The cross-sectional diffusion of jumps and the identification of system(at)ic movements

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Abstract

By analyzing a very large dataset of high-frequency returns, we propose two indexes informative of the cross-sectional diffusion of jumps, having important implications not only in asset allocation and hedging, but also in asset pricing. Notably, the two diffusion indexes capture a part of the variation in stocks' returns which is not explained by the capital asset pricing model's traditional factors. Besides, the empirical results provide evidence of interesting relations between contemporaneous jumps, stock size and capitalization.

Keywords: multiple co-jumps; systemic jumps; systematic jumps; cross-sectional jump diffusion; systemic risk.

JEL codes: C58, G10.

1 Introduction

Every day market operators negotiate tens of thousand of stocks, providing an extremely rich information set useful for studying, for instance, high frequency price dynamics, the price impact of trades, the occurrence of anomalous and unexpected price movements (i.e. price jumps) and the interdependence between price movements and news releases. Research on asset pricing has been boosted thanks to the availability of high-frequency data, which has stimulated the development of non-parametric tests detecting the presence of discontinuous jumps in assets' prices in addition to a continuous diffusion component.

There exists a vast literature that supports the presence of jumps in assets' prices (see, among others Ball and Torous (1983), Jarrow and Rosenfeld (1984), Jorion (1988), Duffie et al. (2000) and Eraker et al. (2003)). Nevertheless, the empirical detection of jumps has been originally challenging as they are rare events and, therefore, their identification requires long time series that are not always available. More recently, the availability of high-frequency data has lead to the development of new methods for detecting jumps. Notably, Barndorff-Nielsen and Shephard (2004b, 2006) represent the cornerstone of this field, focusing on identifying jumps using high-frequency data. In this framework, stocks' volatility is split into a continuous part—driven by continuous price variations—and a component capturing large and isolated prices' changes—the jumps. Starting from the findings in Barndorff-Nielsen and Shephard (2004b, 2006), further studies proposed alternative tests to explicitly identify intraday jumps; see, Andersen et al. (2007), Lee and Mykland (2008), Corsi et al. (2010) and Andersen et al. (2010), among others.

Despite the evidence of jumps in different markets (see, e.g., Huang and Tauchen (2005), Andersen et al. (2007), Lee and Mykland (2008), Evans (2011)), there is still little understanding about their cross-sectional diffusion. In general, the tendency of jumps to occur simultaneously is studied by analyzing either portfolios of stocks (for instance, stocks' indexes) or individual assets. As for the latter, the great majority of the current literature proposes tests which are suitable only to detect common jumps (or co-jumps) between $N = 2$ assets. Nevertheless, generalizing them for $N > 2$ assets is a challenging task. A partial list of recent studies on this topic includes Barndorff-Nielsen and Shephard (2004a), Jacod and Todorov (2009), Mancini and Gobbi (2012), Bibinger and Winkelmann (2015), Bandi and Renò (2016), Caporin et al. (2017) and Ferriani and Zoi (2020).

This paper relates to the strand of literature which investigates co-jumps between $N > 2$ assets. Here, we cite Bollerslev et al. (2008)—who build a test to identify common jumps among multiple stocks—and Gilder et al. (2014)—who propose a simple but still effective co-exceedance rule by intersecting results obtained from univariate jump tests; both methods have a similar power, but the latter is more suitable for empirical studies because of its simplicity. Starting from the work of Gilder et al. (2014), we thus develop a modified version of the co-exceedance rule, which is tailored to the large size of our dataset.

In fact, in contrast to other contributions, we study co-jumps involving a relatively large number of stocks, on a very large dataset of stocks' returns, which is not common in the literature. The database includes 1-minute prices for all $N = 3,509$ stocks belonging to the Russell 3000 index between January 2, 1998 and June 5, 2015 (4,344 days).[1] The potential advantage of having a large dataset of stocks is limited by the market liquidity condition. The liquidity of an asset impacts on the frequency at which the price moves over time. If an asset has a low liquidity level, 1-minute prices might not move for several minutes, thus generating sequences of zero 1-minute returns. In our study, we account for this possibility and focus on stocks with a sufficient number of non-null intraday returns. In general, assets that do not match our liquidity condition are also characterized by small market values.

We follow Boudt et al. (2011) and account for the U-shaped intraday volatility pattern of returns. This enables us to improve the detection of small jumps during low volatility periods and to reduce the spurious detection of jumps at high volatility times and, in particular, in the first minuts of daily market trading (i.e. around market opening). Then, we implement the jump tests at both the daily and the intradaily levels using the sequential procedure suggested by Gilder et al. (2014). Such a procedure identifies in a first step days in which at least a jump occurs and then allocates the

[1]The total number of asset exceeds 3000 as our dataset includes all the assets that have been part of the Russell 3000 index. Therefore, we account for dead assets as well as for asset that, despite being still traded, they do not belong to the Russell 3000 index.

jumps within the day (i.e., we identify at a higher-than-day frequency the occurrence of jumps). Among all possible tests for jumps detection, we use the C-Tz test introduced by Corsi et al. (2010), as it has more power with respect to others tests based on multipower variation; see Corsi et al. (2010).

Further, we use the Gilder et al. (2014)'s coexceedance method to detect contemporaneous jumps in the cross-section of assets. Interestingly, we find a pattern in jumps throughout the day. For instance, around lunch it is possible to observe a large increase of common jumps and a correspondent decrease of single jumps, a phenomenon that we name 'lunch effect'. More importantly, results tell us that, even if co-jumps can involve a large number of assets, up to 956, they are usually small and negligible with respect to the whole stock market.

Then, we classify the detected co-jumps as systematic (due to non-diversifiable events) and non-systematic, by intersecting their occurrence with the occurrence of jumps in the market index. In line with the results of Bollerslev et al. (2008) and Gilder et al. (2014), we find interesting relations between individual stocks and the system. Specifically, systematic co-jumps generally involve a larger number of stocks with respect to non-systematic jumps.

Common jumps which involve only two assets have a limited impact on a huge portfolio (see, e.g., Bollerslev et al. (2008)). As a result, we modify the Gilder et al. (2014) coexceedance method and detect multivariate jumps when at least 20 stocks jump together. We motivate such a choice in light of the large cross-sectional dimension of our dataset. We define these co-jumps as multivariate (or MJs). Similarly to the standard co-jumps, MJs are not often reflected by a jump in the Russell 3000 index, suggesting that the comprised assets are small in size and, consequently, have no impact at the market level. This evidence highlights the need of monitoring the diffusion of jumps in the cross-section of stocks, as the simple analyses of jumps in the market index might not allow to identify large swings in the prices of small cap equities.

The identified common jumps are, in turn, used to build indexes informative of cross-sectional jump diffusion. We propose two indexes which summarize the cross-sectional jump diffusion: the daily diffusion index (DID) and the intraday diffusion index (DII). The DID is proportional to the maximum number of assets involved in a MJ per day, whereas the DII measures the percentage of assets comprised in the MJs for each intraday interval. To make the information content more clear, we isolate the trend from the noise component by filtering each index with the Local Level Model (LLM), see, e.g., Durbin and Koopman (2001). Both trends and residuals show relevant peaks during important economic moments, such as the years 2008-2010.

Collective jumps might represent a relevant fraction of the market, and to analyze this possible association we investigate MJs involving at least 20 stocks among the top 100 size stocks. We define them as systemic MJs, being infrequent events correlated across a relatively large number of large cap stocks (Das and Uppal, 2004). The threshold we adopt is consistent with the findings in Caporin et al. (2017), who prove that 20 assets are sufficient to identify systemic co-jumps. Notably, the systemic jumps we detect are rare but significant events.

The relevance of systemic co-jumps both in terms of their size and the population (i.e. the assets involved), suggest why they are almost always associated with jumps in the Russell 3000 Index. However, the presence of many non-systemic MJs correlated to a market jump (e.g., September 29, 2008) marks the importance of focusing on diffusion indexes together with systemic events. Furthermore, we detect a relationship between systemic MJs and market-level news, consistent with the existing literature; see, among others, Dungey and Hvozdyk (2012), Bollerslev et al. (2008), Lahaye et al. (2011), Gilder et al. (2014) and Caporin et al. (2017). Here, we focus on macroeconomic news, linked to Federal Reserve (or FED) announcements, Federal Open Market Committee (or FOMC) actions, and Associated Press news.

A deeper understanding of the cross-sectional diffusion of jumps can be helpful not only for asset allocation and hedging, but also for pricing. Consequently, we analyze the impact of multivariate jumps on asset returns by including our diffusion indexes in the Capital Asset Pricing Model (CAPM)—see Sharpe (1964), Lintner (1965), Mossin (1966) and Black (1972). Our estimates show evidence that DID and DII are associated with a variation in the exposure to the market returns, thus establishing a link between the cross-sectional diffusion of jumps and the stock sensitivity to market movements. Notably, this relation is not just associated with market jumps given that, as we already noted, the occurrence of jumps in the cross-section is not always reflected in a market jump.

The paper is structured as follows. Section 2 analyses multiple co-jumps. Section 3 discusses the implications of MJs in terms of asset pricing, whereas Section 4 concludes. Appendix A provides a description of the dataset we used, focusing on the companies' size and on the stocks' liquidity. Appendix B describes the tests we implemented to detect jumps for each stock, at both a daily and an intradaily level, providing a discussion on the main empirical findings. Appendix C studies the presence of co-jumps and their link with the market, whereas Appendix D provides a set of robustness checks. Additional results are provided in the supplementary material available online.

2 Multiple co-jumps

We implement our analysis on the 1-, 5- and 11-minute returns of the Russell 3000 index constituents, where we deal with $N = 3509$ companies (dead companies included); all data have been recovered from Kibot.com.[2] Our dataset spans the period January 2, 1998—June 5, 2015, including 4,344 business days and, for each of them, we collect $M+1$ stock prices from 09:30 am to 04:00 pm. We adjust the original data to address typical issues arising from high frequency returns, such as the erratic price behaviour due to the market opening, the liquidity degree of stocks and their volatility periodicity. A complete description of the dataset, along with an exploratory analysis, is provided in Appendix A.

For each series of high-frequency returns retrieved from each trading day and from each company, we check for the presence of jumps by using the C-Tz test of Corsi et al. (2010), because it has a greater power with respect to other tests based on multipower variation. Moreover, we also use the s-BNS test of Barndorff-Nielsen and Shephard (2004b, 2006) as a robustness check. The two tests are described in Appendix B. We are aware that, in a given day, a single stock can exhibit more than one jump. We then deepen the analysis at an intradaily level by using the sequential procedure suggested by Gilder et al. (2014). As a result, we gain knowledge about the number of jumps and their location within the day for a single stock. The results obtained by analyzing univariate jumps are reported and described in Appendix B.

[2]The data quality of is comparable to that of TAQ. A graphical comparison is included in the supplementary material.

Starting from univariate jumps, we compute common jumps (co-jumps) to study their cross-sectional diffusion, following Gilder et al. (2014)—see Appendix C. We checked that the majority of co-jumps involves a restricted number of companies, having a weak or null impact on the entire market. In contrast, we focus here on multivariate jumps that involve a large number of assets and, hence, should impact more strongly on a huge portfolio. Our goal is to use this information to build an index informative about the cross-sectional diffusion of large collective events.

We compute a multivariate jump (or MJ) for the t-th trading day and the i-th intradaily interval as follows:

$$MJ_{t,i} = \begin{cases} \sum_{j=1}^{N} \mathbb{I}\{\text{Jump}_{t,i,j} > 0\} & \text{if } \sum_{j=1}^{N} \mathbb{I}\{\text{Jump}_{t,i,j} > 0\} \geq K \\ 0 & \text{otherwise} \end{cases}, \quad (1)$$

where $K > 2$, $t = 1, ..., T$ and $i = 1, ..., M$.
Using the information about MJs, it is then possible to build an index x_t/N_t, which can either provide daily or intradaily information about large collective events. As for the daily case, for each day in the sample, N_t equals the number of quoted stocks, while x_t takes a value equal to the largest number of stocks simultaneously jumping within a day if at least a MJ is present, and a value of zero otherwise:

$$x_t = \begin{cases} \max_i MJ_{t,i} & \text{if } \sum_{i=1}^{M} MJ_{t,i} > 0 \\ 0 & \text{otherwise} \end{cases}. \quad (2)$$

In contrast, in the intradaily case, x_t conveys information for each intraday interval in the sample, being equal to the number of stocks involved in MJ, if present, and to zero otherwise.

It is now clear how x_t relies on the value of the threshold K. In order to determine the value of K, we study how correlations between RUA jumps and x_t change for different values of the threshold.[3] Figure 1 shows correlation values for K equal to 5, 10, 15, 20, 25, 30, 35, 40, 45 and 50 stocks. The choice of the threshold seems to be relevant when using 11-minute observations, especially for the daily x_t. Instead, for the other two frequencies, that is, 1 and 5 minutes, correlations tend to stabilize around $K = 20$. On

[3]We label as RUA jumps the jumps occurring on the Russell 3000 index (RUA); that is, our proxy for the market index.

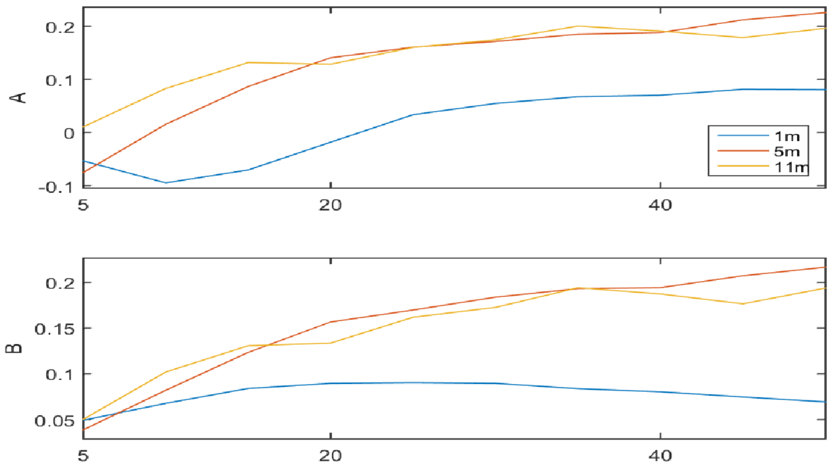

Figure 1 Threshold MJs. The figure shows, for different observation intervals, the correlation between RUA jump days and either the daily x_t index (Panel A) or the intradaily x_t index (Panel B) for different thresholds and for the time-window between January 1998 and June 2015. We use the following thresholds values: $K = 5, 10, 15, 20, 25, 30, 35, 40, 45,$ and 50 stocks.

the basis of the evidence in Figure 1 and following Caporin et al. (2017), who used $N = 20$ stocks to detect systemic co-jumps, we set the threshold equal to 20.

Table 1 presents the number of days with at least one MJ with $K = 20$ and the days with both a RUA jump and a MJ—the systematic MJ days. We first check that the days with at least a multivariate jump are lower than the days with at least a co-jump or a jump. Furthermore, differently from the jump and co-jump statistics, when focusing on MJ days using the C-Tz test, we identify almost two times the days we detect using the s-BNS test. In contrast to the jump and co-jump cases, we not only observe a negative relationship between the interval frequency and Russell 3000 MJ days, but also that RUA jump days (that are almost always also jump and co-jump days) are usually not associated with a multivariate jump in the underlying assets.

Figure 2 shows the trend of MJ along our time window (top panel) and during the 77 5-minute observation intervals of a trading day (bottom panel). Relative to the former, we detect on May 3, 2012 more than one MJ using both the s-BNS test (more details are given in the supplementary material—Section A.9.2) and the C-Tz test. In addition to that day, we

Table 1 Asset jumps and market jumps. The table reports, for the time-window between January 1998 and June 2015, RUA jump days (N_{RUA}), the amount of days in which we observe at least one multivariate jump (N_{mj}) in the constituents of the Russell 3000 and the days with both a jump in the index and a multivariate jump ($N_{RUA} \cap N_{mj}$) in the underlying assets. Results are presented separately for three observation intervals: 1, 5 and 11 minutes, and for the two jump tests (*s-BNS*, and *C-Tz*).

Frequency	N_{RUA}	N_{mj}	$N_{RUA} \cap N_{mj}$
		s-BNS	
1 min	847.00	571.00	113.00
5 min	97.00	84.00	10.00
11 min	19.00	25.00	2.00
		C-Tz	
1 min	1,512.00	1,291.00	422.00
5 min	176.00	643.00	65.00
11 min	57.00	275.00	21.00

detect 140 additional multiple MJ days using the *C-Tz* test. Particularly, we observe three days with more than six MJs: September 19, 2008, September 29, 2008 and May 6, 2010. Details on the days with more than 3 MJs are reported in Table 2. We first notice that all multiple MJ days are associated with important economic and financial events. Moreover, Table 2 suggests that the majority of these MJ days is not reflected by a jump in the RUA index. A possible explanation is that the stocks comprised in the co-jumps are small in size, thus their impact is negligible when considering the whole market and no jumps appear in the Russell 3000 index. We further analyze this point in Section 2.2. Regarding the distribution of MJs during the day, bottom panel of Figure 2 shows that, using the *C-Tz* test, MJs are concentrated around lunch time, similarly to the co-jump case (see Appendix C).

2.1 Diffusion indexes

To summarize the information about the cross-sectional diffusion of MJ, we build two indexes: a daily diffusion index (or DID) and an intraday diffusion index (or DII). The DID, for each day from January 2, 1998 to June 5, 2015, equals the ratio between the the largest number of stocks simultaneously jumping and the number of quoted stocks within the day. Note that the index might also take a zero value when no MJ occurs in a given day. The DII, in contrast, has an intradaily frequency and, for each day, we have M

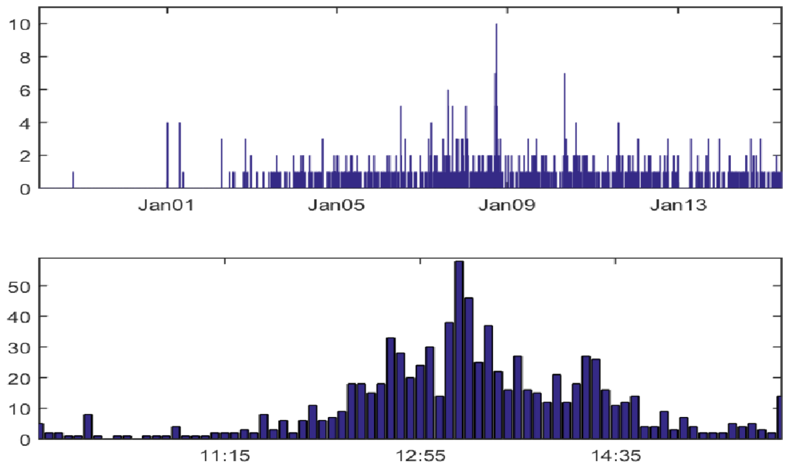

Figure 2 Intervals and days with multivariate jumps. The figure shows in the top panel, for each of the 4,344 days between January 2, 1998 and June 5, 2015, the number of daily intervals (77 daily intervals of 5 minutes) with at least a multivariate jump. Bottom panel, instead, reports, for each of the 77 5-minute intradaily intervals, the number of days for which we observe at least one multivariate jump.

observations. Each of them points out the fraction of stocks involved in a multivariate jump, if present, and 0 otherwise.

