German Economic Review 16(4): 408–421 doi: 10.1111/geer.12058

Tax Competition, Investment Irreversibility and the Provision of Public Goods

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Abstract. *This article studies the effects of tax competition on the provision of public goods under business risk and partial irreversibility of investment. As will be shown, the provision of public goods changes over time and also depends on the business cycle. In particular, under source-based taxation, in the short term, public goods can be optimally provided during a downturn. The converse is true during a recovery: in this case, they are underprovided. In the long term, however, tax competition does not affect capital accumulation. This means that the provision of public goods is unaffected by taxation.*

JEL classification: *H25, H32.*

Keywords: Irreversibility; risk; short- and long-term effects; tax competition.

1. INTRODUCTION

Capital mobility is relatively high (see the comprehensive survey by Zodrow, 2010). However, it is not fully cost free. For example a greenfield and even a brownfield investment are characterized by some irreversibility, which reduces mobility after the investment has been undertaken. Another related cause of partial mobility is the existence of 'location-specific capital', which may be relevant when a resident resides in a place for some time (see, e.g., Wildasin and Wilson, 1996).

Despite these well-known characteristics, most of the existing literature on tax competition treats capital as fully mobile. If this assumption well fits with paper profits and intangible assets (see Devereux, 2007), it is less realistic when tangible assets are considered.

There are a few articles that have dealt with the partial mobility of investment. Among these, Lee (1997) uses a two-period framework where at time 1 firms are free to make an investment abroad and at time 2 they face exit costs. This induces competing governments to intensify tax competition at time 1 and then raise tax rates at time 2. Lee (1997) also shows that time 2's tax rate increase is positively related to the amount of exit costs. Becker and Fuest (2011) assume two types of firms: mobile and immobile. They show that the optimal tax policy depends on whether the mobile firms are more or less profitable than the average firm in the economy.

Both articles use a deterministic framework to derive policy implications, although risk is shown to affect the interaction between taxation and investment (see, e.g., Ghinamo et al., 2010). Like partial mobility, volatility is an important characteristic which is seldom considered. Risk is usually introduced for welfare analysis: the main question raised by the relevant literature is to what extent volatility undermines the welfare state. For instance, Wildasin (2000) argues that increased capital mobility reduces the Government's ability to redistribute resources.[1] On the other hand, Lee (2004) states that capital taxation can be used as an insurance against wage fluctuations. To our knowledge, however, no tax competition article studies strategic interactions when business conditions change over time because of volatility.

The aim of this article is to investigate fiscal policies under both volatility and partial irreversibility (mobility). To do so, we will use an intertemporal neoclassical model with investment irreversibility and depreciation. By letting capital depreciate, we make irreversibility partial, in that obsolescence gives some degree of flexibility to firms that can decide whether and when to re-invest.

Moreover, we will apply this investment framework to the well-known tax competition models, developed by Zodrow and Mieszkowski (1986) and Wilson (1986).[2] We will then show that, when a Government raises revenue by means of a source-based tax on capital, the provision of public goods depends on the state of nature and the time horizon. In particular, we will show that in the short-medium term, during a downturn, public goods can be optimally provided. The reasoning behind this result is simple: when business conditions get worse, firms cannot disinvest because of irreversibility (they can only wait for obsolescence). Since capital is given, the source-based tax is equivalent to a lump-sum one. When however a recovery takes place, taxation discourages capital accumulation and the use of a distortive source-based tax leads to the underprovision of public goods. Results change in the long term. In this case, the distortive effects of taxation vanish, and therefore, public goods can be optimally provided.

The structure of this paper is as follows. In section 2, we introduce a standard neoclassical model with investment irreversibility and depreciable capital. Section 3 examines the provision of public goods, in the short term. Section 4 focuses on the long term. Section 5 summarizes our findings and discusses some possible extensions.

2. THE MODEL

Let us focus on a representative firm, which is subject to a unit tax.[3] For simplicity, we assume that the price of capital is equal to 1. Denoting capital as K_t, we

1. On this point see also Wilson and Wildasin (2004).
2. A clear discussion of this 'basic model of tax competition' is provided by Wilson (1999).
3. For simplicity, we just focus on one representative firm. Alternatively, we could deal with a large number of small identical firms. If each single firm has no relevant market power, results would not change. This is clearly shown by Leahy (1993) and Grenadier (2002).

assume that the production function is $\Theta_t \Psi(K_t)$, where Θ_t is a stochastic productivity variable that follows a geometric Brownian motion,[4]

$$d\ln\Theta_t = [\mu_\Theta - (1/2)\sigma^2]dt + \sigma dz_t, \tag{1}$$

where μ_Θ and σ are the drift and volatility parameter for Θ_t, and dz_t is the increment of a Wiener process, with $E(dz_t) = 0$ and $E(dz_t^2) = dt$. Moreover, the function $\Psi(K_t)$ follows the Inada conditions. Finally, the installment of capital is assumed to be irreversible.[5]

