LQR Temperature Control in smart building via real-time weather forecasting

Marco Barbiero[1], Alessandro Rossi[2], and Luca Schenato[3]

Abstract— **In this work we consider the problem of climate control within a smart building instrumented with multiple temperature sensors and controllable HVACs. The main contribution is to use real-time weather forecasting readily available via internet connection to obtain real-time information about external temperature and solar insulation to reject external disturbances and anticipate temperature changes. The control system is based on MIMO LQR control with a reduced order observer and integral control applied to a model of the building dynamics obtained from construction data. The proposed architecture shows substantial improvements in numerical simulations as compared to PID-based standard controllers in terms of improved comfort, while being computationally simpler than more advanced solutions such as MPC or AI-based control.**

I. Introduction

Over the last decade, the development of Internet of Things devices and wireless technologies has reduced the monitoring cost in buildings, allowing constructors to install sensors even in residential buildings. The spread of Energy Performance Contracting (EPC) in some European countries, including Italy, has also boosted the demand for increasingly effective controls that can reduce operating costs while maintaining comfort. The remuneration of ESCos promoting these contracts depends on the achieved amount of energy savings and, hence, they are interested in tools that allows to maximize it.

To address this problem, in particular for residential units, we propose a standard procedure based on two ideas. The first is to use building construction data to model the thermal behaviour of the building. The second idea concerns the use of real-time data, not only from sensors, but also weather prediction and sun insulation from internet connectivity, that can be used to control the thermal comfort of the building and compensate external disturbances. This data is also used to compensate for any model design errors. The challenge that this work aims to solve is to use and exploit all the data that a modern house provides without requiring anything else. We identify all the available data: building plans and materials used are usually known, while, regarding real time data, internal temperatures and solar radiation (inferred from the production of solar panels, mandatory in Italian houses newly built or renovated since June 2012) can be logged. Furthermore, all the environment data are available from the regional weather service.

The target of this work is to exploit all the available information about a building, both that obtained in real time from sensors and predictions, and the properties of building elements from the designs, to improve energy saving without loss of thermal comfort. We desire to achieve this result by assuming to control directly the actuators: we assume to control the heating power injected or extracted by the HVAC devices. This assumption is not always valid in real environment: sometimes devices have already control loops, allowing to control only set points and not low-level inputs [1]. However, it is possible to extend the methodology of this work by modelling set points as control variables: it is sufficient to add their dynamics into the model.

For buildings thermal modelling, the most spread methodologies are the white box, grey box, and black box methods [2]. White box models are built by means of heat physic equations whereas the black box ones are created by exploiting the thermal data logged during tests in the real buildings we want to model. Grey box models, instead, try to merge these two approaches and shall be preferred, although they always require testing in real buildings. The most widely used in recent years is the black box approach: Machine Learning and Artificial Intelligence techniques are becoming more precise allowing to create a model without knowing anything about the building. Indeed, it is only necessary to collect data from sensors about building temperature, inputs, and habits of the occupants. Hong [3] reports the state-of-the-art about these technologies. Anyway, we want to exploit all available data and provide a fully functional system from the day of delivery to the customer, so we follow the white box methodology. Although white box approaches are widely used in literature for more than a decade, this work wants to combine those together with real time data and weather forecasts.

Developing a white box model can be carried on by means of two different approaches: the Finite Element Method (FEM) and the lumped parameter one. The first is a numerical method that try to solve numerically the partial differential equations that describe the heat behaviour in the building. It is widely used for very precise simulations because it allows to determine accurately the temperature value at any point in the space. For example, [4] uses this approach to test energy demand and performances of different building envelopes. The second approach, instead, assumes that the

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heat flux between volumes which are separated by surfaces can be approximated by first order differential equations [5]. Using the electrothermal analogy, we can simulate buildings as easily as we analyse the dynamic response of RC electronic circuits. It is clear that the latter is the easiest and less accurate method, but this shall not be mislead since it has great potential, as reported for example in [6] where a multi-layered building is simulated: this methodology is reliable [7] and it is used in ISO 13790 too. Furthermore, this approach is much more useful for control, as it allows the model to be written in state space. Several RC-approaches are available depending on the complexity: 2R1C, 3R2C, and 3R4C [8], [9]. In this work, we use the 2R1C approach.

