their mathematical skills. The battery is composed of four subscales (i.e. lexical,

semantic, syntactic, counting) which assess different aspects of math performance in preschoolers. The two groups obtained a similar performance for the total score of the battery and for five out of six subscales. The only difference was the lexical subscale in which DS outperformed MA, $t(36) = 2.93$, $p = 0.006$. The lexical subscale measures the ability to correctly writing and naming Arabic digits, and the ability to individuate among a triplet of Arabic digits that one named by the clinician.

	Chronological age		Mental age - Peabody		Mental age - Raven	
	M	SD	M	SD	M	SD
DS ($n = 21$)	14;2	3;6	5;0	0;11	5;1	1;5
MA ($n = 21$)	5;6	0;10	5;2	0;10	5;6	1;2
CA ($n = 21$)	14;2	3;6	-	-	-	-

Table 1. Group characteristics.

Tasks. Participants completed two delayed match-to-sample tasks.

In the dots-to-dots match-to-sample task (Figure 1, *panel a*), each trial began with a fixation cross in the middle of the screen for 500 ms immediately followed by a blank screen for 150 ms. Thereafter, a sample set of dots was shown in the middle of the screen for 200 ms and immediately replaced by a mask for 100 ms. The size and the area of dots was randomly manipulated in order to prevent participants to base their matching on physical size (e.g. area and perimeter) instead of extracting numerical information from the set. Then, after 1000 ms of black screen, a target set appeared and participants reported whether the target set had the same or a different numerosity with respect to the sample set by pressing the left or the right button of the keypad, respectively. The time allowed to provide a response was 8000 ms, otherwise the program skipped to the next trial and the response was categorized as missing. The target set had the same numerosity of the sample set (match condition) in half of the trials whereas in the other half the target numerosity was minus one or plus one dot in respect of the sample set (non-match condition). When the sample set numerosity was one dot or nine dots, the target in the non-match condition was two dots or eight dots, respectively. There were 12 trials for each numerosity from 1 to 9 in the sample set, thus resulting in a total of 108 trials. The digit-to-dots match-to-sample task (Figure 1, *panel b*) had the same structure of the dots-to-dots match-to-sample task except for one feature: instead of a sample set, a digit ranging from 1 to 9 was shown in the middle of the screen for 200 ms and immediately replaced by a

mask for 100 ms. Participants reported whether the target set had the same or a different numerosity of the digit by pressing the left or the right button of the keypad, respectively.

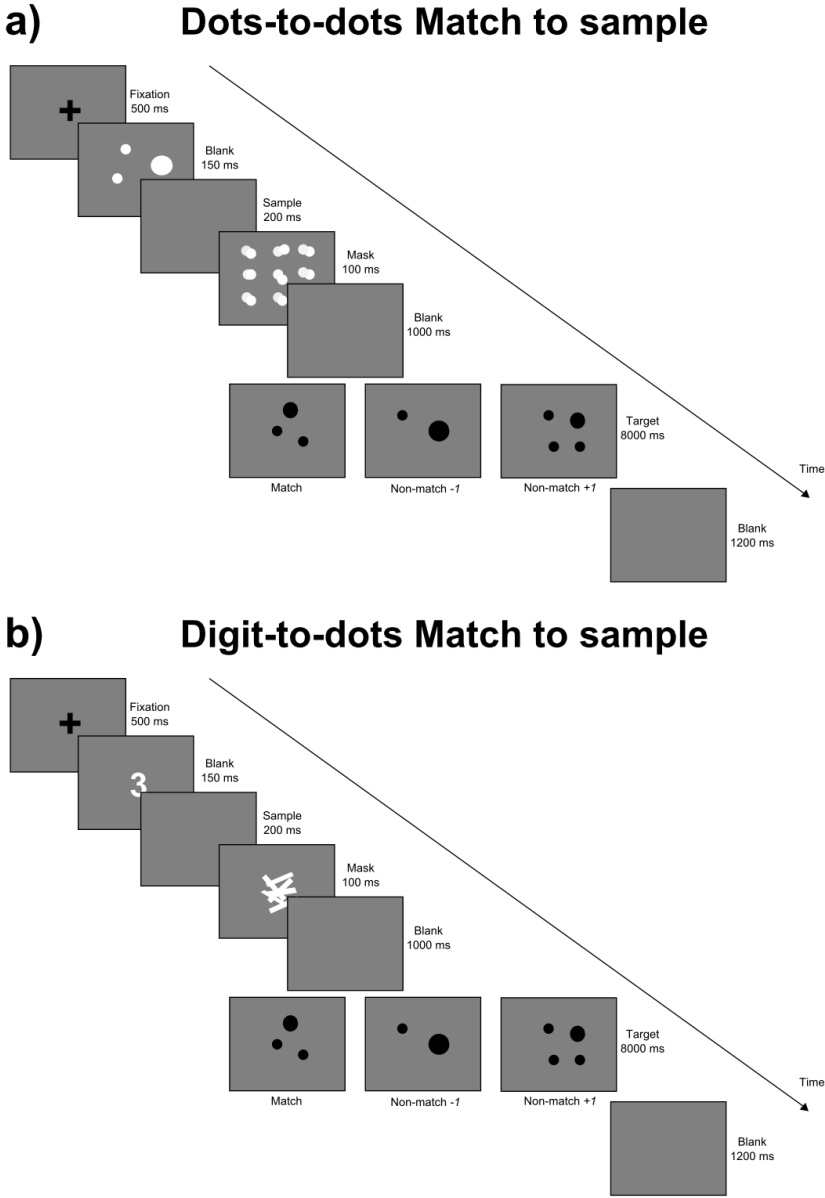


Figure 1. In both tasks, participants decided whether the numerosity in the sample set was the same (match condition) or different (non-match condition) as compared to the target set. The format of the numerosity in the sample varied across tasks, whereas the numerosity of the target set was constantly represented with dots. In the dots-to-dots match-to-sample task (*panel a*), the sample was composed of dots which remained on the screen for 200 milliseconds in order to prevent serial counting of the elements. In the digit-to-dots match-to-sample task (*panel b*), the numerosity of the sample was represented by an Arabic digit which appeared on the screen for 200 milliseconds.

Procedure. Participants sat in a quiet room approximately 60 centimeters from an 16-inch monitor. Children met one to one with the experimenter for three time of approximately 30 minute each. During the first section, they completed the tests for the assessment of mental age, Raven’s Colored Matrices and PPVT-R, during the second section they completed one of the two computerized tasks, and during the third section the other computerized task. The order of administration of the computerized tasks was counterbalanced across participants. A typically developing child was included in the mental age control group when his/her raw scores on the PPVT lay within 4 points (in either direction) of the score of the corresponding kid with DS. Similarly, a typically developing kids was included in the chronological age group when his/her chronological age lay within 4 months (in either direction) of the score of corresponding kid with DS.

Results

We categorized the trials into nine different conditions basing on the number of dots (or digit) in the sample set and in the target set, as reported in Table 2.

Table 2. Trials categorization in nine conditions.