Figure 3 shows the DID and the DII we obtain employing 5-minute intervals ($M = 77$). There are no evident differences between the diffusion indexes at daily and intradaily levels. However, we note that many MJs involve just a restricted number of assets and that the series exhibit relevant peaks. We thus filter each diffusion index series to isolate the trend from the noise component. For this purpose, we use the following Local Level Model (LLM) (see, e.g., Durbin and Koopman (2001)):

$$\begin{aligned} x_t/N_t\% &= \mu_t + \epsilon_t, \\ \mu_{t+1} &= \mu_t + z_t \end{aligned} \tag{3}$$

where $x_t/N_t\%$ is a diffusion index series with trend μ_t, whereas $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$ and $z_t \sim \mathcal{N}(0, \sigma_z^2)$ are the residuals. Smoothing the series could then be useful for identifying special events such as financial crises.

Figure 4 shows the time series of μ_t obtained from (3) for the DID—top panel—and the DII—bottom panel. Focusing on the daily index, the maximum and the average values of μ_t are equal to 0.55 and 0.21, respectively,

Table 2 Multivariate jump days. The table shows, using a 5-minute observation interval, the dates (Days) in which we detect more than 3 multivariate jumps, if RUA jumps are present for the same days (RUA days) and the relevant Economic/Financial events during those days. January 2, 1998—June 5, 2015.

Days	RUA days	Economic/Financial event
03-Jan-01	no	FED cut fed funds rate
18-Apr-01	yes	FED cut short-term interest rates
29-Jun-06	no	FOMC statement
21-Mar-07	no	FOMC statement
09-Aug-07	no	BNP Paribas freeze three of their funds
10-Aug-07	no	FOMC statement
18-Sep-07	yes	FOMC lowers target for federal funds rate (50 bps)
10-Jan-08	no	FED chairman Ben Bernanke statement
18-Sep-08	no	FED measures against pressure in funding markets
19-Sep-08	no	FED announce liquidity programs
29-Sep-08	yes	FOMC meeting unscheduled, Emergency Economic Stabilization Act not approved
03-Oct-08	no	Sign of the Emergency Economic Stabilization Act
06-May-10	yes	The Flash Crash
10-Aug-10	no	FOMC statement
05-Aug-11	no	S&P downgrades US sovereign debt

while the maximum and the average volatility values equal 2.18 and 0.76.[4] Moreover, μ_t shows large time variations, with a clear increase in its level in correspondence of market turmoils, as in 2008 and 2010. The pattern is less evident for DII, which is more volatile. Nevertheless, the latter shows higher peaks in correspondence of market turmoils.

In addition to the trend component, also the irregular component of the LLM conveys relevant information in showing larger and more frequent peaks in correspondence of special market events.[5] Indeed, Table 3 collects the ten largest peaks of the DID's residuals and associates each of them with specific events. Notably, in the great majority of the cases, days with larger peaks are associated with relevant economic phases, thus highlighting the importance of using diffusion indexes to detect events impacting on market movements in a significant way.

Results in Table 3 show that the days detected by the C-Tz test are usually associated with a jump in the RUA index. Particularly, we observe that the days September 18, 2007, September 29, 2008 and May 6, 2010 were also present in Table 2, where we focused on days with more than 3 MJs.

[4]For each day t, the volatility is computed using previous year daily DID observations.

[5]A graphical representation of DID's and DII's residuals is given in the supplementary material, Section A.6.

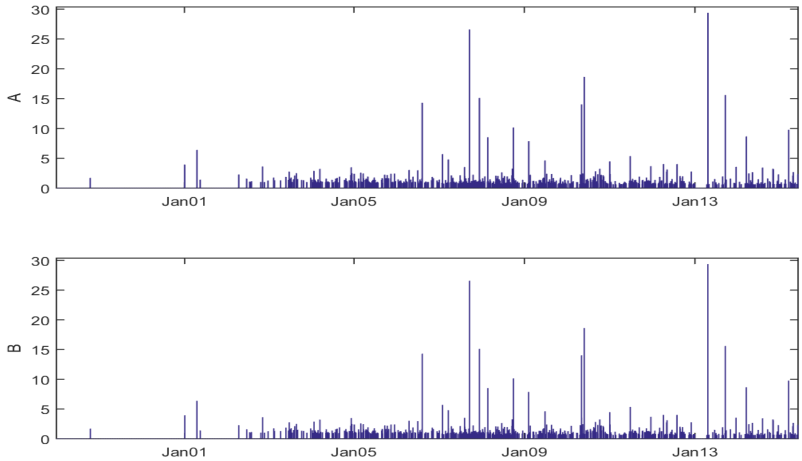

Figure 3 Diffusion indexes. The figure displays the time evolution of the diffusion indexes. Panel A shows, for each of the 4,344 days, the percentage of stocks involved in a multivariate jump, if present, and 0 otherwise (the DID index). Panel B reports, for each 5-minute intraday interval between January 2, 1998 and June 5, 2015, the fraction of stocks involved in a multivariate jump, if present, and 0 otherwise (the DII index).

Besides, the correlations highlight the existence of a positive relationship between RUA jumps and diffusion indexes, which is also stronger than in the jump and co-jump cases (see Appendix C.1).[6]

2.2 Systemic events

In this section we study systemic co-jumps that, following Das and Uppal (2004), we define as infrequent events correlated across a large number of stocks. Among all co-jumps, we first focus on rare and extreme events hitting a large part of the market. We then detect a systemic jump (systemic MJ or SJ) when at least 20 out of the top 100 size stocks jump together, consistent with the empirical evidences in Caporin et al. (2017).

Figure 5 reports the location over time of systemic (and systematic) jumps along with their dimension, that is expressed as the number of assets involved in the systemic co-jump.

Figure 5 supports the evidence that systemic jumps are rare events. In fact, using the C-Tz test, we identify just 6 systemic jump days, but 7

[6]Using 5-minute observations, MJ correlations range from 0.13 for the DID to 0.15 for the DII (both statistically significant at the 1% confidence level), while jump and co-jump correlations are about 0.01.

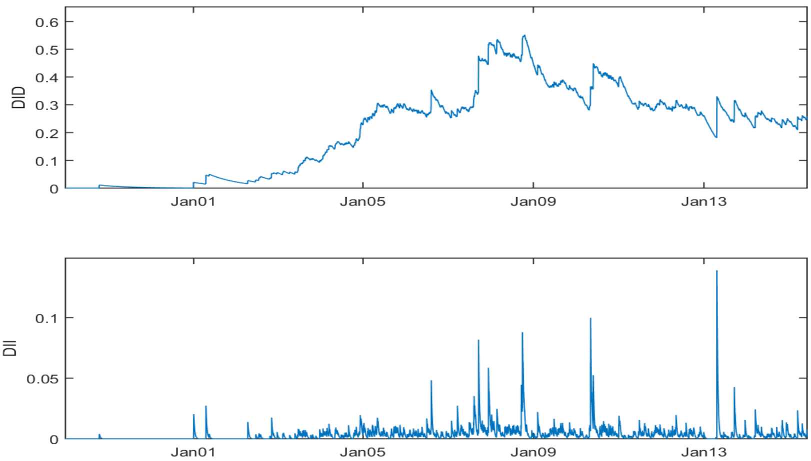

Figure 4 DID and DII trends. The top panel shows, for each of the 4,344 days between January 1998 and June 2015, the DID trend (μ_t). Bottom panel, instead, reports for each 5-minute intraday interval between January 2, 1998 and June 5, 2015, the DII trend (μ_t). μ_t is estimated respectively for each day and intraday interval from (3). Each panel reports the results using 5-minute observation intervals.

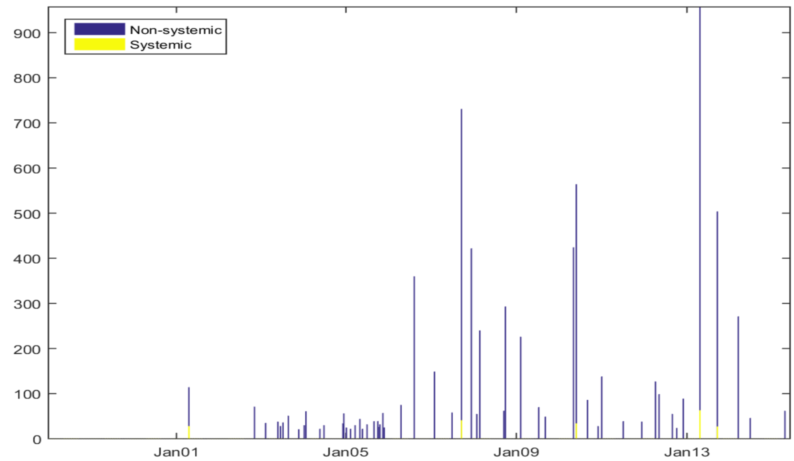

Figure 5 Systemic and non-systemic systematic MJs. The figure presents, for each of the 4,344 days between January 1998 to June 2015, the maximum number of stocks involved in a systematic multivariate jump (MJs among the individual stocks that also involve the market index), if present, and distinguishes between systemic (yellow line) and non-systemic (blue line) MJs. All jumps are detected focusing on 5-minute observation intervals and using the C-Tz test.

Table 3 Residual peaks. The table shows, for the time-window January 1998—June 2015 and using the 5-minute interval, the dates (Days) with the ten larger peaks of DID's residuals, if RUA jumps are present in the same days (RUA days) and the relevant Economic/Financial events during those days.

	Days	RUA days	Economic/Financial event
1	23-Apr-13	yes	AP fake tweet about explosions at the White House
2	18-Sep-07	yes	FOMC lowers target for federal funds rate (50 bps)
3	28-May-10	yes	FED announces three small auctions
4	18-Sep-13	yes	FOMC statement
5	11-Dec-07	yes	FOMC lowers target for federal funds rate (25 bps)
6	08-Aug-06	yes	FOMC keeps its target for the federal funds rate
7	06-May-10	yes	The Flash Crash
8	29-Sep-08	yes	FOMC meeting unscheduled, Emergency Economic Stabilization Act not approved
9	18-Mar-15	no	FOMC statement
10	19-Mar-14	yes	FOMC statement

extreme events as we detect two consecutive systemic jumps on April 23, 2013. Systemic jump days represent about 1% of the total of 643 5-minute MJ days in Table 1. This suggests that many MJs mostly involve small size stocks and, although they sometimes cover a large number of assets, might not be relevant when considering the full market. In contrast, given the size of the involved stocks, systemic jumps represent an important fraction of the market regardless of the number of assets included in the co-jumps.

From Table 4, it is also clear that systemic jumps are economically significant events. Table 4 reports the dates and times of systemic jumps and associates them with macroeconomic/financial information. Table 4 also reports, when present, the times of RUA jumps for the same days. We note that all systemic jumps are associated with important economic events, such as Federal Reserve (or FED) announcements, Federal Open Market Committee (or FOMC) actions and Associated Press (or AP) news. As for the AP releases, the two systemic jumps on April 23, 2013 reflect the market reaction to a false claim of an attack to the White House. In fact, the AP announcement and the consequent retraction caused a fall and a rebound of the markets within a few minutes.

Systemic events during the days September 18, 2007 and May 28, 2010 are in line with the findings of Caporin et al. (2017), who investigated multi-

Table 4 Systemic jump days. The table shows, using the 5-minute interval and data from January 1998 to June 2015, the dates (Days) and times (Time) in which we detect a systemic jump, if RUA jumps are present in the same days (RUA days) and the corresponding times (RUA time) along with the relevant Economic/Financial events during those days (Eco./Fin. event).

Days	Time	RUA days	RUA time	Eco./Fin. event
03-Jan-01	13:15-13:20	no		FED cut fed funds rate
18-Apr-01	10:55-11:00	yes	10:55-11:00	FED cut short-term interest rates
18-Sep-07	14:15-14:20	yes	14:15-14:20	FOMC lowers target for federal funds rate (50 bps)
28-May-10	12:35-12:40	yes	12:35-12:40	FED announces three small auctions
23-Apr-13	13:05-13:10	yes	13:05-13:10	AP fake tweet about explosions at the White House
23-Apr-13	13:10-13:15	yes	13:10-13:15	AP fake tweet about explosions at the White House
18-Sep-13	14:00-14:05	yes	14:00-14:05	FOMC statement

jumps between January 2, 2003 and June 29, 2012. Moreover, we observe in Table 4 that all but one of the systemic jumps are also reflected in a jump of the RUA. We noticed in Table 1 that systematic MJ days represent about 10-12% of all the MJ days (using the intervals of 5 minutes).

Back to Figure 5, it is clear that even if systemic jumps involve large size stocks, they are not among the largest co-jumps among all MJs (in particular if we take a local, in time, perspective). Given their relevance in size and the number of involved assets, we expect systemic jumps to be reflected also in a jump of the market index. Results in Table 4 and Figure 5 confirm our expectations. However, there are also many MJs that, despite not being systemic events, are connected to a jump in the RUA. Note, for instance, that in correspondence to the subprime crisis on September 29, 2008, using the $C\text{-}Tz$ test we detect no systemic jumps but multiple multivariate jumps (Table 2), a large peak in the DID residuals (Table 3) and a jump in the RUA (jump detected in the interval 13:40-13:45, -6.80%). In contrast to our results, Caporin et al. (2017) found no jump in the market index on September 29, 2008. A possible reason of this discrepancy can be found in the different market index in use. In fact, Caporin et al. (2017) used the S&P 500 index, whereas we employ the RUA index.

This example emphasizes the importance of studying diffusion indexes, in addition to systemic events, to detect relevant market movements. Summing up, Table 5 highlights which of the overall 858 detected 5-minute MJs are

Table 5 MJs. The table reports, using the 5-minute interval and data between January 1998 to June 2015, the distribution of detected MJs. Particularly, it distinguishes between systemic (MJs involving at least 20 stocks among the top 100 size stocks) and non systemic jumps, and between systematic (MJs reflected in a market index jump) and non-systematic jumps.

	non-systematic	systematic	Tot
non-SJ	791	60	851
SJ	1	6	7
Tot	792	66	858

systemic, MJs involving at least 20 stocks among the top 100 size stocks, and systematic, MJs reflected in a market index jump. Systemic jumps are rare events, less than 1% of all MJs, usually linked to a jump in the market index, in 86% of the cases. Moreover, just 7.7% of the overall 858 detected MJs are connected with a jump in the RUA index. Furthermore, just 66 out of the overall 229 5-minute jumps detected in the RUA, happen in correspondence of a MJ, while the remaining 163 seem not to reflect a cross-sectional market jump.

3 Market sensitivity and MJs

We now study the impact of multivariate jumps on the stocks sensitivity to the market movements, by extending the standard capital asset pricing model (CAPM); see Sharpe (1964), Lintner (1965), Mossin (1966) and Black (1972). By conditioning on the daily or intraday diffusion indexes, we evaluate if the asset sensitivity to market movements, i.e. the beta, changes in the presence of jumps in the cross-sections of equities.

We remind the reader that the DID is a daily index. Therefore, we evaluate its impact by estimating the CAPM model and its extension at the daily frequency. For simplicity, we we denote the resulting model as MJ-CAPM:

$$R_{t,j} - R_{t,F} = \alpha_j + \beta_j MKT_t + e_{t,j}, \tag{4}$$

$$R_{t,j} - R_{t,F} = \alpha_j + \beta_j MKT_t + \beta_{DID,j}\mathbb{I}\{\mathrm{DID}_t > \tau\} MKT_t + e_{t,j}, \tag{5}$$

where $R_{t,j}$ is the daily return of the j-th asset, computed excluding the first 5 minutes of each trading day, $R_{t,F}$ is the risk-free rate, DID_t is the daily diffusion index, $e_{t,j}$ is a zero-mean residual, $MKT_t = (R_{t,M} - R_{t,F})$ is the market excess return and $\mathbb{I}\{\cdot\}$ is an indicator function, which takes a value of one if the condition in $\{\cdot\}$ is true, and zero otherwise, for $t = 1, \ldots, T$.[7]

We use the daily RUA Index return as market proxy, coherently with our previous analyses. Notably, the RUA index and the K. French's market portfolio exhibit a similar behaviour. Indeed, their correlation equals 0.9967 during the period 01/02/1998—06/05/2015.

We focus here on the coefficient $\beta_{DID,j}$ in Equation (5), which captures the non-linear relation between the excess return of the j-th asset and the market excess return MKT_t. A positive $\beta_{DID,j}$ suggests that, in days when the DID index is above the threshold τ (i.e., with a high intensity in the cross-sectional diffusion of jumps), the market sensitivity increases, thus coherently with an increase of the systematic exposure. In contrast, a negative $\beta_{DID,j}$ might be read in terms of a contraction in the systematic exposure when there is a large cross-sectional jump diffusion. This would allow us to separate stocks on the basis of their relationship with the diffusion indexes. Stocks become riskier in case of a positive $\beta_{DID,j}$, while we have a contraction in the stock risk when observing a negative $\beta_{DID,j}$. Notably, this might provide useful information for portfolio allocation and risk monitoring as the occurrence of a large cross-sectional diffusion of jumps is generally associated with negative market returns.

The estimates of $\beta_{DID,j}$ clearly depend on the threshold τ. In our analyses, we define τ as a static or as a time-varying quantity. In the former case, it equals the 95-th percentile of the full-sample DID, i.e. a quantity that we compute ex-post from the estimated time series of DID. In the latter case, τ depends on time and, at time t, it corresponds to the 75-th percentile of the DID values observed on the time interval $[t - 252 + 1, t]$; that is, the interval spanning the latest 252 trading days (about one year). We then set $\tau = \bar{\tau}$ and $\tau = \tau_t$ to denote a static and a time-varying threshold, respectively.[8]

Table 6 reports the sign and the statistical significance of estimated

[7] We recover the risk-free rate from the Kenneth R. French data library, available at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html. We remind that we compute the daily diffusion index using the C-Tz test.

[8] In case of zero simultaneous jumps during the interval $[t - 252 + 1, t]$, we assume that $\tau_t = \tau_{t-1}$.

Table 6 Coefficients sign and significance and DID R^2_{adj} variation. We compare in this table the models defined in Equations (4) and (5), setting $\tau = \bar{\tau}$. Regressions are subject to the condition that stocks present at least 251 days (about one year) of non-null returns in the window of interest. Column $\hat{\beta}_{DID}$ reports the percentage of times in which $\hat{\beta}_{DID}$ in Equation (5) is statistically significant at the 10% level in the window of interest. Columns three and four present the percentage of negative (< 0) and positive (> 0) significant coefficients (90% confidence interval). From the fifth to the ninth column we focus on the variation in the coefficient of determination (R^2_{adj}) we observe by moving from the CAPM to the MJ-CAPM model over the cross-section. Min and Max are respectively the minimum and the maximum difference values whereas $Q(0.10)$, Median and $Q(0.90)$ are the 10th, 50th, and 90th percentiles of the same differences. We scale the R^2_{adj} values by 100, so that the maximum estimated R^2_{adj} value is 100.

