Available online at www.sciencedirect.com

ScienceDirect

Procedia Structural Integrity 28 (2020) 19–25

Structural Integrity Procedia

www.elsevier.com/locate/procedia

1st Virtual European Conference on Fracture

The strain energy density approach applied to bonded joints

Paolo Ferro[a]*, Filippo Berto[b]

[a]University of Padova, Depatement of Engineering and Management, Stradella San Nicola 3, 36100 Vicenza, Italy
[b]NTNU, Department of Mechanical and Industrial Engineering, Richard Birkelands vei 2b, 7491 Trondheim, Norway

Abstract

Adhesively bonded joints can be treated as bi-materials characterized by a stress singularity promoted by both geometrical and constitutive discontinuities, very close to the point where the interface between two elastic solids intersects a traction-free edge. Different approaches have been proposed to predict their both static and fatigue strength. Most of them are based on the stress distribution at the interface between the adherent and the adhesive. More recently, energy density-based criteria have been developed to predict the static and fatigue strength of mono-material notched components, like welded joints. The most promising one is the strain energy density approach in which it is assumed that failure occurs when the strain energy density, averaged over a control volume of critical radius Rc surrounding the singularity point, will reach a critical value. Advantages are different. Among the others, the strain energy density doesn't depend on the singularity order and doesn't require a fine mesh.
This contribution is aimed at applying the strain energy density criterion to bonded joints.

Keywords: Bonded Joints; Strain Energy Density; Stress Intensity Factor; Design; Modelling

1. Introduction

The use of adhesives as fixing elements has a thousand-year history, but it first use for industrial applications dates back to the 1940s with the introduction of bonded joints in aeronautical field. Compared to traditional joints – welding, fastening, bolting – bonded joints guarantee less weight, less cost, thermal and electric insulation, absence of

* Corresponding author.
E-mail address: paolo.ferro@unipd.it

10.1016/j.prostr.2020.10.004

distortions, less stress concentration effects, absence of galvanic interaction between dissimilar alloys and finally they are the only choice when one or both the parts to be joined are not metallic materials (say, composite). On the other hand, the preparation of adherents must be very meticulous and the environmental interaction (say, working temperature or contact with solvents) must be carefully taken into account.

The need to predict the mechanical strength of a bonded joint leads to analyzing the stress distribution within the joint itself. This study can be done using an analytical or a numerical approach such as the finite element method (FEM). When the domain of investigation is very close to the point where the interface between two elastic solids intersects a traction-free edge (point A in Fig. 1), a stress singularity is captured by both analytical and numerical models.

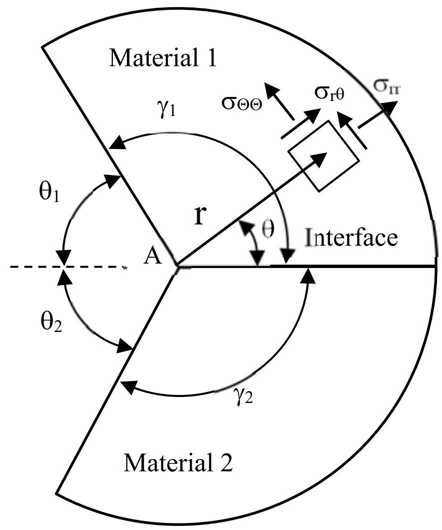

Fig.1. Singularity zone in bi-materials and polar coordinate system

It is worth mentioning that in bi-materials, like bonded joints, the stress singularity is due to both geometrical discontinuities (say, sharp V notches) (Williams, 1952) and constitutive discontinuities at the interface due to the different elastic properties of the two materials coupled.

Over the years, various failure criteria have been developed. The reader can refer to Greenwood et al. (1969), Hart-Smith (1973), Crocombe and Tatarek (1985), Adams and Panes (1994), Ikegami et al. (1989), Crocombe and Adams (1982), Towse et al. (1997a), Towse et al. (1997b), Trantina (1972), Chow and Lu (1992), Zhao, (1991), Schmit and Fraisse (1992). Among these, a local approach based on the intensity of the stress singularity quantified by the stress intensity factor (SIF) has been proposed by Reedy (1993), Reedy and Guess (1993), Lafebvre and Dillard (1999). Unfortunately, SIF parameter requires a very fine mesh near the singularity point and depends on the singularity order, making impossible to compare the strength of bonded joints with different geometries.

