

# Do Structural Breaks in Volatility cause Spurious Volatility Transmission?

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## Abstract:

We show through extensive Monte Carlo simulations that structural breaks in volatility (volatility shifts) across two independently generated return series cause spurious volatility transmission when estimated with popular bivariate GARCH models. This bias is exacerbated when the two series are correlated or when the sample size is small. However, using a dummy variable for the induced volatility shift virtually eliminates this bias. We also show that structural breaks in volatility have a substantial impact on the estimated hedge ratios. We confirm our simulation findings using the US stock market data.

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## 1. Introduction

Volatility transmission (i.e. volatility spillover) is the impact on the variance of a given asset (or market) from past variance or shocks of a different asset (or market). The presence of volatility transmission would imply that shocks (i.e. ‘news’) significantly impact the volatility not only in the asset or market from which the shock originated but in other related assets or markets as well. Ross (1989) showed that volatility in returns of an asset depends upon the rate of information flow, suggesting that information from one market will affect the volatility generating process of other related markets. Since the flow of information and the speed of processing that information varies across markets, one should expect different volatility dynamics across markets over time. Volatility spillovers are also attributed to changes in common information and cross-market hedging, which may simultaneously change expectations across markets. Fleming, Kirby, and Ostdiek (1998) show how cross-market hedging and sharing of common information creates volatility transmission across markets over time. Significant volatility transmission across assets or markets have important implications in decisions regarding optimal portfolio allocation (Kroner and Ng, 1998), risk management (Christoffersen, 2009) and dynamic hedging (Haigh and Holt, 2002). Clearly, policy makers and financial markets participants are interested in knowing if and how shocks and volatility are transmitted across different assets or markets over time. Consequently, this line of research has seen an explosion in the literature over the last thirty years.<sup>1</sup>

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<sup>1</sup> Earlier major papers include Baillie and Bollerslev (1990); Engle, Ito and Lin (1990); Hamao, Masulis and Ng (1990); King and Wadhvani (1990); Engle and Susmel (1993); King, Sentana and Wadhvani (1994); Lin, Engle and Ito (1994); and Karolyi (1995).

There are essentially two main methods used in the literature to examine if volatility is significantly transmitted across two series over time.<sup>2</sup> One simple method is the two-step methodology introduced by Cheung and Ng (1996) that concentrates on the cross correlation function of squared univariate GARCH residual estimates. Hong (2001) extended this method by introducing flexible weighting scheme for the sample cross-correlation which provides non-uniform weighting at each lag to improve power of the test based on the argument that economic agents normally discount past information. However, using Monte Carlo simulations, Van Dijk, Osborn and Sensier (2005) document severe size distortions in both of these tests in the presence of structural breaks in variance. They note that size problems are particularly large when both series exhibit volatility shifts in close temporal proximity, in which case both tests frequently and incorrectly attribute this occurrence to an underlying causality in variance. Rodrigues and Rubia (2007) give a theoretical justification for these findings as a particular case of a general class of non-stationary volatility processes. They show that the size departures are not only a small-sample effect but persist asymptotically because of the failure to consistently estimate cross-correlations.

The alternative approach to see if significant volatility transmission exists across series is by estimating a parametric multivariate GARCH model. This approach is significantly more popular as it yields specific formulations for the volatility spillover effects. However, virtually all studies in the existing literature assume that the unconditional variance of the underlying series is constant implying that volatility is generated by a stable GARCH process. This is surprising as we know that markets often experience structural breaks in the unconditional variance due to political, social, or economic reasons, which causes breaks in the GARCH

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<sup>2</sup> There is a related literature called financial contagion, often detected as changes in the correlations' level during crisis periods (Forbes and Rigobon, 2002). But our focus is on the transmission of time-varying volatility not on time-varying correlations.

parameters. Lamoureux and Lastrapes (1990) show that volatility persistence is biased upward when univariate GARCH models are applied to a series with structural breaks in variance. Mikosch and Starica (2004) provide theoretical explanation with supporting evidence from simulations and US stock market data indicating that ignoring structural breaks in variance results in higher estimated volatility persistence within a univariate GARCH model. Starica and Granger (2005) document that daily US stock market returns are comprised of shifts in the unconditional variance and find that forecasts are improved if this non-stationary behavior of volatility is taken into account. Thus, there is robust evidence which suggests that a properly specified univariate GARCH model should account for structural breaks, if such breaks exist.<sup>3</sup>

If volatility shifts can affect persistence in “own” volatility, then one can argue that allowing for volatility shifts in individual series can also affect the persistence of volatility across two series. Consequently, a finding of volatility spillover may be due to an inaccurate measurement in persistence. This finding of significant volatility transmission across series in the multivariate GARCH context could be triggered by a common volatility shift across the two series. Thus, ignoring structural breaks can result in overestimation of volatility spillover effects.<sup>4</sup>

Ewing and Malik (2005) was the first paper to examine volatility transmission across series under structural breaks. They endogenously detected structural breaks in small cap and large cap US stock returns, and incorporated this information in a bivariate GARCH model. They report that accounting for volatility shifts considerably reduces the transmission in volatility and

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<sup>3</sup> Structural breaks in variance are reported in international stock returns (Aggarwal, Inclan and Leal, 1999), exchange rates (Rapach and Strauss, 2008), oil prices (Ewing and Malik, 2010), among numerous other series.