Condition	Numerosity (or digit) of the Sample set	Numerosity of the Target set	Number of trials
1 vs. 2	1	1	6
	1	2	3
	2	1	6
2 vs. 3	2	2	6
	2	3	3
	3	2	3
3 vs. 4	3	3	6
	3	4	3
	4	3	3
4 vs. 5	4	4	6
	4	5	3
	5	4	3
5 vs. 6	5	5	6
	5	6	3
	6	5	3
6 vs. 7	6	6	6
	6	7	3
	7	6	3
7 vs. 8	7	7	6
	7	8	3
	8	7	3
8 vs. 9	8	8	6
	8	9	3
	9	8	6
9 vs. 9	9	9	6

We analysed the data in a series of mixed and one-way ANOVAs. The Greenhouse-Geisser correction was applied in case of missing sphericity in the data, we corrected the degrees of freedom with formula. In following analyses, departed from normality and variance inhomogeneity was corrected using non-parametric analysis. The planned contrasts were one-tailed and the p -values were corrected for multiple comparisons analysis using the Bonferroni formula. The results are presented divided for the (a) Dots-to-dots match-to-sample task and (b) Digit-to-dots match-to-sample task.

a) Dots-to-dots match-to-sample task. In the examination of data, we discarded from the analysis participants who produced more than 15% of missing responses. An excessive number of missing response may denote poor attention, which could undermine the validity and reliability of the administered task. Moreover, given the dichotomous modality of response, we expected that participants exceeded the chance level at least in the easiest condition, namely 1vs2 dots. Then, we removed those participants who yielded a mean percentage of correct responses below the binomial chance level (11 correct responses out of 15) in condition 1vs2. This procedure reduced our original samples to 14 DS participants (8 males; $M_{age} = 14;9$ years, $SD = 3;0$ years; $M_{verbalMA} = 5;2$, $SD = 0;11$ year; $M_{visuospatialMA} = 5;4$ years, $SD = 1;5$ months) whereas MA children remained the same. One CA kid did not completed the task thus the resulting sample was composed 20 individuals (8 males; $M_{age} = 5;4$ years, $SD = 7$ months). Nevertheless, DS and MA group were still matched for mental age, while DS and CA groups remained matched for chronological age ($p_s > 0.05$). We then calculated the mean percentage of correct responses for each condition removing the missing responses from the computation. Given the reduced number of trials, condition 9vs9 was excluded from the subsequent analysis. We analysed percentage of correct responses in a 8 [Condition: 1vs2, 2vs3, 3vs4, 4vs5, 5vs6, 6vs7, 7vs8, 8vs9] x 3 [Group: DS, MA, CA] mixed ANOVA with Group as between factor (Figure 2, *panel a*). Both the main effect of the Condition, $F(5.85, 304.09) = 119.76$, $p < 0.001$, $\eta^2_p = .697$, and the main effect of the Group, $F(2, 52) = 21.79$, $p < 0.001$, $\eta^2_p = .456$, were significant. The interaction Condition x Group also reached significance, $F(11.7, 304.09) = 4.69$, $p < 0.001$, $\eta^2_p = .153$. For each condition, we analysed the mean percentage of correct responses in a series of one-way Kruskal-Wallis ANOVAs with Group as factor. We found significant differences for conditions 2vs3, 3vs4, 4vs5, 5vs6, 6vs7, and 7vs8 (respectively, $\chi^2(2, N = 55) = 15.1$, $p = 0.001$; $\chi^2(2, N = 55) =$

19.34, $p < 0.001$; $\chi^2(2, N = 55) = 23.39, p < 0.001$; $\chi^2(2, N = 55) = 18.17, p < 0.001$; $\chi^2(2, N = 55) = 9.01, p = 0.011$; $\chi^2(2, N = 55) = 10.35, p = 0.006$.

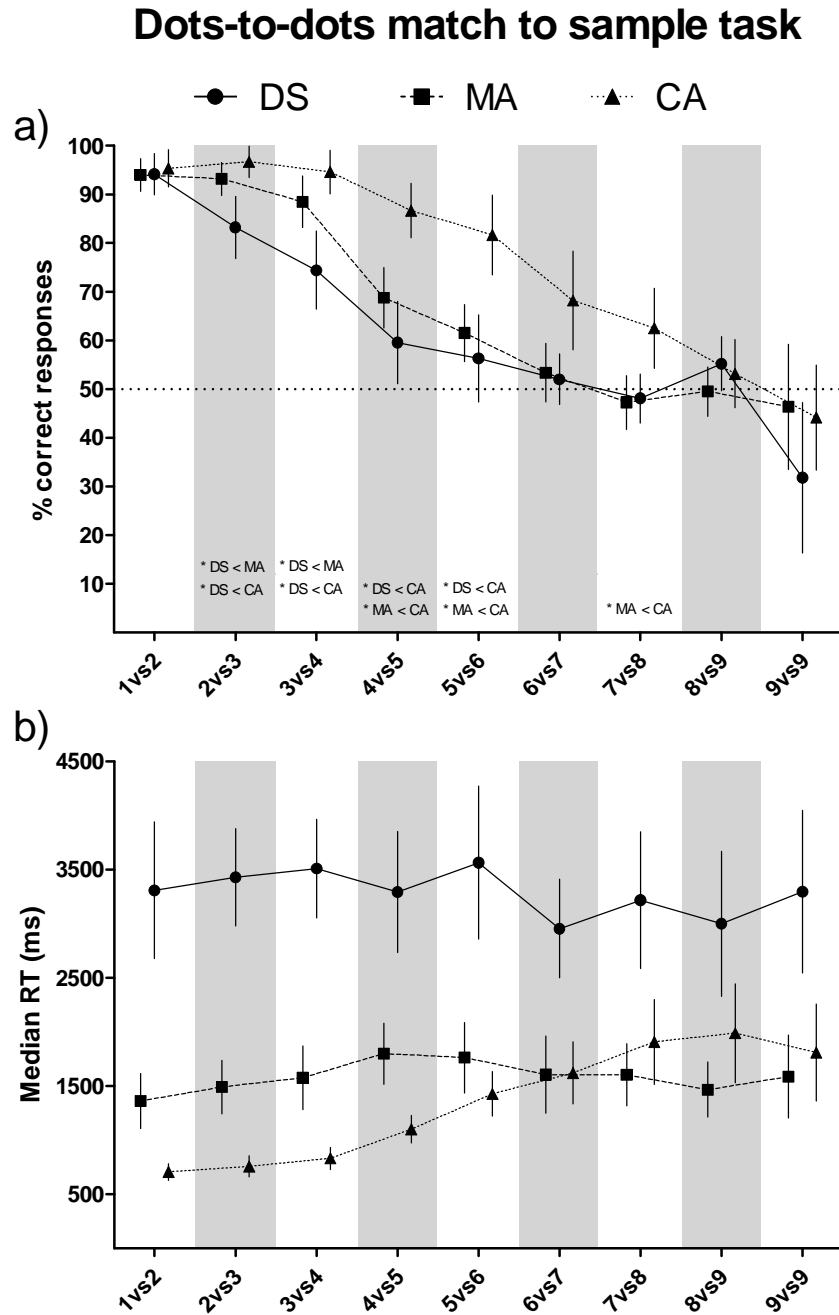


Figure 2. *Panel a)* Mean percentage of correct responses as a function of the conditions (error bars indicate 95% CI, dashed line indicates the chance level; contrasts DS vs. MA, DS vs. CA and MA vs. CA are reported at the bottom of the graph, * $p < 0.05$). *Panel b)* Median reaction times as a function of the conditions (error bars indicate 95% CI).