The topic of thermal control in buildings is also very wide: it ranges from the oldest and most reliable technologies presented in control theory manuals like PID [10], state space techniques like LQ [11], and the most advanced ones based on Model Predictive Control [12]. Our choice fell on an LQR-based optimal control along with some techniques of mitigation of design errors. Furthermore, inspired by [13], we implement an external disturbances mitigation by means of a feed-forward system to improve overall performances. The main contribution is to show that this methodology greatly improve over standard PI control: in particular, there is an improvement of 90% in temperature tracking precision. To achieve more realistic results, we assume to have a constructive variability of up to 20% of the nominal values. The major benefit comes from internet data about external temperature and solar irradiance, that allows us to almost completely reject these disturbances, achieving significantly better comfort and energy savings.

This report can be summarized as follows. In Section II, we describe the 2R1C methodology and how to develop a lumped parameters model using the building construction data. Subsequently, uncontrollable external inputs are modelled using the physical laws and meteorological data. Section III describes the control design: we introduce a reduced order observer for non-observable states and apply the LQR control together with the integral one. In Section IV, we evaluate the proposed solution for control against a standard PI controller in different scenarios: for each of these, we simulate the behaviour using the two controllers and compare their performance. Final conclusions and future improvements are drawn in Section V.

II. Problem Formulation

The first goal of our work is to create the digital twin of a real building. Since we want to control it by means of LQR, it is necessary to define the model in state space. As reported in [14], the best modelling approach is the lumped parameters method as it produces a low order model, even if the building has a considerable number of zones. This feature is perfect for performing LQR control and MPC. The lumped parameters method exploits the mathematical analogy between different physical phenomena that can be described by the same equations, net of constants. Table I reports the analogy between thermal and electrical quantities used in this work.

TABLE I

Analogy between thermal and electrical quantities

Thermal			Electric		
Quantity		m.u.	Quantity		m.u.
Temperature	T	K	Potential	V	V
Temperature Diff.	ΔT	K	Voltage	ΔV	V
Thermal Flux	$\dot{Q}$	J/s = W	Electric current	I	A
Resistance	R	K/W	Resistance	R	Ω
Capacitance	C	J/K	Capacitance	C	F

A. RC model

From Table I, we can write down the electrical analogy of the Newton's law of cooling:

$$\dot{Q} = hA(T - T_{ext}) = hA\Delta T \leftrightarrow I = hA\Delta V = \frac{\Delta V}{R}$$

where T is the surface object temperature, T_{ext} the surrounding temperature, A is the heat transfer surface area while h is the heat transfer coefficient in W/m^{2}K. Exploiting the first law of thermodynamics, it is possible to describe the dynamic of that heat flow with a first order differential equation:

$$\dot{T} = -\frac{hA}{C}(T - T_{ext})$$

analogue to a capacitor discharge behaviour. This result inspired the 2R1C model. Assume that two zones, with temperature T_0 and T_1 respectively, are divided by a wall with thermal resistance R_w and thermal capacitance C_w. Furthermore, assume that the wall temperature T is spatially uniform at any time during the heat transfer process. Then:

$$\dot{Q}_w = C_w\frac{dT}{dt} = \frac{(T_0 - T)}{R_0} + \frac{(T_1 - T)}{R_1} \tag{1}$$

where R_0 and R_1 such that $R_w = \left({R_0}^{-1} + {R_1}^{-1}\right)^{-1}$. Usually, $R_0 = \alpha R_w$ and $R_0 = (1-\alpha)R_w$, where $\alpha \in (0, 1)$. In this work, we use $\alpha = 0.5$ as suggested in [8]. Fig. 1 shows (1) in the electrical domain.

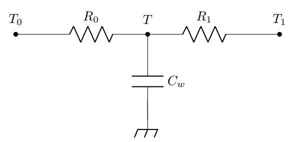

Fig. 1. A wall block in 2R1C approach.

