Redefining the Axion Window

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A major goal of axion searches is to reach inside the parameter space region of realistic axion models. Currently, the boundaries of this region depend on somewhat arbitrary criteria, and it would be desirable to specify them in terms of precise phenomenological requirements. We consider hadronic axion models and classify the representations R_Q of the new heavy quarks Q. By requiring that (i) the Q's are sufficiently short lived to avoid issues with long-lived strongly interacting relics, (ii) no Landau poles are induced below the Planck scale; 15 cases are selected which define a phenomenologically preferred axion window bounded by a maximum (minimum) value of the axion-photon coupling about 2 times (4 times) larger than is commonly assumed. Allowing for more than one R_Q, larger couplings, as well as complete axion-photon decoupling, become possible.

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Introduction.—In spite of its indisputable success, the standard model (SM) is not completely satisfactory: it does not explain unquestionable experimental facts like dark matter (DM), neutrino masses, and the cosmological baryon asymmetry, and it contains fundamental parameters with highly unnatural values, like the Higgs potential term μ^2, the first generation Yukawa couplings $h_{e,u,d}$, and the strong CP violating angle $|\theta| < 10^{-10}$. This last quantity is somewhat special: its value is stable with respect to higher order corrections [1] (unlike μ^2) and (unlike $h_{e,u,d}$ [2]) it evades explanations based on environmental selection [3]. Thus, seeking explanations for the smallness of θ independently of other "small value" problems is theoretically motivated. Basically, only three types of solutions exist. The simplest possibility, a massless up quark, is now ruled out [4,5]. The so-called Nelson-Barr type of models [6,7] require either a high degree of fine-tuning, often comparable to setting $|\theta| \lesssim 10^{-10}$ by hand, or rather elaborate theoretical structures [8]. The Peccei-Quinn (PQ) solution [9–12], although it is not completely free from issues [13–15], arguably stands on better theoretical ground.

Setting aside theoretical considerations, the question as to whether the PQ solution is the correct one could be set experimentally by detecting the axion. In order to focus axion searches, it is then very important to identify as well as possible the region of parameter space where realistic axion models live. The vast majority of search techniques are sensitive to the axion-photon coupling $g_{a\gamma\gamma}$, which is inversely proportional to the axion decay constant f_a. Since the axion mass m_a has the same dependence, theoretical predictions and experimental exclusion limits can be conveniently presented in the m_a-$g_{a\gamma\gamma}$ plane. The commonly adopted *axion band* corresponds roughly to $g_{a\gamma\gamma} \sim m_a \alpha/(2\pi f_\pi m_\pi) \sim 10^{-10}(m_a/\mathrm{eV})\ \mathrm{GeV}^{-1}$, with a somewhat arbitrary width, chosen to include representative models [16–18]. In this Letter we put forth a definition of a *phenomenologically preferred* axion window as the region encompassing *hadronic* axion models which (i) do not contain cosmologically dangerous relics, and (ii) do not induce Landau poles (LPs) below some scale Λ_{LP} close to the Planck mass $m_P = 1.2 \times 10^{19}$ GeV. While all of the cases we consider belong to the Kim-Shifman-Vainshtein-Zakharov (KSVZ) type of models [19,20], the resulting window encompasses also the Dine-Fischler-Srednicki-Zhitnitsky axion [21,22] and many of its variants [17].

Hadronic axion models.—The basic ingredient of any renormalizable axion model is a global $U(1)_{\mathrm{PQ}}$ symmetry. The associated Noether current J_μ^{PQ} must have a color anomaly and, although not required for solving the strong CP problem, it also has, in general, an electromagnetic anomaly:

$$\partial^\mu J_\mu^{\mathrm{PQ}} = \frac{N\alpha_s}{4\pi} G_{\mu\nu}^a \tilde{G}^{a\mu\nu} + \frac{E\alpha}{4\pi} F_{\mu\nu}\tilde{F}^{\mu\nu}, \quad (1)$$

where $G_{\mu\nu}^a(F_{\mu\nu})$ is the color (electromagnetic) field strength tensor, $\tilde{G}^{a\mu\nu}(\tilde{F}^{\mu\nu}) = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}G_{\rho\sigma}^a(F_{\rho\sigma})$ its dual, and N and E the respective anomaly coefficients. In a generic axion model of the KSVZ type [19,20], the anomaly is induced by pairs of heavy fermions Q_L, Q_R which must transform nontrivially under $SU(3)_C$, and chirally under $U(1)_{\mathrm{PQ}}$. Their mass arises from a Yukawa interaction with a SM singlet scalar Φ which develops a PQ breaking vacuum expectation value. Thus, their PQ charges $\mathcal{X}_{L,R}$, normalized to $\mathcal{X}(\Phi) = 1$, must satisfy $|\mathcal{X}_L - \mathcal{X}_R| = 1$. We denote the (vectorlike) representations of the SM gauge group $G_{\mathrm{SM}} = SU(3)_C \times SU(2)_I \times U(1)_Y$, to which we assign the Q as $R_Q = (\mathcal{C}_Q, \mathcal{I}_Q, \mathcal{Y}_Q)$, so that