In order to make our model more realistic, we introduce capital risk. By assumption, therefore, capital lifetime will follow a Poisson process. This means that, over any short period dt, there is a probability λdt that the activity dies. The importance of this assumption is twofold. On the one hand, it makes our analysis more realistic, by adding an important source of uncertainty, i.e., capital risk (e.g., related to obsolescence). On the other hand, depreciation allows us to make the irreversibility assumption weaker. In other words, we state that, as long parameter λ is positive, irreversible investments is not eternal, and that it may be 'made' reversible by technical obsolescence. When the investment project expires, the firm owns a non-depreciable option to restart. As immediate restarting may not be profitable, the firm may find it profitable to wait until Π rises. With such an option, therefore, at the expiration of the project the firm regains a limited degree of reversibility.[6]

Given these assumptions, our representative firm chooses the stock of capital that maximizes its after-tax profit function:

$$\Pi(K_t, \Theta_t) = \Theta_t \Psi(K_t) - \tau K_t, \tag{2}$$

where τ is a constant unit tax on capital. Denoting r as the risk-free interest rate, the firm's investment activity is described by the following:

Lemma 1. The firm invests when the following marginal condition holds:

$$\Theta_t^* \Psi_K(K_t) \equiv \frac{\beta_1}{\beta_1 - 1} \frac{r + \lambda + \tau}{r + \lambda} (r + \lambda - \mu_\Theta), \tag{3}$$

where Θ_t^* is the maximum value of the stochastic variable reached until time t, i.e., $\Theta_t^* = \{\max_{0 \le s \le t} \Theta_s\}, \Psi_K(K_t) \equiv \frac{\partial \Psi(K_t)}{\partial K_t}$ and $\beta_1 = \left(\frac{1}{2} - \frac{\mu_\Theta}{\sigma^2}\right) + \sqrt{\left(\frac{1}{2} - \frac{\mu_\Theta}{\sigma^2}\right)^2 + 2\frac{r+\lambda}{\sigma^2}} > 1$.

4. For simplicity, we assume a productivity shock and disregard the demand side of the market. As shown in Bertola (1998), the quality of results does not change when firms face not only supply- but also demand-side shocks. When different sources of shocks are considered and all of them follow a geometric Brownian motion, overall uncertainty is still described by a geometric Brownian motion.
5. This means that a firm owns a compound option to invest, which consists of a continuum of American call options. For any increment dK the firm can exercise a call option to expand capital. After this exercise, the firm obtains another American call option allowing it to undertake a further increment.
6. Notice that we have assumed uncertain obsolescence. However, the quality of results would not change if we used a deterministic exponential depreciation rate. The effect of allowing for a constant capital depreciation rate is equivalent to that obtained by raising the discount factor.

Proof. See Appendix A.■

Lemma 1 derives the optimal investment policy under irreversibility. As can be seen, investment is optimal when the marginal product $\Theta_t^* \Psi_K(K_t)$ (see the LHS of (3)) equates the marginal cost $\frac{\beta_1}{\beta_1 - 1} \frac{r+\lambda+\tau}{r+\lambda}(r + \lambda - \mu_\Theta)$ (see the RHS).

It is worth noting that, under full reversibility, the marginal cost would be equal to $\frac{r+\lambda+\tau}{r+\lambda}(r + \lambda - \mu_\Theta)$. When however investment is irreversible the marginal cost is higher. In this case, term $\frac{r+\lambda+\tau}{r+\lambda}(r + \lambda - \mu_\Theta)$ is multiplied by $\frac{\beta_1}{\beta_1 - 1} > 1$, which accounts for the effect of uncertainty on investment irreversibility.

Notice that under investment irreversibility, the effects of the business cycle are asymmetric. To explain this point, let us rearrange the investment rule (3) as follows:

$$\xi_t(\Theta_t, K_t) = \Theta_t \Psi_K(K_t) \text{ for } \xi_t < \hat{\xi} \equiv \frac{\beta_1}{\beta_1 - 1} \frac{r + \lambda + \tau}{r + \lambda}(r + \lambda - \mu_\Theta). \quad (4)$$

According to (4), the marginal product ξ_t can be seen as a regulated process with an upper reflecting barrier at point $\hat{\xi}$ (see Harrison, 1985). In other words, when Θ_t rises (typically during a market expansion), ξ_t reaches $\hat{\xi}$ and the firm finds it optimal to invest. Notice that the increase in capital, i.e., $dK_t > 0$, reduces the marginal product $\Psi_K(K_t)$: this countereffect prevents ξ_t from exceeding the upper barrier $\hat{\xi}$. Therefore, under decreasing marginal productivity, $\hat{\xi}$ is an upper reflecting barrier, which cannot be crossed.

Suppose now that, due to a recession, the inequality $\xi_t < \hat{\xi}$ holds. In this case, Θ_t is too low and firms do not invest. In this case, the installed capital exceeds the optimal one, since it cannot be dismantled.

