

# A Glance at Circuit Theory Development

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**A**lthough circuit theory is fundamental in the study and design of electric systems for both information and power applications, its history has seldom been told in the scientific literature. This article is an attempt to fill that gap, at least partially, by surveying its development up to circa 1924, i.e., mainly in the preclassical period, according to Herbert Carlin's definition [1]. Short biographical notes have been added to delineate the merits of the major authors.

## The Early Electric Components

Electric components started appearing long before any circuit analysis was conceived. The resistor can be traced back to the conductors used in the electrostatic experiments performed by eminent English scientist Stephen Gray around 1731 [2]. Capacitor archetypes consisted of the Kleist bottle, conceived by German physicist Ewald Georg von Kleist in 1745, and the Leyden jar, created independently by Dutch professor Pieter van Musschenbroek and coworkers in 1745–1746. For those early devices, the two conductors consisted of the experimenter's hands holding a glass bottle and the filling water.

In 1778, Alessandro Volta (1745–1827) perfected the metal capacitor, defined capacity  $C$ , and determined the constitutive equation,  $q = Cv$ , in modern symbols. Twenty-one years later, Volta invented the electrochemical cell, namely the first voltage source capable of generating steady currents,

thus allowing the transition from electrostatics to electrodynamics, a crucial step forward in electric circuits [3]. Volta received several honors for his achievements, and the term *voltage*, its symbol  $v$ , and its unit volt,  $V$ , were named after him.

## Experimenting With Electric Current

In 1820, soon after Danish physicist Hans Christian Ørsted discovered electromagnetism by using electrodynamics, French scientist André-Marie Ampère (1775–1836) carried out extensive studies on electric current, which he named *intensité de courant*, from which the symbol  $i$  was derived, whereas the unit ampere,  $A$ , honor the author. In the same year, the German scientist Johann Salomo Christoph Schweigger invented the electromagnetic multiplier, which was capable of producing magnetic actions much stronger than those generated by the straight conductor of Ørsted's experiment. This was the first inductor, but its self-induction effect was identified only 11 years later, in 1831, independently by American Joseph Henry and Englishman Michael Faraday [4].

In 1831, Faraday also discovered mutual induction between coupled inductors and invented the eponymously named *Faraday disk*, which was the archetype of an electromagnetic generator and a new voltage source. The principle of this disk had been anticipated the year before by Italian priest and physicist Francesco Zantedeschi, later a professor at the University of Padua. Faraday had worked as an apprentice bookbinder in his teenage years but

became one of the greatest experimenters of all times. A survey of the countless scientific results he produced during 50 years at the Royal Institution is beyond the scope of these few pages. All of these results of 1830–1831 were the effects of electromagnetic induction, but none of the experimenters managed to propose a mathematical formulation.

## Decisive Steps Toward Circuit Theory

A few years prior, in 1826–1827, the resistor constitutive equation,  $v = R i$ , was enunciated by German gymnasium teacher Georg Simon Ohm [1789–1854, Figure 1(a) [5] and, a little earlier, in 1825, but with less visibility, by Italian high school teacher Stefano Marianini (1790–1866). Ohm's law constituted a decisive step in circuit theory, but at that time, the properties of electricity (voltage, intensity, and so on) were ill-defined concepts, studied in nonmathematical forms. Furthermore, the common belief considered electrostatics and electrodynamics not to be directly related. Thus, Ohm's discovery was initially greeted with skepticism. As his work began to be recognized, firstly abroad, Ohm became a professor in Nuremberg, was elected a member of the Royal Society, and finally, was appointed professor of physics at the University of Munich in 1852, realizing his life's dream. Marianini became a professor at the University of Modena in 1835.

German physicist and engineer Moritz Hermann von Jacobi (1801–1874) built a crude electric motor capable of practical use in 1834. Investigating the energy exchanges in such devices, he

enunciated the eponymous theorem on the maximum power transfer from a source in 1837. These studies were later instrumental to James Prescott Joule (1818–1889) in his research on heat generation in resistors [6].