$$N = \sum_Q (\mathcal{X}_L - \mathcal{X}_R) T(\mathcal{C}_Q), \tag{2}$$

$$E = \sum_Q (\mathcal{X}_L - \mathcal{X}_R) \mathcal{Q}_Q^2, \tag{3}$$

where the sum is over irreducible color representations (for generality, we allow for the simultaneous presence of more R_Q's). The color index is defined by $\mathrm{Tr} T_Q^a T_Q^b = T(\mathcal{C}_Q)\delta^{ab}$, with T_Q representing the generators in $\mathcal{C}_Q$ and $\mathcal{Q}_Q$ being the $U(1)_{\mathrm{em}}$ charge. The scalar field Φ can be parametrized as

$$\Phi(x) = (1/\sqrt{2})[\rho(x) + V_a] e^{ia(x)/V_a}. \tag{4}$$

The mass of $\rho(x)$ is of the order $V_a \gg (\sqrt{2}G_F)^{-1/2} = 247$ GeV, while a tiny mass for the axion $a(x)$ arises from nonperturbative QCD effects which explicitly break $U(1)_{\mathrm{PQ}}$. The SM quarks $q = q_L, d_R, u_R$ do not contribute to the QCD anomaly, and thus their PQ charges can be set to zero. The renormalizable Lagrangian for a generic hadronic axion model can be written as

$$\mathcal{L}_a = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\mathrm{PQ}} - V_{H\Phi} + \mathcal{L}_{Qq}, \tag{5}$$

where $\mathcal{L}_{\mathrm{SM}}$ is the SM Lagrangian,

$$\mathcal{L}_{\mathrm{PQ}} = |\partial_\mu \Phi|^2 + \bar{Q} i \not{D} Q - (y_Q \bar{Q}_L Q_R \Phi + \mathrm{H.c.}), \tag{6}$$

with $Q = Q_L + Q_R$. The new scalar terms are

$$V_{H\Phi} = -\mu_\Phi^2 |\Phi|^2 + \lambda_\Phi |\Phi|^4 + \lambda_{H\Phi} |H|^2 |\Phi|^2. \tag{7}$$

Finally, $\mathcal{L}_{Qq}$ contains the possible renormalizable terms coupling $Q_{L,R}$ to SM quarks which can allow for Q decays [23]. Note, however, that SM gauge invariance allows for $\mathcal{L}_{Qq} \neq 0$ only for a few specific R_Q's.

PQ quality and heavy Q stability.—The issue concerning whether the Q's are exactly stable or metastable or decay with safely short lifetimes is of central importance in our Letter, so let us discuss it in some detail. The gauge invariant kinetic term in $\mathcal{L}_{\mathrm{PQ}}$ features a $U(1)^3 \equiv U(1)_{Q_L} \times U(1)_{Q_R} \times U(1)_\Phi$ symmetry corresponding to independent rephasings of the $Q_{L,R}$ and Φ fields. The PQ Yukawa term ($y_Q \neq 0$) breaks $U(1)^3$ down to $U(1)^2$. One factor is the anomalous $U(1)_{\mathrm{PQ}}$, the other one is a nonanomalous $U(1)_Q$—that is, the Q-baryon number of the new quarks [19]—under which $Q_{L,R} \to e^{i\beta} Q_{L,R}$ and $\Phi \to \Phi$. If $U(1)_Q$ were an exact symmetry, the new quarks would be absolutely stable. For the few R_Q's for which $\mathcal{L}_{Qq} \neq 0$ is allowed, $U(1)_Q \times U(1)_B$ is further broken down to $U(1)_{B'}$, a generalized baryon number extended to the Q, which can then decay with unsuppressed rates. However, whether $\mathcal{L}_{Qq}$ is allowed at the renormalizable level depends not only on R_Q but also on the specific PQ charges. For example, independently of R_Q, the common assignment $\mathcal{X}_L = -\mathcal{X}_R = \frac{1}{2}$ would forbid PQ invariant decay operators at all orders. $U(1)_Q$ violating decays could then occur only via PQ-violating effective operators of dimension $d > 4$. Both $U(1)_{\mathrm{PQ}}$ and $U(1)_Q$ are expected to be broken at least by Planck-scale effects, inducing PQ-violating contributions to the axion potential $V_\Phi^{d>4}$, as well as an effective Lagrangian $\mathcal{L}_{Qq}^{d>4}$. Specifically, in order to preserve $|\theta| < 10^{-10}$, operators in $V_\Phi^{d>4}$ must be of dimension $d \geq 11$ [13–15]. Clearly, if $\mathcal{L}_{Qq}^{d>4}$ had to respect $U(1)_Q$ to a similar level of accuracy, the Q's would behave as effectively stable. However, a scenario in which $U(1)_Q$ arises as an accident because of specific assignments for the charges of another global symmetry $U(1)_{\mathrm{PQ}}$ seems theoretically untenable. A simple way out is to assume a suitable discrete (gauge) symmetry $\mathbb{Z}_{\mathbb{N}}$, ensuring that (i) $U(1)_{\mathrm{PQ}}$ arises accidentally and is of the required *high quality*, and (ii) $U(1)_Q$ is either broken at the renormalizable level or can be of sufficient *bad quality* to allow for safely fast Q decays.