Notice that the existence of a reflecting barrier $\hat{\xi}$ does not mean that there is a finite rate of accumulation over time. Rather, it simply means that no investment will be undertaken for long periods: in this case, the average rate of capital accumulation will be nil. When, however, ξ_t reaches $\hat{\xi}$, investment is suddenly made, and therefore the rate of capital accumulation is infinite.[7]

3. OPTIMAL PROVISION OF PUBLIC GOODS

Let us now analyze the provision of public goods. To do so, we will use the well-known models developed by Zodrow and Mieszkowski (1986) and Wilson (1986) where many small countries compete to attract capital but need to use a source-based tax to finance the provision of public goods. By assumption, each competing Government chooses its optimal fiscal policy by maximizing the utility function of a representative citizen, i.e., $U(C_t, G_t)$, where C_t and G_t are a private and public good respectively. Although the model is dynamic, we assume that Governments can choose the tax rate and commit themselves not to change τ in the future.

7. Investment is instantaneous because there are no instalment costs. This means that the investment rate is infinite at point $\hat{\xi}$. Technically speaking we can say that this is due to the fact that, at point $\hat{\xi}$, neither ξ_t nor K_t are differentiable with respect to time t (see Dixit, 1993; Harrison, 1985).

The private budget constraint is equal to

$$C_t = \Pi(K_t, \Theta_t) - rK_t + r\Sigma, \tag{5}$$

where Σ is the capital endowment of our representative citizen. Assuming a balanced public budget, the condition

$$G_t = \tau K_t \tag{6}$$

always holds. In order to address the Government's policy, let us first analyze the effect of taxation on capital accumulation. If the business cycle is expanding and therefore the optimal condition (3) holds, taxation affects investment. Using comparative statics and differentiating (3) we obtain:

$$\frac{\partial K_t}{\partial \tau} = \frac{\frac{\Psi_K(K_t)}{r+\lambda+\tau}}{\Psi_{KK}(K_t)} = \frac{\Psi_K(K_t)}{\Psi_{KK}(K_t)(r+\lambda+\tau)} < 0. \tag{7}$$

Since $\Psi_{KK} < 0$, the higher the tax rate the lower the capital accumulation is.

If $\Theta_t = \Theta_t^*$ (i.e., during a recovery), a change in public spending, caused by a change in τ, is equal to $dG = K_t d\tau + \tau dK_t$. If however a downturn occurs and therefore the inequality $\Theta_t < \Theta_t^*$ holds neither investment nor disinvestment is made (because of irreversibility). Since irreversibility makes capital immobile, we have $\frac{\partial K_t}{\partial \tau} = 0$. Therefore, the change in public spending is equal to $dG = K_t d\tau$ and we can say that, in this latter case, a source-based tax has the same effect as a lump-sum one. To sum up, we can write the following

$$\begin{cases} dG = K_t d\tau + \tau dK_t & \text{if} \quad \Theta_t = \Theta_t^*, \\ dG = K_t d\tau & \text{if} \quad \Theta_t < \Theta_t^*. \end{cases} \tag{8}$$

More precisely, in the former case (when $\Theta_t \geq \Theta_t^*$) new capital, $dK_t > 0$, is invested and, due to the absence of installment costs, the equality $\Theta_t = \Theta_t^*$ is immediately reached. In the latter case, the productivity variable Θ_t is less than Θ_t^*. This means that taxation cannot affect investment (and therefore does not affect the tax base) and the revenue change is simply due to the tax rate change $d\tau$. Substituting (7) into (8) therefore gives

$$\begin{cases} \frac{dK}{dG} = \frac{1}{\left[\Theta_t^* \Psi_{KK}(K_t) K_t + \tau\right]} & \text{if} \quad \Theta_t = \Theta_t^*, \\ \frac{dK}{dG} = 0 & \text{if} \quad \Theta_t < \Theta_t^*. \end{cases} \tag{9}$$

Given these results, we can now analyze the Governments' strategies. In our model, each Government will have to solve the following problem, by using its tax tool τ:

$$\begin{array}{c} \max_{C_t, G_t} U(C_t, G_t) \\ \text{s.t. (5) and (6)} \end{array} \tag{10}$$

Solving (10) we thus obtain the following:

Proposition 1. Under investment irreversibility and uncertain obsolescence the marginal rate of substitution between the public and the private good will be equal to:

$$MRS \equiv \frac{U_{G_t}(C_t, G_t)}{U_{C_t}(C_t, G_t)} = \begin{cases} 1 - \frac{[\Theta_t \Psi_K(K_t) - r]}{\tau} \frac{\epsilon}{1+\epsilon} > 1 & \text{if} \quad \Theta_t = \Theta_t^*, \\ 1 & \text{if} \quad \Theta_t < \Theta_t^*, \end{cases} \tag{11}$$

where $\varepsilon = \frac{\partial K_t}{\partial \tau} \frac{\tau}{K_t} < 0$ with $|\varepsilon| < 1$.

Proof. See Appendix B. ■

The reasoning behind Proposition 1 is straightforward. If $\Theta_t < \Theta_t^*$, no investment is undertaken and, due to irreversibility, no disinvestment occurs. Thus, capital is fixed. In this case, tax rate changes have no impact on capital accumulation. Since the source-base tax has the same effect as a lump-sum one, public good provision is undistorted. If $\Theta_t = \Theta_t^*$, namely Θ_t reaches or overcomes its previous maximum value, investment is undertaken. In this case, taxation discourages capital accumulation and therefore leads to the underprovision of G_t.

