# Particle Beams as Controllable Complex Systems: Application of the Network Theory[*)]

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High power neutral beam injectors, like those for ITER, must satisfy very demanding parameters (40 A of negative ion current accelerated up to 1 MV for one hour). They are made of various components, which influence each other, so that the global performances eventually require the simultaneous control of several interacting parameters: thus the NBI is an example of a complex system. In the present paper, complex network theory is applied to verify the controllability conditions of the NIO1 experiment, a particle beam source operating at Consorzio RFX (Padova, Italy). Previous work on the subject is adapted to NIO1, the controllability conditions are assessed and the driver nodes are identified; first comparison between theoretical predictions and experimental data is also discussed.

## 1. Introduction

High power neutral beam injectors (NBIs) are necessary for plasma heating and current drive in present nuclear fusion experiments. In the case of ITER the NBIs must satisfy very demanding parameters: 40 A of negative ion current accelerated up to 1 MV for one hour [1]. These parameters have never been simultaneously attained yet, so that a dedicated test facility is under preparation at Consorzio RFX (Padua, Italy) to optimise the NBI in view of its operation in ITER [2]. NBIs are modular systems involving several components that affect each other, so that the global performances require simultaneously controlling many interacting parameters and a NBI can be regarded as an instance of a complex network.

The present paper is devoted to the application of network control theory to NBIs. In the first treatment of this kind [3] the correspondence was established between the parameters of a generic NBI and the network parameters. In this paper the method is applied to the NIO1 experiment, a small and flexible negative ion beam source operated at Consorzio RFX for the investigation of source and beam physics, whose latest results are described in [4].

The next section is devoted to the description of NIO1 and to establishing a correspondence between NIO1 pa-

rameters and the corresponding complex network as well as to the characterisation of NIO1 network; then the controllability of such a network will be studied.

## 2. NIO1 and Its Network

A view of the NIO1 experiment is given in Fig. 1. The main components are described in [5]: the system includes a source which is made of a driver region, where a RF-coupled hydrogen plasma is generated, and an expansion region, allowing the plasma to uniformly reach the zone from which the existing negative ions are extracted through 9 apertures drilled in the plasma grid. Negative hydrogen ions are generated by dissociative attachment of electrons to excited molecules; in the vicinity of the plasma grid, parallel to it, a magnetic field is present to cool down the electrons, as high-energy electrons would destroy the negative ions. A two-stages accelerator follows, the first stage devoted to dumping the co-extracted electrons, whereas the second one imparts the final acceleration to negative ions. In the beam line the repulsion of negative charges is reduced thanks to the space charge compensation due to the formation of plasma by interaction of the beam itself with the background gas.

With respect to the previous investigation of a whole NBI [3], NIO1 is characterised by fewer parameters, as it features no neutraliser and residual ion dump. However the
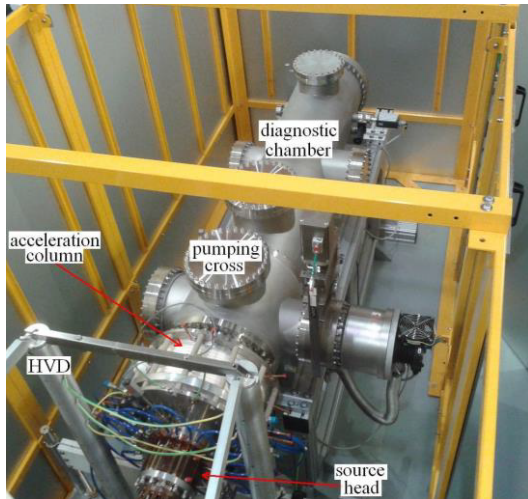
Fig. 1　View of the NIO1 experiment: plasma source, accelerator column and beam line can be seen.