Table I gives a neat example of how such a mechanism can work (see also Ref. [23]). We choose $R_Q = R_{d_R} = (3, 1, -1/3)$ so that G_{SM} invariance allows for $\mathcal{L}_{Qq} \neq 0$, and we assume the following transformations under $\mathbb{Z}_{\mathbb{N}}$: $Q_L \to Q_L$, $Q_R \to \omega^{\mathbb{N}-1} Q_R$, and $\Phi \to \omega \Phi$, with $\omega \equiv e^{i2\pi/\mathbb{N}}$. This ensures that the minimum dimension of the PQ breaking operators in $V_\Phi^{d>4}$ is $\mathbb{N}$. The dimension of the $U(1)_Q$ breaking decay operators depends on the $\mathbb{Z}_{\mathbb{N}}$ charges of the SM quarks. Table I lists different possibilities for $d \leq 4$ and $d = 5$. The last column gives the PQ charges that one has to assign to $Q_{L,R}$ so that $U(1)_{\mathrm{PQ}}$ can be defined also in the presence of the operators in columns 2 and 3.

Cosmology.—We assume a postinflationary scenario [$U(1)_{\mathrm{PQ}}$ broken after inflation]. Then, requiring that the axion energy density from vacuum realignment does not exceed Ω_{DM} implies $V_a/N_{\mathrm{DW}} \equiv f_a \lesssim f_a^{\max}$, with $f_a^{\max} = 5 \times 10^{11}$ GeV [24–26], where $N_{\mathrm{DW}} = 2N$ is the vacuum degeneracy corresponding to a $\mathbb{Z}_{2N} \subset U(1)_{\mathrm{PQ}}$ left unbroken by nonperturbative QCD effects. We further assume that $m_Q < T_{\mathrm{reheating}}$ so that a thermal distribution of Q provides the initial conditions for their cosmological history, which then depends only on the mass m_Q and

TABLE I. $\mathbb{Z}_{\mathbb{N}}$ charges for the SM quarks q which allow for $d \leq 4$ and $d = 5$ operators for $R_Q = (3, 1, -1/3)$.

$\mathbb{Z}_{\mathbb{N}}(q)$	$d \leq 4$	$d = 5$	$(\mathcal{X}_L, \mathcal{X}_R)$
1	$\bar{Q}_L d_R$	$\bar{Q}_L \gamma_\mu q_L (D^\mu H)^\dagger$	(0, −1)
ω	$\bar{Q}_L d_R \Phi^\dagger$		(−1, −2)
$\omega^{\mathbb{N}-2}$	…	$\bar{Q}_L d_R \Phi^2$, $\bar{Q}_R q_L H^\dagger \Phi$	(2,1)
$\omega^{\mathbb{N}-1}$	$\bar{q}_L Q_R H$, $\bar{Q}_L d_R \Phi$	…	(1,0)

representation R_Q. For a given R_Q, only fractionally charged Q hadrons can appear after confinement, which also implies that decays into SM particles are forbidden [27]. These Q hadrons must then exist today as stable relics. However, dedicated searches constrain the abundances of fractionally charged particles relative to ordinary nucleons to $n_Q/n_b \lesssim 10^{-20}$ [28], which is orders of magnitude below any reasonable estimate of the relic abundance and of the resulting concentrations in bulk matter. This restricts the viable R_Q to the much smaller subset for which Q hadrons are integrally charged or neutral. In this case decays into SM particles are not forbidden, but the lifetime τ_Q is severely constrained by cosmological observations. For $\tau_Q \sim (10^{-2}\text{–}10^{12})$ s, Q decays would affect big bang nucleosynthesis (BBN) [29,30]. The window $\tau_Q \sim (10^6\text{–}10^{12})$ s is also strongly constrained by limits on cosmic microwave background (CMB) spectral distortions from an early energy release [31–33], while decays around the recombination era ($\tau_Q \gtrsim 10^{13}$ s) would leave clear traces on CMB anisotropies. Decays after recombination would produce free-streaming photons visible in the diffuse gamma ray background [34], and Fermi LAT limits [35] allow us to exclude $\tau_Q \sim (10^{13}\text{–}10^{26})$ s. For lifetimes longer than the age of the Universe, $\tau_Q \gtrsim 10^{17}$ s, the Q would contribute to the present energy density, and we must require $\Omega_Q \le \Omega_{\rm DM} \approx 0.12h^{-2}$. However, estimating Ω_Q is not so simple. Before confinement, the Q's annihilate as free quarks. Perturbative calculations are reliable, giving, for n_f final state quark flavors,