Window	$\hat{\beta}_{DID}$	$\hat{\beta}_{DID}$	$\hat{\beta}_{DID}$	R^2_{adj}(MJ-CAPM) – R^2_{adj}(CAPM)				
		< 0	> 0	Min	$Q(0.10)$	Median	$Q(0.90)$	Max
1998—2015	19%	81%	19%	-0.032	-0.021	0.000	0.124	10.699
2002—2006	50%	21%	79%	-0.080	-0.074	-0.010	0.095	2.645
2007—2011	39%	89%	11%	-0.127	-0.054	0.030	0.307	15.661
2012—2015	21%	68%	32%	-0.362	-0.110	-0.050	0.282	17.333

coefficients along with the variations in the values of the adjusted R-squared, R^2_{adj}, when we set $\tau = \bar{\tau}$, i.e. the static threshold, that in our sample equals 3.12. We estimate the models in Equations (4) and (5) focusing on the entire period 1998—2015 as well as on three subperiods: 2002—2006, 2007—2011 and 2012—2015. Note that we exclude the years until 2001 because up to 2001 our database behaves in a quite different way and presents an extremely low number of MJs (see Appendix C).

The second column of Table 6 reports the percentage (over the cross-section) of $\hat{\beta}_{DID}$ with an absolute t-statistic greater than 1.645 (10% significance level).[9] When considering the entire period (i.e., January 2, 1998—June 5, 2015), we observe that 19% of $\hat{\beta}_{DID}$s are statistically significant. This suggests that the diffusion index could be an important driver for the evaluation of market risk exposure. Table 6 also highlights how the impact of DID on the betas changes over time with sensible differences over the sub-samples. DID slopes are significantly different from zero in a relevant number of cases for all sub-periods, with larger values compared to the full-sample regression. Moreover, it appears that DID slopes are most frequently significant during the pre-2008 economic crisis, namely from 2002 until 2006, and with the smallest frequency from 2012 to 2015. These empirical evidences suggest

[9]Information on the distribution of estimated β_{DID} is available in the supplementary material—Section A.7.

the relevance of the diffusion index in capturing the market sensitivity of quotes equities. This might have implications for asset pricing and is in line with the theoretical approach put forward by Bentzen and Sellin (2003).

Table 6 also reports the variations in the values of R^2_{adj} when moving from CAPM to MJ-CAPM. Although some variations take negative values,[10] the median and the upper quantile suggest that only a fraction of models do show a sizeable increase in the fit to the data for MJ-CAPM. However, we remark that we are focusing on an index monitoring the cross-sectional diffusion of jumps, the latter being concentrated on a limited number of days. Therefore, the limited changes in the R-squared are not much surprising. We observe that the improvements in the R-squares are particularly pronounced during the periods 2007—2011 and 2012—2015.[11]

The evaluation of the $\hat{\beta}_{DID}$ sign is more interesting, in particular when focusing on the sub-sample estimates. While during the pre-crisis period the sensitivity to the DID index, when present, is mostly positive, thus increasing the reaction of the stock to market movements, in the crisis and in the post-crisis windows, the impact becomes predominantly negative. Thus, from 2007 onward, large values of the DID index are associated with a contraction in market risk exposure and, in relative terms, lead to a more relevant role for the idiosyncratic risk component.

We obtain similar results when using a time-varying threshold $\tau = \tau_t$ in Equation (5); see Table 7, with some differences when focusing on the coefficient signs. In fact, there is now a predominance of negative signs, apart in the full sample estimation.[12]

To gain a closer look on the impact of MJs on asset prices, we move our focus to intraday data. Therefore, we study the contribution of DII in explaining the variation of intraday stock returns, using 5-minutes data. Particularly, the MJ-CAPM model for intraday data is defined as follows:

$$R_{t,i,j} - R_{t,i,F} = \alpha_j + \beta_j MKT_{t,i} + \beta_{DII,j}\mathbb{I}\{\mathrm{DII}_{t,i} > \tau_{DII}\}MKT_{t,i} + e_{t,i,j}, \quad (6)$$

where $R_{t,i,j}$ is the return of the j-th asset, on day t and for the intraday in-

[10]We remind that negative R-squared changes depends on the use of the adjusted R-squared.

[11]See the supplementary material—Section A.7—for further details on the R^2_{adj} variation.

[12]τ_t assume values from a minimum of 0 to a maximum of 6.379, and a median of 1.475; the τ_t mean equals 1.8142.

Table 7 Coefficients sign and significance and DID R^2_{adj} variation with time-varying τ. We compare in this table the models defined in Equations (4) and (5), setting $\tau = \tau_t$. Regressions are subject to the condition that stocks present at least 251 days (about one year) of non-null returns in the window of interest. Column $\hat{\beta}_{DID}$ reports the percentage of times in which $\hat{\beta}_{DID}$ in Equation (5) is statistically significant at the 10% level in the window of interest. Columns three and four present the percentage of negative (< 0) and positive (> 0) significant coefficients (90% confidence interval). From the fifth to the ninth column we focus on the variation in the coefficient of determination (R^2_{adj}) we observe by moving from the CAPM to the MJ-CAPM model over the cross-section. Min and Max are respectively the minimum and the maximum difference values whereas $Q(0.10)$, Median and $Q(0.90)$ are the 10th, 50th, and 90th percentiles of the same differences.

Window	$\hat{\beta}_{DID}$	$\hat{\beta}_{DID}$	$\hat{\beta}_{DID}$	R^2_{adj}(2-factor) $-$ R^2_{adj}(CAPM)				
		< 0	> 0	Min	$Q(0.10)$	Median	$Q(0.90)$	Max
1998—2015	23%	19%	81%	-0.031	-0.021	0.010	0.234	7.141
2002—2006	49%	56%	44%	-0.080	-0.068	0.075	0.362	7.065
2007—2011	15%	69%	31%	-0.127	-0.061	-0.016	0.249	8.631
2012—2015	19%	57%	43%	-0.397	-0.111	-0.041	0.281	7.504

terval i, $R_{t,i,F}$ is the risk-free return that we approximate equal to 0, $DII_{t,i}$ is the C-Tz intraday diffusion index, $MKT_{t,i} = (R_{t,i,M} - R_{t,i,F})$ is the excess return of the Russell 3000 market portfolio and $e_{t,i,j}$ is a zero-mean residual.

Again, we use both a static and a time-variant threshold τ_{DII} in Equation (6). To account for the specific distributional characteristics of the intraday index, we use lower percentiles, compared to the daily framework, to define the threshold. In the static setting, $\tau_{DII} = \bar{\tau}_{DII}$ equals the 75th percentile of the DII over the full sample; that is, a value equal to 1.3569. In the time-varying case, instead, at each day t we compute $\tau_{DII} = \tau_{DII,t}$ as the 50th percentile of the latest 252 days (about one year); as we have 77 observations per day, one year corresponds to $19,404$ DII values. The median (mean) value of τ_t equals 1.342 (1.684).[13]

The use of high-frequency data leads to very long time series of equity and stock market returns. Indeed, we have 334,488 5-minutes observations in our dataset, spanning 4,344 trading days. Consequently, it is possible to estimate several regression models over time, using data from a reduced number of days. By doing so, we can assess the trend over time of the statistical significance of the coefficients associated with MJs. Taking advantage

[13] In case of zero simultaneous jumps during the interval $[t - 252 + 1, t]$, we assume that $\tau_t = \tau_{t-1}$.

of these long time series, we estimate the parameters of our model using non-overlapping rolling windows with a size of 22 days, corresponding to 1,694 5-minutes observations; that is, about one month of data. Figure 6 displays, in panel A, the evolution of the percentage of statistically significant $\hat{\beta}_{DII}$ over the resulting 22-days intervals (we do have 198 intervals).

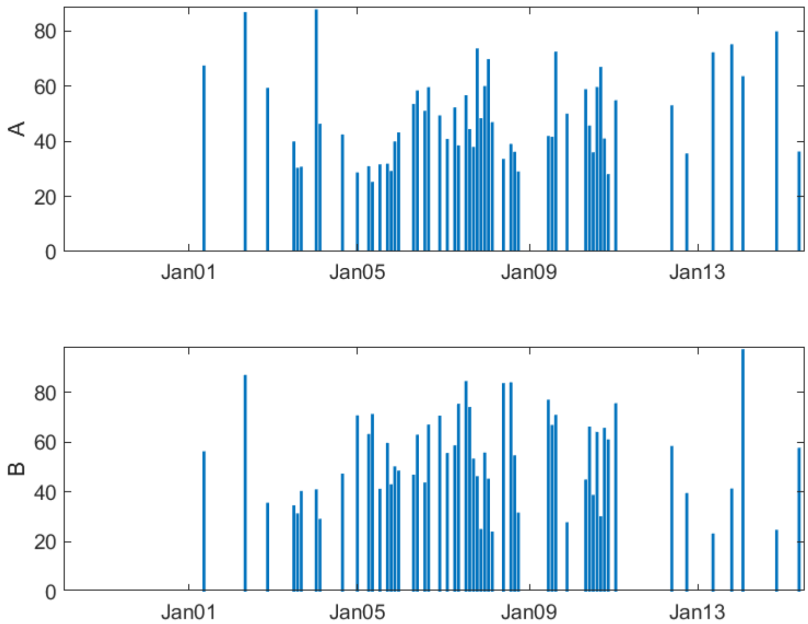

Figure 6 DII coefficients sign and significance. Regressions of 5-minute excess stock returns on both the excess market return (MKT) and the intraday diffusion index (DII) indicator function, using non-overlapping rolling windows with a size of 1,694 observations (Equation (6)). Regressions are subject to the conditions that, in the window of interest, stocks have at least 75% of non-null returns and that the indicator function shows at least 2 non-null values. For each 22-days interval between January 2, 1998 and June 5, 2015, Panel A reports the fraction of $\hat{\beta}_{DII}$ with absolute t-statistic greater than 1.645 (10% significance level). Panel B, instead, shows the percentage of positive estimated coefficients for $\hat{\beta}_{DII}$.

We observe from Figure 6 high percentages of significant betas for a relevant number of intervals from 2004 to 2015. Interestingly, the statistically significant values of $\hat{\beta}_{DII}$ are often positive (see Panel B of Figure 6). The significance of $\hat{\beta}_{DII}$ increases when using a time varying threshold $\tau_{DII,t}$, as Panel A in Figure 7 becomes denser with respect to Figure 6, starting from January 2005. Focusing on the sign of the coefficients, we confirm the findings of the static threshold.

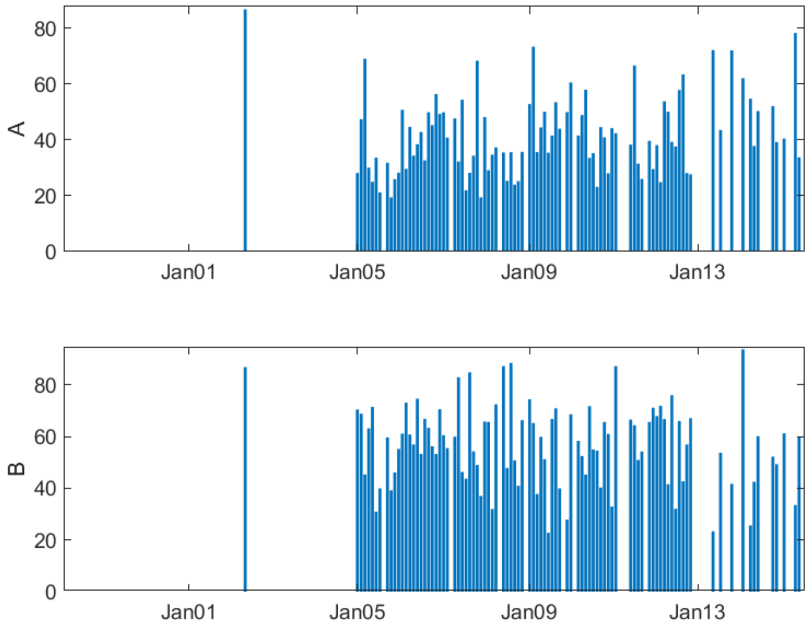

Figure 7 DII coefficients sign and significance with time-varying τ. Regressions of 5-minute excess stock returns on both the excess market return (MKT) and the intraday diffusion index (DII) indicator function, using non-overlapping rolling windows with a size of 1,694 observations (Equation (6)). Regressions are subject to the conditions that, in the window of interest, stocks have at least 75% of non-null returns and that the indicator function shows at least 2 non-null values. For each 22-days interval between January 2, 1998 and June 5, 2015, Panel A reports the fraction of $\hat{\beta}_{DII}$ with absolute t-statistic greater than 1.645 (10% significance level). Panel B, instead, shows the percentage of positive estimated coefficients for $\hat{\beta}_{DII}$.

The results displayed in Figures 6 and 7 support the relevance of multivariate jumps in driving the systematic exposure of quoted equities, capturing information which is missed by the market factor of the standard CAPM and, then, acquiring economic relevance within a high-frequency data framework.

Interestingly, we observe high peaks clustered in 2007, 2008, 2010 and 2013. These are periods of financial turmoil in both the U.S.—we refer to the U.S. subprime crisis during the years 20072009, peaked with the default of Lehman Brothers in September 2008—and the European—we refer to the European sovereign debt crisis, starting with the Greek government-debt crisis, in the fall of 2009—markets.

4 Conclusions

We identify and analyze common jumps which involve a relatively large number of stocks—the multivariate jumps (or MJs)—that we use to build indexes informative of the diffusion of jumps in the cross-section.

We start from the detection of jumps in the returns of the Russell 3000 constituents, employing the C-Tz test proposed by Corsi et al. (2010). Moreover, we also use the BNS test of Barndorff-Nielsen and Shephard (2004b, 2006) as a benchmark for a robustness check. Results are then combined using the Gilder et al. (2014) coexceedance method, which allows us to detect contemporaneous jumps in the cross-section. The co-jumps we identify involve up to 956 assets, but are usually small and show a weak association with market jumps. The distribution of systematic and non-systematic co-jumps also suggest the existence of a positive relationship between jumps in the market index and large co-jumps in the stocks. Indeed, systematic common jumps generally involve more stocks than non-systematic ones.

We then move to common jumps which involve a large number of assets, thus focusing on co-jumps which should have a relevant impact on a huge portfolio. Using a modified version of the Gilder et al. (2014) coexceedance method, we identify a multivariate jump if at least 20 stocks jump together. This information represents the starting point to derive our two diffusion indexes: the daily diffusion index (or DID) and the intraday diffusion index (or DII). Both indexes are informative of the cross-sectional diffusion of jumps, but focus, respectively, on the daily and 5-minute intraday level. Results show that such indexes tend to be subject to more frequent and greater peaks in correspondence of important economic moments, as the years 2008 and 2010, but also highlight the existence of a positive association with the market.

We further investigate MJs representing a relevant fraction of the market—the systemic MJs, that is, MJs involving at least 20 out of the top 100 size stocks. They are significant but rare events. In fact, we identify just 7 systemic MJs using the C-Tz test. Systemic jumps are usually also systematic events. Meanwhile, there exist also multiple systematic but non-systemic events, which suggest how limiting the analysis exclusively on systemic events might be incomplete and misleading.

We also detect a relationship between MJs and market-level news, as well as between systemic co-jumps and important economic and financial news. Notably, they are linked to Federal Reserve (or FED) announcements, Federal Open Market Committee (or FOMC) actions and Associated Press news. The relevance of these relationships is particularly evident in portfolio selection and risk management activities (see, among others, Dungey and Hvozdyk (2012), Bollerslev et al. (2008), Lahaye et al. (2011), Gilder et al. (2014) and Caporin et al. (2017)).

The importance and usefulness of our indexes are also evident when considering their contribution to asset-pricing models. Using a modified version of the CAPM model, which also includes our diffusion indexes, we register a relevant impact of multivariate jumps on the exposure to the market factor. When focusing on the entire 1998—2015 window as well as on various sub-periods, we observe that the impact of DID and DII on the market beta is significantly different from zero in a relevant number of cases. Moreover, the R^2_{adj} values usually increase when one of these two indexes is included in the CAPM.

Our results have important implications not only for asset allocation and hedging, but also in asset pricing. As for the last point, the information derived from mulivariate jumps can be used for identifying a factor capturing the cross-sectional jump risk and could be an interesting topic for future research.

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Appendix

A Data description

We use a dataset including $N = 3,509$ assets belonging to the basket of the Russell 3000 index in the period January 2, 1998—June 5, 2015. The dataset is provided by Kibot and the details are available at http://www.kibot.com. We note that the dataset includes also dead stocks or stocks which are no more included in the Russell 3000 index. Furthermore, we point out that the data quality is comparable to that of TAQ. See the supplementary material for a graphical comparison. The stocks prices are sampled with a frequency of 1 minute from 09:30 am to 04:00 pm, for each of the 4,344 business days.[14] As a result, we record at the t-th day $M = 390$ 1-minute closing prices for each stock, denoted as $p_{t,i}$, for $t = 1, ..., T$ and $i = 1, ..., M$. Following a common practice—see Gilder et al. (2014) and Caporin et al. (2017) among others—we discard the first 5 minutes of each day to avoid potentially erratic price behaviour due to market opening. In general, the shorter the sampling frequency the higher the accuracy of the realized measures of volatility (Barndorff-Nielsen and Shephard (2004b), Andersen et al. (2010) and Corsi et al. (2010)). Nevertheless, this relation could be altered by microstructure noise (Hasbrouck, 2006; Roll, 1984), leading to biased estimates of the integrated volatility. In general, there is not a unique optimal frequency to choose, but, rather, we need to take into account several pros and cons.

Therefore, in addition to 1-minute returns, we also use 5- and 11-minutes returns—two frequencies commonly used in the literature (Corsi et al. (2010) and Gilder et al. (2014)). We obtain the 5- and 11-minutes returns by aggregating the 1-minute returns. However, the potential advantage of having a large dataset of stocks, an important source of information for the jumps analysis, is limited by the market liquidity condition. To cope with that we focus on stocks with a sufficient number of non-null intraday returns to guarantee accurate estimates of the integrated volatility and jumps.

As a result, we implement a set of testing methodologies (see the supplementary material—Section A.3) under the condition that, in a given trading day, the percentage of non-zero intraday returns is greater or equal to the 75%; in contrast, we treat the days in which the percentage of non-null returns is lower than 75% as days where no jumps occur. With some abuse of wording, we define the assets having more than 25% of intradaily returns equal to zero as illiquid. Clearly, the absence of price movements might be due to illiquidity but also to other reasons. Moreover, our definition of illiquidity applies on a daily basis. Consequently, a given stock, say a small

[14] The original number of business days in our sample is equal to 4,384. Nevertheless, we discard 40 days in which we observe particular tight liquidity conditions. See the supplementary material—Section A.2—for details on the excluded days.

cap stock, might be illiquid in most days of our sample, but it will enter our analyses in those days when we do observe a sufficiently large number of price variations.