In the last years, a powerful approach based on the strain energy density (SED) averaged over a control volume around the singularity has been proposed for static and fatigue design of notched components (Lazzarin, Zambardi, 2001; Berto et al., 2016). It is assumed that the critical radius (Rc) of the control volume is a material characteristic only. The SED criterion was successfully used to quantify the effect of residual stress on high cycle fatigue strength of welded joints, as well (Ferro, 2014; Ferro and Berto, 2016; Ferro et al., 2016). The SED parameter can be calculated with a coarse mesh and due to its scalar nature it is independent from the order of the singularity.

In this work the SED criterion is applied to bonded joints. To this aim, some hypotheses were assumed: the elastic properties of the adherents are much higher than those of the adhesive and the control volume is restricted to the sector belonging to the adhesive only (material 1 in Fig. 1) that will behave as a brittle material. Model parameters are calibrated using experimental data coming from literature for adhesively butt joints made out of steel and epoxy resin.

2. Analytical frame

The strain energy density, under plain stress or plain strain conditions, is given by the following Beltrami's equation:

$$W(r,\theta)=\frac{1}{2E}\left\{\sigma_r^2+\sigma_\theta^2-2\nu\sigma_r\sigma_\theta+2(1+\nu)\tau_{r\theta}^2\right\} \tag{1}$$

where E and ν are the elastic modulus and Poisson's ratio of the adhesive, respectively. On the other hand, the stress distribution near the singularity point is described by (Lazzarin, Zambardi, 2001; Berto et al., 2016):

$$\sigma_{ij}(r,\theta)=\frac{K_1 r^{\lambda-1} f_{i,j}(\theta)}{\sqrt{2\pi}}=H_0 r^s f_{ij}^{(0)}(\theta) \tag{2}$$

where K_1 is the mode I SIF, H_0 is the generalized SIF and λ is the eigenvalue (Reedy, 1990). Now, by substituting Eq. (2) in Eq. (1), the following equation holds true:

$$\begin{gathered}W(r,\theta)=\frac{r^{2(\lambda-1)}}{4\pi E}K_1^2\left\{f_{rr}^2(\theta)+f_{\theta\theta}^2(\theta)-2\nu f_{rr}(\theta)f_{\theta\theta}(\theta)+2(1+\nu)f_{r\theta}^2(\theta)\right\}=\frac{r^{2(\lambda-1)}}{4\pi E}K_1^2\beta(\theta)\\ \text{with } \beta(\theta)=f_{rr}^2(\theta)+f_{\theta\theta}^2(\theta)-2\nu f_{rr}(\theta)f_{\theta\theta}(\theta)+2(1+\nu)f_{r\theta}^2(\theta)\end{gathered} \tag{3}$$

Eq. (3) is integrated over the circular sector belonging to the adhesive only, having radius R and $\gamma_1 = \pi/2$ (Fig. 1),

$$E_R=\int_0^R\int_0^{\pi/2}W(r,\theta)dA=\frac{K_1^2}{4\pi E}\int_0^R r^{2(\lambda-1)}r\,dr\int_0^{\pi/2}\beta(\theta)d\theta=\frac{K_1^2}{4\pi E}\frac{R^{2(\lambda-1)+2}}{2\lambda}\int_0^{\pi/2}\beta(\theta)d\theta \tag{4}$$

and then averaged on the adhesive sector:

$$\bar{W}_R=\frac{E_R}{R^2\frac{\pi}{4}}=\frac{K_1^2}{\pi^2 E}\frac{R^{2(\lambda-1)}}{2\lambda}\int_0^{\pi/2}\beta(\theta)d\theta=\frac{H_0^2}{\pi E}\frac{R^{2(\lambda-1)}}{\lambda}\int_0^{\pi/2}\beta(\theta)d\theta \tag{5}$$

It is assumed that failure occurs when the strain energy density averaged over a control volume of critical radius Rc surrounding the singularity point will reach a critical value. The Rc value is obtained by equating the energy density to failure of the adhesive with that of the adhesively bonded butt joint:

$$\frac{\sigma_L^2}{2E}=\frac{H_{0,C}^2}{\pi^2 E}\frac{R_C^{2(\lambda-1)}}{\lambda}\int_0^{\pi/2}\beta(\theta)d\theta \tag{6}$$

In Eq. (7), σ_L and $H_{0,C}$ are the stress to failure and the critical GSIF of the adhesive, respectively.