Topological laws are a major milestone in circuit theory. German mathematical physicist Gustav Robert Kirchhoff [1824–1887, Figure 1(b)] was a 21-year-old student in 1845 when he established the laws of current and voltage in circuits, today universally known by his name. He derived them by applying, for the first time, graph theory to electrical problems [7]. He perfected the formulation in 1847, reconnecting to Ohm's law. Kirchhoff later achieved fundamental results in other fields of physics. Together with Robert Bunsen in 1859, he founded spectroscopy and enunciated the law on the emissive power of the blackbody, which paved the way for the epochal discovery of the quantization of energy by Max Planck in 1900. The mathematical formulation of electromagnetic induction, discovered by Faraday and Henry in 1831, was identified by German physicist Franz Ernst Neumann (1798–1895) in 1845, when he also proposed a formula for mutual induction coefficients. Not to be confused with his son Carl Gottfried, Neumann

studied several problems in mathematical physics.

The superposition principle and the equivalent voltage source theorem were first described in a report on the effects of electric currents on animals by German scientist Hermann Helmholtz (later von; 1821–1894) in 1853 [8], who actually credited the former to his friend Emil du Bois-Reymond (1818–1896). However, the article was overlooked for a long time and was highlighted only in 1948 [9]. Helmholtz excelled in physiology, mathematics, and physics. To mention just one achievement, in a report on biological heat published in 1847, he affirmed on the one hand the physical-chemical nature of animal heat (refuting vitalist theories) and on the other established one of the deepest foundations of the theory of energy conservation [10]. Helmholtz stands as one of the greatest scientists of the 19th century and the most illustrious German scientist of his time, one of the last scholars capable of dominating every field of knowledge. Kaiser Wilhelm I ennobled Helmholtz in 1883, adding “von” to the family name. French telegraph engineer Léon Charles Thévenin (1857–1926) independently enounced the equivalent voltage source theorem in 1883 [11]. This theorem also was initially greeted

with skepticism by the scientific world. In addition, circuit duality had not yet been established, and it took several decades for the current-source equivalent theorem to be conceived, as we will see further on.

### Completing the Framework of Fundamental Circuit Concepts

The capacitor constitutive equation  $i = C dv/dt$  was used by William Thomson (later Lord Kelvin; 1824–1907) in 1854 in developing the second-order partial differential equation  $\partial^2 v/\partial x^2 = RC\partial v/\partial t$ , which he used to study propagation in a telegraph line due to the resistance  $R$  along the conductors and the capacitance  $C$  between them [12]. These investigations were instrumental in the laying of early transatlantic telegraph cables in 1857–1866 [13]. Thomson's contributions to the feat included the highly sensitive mirror galvanometer used as a telegraph receiver since 1858. Several other scientific achievements, including the absolute scale of temperature (1848) [14] and the second law of thermodynamics (1851) [15], entitle him as one of the greatest scientists of the 19th century, the British equivalent of Helmholtz. These two men became friends and, on the occasion of the first International Exhibition of Electricity in Paris of 1881, promoted the first international conference on electricity, in which unified units of measurement for electrical quantities (volt, ohm, and ampere) were introduced. Thomson was knighted in 1866 and ennobled in 1892. Kirchhoff, and independently Wilhelm Weber (1804–1891), took into account inductive effects along the wires to obtain a more accurate form of the propagation equation in 1857, thus identifying that, in a lossless line, signals propagate at the speed of light [16].

James Clerk Maxwell (1831–1879) formulated the constitutive equations of the inductor,  $v = L di/dt$ , and mutual inductors,  $v_1 = L_1 di_1/dt + M di_2/dt$  and  $v_2 = M di_1/dt + L_2 di_2/dt$  (in modern symbols) in 1864 [17]. However, the abstraction of the latter as an ideal transformer was introduced only in the 1920s. Needless to say, Maxwell stands as one of the greatest theoretical physicists for his achievements in electromagnetic fields,

