



Financing flexibility: The case of outsourcing



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ABSTRACT

We investigate the relationship between the extent and timing of vertical flexibility and the financial choices of a firm. By vertical flexibility we mean partial/total and reversible outsourcing of a necessary input. A firm simultaneously selects the vertical setting and the financial sources of investment in flexibility, in particular debt and venture capital. A loan may come from a lender that requires the payment of a fixed coupon over time and an option to buy out the firm in certain circumstances. Debt leads to the same level of flexibility of an unlevered firm. Yet investment occurs earlier. The injection of venture capital reduces the quest for vertical flexibility and speeds up investment. Then, there arises a fresh substitutability between a financial (venture capital) and a real variable (vertical flexibility).

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1. Introduction

Our intent is to analyze the influence of external financial sources on extent and timing of investment in vertical flexibility by a corporate enterprise that buys inputs from the market (outsourcing) in a variable and reversible manner, going back to internal production whenever economically convenient.

Outsourcing and flexibility are crucial for most firms which apparently buy inputs in variable proportions changing often the span of activity along the vertical chain of production. Vertical flexibility improves the ability to cope with uncertain scenarios and impinges on competitiveness, scale of production and social efficiency. Unfortunately flexibility never comes for free. Procurement of inputs from the market calls for the set-up of a supply chain with specific logistic investment. In addition to that a vertically flexible firm must be ready to substitute an internally produced input with an externally acquired one and viceversa. In other words it must be properly equipped to bring back in-house input production partly or entirely at any time (backsourcing or reshoring). Therefore, vertical flexibility entails keeping alive a dedicated internal facility and the associated know-how. All that may turn out to be quite dear.

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The costs of flexibility change over time and industries. They depend on technical progress in production, information and logistic services, efficiency of external input markets and the financial sources adopted. An enterprise may choose among many financial avenues in order to invest in vertical flexibility. Equity, debt and other external sources such as venture capital are among the most common. Unfortunately, the financial side of flexibility is most of the times sidestepped in current studies. Funding and organizational issues are examined separately in financial,¹ managerial, industrial organization and operations research literatures.² Our purpose is to contribute to bridge the gap by jointly analyzing finance and corporate organization to see whether the commitment to vertical flexibility is affected by the specific financial arrangements adopted. On the real side we shall explore extent and type of vertical flexibility that can be secured by arms' length outsourcing of inputs while maintaining in-house production facilities. On the financial side we shall see how equity,³ convertible debt⁴ and venture capital⁵ impact on size and timing of investment in flexibility.

Our investigation is solicited by broad casual observation, literature and press reports.⁶ showing (a) that firms change over time their vertical production structure, expanding and/or subsequently contracting (or the other way round) the extent of outsourcing and (b) that partial outsourcing (in some contributions⁷ called "concurrent sourcing") is quite popular. In the automotive industry most brands adopt partial outsourcing, i.e., concomitant internal production and purchase, for instance, of engines and other intermediate products from external manufacturers. Moreover, the extent of outsourcing is frequently revised as witnessed by the variable level of value of purchased inputs over revenue found in most balance sheets.⁸

From the point of view of the value of a firm, different degrees of outsourcing and vertical flexibility may be associated with distinct degrees of risk born and, hence, different stock values. Then, it seems consequential examining how financial choices affect the riskiness and the quantity of flexibility acquired. As flexible technologies reduce risk (profit volatility) they may be considered a kind of (real) option whose price should reflect their (option) value (Amran and Kulatilaka, 1999, Ch. 16). As a result, a vertically flexible firm may have a value larger than the corresponding non flexible enterprise. However, as we shall see, this is not always the case if the cost of flexibility and the related financial aspects are properly accounted for.

In the ensuing pages we concentrate on two alternative cases: debt and venture capital. In the first the control right over the investment decision in flexibility is allocated to the firm (i.e., the shareholders), while in the second the control belongs with an outside investor, a venture capitalist. While the timing of the investment is set by one party the terms of the investment are determined by both parties. In both circumstances the level of outsourcing is set by the operating party. As to the financial sources, in the first case we deal with debt financing, while the second features a pure equity offer where risk, profits and, finally, ownership are shared with an outside investor without side payments (i.e., no debt service by the firm). We consider venture capital since it represents an important tool for firms, in particular in fast growing and innovation intensive sectors⁹ and, unlike debt, it provides a risky capital involvement that, as we shall see, makes the difference.

Our investigation assumes that debt is warranted to reflect the fairly popular practice of associating loans to innovative firms with call options on part of the capital of the borrowing firm. After all it may be hard to finance an investment unless the lender gets a handsome collateral. In our case this is represented by an option to buy the firm in case outsourcing makes production in-house relatively less convenient. The result is that debt makes a firm invest in the vertically flexible technology earlier than in the pure equity case. Nonetheless, when debt is insured by the associated option, the extent of flexibility acquired is equal to that adopted with equity. Only the timing can be affected since shareholders rush to reap profits from flexibility as soon as possible since they know that future may be gray due to the Damocles' sword of the buyout. When we move to the alternative case of venture capital outsourcing becomes lower than with equity. Risk sharing provided by venture capital makes a firm less willing to adopt outsourcing as insurance against cost uncertainty. Venture capital (a financial resource) appears to be a substitute for outsourcing (a real variable). The firm gets outside capital instead of externalizing a share of the vertical chain of production.¹⁰ Then, we establish a novel substitutability between a real internal organization choice and a financial variable, proving that finance and industrial setting are intertwined decisions.

¹ See, for a good survey of main related issues, Tirole (2006).

² See Van Mieghen (1999), Wang et al. (2007), Moretto and Rossini (2012).

³ We exclude from our investigation new equity raised through a capital increase since it tends to reduce the price of existing stock and may open the way to a loss of control. See Eckbo et al. (2007).

⁴ In general, issuing convertible bonds is one way for a firm to favor investors willingness to fund the investment (<http://www.entrepreneur.com/article/159520>).

⁵ Recent empirical literature emphasizes the weight of venture capital in the growth of infant firms (Hellmann and Puri, 2002; Jørgensen et al., 2006; Da Rin et al., 2011).

⁶ For instance Apple has recently increased the outsourcing of some inputs while reducing and bringing back home other inputs. See for further examples: The Economist (2011, 2013), Forbes (2012). See also empirical assessments in Klein (2005) and Rossini and Ricciardi (2005). Specific examples may be found in Benaroch et al. (2012). An interesting textbook case relating to the vertical flexibility story of toy giant Lego is narrated in Larsen et al. (2010). The historical evolution of outsourcing in Ford and GM may be read in Shih (2013). A web site containing updated news about reshoring in the U.S. is <http://www.reshorennow.org/news/>.

⁷ See for instance Lambrecht et al. (2016).

⁸ This ratio may change also for technology reasons – a new input is added or an existing input is abandoned – and because of changes in relative prices along the vertical chain of production.

⁹ See, in the vast literature on venture capital, Gompers and Lerner (2001) and Metrick and Yasuda (2010).

¹⁰ A growing empirical literature shows the importance that venture capital has for new firms (Hellmann and Puri, 2002; Jørgensen et al., 2006; Kaplan and Lerner, 2010; Da Rin et al., 2011). See also the Venture Capital Yearbook (National Venture Capital Association, 2014).

The paper roadmap is the following: in [Section 2](#) we go through some literature, in [Section 3](#) we see the basic model, in [Section 4](#) we study the value of a vertically flexible firm in the control case without debt, in [Section 5](#) we introduce debt with collateral, in [Section 6](#) we examine the case of venture capital. The epilogue is in [Section 7](#). The Appendix contains the proofs omitted from the text and some numerical background.

2. Literature review

Literature has examined vertical flexibility ([Shy and Stenbacka, 2005](#); [Alvarez and Stenbacka, 2007](#); [Moretto and Rossini, 2012](#); [Yoshida, 2012](#)) scantily going into the relationship with capital budgeting. Contributions on the link between industrial decisions and financial structure may be found in [Lederer and Singhal \(1994\)](#), in [Leland \(1998\)](#), in [Mauer and Sarkar \(2005\)](#), [Benaroch et al. \(2012\)](#), [Nishihara and Shibata \(2013\)](#), [Banerjee et al. \(2014\)](#), [Teixeira \(2014a,b\)](#), [Lambrecht et al. \(2016\)](#). A first point concerns the inefficiency of organization-strategic decisions when they are not taken simultaneously with financial choices. [Mauer and Sarkar \(2005\)](#) focus on the agency cost of financing investment with debt in a dynamic stochastic framework. In a similar framework [Leland \(1998\)](#) digs up the same topic raised in the seminal paper of [Jensen and Meckling \(1976\)](#) without examining flexibility. Unlike [Leland \(1998\)](#); [Mauer and Sarkar \(2005\)](#) emphasize the inefficiency of debt. In the traditional [Modigliani and Miller \(1958\)](#) scenario the value of a firm is given by the sum of its liabilities. Equity and debt turn out to be almost perfect substitutes. However, equity holders and debtholders never coincide and each group has a different objective function. Shareholders maximize the equity value while debtholders maximize the debt value. Equity holders, due to limited liability, tend to overinvest if they do not face the proper agency cost of debt confirming the old [Jensen and Meckling \(1976\)](#) wisdom. The consequence is a sub additive result. Only a “social planner” would rather maximize the sum of debt and equity pursuing a first best. [Mauer and Sarkar \(2005\)](#) calculate the agency cost of debt as the difference between the total value of a firm where each group of stakeholders optimizes separately and the case where the whole value of the firm is jointly maximized. Mainstream literature has partially investigated the relationship between the financial structure and the vertical setting. [Benaroch et al. \(2012\)](#) analyze the particular case of service production. Outsourcing is a shield for a firm facing volatile demand against the risk of bearing fixed costs that cannot be easily covered. By externalizing capital intensive services the firm translates a fixed into a variable cost, reducing risk. As a matter of fact the investment in a new technology, such as a flexible vertical process, financed by an external subject, may be seen as a joint option ([Banerjee et al., 2014](#)). Timing of the exercise of the option and the rule concerning the sharing of returns of the investment have to be established jointly by the firm and by the financial investor and it is inefficient to specify a sharing mode before the venture is carried out ([Banerjee et al., 2014](#)).¹¹ [Teixeira \(2014a,b\)](#) moves forward the joint analysis of financial and real aspects of vertical organization and associates highly leveraged outsourcing to high profit. Moreover, the effect of product market competition and capital structure is examined when a firm can choose between outsourcing based either on spot or long-term contracts. Competition between buyers may make outsourcing less desirable and more costly. In [Yoshida \(2012\)](#) the decisions of one agent affect the level of uncertainty of the scenario. In a two-players symmetric framework an increase in flexibility by one agent calls for a similar move by the rival. Flexibility appears as a strategic complement. The increase in (endogenous) uncertainty induces an investment delay. Unlike [Yoshida \(2012\)](#) in our framework the extent of flexibility is set (asymmetrically) only by the operating party to hedge against cost uncertainty. A fresh contribution by [Lambrecht et al. \(2016\)](#) is a clear leap forward in the relationships between finance and vertical setting. They go through the optimal vertical organization in a dynamic uncertain environment adopting a real options approach for outsourcing and in house production with decreasing returns under product demand uncertainty. The interesting result is that the extent of vertical integration affects the financial beta, a crucial component of a firm's stock value. Our way to model convertible debt to finance investment in vertical flexibility is new. However, there are many examples in the literature where convertible securities are used to enhance adoption of new technologies. Seminal papers are [Green \(1984\)](#), [Stein \(1992\)](#) and [Hellmann \(2006\)](#). [Kaplan and Stromberg \(2001, 2003, 2004\)](#) provide detailed evidence on the extensive use of these convertible securities. As far as the venture capital involvement in the funding of an innovative flexible production organization a large set of contributions provides evidence, contractual details and performances ([Hewitt, 2008](#); [Kaplan and Lerner, 2010](#); [Metrick and Yasuda, 2010](#)).

3. The model

In our endeavor we couple two streams of contributions: the first on vertical flexibility and the timing of adoption of a specific technology to carry out outsourcing ([Shy and Stenbacka, 2005](#); [Alvarez and Stenbacka, 2007](#); [Moretto and Rossini, 2012](#); [Alipranti et al., 2015](#)); the second on financial choices of a firm in an uncertain dynamic framework ([Leland, 1994](#);

¹¹ In a tiny empirical literature, [Bakhtiari and Breunig, \(2014\)](#) assess the role of outsourcing as a device to smooth demand uncertainty at firm level on longitudinal data. They find an asymmetric link with demand fluctuations, i.e., outsourcing increases substantially during slumps while does not respond much to demand increases. Some scanty data investigation on the financial counterpart of outsourcing is attempted but it is fairly inconclusive. In [Moon and Phillips \(2014\)](#) a higher level of outsourcing makes the firm less risky in terms of cash flows. The result is a capital structure with less debt and more equity mainly in high value-added industries.

Lederer and Singhal, 1994; Triantis and Hodder, 1990; Banerjee et al., 2014). We investigate the internal organization of a firm that manufactures a final good at a constant pace. A unit of a perfectly divisible input is needed for producing each unit of output (perfect vertical complementarity). For the input provision, the firm relies on a vertically flexible technology allowing for¹²:

- (i) internal production,
- (ii) (total or partial) outsourcing when the market price of the input decreases,
- (iii) back-sourcing if market conditions change.

Once the technology has been installed, the input may be produced in-house at the marginal cost d , or purchased on the market. The input market price, c_t , fluctuates according to the following geometric Brownian motion¹³:

$$dc_t/c_t = \gamma dt + \sigma d\omega_t, \text{ with } c_0 = c \quad (1)$$

where γ is the drift parameter, $\sigma > 0$ is the instantaneous volatility of the market input price and $d\omega_t$ is the standard increment of a Wiener process (or Brownian motion) uncorrelated over time.¹⁴

By using the technology introduced above the firm may, at any t , partially or fully purchase the input at the price c_t . Hence, denoting by $\alpha \in (0, 1]$ the outsourced share, it follows that the firm:

1. produces the input totally in-house when $d < \hat{c}_t = \alpha c_t + (1 - \alpha)d$, where $(1 - \alpha)$ is the share of input produced in-house,
2. buys the input when $d \geq \hat{c}_t$.

As $\alpha > 0$, the condition $\hat{c}_t > d$ holds whenever $c_t > d$, the instantaneous profit function is:

$$\begin{aligned} \pi_t &= \max \left[0, p - d + \max(d - \hat{c}_t, 0) \right] \\ &= \max \left[0, p - d + \alpha * \max(d - c_t, 0) \right] \end{aligned} \quad (2)$$

where p is the output market price. With $\alpha < 1$ we have partial (or concurrent) outsourcing. The firm uses a linear combination of produced and procured input when $c_t \leq d$. It can go back to vertical integration if $c_t > d$. Notice that, with $\alpha = 1$, the firm buys the input entirely, while keeping the option of returning to complete internal manufacturing. Finally, to exclude default, we assume that $p - d > 0$.¹⁵

Let us consider now the cost of the flexible technology. This is given by¹⁶:

$$I(\alpha) = k_1 + (k_2/2)\alpha^2 \text{ for } \alpha \in (0, 1] \quad (3)$$

where k_1 is the direct cost to install and to keep internal facilities working (i.e., the cost of maintaining and updating the process for the internal production of the input) with total or partial outsourcing. The term $(k_2/2)\alpha^2$ is the organizational cost to design and run a flexible system combining in-house production and outsourcing of a specific input (Simester and Knez, 2002). This cost may then cover, for instance, the set-up of a logistically sustainable supply chain of subcontractors, monitoring input quality, contract enforcement, etc.¹⁷ Most of the organizational arrangements are tailored in order to reduce or eliminate the switching costs that the firm may face when moving from an operative regime to the other. Then,

¹² For the sake of exposition, in the following, we will consider as “technology” the arrangement chosen by the firm in terms of optimal combination of in-house and potentially outsourceable input.

¹³ The dynamic setting adopted implies that the input market is perfectly competitive or that the forces moving the price over time do not depend on the market structure. A different approach is adopted by Bilette de Villemeur et al. (2014) where an imperfect market for the input in the upstream section of production makes the firm delay entry.

¹⁴ When the input is purchased abroad, the price uncertainty may be due to the fluctuating exchange rate (see Kogut and Kulatilaka, 1994; Dasu and Li, 1997; Kouvelis et al., 2001).