The $H_{0,C}$ value is calculated by combining numerical and experimental results taken from literature (Suzuki, 1985). In particular, the butt joint steel-epoxy (Epikote 828) was taken into account, with the adhesive having the following material properties: E = 3140 MPa, ν = 0.37 and σ_L = 65 MPa. The steel elastic properties are: E = 206 GPa, ν = 0.3.

3. Critical radius assessment

The geometry of the adhesively bonded butt joint follows that of Standard ISO 6922. In particular, the joint diameter is 15 mm while the adhesive thicknesses (h) modelled are: 0.12, 0.4, 2.0, 7.5 mm. By taking advantage of the load and geometry symmetry, only a quarter of the geometry has been modelled using PLANE 183 elements and axisymmetric condition. The size of the smallest element was about $1.6 \cdot 10^{-5}$ mm (Fig. 2).

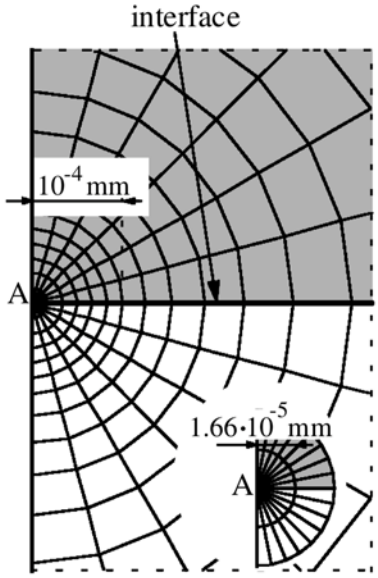

Fig. 2. A detail of the mesh near the singularity point A

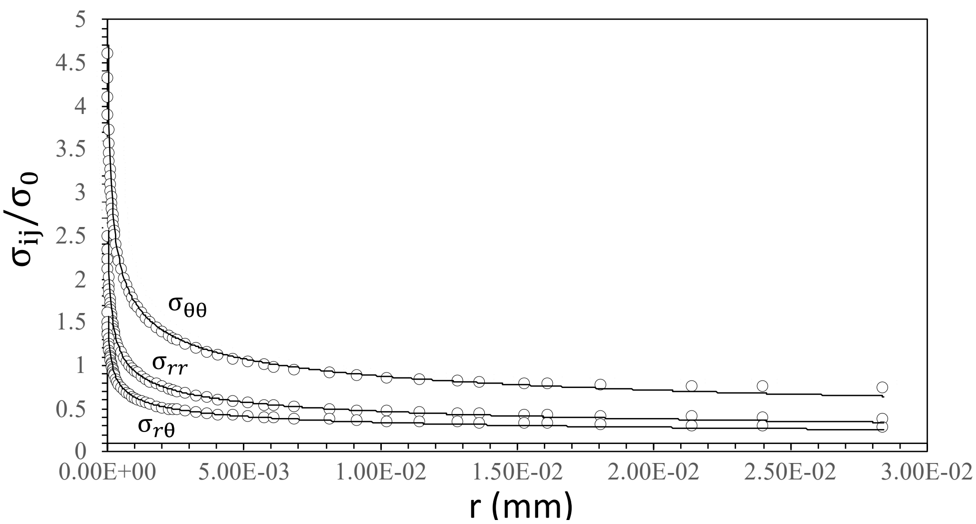

Fig. 3. Asymptotic stress distribution at steel-epoxy interface ($\theta = 0$), adhesive side. $\sigma_0 = 1$ MPa, nominal applied stress; h = 0.12 mm

The order of the singularity was found -0.3 (λ = 0.7), in good agreement with the theoretical value of -0.301 (Akisanya, 1997) (Fig. 3). The GSIF is obtained by using numerical results and Eq. (7):

$$H_0 = \frac{\sigma_{\theta\theta} r^{(\lambda-1)}}{f_{\theta\theta}(0)} = \sigma_{\theta\theta} r^{(\lambda-1)} \tag{7}$$

In particular, by using experimental tensile stress to failure as a function of h, the $H_{0,C}$ value was found to be 10.3 MPa $mm^{0.3}$, irrespective of adhesive thickness size. It is noted that $f_{\theta\theta}(0)$ is set equal to 1 (Eq. 7) since the angular stress distribution functions are always defined within a constant value. Ones these functions are obtained via numerical analysis (Fig. 4), the integral of $\beta(\theta)$ in Eq. (6) is calculated. Finally, by using Eq. (6) the critical radius resulted to be $6.2 \cdot 10^{-3}$ mm. That low value agrees well with the process zone size of bonded joints (Mintzas and Nowell, 2012; Reedy, 2000), which is much lower than that of welded joints, and is considered and intrinsic property of the bi-material system under investigation, rather than a mere adhesive property only.