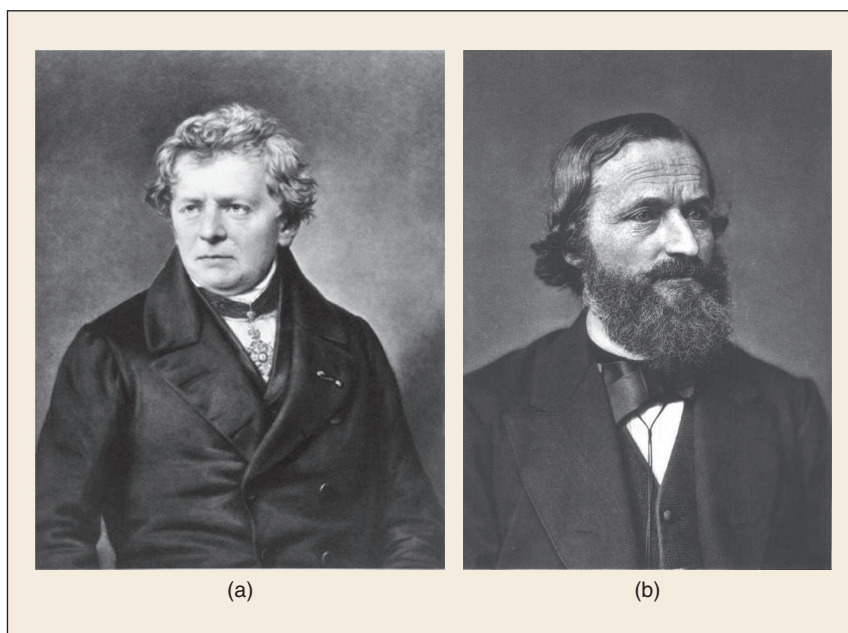


FIGURE 1 – German scientists (a) Georg Simon Ohm (1789–1854) and (b) Gustav Robert Kirchhoff (1824–1887) are often regarded as the founders of circuit theory. (Source: Wikimedia Commons.)

thermodynamics, and optics, despite his short life.

### **The Expansion of Electric Systems**

By the 1880s, two electric technologies were expanding, the electromagnetic telephone (1876) and electric power distribution (1882) [18], and early mathematical tools for investigating dynamic circuits appeared. Oliver Heaviside (1850–1925) was an eccentric, self-taught British physicist and mathematician who developed his research alone. His results, published in a series of articles in *The Electrician*, were initially rejected by the scientific community, to be accepted only after a careful revision [19], [20], [41], [42]. In 1886–1887, he introduced the operational calculus, based on expressing differentiation by an operatorial symbol  $p$ . In this work, he introduced impedance as a generalization

of the resistance and, for the first time, made use of the unit step function, named after him, and the pulse function, later named after Paul Dirac. The operational method was rigorously formalized in the framework of a Laplace transform by Thomas Bromwich (1875–1929). In 1886–1887, Heaviside perfected previous results on propagation in lines by Thomson, Kirchhoff, and Weber by adding the conductance between wires and thus obtaining the telegrapher's equations, deducing the distortionless condition  $G/C = R/L$ , and conceiving loading coils to match it. This idea was taken up by Michael Pupin (1858–1935) in 1899, who registered a patent on it in 1900, to the great disappointment of Heaviside. Oliver Heaviside also provided major contributions to electromagnetic field theory.

Mathematical tools for studying harmonic circuits emerged when ac systems started to spread [21]. An early contribution was made by English mathematician and electrical engineer Thomas Blakesley (1847–1929), who introduced the graphic representation of sinusoidal time functions by means of rotating vectors in 1884. German-born American Charles Proteus (Karl Augustus) Steinmetz (1865–1923) developed the concept into the phasor method, based on complex algebra. The presentation of these methods at the 1893 International Electrical Congress in Chicago was a milestone in electrical engineering [22]. Steinmetz extended his analyses in the following years, founding a systematic theory [23]. Steinmetz worked as design engineer, chief researcher, and then top manager at the General Electric Company. He also studied magnetic hysteresis

and electrical machines and led rich associative and teaching activities.

Building on Steinmetz's theory, Irish-born American electrical engineer Arthur Edwin Kennelly (1861–1939) defined impedance as the complex operator correlating voltage and current phasors in 1893 [24], and he proposed the star-delta transformation in 1899. Kennelly was a coworker of Edison's from 1887, a consultant to Edison General Electric Company (1889) and General Electric Company (1892), and later, a professor of electrical engineering at Harvard University and the Massachusetts Institute of Technology. He also had a rich associative life. Canadian-born American electrical engineer Charles Le Geyt Fortescue (1876–1936) worked at the Westinghouse research center in Pittsburgh from 1898. He introduced the method of symmetrical components for analyzing unbalanced three-phase systems in 1919, building on an intuition of L.G. Stokvis of 1914 [25].