<sup>4</sup> In this paper, we focus on the effect on volatility transmission by ignoring the ‘shifts’ in unconditional variance. However, there is a related line of inquiry focusing on the simultaneous ‘jumps’ across series. Caporin, Kolokolov, and Reno (2017) argue that major financial news triggers a simultaneous occurrence of jumps in several stocks. They propose a novel test procedure to exploit this information which yields short-term predictability in stock returns and determines a persistent pattern of stock variances and correlations.

essentially removes the spillover effects. There are only a handful of studies which have followed Ewing and Malik (2005) to show that significance of volatility transmission is overestimated when breaks are ignored in the bivariate GARCH model using specific data sets.<sup>5</sup> However, none of these studies give conclusive insight on the impact of a structural break on volatility transmission across assets or markets under different scenarios. Given the explosion in literature on the studies documenting volatility transmission and the widely documented breaks in different markets, a key question is: what is the general impact of structural breaks (if any) on volatility transmission mechanism and what are the possible practical economic implications? This is the research question we attempt to answer in this paper.

In this paper, we show through extensive Monte Carlo simulations that structural breaks in volatility (i.e. volatility shifts) cause spurious volatility spillover effects when estimated using popular bivariate GARCH models. For example, using a large sample of 8000 returns, we show that a common break in the unconditional variance of reasonable magnitude across two independently generated series results in average rejection rates (of the null hypothesis of no significant shock and volatility transmission across series at 1% significance level) of 45% using the BEKK model of Engle and Kroner (1995). However, accounting for this induced volatility shift with dummy variable reduces this average rejection rate to the expected value of 1%. Our results further show that this bias of overestimation in the statistical significance of the volatility transmission becomes larger when CCC model of Bollerslev (1990) is used for estimation, when variables are correlated or when small samples are used. Our overall conclusions are robust across a wide variety of specifications and scenarios. Since volatility shifts are widely

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<sup>5</sup> For example, Marcelo, Quiros and Quiros (2008) document that volatility spillover effects are reduced when breaks are incorporated in the small and large cap portfolios of the Spanish stock market. Huang (2012) show that there is bidirectional cross market volatility transmission between the UK and the US but this relation does not hold after controlling for structural breaks in the bivariate GARCH model.

documented in different markets and virtually all previous studies have ignored these phenomena, our findings cast doubt on the vast empirical studies documenting significant shock and volatility transmission across markets.

Our results have important practical implications for building accurate asset pricing models, forecasting volatility of market returns, devising optimal strategies for hedging and risk management. As an example, we show via simulations that structural breaks in volatility have a substantial impact on the estimated hedge ratios. We also confirm our major simulation findings by using publicly available daily data from small cap and large cap indices from the US stock market ranging from January 1995 to June 2018. We conclude that researchers should endogenously detect volatility breaks and incorporate these breaks in the estimation model to get an accurate estimate of the volatility transmission dynamics across series over time.

The next section describes the simulation design and section 3 provides the corresponding simulation results. Section 4 shows a hedging application as an example of economic implications of our study. Section 5 shows empirical results using the US stock market data confirming our simulation findings and section 6 concludes.

## 2. Simulation Design

We simulate returns using the bivariate Gaussian CCC model of Bollerslev (1990), with zero mean, where the variances and the covariance are given as

$$\sigma_{1,t}^2 = \omega_1 + a_1 \varepsilon_{1,t-1}^2 + b_1 \sigma_{1,t-1}^2 \quad (1)$$

$$\sigma_{2,t}^2 = \omega_2 + a_2 \varepsilon_{2,t-1}^2 + b_2 \sigma_{2,t-1}^2 \quad (2)$$

$$\sigma_{12,t} = \rho \sigma_{1,t} \sigma_{2,t} \quad (3)$$

Note here that the conditional correlation ( $\rho$ ) is constant, while the conditional covariance ( $\sigma_{12,t}$ ) dynamics depends on the conditional variances ( $\sigma_{1,t}^2, \sigma_{2,t}^2$ ) in a non-linear fashion. We consider both the case of zero correlation and a correlation of 0.5.

In this model, there is a complete absence of variance spillovers as shocks of the first asset impact only on the variance evolution of the first asset, and lagged variances of the first asset impact only on the variance evolution of the first asset. The same holds for the second asset. Moreover, if the correlation is set to zero, the two assets are linearly independent. This also leads to a true hedge ratio of zero, irrespective of the presence of structural breaks in the volatility. In fact, under a CCC model, the hedge ratio (HR) between our two variables equals the ratio between the covariance and one of the variances (depending on which variable we want to hedge)

$$HR_t = \frac{\sigma_{12,t}}{\sigma_{1,t}^2} = \rho \frac{\sigma_{2,t}}{\sigma_{1,t}} \quad (4)$$

We generate the two series from the model given in Equations 1-3 based on the different parametrizations reported in Table 1. The GARCH parameters and unconditional variance levels that we used are in line with empirical estimates commonly reported across a wide range of studies in different research fields (not only pure financial assets but also energy or commodity markets). Furthermore, we consider a wide variety of possible combinations of parameter values. We used different GARCH parameters and unconditional variance levels in the first five cases (P1 to P5) across the two series. This different levels of volatility persistence and different unconditional variance levels across the two series may cause different responses when structural breaks are induced. In turn, these elements might also impact on the dynamic evolution of the hedge ratios. To control for this possibility, we also report a sixth case (P6) and seventh case

(P7) where we keep the unconditional variances at the same level across the two series, and we set the model parameters very close to each other (P6) or identical (P7).

We use four sample sizes (1000, 2000, 4000 and 8000) to closely analyze the role of the length of the time series in driving the results. Researchers in recent studies have used a sample size of about 2000 observations, which is equivalent to 8 years of daily data. Using a longer time sample increases the likelihood of a structural break and consequently you have to account for it, but shorter sample reduces the possibility of having a break. We run 1000 simulations in all of our cases. We repeated our major simulations with 5000 runs and found the results to be almost identical.<sup>6</sup>

In our simulation setting of Table 1, the series do not include a break. We induce break in the middle of the sample by increasing the value of  $\omega$  by 4, which leads to an increase in the unconditional variance by 4.<sup>7</sup> We also considered different ways in which breaks are introduced. First, break is induced in the two variables but of the same magnitude. The unconditional hedge ratio is proportional to the ratio of unconditional volatilities. Therefore, if the structural break leads to an increase of the unconditional variances by a common scaling factor on both variances, the unconditional hedge ratio will be unaffected. However, if we scale up unconditional variances by different factors, the unconditional hedge ratio will change, depending on the ratio of the scaling factors. Consequently, in the second case, break is induced in the two variables but the break size is different. In this case, we scale up the variance intercept

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<sup>6</sup> Table A1 in the Appendix shows one of these results highlighting the fact that using 1000 or 5000 number of simulations gives almost identical results.