The post-hoc Mann-Whitney U comparisons revealed that participants with DS obtained a worse performance for condition 2vs3 and 3vs4 as compared to MA children (respectively, $Z = 2.84, p < 0.05$; $Z = 2.98, p < 0.05$). The group with DS exhibited a less accurate performance as compared to CA kids for condition 2vs3, 3vs4, 4vs5, 5vs6 (in order, $Z = 3.51, p < 0.05$; $Z = 3.98, p < 0.05$; $Z = 4.18, p < 0.05$; $Z = 3.59, p < 0.05$). Finally, the CA kids were more accurate in discrimination as compared to MA children for condition 4vs5, 5vs6 and 7vs8 (in order, $Z = 3.79, p < 0.05$; $Z = 3.53, p < 0.05$; $Z = 2.89, p < 0.05$).

b) Digit-dots match-to-sample task. As first step, we excluded from the analyses the three conditions with the largest numerosity (i.e. 7vs8, 8vs9, and 9vs9) because children with DS tried to count all the element in the target set but the time at their disposal was insufficient resulting, in an excessive number of missing responses. We then used the same procedure adopted in the dots-to-dots match-to-sample task. This procedure reduced our samples of DS kids to 14 participants (7 males; $M_{\text{age}} = 14;0$ years, $SD = 3;3$ years; $M_{\text{verbalMA}} = 5;2, SD = 6$ months; $M_{\text{visuospatialMA}} = 5;1$ years, $SD = 1;3$ year) and the MA children group to 19 participants (7 males; $M_{\text{age}} = 5;3$ years, $SD = 8$ months). Conversely, the CA children remained the same. Nevertheless, DS and MA group were still matched for mental age, while DS and CA group remained matched for chronological age. We then calculated the mean percentage of correct responses for each condition removing the missing responses from the computation. We analysed mean percentage of correct responses in a 6 [Condition: 1vs2, 2vs3, 3vs4, 4vs5, 5vs6, 6vs7] x 3 [Group: DS, MA, CA] mixed ANOVA with Group as between factor (Figure 4, *panel a*). Both the main effect of Condition, $F(3.74, 194.29) = 9.43, p < 0.001, \eta^2_p = .154$, and the main effect of Group, $F(2, 52) = 20.83, p < 0.001, \eta^2_p = .445$, were significant. The interaction Condition x Group also approached significance, $F(7.47, 194.29) = 2.02, p = 0.051, \eta^2_p = .072$. For each condition, we analysed the mean percentage of correct responses in a series of one-way ANOVAs with Group as factor. We found significant differences for all the conditions (in order, 1vs2, $\chi^2(2, N = 55) = 14.28, p = 0.001$; 2vs3, $\chi^2(2, N = 55) = 17.13, p < 0.001$; 3vs4, $\chi^2(2, N = 55) = 17.9, p < 0.001$; 4vs5, $\chi^2(2, N = 55) = 19.6, p < 0.001$; 5vs6, $\chi^2(2, N = 55) = 15.65, p < 0.001$; 6vs7, $\chi^2(2, N = 55) = 19.09, p < 0.001$). The post-hoc Mann-Whitney comparisons revealed that DS group achieved a performance similar to MA children with the only significant difference for the condition 4vs5, $Z = 2.8, p < 0.05$. Conversely, the DS group obtained a less accurate performance as compared to CA kids for all the conditions (in order, 1vs2, $Z = 3.61, p < 0.05$; 2vs3, $Z = 3.91, p < 0.05$; 3vs4, $Z = 4.23,$

$p < 0.05$; 4vs5, $Z = 4.62$, $p < 0.05$; 5vs6, $Z = 3.69$, $p < 0.05$; 6vs7, $Z = 4.12$, $p < 0.05$. Finally, the MA children had a less accurate performance as compared to CA kids for the condition 6vs7, $Z = 2.91$, $p < 0.05$).

Digit-to-dots match to sample task

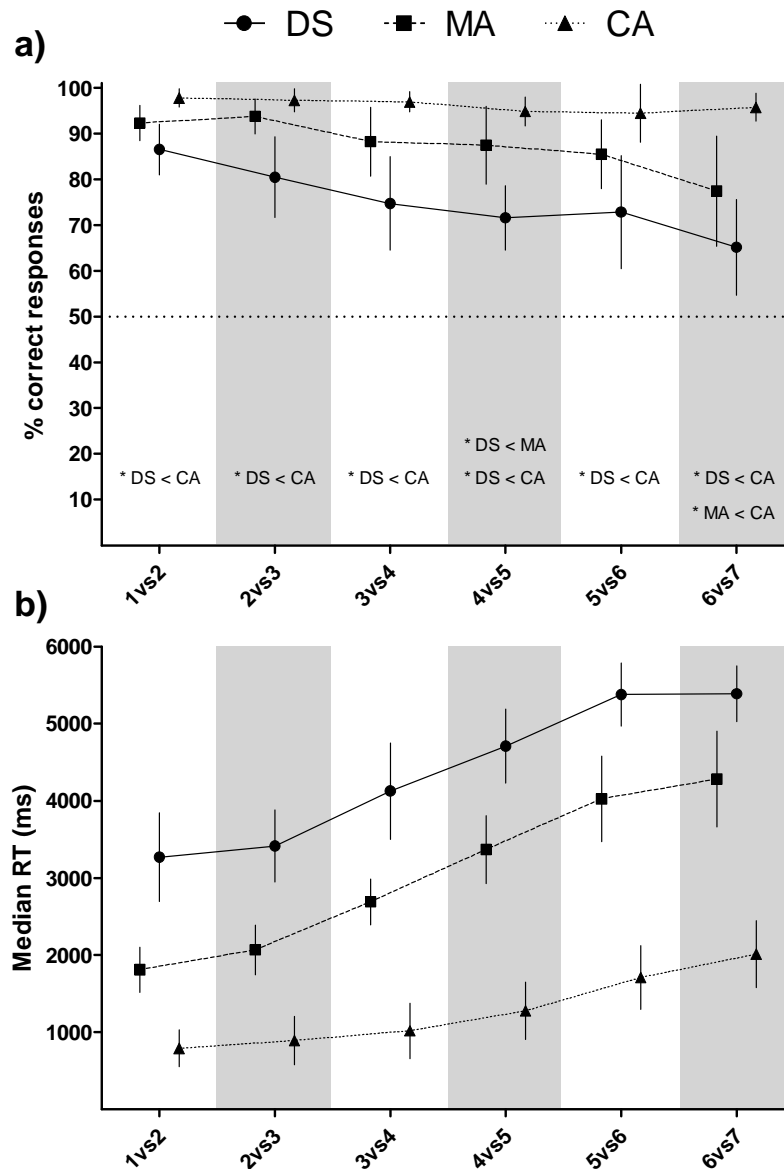


Figure 2. *Panel a)* Mean percentage of correct responses to the target set as a function of the conditions (error bars indicate 95% CI; dashed line indicates the chance level; contrasts DS vs. MA, DS vs. CA and MA vs. CA are reported at the bottom of the graph, * $p < 0.05$). *Panel b)* Median reaction times as a function of the conditions (error bars indicate 95% CI).