### Circuit Theories of the 20th Century

Circuit duality was introduced by Scottish engineer Alexander Russell in 1904 [26]. But progress was not fast, at least if we look at the current-source equivalent theorem, which was enunciated 43 years after Thévenin's theorem and 25 years after the enunciation of duality. American engineer Edward Lawry Norton (1898–1983) published the current-source equivalent theorem in 1926 [27]. Norton worked at Bell Laboratories on long-distance telephony and other topics. Like the voltage-source equivalent concept, the current-source equivalent theorem had a second deviser, Hans Ferdinand Mayer (1895–1980), who independently developed it in 1926 [28]. A research director at Siemens & Halske in Berlin, Mayer was the uncompromising opponent of Nazism who authored the secret *Oslo Report* in 1939, a major intelligence case during the war. Although this report remained unknown in Germany, Mayer was arrested and interned in Dachau in 1943. After the war, while working at Cornell University in the United States in 1948, he pointed out the original contribution of Helmholtz to the voltage-source equivalent theorem.

Notable contributions to circuit theory in the 20th century came from Bernard Dominicus Hubertus Tellegen (1900–1990), a Dutch electrical engineer at Philips Research Laboratories in Eindhoven and a professor at Delft University of Technology. In 1948, he theorized the gyrator, i.e., the first ideal nonreciprocal two-port element [29]. In 1952, Tellegen published the theorem on electrical circuits named after him [30], which indeed had already been enunciated by K. Posthumus and T. Douma, also researchers at Philips, in 1936. Other ideal elements were conceived in the 1960s. Herbert Carlin and Dante C. Youla introduced the nullator ( $v = 0, i = 0$ ) and the norator ( $v, i = \text{arbitrary}$ ) as pathologic one-ports in 1961 [31]. Curiously, the nullor, i.e., the two-port combination of a nullator and a norator, had already been conceived by other authors, such as Vitold Belevitch [32] and Yoshiro Oono [33]. Also in the 1960s, controlled sources became common in representing active devices [34]. However, these developments are well beyond the scope of this article.

Electrical filters stand on a special pedestal in the history of circuit theory. The first one was built by Belgian engineer François Van Rysselberghe (1846–1893) in 1882 to separate telegraphic and telephone signals transmitted on the same line. It consisted of a capacitor and an inductor. High-pass, low-pass, and bandpass passive electrical filters made of inductors and capacitors were studied by K.W. Wagner in Germany from 1915 and by George Ashley Campbell (1870–1954) in the United States at about the same time. Ronald Martin Foster published the eponymous theorem on the monotonicity of the reactance of a lossless passive one-port in 1924 [35]. This paper paved the way for the research of German scientist Wilhelm Cauer (1900–1945), whose work on filters marked the birth of circuit synthesis. His book of 1941 remains a milestone [36]. These advances, which also exceed the scope of this article, have been described by other influential authors [37].

### Conclusion

Some last words are to be reserved for the mathematical tools upon which circuit theory was developed. After being

founded by Euler in the 18th century, topology progressed greatly in the following one, thanks to Augustin-Louis Cauchy, Simon Antoine Jean L'Huilier, and Johann Benedict Listing, who introduced the term *topologie* in 1847 [38], having used it for 10 years in correspondence, and provided major contributions to pertinent algebra. Arthur Cayley introduced the tree concept in 1857, while James Joseph Sylvester introduced the term *graph* in 1878. It took time to fully exploit these ideas in circuit theory, with major results appearing only in the 1940s. In the Western world, the theory of linear equations began in the 17th century, mainly with Isaac Newton, though credit is often given to later authors, as is the case with Gaussian elimination. The theory progressed greatly in the 19th century, starting with German mathematician Hermann Grassmann [39] and then progressing with the work of British and French scholars, such as James Joseph Sylvester, who introduced the term *matrix*, Arthur Cayley, Augustin-Louis Cauchy, and William Rowan Hamilton, who took matrix and determinant concepts to maturity. Matrix analysis was introduced in engineering in the 1930s, starting with aeroelasticity research, and a great push forward came with the advent of digital computing, starting from the following decade. To give just a couple of examples, the incidence matrix was proposed by Theodore R. Bashkow in 1957 [40], and matrix analysis began to disseminate in the 1960s.

I would like to close this column by remarking that none of the authors presented here were awarded a Nobel Prize or a Fields Medal. However, the Nobel Prize was established in 1901 and the Fields Medal in 1936. Before then, the highest honor in science was most likely the Copley Medal of the Royal Society, first established in 1731, and it was awarded to several scientists who contributed to the topics of interest in this article: Gray (1731 and 1732), Volta (1794), Ørsted (1820), Faraday (1832), Ohm (1841), Weber (1859), Joule (1870), Helmholtz (1873), Sylvester (1880), Cayley (1882), Thomson (1883), and Neuman (1886).