In our model, the stochastic variable Θ_t can be considered as an economic shock that not only affects a firm's decision but also the economy as a whole. For this reason we wonder what happens when economic volatility changes. It is easy to show that:

Proposition 2. The derivative $\frac{\partial MRS}{\partial \sigma}$ is positive.

Proof. See Appendix C. ■

The reasoning behind this result is as follows: volatility has a twofold effect. On the one hand, it raises the threshold value: this means that, given an initial value Θ_t, the inequality $\Theta_s < \Theta_s^*$ (with $s \geq t$) holds for longer time: in other words, the public good is optimally provided for longer time. On the other hand, for a given threshold value Θ_t^*, the increase in σ makes Θ_s (with $s \geq t$) more volatile. This implies that the equality $\Theta_s = \Theta_s^*$ (with $s \geq t$) is expected to hold for longer time: in this case, the public good is underprovided. Proposition 2 shows that the latter effect dominates the former and that an increase in volatility raises the marginal rate of substitution, thereby making public good underprovision worse.

4. LONG-TERM TAX EFFECTS

So far, we have focused on the provision of public goods for a given value of the marginal product $\xi_t = \Theta_t \Psi_K(K_t)$. This implicitly means that we have analyzed the tax effect in the short/medium term.

Let us next focus on the long-term tax effects. As shown in section 2, it is impossible to find a finite rate of investment. However, we can find the long-term expected rate of capital accumulation. To do so, we first check whether a long-term distribution of the marginal product ξ_t exists within the range $(-\infty, \hat{\xi})$. If this is the case, when $\xi_t < \hat{\xi}$, the stock of capital is constant and it is thus possible to calculate the marginal probability distribution for K_t. This, in turn, allows us to determine the long-run rate of growth of capital stock. Following

Hartman and Hendrickson (2002) and Di Corato et al. (2014) we can thus prove that:

Proposition 3. For any value of initial stock of capital $\hat{K} > 0$, such that the marginal product $\xi_t(\Theta_t, \hat{K}) \leq \hat{\xi}$, the expected long-term average rate of capital accumulation can be approximated as follows:

$$\frac{1}{dt}E[d\ln K_t] \simeq \begin{cases} -(\mu_\Theta - \frac{1}{2}\sigma^2)\frac{\Psi_K(\hat{K})}{\Psi_{KK}(\hat{K})\hat{K}} & \text{for } \mu_\Theta > \frac{1}{2}\sigma^2, \\ 0 \text{ for } \mu_\Theta \leq \frac{1}{2}\sigma^2. \end{cases} \tag{12}$$

Proof. See Appendix D.■

Proposition 3 shows that, for any initial stock of capital $\hat{K}$, the long-term average rate of capital accumulation is only affected by the dynamics of Θ_t. In particular, if the production function $\Psi(K_t)$ follows the Inada conditions and the drift parameter is high enough (i.e., $\mu_\Theta > \frac{1}{2}\sigma^2$), the expected long-term growth rate of capital is proportional to $(\mu_\Theta - \frac{1}{2}\sigma^2)$. Otherwise, is nil.

To explain this result, let us take the log of (4), when $\xi_t = \hat{\xi}$:

$$\ln \Psi_K(K_t) + \ln \Theta_t = \ln \hat{\xi}.$$

Since $\ln \hat{\xi}$ is constant, this means that any change in K_t (from t onward) will be driven only by changes in $\ln \Theta_t$. In other words, we obtain that the logs of $\Psi_K(K_t)$ and Θ_t are cointegrated, where the marginal product of capital is the stationary cointegrating function (see Bentolila and Bertola, 1990). Therefore, if the barrier $\hat{\xi}$ plays a role for the firm's investment policy in the short term, it is irrelevant in the long term. Using a Taylor expansion of $\Psi_K(K_t)$ around $\hat{K}$ and taking expectations, it is easy to obtain (12).[8]

Notice also that the rate in (12) is decreasing in the volatility of future values of Θ_t. A higher value of σ has two effects on capital accumulation. Firstly, the higher the parameter σ, the higher the barrier $\hat{\xi}$ is. Secondly, the higher the volatility σ the greater the negative skewness of the distribution of ξ_t is. This means that the probability of reaching the barrier $\hat{\xi}$ is lower.[9] Both effects reduce the rate of capital accumulation in both the short and long term. Moreover, if $\alpha \leq \frac{1}{2}\sigma^2$, the process ξ_t deviates from $\hat{\xi}$ and the probability of meeting point $\hat{\xi}$ goes to zero. This implies that the long rate of capital accumulation falls to zero.