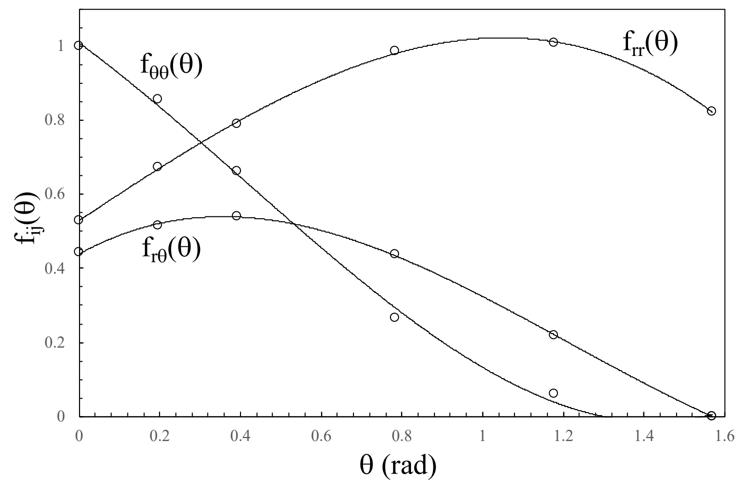

Fig. 4 – Angular stress distribution functions obtained via numerical analysis

4. ASED static failure criterion verification

Table 1 summarizes the results of the simulations in terms of SED values corresponding to the experimental stress to failure as a function of adhesive thickness.

Table 1. Critical SED values derived from experimental results and comparison with the analytical value

Adhesive thickness, h (mm)	Stress to failure σ_I (MPa)*	$\bar{W}^{C}_{R_C}$ (mJ/mm^3)**	$\bar{W}^{C}_{R_C}$ (mJ/mm^3) (Eq. 5)	Δ(%)
0.12	47.1	0.595	0.672	11.4
0.4	31.7	0.592	0.672	11.9
2.0	18.7	0.647	0.672	3.7
7.5	12.1	0.688	0.672	2.3

*(Suzuki, 1985)
**FEM

It is observed that the critical SED value is maintained constant as a function of adhesive thickness, with a maximum deviation from the theoretical value of 11.9% that may be caused by uncertainties on experimental data.

It can be concluded that the SED criterion can be applied to adhesively bended joint brittle fracture assessment.

5. Conclusions

The strain energy density approach was applied to bonded joints brittle fracture assessment. Under the simplifying assumption of infinitely rigid substrates, the control volume of critical radius Rc is restricted to adhesive only. The Rc value was obtained using experimental data taken from literature concerning the tensile tests carried out on steel/epoxy butt joints. It was found that the SED critical value averaged over the control volume of radius Rc is maintained constant despite the adhesive thickness variation, therefore representing a bi-material property that can be used as failure parameter for adhesively bonded joints design.

References

Adams, R. and Panes, G. (1994). Effect of three dimensional stresses on the failure of single lap joints. In Proceedings of Euradh 94.

Akisanya, A. R. (1997). On the singular stress field near the edge of bonded joints. J. Strain Analysis, 32(4), pp. 301–311.

Berto, F., Ayatollahi, M.R., Borsato T. and Ferro, P. (2016). Local strain energy density to predict size-dependent brittle fracture of cracked specimens under mixed mode loading. Theoretical and Applied Fracture Mechanics, 86B, pp. 217–224.

Charalambides, M., Kinloch, A. and Matthews, F. (1997). Strength prediction of bonded joints. In AGARD Conference on Bolted/Bonded Joints in Polymeric Composites. AGARD Conference Proceedings CP-590.

Chow, C. and Lu, T. (1992). Analysis of failure properties and strength of structural adhesive joints with damage mechanics". International Journal of Damage Mechanics, 1(4), pp. 404–434.

Crocombe, A. and Adams, R. (1982). An elastoplastic investigation of the peel test. The Journal of Adhesion, 13, pp. 241–267.