¹⁵ Vertical flexibility, as maintained in the introduction, is an insurance against risk and requires updating the know-how and keeping the facilities to produce the input in house. This assumption allows us to focus on differential financial arrangements and see how they affect the decision as to whether, when and how much to invest in the flexible technology and outsourcing. The consideration of the option to default would not affect the quality of our conclusions. Moreover, the profit function draws on a linear technology with one input. Again, this is a simplifying assumption so as to concentrate on the effect that various financial sources have on the firm's cost minimization. This assumption may reflect a price-taker firm facing constant order rates over time. The assumption of a constant p can be relaxed. Handling two stochastic variables would however make heavier the analytical apparatus without having any substantial impact on the quality of our results.

¹⁶ The investment cost is assumed quadratic only for the sake of simplicity. None of the results is altered if one allows for a more general functional form such as $I(\alpha) = k_1 + (k_2/\delta)\alpha^\delta$ with $\delta > 1$ (see Alvarez and Stenbacka, 2007).

¹⁷ The increasing cost of recurring to outsourcing may be seen as the mirror image of a (specificity based) hold-up which grows with the share of outsourcing as in Transaction Cost Economics (TCE) that emphasizes how hold-up in outsourcing relationships make input markets less efficient than internal production (Williamson, 1971; Joskow, 2005; Whinston, 2003). Of course generic inputs like, for instance, janitorial services do not require specific know how and cannot be modeled in this way (see e.g. Anderson and Parker, 2002; Holmes and Thornton, 2008) while for other services flexibility of outsourcing may matter a lot (Benaroch et al., 2012).

we assume that the decision on the level of outsourcing is irreversible, i.e., α , once chosen, cannot be changed, while a specific operative regime may be changed at no cost.¹⁸

Finally, we do not consider investment in capacity expansion and assume that the new facility is already optimally employed to meet demand producing the input in-house. This requires $(p - d)/r > k_1$ where r is a positive constant risk free interest rate. Therefore, we explicitly exclude the case $\alpha = 0$ with $k_1 > 0$.¹⁹

As anticipated in the introduction, the firm may finance the adoption of the flexible technology in two alternative ways, that is: (1) by issuing a perpetual debt paying a yearly coupon D that debtholders may convert into equity at certain times or (2) by venture capital. In both cases we suppose that the capital markets are frictionless and that there are no information asymmetries between shareholders, lenders and venture capitalists. All agents are assumed to be risk neutral. Last, to assure convergence, we require that $r > \gamma$.²⁰

4. The benchmark case: the unlevered vertically flexible firm

In this section we derive the value of the operating facility, the optimal outsourcing share and the optimal investment policy of a firm entirely financed by equity holders.

4.1. The operating value

We examine the firm's operating value allowing for two potential scenarios. First, if $c_t > d$ we have a vertically integrated firm manufacturing the input in house while keeping the option of buying it in the market whenever convenient, i.e., when $c_t < d$. Second, if $c_t < d$ the firm outsources a share α of the input while making in-house the remaining $1 - \alpha$, keeping the option to manufacture the whole input in-house whenever convenient.

A standard pricing argument leads to the following general solution for the unlevered operating firm's value (see Appendix A):

$$F^U(c_t; \alpha) = \begin{cases} \frac{p-d}{r} + \tilde{A}c_t^{\beta_2} & \text{if } c_t > d, \\ \left[\frac{p-(1-\alpha)d}{r} - \alpha \frac{c_t}{r-\gamma} \right] + \tilde{B}c_t^{\beta_1} & \text{if } c_t < d. \end{cases} \quad (4)$$

where $\beta_2 < 0$ and $\beta_1 > 1$ are, respectively, the negative and the positive roots of the characteristic equation $\Phi(\beta) = (1/2)\sigma^2\beta(\beta - 1) + \gamma\beta - r$ and

$$\begin{aligned} \tilde{A}(\alpha) &= \alpha A \equiv \alpha \frac{r - \gamma\beta_1}{r(\beta_1 - \beta_2)(r - \gamma)} d^{1-\beta_2}, \\ \tilde{B}(\alpha) &= \alpha B \equiv \alpha \frac{r - \gamma\beta_2}{r(\beta_1 - \beta_2)(r - \gamma)} d^{1-\beta_1}. \end{aligned} \quad (4.1)$$

Notice that $F^U(c_t; \alpha)$ is a convex function of c_t , with $\lim_{c \rightarrow \infty} F^U(c_t; \alpha) = (p - d)/r$ and $\lim_{c \rightarrow 0} F^U(c_t; \alpha) = [p - (1 - \alpha)d]/r$. The terms, $\frac{p-d}{r}$ and $[\frac{p-(1-\alpha)d}{r} - \alpha \frac{c_t}{r-\gamma}]$ are the present values of the firm associated with the two distinct vertical arrangements. As it appears from Eq. (4), viable in-house production rules out any closure option or default. The additional terms $\tilde{A}c_t^{\beta_2}$ and $\tilde{B}c_t^{\beta_1}$ represent the value of the option to switch from vertical integration to outsourcing and the other way round, respectively. Notice that if $\alpha \rightarrow 0$, i.e., the firm is vertically integrated, both $\tilde{A}(\alpha)$ and $\tilde{B}(\alpha)$ tend to 0. In this limit case, due to the extreme level of vertical integration, the value of productive flexibility is, as illustrated by the corresponding two options, null. In contrast, if $\alpha \rightarrow 1$, as the input may be purchased entirely from an independent provider, the value associated with the underlying productive flexibility is, ceteris paribus, as high as possible.

4.2. Optimal outsourcing share and investment timing

Since equity is a perpetual claim, the optimal investment timing can be expressed equivalently in terms of the optimal input market price, c^{*U} , triggering investment in the flexible technology by the equity holders. Working backward, we first determine the optimal α , that is, the outsourcing share maximizing the firm's net present value (NPV) once the new

¹⁸ We share this assumption with Lambrecht et al. (2016). Note that the consideration of switching costs would not impact on the quality of our conclusions. They would in fact only produce a hysteresis interval affecting the timing of switching from in-house input production to outsourcing and vice versa. See Benaroch et al. (2012) for a model including switching costs.

¹⁹ The case where $k_1 > 0$ and $\alpha = 0$ represents the standard case where the firm invests in a plant with exclusive in-house input provision. Note that in this paper we will abstract from the analysis of this case (see also footnote 22).

²⁰ Alternatively, under the assumption of complete capital markets, we may assume that there are some traded assets that can be used to hedge the input cost uncertainty z_t of (1). These traded assets together with a riskless asset allow to construct a continuously re-balanced self-financing portfolio that replicates the value of the firm (Cox and Ross, 1976; Constantinides, 1978; Harrison and Kreps, 1979).

technology has been installed. Then, by maximizing the ex-ante value of the firm, we get the optimal investment threshold c^{*U} .

Consider the firm manufacturing the input in-house, while holding the option to switch to outsourcing, at a future date, if c_t becomes lower than d .²¹ We determine the optimal α by solving the following problem:

$$\alpha^{*U} = \operatorname{argmax} \left[\frac{p-d}{r} + \alpha A c_t^{\beta_2} - I(\alpha) \right], \tag{5}$$

i.e., by maximizing the value of the firm in Eq. (4) minus the cost of setting up a dedicated production organization allowing for outsourcing.

Solving Problem (5) we obtain:

$$\alpha^{*U}(c_t) = \begin{cases} 1 & \text{if } c_t \leq \tilde{c}^U \\ (A/k_2)c_t^{\beta_2} & \text{if } c_t > \tilde{c}^U \end{cases} \tag{6}$$

where $\tilde{c}^U = (k_2/A)^{1/\beta_2}$. Note that the optimal α is decreasing in c_t , i.e., $\partial\alpha^{*U}/\partial c_t < 0$. This relationship reads the current and future value that one may attach to flexibility. In fact, the higher is c_t the less likely is its fall. As a consequence, the less likely one may benefit from having invested in flexibility. On the contrary, as c_t decreases, α rises and one may find optimal, for relatively low values of c_t , investing massively in flexibility, i.e., $\alpha^{*U} = 1$.

Let's now turn to the optimal investment policy. The value of the option to invest, i.e., the ex-ante value of the firm, is given by:

$$O^U(c_t, c^{*U}) = \max_{T^{U*}} E_t \left[e^{-r(T^{U*}-t)} \right] \left[F^U(c^{*U}, \alpha^{*U}(c^{*U})) - I(\alpha^{*U}(c^{*U})) \right] \tag{7}$$

where $T^{U*} = \inf\{t \geq 0 \mid c_t = c^{*U}\}$ is the optimal investment timing and $\alpha^{*U}(c^{*U})$ is the optimal outsourcing share to be chosen at $t = T^{U*}$.²²

The standard pricing arguments used for determining $F^U(c_t; \alpha)$ can be applied to solve Problem (7) and determine the optimal investment threshold c^{*U} . Consider a productive organization allowing for partial or even total outsourcing, i.e., $\alpha^{*U} \leq 1$. If with the current input market price, $c_0 = c$, immediate investment is not optimal, we can show that²³ :

Proposition 1. *Provided that $c_0 = c \geq \tilde{c}^U$ and $\tilde{c}^U \geq d$ (or $Ad^{\beta_2} \geq k_2$), the optimal investment thresholds and the corresponding levels of outsourcing are*

$$c^{*U} = \begin{cases} \left\{ \frac{[2k_2(\frac{p-d}{r} - k_1)]^{1/2}}{A} \right\}^{1/\beta_2} & \geq \tilde{c}^U \quad \text{if } \frac{p-d}{r} \leq k_1 + \frac{k_2}{2} \\ c, & \text{if } \frac{p-d}{r} > k_1 + \frac{k_2}{2} \end{cases} \tag{8}$$

and

$$\alpha^{*U}(c^{*U}) = \begin{cases} \left(\frac{2\frac{p-d}{r} - k_1}{k_2} \right)^{1/2} \leq 1, & \text{if } \frac{p-d}{r} \leq k_1 + \frac{k_2}{2} \\ Ac^{\beta_2}/k_2 \leq 1 & \text{if } \frac{p-d}{r} > k_1 + \frac{k_2}{2} \end{cases} \tag{9}$$

Proof. See Appendix A.

The balance between the expected present value of the net cash flows from manufacturing the input in house, $\frac{p-d}{r}$, and the investment cost of a flexible technology, $k_1 + \frac{k_2}{2}$, is crucial for the optimal timing. The investment is postponed when

²¹ In this paper we focus on the problem of adopting a new technology. Taking a different starting point would not make sense, as the option to switch to outsourcing in the future exists only if the firm is not outsourcing now.

²² The model can be easily adapted to the case of a firm already producing output with an input made in-house. One may then consider an investment project that, through a vertical reorganization, provides, as alternative, the option to outsource the input. In this case, the value of the option to invest is:

$$\hat{O}^U(c_t) = \max \left[O^U(c_t), \frac{p-d}{r} \right]$$

where $O^U(c_t)$ is given by Eq. (10). Last, the fixed investment component k_1 should be lowered in order to take into account that in-house facilities are in place yet.

²³ In Appendix A (Scenario 2.A), similar results are obtained when $\tilde{c}^U < d$ (or $Ad^{\beta_2} < k_2$). The firm always opts for a production organization allowing for partial outsourcing. In Scenario 1.2.A the current value of c is such that $\tilde{c}^U \geq c \geq d$. The degree of flexibility degenerates to $\alpha^{*U} = 1$ and the investment timing problem has no interior solution.

$\frac{p-d}{r} \leq k_1 + \frac{k_2}{2}$, otherwise it is optimal investing immediately. The firm puts off investment only when the net present value of future cash flows (from in-house input production) does not cover the investment cost. By waiting, the gap is bridged thanks to the increased value of the option to outsource as c_t goes down.

Once identified the investment threshold, the value of the option to invest can be determined by plugging c^{*U} into Eq. (7), yielding:

$$O^U(c_t; c^{*U}) = (c_t/c^{*U})^{\beta_2} [F^U(c^{*U}, \alpha^{*U}(c^{*U})) - I(\alpha^{*U}(c^{*U}))], \quad \text{for } c_t \geq c^{*U} \tag{10}$$

where $E_t[e^{-r(T^{*U}-t)}] = (c_t/c^{*U})^{\beta_2}$ is the stochastic discount factor based on the probability that the investment will be carried out.²⁴ If $\alpha^{*U}(c^{*U}) \leq 1$, as the firm maintains the ability to produce the input in-house, the value of the option to invest coincides with the value of the option to outsource, i.e.,

$$O^U(c_t; c^{*U}) = \alpha^{*U}(c^{*U}) A c_t^{\beta_2}. \tag{10.1}$$

The intuition? Once the investment is undertaken, the new flexible technology allows to produce the input in-house holding the option to outsource. The firm will find optimal to invest when the profit from producing the input in-house is sufficiently high to cover the investment cost adjusted by the degree of flexibility that maximizes the net present value of the investment project. Ex-ante, this explains why the value of the option to invest coincides with the value of the option to outsource.

5. Debt funding with a takeover option (warrant)

The firm negotiates a contract with an (financial) investor to get the funds to cover part of the cost of the initial investment paying a fixed coupon $D > 0$ per year. The contract makes a provision for a call option to be handed over to debtholders who can exercise it to buy out the equity should outsourcing make it profitable. As said in the introduction, there are several cases reported in the literature and in the specialized press where convertible securities or securitized debt are adopted as means to finance the introduction of new and risky technologies which usually require changes in firm organization.²⁵ This call option is a collateral for debt, i.e., a kind of (costly) “sweetener” for the investor.²⁶ The contract indicates a specific covenant allowing the lender to buy out the firm. Then, a rational shareholder signs the contract only if the coupon $D < p - d$.

Further, if a takeover occurs, the lender keeps producing shutting down the in-house input production facility. This restructuring decision is costly, yet it may entail a potential revenue, through the recovery of part of the initial fixed investment. We denote the relative cash flow by k_3 and assume that $k_3 > -k_1$.²⁷

The sequence of moves, with debt funding, is the following: first the firm and the lender decide the terms of the deal (i.e., the coupon and the buyout option covenant). Then, the firm optimally sets both the level of flexibility α and the investment time schedule while the lender chooses the size of the loan and the buyout timing.²⁸

5.1. The operating value

Lemma 1. *The value of debt is:*

$$D(c_t; \alpha) = \begin{cases} \frac{D}{r} + \left(\frac{c_t}{c^l}\right)^{\beta_2} \left[\left(\frac{p}{r} - \frac{c^l}{r-\gamma}\right) - \left(k_3 + \frac{D}{r}\right) \right] & \text{if } c_t > c^l, \\ \frac{p}{r} - \frac{c_t}{r-\gamma} - k_3 & \text{if } c_t \leq c^l. \end{cases} \tag{11.1}$$

²⁴ The expected present value $E_t[e^{-r(T^{*U}-t)}] = (c_t/c^{*U})^{\beta_2}$, can be determined by using dynamic programming (see e.g. Dixit and Pindyck, 1994, pp. 315–316).

²⁵ See Green (1984), Stein (1992) and Hellmann (2006). A great deal of evidence on the use of convertible debt for innovative investment is in Kaplan and Stromberg (2001, 2003, 2004).

²⁶ The loan is a convertible (into equity) debt. Almost any debt is liable to be convertible into a collateral since each debt implies a kind of pawn. There are several types of conversion of debt according to the financial rules and the legal framework of the contract.

²⁷ There are several ways to restructure the firm after the buyout. We opt for the simplest mode given that the buyout occurs when outsourcing is far more profitable than in-house production. In those circumstances the lender considers internal production of the input unnecessarily expensive and sets $\alpha = 1$. Nonetheless, none of our results depends on this assumption.

²⁸ The evaluation of debt may take place in different scenarios. We confine to a simple framework where the lender buys out the entire equity and adopts the outsourcing setting chosen by incumbent shareholders. We may alternatively consider an option to buy just a chunk of the firm. In Appendix B, at no loss in terms of quality of results, we present the general case where the takeover does not involve the entire equity. Moreover, the Yoshida (2012) symmetry is absent. The extent of flexibility is set (asymmetrically) only by the operating party.

where

$$c^l = \frac{\beta_2}{\beta_2 - 1} (r - \gamma) \left(\frac{p - D}{r} - k_3 \right) \tag{11.2}$$

is the buyout threshold.

Proof. See Appendix B.