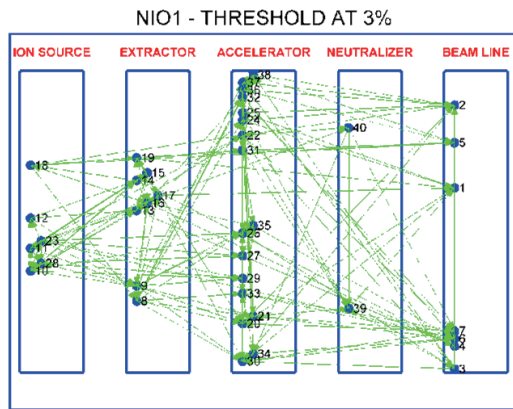Fig. 2　NIO1 network with 3% threshold on link strength.



Fig. 3　NIO1 network with 3% threshold on link strength.

source and accelerator physics are analogous.

NIO1 can be viewed as a network whose processes are represented by nodes and interactions among them are represented by links of a directed graph [6]. A network with $N$ nodes can be represented by an adjacency matrix $A \in N \times N$, whose $a_{ij}$ entry is non-zero if there is a link starting from node $j$ and ending in node $i$ (Fig. 2). The value of $a_{ij}$ represents the strength of the link between nodes. The number of links that are not zero is $N_E$; they represent the existing relationships among parameters in the network under study. These $N_E$ relationships between parameters can be regarded as the links (or edges) of the aforementioned directed graph (Fig. 2). The network also has external inputs acting on the system; the number of the external inputs is $M$. The action of the external inputs can be represented by an input matrix $B \in N \times M$ whose $b_{ik}$ entry is non-zero if input $k$ is acting on node $i$. The state $x$ of the $N$ nodes of the system is captured by the state-space equation $x' = Ax + Bu$, where $x'$ is the time derivative of $x$ and $u$ is the input vector.
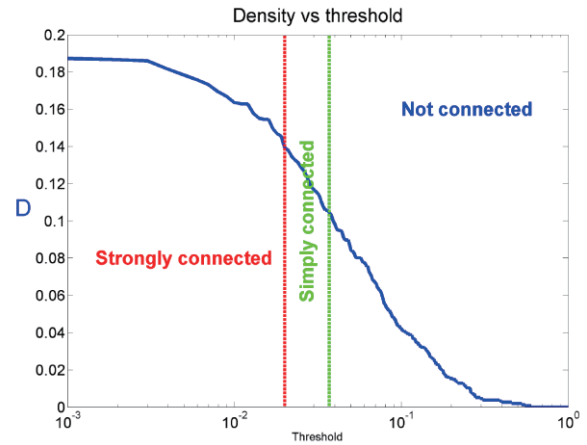
The strength of the links, which is normalised to 1 to reduce different parameters to the same footage, is very variable, so that a suitable thresholding is in order to get rid of too weak links, whose effect on the network operation is very small or which are prone to experimental error. As in [3], the strength of the links is represented by the elements of an $N \times N$ matrix $A$, whereas the strength of the external inputs is contained in an $N \times M$ matrix $B$. Thresholding was used to tune to the desired value the network density [7], $D$, defined as the portion of the potential edges in a network that are actual edges: $D = N_E/[N(N-1)]$. Thresholding affects the number of nodes required to control the network: the denser the network, the fewer nodes are required to fully control it. For NIO1 the threshold was chosen at 3 %, the strength value around which the network loses its strong connectivity (Fig. 3), namely there no longer exists a directed path both from any i-th to any j-th node and vice versa: a compromise is found since on the one hand weak connections are unrealistic to be useful in carrying signals in a real system and on the other hand NIO1 is a deeply interconnected system, so connectivity must be preserved.

## 3. Controllability of Networks

A system is said to be controllable if it can be driven from any initial state to any final state in finite time [8]. Controllability of a system together with its inputs, represented by $(A, B)$, can be checked with Kalman's criterion of controllability. The criterion states that a system is controllable if and only if its $N \times NM$ controllability matrix C $= [B A B A^2 B \dots A^{N-1}B]$ has full rank [8]. For large systems the evaluation of matrix $C$ is susceptible to round-off errors, thus preventing an accurate evaluation of its rank. Besides, Kalman's criterion requires the system parameters (the entries of $A$ and $B$) to be precisely known, which is not always the case in real systems, and the strength of the links may vary over time or be affected by measurement errors. Finally, the matrix $B$ is to be known since the beginning whereas in most cases a method is required to

identify the minimal set of inputs that guarantees complete control over the system.