Finally, it is worth noting that Proposition 3 has an interesting policy implication. As we know, taxation raises the upper barrier $\hat{\xi}$. However, our results show that the expected rate of capital accumulation is unaffected by this upper bound. This means that taxation is neutral in the long term. As a consequence, given the public budget constraint (6), the long-term level of public goods provision is

8. It is worth noting that if the production function is isoelastic, i.e., $\Psi(K_t) = K_t^\gamma$ with $\gamma \in (0,1)$, the long-term growth rate reduces to $\frac{1}{dt}E[d\ln K_t] = \frac{(\mu_\Theta - \frac{1}{2}\sigma^2)}{1-\gamma}$. As can be seen, this rate does not depend on the initial stock of capital.

9. Appendix D shows that, for any time, the higher the parameter σ the lower the probability that the equality $\xi_t = \hat{\xi}$ holds.

unaffected by tax competition. We can thus say that, contrary to previous work, if $\mu_\Theta > \frac{1}{2}\sigma^2$, public goods are optimally provided. If however $\mu_\Theta < \frac{1}{2}\sigma^2$, the long-term capital (tax base) is nil and thus taxation cannot finance the provision of public goods. In neither case, taxation matters.

5. CONCLUSION

In this article, we have analyzed the provision of public goods over time, by assuming that capital is partially mobile (it can be freely installed, but cannot be dismantled). More precisely, we have assumed that investment is irreversible, though it is subject to stochastic obsolescence. When the investment project expires, in fact, the firm owns a non-depreciable option to restart.

As we have shown, the tax effects on the provision of public goods changes over time. In the short term, public goods can be optimally provided during a downturn. In this case, the capital stock is fixed and our source-base tax has the same effect as a lump-sum one. Only during expansions, the growth of capital is discouraged by taxation: this leads to public goods underprovision. In the long term, results are different. Since taxation is neutral, tax competition affects neither capital accumulation nor public good provision.

In this article, we have assumed that competing Governments initially choose the tax rate and commit not to change it in the future. Of course, it would be interesting to investigate the effects of policy uncertainty when Governments do not precommit and can change taxation. This topic is left for future research.

APPENDIX

A. PROOF OF LEMMA 1

The firm's problem is one of choosing the optimal amount of capital:

$$V(K_t, \Theta_t) = \max_{K_t} E_0\left[\int_0^\infty (1-\lambda dt)e^{-rt}[\Pi(K_t,\Theta_t) - dK_t]dt + 0\cdot \lambda dt | K_0 \geq 0,\ \Theta_0 \geq 0\right], \tag{13}$$

with $dK_t \geq 0$ for all t. Without installation costs, the growth rate of capital is unbounded and dK is therefore the investment process. The expectation in equation (13) is conditional on the information available at time zero, accounts for the joint distribution of K_t and Θ_t, as well as the irreversibility constraint.[10]

Assuming that $V(\cdot)$ is twice continuously differentiable, a solution can be obtained starting within a time interval where no new investment occurs. Applying dynamic programming to (13), and rearranging we can write the firm's value as

$$V(K_t, \Theta_t) = \Pi(K_t, \Theta_t)dt + e^{-(r+\lambda)dt}E_0[V(K_t, \Theta_t + d\Theta_t)].$$

10. As we know, at any interval dt, there is a probability λdt that the business value goes to zero. In this case, the firm can decide whether and when to invest.

Expanding the right-hand side and using Itô's lemma gives

$$(r+\lambda)V(K_t,\Theta_t)=\Pi(K_t,\Theta_t)+\mu_\Theta\Theta_t\frac{\partial V(K_t,\Theta_t)}{\partial\Theta_t}+\frac{\sigma^2}{2}\Theta_t^2\frac{\partial^2 V(K_t,\Theta_t)}{\partial\Theta_t^2}. \tag{14}$$

Differentiating (14) with respect to K_t we obtain

$$(r+\lambda)v(K_t,\Theta_t)=[\Psi_K(K)\Theta-\tau]+\mu_\Theta\Theta_t\frac{\partial v(K_t,\Theta_t)}{\partial\Theta_t}+\frac{\sigma^2}{2}\Theta_t^2\frac{\partial^2 v(K_t,\Theta_t)}{\partial\Theta_t^2}, \tag{15}$$

where $v(K_t,\Theta_t)\equiv V_K(K_t,\Theta_t)$. The solution of (15) has the following form:

$$v(K_t,\Theta_t)=c+\Theta_t f(K_t)+\sum_{i=1}^{2}a_i(K_t)\Theta_t^{\beta_i}, \tag{16}$$

where c is a constant to be found and

$$\beta_1=\left(\tfrac{1}{2}-\tfrac{\mu_\Theta}{\sigma^2}\right)+\sqrt{\left(\tfrac{1}{2}-\tfrac{\mu_\Theta}{\sigma^2}\right)^2+2\tfrac{r+\lambda}{\sigma^2}}>1,$$
$$\beta_2=\left(\tfrac{1}{2}-\tfrac{\mu_\Theta}{\sigma^2}\right)-\sqrt{\left(\tfrac{1}{2}-\tfrac{\mu_\Theta}{\sigma^2}\right)^2+2\tfrac{r+\lambda}{\sigma^2}}<0$$

are the roots of the characteristic equation $\frac{\sigma^2}{2}\beta(\beta-1)+\mu_\Theta\beta-(r+\lambda)=0$. The interpretation of equation (16) is then transparent. The contribution of the Kth unit of capital to profitability is equal to

$$\Pi_K(K_t,\Theta_t)\equiv\Psi_K(K)\Theta-\tau.$$

Calculating the expected present value of this marginal contribution thus gives:

$$v(K_t,\Theta_t)=\frac{\Theta_t\Psi_K(K)}{r+\lambda-\mu_\Theta}-\frac{\tau}{r+\lambda}+\sum_{i=1}^{2}a_i(K_t)\Theta_t^{\beta_i}.$$