$$\langle \sigma v \rangle_{Q\bar{Q}} = \frac{\pi \alpha_s^2}{16 m_Q^2}(c_f n_f + c_g), \tag{8}$$

with, e.g., $(c_f, c_g) = (\frac{2}{9}, \frac{220}{27})$ for triplets, and $(\frac{3}{2}, \frac{27}{4})$ for octets. Free Q annihilation freezes out around $T_{\rm fo} \sim m_Q/25$ when (for $m_Q >$ a few TeV) there are $g_* = 106.75$ effective degrees of freedom in thermal equilibrium. Together with Eq. (8), this gives

$$(\Omega_Q h^2)^{\rm Free} \approx 8 \times 10^{-3} \left(\frac{m_Q}{\rm TeV}\right)^2. \tag{9}$$

The upper lines in Fig. 1 give $(\Omega_Q h^2)^{\rm Free}$ as a function of m_Q for $SU(3)_C$ triplets (dotted line) and octets (dashed line). Only a small corner at low m_Q satisfies $\Omega_Q \le \Omega_{\rm DM}$, and future improved LHC limits on m_Q might exclude it completely. However, after confinement ($T_C \approx 180$ MeV), because of finite size effects of the composite Q hadrons, annihilation could restart. Some controversy exists about the possible enhancements for annihilations in this regime. For example, a cross section typical of inclusive hadronic scattering $\sigma_{\rm ann} \sim (m_\pi^2 v)^{-1} \sim 30 v^{-1}$ mb was assumed in Ref. [36], yielding $n_Q/n_b \sim 10^{-11}$. It was later remarked [37] that the relevant process is exclusive (no Q quarks in the final state) with a cross section that is quite likely

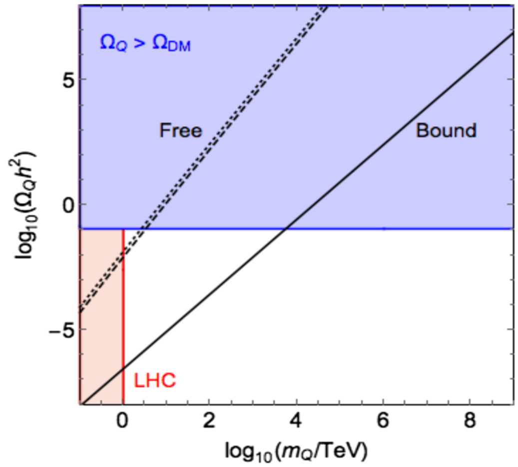

FIG. 1. Heavy Q contribution to the cosmological energy density versus m_Q. The dotted (dashed) line corresponds to free annihilation for color triplets (octets). The solid line corresponds to annihilation via bound state formation.

smaller by a few orders of magnitude. Arvanitaki *et al.* [38] suggested that bound states formed in the collision of two Q hadrons could catalyze annihilations. This mechanism was reconsidered in Refs. [39,40], which argued that Ω_Q could indeed be efficiently reduced. Their results imply

$$(\Omega_Q h^2)^{\rm Bound} \approx 3 \times 10^{-7} \left(\frac{m_Q}{\rm TeV}\right)^{3/2}, \tag{10}$$