<sup>7</sup> We used the value 4 because the ratio between the 90% quantile and the 10% quantile of the unconditional variance of S&P 500 Index returns is 4. We also detected variance breaks in daily S&P 500 Index returns over the last twenty years using the method of Sanso, Arrago and Carrionet (2004) based on the original algorithm given by Inclan and Tiao (1994). We found the average upward shifts in variance to be of the magnitude of 4.5. Thus our magnitude of induced break is very sensible. Out of curiosity, we induced a small break of magnitude 2 and found that the size of break does have much effect on the reported spurious spillover and thus our overall conclusions reported in this paper remain unchanged (see Table A2 in Appendix).



by 4 in the first series and by 2 in the second series. In the third case, break is induced in only one of the two variables.<sup>8</sup>

Our simulated series do not have volatility spillovers but we may have introduced structural breaks in the unconditional variance levels. Thus, we estimate using two specifications which allow for spillovers and their extended versions allow for the presence of a structural break in the model intercepts. In the latter case, we assume a perfect knowledge of the break date. The first specification we consider belongs to the VARMA-GARCH model class of Ling and McAleer (2003) which is given as

$$\sigma_{1,t}^2 = \omega_1 + a_1 \varepsilon_{1,t-1}^2 + b_1 \sigma_{1,t-1}^2 + c_1 \varepsilon_{2,t-1}^2 + d_1 \sigma_{2,t-1}^2 \quad (5)$$

$$\sigma_{2,t}^2 = \omega_2 + a_2 \varepsilon_{2,t-1}^2 + b_2 \sigma_{2,t-1}^2 + c_2 \varepsilon_{1,t-1}^2 + d_2 \sigma_{1,t-1}^2 \quad (6)$$

$$\sigma_{12,t} = \rho \sigma_{1,t} \sigma_{2,t} \quad (7)$$

This corresponds to a CCC model of Bollerslev (1990) augmented with the possible presence of variance spillovers. The occurrence of variance spillovers is associated with the presence of statistically significant coefficients in the set  $\theta = \{c_1, c_2, d_1, d_2\}$ . In this model, we test for the presence of variance spillover by verifying the null hypothesis that the parameter set  $\theta$  has all elements equal to zero. The asymptotic distribution of the Wald test statistic follows from the results in Ling and McAleer (2013). We report tests at the 1% level of significance.<sup>9</sup>

If we have a structural break, we estimate the augmented model as

$$\sigma_{1,t}^2 = \omega_{1,B} D_t + \omega_{1,A} (1 - D_t) + a_1 \varepsilon_{1,t-1}^2 + b_1 \sigma_{1,t-1}^2 + c_1 \varepsilon_{2,t-1}^2 + d_1 \sigma_{2,t-1}^2 \quad (8)$$

$$\sigma_{2,t}^2 = \omega_{2,B} D_t + \omega_{2,A} (1 - D_t) + a_2 \varepsilon_{2,t-1}^2 + b_2 \sigma_{2,t-1}^2 + c_2 \varepsilon_{1,t-1}^2 + d_2 \sigma_{1,t-1}^2 \quad (9)$$

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<sup>8</sup> Note that we focus on cases where the structural break affects only the conditional variance intercepts and does not impact on the parameters governing the dynamic. We expect that a more diffused impact of structural breaks on models parameters will lead to even stronger results compared to those reported in this work. We leave this further generalization of our analysis to future work.

<sup>9</sup> We also calculated all of our results at the 5% significance level, which showed the same overall results but those results are not reported to conserve space but are available on request.

$$\sigma_{12,t} = \rho\sigma_{1,t}\sigma_{2,t} \quad (10)$$

where we introduce a step dummy variable  $D_t$  taking value 1 before the break date and 0 afterwards. Thus we have two intercept values for each equation, one for the pre-break period (B=before) and one for the post-break period (A=after).

The second specification we consider is the popular BEKK model of Engle and Kroner (1995) given as

$$H_{t+1} = C'C + B'H_tB + A'\varepsilon_t\varepsilon_t'A \quad (11)$$

where, in our bivariate case,  $C$  is a  $2 \times 2$  lower triangular matrix with three parameters.  $B$  is a  $2 \times 2$  square matrix of parameters and shows how current levels of conditional variances are related to past conditional variances.  $A$  is also a  $2 \times 2$  square matrix of parameters and measures the effects of shocks on volatility (conditional variance). In the case of structural break, we augment the above model similarly to the CCC case, by introducing a step dummy in equation (11). Note that the step dummy induces a change in all the elements of the matrix  $C$ .