Panel A of Figure 8 shows, for each day of our dataset, the number of

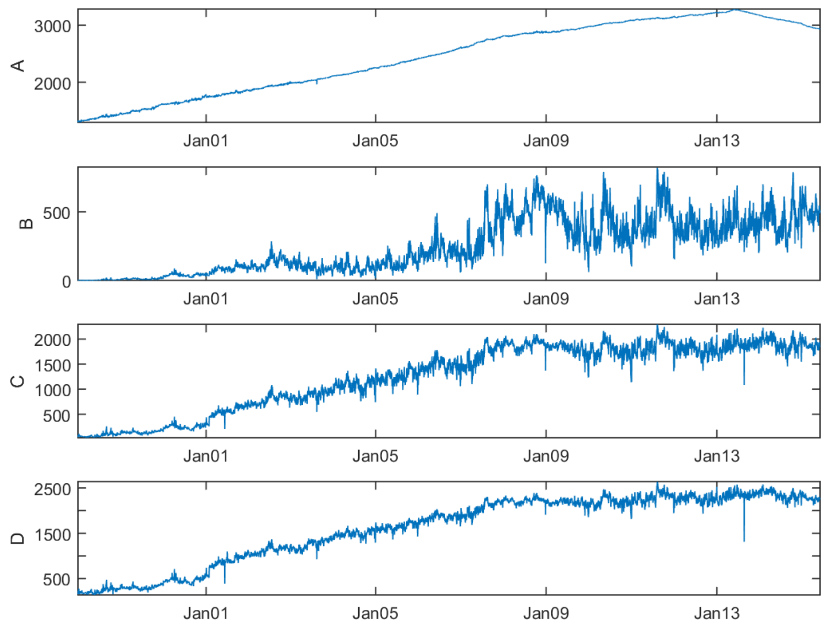

Figure 8 Quoted and liquid assets. Panel A displays, for each day between 01/02/1998 and 06/05/2015, the number of the Russell 3000 quoted assets. The following three panels present, for each day, the number of assets with less than 25% null intraday returns, considering respectively intervals of 1 minute (panel B), 5 minutes (panel C), and 11 minutes (panel D).

the quoted constituents of the Russell 3000 index as well as the number of liquid assets across different sampling frequencies. As expected, the higher the frequency the larger the fraction of illiquid assets. Notably, the patterns of the 5- and 11-minutes frequencies are similar.

To shed further light on illiquid stocks, we study the trend over time of the percentage of stocks that do not match the liquidity condition, clustered according to their capitalization. Thus, we compute the average capitalization of the Russell 3000 index constituents twice a year—at the beginning of January and at the beginning of July, for a total of 35 semesters. For each semester, we allocate the stocks for which we have availability of data into four classes, determined according to the cross-sectional quartiles of the average capitalizations recorded in that semester. Then, we compute the percentage of illiquid stocks belonging to each of the four classes, that

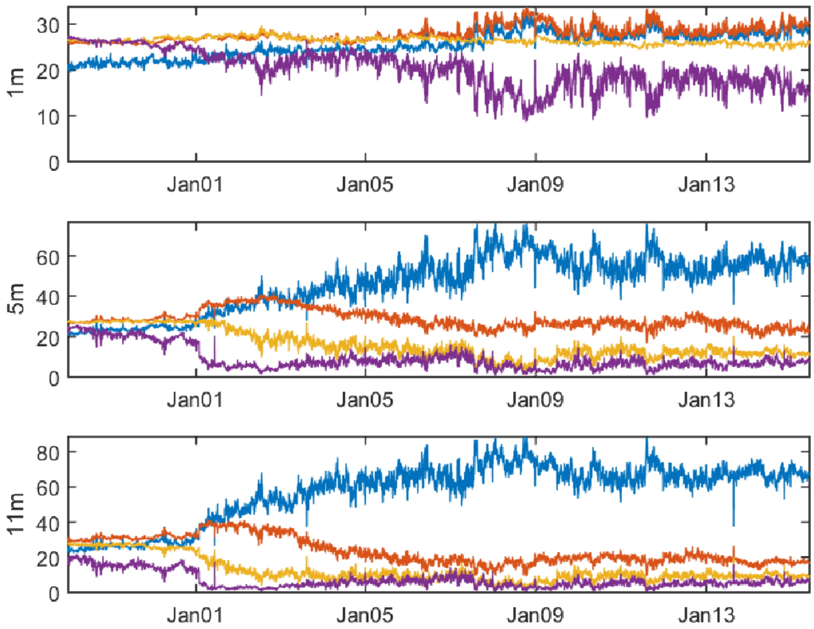

Figure 9 Illiquid assets. The figure shows the behaviour, for each capitalization class, of the quoted assets for which there are more than 25% daily null returns, using either 1- (top panel), 5- (middle panel), or 11-minute (bottom panel) intervals. The four capitalization classes, built at the beginning of January and at the beginning of July of each year using the average capitalization recorded in the previous semester, are the following: Low (blue line), Low-medium (red line), Medium-high (yellow line) and High capitalization (purple line). For each day from 1998 to 2015 the figure reports the percentage of illiquid assets that belongs to each class.

is, low, low-medium, medium-high and high.

As expected, Figure 9 shows that the great majority of assets that do not match the liquidity condition belongs to the low and low-medium capitalization classes. This phenomenon, consistent with the pattern in Figure 8, is more evident with the 5- and the 11-minutes frequencies, where we observe that more than 50% of the illiquid assets are also low capitalized.[15]

B Jump identification

Our study grounds on the occurrence of price jumps and on their location within each trading day in the cross-section of the Russell 3000 constituents. To identify the presence of price jumps, we make use of non-parametric tests

[15]We provide in the supplementary material—Section A.5—further results on the relationship between capitalization and jump detection.

which exploit the discrepancy between realized variance (RV) and bipower variation (BV) in estimating the integrated volatility(IV). Among all the possible tests available in the literature (see Aït-Sahalia and Jacod (2014)) we choose the C-Tz test of Corsi et al. (2010), as it provides a greater power with respect to other tests based on multipower variation. Moreover, for robustness checks, we use the s-BNS test of Barndorff-Nielsen and Shephard (2004b, 2006).[16]

We implement both tests after standardizing the returns following Boudt et al. (2011), which corrects volatility periodicity. The approach of Boudt et al. (2011) enables us to improve the detection of small jumps during low volatility periods and to reduce the spurious detection of jumps at high volatility times.[17]

We implement the jump detection tests at both the daily and the intradaily levels. As a results, we can check whether a jump occurs in a given trading day and gain knowledge about the number of jumps and their location during the day. There exist only few works that propose non-parametric tests to explicitly detect intraday jumps. Among others, we mention the tests of Andersen et al. (2007), Lee and Mykland (2008) and Andersen et al. (2010). In contrast to daily jump tests, which only detect days with at least one jump, we use a broader information resulting from the identification of intraday jumps (see the details in the supplementary material—Section A.3). In particular, it includes the possibility of observing multiple jumps in a given stock for a given day, along with their exact position throughout the day.

Notably, in testing for the presence of jumps, we follow Corsi et al. (2010) and set the significance level α equal to 0.01%. Further details about the tests, the standardization procedure and the procedure for jumps location identification are given in the supplementary material—Section A.3.

Table 8 presents the number of days in which we detect jumps for the Russell 3000 index and its constituents (i.e. jump days), as identified by means of the C-Tz test. Moreover, the Table also reports the percentage of positive jumps and the mean jump size.[18] A comparison with the cor-

[16]As documented, among others, by Dumitru and Urga (2012) and Schwert (2011), different non-parametric jump tests might lead to different timing of jump arrivals. This phenomenon can be due to different capabilities of jump identification as well as to spurious jump detection. Here we use the C-Tz test, that is an upgrade of the s-BNS test and therefore we expect that the differences are mainly driven by the greater capability of the former with respect to the latter. For this reason, in the following, we focus on the results provided by the C-Tz test and, only when relevant, we also report the results obtained with the s-BNS test. All the results obtained with the s-BNS are available in the supplementary material—Section A.9.

[17]Note that, by removing from the analyses the first 5-minutes of each trading day we avoid the false detection of jumps associated with market opening. The approach of Boudt et al. (2011) takes into account the higher volatility present in the first part of each trading day and reduces the detection of false jumps in the first hours of the day.

Table 8 Jumps summary statistics. The table reports summary statistics for the market index—the Russell 3000 (RUA)—and for the RUA constituents in the period January 2, 1998—June 5, 2015. N_{RUA} is the number of days in which we observe at least one intraday jump of RUA; % $J_{RUA} > 0$ is the percentage of positive RUA jumps; Mean_{RUA} is the average RUA jump return; N_j is the number of days in which we observe at least one intraday jump in the RUA constituents; % J$>$ 0 is the percentage of constituents of the Russell 3000 for which we observe a positive jump, and Mean is the average jump return for RUA constituents. We report the results for three observation intervals (1, 5 and 11 minutes).

Frequency	N_{RUA}	% $J_{RUA} > 0$	Mean_{RUA}	N_j	% J$>$ 0	Mean
1 min	1,512.00	51.05	0.09	4,119.00	50.23	-1.24
5 min	176.00	51.09	0.06	4,333.00	51.20	0.69
11 min	57.00	51.43	0.35	4,344.00	52.25	0.78

respondent results for the s-BNS test, see Table S10 in the supplementary material, shows that the C-Tz test detects more jump days for all frequencies.

The average size and the sign of jumps provide information about the distribution of jump size. The similar percentages of positive and negative jumps and the small values of the mean jump size suggest that jumps are symmetrically distributed. This result is consistent with the findings of Lee and Mykland (2008) and Gilder et al. (2014). As for the mean jumps size, we note that most values are slightly positive, with the only exception of the average jump size for the 1-minute frequency in the cross-section of stocks. By comparing the index and the stocks results, we observe much lower values for the index, a somewhat expected finding.[19] However, the small jump occurrence observed for the index at the 11-minutes frequency represents a possible risk, as we might be unable to identify all jumps affecting the market when focusing on the index. This is consistent with Caporin et al. (2017), who show how the detection of co-jumps in the index and in the cross-section might lead to different findings.

We report in Figure 10 the time evolution of the number of assets that jump in a given day, using three different observation intervals—1, 5 and 11 minutes. For each of the 4,344 days, the figure reports the number of assets for which we register at least one intraday jump using the C-Tz test.

Notably, by using 5- and 11-minutes frequencies, we note spikes in correspondence of days in which a large number of assets jump. In contrast, this phenomenon is less clear using 1-minute intervals. A possible cause is the impact of microstructure noise on the jump detection procedure. This

[18] See the supplementary material—Section A.4—for the approach we use to compute the jump sign and the mean jump size.

[19] In fact, we might easily observe false positives in a dataset including about 3,500 stocks when using a 0.01% confidence level.

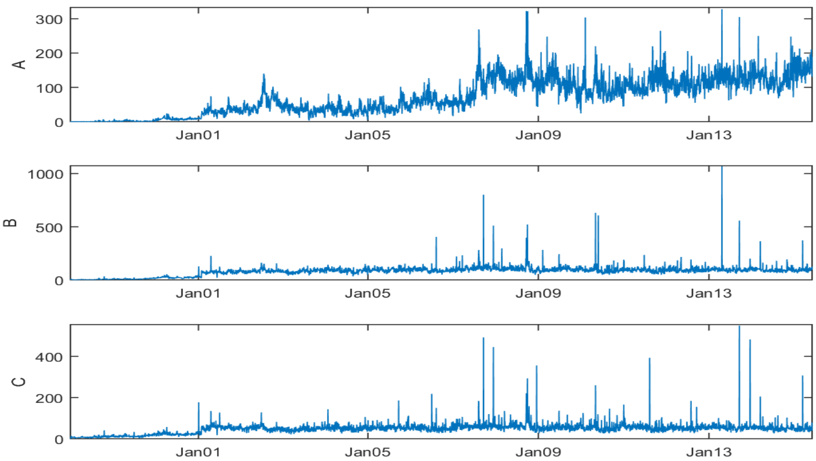

Figure 10 Assets that jump per day. The figure shows, for each day between January 2, 1998 and June 5, 2015 and for three frequencies (1 minute—panel A, 5 minutes—panel B, and 11 minutes—panel C), the number of assets belonging to the Russell 3000 for which we detect at least one intraday jump using the C-Tz test.

is in line with the findings of Christensen et al. (2014), pointing out that the use of 5-minute frequency is necessary to mitigate microstructure effects, but also highlighting how most jumps identified with a 5-minute frequency vanish when employing tick-by-tick data.

Consequently, for the following analyses, we chose to work with the 5-minutes frequency.[20]

C Common jumps detection

By using intradaily returns with a frequency of 5-minutes, we point at analyzing the cross-sectional diffusion of jumps.

Our study relates to Bollerslev et al. (2008), that develop an intraday co-jump for a large panel of N securities (BLT test), and to Gilder et al. (2014) that propose a co-exceedance rule that detects intraday co-jumps from the intersection of univariate jump tests.

In our work we follow the latter approach. The coexceedance rule of Gilder et al. (2014) requires in a first step to detect intraday jumps using a non-parametric univariate test—here we use the C-Tz and the s-BNS tests. In a second step, it is possible to verify if two or more assets record a jump in the same interval using the following variable:

[20]Results for other frequencies are available upon request.

$$C_{t,i} = \sum_{j=1}^{N} \mathbb{I}\{\text{Jump}_{t,i,j} > 0\} \begin{cases} \geq 2 & \text{Co-jump} \\ \leq 1 & \text{Single jump} \end{cases} \tag{7}$$

where $\mathbb{I}$ is an indicator function taking the value of 1 when a jump is detected for asset j ($j = 1, \ldots, N$, $N = 3509$) at the intraday interval i ($i = 1, \ldots, 77$, using 5 minutes intervals) on day t ($t = 1, \ldots, 4344$) and the value of 0 otherwise.

Table 9 reports the results obtained using the coexceedance rule with

Table 9 Jumps and co-jumps summary statistics. The table reports the main statistics for jumps and co-jumps using the 5-minutes returns of the Russell 3000 index constituents recorded in the period: January 1998—June 2015. N_j is the number of days in which we observe at least one intraday jump, N_{sj} is the number of days with at least one intraday single jumps (if in the same interval there are no other assets that jump), N_{cj} is the number of days with at least one intraday co-jump, and Max_{cj} and Mean_{cj} are, respectively, the maximum and average number of assets involved in a co-jump. Results are presented separately for the two jump tests: $s\text{-}BNS$ and $C\text{-}Tz$. The table also splits the number of days in which we register at least a jump (N_j), a single jump (N_{sj}) and a co-jump (N_{cj}) between those detected using only the $s\text{-}BNS$ test (only $s\text{-}BNS$), using only the $C\text{-}Tz$ test (only $C\text{-}Tz$) or using both methods ($s\text{-}BNS \cap C\text{-}Tz$).

Jump test	N_j	N_{sj}	N_{cj}	Max_{cj}	Mean_{cj}
$s\text{-}BNS$	4,280.00	4,279.00	3,882.00	311.00	3.11
$C\text{-}Tz$	4,333.00	4,333.00	4,109.00	956.00	4.38
only $s\text{-}BNS$	0.00	0.00	0.00		
only $C\text{-}Tz$	53.00	54.00	227.00		
$s\text{-}BNS \cap C\text{-}Tz$	4,280.00	4,279.00	3,882.00		

either the $s\text{-}BNS$ or the $C\text{-}Tz$ test. It shows the number of jump and co-jump days, along with the maximum and the mean numbers of stocks involved in the co-jump rule in (7). It also presents the number of days in which we register at least one event using only one between the $s\text{-}BNS$ and the $C\text{-}Tz$ test, and the number of event days detected using both methods (i.e. their intersection).

Table 9 shows that jump and co-jump days are usually detected using both tests. However, the $C\text{-}Tz$ test identifies jumps and co-jump days that are not detected using the $s\text{-}BNS$ test. In terms of assets that are jumping at a given day, the $C\text{-}Tz$ test seems to identify more diffuse jump occurrences as detected by the average number of assets jumping and the maximum number of assets jumping. The latter is particularly different between the two tests, 956 for $C\text{-}Tz$ compared to 311 by $s\text{-}BNS$. However, we note that, despite the presence of co-jumps that involve a large number of assets, the low mean values suggest that they usually have low occurrence. This is coherent with the existence of a limited number of systemic events. From a different viewpoint, the average number of jumping assets might suggest the occurrence of spurious detection of co-jumps. In fact, Gilder et al. (2014) show that the coexceedance criterion produces small spurious co-jumps, with a median number of stock involved equal to 2. In our case, we do have slightly higher average number of jumping assets but this does not allow us to exclude the occurrence of spurious co-jump detection.

For each day between January 2, 1998 and June 5, 2015, Figure 11 reports the percentage of daily intervals (77 daily intervals of 5 minutes) with

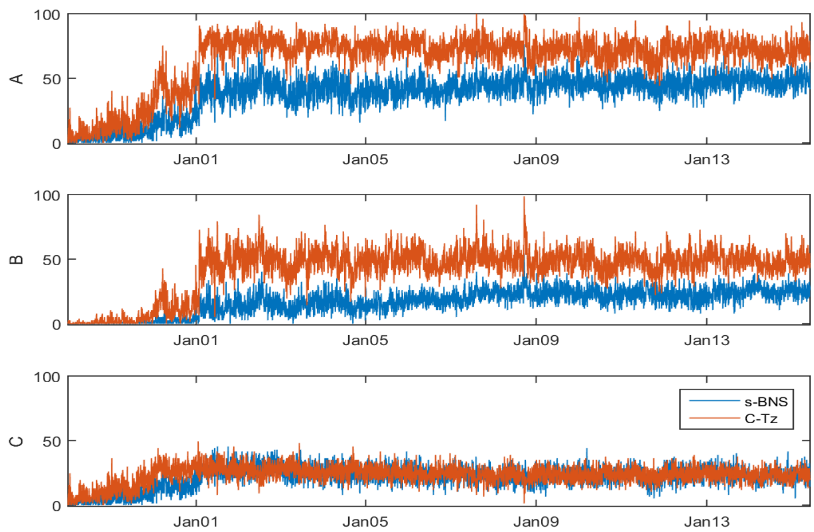

Figure 11 Intervals with jumps per day. The figure shows, for each of the 4,344 days between January 2, 1998 and June 5, 2015, the percentages of daily intervals (77 daily intervals of 5 minutes) with at least one jump (panel A), a co-jump (panel B) and a single jump (panel C) using the $s\text{-}BNS$ and the $C\text{-}Tz$ tests.

at least a jump, a co-jump and a single jump. Figure 11 reinforces the idea of the greater capability of the $C\text{-}Tz$ test to detect common jumps. Figure 11 also highlights structural changes from 2001. Figure 12 investigates the behaviour of jumps and co-jumps during the day. For each intraday interval of 5 minutes, it shows the number of days with at least a jump (panel A), a co-jump (panel B) and a single jump (panel C). The $C\text{-}Tz$ test detects more jumps and co-jump days in all intraday intervals, but the same does not hold for single jumps. A possible explanation is that the greater efficiency of the $C\text{-}Tz$ test corrects for the fact that many $s\text{-}BNS$ single jumps are instead co-jumps.