Let us next introduce the boundary conditions for (16):

$$v\left(K_t,\Theta_t^*\right)=1, \tag{17}$$

$$v_\Theta\left(K_t,\Theta_t^*\right)=0, \tag{18}$$

$$a_2(K_t)=0, \tag{19}$$

where $\Theta_t^*=\{\max_{0\le s\le t}\Theta_s\}$. Equations (17) and (18) are the Value Matching and Smooth Pasting Condition for the firm's optimal policy respectively.[11] Moreover, (19) imposes the irreversibility constraint on capital $dK_t\ge 0$.[12] Substituting (16) into (17) and (18), we have the following two-equation system:

$$\tfrac{\Theta_t\Psi_K(K)}{r+\lambda-\mu_\Theta}-\tfrac{\tau}{r+\lambda}+a_1(K_t)(\Theta_t^*)^{\beta_1}=1,$$
$$\tfrac{\Theta_t\Psi_K(K)}{r+\lambda-\mu_\Theta}+\beta_1 a_1(K_t)(\Theta_t^*)^{\beta_1}=0.$$

11. See Dixit and Pindyck (1994).
12. In other words, when Θ_t is very small, the expected present value of the last unit of capital installed is close to zero. Therefore, the value of the marginal option to scrap it is almost infinite. For further details see Dixit and Pindyck (1994, Ch. 6).

Rearranging gives the following investment rule:

$$\Theta_t \Psi_K(K) = \frac{\beta_1}{\beta_1 - 1} \frac{r + \lambda + \tau}{r + \lambda} (r + \lambda - \mu_\Theta).$$

This concludes the proof of Lemma 1. ■

APPENDIX

B. PROOF OF PROPOSITION 1

Solving (10) gives the following f.o.c.

$$\frac{\partial U(C,G)}{\partial \tau} = U_C(C,G) \frac{\partial C}{\partial \tau} + U_G(C,G) \frac{\partial G}{\partial \tau} = 0, \tag{20}$$

where the derivative

$$\frac{\partial G_t}{\partial \tau} = K_t + \tau \frac{\partial K_t}{\partial \tau}$$

is positive by assumption (i.e., no Laffer curve exists) and

$$\frac{\partial C_t}{\partial \tau} = [\Theta_t \Psi_{K_t}(K_t) - (\tau + r)] \frac{\partial K_t}{\partial \tau} - K_t \tag{21}$$

with

$$\frac{\partial K_t}{\partial \tau} = \frac{\frac{\Psi_K(K_t)}{r+\lambda+\tau}}{\Psi_{KK}(K_t)} = \frac{\Psi_K(K_t)}{\Psi_{KK}(K_t)(r + \lambda + \tau)} < 0. \tag{22}$$

If $\Theta_t = \Theta^*$, using (22) we can write the MRS as follows:

$$\begin{aligned} MRS = \frac{U_G(C,G)}{U_C(C,G)} &= -\frac{\frac{\partial C_t}{\partial \tau}}{\frac{\partial G_t}{\partial \tau}} \\ &= \frac{\left(K_t + \tau \frac{dK_t}{d\tau}\right) - [\Theta_t \Psi_K(K_t) - \tau] \frac{dK_t}{d\tau}}{K_t + \tau \frac{dK_t}{d\tau}} \\ &= 1 - \frac{[\Theta_t \Psi_K(K_t) - r] \frac{dK_t}{d\tau}}{K_t + \tau \frac{dK_t}{d\tau}} \\ &= 1 - \frac{[\Theta_t \Psi_K(K_t) - r]}{\tau} \frac{\frac{\partial K_t}{\partial \tau} \frac{\tau}{K_t}}{1 + \frac{\partial K_t}{\partial \tau} \frac{\tau}{K_t}}. \end{aligned} \tag{23}$$

Notice that the gross marginal product $\Theta_t \Psi_K(K_t)$ is higher than r. Otherwise, our representative firm would not invest, even in the absence of taxation, would save its resources by earning the risk-free rate r. This implies that the term $\frac{[\Theta_t \Psi_{K_t}(K_t) - r]}{\tau}$ is positive. Also, remember that:

$$\frac{\partial K_t}{\partial \tau} \frac{\tau}{K_t} = \varepsilon < 0 \text{ with } |\epsilon| < 1.$$