Structural controllability was developed to bypass such limitations [9]. A structured system ($A$, $B$) is a system represented by structured matrices (their elements are either fixed zero or independent free parameters). Such a system is structurally controllable if the non-zero elements of its matrices can be set so that it is controllable according to Kalman. If a system is structurally controllable, then it is controllable for almost any parameter realisation, except for some pathological cases with Lebesgue measure zero [9]. To assess the structural controllability of a system, both analytical and graph-theoretical tests are available [10]. A network is spanned by a sub-network if they both share the same node set. From the graph-theoretical point of view, it was shown [9] that a system is structurally controllable if its network can be spanned by a cacti structure, namely a network composed only by stems (straight paths) and buds (cycles with an additional link that connects them to a stem) that do not intersect. In other words, accurate knowledge of link weights is no longer required, and an accurate map of the network is enough to check its structural controllability.

The cacti structure problem can be mapped into the maximum matching problem [11]. A matching in a network is a collection of its links that do not share start nodes or end nodes. A node is matched if a link of the matching ends on it; otherwise it is unmatched. A matching of maximum cardinality is a maximum matching. A maximum matching of a network is in correspondence to a cacti structure spanning it. Besides, it can be shown that controlling the state of the unmatched nodes (i.e., the starting nodes of the stems) via external inputs guarantees the ability to steer the entire system towards a desired state [11]. The unmatched nodes are hence called driver nodes and their set is called minimum driver node set (MDNS). The number of driver nodes is $N_D$.

A maximum matching of a network can be found using the Hopcroft-Karp algorithm [12]. However, the algorithm provides only one of the many possible maximum matchings of a network; different maximum matchings may lead to completely different control structures and MDNS. In these conditions, preferential matching [13] was adopted, which is an algorithm taking as input a network and an ordered sequence of its nodes (named matching queue), and generating a maximum matching of the network whose driver nodes are possibly the last ones in the matching queue. As a side effect, preferential matching can be used (and was used by us) to find multiple maximum matchings of the same network by varying the matching queue. It is worth pointing out that since the maximum matching cardinality depends only on the network structure, $N_D$ remains constant in the different maximum matchings of the same network.

# 4. Controllability of NIO1 Network

First the Hopcroft-Karp algorithm was used to detect a maximum matching and $N_D$ of NIO1 network for various threshold values (Fig. 4). When varying the threshold, $N_D$ remains fairly constant in the initial phase (plateau zone), since only unimportant links are removed. The sudden rise of $N_D$ beyond 3 % confirms the chosen threshold value. For NIO1 with this threshold $N_D = 4$, so that controlling 4 nodes (that is, processes) is enough to steer the entire NIO1 network. As already mentioned, structural controllability technique applies to non-zero elements of the $A$ matrix, so in the rest of the work the non-zero elements are set to 1.

To enumerate different maximum matchings of the NIO1 network, preferential matching was used. The key idea is that if a node recurs several times in different maximum matchings of the same network, this may suggest that the associated physical phenomenon is very important in the overall network operation. Some statistics over 1000 samples of NIO1 maximum matchings are shown in Fig. 5. It is worth noting that: (a) node 36 (deflection of $H^-$ ions) is always a driver node in all maximum matchings, suggesting that further analysis on the matter is in order; (b) some driver node quadruplets are more common than others; (c) the probability distribution of having a certain amount of buds peaks at 2 and 2/3 of the samples exhibit only 1 or 2 buds, suggesting that NIO1 network has some relevant self-dynamics, but also that it is not prevalent. Figure 6 shows one of the maximum matchings fea-
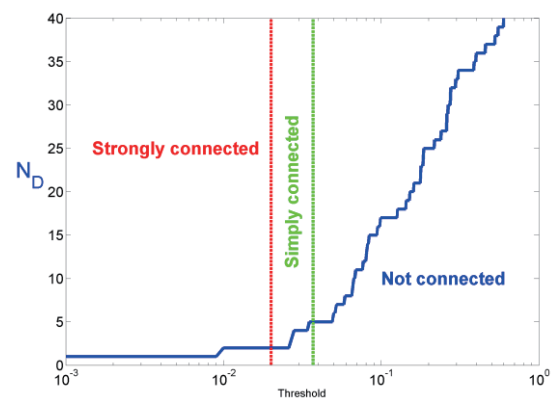