After describing the wall block, we can use this technique to define all the elements of a building envelope and, then, develop a methodology to create the digital twin of any building. Fig. 2 reports equivalent models of rooms, windows, and internal walls in the electrical domain that are discussed in the following paragraphs whereas external wall model can be found in Fig. 1. The various blocks, interconnected according to the following rules, form an electrical network whose behaviour simulates the real building one.

a) External wall, roofs and floors: An external wall, a roof and a floor connect an external T_0-temperature environment with an internal T_1-temperature zone with a noticeable inertia. To describe them it is sufficient to use the definition of a wall, i.e., (1) or Fig. 1. We can assume the heat flux takes place only along the transverse direction and, therefore, the description shall be the same in those scenarios: they can be modelled using two resistors, R_0 and R_1, and a capacitor C_w. The equivalent resistance R_w is given by the sum of the resistances of each single layer as well as the capacitance C_w is equal to the sum of the thermal capacitance of the individual layers. The total resistance shall be split between R_0 and R_1.

(a) Room (b) Window/Internal wall

Fig. 2. A room and a window (or internal wall) blocks in 2R1C approach.

b) Window: The description of a window can also be traced back to the one of a wall that does not retain heat. Indeed, a window links an external T_0-temperature environment with an internal T_1-temperature zone with a negligible inertia. In other words, the thermal capacitance of a window can be approximated to zero. Therefore, the heat flow $\dot{Q}_f^{T_1 \to T_0}$ through the window with resistance R_f can be defined as follows:

$$\dot{Q}_f^{T_1 \to T_0} = \frac{T_1 - T_0}{R_f} \tag{2}$$

where T_0 and T_1 are the temperatures of the two zones adjacent to the window. From an electrotechnical point of view, the same situation is reported in Fig. 2b.

c) Rooms and Internal walls: The description of a building internal wall is controversial: most of the authors, like [15], model it with one capacitor while others omit it. Other ones, like [16], model it using 2R2C approach. Since the purpose of the work is to create a digital twin of a building and then control it precisely, we are forced to find a reliable way to shape rooms individually. Actually, rarely the rooms in a house are all at the same temperature: an example is the spatial division between living and sleeping areas, which are heated alternately. For this reason, we decided to model a room as a capacitor of the same size as the thermal capacitance of the air contained in the room and connected to the others by a resistor that describes the internal walls. In a mathematical way, a room with volume V_r can be described by a capacitor with a capacitance in $\mathrm{J/K}$ equal to

$$C_r = \rho_a c_{p_a} V_r \tag{3}$$

considering $\rho_a = 1.204\,\mathrm{kg/m^3}$ as the air density at $20\,°\mathrm{C}$ and $1\,\mathrm{atm}$ pressure and $c_{p_a} = 1007\,\mathrm{J/kg/K}$ as the air specific heat at $20\,°\mathrm{C}$ and $1\,\mathrm{atm}$ pressure. To simplify the notation, we assume the room is empty and consequently the value of C_r depends from air properties only. However, it is possible to adapt the formula to consider furniture, thus making the estimate more accurate: simply add the thermal capacitance of each furniture element to (3).

B. Input and disturbance modelling

To predict the behaviour of the real building in a useful way, it is necessary to connect the created network to a model of the surrounding environment. There exist three main causes of a temperature variation within a building: external temperature, sunshine, and internal loads. The former tries to impose a temperature on the building and, when the external temperature is the same as the internal one, the heat exchange stops. The other two, instead, try to change the temperature of the building by directly supplying heat to it, regardless of its actual temperature. These two behaviours can be described, by means of the electrothermal analogy, by an ideal voltage or current sources.