The conditional variance and conditional covariance for each equation can be expanded, for the bivariate GARCH (1,1), as follows

$$h_{11,t+1} = c_{11}^2 + a_{11}^2\varepsilon_{1,t}^2 + 2a_{11}a_{12}\varepsilon_{1,t}\varepsilon_{2,t} + a_{21}^2\varepsilon_{2,t}^2 + b_{11}^2h_{11,t} + 2b_{11}b_{12}h_{12,t} + b_{21}^2h_{22,t} \quad (12)$$

$$h_{22,t+1} = c_{12}^2 + c_{22}^2 + a_{12}^2\varepsilon_{1,t}^2 + 2a_{12}a_{22}\varepsilon_{1,t}\varepsilon_{2,t} + a_{22}^2\varepsilon_{2,t}^2 + b_{12}^2h_{11,t} + 2b_{12}b_{22}h_{12,t} + b_{22}^2h_{22,t} \quad (13)$$

$$h_{12,t+1} = c_1c_{12} + a_{11}a_{21}\varepsilon_{1,t}^2 + (a_{11}a_{22} + a_{12}a_{21})\varepsilon_{1,t}\varepsilon_{2,t} + a_{12}a_{22}\varepsilon_{2,t}^2 + b_{11}b_{21}h_{11,t} + (b_{11}b_{22} + b_{12}b_{21})h_{12,t} + b_{12}b_{22}h_{22,t} \quad (14)$$

equations 12-14 show how shocks and volatility are transmitted across the two series over time.

The absence of variance spillovers corresponds to a null hypothesis of having all parameters in the set  $\theta = \{a_{12}, a_{21}, b_{12}, b_{21}\}$  being all equal to zero. The distribution of Wald test statistic follows from the results in Comte and Lieberman (2003).

It should be noted that the BEKK model does not nest the CCC model. Therefore, the estimation of a BEKK model on data generated from a CCC model is exposed to model specification error. To control for this issue, we compare the estimation performances of the BEKK model (in terms of tests for absence of variance spillover) with those of the CCC model, when breaks are not present. This will allow us to verify the role of model misspecification in spurious variance spillover detection (i.e. to verify if a BEKK model estimated on data simulated from CCC data generating process show evidence of spurious spillovers).

A further case covers the data generating processes where the correlation is equal to zero. In this case, the conditional covariances of the CCC model are (almost) zero by construction. In the BEKK case, testing for the absence of variance spillover might require a comparison of the cases where the set of tested parameters include the intercept  $c_{12}$ . In fact, if we drop all variance spillover dynamic parameters, the conditional covariance dynamic becomes

$$h_{12,t+1} = c_1 c_{12} + a_{11} a_{22} \varepsilon_{1,t} \varepsilon_{2,t} + b_{11} b_{22} h_{12,t} \quad (15)$$

and if the true model has a null correlation, and therefore a null conditional covariance, the only possibility of having a null covariance in the BEKK case is to restrict the intercept to zero. We note that a non-null intercept will lead to non-null hedge ratios even when the two series are linearly independent. In the simulation analysis we report both cases, also assessing the role of the intercept when the data generating process includes a non-null correlation. Note that the need for testing including the intercept is indirectly related to the occurrence of variance spillovers. In fact, a distortion in the covariance intercept might lead to a spurious occurrence of statistically significant parameters in the off-diagonal elements of A and B. Furthermore, in the case of a data generating process with non-null correlation, the dynamics of the conditional covariance might be the source of spurious spillover effects.

### 3. Simulation Results

Before we start to examine the impact of breaks on shock and volatility transmission, we run the benchmark case of no structural breaks to document any existing estimation bias associated with model misspecification. Panel A of Table 2 reports the rejection frequencies of the Wald test for verifying the null hypothesis of zero coefficients in the off-diagonal terms of the parameter matrices A, B and C (with and without intercept) of the BEKK model at the 1% significance level, when no breaks are induced. The average for all the cases for the sample size of 1000 is 0.397 (see last row), which is substantially greater than 0.01, indicating a substantial bias. However, this small sample bias is essentially eliminated as sample size becomes large, as the average of all cases for the sample size of 8000 is 0.023. Clearly a sample size of 8000 observations, which is equivalent to 32 years of daily data, is the best choice among the four alternatives considered. It is interesting to note that most papers studying volatility transmission using daily data typically use a much smaller sample than 32 years.

Next, we induce a common volatility break in both series and Panel B of Table 2 reports the rejection frequencies for the same Wald test. The results are quite strong. In all cases, the rejection rates go up substantially. For example, using 8000 observations which yield most accurate estimates, we find that inducing breaks results in rejection frequencies ranging from 10.5% to 66.6 % across different cases (P1-P7). The average across different cases for the sample size of 8000 is 0.455, which is substantially greater than 0.01. This means on average, a researcher using 8000 (1000) observations will spuriously find significant volatility or shock transmission 45.5% (65%) of the time, when none is present as the two series are independently generated. Since common volatility breaks are a frequent occurrence in empirical data, this

finding casts doubt on numerous existing studies which ignore breaks and report significant shock and volatility transmission.<sup>10</sup>

In order to solve this problem of spurious volatility transmission, we account for induced breaks via dummy variables and document the results in Panel C of Table 2. We see that in all cases, when breaks are accounted for, the rejection rates go back to the ‘no break’ level. For example, the average rejection rate across the different cases for the sample size of 8000 is 0.012, which is almost identical to the expected value of 0.01. Thus a practical solution for researchers’ is to endogenously detect volatility breaks and then account them in the estimation process through dummy variables. We use this methodology in our empirical exercise in section 5 as well.<sup>11</sup>

In practice, we may observe two series that experience a simultaneous break but the size of the break is different across the series. In order to study that case, we scale up unconditional variance by 4 in the first series and scale up by 2 in the second series. The results are presented in Table 3. Our previous result of spurious volatility transmission is confirmed with this alternative volatility break dynamic (see Panel A, B and C). However, in real life most economic variables are correlated, so we introduce correlation of 0.5 between the two simulated series. Panel D of Table 3 shows that introducing correlation among the innovations in the data generating process lead to a clear worsening of the rejection frequencies. It is also interesting to note that these rejection frequencies increase with the sample size, thus longer sample period

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<sup>10</sup> Recent studies have widely documented common structural breaks in variance among popular series. For example, breaks caused by the 2007-08 financial crisis are commonly reported across all major financial series (i.e. oil, gold, exchange rates, global stock market indices, etc.).