In order to investigate the speed of responses, we calculated the individual slope of the linear regression with median reaction times as dependent variable and the conditions as predictor. We restricted the analysis only to the subitizing range (i.e. conditions 1vs2, 2vs3, 3vs4). Then, we run one-way Kruskal-Wallis ANOVA with mean of the slopes as dependent variable and Group as between factor. The effect of the Group, $\chi^2(2, N = 55) = 21.6, p < 0.001$, suggests a different trend in median reaction times as function of the condition. The CA group had a smaller mean slope ($M = 114$ ms, $SD = 152$) as compared to MA group ($M = 441$ ms, $SD = 230$), $Z = 4.43, p < 0.05$, and DS group ($M = 428$ ms, $SD = 485$), $Z = 3.27, p < 0.05$. The DS group obtained a slope similar to MA group, $Z = 1.19, p > 0.05$.

Discussion

One previous study (Paterson et al., 2006) implemented a preferential looking paradigm to evaluate the ability of DS and typically developing infants to identify numerical differences between small sets. In contrast to typically developing infants, individuals with DS failed to detect changes in numerosity between sets with two and three elements. It was concluded that DS infants failed to create an object memory file for each element in the set thus suggesting a specific deficit in the OTS (Paterson, 2001; Paterson et al., 2006). For larger numerosities, both children with DS (age range: 6-11 year-old) and typically developing children correctly discriminated between numerical sets when their ratio was 1:2 (i.e. 8 vs. 16) but failed with 2:3 (i.e. 8 vs. 12) (Camos, 2003). Therefore, the ability of children with DS to discriminate between larger sets seems equivalent to typically developing children, signifying a unimpaired ANS. Nevertheless, it is arguable that a broad range of numerical ratios might highlight significant difference between DS and typical development children regarding the ANS acuity. The main concern regarding counting skills is whether kids with DS possess the conceptual knowledge of the cardinality principle (Caycho, Gunn, & Siegel, 1991; Bashash, Outhred, & Bochner, 2003) or they learn to count by rote without a deep understanding of counting (Gelman & Cohen, 1988; Porter, 1999).

The aim of the present study was to explore enumeration abilities in kids with DS comparison to typically developing individuals matched for both mental and chronological age. We employed two delayed match-to-sample tasks in order to evaluate subitizing, estimation and counting processes. In the dots-to-dots match-to-sample task, participants compared sets with small number of elements, within the subitizing range, and with large number of elements, beyond the subitizing range, in order to evaluate both the OTS and the

ANS. To our knowledge, this is the first study that assessed whether the OTS deficit persists in older kids with DS with respect to typically developing individuals. Moreover, we asked participants to compare sets whose ratios between numerosities covered a wide range in order to obtain an adequate evaluation of ANS acuity. In the subitizing range (i.e. 1vs2, 2vs3 and 3vs4 conditions), the performance of MA and CA group was almost at the ceiling level whereas DS kids' performance immediately decreased as the number of elements to be compared increased. MA and CA groups tracked the number of objects in the sample set and compared them to the target set, whereas DS children seems to adopt an estimation strategy also for fewer elements instead of performing an accurate individuation of the items. This result supports the hypothesis of an impaired OTS in kids with DS. A further support to the hypothesis of a OTS impairment in individuals with DS comes from studies on memory, and specifically from the finding of a specific deficit in visual short term memory in DS (Lanfranchi et al., 2009; Carretti & Lanfranchi 2010; Carretti, Lanfranchi & Mammarella, 2013). Recent studies have highlighted the relationship between subitizing and visual working memory, which is the expression of their reliance on the OTS (Cutini, Sella & Zorzi, Study 3 of the present thesis; Cutini & Bonato, 2012; Piazza, Fumarola, Chinello, & Melcher, 2011).

The DS and MA individuals yielded a similar performance in discriminating between numerical quantities with a ratio of 4vs5 onto. Despite the evidence that ANS acuity followed the characteristic ratio dependent effect in DS as well as in the typically developing groups (Camos, 2003; Paterson et al., 2006), the use of several ratios in the present study highlights that DS individuals' ANS acuity is less efficient as compared to CA kids but similar to MA. This result supports the idea of a typically developing but less efficient ANS in DS.

In the digit-to-dots match-to-sample task, the numerosity of the sample set was represented by an Arabic digit and participants could directly compare this value with the cardinality of the target set. MA and DS individuals used a serial counting procedure to identify the numerosity in the target set as suggested by the constant increase of reaction time also for small numerosities. Children with DS recognized the numerosity entailed by the digit and compared it with the cardinality of the target set in a fashion similar to the MA group. Therefore, our results support the idea of a slower automaticity but a preserved knowledge of cardinality in DS (Nye, Fluck, & Buckley, 2001; Bashash, Outhred, & Bochner, 2003; Caycho, Gunn, & Siegel, 1991).

Taken together our results suggest that children with DS have a specific deficit in subitizing as a signature of an impaired OTS (Paterson, 2001; Paterson et al., 2006) whereas their estimation ability, similar to MA (Camos, 2003) but lower as compared to CA controls, suggest an ANS acuity is determined by mental age. The automaticity of counting routine appeared to be weaker in kids with DS as suggested by a large amount of missing responses and slower reaction times. Nevertheless, the understanding of cardinality seems to be preserved in DS.

Conclusion

The scope of the thesis was to provide a better description of the developmental trends of numerical processes considering both the typical and atypical conditions. We designed and implemented a series of different studies, each with specific hypotheses, in order to respond to relevant experimental questions. We answered to these untried questions assuming different theoretical perspectives (e.g. developmental psychology, experimental, clinical developmental psychology) to yield a broader understanding of these issues.

We demonstrated that 2-3 year-old children are able to accomplish specific estimation and quantification tasks implementing basic pre-verbal mechanisms for numerical representation. It is worthwhile to notice that when children are explicitly requested to accomplish a numerical task they proficiently deploy both mechanisms, OTS and ANS, to represent numerical quantities. Conversely, when children spontaneously focus on numerosity they seem to mainly rely on the ANS to represent numerosity. Nevertheless, children may encode also other magnitudes (e.g., time and total size) which positively covaries with numerosity, thereby mimicking an analogue magnitude representation signature.

We highlighted how preschool and school children can translate symbolic and non-symbolic quantities onto a spatial position on a line. In particular, we found that discrete quantities are estimated adopting an intuitive logarithmic representation, a signature of ANS, despite the presence of other physical cues (i.e. total occupied area). The estimation of continuous quantities seems to follow a different developmental trajectory as compared to discrete and symbolic quantities. Continuous quantities appear to be accurately estimated and their pattern of estimation slightly changes from preschool to school children. Conversely, discrete and symbolic estimations seem more influenced by a specific numerical bias and undergo an evident improvement between preschool and school years.