### References

- [1] H. Carlin, "Unconventional network theory," *IEEE Trans. Circuit Theory*, vol. 11, no. 3, pp. 324–326, Sept. 1964. doi: 10.1109/TCT.1964.1082314.

- [2] M. Guarnieri, "Electricity in the age of Enlightenment," *IEEE Ind. Electron. Mag.*, vol. 8, no. 3, pp. 60–63, Sept. 2014. doi: 10.1109/MIE.2014.2335431.
- [3] M. Guarnieri, "The big jump from the legs of a frog," *IEEE Ind. Electron. Mag.*, vol. 8, no. 4, pp. 59–61, 69, Dec. 2014. doi: 10.1109/MIE.2014.2361237.
- [4] M. Guarnieri, "Who invented the transformer?" *IEEE Ind. Electron. Mag.*, vol. 7, no. 4, pp. 56–59, Dec. 2013. doi: 10.1109/MIE.2013.2283834.
- [5] G. S. Ohm, *Die Galvanische Kette, Mathematisch Bearbeitet [The Galvanic Circuit Investigated Mathematically]*. Berlin, Germany: T.H. Riemann, 1827.
- [6] J. P. Joule, "On the production of heat by voltaic electricity," in *Proc. Roy. Soc.*, vol. 4, January 1, 1843, pp. 280–281.
- [7] G. R. Kirchhoff, "Ueber den Durchgang eines elektrischen Stromes durch eine Ebene, insbesondere durch eine kreisförmige [About the passage of an electric current through a plane, in particular through a circular one,]" *Annalen Physik Chemie, Band LXIV*, vol. 140, no. 4, pp. 497–514, 1845. doi: 10.1002/andp.18451400402.
- [8] H. Helmholtz, "Über einige Gesetze der Verteilung elektrischer Ströme in körperlichen Leitern mit Anwendung auf die tierisch-elektrischen Versuche [Some laws concerning the distribution of electrical currents in conductors with applications to experiments on animal electricity]," *Annalen Physik Chemie, Band LXXXIX*, no. 7, pp. 211–233, 1853.
- [9] D. H. Johnson, "Origins of the equivalent circuit concept: The voltage-source equivalent," *Proc. IEEE*, vol. 91, no. 4, pp. 636–640, 2003. doi: 10.1109/JPROC.2003.811716.
- [10] H. Helmholtz, *Über die Erhaltung der Kraft [On the Conservation of Force]*. Berlin, Germany: Druck und Verlag von G. Reiner, 1847.
- [11] L. C. Thévenin, "Sur un nouveau théorème d'électricité dynamique [On a new theorem of dynamic electricity]," *C. R. des Séances de l'Académie des Sci.*, vol. 97, pp. 159–161, Dec. 1883.
- [12] W. Thomson, "On the theory of the electric telegraph," *Proc. Roy. Soc. London*, vol. 7, pp. 382–399, 1856. doi: 10.1098/rspl.1854.0093.
- [13] M. Guarnieri, "The conquest of the Atlantic," *IEEE Ind. Electron. Mag.*, vol. 8, no. 1, pp. 53–55, 67, 2014. doi: 10.1109/MIE.2014.2299492.
- [14] W. Thomson, "On an absolute thermometric scale founded on Carnot's theory of the motive power of heat, and calculated from Regnault's observations," *Philos. Mag.*, vol. 33, pp. 313–317, Dec. 1848.
- [15] W. Thomson, "On a dynamical theory of heat, with numerical results deduced from Mr Joule's equivalent of a thermal unit, and M. Regnault's observations on steam," *Proc. Roy. Soc. Edinburgh*, vol. 3, pp. 48–52, 1851. doi: 10.1017/S0370164600027140.
- [16] G. Kirchhoff, "On the motion of electricity in wires," *Philos. Mag.*, vol. 13, no. 88, pp. 393–412, 1857. doi: 10.1080/14786445708642318.
- [17] J. C. Maxwell, "Dynamical theory of the electromagnetic field," *Philos. Trans. Roy. Soc. London*, vol. 55, pp. 459–512, Jan. 1, 1864.
- [18] M. Guarnieri, "The beginning of electric energy transmission: Part one," *IEEE Ind. Electron. Mag.*, vol. 7, no. 1, pp. 57–60, 2013. doi: 10.1109/MIE.2012.2236484.