The multiple $\frac{\beta_2}{\beta_2 - 1}$ is the wedge between the debtholders' actual investment cost and the benefit. The cost is made by the foregone flow of coupons plus the switching cost ($\frac{D}{r} + k_3$). The gross benefit is the cash flow ($\frac{p}{r}$). The ratio $\frac{\beta_2}{\beta_2 - 1} < 1$ embodies the uncertainty and the irreversibility of the decision to restructure and says that the buyout occurs when the market input price has gone substantially low. Hence it is better to buy the input forever and scrap the option to backsource. Some comparative statics say that:

$$\frac{\partial c^l}{\partial k_3} = -\frac{rc^l}{p - D - rk_3} < 0 \quad \text{and} \quad \frac{\partial c^l}{\partial D} = -\frac{c^l}{p - D - rk_3} < 0.$$

In the first the higher is the cost ($k_3 > 0$), or the lower is the associated benefit ($0 \geq k_3 > -k_1$), the later the buyout occurs. The second maintains that an increase in the coupon (the benefit for the lender) induces a decrease in the threshold, i.e., making the takeover less likely. In other words, a larger coupon makes the lender less eager to buy out the firm by converting debt into equity. The assumption $p - D - rk_3 > 0$ guarantees that c^l is positive. For $p - D - rk_3 \leq 0$ the buyout never materializes (i.e., $c^l \leq 0$) and, consequently, the option is worthless. Hence, we assume that $p - D - rk_3 > 0$.

5.1.1. The value of equity

Letting $E(c_t; D)$ be the market value of the levered equity, standard pricing arguments yield:

Lemma 2. The value of levered equity (for incumbent shareholders) is:

$$E(c_t; \alpha) = \begin{cases} \frac{p - d - D}{r} + \tilde{A}c_t^{\beta_2} - \left(\frac{c_t}{c^l}\right)^{\beta_2} M(c^l, \alpha) & \text{if } c_t > d, \\ \left[\frac{p - (1 - \alpha)d - D}{r} - \alpha \frac{d}{r - \gamma} \right] + \tilde{B}c_t^{\beta_1} - \left(\frac{c_t}{c^l}\right)^{\beta_2} M(c^l, \alpha) & \text{if } c^l < c_t < d \\ 0 & \text{if } c_t \leq c^l \end{cases} \tag{12}$$

where

$$M(c^l, \alpha) = \left[\frac{p - (1 - \alpha)d - D}{r} - \alpha \frac{c^l}{r - \gamma} + \tilde{B}c^{l\beta_1} \right].$$

Proof. See Appendix B.

As above, the terms $\tilde{A}c_t^{\beta_2}$ and $\tilde{B}c_t^{\beta_1}$ indicate the value of the option to switch from vertical integration to outsourcing and the other way round, respectively. The term $E_t[e^{-r(T^l-t)}] * M(c^l, \alpha)$, where $E_t[e^{-r(T^l-t)}] = (c_t/c^l)^{\beta_2}$ is the stochastic discount factor and $T^l = \inf\{t \geq 0 \mid c_t = c^l\}$ is the buyout timing,²⁹ indicates the loss for incumbent shareholders due to the potential buyout. Thus, the presence of this option reduces the market value of equity. This loss can be interpreted as a kind of agency cost that the equity has to pay to the lender since shareholders maximize only the equity value and not the entire value of the firm made by debt plus equity (Mauer and Sarkar, 2005).³⁰

²⁹ Note that c^l must be lower than the internal cost of production d , otherwise, a buyout does not make sense. Why should equityholders borrow money to invest in a flexible technology which would be bought out before it pays off? This holds if (See Appendix B):

$$c^l \leq d \rightarrow \frac{p - D}{r} \leq \left(1 - \frac{1}{\beta_2}\right) \frac{d}{r - \gamma} + k_3.$$

³⁰ In the absence of any agency fee shareholders would excessively increase debt since they are protected by limited liability that sets a boundary on losses which never exceed equity while leaving to shareholders the opportunity of getting the upside cream, i.e., profits, in bonanza times. This occurs in markets in which there is some degree of asymmetric information.

5.1.2. The value of the levered firm

By Lemmas 1 and 2, the market value of the levered firm is given by:

$$F^L(c_t; \alpha) = E(c_t; \alpha) + D(c_t; \alpha) = \begin{cases} F^U(c_t; \alpha) - Z(c_t; c^l) & \text{if } c_t > c^l, \\ \left(\frac{p}{r} - \frac{c_t}{r - \gamma}\right) - k_3 & \text{if } c_t \leq c^l, \end{cases} \tag{13}$$

where $Z(c_t; c^l) = (c_t/c^l)^{\beta_2} [\tilde{B}c^{\beta_1} - (1 - \alpha)(\frac{d}{r} - \frac{c^l}{r - \gamma}) + k_3]$.

Notice that the Modigliani–Miller irrelevance theorem does not hold here. The value of the levered firm is equal to the value of the unlevered firm, $F^U(c_t; \alpha)$, minus the present value of the payoff associated with the buyout, which accrues to the debtholders converting their debt. This payoff comes from the restructuring of the firm's operations. It is the present value of future incremental cash flows, due to the decision of fresh owners to set $\alpha = 1$, $-(1 - \alpha)(\frac{d}{r} - \frac{c^l}{r - \gamma})$, minus the implicit cost of the operation, i.e., the foregone value of the option to produce the input in-house, $\tilde{B}c^{\beta_1}$, plus the switching cost $k_3 > 0$ (or benefit if $0 \geq k_3 > -k_1$).

If $c^l \rightarrow 0$ the firm is never bought by the lender. Then $Z(c_t; c^l) \rightarrow 0$. We are back to the unlevered firm as illustrated in Section 4. The value of the firm does not depend on debt but on the collateral. Without it, the value of the firm would be the sum of debt and equity and the use of debt would not reduce the value of the firm, i.e., $F^L(c_t; \alpha) = F^U(c_t; \alpha)$.

Once again, the term $Z(c_t; c^l)$ can be viewed as the agency cost of debt and it may mark the distance between a perfectly competitive complete debt market and an imperfect one where collaterals are required.

5.2. The optimal outsourcing share and the investment timing

Since equity holders control both the decision about the outsourcing share and the timing of the investment, we proceed as above by determining first α^{*L} and then c^{*L} . To get α^{*L} , equity holders maximize Eq. (12) minus the cost of setting up the production organization:

$$\alpha^{*L} = \operatorname{argmax} [E(c_t; \alpha) - (I(\alpha) - K^L)] \tag{14}$$

where $K^L \leq I(\alpha)$ is the share of the investment expenditure paid by the lender who controls the amount to loan and the buyout timing. Since a rational investor will not agree to finance the firm unless K^L is a (financially) fair price for the debt, we set $K^L = D(c_t; \alpha)$ for $c_t > c^l$.³¹ Then, substituting in Eq. (14), we obtain:

$$\alpha^{*L} = \operatorname{argmax} [F^L(c_t; \alpha) - I(\alpha)] \tag{15}$$

where $F^L(c_t; \alpha)$ is given by Eq. (13).

As before, let's consider a firm manufacturing in-house the input, while holding the option to switch to outsourcing. By the first-order condition for Problem (15), the optimal outsourcing share is the solution of the following equation:

$$Ac_t^{\beta_2} - \left(\frac{c_t}{c^l}\right)^{\beta_2} \left[Bc^{\beta_1} + \left(\frac{d}{r} - \frac{c^l}{r - \gamma}\right) \right] - k_2\alpha^{*L}(c_t) = 0 \tag{16}$$

where A and B are as in Eq. (4.1).

Since $\partial\alpha^{*L}/\partial c_t < 0$, if c_t is low it is optimal to choose $\alpha^{*L} = 1$, while, as c_t increases α goes down and tends to zero for high values of c_t . Further, if $c^l \rightarrow 0$, then $\alpha^{*L} \rightarrow \alpha^{*U}$. The value of the option to invest in the vertically flexible technology is equal to:

$$O^L(c_t, c^{*L}) = \max_{T^{*L}} E_t \left[e^{-r(T^{*L}-t)} \right] [F^U(c^{*L}, \alpha^{*L}(c^{*L})) - I(\alpha^{*L}(c^{*L}))].$$

where $T^{*L} = \inf\{t \geq 0 \mid c_t = c^{*L}\}$ is the optimal investment timing.

Then, going through the same steps as before, we can prove that:

Proposition 2. *Provided that $c_0 = c \geq \tilde{c}^L$ and $\tilde{c}^L \geq d$ (or $Ad^{\beta_2} \geq k_2$), the optimal investment thresholds and the corresponding levels of outsourcing are³²:*

$$c^{*L} = \begin{cases} \left\{ \frac{[2k_2(\frac{p-d}{r} - k_1)]^{1/2}}{A - (\frac{1}{c^l})^{\beta_2} [Bc^{\beta_1} + (\frac{d}{r} - \frac{c^l}{r - \gamma})]} \right\}^{1/\beta_2} & \text{if } \frac{p-d}{r} \leq k_1 + \frac{k_2}{2} \\ c, & \text{if } \frac{p-d}{r} > k_1 + \frac{k_2}{2} \end{cases} \tag{17}$$

³¹ Note that the lender chooses the amount of the loan as a function of c_t . That is, as in Mauer and Sarkar (2005), the contract may be seen as a revolving credit line where the firm decides when to use it.

³² We propose in Appendix B the analysis of two cases: (i) $\tilde{c}^L < d$ (or $Ad^{\beta_2} < k_2$) and $\alpha^{*L} < 1$ (Scenario 2.B) and (ii) c is such that $\tilde{c}^L \geq c \geq d$ and $\alpha^{*U} = 1$ (Scenario 1.2.B).

and

$$\alpha^{*L}(c^{*L}) = \begin{cases} \left(2 \frac{p-d}{r} - k_1\right)^{1/2} / k_2 \leq 1, & \text{if } \frac{p-d}{r} \leq k_1 + \frac{k_2}{2} \\ Ac^{\beta_2} / k_2 \leq 1 & \text{if } \frac{p-d}{r} > k_1 + \frac{k_2}{2} \end{cases} \quad (18)$$

Proof. See Appendix B.

Substituting Eqs. (17)–(18) in $O^L(c_t)$, we can rearrange the value of the option to invest as follows:

$$O^L(c_t; c^{*L}) = \alpha^{*L}(c^{*L})Ac_t^{\beta_2} - Z(c_t; c^L) \text{ for } c_t > c^{*L}. \quad (19)$$

Unlike the case of pure equity, the value of investing is reduced by the term that captures, among others, the value of the option to buy out held by debtholders. We sum up the comparison with the unlevered firm in the following:

Proposition 3. *The levered firm invests earlier than the unlevered, i.e.,*

$$c^{*L} \geq c^{*U} \quad (20.1)$$

but adopts the same proportion of outsourced input, i.e.,

$$\alpha^{*L} = \alpha^{*U}. \quad (20.2)$$

The value of the option to invest is lower for the levered firm than for the unlevered, i.e.,

$$O^L(c_t) < O^U(c_t), \quad \text{for } c_t > c^{*L} \geq c^{*U} \quad (20.3)$$

Since the levered firm decides both α^{*L} and c^{*L} by maximizing only the value of equity, it has no reason to change the level of outsourcing. Part of the investment is paid by the lender and the risk born by the equity holders is just represented by the buyout option in the hands of the lender. Then, the equity holders have an incentive to invest earlier to reap profits as soon as possible.

6. Venture capital

In this section we examine a second financial arrangement involving risk capital. We consider a firm offering to an outside investor, a venture capitalist, a share of profits $\psi \in (0, 1)$ in exchange for partially funding the investment in the flexible technology. The venture capitalist decides when the deal should be implemented by setting the optimal investment threshold c^{*V} . The equity holders control the investment technological design by tuning the optimal outsourcing share α^{*V} .³³

The decision frame is modified with respect to debt where equity holders set both investment timing and the share of outsourced input. Now the sequence of moves is the following:

- (i) the equity holders offer ψ ,
- (ii) the venture capitalist observes the realizations of c_t and decides when to accept the offer ψ and invest,
- (iii) the equity holders set the optimal share of outsourcing.

As before we proceed backwards. First, the equity holders decide $\alpha^{*V}(c_t)$. Then, the venture capitalist, given the optimal reaction function $\alpha^{*V}(c_t)$, sets the optimal investment threshold c^{*V} . Equity holders offer ψ before investment takes place, i.e., at $t < T^{*V}$, and commit to carry it out.³⁴

Equity holders select the optimal α as follows:

$$\alpha^{*V} = \operatorname{argmax}[(1 - \psi)F^U(c_t; \alpha) - (I(\alpha) - K^V)] \quad (21)$$

where $K^V \leq I(\alpha)$ is the transfer set by the venture capitalist in order to co-fund the investment and $(1 - \psi)F^U(c_t; \alpha)$ is the portion of project value compensating equity holders for their contribution to the coverage of initial investment costs, i.e., $I(\alpha) - K^V \geq 0$.

³³ For an exhaustive presentation of the variety of venture capital arrangements see [Hewitt \(2008\)](#) and [Metrick and Yasuda \(2010\)](#) and the example reported in the introduction.

³⁴ We can model the above framework as a sequential game where, at each time $s \geq t$, equity holders offer ψ and the venture capitalist may accept or reject the offer. Thus, at every point of time, the external investor has the action set [Accept, Reject] that can be seen as a perpetual call option. See [Lukas and Welling \(2014\)](#) for an application of this game to supply chains.

Solving Problem (21) yields:

$$\alpha^{*V}(c_t) = \begin{cases} 1 & \text{if } c_t \leq \bar{c}^V \\ (1 - \psi)(A/k_2)c_t^{\beta_2} & \text{if } c_t > \bar{c}^V \end{cases} \quad (22)$$

where $\bar{c}^V = [k_2/(1 - \psi)A]^{1/\beta_2} \leq \bar{c}^U$ for $\psi \in (0, 1]$.

Let's now turn to the optimal investment policy set by the venture capitalist. The value of the option to invest held by the venture capitalist is:

$$O^V(c_t) = \max_{T^*} E_t \left[e^{-r(T^* - t)} \right] [\psi F^U(c^{*V}, \alpha^{*V}(c^{*V})) - K^V] \quad (23)$$

where $T^{*V} = \inf\{t \geq 0 \mid c_t = c^{*V}\}$ is the optimal investment timing and $\alpha^{*V}(c^{*V})$ is the corresponding optimal outsourcing share. Then, going through the same steps as before, we can prove that:

Proposition 4. *Provided that $c_0 = c \geq \bar{c}^V$ and $\bar{c}^V \geq d$ (or $Ad^{\beta_2} \geq k_2/(1 - \psi)$), the optimal investment thresholds and the corresponding levels of outsourcing are ³⁵ :*

$$c^{*V} = \begin{cases} \left\{ \left[\frac{k_2}{1 - \psi} \left(\frac{p-d}{r} - \frac{K^V}{\psi} \right) \right]^{1/2} \right\}^{1/\beta_2} & \text{if } \frac{p-d}{r} \leq \frac{K^V}{\psi} + \frac{k_2}{1 - \psi} \\ c, & \text{if } \frac{p-d}{r} > \frac{K^V}{\psi} + \frac{k_2}{1 - \psi} \end{cases} \quad (24)$$

and

$$\alpha^{*V}(c^{*V}) = \begin{cases} \left[\frac{1 - \psi}{k_2} \left(\frac{p-d}{r} - \frac{K^V}{\psi} \right) \right]^{1/2} \leq 1, & \text{if } \frac{p-d}{r} \leq \frac{K^V}{\psi} + \frac{k_2}{1 - \psi} \\ (1 - \psi)Ac^{\beta_2}/k_2 \leq 1 & \text{if } \frac{p-d}{r} > \frac{K^V}{\psi} + \frac{k_2}{1 - \psi} \end{cases} \quad (25)$$

Proof. See Appendix C.

Substituting Eqs. (24) and (25) into Eq. (23) the value of the option to invest is equal to

$$O^V(c_t; c^{*V}) = 2 \left(\frac{c_t}{c^{*V}} \right)^{\beta_2} \left(\psi \frac{p-d}{r} - K^V \right) \quad (23.1)$$

Note that the value associated with this option is positive if and only if $\psi > \underline{\psi} = K^V/(p-d)$. That is, the share on the periodic profits accruing when the firm is operating by producing the input in-house, $\psi(p-d)$, must cover the rental cost of the capital contribution provided by the venture capitalist, rK^V . Hence, as immediate consequence, this condition secures also the existence and the finiteness of the optimal trigger c^{*V} . The characterization of the optimal investment policy is then completed by imposing that $\frac{p-d}{r} \leq \frac{K^V}{\psi} + \frac{k_2}{1-\psi}$. Note that this is in fact the necessary condition for having $c^{*V} \geq \bar{c}^V$ and then $\alpha^{*V} \leq 1$.