This means that $(\frac{\partial K_\tau}{\partial_\tau}\frac{\tau}{K_t})/(1+\frac{\partial K_t}{\partial \tau}\frac{\tau}{K_t}) < 0$ and therefore $MRS > 1$ if $\Theta_t = \Theta^*$. Proposition 1 is thus proven. ■

APPENDIX

C. PROOF OF PROPOSITION 2

Let us set $\Theta_t = \Theta_t^*$ and differentiate (23) with respect to σ:

$$\begin{aligned}\frac{\partial MRS}{\partial \sigma} &= \frac{\partial}{\partial \sigma}\left\{1 - \frac{\left[\Theta_t^* \Psi_{K_t}(K_t) - r\right]}{\tau}\frac{\frac{\partial K_t}{\partial \tau}\frac{\tau}{K_t}}{1+\frac{\partial K_t}{\partial \tau}\frac{\tau}{K_t}}\right\} \\ &= \frac{\partial}{\partial \beta_1}\left\{1 - \frac{\left[\Theta_t^* \Psi_{K_t}(K_t) - r\right]}{\tau}\frac{\varepsilon}{1+\varepsilon}\right\}\frac{\partial \beta_1}{\partial \sigma},\end{aligned} \tag{24}$$

with $\frac{\partial \beta_1}{\partial \sigma} < 0$. Using (3) we can write

$$\frac{\partial}{\partial \beta_1}\left\{1 - \frac{\left[\Theta_t^* \Psi_{K_t}(K_t) - r\right]}{\tau}\frac{\varepsilon}{1+\varepsilon}\right\} = \frac{\partial}{\partial \beta_1}\left\{1 - \frac{\frac{\beta_1}{\beta_1 - 1}\frac{r+\lambda+\tau}{r+\lambda}(r+\lambda-\mu_\Theta)}{\tau}\frac{\varepsilon}{1+\varepsilon}\right\}. \tag{25}$$

Solving (25) gives

$$\frac{\partial}{\partial \beta_1}\left\{1 - \frac{\frac{\beta_1}{\beta_1 - 1}\frac{r+\lambda+\tau}{r+\lambda}(r+\lambda-\mu_\Theta)}{\tau}\frac{\varepsilon}{1+\varepsilon}\right\} = -\frac{1}{\beta_1(\beta_1 - 1)}\Theta_t^*\frac{\Psi_{K_t}(K_t)}{\tau}\frac{\varepsilon}{1+\varepsilon} < 0. \tag{26}$$

Given $\frac{\partial \beta_1}{\partial \sigma} < 0$ and the inequality (26) we thus obtain $\frac{\partial MRS}{\partial \sigma} > 0$. ■

APPENDIX

D. PROOF OF PROPOSITION 3

D.1. LONG-TERM DISTRIBUTION

Let h_t be a linear Brownian motion with parameters μ and σ that evolves according to $dh_t = \mu dt + \sigma dz_t$. Following Harrison (1985, pp. 90–91) and Dixit (1993, pp. 58–68)), the long-term density function for h fluctuating between a lower reflecting barrier, $a \in (-\infty,\infty)$, and an upper reflecting barrier, $b \in (-\infty, \infty)$, is given by the following truncated exponential distribution:

$$f(h_t) = \begin{cases} \frac{2\mu}{\sigma^2}\frac{e^{\frac{2\mu}{\sigma^2}h_t}}{e^{\frac{2\mu}{\sigma^2}b} - e^{\frac{2\mu}{\sigma^2}a}} & \mu \neq 0, \\ \frac{1}{b-a} & \mu = 0. \end{cases} \tag{27}$$

Let us next focus on the limit case where $a \to -\infty$. In this case, from (27), a limiting argument gives:

$$f(h_t) = \begin{cases} \frac{2\mu}{\sigma^2} e^{-\frac{2\mu}{\sigma^2}(b-h_t)} & \mu > 0, \\ 0 & \mu \leq 0. \end{cases} \tag{28}$$

for $-\infty < h_t < b$. Hence, the long-term average of h_t can be evaluated as $E[h_t]=\int_\Phi h_t f(h_t)dh_t$, where Φ depends on the distribution assumed. In steady-state, this gives:

$$\begin{aligned} E[h_t] &= \int_{-\infty}^{b} h_t f(h_t) dh_t = \int_{-\infty}^{b} h_t \frac{2\mu}{\sigma^2} e^{-\frac{2\mu}{\sigma^2}(b-h_t)} dh_t \\ &= \frac{2\mu}{\sigma^2} e^{-\frac{2\mu}{\sigma^2} b} \int_{-\infty}^{b} h_t e^{\frac{2\mu}{\sigma^2} h_t} dh_t = b - \frac{2\mu}{\sigma^2}. \end{aligned} \tag{29}$$

D.2. THE EXPECTED LONG-TERM RATE OF CAPITAL ACCUMULATION

Let us next take the logarithm of $\Theta_t \Psi_K(K_t)$:

$$\ln \xi_t = \ln[\Theta_t \Psi_K(K_t)] = \ln \Theta_t + \ln[\Psi_K(K_t)]. \tag{30}$$