- [19] O. Heaviside, *Electrical Papers*, vols. I, London and New York: Macmillan, 1892.
- [20] O. Heaviside, *Electromagnetic Theory*. London: The Electrician Printing and Publishing, vol. I, 1893.
- [21] M. Guarnieri, "The beginning of electric energy transmission: Part two," *IEEE Ind. Electron. Mag.*, vol. 7, no. 2, pp. 52–59, 2013. doi: 10.1109/MIE.2013.2256297.
- [22] C. P. Steinmetz, "Complex quantities and their use in electrical engineering," in *Proc. AIEE Int. Elect. Congr.*, July 1893, pp. 33–74.
- [23] C. P. Steinmetz, *Theory and Calculation of Alternating Current Phenomena*. Boston, MA: Johnston Company, 1897.
- [24] A. E. Kennelly, "Impedance," *Trans. AIEE*, vol. X, pp. 172–232, Jan.–Dec. 1893. doi: 10.1109/T-AIEE.1893.4768008.
- [25] C. L. Fortescue, "Method of symmetrical co-ordinates applied to the solution of poly-phase networks," *Trans. AIEE*, vol. XXXVII, no. 2, pp. 1027–1140, June 1918. doi: 10.1109/T-AIEE.1918.4765570.
- [26] A. Russell, *A Treatise on the Theory of Alternating Currents*, vol. 1, Cambridge, U.K.: Cambridge Univ. Press, 1904, ch. XXI.
- [27] E. L. Norton, "Design of finite networks for uniform frequency characteristic," Bell Labs, Tech. Rep. TM26–0–1860, Nov. 11, 1926.
- [28] D. H. Johnson, "Origins of the equivalent circuit concept: The current-source equivalent," *Proc. IEEE*, vol. 91, no. 5, pp. 817–821, 2003. doi: 10.1109/JPROC.2003.811795.
- [29] B. D. H. Tellegen, "The gyrator, a new electric network element," *Philips Res. Rep.*, vol. 3, pp. 81–101, Apr. 1948.
- [30] B. D. H. Tellegen, "A general network theorem, with applications," *Philips Res. Rep.*, vol. 7, pp. 259–269, Apr. 1952.
- [31] H. J. Carlin and D. C. Youla, "Network synthesis with negative resistors," *Proc. IRE*, vol. 49, no. 5, pp. 907–920, May 1961. doi: 10.1109/JRPROC.1961.287934.
- [32] V. Belevitch, "On the algebraic structure of formal realizability theory," *Revue HF*, vol. 4, pp. 1–12, Aug. 1959.
- [33] Y. Oono, *Formal Realizability of Linear Networks*, Brooklin, N.Y.: Polytechnic Press of the Polytechnic Inst. of Brooklyn, 1960.
- [34] I. W. Sanders, "Synthesis of N-port active RC networks," *Bell Syst. Tech. J.*, vol. 40, no. 1, pp. 329–347, 1961. doi: 10.1002/j.1538-7305.1961.tb03988.x.
- [35] R. M. Foster, "A reactance theorem," *Bell Syst. Tech. J.*, vol. 3, no. 2, pp. 259–267, Nov. 1924. doi: 10.1002/j.1538-7305.1924.tb01358.x.
- [36] W. Cauer, *Theorie der linearen Wechselstromschaltungen [Theory of Linear AC Circuits]*, vol. I, Leipzig, Germany: Akad. Verlags-Gesellschaft Becker und Erler, 1941.
- [37] V. Belevitch, "Summary of the History of Circuit Theory," *Proc. IRE*, vol. 50, no. 5, pp. 848–855, May 1962. doi: 10.1109/JRPROC.1962.288301.
- [38] J. B. Listing, *Vorstudien zur Topologie*. Göttingen, Germany: Vandenhoeck und Ruprecht, 1848.
- [39] H. Grassmann, *Die Lineale Ausdehnungslehre, ein neuer Zweig der Mathematik [The Theory of Linear Extension, a New Branch of Mathematics]*. Leipzig, Germany: Verlag von Otto Wigand, 1844.
- [40] T. R. Bashkow, "The A matrix, new network description," *Trans. IRE Trans. Circuit Theory*, vol. 4, no. 3, pp. 117–119, 1957. doi: 10.1109/TCT.1957.1086361.
- [41] O. Heaviside, *Electromagnetic Theory*. London, U.K.: Electrician Printing and Publishing, vol. II, 1899.
- [42] O. Heaviside, *Electrical Papers*, vol. II, London and New York: Macmillan, 1894.