Note that for $\frac{p-d}{r} > \frac{K^V}{\psi} + \frac{k_2}{1-\psi}$, the option to invest $O^V(c_t, c^{*V})$ is always increasing in c^{*V} which implies that the venture capitalist invests immediately, i.e., at $c_0 = c$, and sets $\alpha^{*V}(c^{*V}) = (1 - \psi)Ac^{\beta_2}/k_2$. In contrast, for $\frac{p-d}{r} \leq \frac{K^V}{\psi} + \frac{k_2}{1-\psi}$ the investment is always postponed, i.e., $c^{*V} < c$.

Rearranging Eq. (23.1) the value of the option to invest is equal to

$$O^V(c_t; c^{*V}) = 2\psi\alpha^{*V}(c^{*V})Ac_t\beta_2 \quad (26)$$

i.e., 2ψ times the value of the option to outsource. Hence, the value of the firm is split in equal parts between the shareholders and the venture capitalist only when $\psi = 1/2$.

In the Appendix we show that:

Proposition 5. *Provided that $K^V \leq k_1$, a financial arrangement (ψ, K^V) is feasible and profitable for both parties for any $\psi \in (\underline{\psi}, 1)$.*

³⁵ We provide in Appendix C the analysis of the cases: (i) $\bar{c}^V < d$ (or $Ad^{\beta_2} < k_2/(1 - \psi)$) and $\alpha^{*V} < 1$ (Scenario 2.C) and (ii) c is such that $\bar{c}^V \geq c \geq d$ and $\alpha^{*V} = 1$ (Scenario 1.2.C).

Otherwise, i.e., $K^V > k_1$, provided that

$$\frac{p-d}{r} \geq K^V + \Theta > K^V, \tag{27}$$

where $\Theta = 2(\sqrt{2\frac{p-d}{r}K^V} - k_1)$, the set of feasible financial arrangements (ψ, K^V) reduces to the deals where $\psi \in [\underline{\psi}_1, \bar{\psi}]$ where $\underline{\psi} < \underline{\psi}_1$ and $\bar{\psi} < 1$ are the positive roots of the equation $K^V = I(\alpha^{*V}(c^{*V}))$.

Proof. See Appendix C.

By Proposition 5 we can easily see that as the contribution of the venture capitalist, K^V , is equal or lower than the minimal amount of investment required, i.e., $\lim_{\alpha^{*V}(c^{*V}) \rightarrow 0} I(\alpha^{*V}(c^{*V})) = k_1$, any deal $\psi \in (\underline{\psi}, 1)$ is feasible and profitable for both parties. In contrast, when higher, the project must secure a minimal profitability as, otherwise, it will not be neither feasible nor profitable for the equity holders. In particular, $\frac{p-d}{r}$, i.e the net present value of returns from operating with in house input production, must be such that the portion of capital investment funded by the venture capitalist plus the wedge $\Theta > 0$ are covered. This allows compensating the venture capitalist for his contribution in terms of capital investment leaving at the same time a sufficient residual for compensating the investment effort undertaken by the equity holders. In addition, note that, even meeting this profitability requirement, the project share to be offered to the venture capitalist, ψ , must belong to the subset $[\underline{\psi}_1, \bar{\psi}]$, basically, it must be neither too low nor too high. The intuition behind this result relates to the role that, given a certain K^V , the profit share ψ plays when the firm sets the optimal degree of outsourcing. Note in fact that $\lim_{\psi \rightarrow \underline{\psi}} \alpha^{*V}(c^{*V}) = \lim_{\psi \rightarrow 1} \alpha^{*V}(c^{*V}) = 0$. This implies that, as $K^V > k_1$, the constraint $K^V \leq I(\alpha^{*V}(c^{*V}))$ would be violated.

Once identified the conditions for the feasibility and profitability of a financial agreement, let's now focus on how the pair (ψ, K^V) impacts on the definition of optimal outsourcing level, $\alpha^{*V}(c^{*V})$, and, consequently, of the investment time threshold, c^{*V} . Taking the derivative of $\alpha^{*V}(c^{*V})$ and c^{*V} with respect to ψ yields:

$$\begin{aligned} \frac{\partial \alpha^{*V}(c^{*V})}{\partial \psi} &= -\frac{1}{2} \frac{\alpha^{*V}(c^{*V})}{\psi \frac{p-d}{r} - K^V} \frac{\psi}{1-\psi} \left(\frac{p-d}{r} - \frac{K^V}{\psi^2} \right) \\ \frac{\partial c^{*V}}{\partial \psi} &= \frac{1}{\beta_2} \frac{c^{*V}}{\alpha^{*V}(c^{*V})} \left(\frac{\partial \alpha^{*V}(c^{*V})}{\partial \psi} + \frac{\alpha^{*V}(c^{*V})}{1-\psi} \right) \end{aligned}$$

We notice that both $\alpha^{*V}(c^{*V})$ and c^{*V} are non-monotonic in ψ and that:

$$\frac{\partial \alpha^{*V}(c^{*V})}{\partial \psi} = \begin{cases} \geq 0, & \text{for } \psi \leq \underline{\psi}^{1/2}, \\ < 0 & \text{for } \psi > \underline{\psi}^{1/2} \end{cases},$$

while

$$\frac{\partial c^{*V}}{\partial \psi} = \begin{cases} \geq 0, & \text{for } \frac{\partial \alpha^{*V}(c^{*V})}{\partial \psi} \leq -\frac{\alpha^{*V}(c^{*V})}{1-\psi}, \\ < 0 & \text{for } \frac{\partial \alpha^{*V}(c^{*V})}{\partial \psi} > -\frac{\alpha^{*V}(c^{*V})}{1-\psi} \end{cases}$$

where $\underline{\psi}^{1/2} > \underline{\psi}$.

This implies that $\alpha^{*V}(c^{*V})$, starting from 0, i.e., the value taken at $\psi = \underline{\psi}$, keeps growing, as ψ increases, up to $\psi = \underline{\psi}^{1/2}$. Then, it starts lowering up to, again, 0, i.e., the value taken at $\psi = 1$. Note that $\alpha^{*V}(c^{*V})$ and c^{*V} are negatively related, that is, the higher the level of outsourcing the later the investment should be undertaken. Hence, the equity holders uses ψ in order to influence the investment timing decision taken by the venture capitalist. Basically, whenever the equity holders deem profitable a higher $\alpha^{*V}(c^{*V})$, a large profit share ψ must be offered to the venture capitalist. This will in turn induce the investment delay which magnifies the chances of exercise and, consequently, the value of the option to outsource the input portion $\alpha^{*V}(c^{*V})$. This mechanism is clearly illustrated by Eq. (23.1). With a fixed ψ , since a higher $\alpha^{*V}(c^{*V})$ is accompanied by a lower c^{*V} , the value of the option to invest, $O^V(c_t; c^{*V})$, is lower, because the investment is postponed. Hence, in order to balance this loss and make the deal convenient for the venture capitalist, a higher share ψ is offered. This is what occurs up to $\psi = \underline{\psi}^{1/2}$. Once reached this level, "buying" investment delay by paying a higher ψ becomes too costly. Then, the opposite strategy of accelerating investment so as to cash profits earlier, becomes more convenient for the

equity holders. Therefore, to induce earlier investment, $\alpha^{*V}(c^{*V})$ is lowered. However, the value of the option to invest is decreasing in $\alpha^{*V}(c^{*V})$, as shown in Eq. (26). Hence, to compensate the venture capitalist for the value lost by anticipating the investment, a higher share ψ must be paid.

To complete the analysis, we take the derivative of $\alpha^{*V}(c^{*V})$ and c^{*V} with respect to K^V :

$$\frac{\partial \alpha^{*V}(c^{*V})}{\partial K^V} = -\frac{1}{2} \frac{\alpha^{*V}(c^{*V})}{\psi \frac{p-d}{r} - K^V} < 0, \quad \frac{\partial c^{*V}}{\partial K^V} = \frac{1}{\beta_2} \frac{\partial \alpha^{*V}(c^{*V})}{\partial K^V} \frac{c^{*V}}{\alpha^{*V}(c^{*V})} > 0$$

The first inequality shows substitutability between outsourcing and the extent of the venture capital involvement, i.e., a substitutability between a real and a financial variable. The intuition? Given a certain ψ , if the K^V paid by the venture capitalist increases, the venture capitalist should postpone investment benefiting from the higher value of the option to outsource. Incumbent shareholders anticipate it and, in contrast, as this would increase the portion of value to be cashed, they prefer earlier investment. Hence, they reduce α^{*V} to induce earlier investment. A smaller α^{*V} implies that (i) their contribution to the investment cost, $I(\alpha^{*V}(c^{*V})) - K^V$, given K^V paid by the venture capitalist, is lower and (ii) as outsourcing is lower the postponement of investment is less profitable for the venture capitalist who will set a higher investment threshold.

6.1. Venture capital and the unlevered firm: a comparison

In this section we use the unlevered firm as benchmark and discuss the case of investment partially financed with venture capital. The comparison is between optimal degree of outsourcing and investment timing in the two scenarios. Our findings are summarized in:

Proposition 6. *If the flexible technology is partially financed by a venture capitalist, then (i) the level of outsourcing adopted is, when $\frac{p-d}{r} > \Gamma$,*

$$\alpha^{*V} < \alpha^{*U} \quad \text{for any } \psi \in (\underline{\psi}, 1), \tag{28.1}$$

when $\frac{p-d}{r} \leq \Gamma$ it is:

$$\begin{aligned} \alpha^{*V} &\geq \alpha^{*U} \quad \text{for } \psi \in [\underline{\psi}_2, \bar{\psi}_1] \\ \alpha^{*V} &< \alpha^{*U} \quad \text{otherwise,} \end{aligned} \tag{28.2}$$

where $\Gamma = K^V - \Theta$ and $\underline{\psi} < \underline{\psi}_1 < \underline{\psi}_2$ and $\bar{\psi}_1 < \bar{\psi} < 1$ are the two positive roots of the equation $\alpha^{*V} = \alpha^{*U}$;

(ii) investment occurs at

$$\begin{aligned} c^{*V} &\geq c^{*U}, \quad \text{for } \underline{\psi} < \psi \leq \hat{\psi} \\ c^{*V} &< c^{*U}, \quad \text{for } \hat{\psi} < \psi < 1 \end{aligned} \tag{28.3}$$

where $\hat{\psi}$ is the positive root of the equation $c^{*V} = c^{*U}$.

Proof. See Appendix C.

The first part of Proposition 6 says that when project profitability is sufficiently high, (i.e., $\frac{p-d}{r} > \Gamma$), the incentive to have high flexibility is lower with venture capital. Equity holders find profitable substituting α^{*V} with K^V . However, two different timing of investment emerge and a crucial role is played by the profit share ψ cashed by the venture capitalist. When this portion is high, i.e., $\psi \in (\hat{\psi}, 1)$, the investment is delayed with respect to the unlevered firm. The intuition? Even if outsourcing is low, the venture capitalist further postpones investment as this increases the value of the project. That leaves to equity holders a sufficiently high compensation for their share on the total invested capital. The venture capitalist is keen on it due to his high profit share. Opposite considerations hold when ψ is relatively low, i.e., $\psi \in (\underline{\psi}, \hat{\psi})$. Postponing investment is less convenient for the venture capitalist. On the other hand procrastinating investment is dearer for equity holders if compared with cashing earlier a large profit portion. Hence, both parties prefer an earlier investment with respect to an unlevered firm.

In contrast, when $\frac{p-d}{r} \leq \Gamma$, the level of technological flexibility adopted may even be higher than the one set by an unlevered firm, i.e., $\alpha^{*V} \geq \alpha^{*U}$. This may occur when $\psi \in [\underline{\psi}_2, \bar{\psi}_1]$. In this case, as in-house input production does not secure a sufficiently high project profitability, both parties aim at increasing the project value by adopting more flexibility. In this case, (i.e., $c^{*V} < c^{*U}$) as shown in the appendix, investment is always postponed if compared to the unlevered firm. A delayed investment magnifies the chances of exercise and, consequently, the value of the option to outsource.

In general, when $\frac{p-d}{r} \leq \Gamma$ the trade-off between vertical flexibility and investment anticipation is less pronounced since:

$$\begin{aligned} c^{*V} &\geq c^{*U}, && \text{when } \alpha^{*V} < \alpha^{*U} - \Lambda \\ c^{*V} &< c^{*U}, && \text{otherwise,} \end{aligned} \tag{28.4}$$

where $\Lambda = 2\psi^2(2 - \psi)(\frac{p-d}{r} - k_1)$. This implies that it may be preferable postponing investment even for $\alpha^{*V} < \alpha^{*U}$. As $\frac{p-d}{r} \leq \Gamma$ a lower cost is associated with waiting time. Hence, earlier investment with respect to the unlevered firm may occur only if the flexibility adopted is sufficiently below α^{*U} .

6.2. Option to invest: the impact of the funding source

Using Eq. (10.1) and Proposition 6, we find that:

$$\Phi(\psi) = O^V(c_t)/O^U(c_t) = 2\psi \frac{\alpha^{*V}(c^{*V})}{\alpha^{*U}(c^{*U})} = \left[2(1 - \psi) \frac{\psi - \frac{\psi}{1 - r \frac{k_1}{p-d}} \psi} \right]^{1/2}, \quad \text{for } c_t > \max(c^{*V}, c^{*U}) \tag{29}$$

With venture capital, unlike debt, the impact of the funding source on the option to invest is ambiguous. The ratio $\Phi(\psi)$ may be higher than 1, which implies that a higher value is associated with this option. However, notice that $O^V(c_t) < O^U(c_t)$ when the venture capitalist's commitment is higher than the fixed cost (i.e., for $K^V \geq k_1$).

In other words the value of the option to invest for the venture capitalist is higher than for the unlevered firm, when $K^V < k_1$ and the share of profits is sufficiently high, vis à vis the capital injection K^V .

This combination provides the conditions for associating a higher value to the option to invest in the firm. If a high share of profits (above the cutoff level) is granted the venture capitalist involvement turns out to be an actual alternative to vertical flexibility in terms of risk for the shareholders. This result shows the difference between the incentives to invest in the flexible technology by two different agents, the lender and the venture capitalist. For the latter the investment commitment and the expected reward may make the value of the option to enter the project higher than for the lender, who is constrained to a fixed coupon and participates to the risk of the project only if flexibility becomes useless.

How large is the gain for the venture capitalist depends on his ability to negotiate a favorable ψ vis à vis a small K^V . The venture capitalist may obtain a large benefit if shareholders are foreclosed from the credit market and badly need funds for investing.

6.3. Optimal sharing rule

An open question is the determination of the share ψ set by the equity holders. This parameter is obtained by maximizing the expected net present value of the project payoff.³⁶ Then, before reaching the investment timing corresponding to c^{*V} , they solve the following problem:

$$\max_{\psi} \left(\frac{c_t}{c^{*V}} \right)^{\beta_2} \left[(1 - \psi) F^U(c^{*V}; \alpha^{*V}(c^{*V})) - \left(k_1 + \frac{k_2}{2} \alpha^{*V}(c^{*V})^2 - K^V \right) \right]. \tag{30}$$

and announce the ψ solving Problem (30).

As shown in Appendix D, we can rearrange Problem (30) and obtain:

$$\max_{\psi} G(\psi) = \left(\frac{1 - \psi}{\frac{p-d}{r} - \frac{K^V}{\psi}} \right)^{1/2} \left[3(1 - \psi) \frac{p-d}{r} + \left(3 - \frac{1}{\psi} \right) K^V - 2k_1 \right]. \tag{30.1}$$

We have to resort to numerical calibrations in order to identify the optimal share ψ^* under different scenarios as reported in the ensuing tables.

In Table 1 we see the parameters used in our numerical exercises.

In Table 2 we find the computed values relative to the optimal share ψ^* , the cutoff $\hat{\psi}$ and $\Phi(\psi^*)$.

From Table 2 we realize that ψ^* lies always above the cutoff value ($\hat{\psi}$) (discriminating between early and delayed investment with respect to the unlevered project) and goes up with the commitment of the venture capitalist K^V . At the same time the value of the option to invest by the venture capitalist decreases with respect to the unlevered firm since it becomes more expensive.