It is also interesting to notice that around lunch time we observe a great increase of co-jumps and a correspondent decrease of single jumps, a phenomenon that is particularly evident using the $C\text{-}Tz$ test. There are various studies—see among others Jain and Joh (1988) and Admati and Pfleiderer

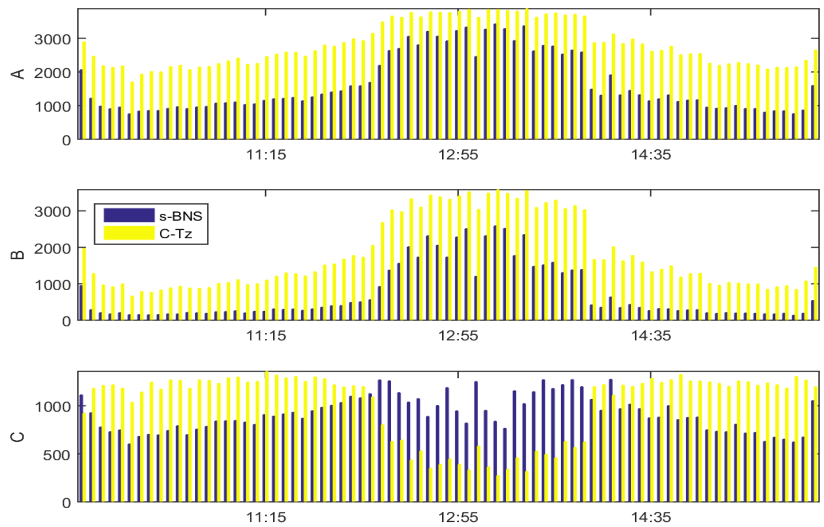

Figure 12 Days with jumps per intraday interval. The figure shows, for each of the 77 5-minutes daily intervals, the number of days for which we observe at least a jump (panel A), a co-jump (panel B) and a single jump (panel C) using the *s-BNS* and the *C-Tz* tests in the period: January 1998—June 2015.

(1988)—which report the existence of a U-shaped pattern for the average volume of traded stocks and, in particular, a relative light trading in the middle of the day. The co-jump intraday pattern, with larger detection in the middle of the day, is coherent with such an evidence and in line with Boudt et al. (2011) and Farmer et al. (2004).

Lastly, it is worth underlying how capitalization has a limited impact on the number of co-jumps. Using 5-minutes observations, it is possible to observe that co-jumps tend to be less present in stocks with low capitalization with respect to stocks with higher capitalization (low-medium, medium, and medium-high). Moreover, a part from the low capitalized stocks, it does not seem that capitalization plays a role in determining the number of common jumps. Indeed, co-jumps are almost uniformly distributed among low-medium, medium, and medium-high capitalization stocks.[21]

C.1 Co-jumps and RUA jumps

Jumps of individual stocks might have effect the at the entire market level. For instance, market-level news causing co-jumps of individual stocks might be also reflected also in jumps of market portfolios. Further, co-jumps of

[21] Further details about the relationship between co-jumps and capitalization are given in the supplementary material—Section A.5.

stocks involving the market index can be seen as non-diversifiable events, with important implications for portfolio selection and hedging. For this reason, in the remainder of the paper, we define as systematic the co-jumps which occur simultaneously with a jump in the market index.

Consequently, we will label as non systematic the co-jump events detected from single assets co-jumps but not reflected in a jump at the market index level. In short, we label as RUA jumps the jumps occurring on the Russell 3000 index (RUA), our proxy for the market index.

Table 10 shows jump days for the market index, jumps and co-jump days

Table 10 Asset jumps and market jumps. The table reports the number of days in which we observe at least one RUA jump (N_{RUA}), the amount of days in which we observe at least one intraday jump (N_j) or one co-jump (N_{cj}) in the constituents of the Russell 3000 and the days with both a jump in the index and a jump ($N_{RUA} \cap N_j$) or a co-jump ($N_{RUA} \cap N_{cj}$) in the underlying assets. These statistics are computed for the period: January 1998—June 2015. We consider three observation intervals—1, 5 and 11 minutes.

Frequency	N_{RUA}	N_j	$N_{RUA} \cap N_j$	N_{cj}	$N_{RUA} \cap N_{cj}$
1 min	1,512.00	4,119.00	1,418.00	3,858.00	1,345.00
5 min	176.00	4,333.00	175.00	4,109.00	172.00
11 min	57.00	4,344.00	57.00	4,269.00	57.00

in the underlying assets and the number of jump and co-jump days that are also RUA jump days. For completeness, we report results for all observation intervals, that is, 1, 5 and 11 minutes.

We already observed in Appendix B that the $C\text{-}Tz$ test detects more jump days than the $s\text{-}BNS$ test, that there is a negative relation between index jump days and sampling frequency and a positive relation between observation interval and Russell 3000 jump days. In contrast from Table 10 we learn that also co-jump days are positively related to the interval length. The number of days with at least an intraday co-jump is always lower than the correspondent number of days with at least an intraday jump, nevertheless it still takes relevant values: from a minimum of 3,858 days (89% of the sample days) to a maximum of 4,269 days (98% of the sample days). Thus, we observe a co-jump for almost each day of the sample. We stress that the definition of co-jumps is not particularly restrictive and, thus, we might have co-jumps involving only a small number of assets.

Moreover, columns 4 and 6 of Table 10 present the intersections between jump days in the market index and jumps, or co-jumps, days detected in the underlying assets. This highlights the capability of RUA in reflecting cross-sectional jump events. Starting from the evidence that the majority of jumps and co-jumps do not occur simultaneously with jumps in the index, it is possible to deduce that the jumps in the index are not really informative of the presence of jumps and co-jumps in the cross-section. Since we are

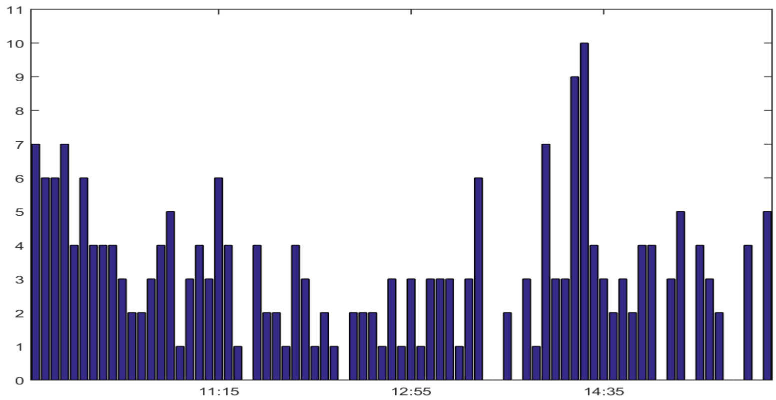

Figure 13 Days with jumps per intraday interval, RUA. The figure shows, for each of the 77 5-minutes daily intervals, the number of days with at least one RUA intraday jump using the C-Tz test: January 1998 to June 2015.

aggregating results to a daily frequency, outcomes derived from on intraday intervals would show even less intersections.

Figure 13 investigates the location of RUA jumps during the day. In contrast to Figure 12, where we observe a pattern in the single assets jumps and co-jumps during the day, RUA jumps do not present a clear behaviour. Nevertheless, at 2 p.m. we can observe a greater density. This reinforces the idea that index jumps are not informative of market co-jumps. In fact, the market lunch effect in not reflected. Such a finding is coherent with the observations in Caporin et al. (2017).

To deeper understand the relation between jumps in the index and jumps in the underlying assets, Table 11 reports the linear correlation (ρ and its p-value) between RUA jumps and Russell 3000 constituents jumps, co-jumps and single jumps.
In addition to the frequency of 5 minutes, Table 11 also presents results using 1 and 11 minutes time intervals, along with aggregations to a daily level of intraday observations. In this last case we observe a jump (co-jump or single jump) in day t, if in at least one intraday interval we detect a jump (co-jump or single jump).

Despite the intraday correlations take low values, their P-values are smaller than 0.1 (10% significance level), highlighting their significance. Moreover, intraday correlations are positive for jumps and co-jumps but negative for single jumps, thus indicating the existence of a positive relation between jumps in the index and jumps and co-jumps in the constituents of the RUA index. However, in line with the findings of Bollerslev et al. (2008), these associations are weak, ranging respectively from 0.01 to 0.03 for jumps, from 0.01 to 0.04 for co-jumps and are equal to -0.01 for single

Table 11 Correlations assets jumps and index jumps. The table reports the correlations (ρ), along with the correspondent p-values (P-val(ρ)), between jumps (j), co-jumps (cj) and single jumps (sj) in the assets and jumps in the market index using data from January 1998 to June 2015. Results are presented separately for six frequency intervals: 1, 5, 11 minutes, daily 1, daily 5 and daily 11 minutes. In case of daily 1 minute, a jump/co-jump is detected in day t if there is at least one intraday 1 minute interval with a jump/co-jump. Daily 5 and daily 11 minutes work exactly as the daily 1 minute with the difference that they are recovered from 5 and 11 minutes intervals.

Jump test	j	cj	sj	j	cj	sj
		ρ			P-val(ρ)	
1 min	0.03	0.04	-0.01	0.00	0.00	0.00
5 min	0.01	0.01	-0.01	0.00	0.00	0.00
11 min	0.01	0.01	-0.01	0.00	0.00	0.03
Daily 1 min	-0.03	0.00	-0.03	0.02	0.83	0.02
Daily 5 min	-0.01	0.03	-0.01	0.40	0.06	0.40
Daily 11 min		0.02	0.00		0.31	0.87

jumps.

Furthermore, when aggregating results to a daily level, correlations are rarely statistically significant (at conventional levels) and, when significant, are small and negative. Consequently, by using daily information we are unable to identify a clear relation between individual stocks and the market index in terms of jumps. We remind the reader that when aggregating we lose the information on jump location within the day. Therefore, the previous results highlight the importance of using intraday detection and intersection methods.

C.2 Dimension of systematic and non-systematic co-jumps

Following Gilder et al. (2014), we call systematic the co-jumps of individual stocks that also involve the market index. In contrast, we define non-systematic the co-jumps which are not linked with jumps in the market index. Table 12 and Figure 14 report the number and proportions of systematic (cj $\cap$ j_{RUA}) and non-systematic (cj $\not\cap$ j_{RUA}) co-jumps involving different numbers of stocks.

Table 12 shows that the majority of co-jumps are not associated with a market jump. However, the mean and median number of stocks involved in systematic co-jumps are significantly greater than those involved in non-systematic co-jumps. Moreover, Figure 14 highlights how systematic co-jumps involve more stocks than non-systematic co-jumps. In particular panel B of Figure 14 confirms that non-systematic jumps are simply co-jumps among a small number of stocks that, due to the aggregation effect, do not show up and do not give rise to a jump in the index. All the previous observations allow us to support the hypothesis of a positive association

Table 12 Co-jumps distribution. The table reports, using data from January 1998 to June 2015, the numbers of detected co-jumps involving different number of stocks, being either systematic (panel A: cj $\cap$ j_{RUA}) or non-systematic (panel B: cj $\not\cap$ j_{RUA}). The last three rows of each panel list the maximum, the average and the median number of stocks detected to participate in the correspondent systematic or non-systematic co-jumps.

No. Stocks	cj $\cap$ j_{RUA}	cj $\not\cap$ j_{RUA}
2	14.00	46,921.00
3	6.00	28,352.00
4	4.00	18,589.00
5	4.00	12,704.00
6-10	24.00	26,015.00
11-15	21.00	4,087.00
16-20	10.00	840.00
>20	66.00	698.00
Max	956.00	290.00
Mean	65.30	4.32
Median	16.00	3.00

between jumps in the index and co-jumps in the underlying stocks.

Even if the number of stocks involved in systematic co-jumps is usually moderate relative to the dimension of our sample, we point out that 65.10% of systematic co-jumps involve more than 10 stocks. Comparatively, the correspondent proportion for non-systematic co-jumps is 4.07%. Consequently, similarly to Gilder et al. (2014), we have evidence for a positive relation between jumps in the market index and large co-jumps in the stocks. In addition, Gilder et al. (2014) and Bollerslev et al. (2008) propose an explanation for the presence of non-systematic co-jumps involving a large number of stocks. They suggest that these non-systematic co-jumps are misclassified due to a lack of power of jump tests for detecting systematic jumps from the market index. A further explanation comes from the effects of the aggregation of single stocks into the equity market index and the different stock market values and liquidity.

D Robustness checks

D.1 Change in liquidity threshold

In Appendix A we said that we focus on sufficiently liquid stocks to obtain accurate estimates. Then, we used liquid assets, that is the assets that, in a given trading day, present 75%, or more, non-zero intraday returns according to our liquidity rule. In this section we report the outcomes from a tighten liquidity rule as robustness check: we consider as liquid the assets with at least 90% non-null intraday returns, in a given day.[22]

[22]Additional details on the results obtained using the 90% liquidity threshold are given in the supplementary material—Section A.8.

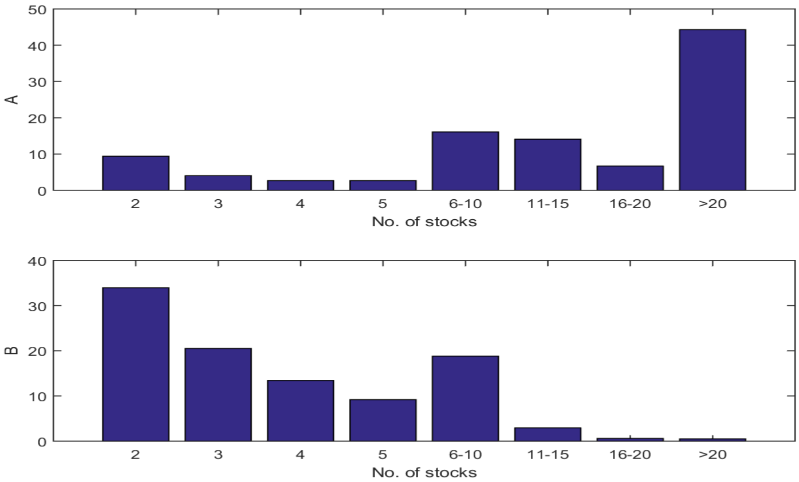

Figure 14 Co-jump distribution. The figure shows co-jump distributions focusing on jumps that involve the market index—or systematic co-jumps (panel A)—and stock jumps dissociated from RUA jumps—or non-systematic co-jumps (panel B)—for the period: January 1998—June 2015. Each panel presents the proportion of co-jumps that involve different numbers of stocks.

A first consequence of the narrower liquidity rule is that we identify more illiquid assets. In fact, on average we find 209 (1 minute), 753 (5 minutes), and 661 (11 minutes) additional liquid assets using the 90% liquidity rule. We also detect fewer jump, co-jump and, especially, MJ days.

Despite the smaller number of events we reach similar conclusions with respect to the 75% liquidity case for jumps and co-jumps. In fact, we observe that the detected jumps are symmetrically distributed and that the C-Tz test identifies more diffuse jumps (in terms of assets involved) than the s-BNS test. Co-jumps involving a large number of assets exist but are infrequent. Furthermore, we detect at least a co-jump in a large part of the days (35%—1 minute, 73%—5 minutes, and 85%—11 minutes). Lastly, index jumps are scarcely informative of the presence of jumps and co-jumps in the cross-section (systematic events are less then 4%) and there exists a significant, positive but weak association between jumps in the index and jumps and co-jumps in the constituents of the Russell 3000 (correlations span from 0.02 to 0.04).

As for the MJs, we observe a large drop in the event days—which decrease runs from 76% (11 minutes) to 99% (1 minute)—and in the number of multiple MJ days—which move from 141 to 9. Results for the 90% liquidity rule are in line with the results for the 75% liquidity case. We find that systematic MJ days are rare events, while RUA jump days are often not linked with a multivariate jump in the underlying assets. Moreover, we observe that days with more than 3 MJs are not only often systematic jump days, but also associated with important economic and financial events. Similarly to the main case (75% liquidity rule), we also observe that correlations between RUA jumps and diffusion indexes suggest the existence of moderate large relation between jumps in the index and diffusion indexes in the constituents of the Russell 3000 (0.24 for the DID and 0.23 for the DII).

Finally, with respect to systemic jumps, the 90% liquidity rule leads to the non-identification of 3 systemic jump days (01/03/2001, 09/18/2001, and 09/18/2013). In fact, we identify just 3 systemic jump days that represent about the 7% of the overall 41 MJ days. They are not only systematic jump days, but also related with economically significant events.

D.2 *s-BNS* test results

Using the *s-BNS* test in place of the *C-Tz* test allows us to detect a lower number of jumps and common jumps that, however, lead to similar conclusions. Indeed, we identify 4,057 (1 minute), 4,280 (5 minutes) and 4,313 (11 minutes) jump days, and 3,739 (1 minute), 3,882 (5 minutes) and 3,818 (11 minutes) co-jump days in the constituents on the Russell 3000 index. They represent, respectively, about 98% and 89%—97% of the correspondent *C-Tz* j and cj days. The impact on RUA jump days is instead considerable, inducing a decrease of more than 40%. They drop to 847, 97 and 19 respectively for 1, 5 and 11 minutes frequencies, respectively.

Using the information retrived from all detected jumps and co-jumps, we highlight further results. *s-BNS* test results confirm that the jumps are symmetrically distributed (percentages of positive jumps and the mean jump values are respectively close to 50% and 0). Nevertheless, as discussed in Appendix C, *s-BNS* co-jumps are less diffusive jumps in terms of maximum and average number of asset involved.

Focusing on the relationships between events in the index and events in the underlying assets, we confirm the reduced presence of systematic jump and co-jump days (event days in the constituents of the Russell 3000 that are contemporaneously RUA jump days). Indeed, in the *C-Tz* case systematic event days represent 1%—35% of all the event days. In contrast, in the *s-BNS* case they decrease to 0.4%—19%. Using the information obtained from linear correlations between RUA jumps and events (jumps and co-jumps) in the underlying assets, it is possible to confirm the limited ability of the jumps in the index to be informative of the events in cross-section. The positive and significant relations between jumps in the index and jumps and co-jumps in the constituents of the Russell 3000 are, however, weak, as suggested by the small correlation values.

Moreover, by comparing the proportions of systematic and non-systematic co-jumps in respect of the number of assets involved, similarly to Appendix C.2, we find that systematic events are less frequent but involve more stocks. 34.00% of systematic co-jumps include more than 10 stocks, compared to

0.72% for the non-systematic common jumps. Making use of the s-BNS test, we not only detect less jump and co-jump days but also fewer multivariate jump days. With respect to the C-Tz case, we identify about half MJ days, do not observe a clear 'lunch effect' for MJs and identify only one multiple MJ day: 05/03/2012. Note that with respect to the last point the differences are particular striking considering that employing the C-Tz test we observe a total of 141 multiple MJ days.

As a consequence of the reduced number of MJs, we obtain diffusion indexes that are often equal to 0. Results from filtering the daily and intraday diffusion indexes allow us to better understand the MJs behaviour. In line with C-Tz results, DID trend clearly varies over time and increases in periods of market turmoil. Furthermore, both DID and DII residuals shows greater spikes in special market moments. Focusing on the days corresponding to the largest spikes of the irregular component, it is possible to underline some similarities and some differences with C-Tz residual results. We observe a link between largest spike days and relevant economic events in both models, while only s-BNS spikes are usually not systematic.

Diffusion indexes are also weakly related with RUA jumps, with significant (1% confidence level) but low linear correlations. DID and DII correlations are slightly lower than in the correspondent C-Tz values—0.09 and 0.10 respectively (0.13 and 0.15 in the C-Tz case).

Focusing on the overall 85 detected 5-minute MJs, 12% of them are systematic events while only one of them is a systemic co-jump. The only systemic MJ day is 05/28/2010, which is also a systematic day, and a day linked with a relevant economic event: the announcement of three small actions by the FED. s-BNS systemic results confirm the findings in Section 2.2, where we observed six event days, in which 05/28/2010 is included, and seven systemic jumps. Indeed, systemic jumps are associated with important economic events. Moreover, the low incidence of systemic events with respect to all MJs confirms that multivariate jumps are often irrelevant when considering the full market since they mainly involve small size stocks.