which corresponds to the continuous line in Fig. 1. Kusakabe and Takesako [41] studied this mechanism more quantitatively and concluded that Eq. (10) represents a lower limit on Ω_Q, but much larger values are also possible. The authors of Refs. [39,40], in fact, did not consider the possible formation of $QQ\ldots$ bound states which, contrary to $Q\bar{Q}$, would hinder annihilation rather than catalyzing it. Then, if a sizable fraction of Q's gets bounded in such states, the free quark result equation (9) would give a better estimate than Eq. (10). If instead the estimate equation (10) is correct, energy density considerations would not exclude relics with $m_Q \lesssim 5.4 \times 10^3$ TeV; nevertheless, present concentrations of Q hadrons would still be rather large, $10^{-8} \lesssim n_Q/n_b \lesssim 10^{-6}$. While it has been debated if concentrations of the same order should be expected also in the galactic disk [42,43], searches for anomalously heavy isotopes in terrestrial, lunar, and meteoritic materials yield limits on n_Q/n_b that are many orders of magnitude below the quoted numbers [44]. Moreover, even a tiny amount of heavy Q's in the interior of celestial bodies (stars, neutron stars, Earth) would produce all sorts of effects, like instabilities [45], collapses [46], and anomalously large heat flows [47]. Therefore, unless an extremely efficient mechanism exists that keeps Q matter completely separated from ordinary matter, stable Q hadrons would be ruled out.

Selection criteria.—The first criterium to discriminate hadronic axion models is the following. (i) Models that allow for lifetimes $\tau_Q \lesssim 10^{-2}$ s are phenomenologically preferred with respect to models containing long-lived or cosmologically stable Q's. All R_Q allowing for decays via renormalizable operators satisfy this requirement. Decays can also occur via operators of higher dimensions. We assume that the cutoff scale is m_P and write $O_{Qq}^{d>4} = m_P^{4-d}\mathcal{P}_d(Q,\varphi^n)$ where $\mathcal{P}_d$ is a d-dimensional Lorentz and gauge invariant monomial linear in Q and containing n SM fields φ. For $d = 5, 6, 7$, the final states always contain $n \geq d-3$ particles. Taking, conservatively, $n = d-3$, we obtain

$$\Gamma_d \lesssim \frac{\pi g_f m_Q}{(d-4)!(d-5)!}\left(\frac{m_Q^2}{16\pi^2 m_P^2}\right)^{d-4}, \qquad (11)$$

with g_f representing the final degrees of freedom, and we have integrated analytically the n-body phase space, neglecting φ masses and taking momentum independent matrix elements (see, e.g., Ref. [48]). For $d = 5, 6, 7$, we obtain $\tau_Q^{(d)} \gtrsim (4\times 10^{-20}, 7\times 10^{-3}, 4\times 10^{15}) \times (f_a^{\max}/m_Q)^{2d-7}$ s. For $d = 5$, as long as $m_Q \gtrsim 800$ TeV decays occur with safe lifetimes, $\tau_Q^{(5)} \lesssim 10^{-2}$ s. For $d = 6$, even for the largest values, $m_Q \sim f_a^{\max}$ decays occur dangerously close to BBN [49]. Operators of $d = 7$ and higher are always excluded. This selects the R_Q's which allow for $\mathcal{L}_{Qq} \neq 0$ (the first seven in Table II), plus other 13 which allow for $d = 5$ decay operators. Some of these representations are, however, rather large, and they can induce a LP in the SM gauge couplings g_1, g_2, g_3 at some uncomfortably low-energy scale $\Lambda_{\rm LP} < m_P$. Gravitational corrections to the running of gauge couplings become relevant at scales approaching m_P, and they can delay the emergence of a LP [50]. We then specify our second criterium, choosing a value of $\Lambda_{\rm LP}$ for which these corrections can presumably be neglected. (ii) R_Q's which do not induce a LP in g_1, g_2, g_3 below $\Lambda_{\rm LP} \sim 10^{18}$ GeV are phenomenologically preferred. We use two-loop β functions to evolve the couplings [48] and set (conservatively) the threshold for R_Q at $m_Q = 5\times 10^{11}$ GeV. The R_Q's surviving this last selection are listed in Table II.

Other features can render some R_Q's more appealing than others. For example, problems with cosmological domain walls [51] are avoided for $N_{\rm DW} = 1$, while specific R_Q's can improve gauge coupling unification [52]. We prefer not to consider these as crucial discriminating criteria since solutions to the domain wall (DW) problem exist (see, e.g., Refs. [23,53]), while improved unification might be accidental because of the many R_Q's we consider. Nevertheless, we have studied both of these issues. The values of $N_{\rm DW}$ are included in Table II, while, as was noted in Ref. [52], gauge coupling unification improves considerably only for R_3.

Axion coupling to photons.—The most promising way to unveil the axion is via its interaction with photons $g_{a\gamma\gamma} a\mathbf{E}\cdot\mathbf{B}$, where [16]

TABLE II. R_Q allowing for $d \leq 4$ and $d = 5$ decay operators ($\sigma G \equiv \sigma_{\mu\nu}G^{\mu\nu}$) and yielding a LP above 10^{18} GeV. The anomaly contribution to $g_{a\gamma\gamma}$ is given in the fourth column, and the DW number in the fifth one.