We confirmed the intimate relation between visual short-term memory and OTS capacity in young adults. We implemented a dual-task paradigm showing that the subitizing range is strongly correlated to the number of elements reliably stored into short-term memory system. This result, in line with previous studies, opens the possibility to investigate VSTM and enumeration abilities in order to investigate their specific developmental trajectories from infants to young adults.

In the atypical development section of the thesis, we deployed different paradigms to investigate the numerical representation in children with developmental dyscalculia and Down syndrome. We replicated and extended data from the previous studies.

Children with developmental dyscalculia showed an intuitive logarithmic representation when translating the magnitude of numbers into a spatial position on the line. These results might suggest the adoption of the number-line task as a proficient diagnostic tool. We also investigated the OTS and ANS system functioning in children with diagnosis of developmental dyscalculia in comorbidity with a profile of non-verbal syndrome. We found a specific deficit for the processing of numerosity inside and outside the OTS capacity. Given that the presented stimuli were all analogical (i.e., set of dots), we avoided the possibility that the observed deficit may be related to an impaired access to magnitude from symbolic representation.

Children with Down syndrome displayed a specific deficit in the OTS capacity whereas the ANS acuity appear to be only delayed as compared to mental age match typical developing children.

In summary, the thesis suggests that in the early stage of development, at 2-3 years of age, children are able to proficiently exploit pre-verbal mechanisms for estimating quantity. At age 5-6 we demonstrated accurate estimation of non-numerical continuous quantities, but this was not true for numerical quantities, both discrete and symbolic. The mechanism related to the visual continuous estimation seems to mature earlier and to be less influenced by development and education. Conversely, discrete and symbolic estimation slowly improve between age 5-6 and age 8-9 under the influence of time and formal education. It appears that discrete and symbolic estimations are coupled, showing the signature of common underlying mechanism that in dyscalculia is less precise. Indeed, 10 year-old children with developmental dyscalculia failed to reach a precise linear estimation of symbolic magnitude in the interval 0-100 and 0-1000 thus suggesting a specific deficit in translating the representation of symbolic quantities onto a spatial position. At the age of 5-6, children also showed a finely developed OTS mechanism because they are able to accurately discriminate between small quantities (3 vs. 4). This ability remains stable during development: in our studies, typically developing participants of approximately 5, 8, and 14 years of age showed a ceiling effect in discriminating between analogical small quantities. Conversely, the OTS capacity seems to be compromised in children with Down syndrome and developmental

dyscalculia, thereby suggesting that an impaired individuation of small quantities might be detrimental in the math achievement in these individuals. Additionally, we provided supplementary data which confirms the intimately relation between visual-short term memory capacity and the enumeration of small quantities. This finding might open a door to the developmental study which will consider both the trajectories of visual-short term memory and enumeration ability in order to better understand the interaction of the two systems.

Point by point, on the basis of the reported studies we offered evidence of: a) developmental trend of the investigated numerical systems; b) different developmental trajectories between numerical and non-numerical quantities estimations; c) impaired or delayed cognitive profiles in the atypical development of numerical representation.

References

- Abdelahmeed, H. (2007). Do children with Down syndrome have difficulty in counting and why. *International Journal of special education*, 22, 1-11.
- Agrillo, C., Piffer, L., & Bisazza, A. (2010). Large Number Discrimination by Mosquitofish. *PLoS ONE*, 5(12), e15232. doi:10.1371/journal.pone.0015232
- Antell, S. E., & Keating, D. P. (1983). Perception of numerical invariance in neonates. *Child development*, 54(3), 695–701.
- Barth, H. C., & Paladino, A. M. (2011). The development of numerical estimation: evidence against a representational shift. *Developmental science*, 14(1), 125–35. doi:10.1111/j.1467-7687.2010.00962.x
- Bashash, L., Outhred, L. & Bochner, S. (2003). Counting skills and number concepts of students with moderate intellectual disabilities. *International Journal of Disability, Development and Education*, 50, 3, 325-345.
- Beran, M. J. (2007). Rhesus monkeys (*Macaca mulatta*) enumerate large and small sequentially presented sets of items using analog numerical representations. *Journal of experimental psychology. Animal behavior processes*, 33, 42–54.
- Berteletti, I., Lucangeli, D., & Zorzi, M. (2012). Representation of numerical and non-numerical order in children. *Cognition*, 124(3), 304–313. doi:10.1016/j.cognition.2012.05.015
- Berteletti, I., Lucangeli, D., Piazza, M., Dehaene, S., & Zorzi, M. (2010). Numerical estimation in preschoolers. *Developmental psychology*, 46(2), 545-51. doi:10.1037/a0017887
- *Big Fish* (2003), directed by T. Burton, Columbia Pictures, DVD.
- Bonato, M., Sella, F., Berteletti, I., & Umiltà, C. (2012). Neuropsychology is nothing without control : A potential fallacy hidden in clinical studies. *Cortex*, 12–14. doi:10.1016/j.cortex.2011.06.017
- Booth, J. L., & Siegler, R. S. (2006). Developmental and individual differences in pure numerical estimation. *Developmental psychology*, 42(1), 189-201. doi:10.1037/0012-1649.41.6.189

- Booth, J. L., & Siegler, R. S. (2008). Numerical magnitude representations influence arithmetic learning. *Child development*, 79(4), 1016–31. doi:10.1111/j.1467-8624.2008.01173.x
- Brannon, E. M. (2002). The development of ordinal numerical knowledge in infancy. *Cognition*, 83(3), 223–240.
- Brannon, E. M., & Terrace, H. S. (1998). Ordering of the Numerosities 1 to 9 by Monkeys. *Science*, 282, 746–749.
- Briars, D. J., & Siegler, R. S. (1984). A featural analysis of preschooler's counting knowledge. *Developmental Psychology*, 20, 607-618.
- Brigstocke, S., Hulme, C., & Nye, J. (2008). Number and arithmetic skills in children with Down Syndrome. *Down Syndrome Research and Practice*. doi10.3104/reviews2070
- Bryan, T. (1974). Peer popularity of learning disabled children. *Journal of Learning Disabilities*, 7, 261–268.
- Buckley, S. & Sacks, B. (1987). *The Adolescent with Down syndrome: Life for the teenager and for the family*. Portsmouth UK: Portsmouth Polytechnic.
- Bull, R., & Scerif, G. (2001). Executive functioning as a predictor of children's mathematics ability: inhibition, switching, and working memory. *Developmental Neuropsychology*, 19, 273-293.
- Bull, R., Johnston, R.S., & Roy, J.A. (1999). Exploring the roles of the visual-spatial sketch pad and central executive in children's arithmetical skills: Views from cognition and developmental neuropsychology. *Developmental Neuropsychology*, 15, 421-442.
- Burr, D. C., Turi, M., & Anobile, G. (2010). Subitizing but not estimation of numerosity requires attentional resources. *Journal of Vision*, 10, 1–10. doi:10.1167/10.6.20.
- Burr, D., & Ross, J. (2008). A visual sense of number. *Current biology : CB*, 18(6), 425–8. doi:10.1016/j.cub.2008.02.052.
- Camos, V. (2009). Numerosity discrimination in children with Down syndrome. *Developmental Neuropsychology*, 34, 435–447. doi:10.1080/87565640902964557.