<sup>11</sup> Also sometimes in real life, markets do not experience breaks simultaneously but experience with a time lag (may be due to different time zones or other reasons). In order to explore that possibility, we computed Wald test ratios when second series experience a break after time lag of 5 periods. The results are presented in Table A3 of Appendix and show almost identical results to the case when the two series experience synchronous break.

yields more frequent detection of spurious volatility spillover (even when we exclude the intercept from the analysis).

In order to confirm that our results are not driven by the model choice, we re-estimated using the CCC model rather than BEKK model on the same simulated data. This is also important to evaluate since the estimation of a BEKK model on data generated from a CCC model is exposed to model specification error as the BEKK model is not nested in the CCC model. The results are even stronger than before and are presented in Table 4. Panel A reports the case of no break which shows no spurious spillover; as a matter of fact, the average rejection rate percentage for all cases is even smaller than the expected value of 0.01. Panel B shows that when a break of equal size is induced in both series, the average rejection rate percentage for all cases becomes very high (0.970 for a sample size of 8000). This shows that a researcher will find spurious volatility transmission 97% of the times using a sample of 8000, when this should be 1%. Similar to the case of correlated variables documented earlier, we find that the spurious spillover rejection frequencies increase with the sample size. Panel C shows the problem of spurious volatility transmission persists even when you have a different size of break across the two series. However, Panel D reports that once breaks are accounted for with dummy variables then there is no detection of spurious spillover effect. Thus, our results show that the detection of spurious spillover is much larger when breaks are present, but also depend on model misspecification. How much is the contribution of each source can be evaluated by comparing Panel A of Table 4 (no break case estimated using CCC model) with Panel A of Table 2 (no break case estimated using BEKK model). This comparison allows us to verify the role of model misspecification in spurious variance spillover detection. This comparison shows that a substantial portion of the total bias is due to breaks while model misspecification bias is small.

More importantly, this model misspecification bias diminishes with increasing sample size and is almost zero in large samples ( $n=8000$ ).

We also consider a case where we induce an upward shift in unconditional variance in only one (the first) variable. Table 5 reports the rejection frequencies of the Wald test for the null hypothesis of zero coefficients in the off-diagonal terms of the parameter matrices A, B and C (with and without intercept) at the 1% significance level. We find that the spurious volatility spillover effect is substantially less than before and the rejection frequencies appear to be much closer to the expected values. This indicates that the spurious spillover detection depends also on the diffusion of the break among the considered series.

Finally, we report the significance of the individual elements which comprise the ARCH and GARCH coefficients as shown in Equations 12-14 of the BEKK model, which dictate how shocks and volatility are transmitted across the two series over time. These estimates are documented in Tables A4-A6 given in the Appendix. Under the BEKK model the volatility transmission is associated with the coefficients  $c_{12}$ ,  $b_{12}$ ,  $b_{21}$ ,  $a_{12}$  and  $a_{21}$ , which should all be equal to 1% if there is no volatility transmission. Not surprisingly, in Table A4, we see that in the case of no break, the frequencies converge to the expected values (1%). However, Table A5 shows clear evidence of spurious spillover as the elements ( $c_{12}$ ,  $b_{12}$ ,  $b_{21}$ ,  $a_{12}$  and  $a_{21}$ ) are substantially larger than 1%, when breaks are present. Table A6 shows that adding a dummy variable for the induced variance shift solves the problem as the rejection frequencies converge back to 1%.<sup>12</sup>

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<sup>12</sup> Similar results (not reported) were obtained using the CCC model. Also, we have tables (not reported) showing that average estimated coefficients change when breaks are induced and then accounted for via dummies. Finally, throughout our simulation exercises, we have looked at the case of an increase in volatility. However, in real life, sometimes unconditional variance can suddenly decrease due to many reasons (for example, due to a stabilizing speech of a central bank governor). Out of curiosity, we also repeated some simulations where unconditional variance was decreased and we found similar results of spurious volatility transmission as documented earlier. Results are not reported but are available on request.

#### 4. Economic Implications: A Hedging Application

As mentioned earlier, correctly estimating volatility transmission mechanism across markets has serious economic consequences regarding optimal portfolio allocation (Kroner and Ng, 1998), risk management (Christoffersen, 2009) and dynamic hedging (Haigh and Holt, 2002). Clearly financial markets participants are interested in knowing if shocks and volatility are significantly transmitted across different assets or markets. A complete analysis of all economic implications is beyond the scope of this paper, so here we document only the impact that breaks have on estimated optimal hedge ratios. Haigh and Holt (2002) demonstrate that a bivariate GARCH model that accounts for volatility spillover between spot and futures markets result in a better hedging strategy compared with a bivariate GARCH model which ignores spillover effects. Lien and Yang (2010) show that daily currency risk can be better hedged with currency futures when controlling for unconditional variance breaks using a bivariate GARCH model. Thus, correctly estimating volatility spillover is important in hedging decisions.