Crocombe, A. and Tatarek, A. (1985). A unified approach to adhesive joint analysis. In Proceedings of Adhesives, Sealants and Encapsulants 85. Plastics and Rubber Institute, London.

Ferro, P. (2014). The local strain energy density approach applied to pre-stressed components subjected to cyclic load. Fatigue and Fracture of Engineering Materials and Structures, 37(11), pp. 1268-1280.

Ferro, P., Berto, F. (2016). Quantification of the influence of residual stresses on fatigue strength of Al-alloy welded joints by means of the local strain density approach. Strength of Materials, 48(3), pp. 426–436 doi:10.1007/s11223-016-9781-0.

Ferro, P., Berto, F., James, M.N., Borsato, T. (2016). Review of recent advances in local approaches applied to pre-stressed components under fatigue loading. Procedia Structural Integrity, 2, pp. 3467–3474.

Greenwood, L., Boag, T. and McLaren, A. (1969). Stress distribution in lap joints. In Adhesion: Fundamentals and practice, pp. 273–279..

Harris, J. and Adams, R. (1984). Strength prediction of bonded single lap joints by non-linear finite element methods. International Journal of Adhesion and Adhesives, 4(2), pp. 65–78.

Hart-Smith, L.J., (1973). Adhesive-bonded single-lap joints. Technical report, NASA CR-112236.

Ikegami, K., Takeshita, T., Matsuo, K. and Sugibayashi, T. (1989). Strength of adhesively bonded scarf joints between glass fibre reinforced plastics and metals. In Proceeding of Structural Adhesives in Engineering II.

Lazzarin, P., Zambardi, R. (2001). A finite-volume-energy based approach to predict the static and fatigue behavior of components with sharp V-shaped notches. International Journal of Fracture, 112, pp. 275–298. https://doi.org/10.1023/A:1013595930617.

Lee, S. and Lee, G. (1992). Development of a failure model for the adhesively bonded tubular single lap joint. The Journal of Adhesion, 40, pp. 1–14.

Lefebvre, D. R. and Dillard, D. A. (1999). A stress singularity approach for the prediction of fatigue crack initiation in adhesive bonds. Part 1: theory. J. Adhesion, 70, pp. 119 – 138.

Mintzas, A., Nowell, D. (2012). Validation of an Hcr-based fracture initiation criterion for adhesively bonded joints. Engineering Fracture Mechanics 80, pp.13–27.

Reedy Jr, E. D. (1990). Intensity of stress singularity at the interface-corner between a bonded elastic and rigid layer. Engng. Fracture Mechanics, 36, pp. 575 – 583.

Reedy Jr, E. D. (1993), Asymptotic interface-corner solutions for butt tensile joints. Int. J. Solids Structs, 30, pp. 767– 777.

Reedy Jr, E. D. and Guess, T. R. (1993). Comparison of butt tensile strength data with interface corner stress intensity factor prediction. Int. J. Solids Structs, 30, pp. 2929–2936.

Reedy Jr, E.D. (2000). Connection between interface corner and interfacial fracture analyses of an adhesively-bonded butt joint. International Journal of Solids and Structures, 37, pp. 2429-2442.

Schmit, F. and Fraisse, P. (1992). Fracture mechanics analysis of the strength of bonded joints. In Materiaux et Techniques, 80, pp. 55–63.

Suzuki, Y. (1985). Adhesive tensile strengths of scarf and butt joints of steel plates. Bulletin of JSME, 28(245).

Towse, A., Davies, R., Clarke, A., Wisnom, M., Adams, R. and Potter, K. (1997a). The design and analysis of high load intensity adhesively bonded double lap joints. In Proceedings of the Fourth International Conference on Deformation and Fracture of Composites. Manchester, UK, pp. 479–488.

Towse, A., Potter, K., Wisnom, M. and Adams, R. (1997b). A novel comb joint concept for high strength unidirectional carbon fibre bonded joints. In Proceedings of the 11th International Conference on Composite Materials. Goald Coast, Australia, Vol. 6, pp. 95–101.

Trantina, G. (1972). Fracture mechanics approach to adhesive joints. Journal of Composite Materials, 6(2), pp. 192–207.

Williams, M. L. (1952). Stress singularities resulting from various boundary conditions in angular corners of plates in extension. J. Appl. Mechanics, 19, pp. 526– 528.

Zhao, X. (1991). Stress and failure of adhesively bonded lap joints. Ph.D. thesis, University of Bristol.