Fig. 4   Number of NIO1 driver nodes on varying threshold. Variation of connectivity properties is also shown.
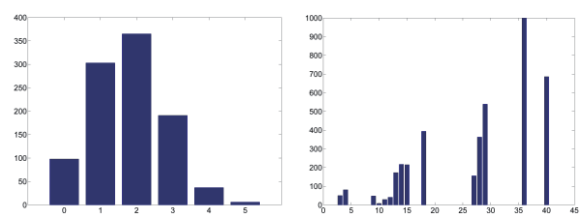


Fig. 5   Statistics of NIO1 MDNSs in 1000 draws: (left) number of maximum matchings with a certain number of buds; (right) occurrence of each node as driver.
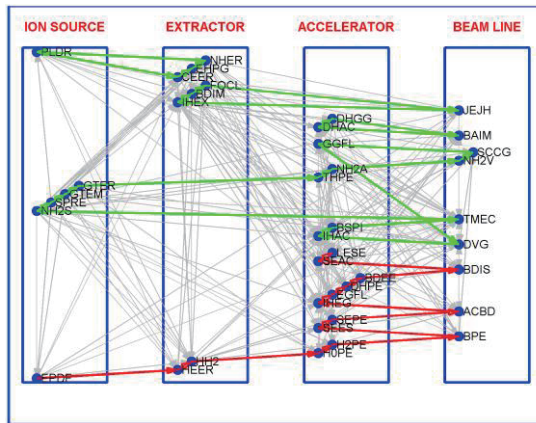
Fig. 6   NIO1 network (grey) and a maximum matching found with Hopcroft-Karp algorithm (with the 4 stems in red, and 4 buds in green).

turing 4 buds.

The two most probable MDNSs for NIO1 are quadruplets [18,29,36,40] and [18,28,36,40], meaning that control over the entire system is guaranteed if the following processes are controlled: plasma drifts (18), gas temperature in expansion region (28) or density of atomic hydrogen in the PG-EG gap (29), deflection of $H^-$ in the PG-EG gap (36), density of molecular hydrogen in the vessel (40).

## 5. Conclusions

The present paper represents a step forward with respect to previous work on the subject: indeed the description of a Neutral Beam Injector (NBI) as a linear time-invariant system has advanced from the graph-theoretical point of view, novel theories were applied and the theory of structural control was applied to an existing system, NIO1. In this work the topology of the graph is fixed, so that major events (accelerator breakdowns) are not considered.

The graph-theoretical approach to NBI is a new field: it aims at identifying the most important parameters for controlling the system. It is supposed to help the exper-

imental activity and to profit from the experimental improvement of the knowledge of the system.

Preferential matching was adopted to identify several maximum matchings for the system, which were statistically analysed, and to identify the most persisting driver nodes. These can be interpreted as very relevant phenomena, ruling the behaviour of the whole network, and give indications regarding the most important parameters to be diagnosed during NBI operation.

Future work will be devoted to the selection of the best minimum driver node set for NIO1 by focussing on the input nodes; diverse criteria can be employed to this purpose, like accessibility of the driver nodes from the inputs (nodes easiest to be accessed in the real system), intensity of the effect of the driver nodes on the outputs, side effects (the influence the inputs have on nodes other than the driver nodes they are supposed to control).

Another line of future work is represented by the experimental check of controllability: a suitable set of experimental data will be identified for NIO1 and the results of the complex network will be compared to the data in order to assess also the suitability of this approach to the prevision of the behaviour of NBIs.

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