Supplementary material to

The cross-sectional diffusion of jumps and the identification of system(at)ic movements

A.1 A comparison of Kibot.com and TAQ data

Our data source is not commonly used in financial analyses. To validate the data we perform a comparison with a more diffused data provider, TAQ. Figure S1 contrasts the evolution of the Kibot.com 1-minute price (continuous line) with the trade prices included in TAQ in a specific minute (dots) for the Ford equity in the year 2014. Notably, the evolution of the two series closely follow, without any deviation.

A.2 Excluded business days

From the 4,384 business days included in our dataset and which span the period January 2, 1998 - June 5, 2015, we exclude 40 days in correspondence of which we report a peculiar low number of assets meeting our liquidity condition. We report below the list of exclueded days:

27-Nov-98	11-Sep-02	24-Nov-06	25-Nov-11
24-Dec-98	29-Nov-02	03-Jul-07	03-Jul-12
26-Nov-99	24-Dec-02	23-Nov-07	23-Nov-12
31-Dec-99	03-Jul-03	24-Dec-07	24-Dec-12
03-Jul-00	28-Nov-03	03-Jul-08	03-Jul-13
24-Nov-00	24-Dec-03	28-Nov-08	29-Nov-13
03-Jul-01	26-Dec-03	24-Dec-08	24-Dec-13
23-Nov-01	26-Nov-04	27-Nov-09	03-Jul-14
24-Dec-01	25-Nov-05	24-Dec-09	28-Nov-14
05-Jul-02	03-Jul-06	26-Nov-10	24-Dec-14

In these days we observe particular market conditions, as for the Fridays after Thanksgiving and the days prior July 4 when markets cloesed early.

A.3 Realized measures of volatility and tests for jumps

Let p_t be the logarithmic price of a financial asset at time t. We can assume that p_t follows the Brownian semi-martingale process:

$$dp_t = \mu_t dt + \sigma_t dW_t, \tag{S1}$$

with μ_t being the drift, locally bounded and predictable, whereas σ_t is a strictly positive process and càdlag, independent of the standard Brownian Motion W_t.

Let $r_{t,i}$ be the i-th intraday return of the t-th trading day, for $t = 1, ..., T$ and $i = 1, ..., M$ ($M = 385$, $M = 77$, and $M = 35$ using, respectively,

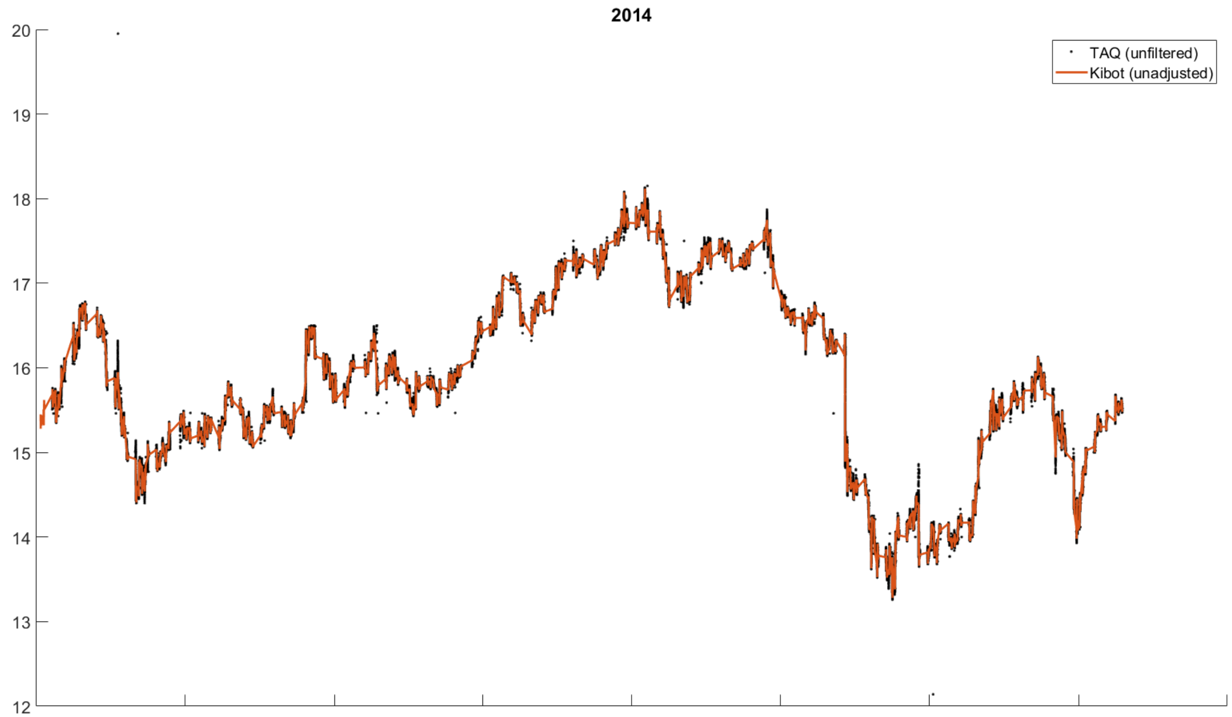

Figure S1 A comparison of Kibot.com and TAQ data for the Ford equity during the year 2014.

intervals of 1, 5, and 11 minutes). Under the process in (S1), the integrated volatility (IV), defined as

$$IV_t = \int_{t-1}^{t} \sigma_u^2 du, \tag{S2}$$

can be estimated through the realized variance (RV):

$$RV_t = \sum_{i=1}^{M} r_{t,i}^2. \tag{S3}$$

Notably, RV_t is a consistent estimator of IV as $M \to \infty$ (Barndorff-Nielsen and Shephard, 2002). Nevertheless, several studies in the literature, see, e.g., Barndorff-Nielsen and Shephard (2004b), highlight the presence of discontinuous jumps, in addition to the continuous diffusion component in Equation (S1). Then, the process in (S1) should be rewritten as follows:

$$dp_t = \mu_t dt + \sigma_t dW_t + k_t dN_t, \tag{S4}$$

with N_t being a finite activity non-explosive Poisson counting process with intensity λ_t, whereas k_t are the random jump sizes.

RV is no longer a consistent estimator of IV in the presence of jumps,

$$\operatorname*{plim}_{M\to\infty} RV_t = \int\limits_{t-1}^{t} \sigma_u^2 \mathrm{d}u + \sum_{(t-1)\leq s\leq t} (\Delta p_s)^2 = IV_t + JP_t, \tag{S5}$$

where JP_t is the sum of the instantaneous changes in the log price due to a jump at time s.

In contrast, Barndorff-Nielsen and Shephard (2004b, 2006) proposed the bipower variation (BV), that is robust to jumps. BV is defined as:

$$BV_t = \mu_1^{-2}\Big(\frac{M}{M-1}\Big)\sum_{i=2}^{M} |r_{t,i}||r_{t,i-1}|, \tag{S6}$$

where $\mu_1^{-2} = (\mathbb{E}[|u|])^{-2} = \pi/2$.[23]

In estimating IV, we expect large differences between RV and BV in the presence of jumps. Building on such discrepancies, many non-parametric jump tests have been developed. Among them we focus on the C-Tz test introduced by Corsi et al. (2010), since it has more power with respect to the tests based on multipower variation. As benchmark, we also use the BNS test (Barndorff-Nielsen and Shephard, 2004b, 2006), that represents a reference in the literature.

[23] $\mu_p = \mathbb{E}[|u|^p]$ and $u \sim \mathcal{N}(0,1)$.

As recommended in Huang and Tauchen (2005), we use the ratio form of the BNS test proposed by Barndorff-Nielsen and Shephard (2006), that is based on the difference between RV and BV:

$$Z_{BNS,t} = \frac{\frac{RV_t - BV_t}{RV_t}}{\sqrt{\left(\left(\frac{\pi}{2}\right)^2 + \pi - 5\right)\frac{1}{M}\max\left(1, \frac{TPV_t}{BV_t^2}\right)}} > \Phi^{-1}_{1-\alpha}, \tag{S7}$$

where

$$TPV_t = M\mu_{4/3}^{-3}\left(\frac{M}{M-2}\right)\sum_{i=3}^{M}|r_{t,i}|^{\frac{4}{3}}|r_{t,i-1}|^{\frac{4}{3}}|r_{t,i-2}|^{\frac{4}{3}},$$

and $\Phi^{-1}_{1-\alpha}$ is the inverse of the standard cumulative distribution function.

The C-Tz test (Corsi et al., 2010) is a modified version of the BNS test, combining BV (Barndorff-Nielsen and Shephard, 2004b, 2006) and the threshold realized variance (TRV) discussed in Mancini (2009).[24] Notably, the small sample bias of the former, affecting in particular big jumps, is counterbalanced by the low effectiveness of TRV, with small jumps but also by its much more effectiveness with large ones. The test statistic proposed by Corsi et al. (2010) reads as:

$$Z_{C-Tz,t} = \frac{\frac{RV_t - C\text{-}TBV_t}{RV_t}}{\sqrt{\frac{1}{M}(\frac{\pi^2}{4} + \pi - 5)\max\left\{1, \frac{C\text{-}TTriPV_t}{C\text{-}TBV_t^2}\right\}}} > \Phi^{-1}_{1-\alpha}. \tag{S8}$$

The C-Tz test substitutes the estimators based on the multipower variation in Equation (S7), that is BV_t and TPV_t, with estimators based on the threshold multipower variation (TMV): C-TBV_t and C-$TTriPV_t$. This corrects for the small sample bias in BV using indicator functions, which guarantee that returns with a jump larger than the threshold vanish asymptotically. In particular, C-TBV_t is a modified version of TBV_t that accounts for returns variations larger than the threshold in absence of jumps:

$$TBV_t = \mu_1^{-2}\sum_{i=1}^{M}|r_{t,i}||r_{t,i-1}|\mathbb{I}_{\{|r_{t,i}|^2 \le \vartheta_i\}}\mathbb{I}_{\{|r_{t,i-1}|^2 \le \vartheta_{i-1}\}}, \tag{S9}$$

where ϑ is the threshold function, $\vartheta_t = c_\vartheta^2 \cdot \hat{V}_t$, with $\hat{V}$ being an auxiliary estimator of σ_t^2 and c_ϑ is a constant.

In contrast, C-$TTriPV_t$ is a special case of threshold multipower variation:

$$C\text{-}TTriPV_t = \mu_{\frac{4}{3}}^{-3} C\text{-}TMV_t^{[\frac{4}{3},\frac{4}{3},\frac{4}{3}]}.$$

[24] $TRV_t = \sum_{i=2}^{M}|r_{t,i}|^2\mathbb{I}_{\{|r_{t,i}|^2 \le \Theta_\delta\}}$, where Θ_δ is a threshold function.

For further details about the C-Tz test and the threshold function specification we refer the reader to Corsi et al. (2010). We apply both the BNS (Barndorff-Nielsen and Shephard, 2006) and the C-Tz (Corsi et al., 2010) tests on standardized returns, to correct their volatility periodicity (among others Wood et al. (1985); Harris (1986); Boudt et al. (2011)). Indeed, Boudt et al. (2011) show that not accounting for the U-shaped intraday volatility pattern of returns leads to non-parametric tests that overdetect (underdetect) jumps at periodically high (low) volatility intraday times. Assuming that the periodicity factor depends on the time of the day, the first step of the procedure proposed by Boudt et al. (2011) consists in computing $\bar{r}_{t,i}$:

$$\bar{r}_{t,i} = \frac{r_{t,i}}{\sqrt{\Delta \cdot BV_t}}, \tag{S10}$$

where $\Delta = 1/M$.

Then, it is possible to compute the shortest half scale estimator ($ShortH_i$) of Rousseeuw and Leroy (1988):

$$ShortH_i = 0.741 \cdot \min\{\bar{r}_{(h_i),i} - \bar{r}_{(1),i}, \ldots, \bar{r}_{(T_i),i} - \bar{r}_{(T_i-h_i+1),i}\}, \tag{S11}$$

where $\bar{r}_{(j),i}$ are the order statistics of $\bar{r}_{j,i}$ and $h_i = \lfloor T_i/2 \rfloor + 1$. For the latter $\lfloor T_i/2 \rfloor$ rounds $T_i/2$ to the lowest integer and T_i represents the total number of observations in intraday interval i. As third step, we define $\hat{s}^2_{ShortH,i}$:

$$\hat{s}^2_{ShortH,i} = \frac{M \cdot ShortH_i^2}{\sum_{i=1}^{M} ShortH_i^2}. \tag{S12}$$

The Weighted Standard Deviation (WSD) estimator is then computed as:

$$WSD_i^2 = 1.081 \frac{\sum_{t=1}^{T} w_{t,i}\bar{r}_{t,i}^2}{\sum_{t=1}^{T} w_{t,i}}, \tag{S13}$$

where $w_{t,i} = w\left(\frac{\bar{r}_{t,i}}{\hat{s}_{ShortH,i}}\right)$ and $w(z) = 1$ if $z^2 \leq 6.635$, 0 otherwise.

Finally, we compute the Boudt et al. (2011)'s standard deviation robust estimator:

$$\hat{s}^2_{WSD,i} = \frac{M \cdot WSD_i^2}{\sum_{i=1}^{M} WSD_i^2}. \tag{S14}$$

Now it is possible to compute Boudt et al. (2011) standardized returns:

$$\ddot{r}_{t,i} = \frac{r_{t,i}}{\sqrt{\hat{s}^2_{WSD,i} \cdot \Delta \cdot BV_t}}. \tag{S15}$$

The Boudt et al. (2011)'s correction enables us to improve the detection of small jumps during low volatility times and to reduce the spurious detection of jumps at high volatility times. After standardizing the returns, we apply the C-Tz test at both the daily and the intradaily levels. In particular, for a given stock, we compute $Z_{C-Tz,t}$, defined in Equation (S8), using all the M intradaily returns, for $t = 1, ..., T$. Then, the t-th trading day has a daily jump if the null hypothesis of the C-Tz test is rejected at the significance level α. For all the days with daily jumps, we also test for intradaily jumps, by using the sequential procedure suggested by Gilder et al. (2014). This sequential procedure requires to firstly detect the jump days using the $Z_{C-Tz,t}$ test, as recommended by Huang and Tauchen (2005); notably, the t-th trading day is classified as jump day if the null hypothesis of the C-Tz test, applied on all the M intraday standardized returns, is rejected at the significance level α. Then, for each jump days, we select the maximum intraday standardized return to be the first intraday jump. The underlined assumption is that the jump size dominates the diffusion component. To detect all the further intraday jumps we repeat the procedure, after setting the previously identified intradaily jump-return equal to zero, until the null hypothesis of no jumps is not rejected at the significance level α. Likewise, we repeat the same procedure by using the BNS test, that we denote as sequential BNS (or s-BNS). We point out that, in testing for the presence of jumps, we use the $\alpha = 0.01\%$ significance level, following Corsi et al. (2010), and set $\vartheta = 3$ for the C-Tz threshold function. We stress that we employ the BNS test just as a benchmark and that our main goal is not showing its inferiority with respect to the C-Tz test. About this point, in fact, Corsi et al. (2010) already compared the two test using simulated data and highlighted several advantages in using the C-Tz test. In particular, they observed that even if the two tests are equally sized under the null, the C-Tz test has more power in the presence of jumps and, especially, in the case of consecutive jumps.

A.4 Jump sign and size

The iterative jump detection methodology described in Section A.3, makes it possible not only to identify the location of the intraday jumps, but also to gain some relevant information about their values. In particular, focusing on the mean jump value and on the proportions of positive and negative jumps we recover important indicators for the symmetry of the jumps distribution. We compute the first moment, $Mean$, by summing the standardized returns, $\ddot{r}_{t,i,j}$, that we identify as jump events, $\text{Jump}_{t,i,j} > 0$:

$$Mean = \frac{1}{N_j} \sum_{j=1}^{N} \sum_{i=1}^{M} \sum_{t=1}^{T} \{\ddot{r}_{t,i,j} | \text{Jump}_{t,i,j} > 0\} \tag{S16}$$

where N_j is the total number of detected jumps.

$$N_j = \sum_{j=1}^{N} \sum_{i=1}^{M} \sum_{t=1}^{T} \mathbb{I}\{\text{Jump}_{t,i,j} > 0\}, \tag{S17}$$

whereas the indicator function $\mathbb{I}$ takes a value of 1 when a jump is detected in asset j ($j = 1, \ldots, N$, $N = 3509$) at the intraday interval i ($i = 1, \ldots, M$, $M = 77$ using 5 minutes intervals) on day t ($t = 1, \ldots, T$, $T = 4344$) and a value of 0 otherwise.

We instead compute the proportion of positive jumps as follows:

$$\%J > 0 = \frac{\sum_{j=1}^{N} \sum_{i=1}^{M} \sum_{t=1}^{T} \mathbb{I}\{\ddot{r}_{t,i,j} > 0, \text{Jump}_{t,i,j} > 0\}}{N_j} \tag{S18}$$

where the indicator function $\mathbb{I}$ takes a value of 1 when a jump is detected in asset j at the intraday interval i on day t and the correspondent standardized return is greater than zero and a value of 0 otherwise.

A.5 Co-jumps and capitalization

In Appendix A we discussed the importance of stock liquidity to guarantee accurate estimates for the univariate jump tests and found that illiquid assets are usually small caps. In this section we further investigate the relation between capitalization and detected jumps.

At the beginning of January and July of each year from 1998 to 2015, we sort all stocks according to their average capitalization in the previous semester; note, we exclude the stocks that are not traded in a given semester. We then determine the quartiles breakpoints (25%, 50%, and 75%) and allocate the assets into four classes according to their average capitalization: low, low-medium, medium-high, and high capitalization. Table S1 shows the number of jump and co-jump days we detect in each capitalization class and for different observation frequencies.

We observe that jump days are positively related to capitalization. As a result, we expect to observe a larger number of jumps as the stock capitalization increases. In contrast, the relation between the number of events and the capitalization depends on the observation interval when focusing on co-jumps. We register a strong positive relation for 1-minute returns, whereas we observe a positive but weaker relation for higher frequencies. In particular we observe an almost uniform distribution among the top three capitalization groups. Consequently, we expect to observe few co-jumps among low capitalization stocks and a similar number of co-jumps among the other groups. Then, co-jumps mainly come from jumps in low-medium, medium and medium-high capitalization stocks.

Table S1 Jumps and co-jumps by capitalization. The table reports the number of jump and co-jump days for four capitalization groups: L - low, L/M - low-medium, M/H - medium-high and H - High capitalization. We focus on the period January 1998 - June 2015. N_j is the number of days for which we observe at least one intraday jump, whereas N_{cj} is the number of days with at least one intraday co-jump.

Frequency	L	L/M	M/H	H
			N_j	
1 min	673.00	1,911.00	3,484.00	4,101.00
5 min	3,528.00	3,926.00	4,205.00	4,320.00
11 min	3,702.00	4,152.00	4,316.00	4,320.00
			N_{cj}	
1 min	6.00	776.00	2,800.00	3,849.00
5 min	2,442.00	3,515.00	3,861.00	3,796.00
11 min	2,950.00	3,708.00	3,801.00	3,229.00

A.6 Diffusion index residuals

Figure S2 shows the time series of ϵ_t that is, the residual defined in (3), for the DID (top panel) and the DII (bottom panel).