(a) External Temperature (b) Internal Load/Sunshine

Fig. 3. An External Temperature and Internal Load/Sunshine 2R1C blocks.

a) External temperature behaviour: Fig. 3 shows how to model the external environment at T_e K and an internal or solar load of I_s W. Voltage sources, one representing external air and one representing the ground, shall be connected to all the blocks in contact with them respectively. Current sources, instead, shall be connected only to the capacitor inside the node.

b) Sun heating behaviour: To describe the Sun's heating behaviour with respect to the envelope, there are three different problems to face: estimate the intensity, the direction, and the amount of heat that actually reaches it. The first problem can be solved by relying on online services, even free of charge, that provide direct sunlight in real time given a location. To solve the second, it is necessary to model the Earth motion of rotation and revolution. We assign to each surface of the envelope a 3D vector representing the outer normal of the surface and model, by means of algorithms such as [17], the vector representing the Sun's rays direction. To determine the amount of radiation incident on the surface, and the incident energy value, we calculate the dot product between the vector of the sun and the vector of the surface. It should not be forgotten, however, that every surface has an albedo, i.e. it reflects part of the incident radiation. Therefore, we weigh each input using $1 - a$, where a represent the surface albedo.

C. Solving the RC circuit via State-Space

The previous sections illustrate a method for modelling a building as an electrical circuit. To evaluate the dynamic behaviour, an efficient method to solve the circuit is needed. Since the relationships between the components are linear, the best idea is to reduce the circuit to a model in state space since it would be very quick to evaluate its properties and develop effective controllers. We take inspiration from the method described in [18]. Without loss of generality, we describe only ideal current sources, with or without shunt resistors, since, in our case, voltage sources always have a resistor in series: they represent external or ground temperatures and they are always connected to a wall block. Therefore, it is possible to apply the Norton theorem and obtain an ideal current source in parallel with a shunt resistor. We also assume that there are not null capacitances or resistances, and resistors connected in series.

Let N_w be the number of external walls, N_r the number of rooms and N_f the number of windows. Since only rooms and walls are modelled by means of capacitors, let $N = N_w + N_r$. Let $C \in \mathbb{R}^{N\times N}$ be the diagonal matrix containing the capacitances, i.e., $C_{ii} = C_i$, $i \in \{1, \dots, N\}$. Since they are the only dynamic parts of the model, we number them, order them, and we refer to them as node n_i. Similarly, let R be the diagonal matrix of resistors, i.e., $R_{jj} = R_j$, $j \in \{1, \dots, N_e\}$, where R_j indicates the j-th non-shunt resistance and N_e is the total number of non-shunt resistors in the circuit. It is immediate to note that resistors behave like the edges of a graph in which the nodes are the capacitors. It is useful to exploit this concept by introducing some specific graph matrices.

Let $\Gamma \in \mathbb{R}^{N\times N_e}$ be the oriented incidence matrix, i.e., the matrix that links nodes and edges: $\Gamma_{ij} = +1$ if R_j starts from n_i, $\Gamma_{ij} = -1$ if R_j ends in n_i, and $\Gamma_{ij} = 0$ otherwise. The orientation of the resistors is completely free, but, when it is defined, it shall be not changed in any phase of the process. Let $\mathcal{L} = \Gamma R \Gamma^\intercal$ be an irreducible Laplacian matrix and let S be the diagonal matrix of shunt conductance, i.e., the conductance parallel to the ideal current sources: they are as many as resistances in series to the voltage sources that represent the outside temperature and the ground temperature, i.e., $S_{kk} = 1/R^s_k$, $k \in \{1, \dots, N_s\}$ where R^s_k is the k-th shunt resistor and N_s the number of them. Finally, let I^* be the column vector describing the current flows into nodes, where I^*_i is the current that flows into node i. I^*_i consists of several contributions, not all of which are always present: the external temperature, i.e., T_e/R^i_0, the ground temperature, i.e., T_g/R^i_0, the sun contribution I^s_i, and internal loads I^l_i. As reported in [18], by exploiting these matrices, the voltage circuit behaviour can be written as:

$$\dot{V}(t) = \underbrace{-C^{-1}(\mathcal{L} + S)}_{F} V(t) + \underbrace{C^{-1}I^*}_{GU}$$

It can be described as a state space model: the state is the voltage of the nodes, F is the state matrix whereas $G = [G_e|G_g|G^s|G^l]$ the input matrix by defining U as the system inputs, i.e., $U = [T_e, T_g, I^s_1, \dots, I^s_N, I^l_1, \dots, I^l_N]$. Indeed, the inputs are linear and can be easily expressed in matrix form: the obtained matrix G is a tall matrix and, therefore, not invertible. Recalling that the voltage of a node represents the temperature of the corresponding room or wall, the 2R1C model of the building in state-space representation is

$$\begin{aligned} \dot{x}(t) &= Fx(t) + Gu(t) \\ y(t) &= Hx(t) \end{aligned} \tag{4}$$

by defining H as selection matrix: this matrix selects only the rooms that can be measured. The model shown also allows the application of controllable inputs: it is sufficient to add to the internal loads the heat flow that the controller desires to introduce in that node.