Consider an investor who is holding an underlying asset but wants to hedge the price risk of the underlying asset by using its corresponding futures contract. At time  $t$ , the investor has to decide an optimal position in futures to minimize the risk of the combined positions of the underlying asset and futures at time  $t+1$ . The returns of the spot ( $r_{s,t}$ ) and futures ( $r_{f,t}$ ) are given at time  $t$ . The hedge ratio at time  $t$  is defined as the number of futures positions taken for each unit of spot position held at time  $t$ . The optimal (minimum variance) hedge ratio is given as  $Covariance(r_{s,t+1}, r_{f,t+1}) / Variance(r_{f,t+1})$ .<sup>13</sup>

Bivariate GARCH models are popularly used to estimate the above time-varying covariance and variance terms. Clearly mis-predicting covariance term between the two series

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<sup>13</sup> See Kroner and Sultan (1993) for a detailed proof on the optimal hedge ratio. Wang, Wu, and Yang (2015) provide a recent literature review on this topic.



will directly impact the optimal hedge ratio. However, two independently generated series will have a hedge ratio almost equal to zero as the covariance (numerator in the optimal hedge ratio formula) term is almost equal to zero. Thus, in this section, we focus on the case when the correlation between the two series is non-zero (0.5). Interestingly those series that have non-zero correlation are more likely to experience a common break, thus our hedging results are very relevant for financial market participants.<sup>14</sup>

Table 6 shows the median and two quantiles of the hedge ratios for three sample periods (full, before break and after break) computed from the BEKK model using two correlated variables experiencing breaks of different sizes. For the case of P7 using a sample size of 8000, we find that the full sample median hedge ratio is 0.558 but median hedge ratio before the break is 0.496 and the median hedge ratio after the break is 0.691. This change in estimated hedge ratio is quite substantial. As shown in the last row and last column the average increase of median hedge ratio for all cases, for the sample size of 8000, is 39.7%. This means, on average, breaks change hedge ratios by almost 40% for the sample size of 8000. This substantial over-estimation of hedge ratio is pretty consistent across different sample sizes.<sup>15</sup>

The two quantiles of estimated hedge ratios in Table 6 shows that there is a considerable increase in the variability of the hedge ratios across the simulations when we have a break. This is an interesting finding which implies that more variance in hedge ratios will result in substantial increase in portfolio rebalancing costs since traders will have to make more portfolio adjustments. It has been argued that when the variability of the estimated time-varying hedge

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<sup>14</sup> We also computed hedge ratios where correlation between the two series is zero. As expected, we found the median hedge ratios to be very close to zero, both before and after the break. However, the hedge ratios had more variability in the post-break case. Results are available on request.

<sup>15</sup> We also generated the similar table (not reported) using the CCC model which shows similar results. It should be noted that these results are based on in-sample forecasts, which has limited use in real life situations. A pure out-of-sample forecast would require estimating the model in-sample and projecting the variances one-step-ahead, with say, hundreds of estimations for each simulation, which is computationally prohibitively expensive and beyond the scope of the current paper.

ratios is large, the hedging performance from using that estimated hedge ratio is not as good relative to an unconditional (constant) hedge ratio (see Fan, Li and Park, 2016; Lien, 2010).

## 5. Empirical Results

In this section, we use empirical data in order to confirm our findings from simulations. Previous researchers have used weekly returns based on the CRSP index (top and bottom deciles) to highlight the estimation differences between different bivariate GARCH models (see Kroner and Ng, 1998; Ewing and Malik, 2005; among others). Following these studies, we use the S&P 500 Index to measure the performance of large (\$6.1 billion worth or more) capitalization (cap) US stocks. S&P 600 Index is used to measure the performance of small cap stocks, as this index is designed to measure the performance of 600 small size (\$1.8 billion worth or less) companies in the US. Our data set is better as it has daily frequency which gives more degrees of freedom and the data itself is publicly available. Also, a significant amount of assets are traded on these indices (exchange traded funds), which gives them market visibility. Specifically, we use daily returns from January 2, 1995 (S&P 600 index starts from this date) to June 15, 2018. This gives us 5905 observations. Descriptive statistics are provided in Table 7. It is important to note that the correlation between the two return series is 0.873. This high value of positive correlation is relevant for hedging decisions as we compute hedge ratios later in this section and the results are then compared with the simulation findings from the last section.

In order to detect breaks in the unconditional variance, we use the methodology of Sanso, Arrago and Carrionet (2004) which is based on the iterative algorithm given by Inclan and Tiao (1994).<sup>16</sup> We find 8 volatility breaks in each of the return series using this methodology and the

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<sup>16</sup> Interested readers are referred to Rapach and Strauss (2008) as they provide details of the methodology that we use.

break dates are shown in Table 8. Some breaks are very close to each other across series, while others are far apart. Even if the breaks are triggered by a common event across markets, we do expect different impact on volatility across markets as the rate of information flows are different across markets as suggested by Ross (1989).

Next, we estimate the BEKK model as given in Equation 11 and report the results in Table 9 (Panel A). We find that all four coefficient terms of the news and volatility spillover (for example, how  $h_{22}$  affects  $h_{11}$ ) are significant at 1%. Overall, the results show very significant shock and volatility transmission across the two series. The average estimated hedge ratio is 0.48 over the whole sample period.

Finally, we estimate the same BEKK model from above but account the breaks with dummy variables. Following Ewing and Malik (2005), the BEKK model with structural breaks is estimated as

$$H_{t+1} = C'C + B'H_tB + A'\varepsilon_t\varepsilon_t'A + \sum_{i=1}^n D_i'X_i'X_iD_i \quad (16)$$

This equation differs from the previous equation as it includes the last term.  $D_i$  is a  $2 \times 2$  square diagonal matrix of parameters and  $X_i$  is a  $1 \times 2$  row vector of break control variables, and  $n$  is the number of break points found in variance. Since we find 8 break points in each series, so  $n$  is equal to 8. First (second) element in  $X_i$  row vector represents the dummy for first (second) series. If the first series undergoes a volatility break at time  $t$ , then the first element will take a value of zero before time  $t$  and a value of one from time  $t$  onwards. The estimation results from the model which incorporates breaks are reported in Table 9 (Panel B).<sup>17</sup> We find that all four coefficient terms which govern if news and volatility

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<sup>17</sup> As Table 9 shows that the log likelihood increased after accounting for structural breaks indicating that the model with structural breaks give a better fit. The importance of structural breaks is further supported by the likelihood ratio statistic. The likelihood ratio statistic is calculated as  $2[L(\Theta_1) - L(\Theta_0)]$  where  $L(\Theta_1)$  and  $L(\Theta_0)$  are the maximum log likelihood values obtained from the GARCH models with and without structural breaks, respectively. This statistic is asymptotically distributed as  $\chi^2$  with degrees of freedom equal to the number of restrictions from the more general model (with breaks) to the more parsimonious model (without breaks). In our case, the likelihood ratio

transmits across markets are not significant at 5% level. Essentially the results show that there is no significant shock and volatility transmission between the two series once we account for breaks. The average estimated hedge ratio is 0.70 over the whole sample period, which has considerably changed from the previous value of 0.48. Thus empirical evidence reported here is in line with our overall conclusion drawn from our simulations that structural breaks cause spurious volatility transmission and estimated hedge ratios are substantially affected by these structural breaks.