A.7 DID and asset prices

We investigate the ability of the DID to capture common variations in stock returns by including it to the CAPM. The resulting 2-factor model, that we report here for simplicity, is defined as:

$$R_{t,j} - R_{t,F} = \alpha_j + \beta_j MKT_t + \beta_{DID,j} DID_t + e_{t,j},$$

where $R_{t,j}$ is the daily return of a security j, $R_{t,F}$ is the risk-free return, DID_t is the daily diffusion index computed using the C-Tz test and $e_{t,j}$ is a zero-mean residual. Lastly, $MKT_t = (R_{t,M} - R_{t,F})$ is the excess return of a capitalization-weighted stock market portfolio.

We report in Figure S3 the boxplots of the estimated β_{DID} (left panel) and of the variation in the coefficient of determination we obtain employing the 2-factor model with respect to the CAPM (right panel).

The left panel of Figure S3 reports the distributions of significant (90% confidence interval) $\hat{\beta}_{DID}$ for different time windows. Since the DID values are sometimes larger than stock returns, we expect correspondent $\hat{\beta}$ to be small. Consistent with our expectations, Figure S3 shows that the median, the upper and the lower quartiles (75^{th} and 25^{th} percentiles), the maximum and the minimum values of $\hat{\beta}$, excluding outliers, are close to 0. However, there are also many outliers, which are more(less) than 3/2 times the upper(lower) quartile values, in the sub-periods 2002-2006 and 2012-2015, for which we register the more extreme values, but almost none in the 2007-2011

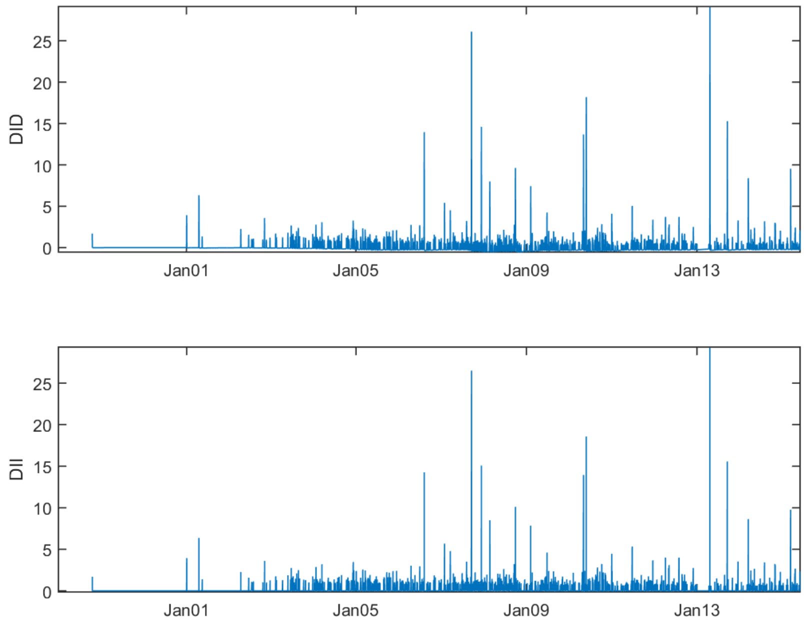

Figure S2 DID and DII residuals. For each of the 4,344 days between January 1998 to June 2015 the top panel shows DID's residuals (ϵ_t). Bottom panel, instead, reports for each 5-minutes intraday interval between January 2, 1998 and June 5, 2015, the DII residuals (ϵ_t). (ϵ_t) is estimated respectively for each day and intraday interval from (3). Each panel reports the results using 5-minutes observation intervals.

sub-period and for the full sample.

Right panel of Figure S3, shows the variations in the values of R^2_{adj} we observe by including the DID in the CAPM model. R^2_{adj} increases for 30% (1998-2015), 47% (2002-2006), and 26% (2007-2011 and 2012-2015) of the stocks. Increases are more pronounced in the first and third sub-periods.

A.8 90% liquidity threshold

All the results presented in the paper rely on the 75% liquidity condition, that is, the condition that each stock, in a given trading day, has less than 25% intradaily null returns. Then, stocks which do not satisfy this condition are defined as illiquid and are excluded from the sample.

In this section we present the results obtained with a narrower liquidity rule, where the threshold of 75% increases to 90%. Figure S4 shows for each day of our dataset, the number of quoted constituents of the Russell 3000 index (panel A) and the number of liquid assets using intervals of 1 minute (panel B), 5 minutes (panel C) and 11 minutes (panel D). Panels B, C, and D report the results both applying the 75% liquidity rule (green line) and

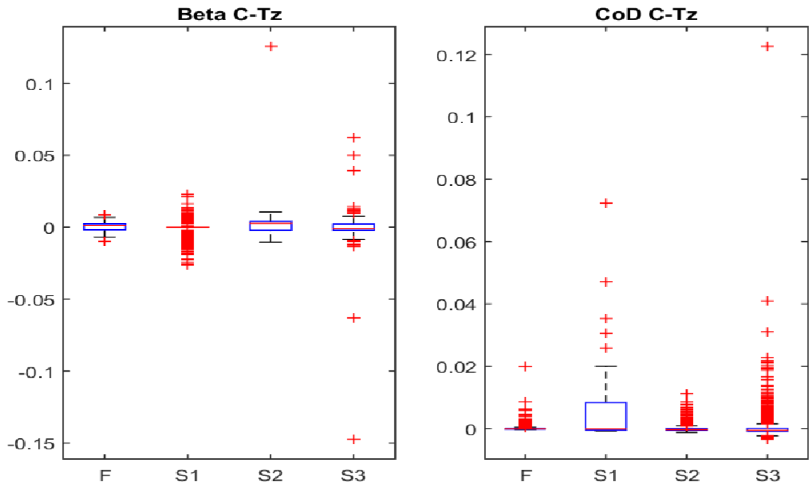

Figure S3 Significant $\hat{\beta}_{DID}$ distribution and DID R^2_{adj} variation. Left panel reports the significant (10% confidence level) estimated βs we obtain from the regressions of daily excess stock returns on the excess market return and the daily diffusion index. We focus on the period: January 1998 - June 2015. Right panel shows the variation in the coefficient of determination (or CoD) we observe using the 2-factor model with respect to the CAPM (R^2_{adj}(2-factor) $-$ R^2_{adj}(CAPM)). Coefficients of determination result from regressions of daily excess stock returns on the excess market return and the daily diffusion index (or 2-factor model), or exclusively on the excess market return (or CAPM). Regressions are subject to the condition that, in the window of interest, stocks presents at least 251 days (about a year) of non-null returns. Each panel shows the distribution of either $\hat{\beta}$s or R^2_{adj} differences resulting from regressions that use the full 1998-2015 sample (F), or 2002-2006 (S1), 2007-2011 (S2), and 2012-2015 (S3) data.

the 90% liquidity rule (blue line).

As expected, using a tighten liquidity rule we discard more assets because of their incapability to meet the liquidity condition. With the 75% rule there are, on average, 209 (1 minute), 753 (5 minutes) and 661 (11 minutes) more liquid assets than with the 90% requirement.

A.8.1 Jump and co-jump identification

Using the 90% liquidity rule leads to a smaller dataset and therefore we also expect to detect fewer jump events. Tables S2 - S3 report jump and co-jump results we obtain using the 90% liquidity rule. Table S2 shows the number of jump days, identified using the C-Tz test, along with the percentage of positive jumps and the mean jump size. Similarly to Appendix B, results suggest that detected jumps are symmetrically distributed around zero. In fact, the percentages of positive and negative jumps are similar and the average jump sizes are close to zero.

In contrast, Table S3 reports the number of jump, single jump and co-

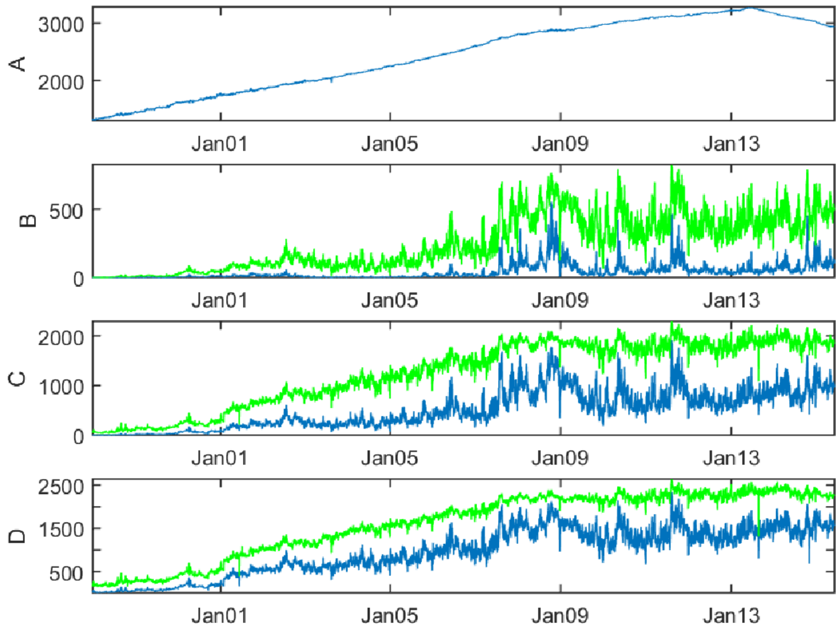

Figure S4 Quoted and liquid assets, 90% liquidity rule. The figure shows in the top panel, for each day between 01/02/1998 and 06/05/2015, the number of the Russell 3000 quoted assets. The following three panels present, for each day, the number of liquid assets considering respectively intervals of 1 (panel B), 5 (panel C) and 11 minutes (panel D). Each of them reports the results applying two different liquidity rules: assets with less than 25% null intraday returns - green line, assets with less than 10% null intraday returns - blue line.

jump days. It also shows the maximum and average numbers of stocks involved in a co-jump and the amount of days in which we detect at least one event (j, cj, or sj) using either the s-BNS or the C-Tz test, or both methods. Again the results are similar to those obtained from the 75% liquidity rule. In fact, many jumps and co-jumps are detected using both methods, but the C-Tz test detects events that are not captured by the s-BNS test. Moreover, the C-Tz test identifies more diffuse jumps in terms of assets involved. As for this last point, increasing the liquidity threshold leads to a relevant decrease in the maximum and average number of assets involved in co-jumps.[25] Despite the relevant maximum values, the low mean values suggest that, even if co-jumps involving a large number of assets exist, they are infrequent. Tables S4 - S5 investigates intersections and correlations between jump days of the Russell 3000 Index and jumps and co-jump days

[25]Maximum and average number of assets involved in co-jumps move from 956 to 391 and from 4.38 to 2.85 in the C-Tz case, and from 311 to 146 and from 3.11 to 2.53 in the s-BNS case, when increasing the liquidity threshold.

Table S2 Jumps summary statistics, 90% liquidity rule. The table reports summary statistics for the market index — the Russell 3000 (RUA) — and for the RUA constituents in the period January 2, 1998 - June 5, 2015. N_{RUA} is the number of days in which we observe at least one intraday jump of RUA; % $J_{RUA} > 0$ is the percentage of positive RUA jumps; $Mean_{RUA}$ is the average RUA jump return; N_j is the number of days in which we observe at least one intraday jump in the RUA constituents; % J> 0 is the percentage of constituents of the Russell 3000 for which we observe a positive jump, and Mean is the average jump return for RUA constituents. We report the results for three observation intervals (1 minute, 5 minutes, and 11 minutes).

Frequency	N_{RUA}	% $J_{RUA} > 0$	$Mean_{RUA}$	N_j	% J> 0	Mean
1 min	1,248.00	50.76	0.06	2,838.00	50.08	1.05
5 min	138	51.43	0.06	3,963.00	51.74	-1.44
11 min	49	51.61	0.24	4,206.00	53.12	1.42

Table S3 Jumps and co-jumps summary statistics, 90% liquidity rule. The table reports the main statistics for jumps and co-jumps using the 5-minutes returns of the Russell 3000 index constituents recorded in the period: January 1998 - June 2015. N_j is the number of days in which we observe at least one intraday jump, N_{sj} is the number of days with at least one intraday single jumps (if in the same interval there are no other assets that jump), N_{cj} is the number of days with at least one intraday co-jump, and Max_{cj} and $Mean_{cj}$ are, respectively, the maximum and average number of assets involved in a co-jump. Results are presented separately for the two jump tests: *s-BNS* and *C-Tz*. The table also splits the number of days in which we register at least a jump (N_j), a single jump (N_{sj}) and a co-jump (N_{cj}) between those detected using only the *s-BNS* test (only *s-BNS*), using only the *C-Tz* test (only *C-Tz*) or using both methods (*s-BNS* ∩ *C-Tz*)..

Jump test	N_j	N_{sj}	N_{cj}	Max_{cj}	$Mean_{cj}$
s-BNS	3,634.00	3,609.00	1,491.00	146	2.53
C-Tz	3,963.00	3,962.00	3,190.00	391	2.85
only *s-BNS*	0	0	0		
only *C-Tz*	329	353	1,699.00		
s-BNS ∩ *C-Tz*	3,634.00	3,609.00	1,491.00		

of the underlying assets. Table S4 shows the number of RUA's jump days, the amounts of jump and co-jump days of the underlying assets and the number of days in which we detect both a jump in the RUA and a jump, or a co-jump, in the constituents of the Russell 3000.

Even if co-jump days significantly decrease when moving from the 75% to the 90% liquidity rule (from 13% for 11 minutes data to 60% for 1 minute data), we still observe at least a co-jump in a large part of the days: 35%, 73% and 85% of the sample days respectively for 1, 5 and 11 minute observation intervals. Again, we observe similar results also with respect to the amount of event days, jump and co-jump days, that are also RUA jump days. In particular they are less than 4% for 5 and 11 minute observation intervals, thus supporting the idea that index jumps are scarcely informative

Table S4 Asset jumps and market jumps, 90% liquidity rule. The table reports the number of days in which we observe at least one RUA jump (N_{RUA}), the amount of days in which we observe at least one intraday jump (N_j) or one co-jump (N_{cj}) in the constituents of the Russell 3000 and the days with both a jump in the index and a jump ($N_{RUA} \cap N_j$) or a co-jump ($N_{RUA} \cap N_{cj}$) in the underlying assets. These statistics are computed for the period: January 1998 - June 2015. We consider three observation intervals — 1, 5 and 11 minutes.

Frequency	N_{RUA}	N_j	$N_{RUA} \cap N_j$	N_{cj}	$N_{RUA} \cap N_{cj}$
1 min	1,248.00	2,838.00	845.00	1,528.00	441.00
5 min	138	3,963.00	130	3,190.00	101
11 min	49	4,206.00	46	3,712.00	43

of the presence of jumps and co-jumps in the cross-section. The existence of a weak association between jumps in the index and jumps and co-jumps in the constituents of the Russell 3000 is further supported by their linear correlation values reported in Table S5.

Table S5 Correlations assets jumps and index jumps, 90% liquidity rule. The table reports the correlations (ρ), along with the correspondent p-values (P-val(ρ)), between jumps (j), co-jumps (cj) and single jumps (sj) in the assets and jumps in the market index using data from January 1998 to June 2015. Results are presented separately for six frequency intervals: 1, 5, 11 minutes, daily 1, daily 5 and daily 11 minutes. In case of daily 1 minute, a jump/co-jump is detected in day t if there is at least one intraday 1 minute interval with a jump/co-jump. Daily 5 and daily 11 minutes work exactly as the daily 1 minute with the difference that they are recovered from 5 and 11 minutes intervals.

Jump test	j	cj	sj	j	cj	sj
		ρ			P-val(ρ)	
1 min	0.02	0.02	0.01	0.00	0.00	0.00
5 min	0.02	0.04	0.00	0.00	0.00	0.98
11 min	0.02	0.04	-0.01	0.00	0.00	0.00

In line with the 75% liquidity rule, in the 90% case we observe, for all frequencies, positive and significant (99% confidence interval) jump and co-jump correlations, thus supporting the existence of a relation, but weak (from 0.02 to 0.04), between the events in the underlying assets and the jumps in the index.

A.8.2 Multiple co-jumps and diffusion indexes

Using a tighten liquidity rule leads to a decrease in the number of jump and co-jump days, but also to a drop in the number of days in which we detect multivariate jumps. Table S6 shows that the decrease of MJ days ranges from 76% (11 minutes) to 99% (1 minute). Furthermore, Table S6 also shows that systematic MJ days are rare and that RUA jump days are, in

Table S6 Asset jumps and market jumps, 90% liquidity rule. The table reports, for the time-window between January 1998 and June 2015, RUA jump days (N_{RUA}), the amount of days in which we observe at least one multivariate jump (N_{mj}) in the constituents of the Russell 3000, and the days with both a jump in the index and a multivariate jump ($N_{RUA} \cap N_{mj}$) in the underlying assets. Results are presented separately for three observation intervals: 1, 5 and 11 minutes, and for the two jump tests (s-BNS, and C-Tz).

Frequency	N_{RUA}	N_{mj}	$N_{RUA} \cap N_{mj}$
1 min	1,248.00	10.00	7.00
5 min	138.00	41.00	19.00
11 min	49.00	67.00	12.00

the majority of the cases, not linked to a multivariate jump in the underlying assets.

In 9 days we also detect more than one MJ and in 2 days (09/29/2008 and 05/06/2010) more than six multiple MJs. Then, we highlight an important decrease with respect to the 75% liquidity rule where we found respectively 141 and 2 days. Table S7 reports details on the days with more than 3 MJs. It is possible to observe that the three days in Table S7 are associated with important economic and financial events and that in two cases they are also RUA jump days.

Table S7 Multivariate jump days, 90% liquidity rule. Using a 5-minutes observation interval, the table reports the dates (Days) in which we detect more than 3 multivariate jumps, if RUA jumps are present for the same days (RUA days) and the relevant Economic/Financial events during these days. We study the period January 1998 - June 2015.

Days	RUA days	Economic/Financial event
19-Sep-08	no	FED announce liquidity programs
29-Sep-08	yes	FOMC meeting unscheduled, Emergency Economic Stabilization Act not approved
06-May-10	yes	The Flash Crash

We summarize the information about MJs in Figure S5 where we present the daily diffusion index (top panel) and the systematic MJs (MJs involving the market index). Furthermore, Table S8 reports additional information on systemic jumps and RUA jumps. From Figure S5 it is evident that MJs are rare events and that few of them are associated with a jump in the market index. Correlations between RUA jumps and diffusion indexes, using the 5 minutes intervals, are 0.24 for the DID and 0.23 for the DII (both statistically significant at the 1% confidence level). Thus there exists a significant, positive and moderate large relation between jumps in the index and diffusion indexes in the constituents of the Russell 3000 index.