R_Q	O_{Qq}	$\Lambda_{\rm LP}^{R_Q}$ (GeV)	E/N	$N_{\rm DW}$
R_1: $(3,1,-\frac{1}{3})$	$\bar{Q}_L d_R$	$9.3\times 10^{38}(g_1)$	2/3	1
R_2: $(3,1,+\frac{2}{3})$	$\bar{Q}_L u_R$	$5.4\times 10^{34}(g_1)$	8/3	1
R_3: $(3,2,+\frac{1}{6})$	$\bar{Q}_R q_L$	$6.5\times 10^{39}(g_1)$	5/3	2
R_4: $(3,2,-\frac{5}{6})$	$\bar{Q}_L d_R H^\dagger$	$4.3\times 10^{27}(g_1)$	17/3	2
R_5: $(3,2,+\frac{7}{6})$	$\bar{Q}_L u_R H$	$5.6\times 10^{22}(g_1)$	29/3	2
R_6: $(3,3,-\frac{1}{3})$	$\bar{Q}_R q_L H^\dagger$	$5.1\times 10^{30}(g_2)$	14/3	3
R_7: $(3,3,+\frac{2}{3})$	$\bar{Q}_R q_L H$	$6.6\times 10^{27}(g_2)$	20/3	3
R_8: $(3,3,-\frac{4}{3})$	$\bar{Q}_L d_R H^{\dagger 2}$	$3.5\times 10^{18}(g_1)$	44/3	3
R_9: $(\bar{6},1,-\frac{1}{3})$	$\bar{Q}_L \sigma d_R G$	$2.3\times 10^{37}(g_1)$	4/15	5
R_{10}: $(\bar{6},1,+\frac{2}{3})$	$\bar{Q}_L \sigma u_R G$	$5.1\times 10^{30}(g_1)$	16/15	5
R_{11}: $(\bar{6},2,+\frac{1}{6})$	$\bar{Q}_R \sigma q_L G$	$7.3\times 10^{38}(g_1)$	2/3	10
R_{12}:$(8,1,-1)$	$\bar{Q}_L \sigma e_R G$	$7.6\times 10^{22}(g_1)$	8/3	6
R_{13}: $(8,2,-\frac{1}{2})$	$\bar{Q}_R \sigma \ell_L G$	$6.7\times 10^{27}(g_1)$	4/3	12
R_{14}: $(15,1,-\frac{1}{3})$	$\bar{Q}_L \sigma d_R G$	$8.3\times 10^{21}(g_3)$	1/6	20
R_{15}: $(15,1,+\frac{2}{3})$	$\bar{Q}_L \sigma u_R G$	$7.6\times 10^{21}(g_3)$	2/3	20

$$g_{a\gamma\gamma} = \frac{m_a}{\rm eV}\frac{2.0}{10^{10}\ {\rm GeV}}\left(\frac{E}{N} - 1.92(4)\right), \qquad (12)$$

with N, E being the anomaly coefficients in Eqs. (2) and (3) (the uncertainty comes from the next-to-leading-order chiral Lagrangian [54]). The last column in Table II gives E/N for the selected R_Q's. We have sketched in Fig. 2 the "density" of preferred hadronic axion models, drawing with oblique lines (only at small m_a) the corresponding couplings. The strongest coupling is obtained for $R_Q^s = R_8$ and the weakest for $R_Q^w = R_3$. They delimit a window $0.25 \leq |E/N - 1.92| \leq 12.75$ encompassing all axion models in Table II. The corresponding couplings $g_{a\gamma\gamma}$ fall within the band delimited in Fig. 2 by the lines $E/N = 5/3$ and $44/3$. With respect to the usual window, $0.07 \leq |E/N - 1.92| \leq 7$ [5] (delimited by the two dashed lines), the upper (lower) limit is shifted upwards approximately by a factor of 2 (3.5). It is natural to ask if $g_{a\gamma\gamma}$ could get enhanced by allowing for more R_Q's ($N_Q > 1$). Let us consider the combined anomaly factor for $R_Q^s \oplus R_Q$:

$$\frac{E_c}{N_c} \equiv \frac{E + E_s}{N + N_s} = \frac{E_s}{N_s}\left(\frac{1 + E/E_s}{1 + N/N_s}\right). \qquad (13)$$

Since by construction the anomaly coefficients of all R_Q's in our set satisfy $E/N \leq E_s/N_s$, the factor in parenthesis is ≤ 1, implying that $E_c/N_c \leq E_s/N_s$. This result is easily generalized to $N_Q > 2$. Therefore, as long as the sign of