³⁶ In a different environment Banerjee et al., (2014) introduce a bargaining on the share parameter and find that it is inefficient to set it before the investment, due to time inconsistency. Only a bargaining carried out after the investment may assure intertemporal efficiency. In our case efficiency comes from the backward induction solution whereby ψ is set at the end of the decision chain and from the fact that ψ does not depend on c_t .

Table 1
Parameters' values.

| Parameters | Value |
|------------|------------------|
| p | 3.5 |
| d | 3.47 |
| k_1 | 0.6 |
| k_2 | 0.8, 1, 1.2 |
| K^V | 0.1, 0.25, 0.5 |
| r | 0.03, 0.04 |
| γ | 0, -0.005, -0.01 |
| σ | 0.05, 0.1, 0.15 |

Table 2
The optimal share of profits ψ^* , the cutoff $\hat{\psi}$ and the ratio $\Phi(\psi^*)$.

| | | $(p-d)/r = 0.75$ | | | $(p-d)/r = 1$ | | |
|-------|------|------------------|------------------|--------------------|---------------|------------------|--------------------|
| | | ψ^* (%) | $\hat{\psi}$ (%) | $\Phi(\psi^*)$ (%) | ψ^* (%) | $\hat{\psi}$ (%) | $\Phi(\psi^*)$ (%) |
| K^V | 0.1 | 83.867 | 19.648 | 97.691 | 88.283 | 25.000 | 63.631 |
| | 0.25 | 87.245 | 43.145 | 77.456 | 91.177 | 44.782 | 51.593 |
| | 0.5 | 95.863 | 74.304 | 34.027 | 97.255 | 67.539 | 25.115 |

Table 3
Optimal outsourcing shares and investment timing for $K^V = 0.25$.

| | | $(p-d)/r = 0.75; \psi^* = 0.87245$ | | | | | |
|--------------------------|--|------------------------------------|--|----------------|--|-----------------|--|
| | | $\sigma = 0.05$ | | $\sigma = 0.1$ | | $\sigma = 0.15$ | |
| | | c^{*U} | | c^{*U} | | c^{*U} | |
| $\alpha^{*U} = 61.237\%$ | | | | | | | |
| $\gamma = 0$ | | 5.897 | | 14.722 | | 48.130 | |
| $\gamma = -0.005$ | | 8.448 | | 21.767 | | 73.375 | |
| $\gamma = -0.01$ | | 13.230 | | 33.812 | | 115.472 | |
| | | c^{*V} | | c^{*V} | | c^{*V} | |
| $\alpha^{*V} = 27.183\%$ | | | | | | | |
| $\gamma = 0$ | | 4.635 | | 8.703 | | 20.375 | |
| $\gamma = -0.005$ | | 6.022 | | 11.668 | | 28.052 | |
| $\gamma = -0.01$ | | 8.376 | | 16.247 | | 39.571 | |
| | | $(p-d)/r = 1; \psi^* = 0.91177$ | | | | | |
| | | $\sigma = 0.05$ | | $\sigma = 0.1$ | | $\sigma = 0.15$ | |
| | | c^{*U} | | c^{*U} | | c^{*U} | |
| $\alpha^{*U} = 100\%$ | | | | | | | |
| $\gamma = 0$ | | 6.368 | | 18.660 | | 77.105 | |
| $\gamma = -0.005$ | | 10.318 | | 31.464 | | 135.380 | |
| $\gamma = -0.01$ | | 18.914 | | 56.841 | | 248.335 | |
| | | c^{*V} | | c^{*V} | | c^{*V} | |
| $\alpha^{*V} = 28.293\%$ | | | | | | | |
| $\gamma = 0$ | | 4.894 | | 10.420 | | 29.383 | |
| $\gamma = -0.005$ | | 6.997 | | 15.499 | | 45.253 | |
| $\gamma = -0.01$ | | 11.006 | | 24.316 | | 72.080 | |

Table 3 contains values for the optimal outsourcing and the corresponding investment thresholds³⁷ for venture capital and the (benchmark) case of an unlevered firm. We study the impact of changes in drift (γ) and volatility (σ) of the input price diffusion process for different optimal ψ^* (taken from Table 2).

In Table 3 we see that with venture capital α^{*V} is always lower than in the unlevered case, i.e., α^{*U} . Equity holders find it profitable, as explained in Section 5, substituting α^{*V} with K^V . The “real” hedging derived by properly combining in-house produced and outsourced input can be substituted by the “financial” funding secured by venture capital. Notice that α^{*V} grows in $(p-d)/r$ since the higher investment cost may be covered by larger profitability of the project. Moreover,

³⁷ See Appendix D for the simulation background.

Table 4Optimal outsourcing shares α^{*V} and α^{*U} for $(p-d)/r = 0.75$, $\gamma = 0$, and $\sigma = \{0.05, 0.1, 0.15\}$.

| | $K^V = 0.1; \psi^* = 83.867\%$ | | $K^V = 0.25; \psi^* = 87.245\%$ | | $K^V = 0.5; \psi^* = 95.863\%$ | |
|-------------|--------------------------------|-------------------|---------------------------------|-------------------|--------------------------------|-------------------|
| | α^{*V} (%) | α^{*U} (%) | α^{*V} (%) | α^{*U} (%) | α^{*V} (%) | α^{*U} (%) |
| $k_2 = 0.8$ | 31.828 | 61.237 | 27.183 | 61.237 | 15.905 | 61.237 |
| $k_2 = 1$ | 28.468 | 54.772 | 24.313 | 54.772 | 14.226 | 54.772 |
| $k_2 = 1.2$ | 25.988 | 50.000 | 22.195 | 50.000 | 12.987 | 50.000 |

$\partial\alpha^{*U}/\partial d < 0$ and $\partial\alpha^{*V}/\partial d < 0$. Two considerations to explain this result. On the one hand, a higher d may make convenient a higher α^{*V} (and α^{*U}). On the other hand, a higher α^{*V} would induce investment delay, because $\partial\alpha^{*V}/\partial c^{*V} < 0$ (and $\partial\alpha^{*U}/\partial c^{*U} < 0$). The negative effect of investment delay dominates, in terms of impact on the value of the project, the positive effect of having a higher α^{*V} (and α^{*U}). The result is a negative relationship between extent of outsourcing and cost of internal input production.

We observe that a higher ψ^* is paid when $(p-d)/r$ goes up. This is needed to induce the timing that allows benefiting from the investment in flexibility. In particular, in the range of values considered, the investment is postponed with respect to the unlevered firm. A delayed investment is required to compensate for the lower level of outsourcing, when it comes to the value of the project to be split between the two parties. The impact on investment timing of a change in parameters γ and σ is similar in the two compared scenarios. Investment occurs earlier as γ gets lower since it becomes higher the probability of exploiting outsourcing. In both cases, earlier investment is the response to more volatility. This is a remarkable result in the context of the literature on real options. The standard effect inducing delay of the exercise of the option to invest is more than balanced by the presence of the option to outsource and backsource. These turn out to be two hedging tools against the fluctuations of the relative profitability of buying rather than making and vice versa.

In Table 4, we check for the impact on flexibility of different levels of commitments of the venture capitalist. In line with our findings in Section 5, α^{*V} is decreasing in K^V . Hence, a higher K^V makes the equity holders reduce α^{*V} (c^{*V}) so that their contribution to the initial capital expenditure is lower and investment occurs earlier. We also observe that α^{*V} is decreasing in k_2 : less flexibility is adopted when its relative impact on the initial capital expenditure increases. Finally, note that ψ^* is increasing in K^V : since setting a lower α^{*V} (c^{*V}) reduces the value of the option to invest, a higher profit share must be paid to the venture capitalist.

7. Epilogue

We have investigated how the financial choices of a firm affect the extent and timing of investment in vertical flexibility. To this purpose we have considered a firm that must decide simultaneously the internal vertical setting and the corresponding financial structure in a dynamic stochastic framework. In our frame the firm is vertically flexible since it has an option to outsource entirely or partially a necessary input and to reverse its choice by going back to in-house production, i.e., vertical integration.

Flexibility calls for a costly investment, partly fixed and partly dependent upon the extent of outsourcing. The goal is to set up a suitable supply chain and to keep alive the know-how and the facilities to backsource the input in case market circumstances require to do so. Two quite common external financial sources for the investment in the vertically flexible firm are: fixed price finance, i.e., debt, and risk capital, i.e., venture capital. So far the latter has never been investigated together with vertical flexibility. In the former case a lender may be willing to finance the project if she gets a collateral. This requirement may be fulfilled by an option to buy the company's equity in case the production in-house becomes worthless. This option makes the lender willing to finance the corporate since limited liability may otherwise induce the incumbent equity holders to overinvest. The levered firm decides the level of outsourcing and the timing of the investment while the lender sets the size of the investment and the buyout time. With collateralized debt the shareholders rush to invest earlier with respect to a pure equity unlevered firm. Debt induces the firm to invest earlier since shareholders are eager to reap expected profits, consistently with common observation suggesting that debt may accelerate innovation in organizational flexibility.

The sharing of risk that the participation of the venture capitalist implies may make the firm less eager to adopt much outsourcing as an insurance against uncertainty. Further, we find that the higher is the commitment in terms of venture capital the lower is the extent of outsourcing. We may then conclude that outsourcing and venture capital may be viewed as a kind of substitute. This result establishes a fresh substitutability between a real and a financial decision of a firm. Finally, if the share of profits ψ is high, the venture capitalist prefers holding longer the option to invest so that investment occurs when higher profits are expected.

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Appendix A. Benchmark case

A.1. The operating value

The standard arbitrage and hedging arguments require that the vertically flexible firm value, $F^U(c_t; \alpha)$, is the solution of the following dynamic programming problems:

$$\Gamma F^U(c_t; \alpha) = -(p - d), \quad \text{for } c_t > d \quad (\text{A.1})$$

and

$$\Gamma F^U(c_t; \alpha) = -[p - \alpha c_t - (1 - \alpha)d], \quad \text{for } c_t < d, \quad (\text{A.2})$$

where Γ is the differential operator: $\Gamma = -r + \gamma c \frac{\partial}{\partial c} + \frac{1}{2} \sigma^2 c^2 \frac{\partial^2}{\partial c^2}$. The solution of Eqs. (A.1) and (A.2) requires the following boundary conditions:

$$\lim_{c \rightarrow \infty} [F^U(c_t; \alpha) - (p - d)/r] = 0 \quad \text{if } c_t > d$$

and

$$\lim_{c \rightarrow 0} \left\{ F^U(c_t; \alpha) - \left[\frac{p - (1 - \alpha)d}{r} - \alpha \frac{c_t}{r - \gamma} \right] \right\} = 0, \quad \text{if } c_t < d$$

where $\frac{p-d}{r}$ is the present value of the firm “making” the input, while $[\frac{p-(1-\alpha)d}{r} - \alpha \frac{c_t}{r-\gamma}]$ is the present value when “buying” a share α of the input. Then, from the assumptions and the linearity of Eqs. (A.1) and (A.2), using the above boundary conditions, we get:

$$F^U(c_t; \alpha) = \begin{cases} \frac{p-d}{r} + \tilde{A}c_t^{\beta_2} & \text{if } c_t > d \\ \left[\frac{p - (1 - \alpha)d}{r} - \alpha \frac{c_t}{r - \gamma} \right] + \tilde{B}c_t^{\beta_1} & \text{if } c_t < d. \end{cases} \quad (\text{A.3})$$

where $\beta_2 < 0$ and $\beta_1 > 1$ are, respectively, the negative and the positive root of the characteristic equation: $\Phi(\beta) \equiv \frac{1}{2} \sigma^2 \beta(\beta - 1) + \gamma \beta - r$. By the value matching and the smooth pasting conditions at $c_t = d$ we obtain the two constants (Dixit and Pindyck, 1994, p. 189):

$$\begin{aligned} \tilde{B} = \alpha B &\equiv \alpha \frac{r - \gamma \beta_2}{r(\beta_1 - \beta_2)(r - \gamma)} d^{1-\beta_1}, \\ \tilde{A} = \alpha A &\equiv \alpha \frac{r - \gamma \beta_1}{r(\beta_1 - \beta_2)(r - \gamma)} d^{1-\beta_2}, \end{aligned} \quad (\text{A.4})$$

which are always nonnegative and linear in α .

A.2. Optimal outsourcing share

Since $\tilde{A} = \alpha A$, the optimal vertical arrangement is given by:

$$\begin{aligned} \alpha^{*U} &= \operatorname{argmax} [F^U(c_t; \alpha) - I(\alpha)] \\ &= \operatorname{argmax} \left[\frac{p-d}{r} + \alpha A c_t^{\beta_2} - \left(k_1 + \frac{k_2}{2} \alpha^2 \right) \right]. \end{aligned} \quad (\text{A.5})$$

Then, the FOC is:

$$A c_t^{\beta_2} - k_2 \alpha = 0 \quad (\text{A.5.1})$$

while the SOC is always satisfied. From Eq. (A.5.1) we obtain:

$$\alpha^{*U}(c_t) = \begin{cases} 1 & \text{if } c_t \leq \tilde{c}^U \\ (A/k_2)c_t^{\beta_2} & \text{if } c_t > \tilde{c}^U \end{cases} \quad (\text{A.6})$$

where $\tilde{c}^U = (k_2/A)^{1/\beta_2}$, which corresponds to Eq. (6) in the text.

A.3. Investment timing

The value function of the option to invest is

$$O^U(c_t) = \max_{T^{U*}} E_t \left[e^{-r(T^{U*}-t)} \right] (F^U(c^{*U}, \alpha^{*U}(c^{*U})) - I(\alpha^{*U}(c^{*U}))) \quad (\text{A.7})$$

where $T^{U*} = \inf\{t \geq 0 \mid c_t = c^{*U}\}$ is equivalent to the optimal investment timing and $\alpha^{*U}(c^{*U})$ is the optimal outsourcing share at $t = T^{U*}$.

Eq. (A.7) is equivalent to

$$O^U(c_t, c^{*U}) = \max_{c^{*U}} \left(\frac{c_t}{c^{*U}} \right)^{\beta_2} (F^U(c^{*U}, \alpha^{*U}(c^{*U})) - I(\alpha^{*U}(c^{*U}))) \quad (\text{A.7.1})$$

Let's solve the maximization problem in two scenarios:

$$\text{Scenario 1.A } \alpha^{*U} \leq 1, \quad \text{if } \tilde{c}^U \geq d \rightarrow Ad^{\beta_2} \geq k_2$$

$$\text{Scenario 2.A } \alpha^{*U} < 1, \quad \text{if } \tilde{c}^U < d \rightarrow Ad^{\beta_2} < k_2$$

A.3.1. Scenario 1.A

At $c_0 = c$, when evaluating the investment decision and the optimal timing, two investment scenarios may arise:

$$\text{Scenario 1.1.A } \alpha^{*U} \leq 1, \quad \text{if } c \geq \tilde{c}^U$$

$$\text{Scenario 1.2.A } \alpha^{*U} = 1, \quad \text{if } \tilde{c}^U \geq c > d$$

Scenario 1.1.A. By substituting Eqs. (3), (A.3) and (A.6) into Eq. (A.7) we have

$$O^U(c, c^{*U}) = \max_{c^{*U}} \left(\frac{c}{c^{*U}} \right)^{\beta_2} \left[\frac{p-d}{r} + \frac{1}{2} \frac{(Ac^{*U\beta_2})^2}{k_2} - k_1 \right]. \quad (\text{A.8})$$

Optimality requires:

$$-\frac{\beta_2}{c^{*U}} \left(\frac{c}{c^{*U}} \right)^{\beta_2} \left[\frac{p-d}{r} - \frac{1}{2} \frac{(Ac^{*U\beta_2})^2}{k_2} - k_1 \right] = 0. \quad (\text{A.8.1})$$

Solving for c^{*U} yields

$$c^{*U} = \left\{ \frac{[2k_2(\frac{p-d}{r} - k_1)]^{1/2}}{A} \right\}^{1/\beta_2}. \quad (\text{A.8.2})$$

Substituting c^{*U} into Eq. (3) gives

$$\alpha^{*U}(c^{*U}) = \left(2 \frac{\frac{p-d}{r} - k_1}{k_2} \right)^{1/2}. \quad (\text{A.8.3})$$

Let's check if the investment threshold is consistently set, that is, if $c^{*U} \geq \tilde{c}^U$. Note that

$$c^{*U} = \alpha^{*U}(c^{*U})^{1/\beta_2} \tilde{c}^U,$$

hence, it follows that

$$c^{*U} \geq \tilde{c}^U \quad \text{for } \alpha^{*U}(c^{*U}) \leq 1$$

or

$$c^{*U} \geq \tilde{c}^U \quad \text{for} \quad \frac{p-d}{r} \leq k_1 + \frac{k_2}{2}. \tag{A.8.4}$$