- Campbell, J.I.D. (2005). *Handbook of mathematical cognition*. New York: Psychology Press.
- Cantlon J.F., Safford, K.E., Brannon, E.M. (2010). Spontaneous analog number representations in 3-year-old children. *Developmental Science*, 13, 289–297.
- Cantlon, J. F., & Brannon, E. M. (2006). Shared system for ordering small and large numbers in monkeys and humans. *Psychological science*, 17(5), 401-6.
doi:10.1111/j.1467-9280.2006.01719.x
- Cantlon, J. F., & Brannon, E. M. (2007). Basic math in monkeys and college students. *PLoS biology*, 5(12), e328. doi:10.1371/journal.pbio.0050328
- Carey, S. (2001). Cognitive foundations of arithmetic: Evolution and ontogenesis. *Mind & Language*, 16, 37–55.
- Carey, S. (2004). Bootstrapping and the origin of concepts. *Daedalus*, 59–68.
- Carr, J. (1988). Six weeks to twenty-one years old: a longitudinal study of children with Down’s syndrome and their families. *Journal of Child Psychology and Psychiatry*, 29, 407– 431.
- Carretti, B., & Lanfranchi, S. (2010). The effect of configuration on VSWM performance of Down syndrome individuals. *Journal of Intellectual Disability Research*, 54, 1058-1056.
- Carretti, B., Lanfranchi, S., & Mammarella, I. (2013). Spatial-simultaneous and spatial-sequential working memory in individuals with Down syndrome: the effect of configuration. *Research in Developmental Disability*, 34, 669-675.
- Carroll, L. (1865). *Alice’s Adventures in Wonderland*.
- Caycho, L., Gunn, P., & Siegal, M. (1991). Counting by children with Down syndrome. *American Journal on mental retardation*, 95(5), 575-583.
- Clearfield, M. W., & Mix, K. S. (2001). Infant use continuous quantity – not number – to discriminate small visual sets. *Journal of Cognition and Development*, 2(3), 243–260.
- Clearfield, M.W., & Mix, K.S. (1999). Number versus contour length in infants’ discrimination of small visual sets. *Psychological Science*, 10, 408–411.
- Cordes, S., & Brannon, E. M. (2008). The difficulties of representing continuous extent in infancy: Using number is just easier. *Child Development*, 79(2), 476–489.

- Cordes, S., & Brannon, E. M. (2009). The relative salience of discrete and continuous quantity in young infants. *Developmental science*, 12(3), 453–63. doi:10.1111/j.1467-7687.2008.00781.x
- Cordes, S., Gelman, R., Gallistel, C. R., & Whalen, J. (2001). Variability signatures distinguish verbal from nonverbal counting for both large and small numbers. *Psychonomic bulletin & review*, 8, 698–707.
- Cornoldi, C., Lucangeli, D., & Bellina, M. (2002). AC-MT: *test di valutazione delle abilità di calcolo* [AC-MT : assessment for calculation abilities]. Trento: Erickson
- counts? Influence of perceptual variables. *Journal of Experimental Child Psychology*, 87(1), 57-84.
- Cowan, N. (2001). The magical number 4 in short-term memory: a reconsideration of mental storage capacity. *Behavioral and brain sciences*, 24, 114–85.
- Cutini, S., & Bonato, M. (2012) Subitizing and visual short-term memory in human and non-human species: a common shared system? *Frontiers in Comparative Psychology*, 3:469. doi: 10.3389/fpsyg.2012.00469.
- Dehaene, S. (1997). *The number sense: How the mind creates mathematics*. New York, NY: Oxford University Press.
- Dehaene, S., Izard, V., Spelke, E., & Pica, P. (2008). Log or linear? Distinct intuitions of the number scale in Western and Amazonian indigene cultures. *Science*, 320(5880), 1217-20. doi:10.1126/science.1156540
- Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology*, 20, 487-506. doi:10.1162/jocn.1993.5.4.39010.1080/02643290244000239
- development of cardinality. In M. Panhuizen-Van Heuvel (Ed.), *Proceedings of 25th conference of the international group for the psychology of mathematics education*, Vol. 3 (pp. 113–120). Amersfoort, TheNetherlands: Drukkerij Wilco.
- Dunn, L. M., & Dunn, L. M. (1997). *Peabody Picture Vocabulary Test* (3rd ed.). Circle Pines, MN: American Guidance Service.
- Dykens, E. M., Hodapp, R. M., & Finucane, B. M. (2000). *Genetics and mental retardation syndromes*. New York: Paulh Brookes.
- Feigenson, L., & Carey, S. (2003). Tracking individuals via object-files : evidence from infants ' manual search. *Developmental Science*, 5, 568 – 584.

- Feigenson, L., Carey, S., & Hauser, M. (2002). The representations underlying infants' choice of more: object files versus analog magnitudes. *Psychological Science*, 13(2), 150–156.
- Feigenson, L., Carey, S., & Spelke, E. (2002). Infants' discrimination of number vs. continuous extent. *Cognitive Psychology*, 44(1), 33–66.
- Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences*, 8(7), 307-314. doi:10.1016/j.tics.2004.05.002
for cognitive science. Course notes available at <http://users.fmg.uva.nl/ewagenmakers/BayesCourse/BayesBook.pdf>.
- Fuson, K. C. (1988). *Children's counting and concepts of number*. London: Springer-Verlag.
- Gallistel, C.R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition*, 44, 43–74.
- Geary, D.C. (1994). *Children's mathematical development*. Washington: American Psychological Association.
- Geary, D.C., Brown, S.C., & Samaranayake, V.A. (1991). Cognitive addition: A short longitudinal study of strategy choice and speed-of-processing differences in normal and mathematically disabled children. *Developmental Psychology*, 27, 787-797.
- Geary, D.C., Hoard, M.K., Byrd-Craven, J., Nugent, L., & Numtee, C. (2007). Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. *Child Development*, 78, 1343–1359.
- Gebuis, T., & Reynvoet, B. (2012a). The interplay between nonsymbolic number and its continuous visual properties. *Journal of Experimental Psychology: General*, 141, 642–648.
- Gebuis, T., & Reynvoet, B. (2012b). The role of visual information in numerosity estimation. *PLoS ONE*, 7(5), doi:10.1371/journal.pone.0037426.
- Gelman, R., & Cohen, M. (1988). Qualitative differences in the way Down syndrome and normal children solve a novel counting problem. In L. Nadel (ed.), *The Psychobiology of Down's syndrome*. Cambridge, MA: MIT Press.
- Gelman, R., & Gallistel, C. R. (1978). *The child's understanding of number*. Cambridge, MA: Harvard University Press.