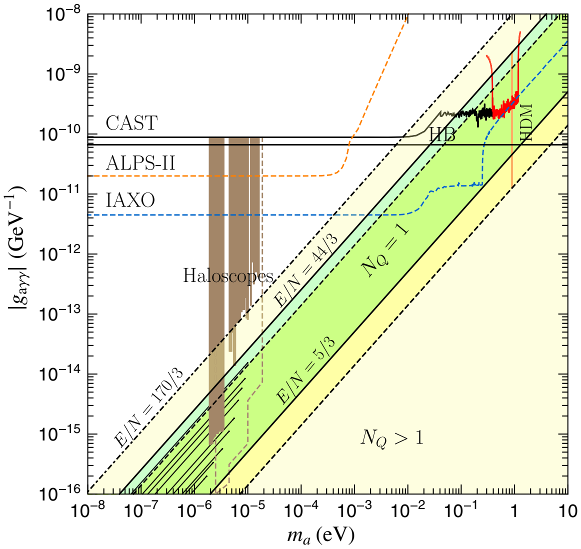

FIG. 2. The window for preferred axion models. The green band encompasses models with a single R_Q. With more R_Q's, the region below the line $E/N = 170/3$ becomes allowed. The two dashed lines enclose the usual window $|E/N - 1.92| \in [0.07, 7]$ [5]. Current (projected) exclusion limits are delimited by solid (dashed) lines.

$\Delta\mathcal{X} = \mathcal{X}_L - \mathcal{X}_R$ is the same for all R_Q's, no enhancement is possible. However, if we allow for R_Q's with PQ charge differences of opposing signs (we use the symbol $\ominus$ to denote reducible representations of this type), E/E_s and N/N_s in Eq. (13) become negative and $g_{a\gamma\gamma}$ can get enhanced. For $N_Q = 2$ the largest value is $E_c/N_c = 122/3$, obtained for $R_Q^s \ominus R_Q^w$. For $N_Q > 2$, even larger couplings can be obtained. However, contributions to the β functions also become large and can induce a LP. This implies that there is a maximum value $g_{a\gamma\gamma}^{\max}$ for which our second condition remains satisfied. We find that $R_Q^s \oplus R_6 \ominus R_9$, giving $E_c/N_c = 170/3$, yields the largest possible coupling. The uppermost oblique line in Fig. 2 depicts the corresponding $g_{a\gamma\gamma}^{\max}$. More R_Q's can also suppress $g_{a\gamma\gamma}$ and can even produce a complete decoupling. This requires an *ad hoc* choice of R_Q's, but no numerical fine-tuning. With two R_Q's, there are three cases yielding $g_{a\gamma\gamma} = 0$ within theoretical errors [27] (e.g., $R_6 \oplus R_9$, giving $E_c/N_c = 23/12 \simeq 1.92$). This provides additional motivation for search techniques which do not rely on the axion coupling to photons [55,56]. Finally, since $T(8) = 3$ and $T(6) = 5/2$, by combining with opposing PQ charge differences R_{12} with R_9 or R_{10}, new models with $N_{\mathrm{DW}} = 1$ can be constructed.

We have classified hadronic axion models using well-defined phenomenological criteria. The window of preferred models is shown in Fig. 2.

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[1] J. R. Ellis and M. K. Gaillard, Nucl. Phys. **B150**, 141 (1979).
[2] L. J. Hall, D. Pinner, and J. T. Ruderman, J. High Energy Phys. 12 (2014) 134.
[3] L. Ubaldi, Phys. Rev. D **81**, 025011 (2010).
[4] S. Aoki *et al.*, arXiv:1607.00299.
[5] C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C **40**, 100001 (2016).
[6] A. E. Nelson, Phys. Lett. **136B**, 387 (1984).
[7] S. M. Barr, Phys. Rev. Lett. **53**, 329 (1984).
[8] M. Dine and P. Draper, J. High Energy Phys. 08 (2015) 132.
[9] R. D. Peccei and H. R. Quinn, Phys. Rev. D **16**, 1791 (1977).
[10] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. **38**, 1440 (1977).
[11] S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978).
[12] F. Wilczek, Phys. Rev. Lett. **40**, 279 (1978).
[13] M. Kamionkowski and J. March-Russell, Phys. Lett. B **282**, 137 (1992).
[14] R. Holman, S. D. H. Hsu, T. W. Kephart, E. W. Kolb, R. Watkins, and L. M. Widrow, Phys. Lett. B **282**, 132 (1992).
[15] S. M. Barr and D. Seckel, Phys. Rev. D **46**, 539 (1992).
[16] D. B. Kaplan, Nucl. Phys. **B260**, 215 (1985).
[17] S. L. Cheng, C. Q. Geng, and W. T. Ni, Phys. Rev. D **52**, 3132 (1995).
[18] J. E. Kim, Phys. Rev. D **58**, 055006 (1998).
[19] J. E. Kim, Phys. Rev. Lett. **43**, 103 (1979).
[20] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B166**, 493 (1980).
[21] A. R. Zhitnitsky, Yad. Fiz. **31**, 497 (1980) [Sov. J. Nucl. Phys. **31**, 260 (1980)].
[22] M. Dine, W. Fischler, and M. Srednicki, Phys. Lett. **104B**, 199 (1981).
[23] A. Ringwald and K. Saikawa, Phys. Rev. D **93**, 085031 (2016).
[24] C. Bonati, M. D'Elia, M. Mariti, G. Martinelli, M. Mesiti, F. Negro, F. Sanfilippo, and G. Villadoro, J. High Energy Phys. 03 (2016) 155.
[25] P. Petreczky, H.-P. Schadler, and S. Sharma, Phys. Lett. B **762**, 498 (2016).
[26] S. Borsanyi *et al.*, Nature (London) **539**, 69 (2016).
[27] L. Di Luzio, F. Mescia, and E. Nardi (to be published).
[28] M. L. Perl, E. R. Lee, and D. Loomba, Annu. Rev. Nucl. Part. Sci. **59**, 47 (2009).
[29] M. Kawasaki, K. Kohri, and T. Moroi, Phys. Rev. D **71**, 083502 (2005).
[30] K. Jedamzik, J. Cosmol. Astropart. Phys. 03 (2008) 008.
[31] W. Hu and J. Silk, Phys. Rev. Lett. **70**, 2661 (1993).
[32] J. Chluba and R. A. Sunyaev, Mon. Not. R. Astron. Soc. **419**, 1294 (2012).