From the bottom panel of Figure S5 it is also possible to observe that

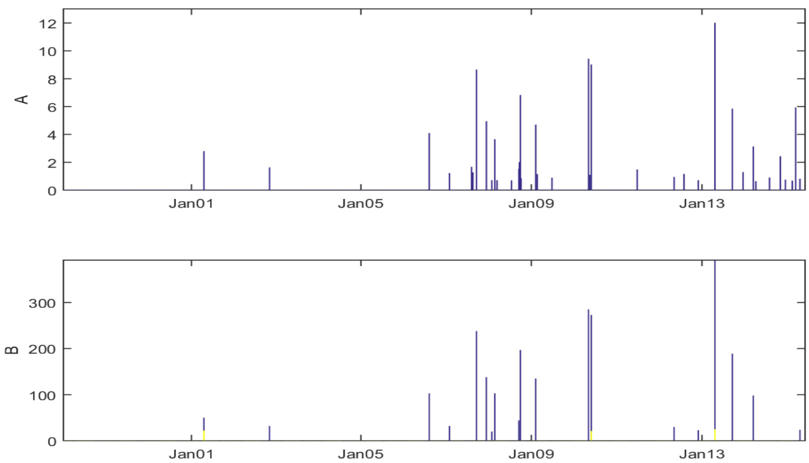

Figure S5 Mjs, 90% liquidity rule. The figure reports in the top panel the time evolution of the daily diffusion index and in the bottom panel the systematic MJs. Panel A shows, for each of the 4,344 days between January 1998 to June 2015, the maximum fraction of stocks involved in a multivariate jump, if present, and 0 otherwise (the DID index). Panel B presents, for each day, the maximum number of stocks involved in a systematic multivariate jump (MJs among the individual stocks that also involve the market index), if present, and distinguishes between systemic (yellow line) and non-systemic (blue line) MJs. All jumps are detected focusing on 5-minutes intervals.

systemic jumps (MJs involving at least 20 stocks among the top 100 size stocks) are even less frequent. We identify just 3 systemic jump days (4 systemic events since we observe 2 systemic MJs on 23 April 2013) that represent about the 7% of the overall 41 5-minutes MJ days (Table S6).

Table S8 reports the dates and times of both the systemic jumps and RUA jumps, if present, and link them with macroeconomic/financial information. It is possible to observe that not only all systemic jumps are related with economically significant events, but all of them are also systematic jumps. In contrast to the 75% liquidity rule, where we obtained similar conclusions, 01/03/2001, 09/18/2001 and 09/18/2013 are no longer systemic jump days with the 90% liquidity rule.

Lastly, Table S9 presents a recap of the overall 62 detected 5-minute MJs, highlighting which are systemic and/or systematic. As discussed above, systemic jumps are always systematic events but represents just about 7% of all MJs. Of all 62 MJs about 40% are connected with an index jump, at the meantime of the overall 175 RUA jumps only 15.9% happen in correspondence of a MJ, thus indicating a limited ability of the index to be informative of the presence of large co-jumps in the constituents of the Russell 3000.

Table S8 Systemic jump days, 90% liquidity rule. The table shows, using the 5-minutes interval and data from January 1998 to June 2015, the dates (Days) and times (Time) in which we detect a systemic jump, if RUA jumps are present in the same days (RUA days) and the corresponding times (RUA time) along with the relevant Economic/Financial events during those days (Eco./Fin. event).

Days	Time	RUA days	RUA time	Eco./Fin. event
18-Apr-01	10:55-11:00	yes	10:55-11:00	FED cut short-term interest rates
28-May-10	12:35-12:40	yes	12:35-12:40	FED announces three small auctions
23-Apr-13	13:05-13:10	yes	13:05-13:10	AP fake tweet about explosions at the White House
23-Apr-13	13:10-13:15	yes	13:10-13:15	AP fake tweet about explosions at the White House

Table S9 MJs, 90% liquidity rule. The table reports, using the 5-minutes interval and data between January 1998 and June 2015, the distribution of detected MJs. In particular, it distinguishes between systemic (MJs involving at least 20 stocks among the top 100 size stocks) and non systemic jumps, and between systematic (MJs reflected in a market index jump) and non-systematic jumps.

	non-systematic	systematic	Tot
non-SJ	38	20	58
SJ	0	4	4
Tot	38	24	62

A.9 s-BNS results

This section presents jump and co-jump outcomes we obtain using the s-BNS test of Barndorff-Nielsen and Shephard (2004b, 2006). As discussed in Appendix B, we employ the s-BNS test just for robustness checks, while we choose the C-Tz test of Corsi et al. (2010) as primary non parametric intraday jump test thanks to its greater power with respect to other tests based on multipower variation.

A.9.1 Jump and cojump identification using the s-BNS test

Table S10 presents the number of jump days we detect using the s-BNS test, plus the percentage of positive jumps and the mean jump sizes. With respect to Table 8, we detect less jump days for all frequencies but similar distributions of the jumps. Even if the percentages of positive jumps and the mean values are further, respectively, from 50% and 0 than in the C-Tz case, it is still possible to observe that detected jumps are symmetrically

Table S10 Jumps summary statistics, *s-BNS* test. The table reports summary statistics for the market index — the Russell 3000 (RUA) — and for the RUA constituents in the period January 2, 1998 - June 5, 2015. N_{RUA} is the number of days in which we observe at least one intraday jump of RUA; % $J_{RUA} > 0$ is the percentage of positive RUA jumps; $Mean_{RUA}$ is the average RUA jump return; N_j is the number of days in which we observe at least one intraday jump in the RUA constituents; % J> 0 is the percentage of constituents of the Russell 3000 for which we observe a positive jump, and Mean is the average jump return for RUA constituents. We report the results for three observation intervals (1, 5 and 11 minutes).

Frequency	N_{RUA}	% $J_{RUA} > 0$	$Mean_{RUA}$	N_j	% J> 0	Mean
1 min	847.00	51.19	0.15	4,057.00	50.19	-1.99
5 min	97.00	51.56	0.21	4,280.00	50.92	1.61
11 min	19.00	45.45	-0.23	4,313.00	52.37	1.42

distributed.

Focusing on 5 minutes observation intervals, Table S11 and Table S12 investigate the relationships between events in the index and events in the underlying assets. The former reports the amounts of jump co-jump days, both for the RUA index and for the underlying assets, and the numbers of days that present an event in the index as well as in the constituents of the Russell 3000. The latter, Table S12, presents the linear correlations (ρ and correspondent P-values for a significance test), between jumps in the index and events (jumps, co-jumps, and single jumps) in the constituents of the Russell 3000.

Table S11 Asset jumps and market jumps, *s-BNS* test. The table reports the number of days in which we observe at least one RUA jump (N_{RUA}), the amount of days in which we observe at least one intraday jump (N_j) or one co-jump (N_{cj}) in the constituents of the Russell 3000 and the days with both a jump in the index and a jump ($N_{RUA} \cap N_j$) or a co-jump ($N_{RUA} \cap N_{cj}$) in the underlying assets. These statistics are computed for the period: January 1998 - June 2015. We consider three observation intervals — 1, 5 and 11 minutes.

Frequency	N_{RUA}	N_j	$N_{RUA} \cap N_j$	N_{cj}	$N_{RUA} \cap N_{cj}$
1 min	847.00	4,057.00	771.00	3,739.00	696.00
5 min	97.00	4,280.00	96.00	3,882.00	84.00
11 min	19.00	4,313.00	19.00	3,818.00	17.00

From Table S11 it is possible to observe that almost all days in the sample are co-jump days, and that a small fraction of jump and co-jump days happen in correspondence of RUA jump days. Intraday correlation values in Table S12, instead, are significant and positive for jumps and co-jumps. The information content in tables S11 - S12 leads to the same conclusion of the *C-Tz* test results: there exists a positive but weak relation between jumps in the index and jumps and co-jumps in the constituents

Table S12 Correlations assets jumps and index jumps, *s-BNS* test. The table reports the correlations (ρ), along with the correspondent p-values (P-val(ρ)), between jumps (j), co-jumps (cj) and single jumps (sj) in the assets and jumps in the market index using data from January 1998 to June 2015. Results are presented separately for six frequency intervals: 1, 5, 11 minutes, daily 1, daily 5 and daily 11 minutes. In case of daily 1 minute, a jump/co-jump is detected in day t if there is at least one intraday 1 minute interval with a jump/co-jump. Daily 5 and daily 11 minutes work exactly as the daily 1 minute with the difference that they are recovered from 5 and 11 minutes intervals.

Jump test	j	cj	sj	j	cj	sj
	ρ			P-val(ρ)		
1 min	0.02	0.02	-0.00	0.00	0.00	0.00
5 min	0.00	0.01	-0.01	0.04	0.00	0.00
11 min	0.01	0.01	-0.00	0.00	0.00	0.97
Daily 1 min	-0.05	-0.06	-0.05	0.00	0.00	0.00
Daily 5 min	0.01	-0.01	0.01	0.71	0.37	0.70
Daily 11 min	0.01	0.00	0.01	0.71	0.83	0.70

of the Russell 3000, and index jumps are only scarcely informative of the events (j and cj) in the cross-section.

We further investigate the relation between index jumps and co-jumps in the underlying assets in Table S13. Our focus, in this case, is on the number and proportions of systematic (cj $\cap$ j_{RUA}) and non-systematic (cj $\not\cap$ j_{RUA}) co-jumps involving different number of stocks. Similarly to the results in Appendix C.2, it is possible to observe that the majority of co-jumps are not systematic but also that systematic co-jumps involve more stocks than non-systematic co-jumps. Indeed, co-jumps involving more than 10 stocks are 34.00% and 0.72% respectively of systematic and non-systematic events.

A.9.2 Multiple cjs and diffusion indexes with the *s-BNS* test

We discussed in Section 2 that by using the *s-BNS* test we detect almost half the MJ days we find using the *C-Tz* test. This difference is more evident considering the diffusion indexes (Figure S6) and the distribution of MJs (Figure S7) along our time window (top panel) and during the 77 5-minutes observation intervals of a trading day (bottom panel).

A comparison of Figure S6 and Figure 3, highlights that *s-BNS* diffusion indexes present values equal to 0 much more often than in *C-Tz* correspondent indexes.

Figure S7 highlights two additional considerations: there is only one day in which we observe more than a MJ (05/03/2012) and there is no evidence of a 'lunch effect' for MJs. Differently, using the *C-Tz* test we identify 141 multiple MJ days (including 05/03/2012) and MJs are clearly concentrated around lunch time. To gain a clearer image of MJs behaviour, Figure S8

Table S13 Co-jumps distribution, s-BNS test. The table reports, using data from January 1998 to June 2015, the numbers of detected co-jumps involving different number of stocks, being either systematic (panel A: cj $\cap$ j_{RUA}) or non-systematic (panel B: cj $\not\cap$ j_{RUA}). The last three rows of each panel list the maximum, the average and the median number of stocks detected to participate in the correspondent systematic or non-systematic co-jumps.

No. Stocks	cj $\cap$ j_{RUA}	cj $\not\cap$ j_{RUA}
2	9.00	28,209.00
3	3.00	13,309.00
4	4.00	6,702.00
5	4.00	3,375.00
6-10	13.00	3,603.00
11-15	4.00	267.00
16-20	3.00	64.00
>20	10.00	69.00
Max	311.00	157.00
Mean	23.88	3.09
Median	8.00	2.00

presents the trend and irregular components resulting from the Local Level Model (LLM) filtering procedure. In contrast to the C-Tz test, the trend exhibits clear variations over time only for the daily index, while the DII trend is almost flat. As expected, the maximum and the average DID's trend and volatility (or σ_t) values are much lower in the s-BNS than in the C-Tz case. They are equal to 0.08 and 0.02, respectively, for μ_t (0.55 and 0.21 with the C-Tz test) and 0.73 and 0.19 for σ_t (2.18 and 0.76 with the C-Tz test). The flat DII's trend is associated with low maximum and average trend and volatility values: 0.0003 and 0.0001 for the former, 0.65 and 0.01 for the latter.

Differently from the trends, both DID and DII residual components have more frequent and larger spikes in special market moments. Table S14 links the days corresponding to the ten largest DID residual spikes of Figure S8 (panel C), with RUA and Economic/Financial events. It is possible to detect some similarities and some differences between s-BNS and C-Tz residual results. While the ten largest spike days are usually associated with relevant economic events for both models, only s-BNS results are usually not associated with a jump in the RUA index.

A further relationship between multivariate jumps and index jumps exists in the linear correlations between RUA jumps and diffusion indexes. DID's and DII's correlations are respectively equal to 0.09 and 0.10 and are both statistically significant (1% confidence level) but smaller than in the C-Tz case. Then, we have a weaker association between jumps in the index and diffusion indexes in the constituents of the Russell 3000 index.

Similarly to the number of MJs, also the amount of systemic jumps drops employing the s-BNS test. Indeed, we identify just 1 systemic jump

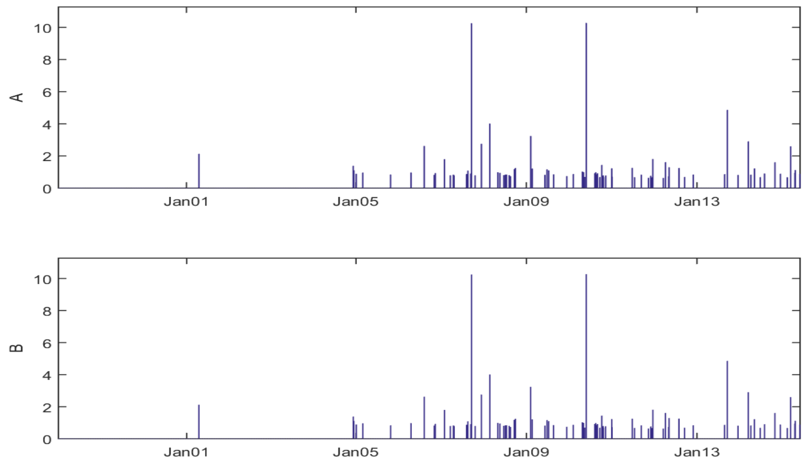

Figure S6 Diffusion indexes, s-BNS test. The figure displays the time evolution of the diffusion indexes. Panel A shows, for each of the 4,344 days, the maximum fraction of stocks involved in a multivariate jump, if present, and 0 otherwise (the DID index). Panel B reports, for each 5-minutes intraday interval between 01/02/1998 and 06/05/2015, the percentage of stocks involved in a multivariate jump, if present, and 0 otherwise (the DII index).

day, that is, 05/28/2010, which represents about 1% of all MJ days. In the same day we also observe a jump in the RUA index and the announcement of three small actions by the FED. This is in line with the findings in Section 2.2, where we observed that systemic jumps are associated with important economic events, and that MJs involve prevalently small size stocks thus becoming negligible when considering the full market.

We report a repartition of MJs in Table S15, where we point out which of the overall 85 detected 5-minute MJs are systemic and/or systematic. As said before, we detect only one systemic MJ which is also a systematic event. Only 12% of all MJs are systematic co-jumps, while a small 8% of all RUA jumps (total of 128 detected RUA jumps) happen in correspondence of a MJ.

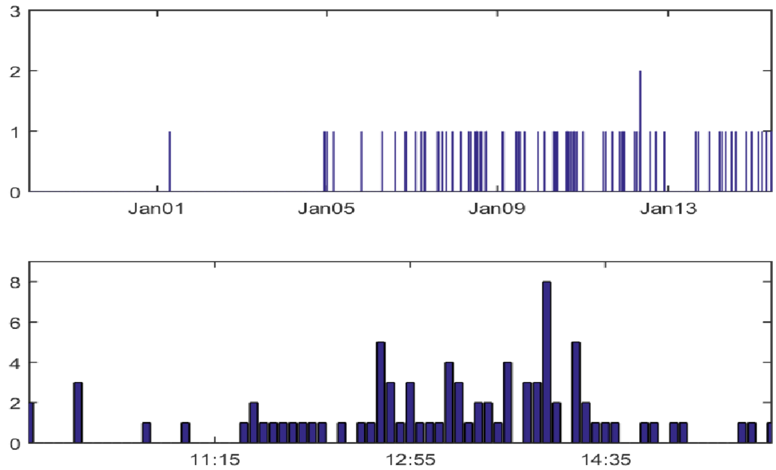

Figure S7 Intervals and days with multivariate jumps, ***s-BNS*** **test**. The figure shows in the top panel, for each of the 4,344 days between January 1998 to June 2015,, the number of daily intervals (77 daily intervals of 5 minutes) with at least a multivariate jump. Bottom panels, instead, reports, for each of the 77 5-minutes daily intervals, the number of days for which we observe at least a multivariate jump.

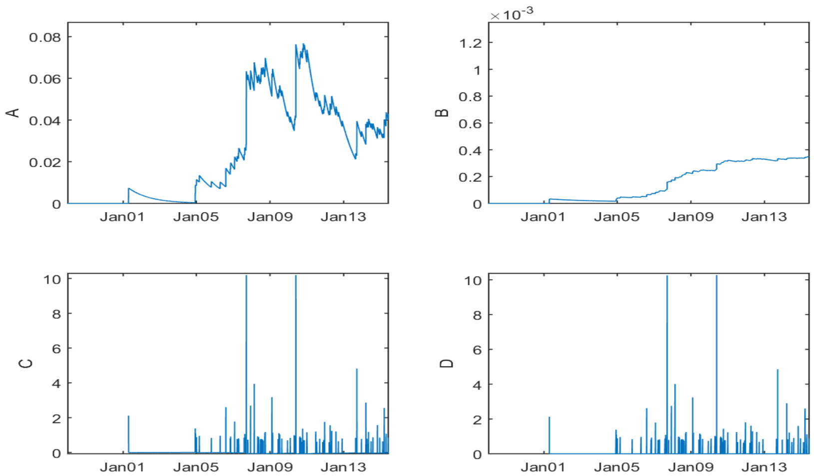

Figure S8 DID and DII trends and residuals, ***s-BNS*** **test**. The figure reports trend and residual components evolutions for the DID, panels A and C, and for the DII, panel B and D. Panels A and C shows respectively, for each of the 4,344 days between January 1998 to June 2015, the DID trend (μ_t) and the DID residuals (ϵ_t). Panels B and D, instead, reports for each 5-minutes intraday interval between January 2, 1998 and June 5, 2015, respectively the DII trend (μ_t) and the DII residuals (ϵ_t). μ_t ϵ_t and are estimated for each day, in the DID case, and intraday interval, in the DII case, from equation 3. Each panel reports the results using 5-minutes observation intervals.

Table S14 Residual spikes, s-BNS test. The table shows, for the time-window January 1998 - June 2015 and using the 5-minutes interval, the dates (Days) with the ten larger spikes of DID's residuals, if RUA jumps are present in the same days (RUA days) and the relevant Economic/Financial events during those days.

	Days	RUA days	Economic/Financial event
1	28-May-10	yes	FED announces three small auctions
2	18-Sep-07	yes	FOMC lowers target for federal funds rate (50 bps)
3	18-Sep-13	no	FOMC statement
4	25-Feb-08	yes	
5	10-Feb-09	no	
6	19-Mar-14	no	FOMC statement
7	11-Dec-07	no	FOMC lowers target for federal funds rate (25 bps)
8	08-Aug-06	no	FOMC keeps its target for the federal funds rate
9	18-Mar-15	no	FOMC statement
10	18-Apr-01	no	FED cut short-term interest rates

Table S15 MJs, s-BNS test. The table reports, using the 5-minutes interval and data between January 1998 to June 2015, the distribution of detected MJs. In particular, it distinguishes between systemic (MJs involving at least 20 stocks among the top 100 size stocks) and non systemic jumps, and between systematic (MJs reflected in a market index jump) and non-systematic jumps.

	non-systematic	systematic	Tot
non-SJ	75	9	84
SJ	0	1	1
Tot	75	10	85