## 6. Conclusion

There has been a proliferation in the literature documenting a significant transmission of shocks and volatility across assets or markets using bivariate GARCH models. Related literature has provided robust evidence to show that shifts in unconditional variance result in biased estimated volatility persistence within a univariate GARCH framework. In this paper, we combine the above two strands of literature by undertaking extensive Monte Carlo simulations showing that structural breaks in volatility cause spurious volatility spillover effects using popular bivariate GARCH models. Our overall conclusions are robust across a wide variety of specifications and scenarios. Moreover, this spurious volatility transmission effect is exacerbated when the variables are correlated or when the sample size is small. However, an important contribution from a practical standpoint, and one that is fairly simple to implement, is that using a dummy variable for the induced volatility shift virtually eliminates this bias.

Since volatility shifts are a frequent occurrence in all assets or markets and virtually all previous studies have ignored them, our findings cast doubt on the vast empirical studies

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statistic is equal to  $2(41529.92 - 41202.50) = 654.84$ . Thus we reject the null hypothesis of no change even at the 1% significance level.

documenting significant shock and volatility transmission across assets or markets. We further show through simulations that this spurious volatility spillover effect is not only statistically significant but has substantial economic implications in terms of hedging implications. We show that induced breaks in volatility substantially change the average estimated hedge ratios.

Finally, we confirm our major simulation findings by using publicly available daily data from small cap and large cap indices from the US stock market ranging from January 1995 to June 2018. We suggest that researchers endogenously detect volatility breaks and incorporate these breaks in the estimation model to get an accurate estimate of the volatility transmission dynamics across series over time. Failure to account for breaks will result in mis-predicting the degree of volatility transmission that actually exists between two assets or markets. Thus our finding has important implications for building accurate asset pricing models, forecasting volatility of market returns, dynamic hedging, and risk management.

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**Table 1: Simulation Design**

	First series				Second series			
	$\omega$	a	b	Unconditional Variance	$\omega$	a	b	Unconditional Variance
P1	0.08	0.10	0.80	0.80	0.08	0.10	0.65	0.32
P2	0.08	0.05	0.90	1.60	0.08	0.10	0.80	0.80
P3	0.08	0.02	0.97	8.00	0.08	0.10	0.80	0.80
P4	0.08	0.05	0.90	1.60	0.08	0.10	0.65	0.32
P5	0.08	0.02	0.97	8.00	0.08	0.05	0.90	1.60
P6	0.08	0.06	0.92	4.00	0.08	0.03	0.95	4.00
P7	0.08	0.05	0.90	1.60	0.08	0.05	0.90	1.60

**Notes:** We simulate returns using the bivariate Gaussian CCC model of Bollerslev (1990) which is given as  $\sigma_{1,t}^2 = \omega_1 + a_1 \varepsilon_{1,t-1}^2 + b_1 \sigma_{1,t-1}^2$ ,  $\sigma_{2,t}^2 = \omega_2 + a_2 \varepsilon_{2,t-1}^2 + b_2 \sigma_{2,t-1}^2$  and  $\sigma_{12,t} = \rho \sigma_{1,t} \sigma_{2,t}$ . P1-P7 refers to different cases of parametrization. The conditional correlation ( $\rho$ ) is constant, while the conditional covariance ( $\sigma_{12,t}$ ) dynamics depends on the conditional variance ( $\sigma^2$ ) in a non-linear fashion. We consider both the case of zero correlation and a correlation of 0.5. The break corresponds to a change in  $\omega$ , which is multiplied by 4.

**Table 2: Effect of a simultaneous same size break in both series on estimated volatility transmission using a BEKK model**

T	Test	Panel A				Panel B				Panel C			
		No break				Both series with break (intercept x 4)				Both series with break (x4) + Dummy			
		1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000
P1	With Intercept	0.371	0.163	0.056	0.010	0.739	0.672	0.604	0.651	0.333	0.165	0.047	0.015
	No Intercept	0.321	0.147	0.050	0.013	0.748	0.680	0.632	0.666	0.296	0.131	0.045	0.013
P2	With Intercept	0.384	0.181	0.041	0.009	0.633	0.503	0.459	0.390	0.268	0.105	0.025	0.004
	No Intercept	0.274	0.138	0.036	0.018	0.601	0.474	0.472	0.428	0.221	0.095	0.025	0.010
P3	With Intercept	0.598	0.396	0.183	0.075	0.739	0.672	0.588	0.618	0.353	0.167	0.060	0.021
	No Intercept	0.274	0.244	0.133	0.049	0.615	0.589	0.561	0.645	0.235	0.147	0.045	0.023
P4	With Intercept	0.537	0.319	0.088	0.024	0.758	0.616	0.560	0.503	0.430	0.215	0.071	0.025
	No Intercept	0.361	0.243	0.067	0.015	0.732	0.621	0.578	0.537	0.327	0.177	0.052	0.015
P5	With Intercept	0.573	0.337	0.148	0.044	0.710	0.559	0.456	0.378	0.406	0.172	0.052	0.008
	No Intercept	0.372	0.220	0.132	0.027	0.643	0.494	0.438	0.429	0.327	0.146	0.050	0.014
P6	With Intercept	0.347	0.161	0.050	0.010	0.437	0.263	0.128	0.105	0.282	0.083	0.018	0.001
	No Intercept	0.324	0.172	0.063	0.014	0.452	0.292	0.177	0.157	0.245	0.092	0.022	0.004
P7	With Intercept	0.440	0.202	0.036	0.006	0.651	0.570	0.491	0.400	0.335	0.128	0.028	0.002
	No Intercept	0.375	0.170	0.037	0.009	0.644	0.582	0.515	0.466	0.303	0.115	0.040	0.006
Average		0.397	0.221	0.080	0.023	0.650	0.542	0.476	0.455	0.312	0.138	0.041	0.012