III. Controller Design

The aim of this work is to define a methodology to reduce the energy consumption of a building without sacrificing comfort. To pursue this objective, after defining a model, it is necessary to develop a controller to govern its heating behaviour. In this work, we adopt a modified Linear-Quadratic-Gaussian controller, consisting of an augmented states LQR and a reduced order LQE, together with a feed-forwarding system. We develop the controller in continuous time and only discretise it at the end of design phase by means of a zero-order hold. Furthermore, thanks to the separation principle, we design controller and observer in two different stages.

1) Integral LQR: The purpose of our controller is to require the minimum power on the inputs in such a way that a reference temperature is reached and maintained within a subset of rooms $\mathcal{R} = \{x^R_1, \dots, x^R_m\}$, selected by matrix H_m. Let $r(t) \in \mathbb{R}^m$ be the reference temperature to be reached and maintained there. Therefore, the controller shall minimize

$$\|e(t)\| = \|y(t) - r(t)\| = \|H_m x(t) - r(t)\|$$

in a robust manner, i.e., we desire $y(t)$ to converge to $r(t)$ with no steady-state error for any constant input and disturbance. We assume the temperature in $\mathcal{R}$ is available: if it is not the case, it is sufficient to develop a state observer, as it is reported in the Subsection III-.2. Let $x_I \in \mathbb{R}^m$ be a vector of additional states such that $\dot{x_I}(t) = e(t)$. Our target can be rewritten as $x_I(t) \to 0$, since that implies $y(t) \to r(t)$. We can define the augmented model:

$$\underbrace{\begin{bmatrix} \dot{x_I} \\ \dot{x} \end{bmatrix}}_{\dot{z}} = \underbrace{\begin{bmatrix} 0 & H_m \\ 0 & F \end{bmatrix}}_{F_z} \underbrace{\begin{bmatrix} x_I \\ x \end{bmatrix}}_{z} + \underbrace{\begin{bmatrix} 0 & -1 \\ G & 0 \end{bmatrix}}_{G_z} \begin{bmatrix} u \\ r \end{bmatrix} \tag{5}$$

and the quadratic cost function:

$$\begin{aligned} J(u) &= \int_0^\infty z^\intercal Q_z z + u^\intercal R u \, dt & (6) \\ &= \int_0^\infty x_I^\intercal Q_I x_I + x^\intercal Q_x x + u^\intercal R u \, dt & (7) \end{aligned}$$

We shall develop a linear state-feedback law $u = -Kz$ that minimize (6). Since (F_z, G_z) are a stabilizable pair by construction, we can solve the associated continuous-time algebraic Riccati equation and obtain the optimal gain matrix $K = [K_I \; K_x]$: this matrix guarantees the closed loop is stabile.

2) Reduced-order LQE: The controller developed in the previous section needs the measurements of each node in the system. Since some nodes cannot be measured easily, like external wall nodes, we develop a reduced state observer whose feedback matrix is designed using the LQE methodology.

Let $T \in \mathbb{R}^{N \times N}$ be the transformation matrix from state-space to the reduce-observer space, i. e., $\begin{bmatrix} x_y \\ x_e \end{bmatrix} = Tx$. First rows, $x_y \in \mathbb{R}^m$, represent the directly measurable nodes whereas the others, $x_e \in \mathbb{R}^{N-m}$, the nodes that shall be estimated. Our target, therefore, is to develop an observer for x_e since $x_y(t) = y(t)$. Using T, we can partition the system and obtain

$$\begin{bmatrix} \dot{x_y} \\ \dot{x_e} \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \begin{bmatrix} x_y \\ x_e \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} u.$$