- Halberda, J., & Feigenson, L. (2008). Developmental change in the acuity of the “Number Sense”: The Approximate Number System in 3-, 4-, 5-, and 6-year-olds and adults. *Developmental psychology*, *44*(5), 1457–65. doi:10.1037/a0012682
- Halberda, J., Ly, R., Wilmer, J. B., Naiman, D. Q., & Germine, L. (2012). Number sense across the lifespan as revealed by a massive Internet-based sample. *Proceedings of the National Academy of Sciences of the United States of America*, *109*(28), 11116–20. doi:10.1073/pnas.1200196109
- Halberda, J., Mazocco, M. M. M., & Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievement. *Nature*, *455*(7213), 665–8. doi:10.1038/nature07246.
- Hannula, M. M., & Lehtinen, E. (2001). Spontaneous tendency to focus on numerosities in the
- Hannula, M. M., & Lehtinen, E. (2003). Spontaneous focusing on numerosity in the development of early mathematical skills. Paper Presented in EARLI 2003, Padova, Italy.
- Hannula, M. M., & Lehtinen, E. (2005). Spontaneous focusing on numerosity and mathematical skills of young children. *Learning and Instruction*, *15*(3), 237–256. doi:10.1016/j.learninstruc.2005.04.005
- Hannula, M. M., Lepola, J., & Lehtinen, E. (2010). Spontaneous focusing on numerosity as a domain-specific predictor of arithmetical skills. *Journal of experimental child psychology*, *107*(4), 394–406. doi:10.1016/j.jecp.2010.06.004
- Hannula, M. M., Rasanen, P., & Lehtinen, E. (2007). Development of Counting Skills: Role of Spontaneous Focusing on Numerosity and Subitizing-Based Enumeration. *Mathematical Thinking and Learning*, *9*(1), 51–57. doi:10.1207/s15327833mtl0901_4
- Hitch, G. J. (1978). The role of short-term working memory in mental arithmetic. *Cognitive Psychology*, *10*, 302–323.
- Hyde, D. C. (2011). Two systems of non-symbolic numerical cognition. *Frontiers in human neuroscience*, *5*, 150.
- Kahneman, D., Treisman, A., & Gibbs, B. (1992). The reviewing of object-files: object specific integration of information. *Cognitive Psychology*, *24*, 175–219.
- Kail, R.V., & Hall, L.K. (1999). Sources of developmental change in children’s word-problem performance. *Journal of Educational Psychology*, *91*, 660-668.

- Kaufman, E. L., Lord, M. W., & Volkman, J. (1949). The discrimination of visual number. *American Journal of Psychology*, 62, 498–525.
- Landerl, K., Bevan, A., & Butterworth, B. (2004). Developmental dyscalculia and basic numerical capacities: A study of 8–9-year-old students. *Cognition*, 93, 99–125.
- Landerl, K., Fussenegger, B., Moll, K., & Willburger, E. (2009). Dyslexia and dyscalculia: two learning disorders with different cognitive profiles. *Journal of experimental child psychology*, 103(3), 309–24. doi:10.1016/j.jecp.2009.03.006
- Lanfranchi, S., Carretti, B., Spanò, G., & Cornoldi, C. (2009). A specific deficit in visuospatial simultaneous working memory in Down syndrome. *Journal of Intellectual Disability Research*, 53, 474–483.
- Le Corre, M., & Carey, S. (2007). One, two, three, four, nothing more: an investigation of the conceptual sources of the verbal counting principles. *Cognition*, 105(2), 395–438. doi:10.1016/j.cognition.2006.10.005.
- Lee, M. D., & Wagenmakers, E.-J. (2009). A course in Bayesian graphical modeling
- LeFevre, J.A., DeStefano, D., Coleman, B., & Shanahan, T. (2005). Mathematical cognition and working memory. In *Handbook of mathematical cognition*, Campbell, J.I.D. (Ed.), 361–378. New York: Psychology Press.
- Lemaire, P., Abdi, H., & Fayol, M. (1996). The role of working memory resources in simple cognitive arithmetic. *European Journal of Cognitive Psychology*, 8(1), 73–103.
- Lepola, J., Niemi, P., Kuikka, M., & Hannula, M. M. (2005). Cognitive–linguistic skills and motivation as longitudinal predictors of reading and arithmetical achievement: A follow-up study from kindergarten to Grade 2. *International Journal of Educational Research*, 43, 250–271.
- Lipton, J. S., & Spelke, E. S. (2003). Origins of number sense. Large-number discrimination in human infants. *Psychological science : a journal of the American Psychological Society / APS*, 14(5), 396–401. Retrieved from <http://www.ncbi.nlm.nih.gov/pubmed/12930467>
- Lourenco, S. F., Bonny, J. W., Fernandez, E. P., & Rao, S. (2012). Nonsymbolic number and cumulative area representations contribute shared and unique variance to symbolic math competence. *Proceedings of the National Academy of Sciences of the United States of America*, 109(46), 18737–18742. doi:10.1073/pnas.1207212109

- Lucangeli, D., & Cabrele, S. (2006). Mathematical Difficulties and ADHD. *Exceptionality*, 14(1), 53-6.
- Luck, S. J., & Vogel, E. K. (1997). The capacity of visual working memory for features and conjunctions. *Nature*, 390, 279–81.
- Mandler, G., & Shebo, B. J. (1982). Subitizing: An analysis of its component processes. *Journal of Experimental Psychology: General*, 111, 1–22.
- Mazza, V., & Caramazza, A. (2011). Temporal brain dynamics of multiple object processing: the flexibility of individuation. *PloS one*, 6, e17453.
- Meck, W. H., & Church, R. M. (1983). A mode control model of counting and timing processes. *Journal of Experimental Psychology: Animal Behavior Processes*, 9, 320-334.
- Melcher, D., & Piazza, M. (2011). The role of attentional priority and saliency in determining capacity limits in enumeration and visual working memory. *PloS one*, 6, e29296.
- Mix, K. S., Huttenlocher, J., & Levine, S. C. (2002). Multiple Cues for Quantification in Infancy : Is Number One of Them ? *Psychological Bulletin*, 128(2), 278 –294. doi:10.1037//0033-2909.128.2.278
- Moeller, K., Neuburger, S., Kaufmann, L., Landerl, K., & Nuerk, H.-C. (2009). Basic number processing deficits in developmental dyscalculia. Evidence from eye-tracking. *Cognitive Development*, 24, 371-386.
- Moeller, K., Pixner, S., Kaufmann, L., & Nuerk, H.-C. (2009). Children’s early mental number line: logarithmic or decomposed linear? *Journal of Experimental Child Psychology*, 103, 503–515.
- Molin, A., Poli, S., Lucangeli, D. (2007). *Batteria Intelligenza Numerica [Battery for Numerical Intelligence]*. Trento: Ed. Erickson.
- Nye, J., Fluck, M. and Buckley, S. (2001). Counting and cardinal understanding in children with Down syndrome and typically developing children. *Down Syndrome: Research and Practice*, 7(2), 68-78.
- Odic, D., Libertus, M. E., Feigenson, L., & Halberda, J. (2012). Developmental Change in the Acuity of Approximate Number and Area Representations. *Developmental psychology*. doi:10.1037/a0029472