Note that for

$$\frac{p-d}{r} > k_1 + \frac{k_2}{2} > k_1 + \frac{k_2}{2} \alpha^{*U} (c^{*U})^2$$

$O^U(c, c^{*U})$ is increasing in c^{*U} and Eq. (A.8.1) has no solution. This implies that the firm invests immediately, i.e., at $c_0 = c$, and sets

$$\alpha^{*U} (c^{*U}) = Ac^{\beta_2} / k_2. \tag{A.8.5}$$

Scenario 1.2.A. By substituting Eqs. (3), (A.3) and (A.6) into Eq. (A.7) we have

$$O^U(c, c^{*U}) = \max_{c^{*U}} \left(\frac{c}{c^{*U}} \right)^{\beta_2} \left[\frac{p-d}{r} + Ac^{*U\beta_2} - \left(k_1 + \frac{k_2}{2} \right) \right]. \tag{A.9}$$

By taking the first derivative of the objective with respect to c^{*U} we have:

$$\frac{\partial O^U(c_t, c^{*U})}{\partial c^{*U}} = -\frac{\beta_2}{c^{*U}} \left(\frac{c}{c^{*U}} \right)^{\beta_2} \left[\frac{p-d}{r} - \left(k_1 + \frac{k_2}{2} \right) \right]. \tag{A.9.1}$$

This implies that

$$c^{*U} = \begin{cases} \tilde{c}^U, & \text{if } \frac{p-d}{r} > k_1 + \frac{k_2}{2} \\ d, & \text{if } \frac{p-d}{r} \leq k_1 + \frac{k_2}{2} \end{cases} \tag{A.9.2}$$

A.3.2. Scenario 2.A

For $\tilde{c}^U < d$, the firm invests in a technological scenario where $\alpha^{*U} < 1$. Then the analysis is identical to scenario 1.1.A. We only need to check if the investment threshold is consistently set, that is, if $c^{*U} \geq d$. It is immediate to show that

$$c^{*U} = \alpha^{*U} (c^{*U})^{1/\beta_2} \tilde{c}^U \geq d \rightarrow \frac{p-d}{r} \leq k_1 + \frac{k_2}{2} \left(\frac{d}{\tilde{c}^U} \right)^{2\beta_2} = k_1 + \frac{k_2}{2} \left(\frac{Ad^{\beta_2}}{k_2} \right)^2 < k_1 + \frac{k_2}{2}.$$

As above, if $\frac{p-d}{r} > k_1 + \frac{k_2}{2} \left(\frac{Ad^{\beta_2}}{k_2} \right)^2$, the firm invests immediately, i.e., at $c_0 = c$, and chooses

$$\alpha^{*U} (c^{*U}) = Ac^{\beta_2} / k_2.$$

Appendix B. Debt and equity

B.1. Debt

The value function for $c_t > c^l$ is

$$\max_{\tau > 0} D(c_t; \alpha) = \frac{D}{r} + E[e^{-r\tau}] \left[\left(\frac{p}{r} - \frac{c^l}{r-\gamma} \right) - \left(k_3 + \frac{D}{r} \right) \right] \tag{B.1}$$

where $\tau = \min\{t > 0 : c_t = c^l\}$ and $k_3 \geq -k_1$. The problem can be rearranged as follows

$$\max_{c^l} D(c_t; \alpha) = \frac{D}{r} + \left(\frac{c_t}{c^l} \right)^{\beta_2} \left[\left(\frac{p}{r} - \frac{c^l}{r-\gamma} \right) - \left(k_3 + \frac{D}{r} \right) \right]. \tag{B.1.1}$$

Optimality requires:

$$-\frac{\beta_2}{\beta_2 - 1} \left(\frac{p-D}{r} - k_3 \right) + \frac{c^l}{r-\gamma} = 0 \tag{B.1.2}$$

which gives

$$c^l = \frac{\beta_2}{\beta_2 - 1} (r-\gamma) \left(\frac{p-D}{r} - k_3 \right) \tag{B.1.3}$$

where

$$\frac{\partial c^l}{\partial D} = -\frac{c^l}{p-D-rk_3} < 0 \quad (\text{B.1.3.1})$$

$$\frac{\partial c^l}{\partial k_3} = -\frac{r}{p-D-rk_3} c^l < 0. \quad (\text{B.1.3.2})$$

Last note that

$$c^l \leq d \rightarrow \frac{p-D}{r} \leq \left(1 - \frac{1}{\beta_2}\right) \frac{d}{r-\gamma} + k_3. \quad (\text{B.1.4})$$

B.2. Equity

The dynamic programming problem underlying the definition of the market value of equity is similar to the one solved above for the determination of the operating value in the benchmark case. One simply needs to adjust for the periodic cash flow which is $p-d-D$ for $c_t > d$ and $p-D-\alpha c_t - (1-\alpha)d$ for $c^l \leq c_t < d$.

Conditions for an optimal switch, i.e., value matching plus smooth pasting condition, at $c_t = d$ between the two productive frames require

$$\frac{p-d-D}{r} + \hat{A}d^{\beta_2} = \left[\frac{p-(1-\alpha)d-D}{r} - \alpha \frac{d}{r-\gamma} \right] + \hat{B}d^{\beta_1} + -\left(\frac{d}{c^l}\right)^{\beta_2} \left[\frac{p-(1-\alpha)d-D}{r} - \alpha \frac{c^l}{r-\gamma} + \hat{B}c^{l\beta_1} \right] \quad (\text{B.2.1})$$

$$\hat{A}\beta_2 d^{\beta_2-1} = -\alpha \frac{1}{r-\gamma} + \hat{B}\beta_1 d^{\beta_1-1} + -\frac{\beta_2}{c^l} \left(\frac{d}{c^l}\right)^{\beta_2-1} \left[\frac{p-(1-\alpha)d-D}{r} - \alpha \frac{c^l}{r-\gamma} + \hat{B}c^{l\beta_1} \right]. \quad (\text{B.2.2})$$

Solving the system (B.2.1)–(B.2.2) yields

$$\hat{A} = \tilde{A} - \left(\frac{1}{c^l}\right)^{\beta_2} \left[\frac{p-(1-\alpha)d-D}{r} - \alpha \frac{c^l}{r-\gamma} + \hat{B}c^{l\beta_1} \right] \quad (\text{B.3.1})$$

$$\hat{B} = \alpha \frac{r-\beta_2\gamma}{r(r-\gamma)(\beta_1-\beta_2)} d^{1-\beta_1} = \tilde{B}. \quad (\text{B.3.2})$$

Note that:

$$\frac{\partial \hat{A}}{\partial c^l} = c^{l-(\beta_2+1)} \left[\beta_2 \frac{p-d-D}{r} (1-\alpha) + \alpha \beta_2 k_3 + (\beta_2 - \beta_1) \tilde{B} c^{l\beta_1} \right] < 0$$

and

$$\lim_{c^l \rightarrow 0} \hat{A} = \tilde{A}, \quad \lim_{c^l \rightarrow d} \hat{A} = -\left(\frac{1}{d}\right)^{\beta_2} \left(\frac{p-d-D}{r}\right).$$

B.3. Value of the levered firm

The market value of the levered firm is given by:

$$V^L(c_t; \alpha) = E(c_t; \alpha) + D(c_t; \alpha)$$

Using our results above, it is immediate to show that

$$E(c_t; \alpha) + D(c_t; \alpha) = \begin{cases} \left[\frac{p-d}{r} + \tilde{A}c_t^{\beta_2} - \left(\frac{c_t}{c^l}\right)^{\beta_2} \left[\tilde{B}c^{l\beta_1} - (1-\alpha) \left(\frac{d}{r} - \frac{c^l}{r-\gamma}\right) + k_3 \right] \right] & \text{if } c_t > d, \\ \left[\frac{p-(1-\alpha)d}{r} - \alpha \frac{c_t}{r-\gamma} \right] + \tilde{B}c_t^{\beta_1} - \left(\frac{c_t}{c^l}\right)^{\beta_2} \left[\tilde{B}c^{l\beta_1} - (1-\alpha) \left(\frac{d}{r} - \frac{c^l}{r-\gamma}\right) + k_3 \right] & \text{if } c^l < c_t < d \\ \left(\frac{p}{r} - \frac{c_t}{r-\gamma} \right) - k_3 & \text{if } c_t \leq c^l \end{cases}$$

which, in turn, implies that

$$F^L(c_t; \alpha) = \begin{cases} F^U(c_t; \alpha) - Z(c_t; c^l) & \text{if } c_t > c^l, \\ \left(\frac{p}{r} - \frac{c_t}{r-\gamma}\right) - k_3 & \text{if } c_t \leq c^l \end{cases} \tag{B.4}$$

where $Z(c_t; c^l) = (c_t/c^l)^{\beta_2} [\tilde{B}c^{\beta_1} - (1-\alpha)(\frac{d}{r} - \frac{c^l}{r-\gamma}) + k_3]$.

B.4. Optimal outsourcing share and investment timing

Since equity holders control both the outsourcing share and the timing of the investment, we proceed as above by determining first α^{*L} and then c^{*L} . To determine α^{*L} , the equity holders solve the problem:

$$\alpha^{*L} = \operatorname{argmax} [E(c_t; \alpha) - (I(\alpha) - K^L)] \tag{B.5.1}$$

where $K^L \leq I(\alpha)$ is the share of investment expenditure paid by the lender who controls the amount to loan and the buyout timing. Since a rational investor will not agree to finance the firm unless k is a (financially) fair price for the debt, we set $K^L = D(c_t; \alpha)$ for $c_t > c^l$. Then, substituting, we obtain:

$$\alpha^{*L} = \operatorname{argmax} [F^L(c_t; \alpha) - I(\alpha)] \tag{B.5.2}$$

where, as shown above, $F^L(c_t; \alpha) = F^U(c_t; \alpha) - Z(c_t; c^l)$.

Substituting for $F^L(c_t; \alpha)$ and $I(\alpha)$ the problem can be rearranged as follows

$$\alpha^{*L} = \operatorname{argmax} \left\{ \frac{p-d}{r} + \alpha A c_t^{\beta_2} - \left(\frac{c_t}{c^l}\right)^{\beta_2} \left[\alpha B c^{\beta_1} - (1-\alpha) \left(\frac{d}{r} - \frac{c^l}{r-\gamma}\right) + k_3 \right] - \left(k_1 + \frac{k_2}{2} \alpha^2\right) \right\}. \tag{B.5.3}$$

The relative FOC is:

$$A c_t^{\beta_2} - \left(\frac{c_t}{c^l}\right)^{\beta_2} \left[B c^{\beta_1} + \left(\frac{d}{r} - \frac{c^l}{r-\gamma}\right) \right] - k_2 \alpha = 0 \tag{B.5.4}$$

while the SOC is always satisfied.

Eq. (B.5.4) yields

$$\alpha^{*L}(c_t) = \left\{ A - c^{l-\beta_2} \left[B c^{\beta_1} + \left(\frac{d}{r} - \frac{c^l}{r-\gamma}\right) \right] \right\} \frac{c_t^{\beta_2}}{k_2}. \tag{B.5.5}$$

Now, we must identify conditions for having

$$0 < \alpha^{*L}(c_t) \leq 1.$$

In order to prove that $\alpha^{*V}(c_t) > 0$ it suffices to show that

$$H(c^l) = A c^{l\beta_2} - \left[B c^{l\beta_1} + \left(\frac{d}{r} - \frac{c^l}{r-\gamma}\right) \right] > 0.$$

Note that $H(c^l)$ is convex in c^l and

$$\lim_{c^l \rightarrow 0} H(c^l) = \infty, \quad H(d) = 0, \quad \left. \frac{\partial H(c^l)}{\partial c^l} \right|_{c^l=0} = 0.$$

It follows that $\alpha^{*L}(c_t) > 0$ for any $c_t \in (0, d]$. Further, we have $\alpha^{*V}(c_t) \leq 1$ if

$$c_t \leq \tilde{c}^L = \left\{ \frac{k_2}{A - c^{l-\beta_2} \left[B c^{l\beta_1} + \left(\frac{d}{r} - \frac{c^l}{r-\gamma}\right) \right]} \right\}^{1/\beta_2}.$$

Summing up:

$$\alpha^{*L}(c_t) = \begin{cases} 1 & \text{if } c_t \leq \tilde{c}^L \\ \frac{A - c^{l-\beta_2} \left[B c^{l\beta_1} + \left(\frac{d}{r} - \frac{c^l}{r-\gamma}\right) \right]}{k_2} c_t^{\beta_2} & \text{if } c_t > \tilde{c}^L \end{cases} \tag{B.6}$$

and it is immediate to show that

$$\tilde{c}^L \leq \tilde{c}^U$$

$$\alpha^{*L}(c_t) = \alpha^{*U}(c_t) - \frac{Bc^{l\beta_1} + \left(\frac{d}{r} - \frac{c^l}{r-\gamma}\right)}{k_2} \left(\frac{c_t}{c^l}\right)^{\beta_2} \leq \alpha^{*U}(c_t).$$

B.5. Investment timing

The value function of the option to invest for the levered firm is

$$O^L(c_t) = \max_{c^{*L}} \left(\frac{c_t}{c^{*L}}\right)^{\beta_2} (F^L(c^{*L}, \alpha^{*L}(c^{*L})) - I(\alpha^{*L}(c^{*L}))) \quad (\text{B.7})$$

where $T^{L*} = \inf\{t \geq 0 \mid c_t = c^{*L}\}$ is the optimal investment timing and $\alpha^{*L}(c^{*L})$ is the optimal outsourcing share at $t = T^{L*}$.

Let's solve the maximization problem in the two potential scenarios:

$$\text{Scenario 1.B } \alpha^{*L} \leq 1, \quad \text{if } \tilde{c}^L \geq d \rightarrow Ad^{\beta_2} \geq k_2 + \left(\frac{d}{c^l}\right)^{\beta_2} \left[Bc^{l\beta_1} + \left(\frac{d}{r} - \frac{c^l}{r-\gamma}\right) \right]$$

$$\text{Scenario 2.B } \alpha^{*L} < 1, \quad \text{if } \tilde{c}^L < d \rightarrow Ad^{\beta_2} < k_2 + \left(\frac{d}{c^l}\right)^{\beta_2} \left[Bc^{l\beta_1} + \left(\frac{d}{r} - \frac{c^l}{r-\gamma}\right) \right].$$

B.5.1. Scenario 1.B

At $c_0 = c$, when evaluating the investment decision and the relative optimal timing, two potential investment scenarios may arise, that is,

$$\text{Scenario 1.1.B } \alpha^{*L} \leq 1, \quad \text{if } c \geq \tilde{c}^L$$

$$\text{Scenario 1.2.B } \alpha^{*L} = 1, \quad \text{if } \tilde{c}^L \geq c < d.$$

Scenario 1.1.B. Using Eqs. (3), (A.3), (B.4) and (B.6), the problem (B.7) can be rearranged as follows

$$\begin{aligned} O^L(c_t, c^{*L}) &= \max_{c^{*L}} \left(\frac{c_t}{c^{*L}}\right)^{\beta_2} \left[F^U(c^{*L}, \alpha^{*L}(c^{*L})) - Z(c^{*L}; c^l) - \left(k_1 + \frac{k_2}{2} \alpha^{*L}(c^{*L})^2\right) \right] \\ &= \max_{c^{*L}} \left(\frac{c_t}{c^{*L}}\right)^{\beta_2} \left[\frac{p-d}{r} + \alpha^{*L}(c^{*L}) Ac^{*L\beta_2} - \left(k_1 + \frac{k_2}{2} \alpha^{*L}(c^{*L})^2\right) \right] + \\ &\quad - \left(\frac{c_t}{c^l}\right)^{\beta_2} \left[\alpha^{*L}(c^{*L}) Bc^{l\beta_1} - (1 - \alpha^{*L}(c^{*L})) \left(\frac{d}{r} - \frac{c^l}{r-\gamma}\right) + k_3 \right]. \end{aligned} \quad (\text{B.8})$$