Using $v(t) = x_e(t) + Ly(t)$, $L \in \mathbb{R}^{(N-m)\times m}$, it is sufficient to estimate $v(t)$ to obtain the $x_e(t)$ estimation. The observer can be developed as

$$\dot{\hat{v}} = F_v \hat{v} + \begin{bmatrix} G_y & G_u \end{bmatrix} \begin{bmatrix} y \\ u \end{bmatrix}$$
$$\dot{\hat{x_e}} = I_m \hat{v} + \begin{bmatrix} L & 0 \end{bmatrix} \begin{bmatrix} y \\ u \end{bmatrix}.$$

The system dynamics is governed by $F_{11} - LF_{21}$. The estimate converges to the real value only if F_v is asymptotically stable: L allows F_v to be stabilised if the pair (F_{22}, F_{12}) is detectable, i.e., the pair (F, H) is detectable. By constriction, F is full rank and asymptotically stable. These considerations allow us to design L by means of Kalman procedure, the dual of LQR, after defining process noise covariance Q and measurements noise covariance R.

3) Weight choice: LQR and Kalman procedures provide control matrices that minimizes the cost indexes. Using Bryson's rule [19], it is possibile to define Q and R matrices as $Q_{ii} = x_{i\_max}^{-2}$ and $R_{jj} = u_{j\_max}^{-2}$, where x_{i_max} represents the max acceptable error on the i-th state whereas u_{j_max} the max acceptable value on the j-th input. In this work, we use $x_{i\_max} = 10^4$ for non controlled nodes, $x_{i\_max} = \sqrt{0.1}$ for controlled ones, $x_{i\_max} = 36$ for integrator states, and $u_{j\_max} = \sqrt{10^3}$ for all the inputs. Some of this values are obtained after a session of try and error: the main issue is to maintain a good condition number of Q. The observer weights, instead, shall describe the reliability of measures and the model. Therefore, the ratio between the measures noise weight and the process noise power is very important: if the ratio is higher than 1, it suggests the observer to trust more the measures than the model outcomes. Since a building model has a lot of uncertainties, we choose to a ratio equal to 10^5. The diagonal matrices chosen are $Q_{ii} = 10^5$ and $R_{jj} = 1$.

4) Feed Forward compensator: The designed controller does not take external inputs into account. However, measurements of external temperature and solar radiation are readily available, even with low sampling times. For this reason, we implement a feed forward compensation system for external temperature, solar radiation, and the temperature references. Remembering Equation (4) for controlling a subset of nodes describes by selection matrix H_m, the system can be written as:

$$\dot{x} = Fx + [G_e|G_g|G^s]d(t) + G^l u(t)$$
$$y = H_m x$$

where $d(t)$ represents the disturbances, i.e., external and ground temperatures and the solar radiation. When the system is in steady-state, i.e, $\dot{x} = 0$, to obtain a feed forward such that $y \to r$, we shall use:

$$-H_m F^{-1}([G_e|G_g|G^s]d(t) + G^l u_{ff}(t)) = r$$
$$-H_m F^{-1}(G^l u_{ff}(t)) = r + H_m F^{-1}([G_e|G_g|G^s]d(t)$$
$$u_{ff}(t) = -(H_m F^{-1} G^l)^{+}(r + H_m F^{-1}([G_e|G_g|G^s]d(t))$$

IV. Case Study

To test our methodology, we create a digital twin from scratch. We obtained the blueprint and project data for a single-storey house with a surface area of $128\,\mathrm{m}^2$, a overall volume of $1130\,\mathrm{m}^3$, and a dispersion surface of $918\,\mathrm{m}^2$. The analysed building is located near Treviso, Italy, climatic area D. The house consists of 9 rooms, all heated except for the garage: therefore, the number of controllable zones is $m = 8$. All these zones are measurable. The equivalent RC model, with the exception of the roof and fundaments, is shown in Fig. 4. In particular, the model consists of 23 capacitors and 25 resistances; therefore, system dimension is $N = 23$.