By Ito's lemma, $\ln \xi_t$ evolves according to $d \ln \xi_t = d \ln \Theta_t = [(\mu_\Theta - \frac{1}{2}\sigma^2)dt + \sigma dz_t]$, where $\ln \hat{\xi}$ is its upper reflecting barrier. Setting $h_t = \ln \xi_t$, the random variable $\ln \xi_t$ follows a linear Brownian motion with a drift parameter equal to $\mu = (\mu_\Theta - \frac{1}{2}\sigma^2)$. Moreover, its long-term distribution is characterized by the density function (28). Solving (30) for $\ln \Psi_K(K_t)$ gives:

$$\ln \Psi_K(K_t) = h_t - \ln \Theta_t. \tag{31}$$

Let us next calculate the expected value of (31):

$$E[\ln \Psi_K(K_t)] = E[h_t] - \left[\Theta_0 + \left(\mu_\Theta - \frac{1}{2}\sigma^2\right)t\right].$$

Using Taylor's theorem, we the expand $\Psi_K(K_t)$ around the point $\hat{K}$, thereby obtaining:

$$\begin{aligned} E\left[\ln(\Psi_K(\hat{K}) - \Psi_{KK}(\hat{K})\hat{K} + \Psi_{KK}(\hat{K})K_t)\right] &= E\left[\ln[\Psi_{KK}(\hat{K})(K_t - \Delta(\hat{K}))]\right] \\ &\simeq E[h_t] - \left[\Theta_0 + \left(\mu_\Theta - \tfrac{1}{2}\sigma^2\right)t\right], \end{aligned} \tag{32}$$

where $\Delta(\hat{K}) = \frac{\Psi_{KK}(\hat{K})\hat{K} - \Psi_K(\hat{K})}{\Psi_{KK}(\hat{K})}$. Given this result we obtain:

$$E\left[\ln[(K_t - \Delta(\hat{K}))]\right] = E[h_t] - \left[\Theta_s + \left(\mu_\Theta - \frac{1}{2}\sigma^2\right)t\right] - \ln \Psi_{KK}(\hat{K}).$$

Rewriting $\ln(K_t - \Delta(\hat{K}))$ as $\ln[x - \hat{x}]$ and using a Taylor expansion around the point $(\ln \hat{x}, \ln x)$ gives:

$$\ln[x - \hat{x}] \equiv \ln\left[e^{\ln x} - e^{\ln \hat{x}}\right] \simeq v_0 + v_1 \ln x + v_2 \ln \hat{x},$$

where

$$v_0 = \ln\left[e^{\widetilde{\ln x}} - e^{\widetilde{\ln \hat{x}}}\right] - \left[\frac{\widetilde{\ln \hat{x}}}{1 - e^{\widetilde{\ln x} - \widetilde{\ln \hat{x}}}} + \frac{\widetilde{\ln x}}{1 - e^{-(\widetilde{\ln x} - \widetilde{\ln \hat{x}})}}\right],$$

$$v_1 = \frac{1}{1 - e^{\widetilde{\ln \hat{x}} - \widetilde{\ln x}}}, v_2 = \frac{1}{1 - e^{(\widetilde{\ln x} - \widetilde{\ln \hat{x}})}}, \frac{v_2}{v_1} = \frac{1 - v_1}{v_1} < 0.$$

Substituting this approximation into (32) we have:

$$E[\ln K_t] = \frac{E[h_t] - \left[\Theta_0 + (\mu_\Theta - \frac{1}{2}\sigma^2)t\right]}{v_1} - \frac{v_0 + v_2 \ln \Delta(\hat{K}) + \ln \Psi_{KK}(\hat{K})}{v_1}. \qquad (33)$$

Since (29) implies that $E(h_t)$ does not depend on t, differentiating with respect to t gives:

$$\begin{aligned} \frac{1}{dt} E[d \ln K_t] &= \frac{-(\mu_\Theta - \frac{1}{2}\sigma^2)}{v_1} \\ &= -(\mu_\Theta - \frac{1}{2}\sigma^2)(1 - e^{\widetilde{\ln \Delta(\hat{K})} - \widetilde{\ln K}}). \end{aligned} \qquad (34)$$

By the monotonicity property of the logarithm, a level $\hat{K}$ must exist such that $\ln \hat{K} = \widetilde{\ln K}$ and $\ln \Delta(\hat{K}) = \widetilde{\ln \Delta(\hat{K})}$. Therefore, we obtain:

$$\begin{aligned} \frac{1}{dt} E[d \ln K_t] &= -(\mu_\Theta - \frac{1}{2}\sigma^2)(1 - \frac{\hat{K}}{\Delta(\hat{K})}) \\ &= -(\mu_\Theta - \frac{1}{2}\sigma^2)\frac{\Psi_K(\hat{K})}{\Psi_{KK}(\hat{K})\hat{K}} \text{ for } \mu_\Theta > \frac{1}{2}\sigma^2. \end{aligned} \qquad (35)$$

This concludes the proof. ■

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