Optimality requires:

$$\begin{aligned} & - \frac{\beta_2}{c^{*L}} \left(\frac{c_t}{c^{*L}}\right)^{\beta_2} \left[\frac{p-d}{r} + \alpha^{*L}(c^{*L}) Ac^{*L\beta_2} - \left(k_1 + \frac{k_2}{2} \alpha^{*L}(c^{*L})^2\right) \right] \\ & + \left\{ \left(\frac{c_t}{c^{*L}}\right)^{\beta_2} (Ac^{*L\beta_2} - k_2 \alpha^{*L}(c^{*L})) - \left(\frac{c_t}{c^l}\right)^{\beta_2} \left[Bc^{l\beta_1} + \left(\frac{d}{r} - \frac{c^l}{r-\gamma}\right) \right] \right\} \frac{\partial \alpha^{*L}(c^{*L})}{\partial c^{*L}} + \\ & + \left(\frac{c_t}{c^{*L}}\right)^{\beta_2} \alpha^{*L}(c^{*L}) Ac^{*L\beta_2} \frac{\beta_2}{c^{*L}} = 0 \end{aligned}$$

which reduces to

$$\frac{p-d}{r} - \left(k_1 + \frac{k_2}{2} \alpha^{*L}(c^{*L})^2\right) = 0. \quad (\text{B.8.1})$$

Solving for $\alpha^{*L}(c^{*L})$ yields

$$\alpha^{*L}(c^{*L}) = \left(2 \frac{p-d}{r} - k_1\right)^{1/2} = \alpha^{*U}(c^{*U}). \quad (\text{B.8.2})$$

The investment threshold is instead given by

$$c^{*L} = \left\{ \frac{[2k_2 \left(\frac{p-d}{r} - k_1\right)]^{1/2}}{A - c^{l-\beta_2} [Bc^{l\beta_1} + \left(\frac{d}{r} - \frac{c^l}{r-\gamma}\right)]} \right\}^{1/\beta_2} \leq c^{*U}. \quad (\text{B.8.3})$$

Note that

$$c^{*L} = \alpha^{*L} (c^{*L})^{1/\beta_2} \tilde{c}^L.$$

Hence,

$$c^{*L} \geq \tilde{c}^L \quad \text{for } \alpha^{*L} (c^{*L}) \leq 1$$

or

$$c^{*L} \geq \tilde{c}^L \quad \text{for } \frac{p-d}{r} \leq k_1 + \frac{k_2}{2}.$$

Notice that for

$$\frac{p-d}{r} > k_1 + \frac{k_2}{2} > k_1 + \frac{k_2}{2} \alpha^{*L} (c^{*L})^2$$

$O^L(c_t, c^{*L})$ is increasing in c^{*L} and Eq. (B.8.1) has no solution. Then the firm invests immediately, i.e., at $c_0 = c$, and sets

$$\alpha^{*L} (c^{*L}) = Ac^{\beta_2} / k_2. \tag{B.8.4}$$

Scenario 1.2.B. By substituting Eqs. (3), (A.3), (B.4) and (B.6) into Eq. (B.7) we have

$$O^L(c_t, c^{*L}) = \max_{c^{*L}} \left(\frac{c_t}{c^{*L}} \right)^{\beta_2} \left[\frac{p-d}{r} + Ac^{*L\beta_2} - \left(k_1 + \frac{k_2}{2} \right) \right] - \left(\frac{c_t}{c^L} \right)^{\beta_2} (Bc^L\beta_1 + k_3). \tag{B.9}$$

By taking the first derivative of the objective with respect to c^{*L} we have:

$$\frac{\partial O^L(c_t, c^{*L})}{\partial c^{*L}} = -\frac{\beta_2}{c^{*L}} \left(\frac{c_t}{c^{*L}} \right)^{\beta_2} \left[\frac{p-d}{r} - \left(k_1 + \frac{k_2}{2} \right) \right]. \tag{B.9.1}$$

This implies that

$$c^{*L} = \begin{cases} \tilde{c}^L, & \text{if } \frac{p-d}{r} > k_1 + \frac{k_2}{2} \\ d, & \text{if } \frac{p-d}{r} \leq k_1 + \frac{k_2}{2} \end{cases} \tag{B.9.2}$$

B.5.2. Scenario 2.B

For $\tilde{c}^L < d$, the firm sets $\alpha^{*L} < 1$. The analysis is identical to scenario 1.1.B. Note that

$$\begin{aligned} c^{*L} &= \alpha^{*L} (c^{*L})^{1/\beta_2} \tilde{c}^L \\ &\geq d \rightarrow \frac{p-d}{r} \leq k_1 + \frac{k_2}{2} \left(\frac{d}{\tilde{c}^L} \right)^{2\beta_2} < k_1 + \frac{k_2}{2}. \end{aligned}$$

In contrast, the firm invests immediately, i.e., at $c_0 = c$, and chooses

$$\alpha^{*L} (c^{*L}) = Ac^{\beta_2} / k_2 \quad \text{for } \frac{p-d}{r} > k_1 + \frac{k_2}{2} \left(\frac{d}{\tilde{c}^L} \right)^{2\beta_2}.$$

B.6. Convertible debt and equity: a generalization

In this subsection we show that the quality of our results holds even allowing for a more general frame where the lender receives only a share of the firm's equity from conversion. To do it we assume, following Egami (2010), that the lender receives a pre-determined fraction $\eta/(1 + \eta)$ with $\eta > 0$ of the firm's equity upon conversion. This is consistent with issuing at the conversion time a new fraction of share η for each unit of pre-existing shares. Hence, once conversion has occurred, a portion $\eta/(1 + \eta)$ of the resulting equity is held by the (previous) debtholders while the residual $1 - \eta/(1 + \eta)$ is held by the (previous) equityholders. Note that by letting $\eta \rightarrow \infty$ we return to the model above where the lender acquires the firm completely upon conversion. Last, for the sake of simplicity, we assume that the conversion cost k_3 is split among the parties on the basis of the portions of equity held.

B.6.1. Debt

The value function for $c_t > \tilde{c}^L$ is

$$\max_{\tau > 0} D(c_t; \alpha) = \frac{D}{r} + E[e^{-r\tau}] \left[\frac{\eta}{1 + \eta} \left(\frac{p}{r} - \frac{\tilde{c}^L}{r - \gamma} - k_3 \right) - \frac{D}{r} \right]$$

where $\tau = \min\{t > 0 : c_t = \bar{c}^t\}$ and $k_3 \geq -k_1$. The problem can be rearranged as follows

$$\max_{\bar{c}^t} D(c_t; \alpha) = \frac{D}{r} + \left(\frac{c_t}{\bar{c}^t}\right)^{\beta_2} \left[\frac{\eta}{1+\eta} \left(\frac{p}{r} - \frac{\bar{c}^t}{r-\gamma} - k_3 \right) - \frac{D}{r} \right].$$

Optimality requires:

$$-\frac{\beta_2}{\beta_2-1} \left(\frac{\eta}{1+\eta} \frac{p}{r} - \frac{\eta}{1+\eta} k_3 - \frac{D}{r} \right) + \frac{\eta}{1+\eta} \frac{\bar{c}^t}{r-\gamma} = 0.$$

which gives

$$\bar{c}^t = \frac{\beta_2}{\beta_2-1} (r-\gamma) \left[\frac{p}{r} - \left(1 + \frac{1}{\eta}\right) \frac{D}{r} - k_3 \right] < c^t \quad (\text{B.10})$$

where

$$\bar{c}^t \leq d \rightarrow \frac{p - (1 + \frac{1}{\eta})D}{r} \leq \left(1 - \frac{1}{\beta_2}\right) \frac{d}{r-\gamma} + k_3$$

and

$$\frac{\partial \bar{c}^t}{\partial D} = -\left(1 + \frac{1}{\eta}\right) \frac{\bar{c}^t}{\frac{p}{r} - \left(1 + \frac{1}{\eta}\right) \frac{D}{r} - k_3} < 0$$

$$\frac{\partial \bar{c}^t}{\partial k_3} = -\frac{r}{p - \left(1 + \frac{1}{\eta}\right)D - rk_3} \bar{c}^t < 0.$$

Note that as $\bar{c}^t < c^t$ the conversion occurs, in expected terms, later than in the case where debtholders acquire the entire enterprise. This makes sense considering the lower net value associated with debt conversion.

B.6.2. Equity

Conditions for an optimal switch, i.e., value matching plus smooth pasting condition, at $c_t = d$ between the two productive frames require

$$\frac{p-d-D}{r} + \hat{A}d^{\beta_2} = \left[\frac{p - (1-\alpha)d - D}{r} - \alpha \frac{d}{r-\gamma} \right] + \hat{B}d^{\beta_1} +$$

$$-\left(\frac{d}{\bar{c}^t}\right)^{\beta_2} \left\{ \left[\frac{p - (1-\alpha)d - D}{r} - \alpha \frac{\bar{c}^t}{r-\gamma} + \hat{B}\bar{c}^{t\beta_1} \right] - \left[\frac{1}{1+\eta} \left(\frac{p}{r} - \frac{\bar{c}^t}{r-\gamma} - k_3 \right) \right] \right\} \quad (\text{B.11.1})$$

$$\hat{A}\beta_2 d^{\beta_2-1} = -\alpha \frac{1}{r-\gamma} + \hat{B}\beta_1 d^{\beta_1-1} +$$

$$-\frac{\beta_2}{\bar{c}^t} \left(\frac{d}{\bar{c}^t}\right)^{\beta_2-1} \left\{ \left[\frac{p - (1-\alpha)d - D}{r} - \alpha \frac{\bar{c}^t}{r-\gamma} + \hat{B}\bar{c}^{t\beta_1} \right] - \left[\frac{1}{1+\eta} \left(\frac{p}{r} - \frac{\bar{c}^t}{r-\gamma} - k_3 \right) \right] \right\}. \quad (\text{B.11.2})$$

Solving the system (B.11.1)–(B.11.2) yields

$$\hat{A} = \tilde{A} - \left(\frac{1}{\bar{c}^t}\right)^{\beta_2} \left\{ \left[\frac{p - (1-\alpha)d - D}{r} - \alpha \frac{\bar{c}^t}{r-\gamma} + \hat{B}\bar{c}^{t\beta_1} \right] - \left[\frac{1}{1+\eta} \left(\frac{p}{r} - \frac{\bar{c}^t}{r-\gamma} - k_3 \right) \right] \right\} \quad (\text{B.12.1})$$

$$\hat{B} = \alpha \frac{r - \beta_2\gamma}{r(r-\gamma)(\beta_1 - \beta_2)} d^{1-\beta_1} = \tilde{B}. \quad (\text{B.12.2})$$

Note that:

$$\frac{\partial \hat{A}}{\partial \bar{c}^t} = \bar{c}^{t-(\beta_2+1)} \left[\beta_2 \frac{p-d-D(1+\frac{1}{\eta})}{r} (1-\alpha) + \alpha\beta_2 k_3 + (\beta_2 - \beta_1) \hat{B}\bar{c}^{t\beta_1} \right] < 0$$

and

$$\lim_{\bar{c}^t \rightarrow 0} \hat{A} = \tilde{A}, \quad \lim_{\bar{c}^t \rightarrow d} \hat{A} = -\left(\frac{1}{d}\right)^{\beta_2} \left[\frac{p-d-D(1+\frac{1}{\eta})}{r} \right].$$

B.6.3. Value of the levered firm

The market value of the levered firm is, as above, given by:

$$V^L(c_t; \alpha) = E(c_t; \alpha) + D(c_t; \alpha).$$

It is then easy to show that

$$V^L(c_t; \alpha) = \begin{cases} \left[\frac{p-d}{r} + \tilde{A}c_t^{\beta_2} - \left(\frac{c_t}{\bar{c}^l} \right)^{\beta_2} \left[\tilde{B}c_t^{\beta_1} - (1-\alpha) \left(\frac{d}{r} - \frac{c^l}{r-\gamma} \right) + k_3 \right] \right] & \text{if } c_t > d, \\ \left[\frac{p-(1-\alpha)d}{r} - \alpha \frac{c_t}{r-\gamma} + \tilde{B}c_t^{\beta_1} - \left(\frac{c_t}{\bar{c}^l} \right)^{\beta_2} \left[\tilde{B}\bar{c}^{\beta_1} - (1-\alpha) \left(\frac{d}{r} - \frac{\bar{c}^l}{r-\gamma} \right) + k_3 \right] \right] & \text{if } c_t \leq \bar{c}^l \end{cases}$$

and

$$F^L(c_t; \alpha) = \begin{cases} F^U(c_t; \alpha) - Z(c_t; \bar{c}^l) & \text{if } c_t > \bar{c}^l, \\ \left(\frac{p}{r} - \frac{c_t}{r-\gamma} \right) - k_3 & \text{if } c_t \leq \bar{c}^l \end{cases} \tag{B.13}$$

where $Z(c_t; \bar{c}^l) = (c_t/\bar{c}^l)^{\beta_2} [\tilde{B}\bar{c}^{\beta_1} - (1-\alpha)(\frac{d}{r} - \frac{\bar{c}^l}{r-\gamma}) + k_3]$.

Eq. (B.13) generalizes Eq. (4) without impacting on the quality of the results concerning optimal outsourcing and timing.

Appendix C. Venture capital

C.1. Optimal outsourcing share

The optimal vertical arrangement is given by:

$$\begin{aligned} \alpha^{*V} &= \operatorname{argmax} \left[(1-\psi)F^U(c_t; \alpha) - \left(k_1 + \frac{k_2}{2}\alpha^2 - K^V \right) \right] \\ &= \operatorname{argmax} \left[(1-\psi) \left(\frac{p-d}{r} + \alpha Ac_t^{\beta_2} \right) - \left(k_1 + \frac{k_2}{2}\alpha^2 - K^V \right) \right] \end{aligned} \tag{C.1}$$

where $K^V \leq k_1 + \frac{k_2}{2}\alpha^2$. The relative FOC is:

$$(1-\psi)Ac_t^{\beta_2} - k_2\alpha = 0 \tag{C.1.1}$$

while the SOC is always satisfied. Solving for α yields:

$$\alpha^{*V}(c_t) = \begin{cases} 1 & \text{if } c_t \leq \bar{c}^V \\ [(1-\psi)A/k_2]c_t^{\beta_2} & \text{if } c_t > \bar{c}^V \end{cases} \tag{C.2}$$

where $\bar{c}^V = [k_2/(1-\psi)A]^{1/\beta_2} < \bar{c}^U$ for $\psi \in (0, 1)$.

C.2. Investment timing

The value function with venture capital is

$$O^V(c_t, c^{*V}) = \max_{c^{*V}} \left(\frac{c_t}{c^{*V}} \right)^{\beta_2} (\psi F^U(c^{*V}, \alpha^{*V}(c^{*V})) - K^V) \tag{C.3}$$

where $T^{V*} = \inf\{t \geq 0 \mid c_t = c^{*V}\}$ is the optimal investment timing and $\alpha^{*V}(c^{*V})$ is the optimal outsourcing share at $t = T^{V*}$.

Let's solve the maximization problem in two potential scenarios:

Scenario 1.C $\alpha^{*V} \leq 1$, if $\bar{c}^V \geq d \rightarrow Ad^{\beta_2} \geq \frac{k_2}{1-\psi}$

Scenario 2.C $\alpha^{*V} < 1$, if $\bar{c}^V < d \rightarrow Ad^{\beta_2} < \frac{k_2}{1-\psi}$.

C.2.1. Scenario 1.C

At $c_0 = c$, when evaluating investment and timing, two potential investment scenarios may arise:

Scenario 1.1.C $\alpha^{*V} \leq 1$, if $c \geq \bar{c}^V$

Scenario 1.2.C $\alpha^{*V} = 1$, if $\bar{c}^V \geq c < d$.