- Opfer, J., & Siegler, S. (2007). Representational change and children's numerical estimation. *Cognitive Psychology*, 55, 169–195.
- Passolunghi, M.C., Vercelloni, B., & Schadee, H. (2006). The precursors of mathematics learning: Working memory, phonological ability and numerical competence. *Cognitive Development*, 22, 165–184.
- Paterson, S. (2001). Language and number in Down syndrome: The complex development trajectory from infancy to adulthood. *Down Syndrome Research and Practice*, 7(2), 79-86.
- Paterson, S. J., Girelli, L., Butterworth, B., & Karmiloff-Smith, A. (2006). Are numerical impairments syndrome specific? Evidence from Williams syndrome and Down's syndrome. *Journal of Child Psychology and Psychiatry*, 47, 190–204.
- Piazza, M. (2010). Neurocognitive start-up tools for symbolic number representations. *Trends in cognitive sciences*, 14(12), 542–51. doi:10.1016/j.tics.2010.09.008
- Piazza, M., Facoetti, A., Trussardi, A. N., Berteletti, I., Conte, S., Lucangeli, D., Dehaene, S., & Zorzi, M. (2010). Developmental trajectory of number acuity reveals a severe impairment in developmental dyscalculia. *Cognition*, 116(1), 33–41. doi:10.1016/j.cognition.2010.03.012
- Piazza, M., Fumarola, A., Chinello, A., & Melcher, D. (2011). Subitizing reflects visuo-spatial object individuation capacity. *Cognition*, 121(1), 147–53. doi:10.1016/j.cognition.2011.05.007
- Pica, P., Lemer, C., Izard, V., & Dehaene, S. (2004). Exact and approximate arithmetic in an Amazonian indigene group. *Science*, 306(5695), 499-503. doi:10.1126/science.1102085
- Porter, J. (1999). Learning to count: a difficult task? *Down Syndrome Research and Practice*, 6(2), 85-94.
- Raven, J., Raven, J. C., & Court, J. H. (1998). *Coloured Progressive Matrices*. Oxford Psychologists Press, Oxford.
- Revkin, S. K., Piazza, M., Izard, V., Cohen, L., & Dehaene, S. (2008). Does subitizing reflect numerical estimation? *Psychological science : a journal of the American Psychological Society / APS*, 19(6), 607–14. doi:10.1111/j.1467-9280.2008.02130.x
- Ross-Sheehy, S., Oakes, L. M., & Luck, S. J. (2003). The development of visual short-term memory capacity in infants. *Child development*, 74(6), 1807–22.

- Rourke, B. (1989). *Nonverbal learning disabilities. The syndrome and the model*. New York: Guilford Press
- Rousselle, L., & Noel, M.P. (2007). Basic numerical skills in children with mathematics learning disabilities: a comparison of symbolic vs. non-symbolic number magnitude processing. *Cognition*, 102 (3), 361–395.
- Rousselle, L., Palmers, E., & Noel, M-P. (2004). Magnitude comparison in preschoolers: What
- Sartori, G., Job, R., Tressoldi, P.E. (1995). *Batteria per la valutazione della dislessia e della disortografia evolutiva*. Florence: Organizzazioni Speciali.
- Schleifer, P., & Landerl, K. (2011). Subitizing and counting in typical and atypical development. *Developmental Science*, 14, 280–291. doi: 10.1111/j.1467-7687.2010.00976.x
- Scholl, B. (2001). Objects and attention: the state of the art. *Cognition*, 80, 1–46.
- Sherman S. L., Allen E. G., Bean L. H. & Freeman S. B. (2007). Epidemiology of Down syndrome. *Mental Retardation and Developmental Disabilities Research Reviews*, 13, 221–7.
- Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child development*, 75(2), 428-44. doi:10.1111/j.1467-8624.2004.00684.x
- Siegler, R. S., & Opfer, J. E. (2003). The development of numerical estimation: evidence for multiple representations of numerical quantity. *Psychological science*, 14(3), 237-43.
- Siegler, R.S. (1996). *Emerging minds: The process of change in children's thinking*. New York: Oxford University Press.
- Siegler, R.S. (2007). Cognitive variability. *Developmental Science*, 10, 104-209.
- Siegler, R.S., & Arraya, R. (2005). A computational model of conscious and unconscious strategy discovery. In R.V. Kail (Ed.), *Advances in child development and behaviour* (pp. 1-42). Oxford: Elsevier.
- Spelke, E. S., & Kinzler, K. D. (2007). Core knowledge. *Developmental science*, 10(1), 89–96. doi:10.1111/j.1467-7687.2007.00569.x
- Starkey, P., & Cooper, R. G., Jr. (1980). Perception of numbers by human infants. *Science*, 210, 1033–1035.1980.

- Stoianov, I., & Zorzi, M. (2012). Emergence of a “ visual number sense ” in hierarchical generative models. *Nature Neuroscience*, 15(2), 194–196.
- Thompson, C. a., & Siegler, R. S. (2010). Linear numerical-magnitude representations aid children’s memory for numbers. *Psychological science*, 21(9), 1274–81. doi:10.1177/0956797610378309
- Trick, L. M., & Pylyshyn, Z. W. (1994). Why are small and large numbers enumerated differently—A limited-capacity preattentive stage in vision. *Psychology Review*, 101, 80–102.
- vanMarle, K., & Wynn, K. (2009). Infants’ auditory enumeration: Evidence for analog magnitudes in the small number range. *Cognition*, 111, 302–316.
- Verguts, T., Fias, W., & Stevens, M. (2005). A model of exact small-number representation. *Psychonomic bulletin & review*, 12(1), 66–80.
- Walsh, V. (2003). A theory of magnitude: common cortical metrics of time, space and quantity. *Trends in Cognitive Sciences*, 7(11), 483–488. doi:10.1016/j.tics.2003.09.002
- Wechsler, D. (1991). WISC-III: Wechsler Intelligence Scale for children. New York: The Psychological Corporation, 1991 [Italian edition Scala di Intelligenza Wechsler per Bambini-Terza Edizione - WISC III, Firenze, Organizzazioni Speciali 2006].
- Wood, J. N., & Spelke, E. S. (2005). Infants’ enumeration of actions: numerical discrimination and its signature limits. *Developmental science*, 8(2), 173–81. doi:10.1111/j.1467-7687.2005.00404.x
- Wynn, K. (1990). Children’s understanding of counting. *Cognition*, 36, 155–193.
- Wynn, K. (1992). Addition and subtraction by 5-month-old human infants. *Nature*, 348, 749-750.
- Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition*, 74(1), B1–B11.
- Xu, F., Spelke, E. S., & Goddard, S. (2005). Number sense in human infants. *Developmental science*, 8(1), 88–101. doi:10.1111/j.1467-7687.2005.00395.x
- Zorzi, M., & Butterworth, B. (1999). A computational model of number comparison. In M. Hahn & S. C. Stoness (Eds.), *Proceedings of the Twenty-First Annual Conference of the Cognitive Science Society* (pp. 778–783). Mahwah, NJ: Erlbaum.