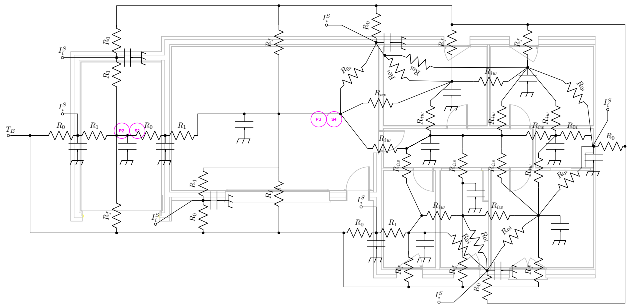

Fig. 4. RC Model on blueprint of case study.

To test our work, we compare the performance of our controller with a PID controller. This type of control is quite advanced in house thermal control area since, usually, the temperature are controlled by means of an on-off thermostat. The comparison is made on a 15-days simulation using weather service data [20] from the 9-th to the 24-th February 2019. The controllers must reach $21\,^{\circ}\mathrm{C}$ in the controlled rooms from 8 a.m. to 10 p.m. and not less than $16\,^{\circ}\mathrm{C}$ during the rest of the day, minimizing the energy consumption.

To carry on this experiment in a more realistic way, we select a subset of 4 rooms and we start the simulation at 6:00 am using the outer temperature as initial condition for all nodes. Furthermore, we enable only the winter mode of HVAC, i.e., only positive inputs can be applied. Fig. 5 shows the Root Mean Square Error in tracking using the two different controllers. We do not report if temperature is higher than reference since Sun energy can improve appreciably the temperature and we don't want to penalize this natural factor.

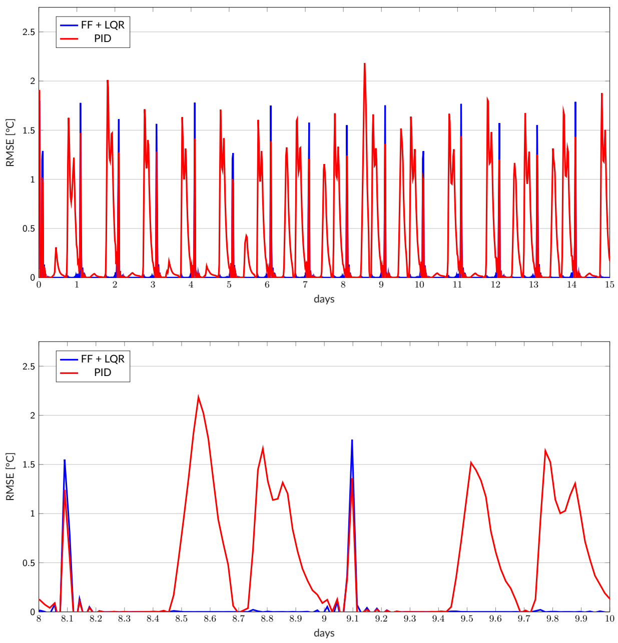

Fig. 5. Temperature error over 15 days (top panel) and zoom for days 8 to 10 (bottom panel).

In Table II the cumulative and synthetic results are reported. These results show that, for the price of an increase of less than 2% in energy consumption, there is a 9-fold reduction in temperature tracking errors. Furthermore, Fig. 5 shows that the PID control commits an error above 1 °C for many hours a day whilst it happens for only a few minutes a day using our control. By checking the inputs defined by the two controllers, it can be seen that the energy saving given by the PID is due to the slow response to the reference change and, therefore, this behaviour shall be condemned.

TABLE II
SIMULATION SYNTHETIC RESULTS

	PID	LQR	FF + LQR
Energy Utilization	295.49 MJ	301.30 MJ	301.27 MJ
Mean RMSE	0.3459 °C	0.0404 °C	0.0388 °C

V. Conclusions

In this work, we have shown how the use of real time weather forecast prediction in terms of external temperature and solar radiation paired with model based control, namely LQR, provides considerably higher comfort than standard PID control while maintaining the same energy consumption. The dynamical model of the building upon which the LQR design is based, can be easily obtained from the 2R1C methodology and can be automated as long as construction material properties and building blueprint are available.

Future improvements include the comparison of the proposed strategy with Model Predictive Control and its validation on the real building described in this work.

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