Scenario 1.1.C. By substituting Eqs. (3), (A.3) and (C.2) into Eq. (C.3) we have

$$O^V(c_t, c^{*V}) = \max_{c^{*V}} \left(\frac{c_t}{c^{*V}} \right)^{\beta_2} \left[\frac{p-d}{r} + (1-\psi) \frac{(Ac^{*V\beta_2})^2}{k_2} - \frac{K^V}{\psi} \right] \psi. \quad (C.4)$$

Optimality requires:

$$-\frac{\beta_2}{c^{*V}} \left(\frac{c_t}{c^{*V}} \right)^{\beta_2} \left[\frac{p-d}{r} - (1-\psi) \frac{(Ac^{*V\beta_2})^2}{k_2} - \frac{K^V}{\psi} \right] \psi = 0 \quad (C.4.1)$$

which reduces to

$$\frac{p-d}{r} - (1-\psi) \frac{(Ac^{*V\beta_2})^2}{k_2} = \frac{K^V}{\psi}. \quad (C.4.2)$$

Solving for c^{*V} yields

$$c^{*V} = \left\{ \frac{\left[\frac{k_2}{1-\psi} \left(\frac{p-d}{r} - \frac{K^V}{\psi} \right) \right]^{1/2}}{A} \right\}^{1/\beta_2}. \quad (C.4.3)$$

Substituting c^{*V} into Eq. (C.2) gives

$$\alpha^{*V}(c^{*V}) = \left[\frac{1-\psi}{k_2} \left(\frac{p-d}{r} - \frac{K^V}{\psi} \right) \right]^{1/2}. \quad (C.4.4)$$

Note that

$$c^{*V} = \alpha^{*V}(c^{*V}) \bar{c}^V,$$

hence,

$$c^{*V} \geq \bar{c}^V \quad \text{for } \alpha^{*V}(c^{*V}) \leq 1$$

or

$$c^{*V} \geq \bar{c}^V \quad \text{for } \frac{p-d}{r} \leq \frac{K^V}{\psi} + \frac{k_2}{1-\psi}.$$

Notice that for

$$\frac{p-d}{r} > \frac{K^V}{\psi} + \frac{k_2}{1-\psi}$$

$O^V(c_t, c^{*V})$ is increasing in c^{*V} and Eq. (C.4.2) has no solution. This implies that the venture capitalist calls for an immediate investment, i.e., at $c_0 = c$, which in turn corresponds to

$$\alpha^{*V}(c^{*V}) = (1-\psi)Ac^{\beta_2}/k_2. \quad (C.4.5)$$

Scenario 1.2.C. By substituting Eqs. (3), (A.3) and (C.2) into Eq. (C.3.1) we have

$$O^V(c_t, c^{*V}) = \max_{c^{*V}} \left(\frac{c_t}{c^{*V}} \right)^{\beta_2} \left(\frac{p-d}{r} + Ac^{*V\beta_2} - \frac{K^V}{\psi} \right) \psi. \quad (C.5)$$

By taking the first derivative of the objective with respect to c^{*V} we have:

$$\frac{\partial O^V(c_t, c^{*V})}{\partial c^{*V}} = -\frac{\beta_2}{c^{*V}} \left(\frac{c_t}{c^{*V}} \right)^{\beta_2} \left(\frac{p-d}{r} - \frac{K^V}{\psi} \right) \psi. \quad (C.5.1)$$

This implies that

$$c^{*V} = \begin{cases} \bar{c}^V, & \text{if } \frac{p-d}{r} > \frac{K^V}{\psi} \\ d, & \text{if } \frac{p-d}{r} \leq \frac{K^V}{\psi} \end{cases} \quad (C.5.2)$$

C.2.2. Scenario 2.C

For $\bar{c}^V < d$, the firm sets $\alpha^{*V} < 1$. The analysis replicates scenario 1.1.C. Note that

$$c^{*V} = \alpha^{*V} (c^{*V})^{1/\beta_2} \bar{c}^V \geq d \rightarrow \frac{p-d}{r} \leq \frac{K^V}{\psi} + \frac{k_2}{1-\psi} \left(\frac{d}{\bar{c}^V}\right)^{2\beta_2} < \frac{K^V}{\psi} + \frac{k_2}{1-\psi}.$$

In contrast, the venture capitalist calls for immediate investment, i.e., at $c_0 = c$, and the firm chooses

$$\alpha^{*V}(c^{*V}) = (1-\psi)Ac^{\beta_2}/k_2 \text{ for } \frac{p-d}{r} > \frac{K^V}{\psi} + \frac{k_2}{1-\psi} \left(\frac{d}{\bar{c}^V}\right)^{2\beta_2}.$$

C.2.3. Partial outsourcing

Comparative statics. Studying the impact of K^V on $\alpha^{*V}(c^{*V}) < 1$ and on c^{*V} we notice that:

$$\frac{\partial \alpha^{*V}(c^{*V})}{\partial K^V} = -\frac{1}{2} \frac{\alpha^{*V}(c^{*V})}{\psi \frac{p-d}{r} - K^V} < 0 \tag{C.6}$$

$$\frac{\partial c^{*V}}{\partial K^V} = \frac{1}{\beta_2} \frac{\partial \alpha^{*V}(c^{*V})}{\partial K^V} \frac{c^{*V}}{\alpha^{*V}(c^{*V})} > 0. \tag{C.7}$$

Taking the derivative of $\alpha^{*V}(c^{*V})$ and c^{*V} with respect to ψ we find that:

$$\frac{\partial \alpha^{*V}(c^{*V})}{\partial \psi} = -\frac{1}{2} \frac{\alpha^{*V}(c^{*V})}{\psi \frac{p-d}{r} - K^V} \frac{\psi}{1-\psi} \left(\frac{p-d}{r} - \frac{K^V}{\psi^2}\right) \tag{C.8}$$

$$\frac{\partial c^{*V}}{\partial \psi} = \frac{1}{\beta_2} \frac{c^{*V}}{\alpha^{*V}(c^{*V})} \left(\frac{\partial \alpha^{*V}(c^{*V})}{\partial \psi} + \frac{\alpha^{*V}(c^{*V})}{1-\psi}\right) \tag{C.9}$$

both $\alpha^{*V}(c^{*V})$ and c^{*V} are non-monotonic in ψ . In particular we note that:

$$\frac{\partial \alpha^{*V}(c^{*V})}{\partial \psi} = \begin{cases} \geq 0, & \text{for } \psi \leq \underline{\psi}^{1/2}, \\ < 0 & \text{for } \psi > \underline{\psi}^{1/2} \end{cases}$$

where $\underline{\psi} = K^V / (\frac{p-d}{r})$ and

$$\frac{\partial c^{*V}}{\partial \psi} = \begin{cases} \geq 0, & \text{for } \frac{\partial \alpha^{*V}(c^{*V})}{\partial \psi} \leq -\frac{\alpha^{*V}(c^{*V})}{1-\psi}, \\ < 0 & \text{for } \frac{\partial \alpha^{*V}(c^{*V})}{\partial \psi} > -\frac{\alpha^{*V}(c^{*V})}{1-\psi} \end{cases}$$

Constraints. We study in this subsection the role of constraints of the optimal investment policy with venture capital. Let's first check the profitability that both parties attribute to the financial arrangement to be set up based on the offer of a profit share ψ in exchange of a capital contribution K^V .

The pair (ψ, K^V) must be such that

$$\psi F^U(c^{*V}; \alpha^{*V}(c^{*V})) \geq K^V \tag{Constraint 1}$$

$$(1-\psi)F^U(c^{*V}; \alpha^{*V}(c^{*V})) \geq I(\alpha^{*V}(c^{*V})) - K^V \tag{Constraint 2}$$

that is, the net value accruing to venture capitalist and to equity holders, respectively, must be non-negative. It is easy to show that the two constraints reduce to the following conditions:

$$\frac{p-d}{r} \geq \frac{K^V}{\psi} \tag{C.10}$$

$$G(\psi) = 3\psi^2 \frac{p-d}{r} - \psi \left(3 \frac{p-d}{r} - K^V + 2k_1\right) + K^V \leq 0. \tag{C.11}$$

Note that condition (C.10) is met by assumption.

A further restriction concerns the maximum contribution allowed to the venture capitalist, i.e.,

$$I(\alpha^{*V}(c^{*V})) \geq K^V. \tag{Constraint 3}$$

Rearranging constraint (3) yields

$$(2/k_2)(K^V - k_1) \leq (\alpha^{*V}(c^{*V}))^2.$$

This implies that $N(\psi) \leq 0$ for any $\psi \in (\underline{\psi}, 1)$ when $K^V \leq k_1$.

Let's then consider the case where $K^V > k_1$. Rearranging constraint (3) we have:

$$N(\psi) = \psi^2 \frac{p-d}{r} - \psi \left(\frac{p-d}{r} - K^V + 2k_1 \right) + K^V \leq 0. \quad (\text{C.12})$$

As $N(\psi)$ is convex in ψ , a necessary and sufficient condition for inequality (C.12) to hold is

$$N(\psi_{\min}^N) \leq 0 \rightarrow \frac{p-d}{r} \geq K^V - 2 \left(k_1 - \sqrt{\frac{p-d}{r} K^V} \right) > K^V - 2(k_1 - K^V) \quad (\text{C.13})$$

where

$$\psi_{\min}^N = \frac{1}{2} \left(\frac{p-d}{r} - K^V + 2k_1 \right) / \left(\frac{p-d}{r} \right) > \underline{\psi}$$

is the minimum of the function $N(\psi)$.

Hence, provided that condition (C.13) holds, $N(\psi) \leq 0$ in the interval $[\psi_1^N, \psi_2^N]$ where

$$\psi_1^N = \psi_{\min}^N - \sqrt{(\psi_{\min}^N)^2 - \underline{\psi}} > \underline{\psi}, \quad \psi_2^N = \psi_{\min}^N + \sqrt{(\psi_{\min}^N)^2 - \underline{\psi}} < 1$$

are the two positive roots of the equation $N(\psi) = 0$.

Last, it is easy to show that

$$N(\psi) - G(\psi) = -2 \frac{p-d}{r} (\psi - 1) \psi > 0 \quad \text{for any } \psi \in (\underline{\psi}, 1).$$

This implies that condition (C.11) holds whenever $N(\psi) \leq 0$.

Finally, note that ψ_1^N and ψ_2^N are reported in Proposition 5 as $\underline{\psi}_1$ and $\bar{\psi}$, respectively.

Optimal outsourcing level: comparison with the benchmark. Let's identify the conditions under which $\alpha^{*V} < \alpha^{*U}$. The inequality holds if:

$$Q(\psi) = \psi^2 \frac{p-d}{r} + \psi \left(\frac{p-d}{r} - 2k_1 - K^V \right) + K^V > 0. \quad (\text{C.14})$$

It is easy to check that $Q(\psi) > 0$ for $\frac{p-d}{r} \geq K^V + 2k_1$. Otherwise, i.e., for $\frac{p-d}{r} < K^V + 2k_1$, the following condition must hold:

$$Q(\psi_{\min}^Q) > 0 \rightarrow \frac{p-d}{r} > K^V + 2 \left(k_1 - \sqrt{\frac{p-d}{r} K^V} \right) \quad (\text{C.15})$$

where

$$\psi_{\min}^Q = -\frac{1}{2} \left(\frac{p-d}{r} - K^V - 2k_1 \right) / \left(\frac{p-d}{r} \right)$$

is the minimum of the function $Q(\psi)$. Note that

$$K^V + 2 \left(k_1 - \sqrt{\frac{p-d}{r} K^V} \right) > K^V + 2k_1.$$

It follows that when

$$\frac{p-d}{r} > K^V + 2 \left(k_1 - \sqrt{\frac{p-d}{r} K^V} \right),$$

we have

$$Q(\psi) > 0 \quad \alpha^{*V} < \alpha^{*U}, \quad \text{for } \psi \in (\underline{\psi}, 1).$$

Let's consider the case of $\frac{p-d}{r} \leq K^V + 2 \left(k_1 - \sqrt{\frac{p-d}{r} K^V} \right)$. Under this scenario, we may have:

$$Q(\psi) \leq 0 \quad \alpha^{*V} \geq \alpha^{*U} \quad \text{for } \psi \in [\psi_1^Q, \psi_2^Q]$$

$$Q(\psi) > 0, \quad \alpha^{*V} < \alpha^{*U} \quad \text{otherwise,}$$

where ψ_1^Q and ψ_2^Q are the two positive roots of the equation $Q(\psi) = 0$. Note that as:

$$Q(\psi) - N(\psi) = 2\psi \left(\frac{p-d}{r} - K^V \right) > 0 \quad \text{for any } \psi > \underline{\psi},$$

then $\underline{\psi} < \psi_1^N < \psi_1^Q$ and $\psi_2^Q < \psi_2^N < 1$. Finally, note that ψ_1^Q and ψ_2^Q are reported in Proposition 6 as $\underline{\psi}_2$ and $\bar{\psi}_1$, respectively.

Investment threshold: comparison with the benchmark. Let's identify the conditions under which $c^{*V} < c^{*U}$. The inequality holds if:

$$\begin{aligned} J(\psi) &= 2\psi^2 \left(\frac{p-d}{r} - k_1 \right) - \psi \left(\frac{p-d}{r} - 2k_1 \right) - K^V \\ &= 2\psi^2 \frac{2-\psi}{1-\psi} \left(\frac{p-d}{r} - k_1 \right) - \frac{Q(\psi)}{1-\psi} > 0. \end{aligned} \tag{C.16}$$

Condition (C.16) is satisfied when $Q(\psi) \leq 0$. Hence, whenever, with venture capital, optimal outsourcing is $\alpha^{*V} \geq \alpha^{*U}$, investment occurs at $c^{*V} < c^{*U}$.

In contrast, when $Q(\psi) > 0$, i.e., $\alpha^{*V} < \alpha^{*U}$, the sign of $J(\psi)$ is ambiguous. In general, we have

$$\begin{aligned} J(\psi) \leq 0 & \quad Q(\psi) \geq 2\psi^2(2-\psi) \left(\frac{p-d}{r} - k_1 \right) \\ J(\psi) > 0 & \quad \text{otherwise.} \end{aligned}$$

We note that

(i)

$$J(\underline{\psi}) = 2\underline{\psi} \left(\frac{p-d}{r} - k_1 \right) (\underline{\psi} - 1) < 0,$$

(ii)

$$J(1) = \frac{p-d}{r} - K^V > 0,$$

Hence, we may conclude that

$$\begin{aligned} J(\psi) \leq 0 & \quad \text{for } \underline{\psi} < \psi \leq \hat{\psi} \\ J(\psi) > 0 & \quad \text{for } \hat{\psi} < \psi < 1 \end{aligned}$$

where $\hat{\psi}$ is the positive root of the equation $J(\psi) = 0$.

Appendix D. The shareholder's problem

In order to identify the optimal ψ , the shareholders maximize the following function:

$$\max_{\psi} \left(\frac{c_t}{c^{*V}} \right)^{\beta_2} \left[(1-\psi)F^U(c^{*V}; \alpha^{*V}(c^{*V})) - \left(k_1 + \frac{k_2}{2} \alpha^{*V}(c^{*V})^2 - K^V \right) \right]. \tag{D.1}$$

under constraints (1)–(3) plus:

$$\alpha^{*V}(c^{*V}) \leq 1 \rightarrow \frac{p-d}{r} \leq \frac{K^V}{\psi} + \frac{k_2}{1-\psi}. \tag{Constraint 4}$$

Plugging α^{*V} and c^{*V} into Problem (D.1) we have

$$\max_{\psi} \left\{ \frac{1}{2} \frac{A}{(k_2)^{1/2}} c_t^{\beta_2} \left(\frac{1-\psi}{\frac{p-d}{r} - \frac{K^V}{\psi}} \right)^{1/2} \left[3(1-\psi) \frac{p-d}{r} + \left(3 - \frac{1}{\psi} \right) K^V - 2k_1 \right] \right\}.$$

Solving Problem (D.1) is equivalent to solve the following:

$$\max_{\psi} G(\psi) = \left(\frac{1-\psi}{\frac{p-d}{r} - \frac{K^V}{\psi}} \right)^{1/2} \left[3(1-\psi) \frac{p-d}{r} + \left(3 - \frac{1}{\psi} \right) K^V - 2k_1 \right].$$

The relative first order condition is:

$$-(1/2) \left(\frac{1-\psi}{\frac{p-d}{r} - \frac{K^V}{\psi}} \right)^{-1/2} \left[3(1-\psi) \frac{p-d}{r} + \left(3 - \frac{1}{\psi} \right) K^V - 2k_1 \right] \frac{\left(\frac{p-d}{r} - \frac{K^V}{\psi} \right) + (1-\psi) \frac{K^V}{\psi^2}}{\left(\frac{p-d}{r} - \frac{K^V}{\psi} \right)^2} +$$

$$-\left(\frac{1-\psi}{\frac{p-d}{r} - \frac{K^V}{\psi}} \right)^{1/2} \left(3 \frac{p-d}{r} - \frac{K^V}{\psi^2} \right) = 0$$

which reduces to:

$$\frac{1}{1-\psi} + \frac{\frac{K^V}{\psi^2}}{\frac{p-d}{r} - \frac{K^V}{\psi}} + 2 \frac{3 \frac{p-d}{r} - \frac{K^V}{\psi^2}}{3(1-\psi) \frac{p-d}{r} + \left(3 - \frac{1}{\psi} \right) K^V - 2k_1} = 0. \quad (D.2)$$

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