[33] J. Chluba, Mon. Not. R. Astron. Soc. **436**, 2232 (2013).
[34] G. D. Kribs and I. Z. Rothstein, Phys. Rev. D **55**, 4435 (1997); **56**, 1822(E) (1997).
[35] M. Ackermann *et al.* (Fermi-LAT Collaboration), Phys. Rev. D **86**, 022002 (2012).
[36] C. B. Dover, T. K. Gaisser, and G. Steigman, Phys. Rev. Lett. **42**, 1117 (1979).
[37] E. Nardi and E. Roulet, Phys. Lett. B **245**, 105 (1990).
[38] A. Arvanitaki, C. Davis, P. W. Graham, A. Pierce, and J. G. Wacker, Phys. Rev. D **72**, 075011 (2005).
[39] J. Kang, M. A. Luty, and S. Nasri, J. High Energy Phys. 09 (2008) 086.
[40] C. Jacoby and S. Nussinov, arXiv:0712.2681.
[41] M. Kusakabe and T. Takesako, Phys. Rev. D **85**, 015005 (2012).
[42] S. Dimopoulos, D. Eichler, R. Esmailzadeh, and G. D. Starkman, Phys. Rev. D **41**, 2388 (1990).
[43] L. Chuzhoy and E. W. Kolb, J. Cosmol. Astropart. Phys. 07 (2009) 014.
[44] S. Burdin, M. Fairbairn, P. Mermod, D. Milstead, J. Pinfold, T. Sloan, and W. Taylor, Phys. Rep. **582**, 1 (2015).
[45] M. P. Hertzberg and A. Masoumi, arXiv:1607.06445.
[46] A. Gould, B. T. Draine, R. W. Romani, and S. Nussinov, Phys. Lett. B **238**, 337 (1990).
[47] G. D. Mack, J. F. Beacom, and G. Bertone, Phys. Rev. D **76**, 043523 (2007).
[48] L. Di Luzio, R. Gröber, J. F. Kamenik, and M. Nardecchia, J. High Energy Phys. 07 (2015) 074.
[49] Since $m_Q \sim y_Q N_{\rm DW} f_a$, if $y_Q N_{\rm DW} > 1$ we can have $m_Q > f_a$ and a window could open up also for some $d = 6$ operators. This case will be addressed in Ref. [27].
[50] S. P. Robinson and F. Wilczek, Phys. Rev. Lett. **96**, 231601 (2006).
[51] P. Sikivie, Phys. Rev. Lett. **48**, 1156 (1982).
[52] G. F. Giudice, R. Rattazzi, and A. Strumia, Phys. Lett. B **715**, 142 (2012).
[53] J. E. Kim, Phys. Rep. **150**, 1 (1987).
[54] G. Grilli di Cortona, E. Hardy, J. P. Vega, and G. Villadoro, J. High Energy Phys. 01 (2016) 034.
[55] D. Budker, P. W. Graham, M. Ledbetter, S. Rajendran, and A. O. Sushkov, Phys. Rev. X **4**, 021030 (2014).
[56] A. Arvanitaki and A. A. Geraci, Phys. Rev. Lett. **113**, 161801 (2014).