**Notes:** This table reports the rejection frequencies of the Wald test for verifying the null hypothesis of zero coefficients in the off-diagonal terms of the parameter matrices A, B and C (with and without intercept) of the BEKK model  $H_{t+1} = C'C + B'H_tB + A'\varepsilon_t\varepsilon_t'A$  at the 1% significance level. P1-P7 refers to different cases of parametrization as given in Table 1.

**Table 3: Effect of a simultaneous different size break in both series on estimated volatility transmission using a BEKK model**

T	Test	Panel A				Panel B				Panel C				Panel D			
		No break				Both series with break (1x4 1x2)				Both series with break (1x4 1x2) + dummy				Both series with break (1x4 1x2) + correlation			
		1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000
P1	With Intercept	0.371	0.163	0.056	0.010	0.602	0.591	0.565	0.699	0.424	0.204	0.097	0.014	0.821	0.898	0.963	0.990
	No Intercept	0.321	0.147	0.050	0.013	0.499	0.523	0.565	0.690	0.348	0.158	0.059	0.010	0.542	0.662	0.879	0.969
P2	With Intercept	0.384	0.181	0.041	0.009	0.587	0.514	0.361	0.286	0.320	0.129	0.022	0.008	0.740	0.841	0.941	0.995
	No Intercept	0.274	0.138	0.036	0.018	0.409	0.291	0.250	0.246	0.240	0.108	0.026	0.012	0.345	0.424	0.683	0.979
P3	With Intercept	0.598	0.396	0.183	0.075	0.737	0.716	0.642	0.548	0.358	0.198	0.089	0.039	0.866	0.967	0.997	1.000
	No Intercept	0.274	0.244	0.133	0.049	0.379	0.332	0.368	0.396	0.219	0.137	0.057	0.027	0.346	0.569	0.921	1.000
P4	With Intercept	0.537	0.319	0.088	0.024	0.683	0.550	0.499	0.511	0.488	0.264	0.088	0.034	0.848	0.878	0.950	0.991
	No Intercept	0.361	0.243	0.067	0.015	0.479	0.405	0.423	0.500	0.357	0.204	0.055	0.013	0.531	0.542	0.747	0.944
P5	With Intercept	0.573	0.337	0.148	0.044	0.680	0.609	0.576	0.502	0.436	0.207	0.065	0.022	0.699	0.749	0.834	0.950
	No Intercept	0.372	0.220	0.132	0.027	0.491	0.401	0.414	0.432	0.325	0.163	0.049	0.015	0.391	0.388	0.569	0.874
P6	With Intercept	0.347	0.161	0.050	0.010	0.542	0.411	0.347	0.403	0.367	0.174	0.054	0.006	0.721	0.763	0.898	0.996
	No Intercept	0.324	0.172	0.063	0.014	0.503	0.391	0.399	0.483	0.296	0.156	0.052	0.010	0.450	0.432	0.483	0.782
P7	With Intercept	0.440	0.202	0.036	0.006	0.613	0.550	0.498	0.536	0.377	0.183	0.030	0.009	0.721	0.762	0.909	0.982
	No Intercept	0.375	0.170	0.037	0.009	0.549	0.475	0.466	0.551	0.319	0.165	0.032	0.013	0.496	0.466	0.603	0.782
	Average	0.397	0.221	0.080	0.023	0.554	0.483	0.455	0.485	0.348	0.175	0.055	0.017	0.608	0.667	0.813	0.945

**Notes:** This table reports the rejection frequencies of the Wald test for verifying the null hypothesis of zero coefficients in the off-diagonal terms of the parameter matrices A, B and C (with and without intercept) of the BEKK model  $H_{t+1} = C'C + B'H_tB + A'\varepsilon_t\varepsilon_t'A$  at the 1% significance level. P1-P7 refers to different cases of parametrization as given in Table 1.

**Table 4: Effect of a simultaneous same and different size break in both series on estimated volatility transmission using a CCC model**

		Panel A				Panel B				Panel C				Panel D			
		No break				Both series with break (intercept x 4)				Both series with break (1x4 1x2)				Both series with break (1x4 1x2) + dummy			
T	Test	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000
P1	With Correlation	0.000	0.000	0.000	0.000	0.064	0.269	0.803	0.999	0.025	0.126	0.528	0.978	0.001	0.001	0.000	0.000
	No Correlation	0.001	0.000	0.000	0.000	0.133	0.445	0.911	0.999	0.048	0.221	0.719	0.992	0.002	0.000	0.000	0.000
P2	With Correlation	0.005	0.001	0.000	0.000	0.038	0.183	0.670	0.996	0.008	0.031	0.483	0.991	0.000	0.000	0.000	0.000
	No Correlation	0.005	0.001	0.000	0.000	0.068	0.330	0.858	1.000	0.012	0.085	0.725	0.998	0.000	0.000	0.000	0.000
P3	With Correlation	0.006	0.001	0.000	0.000	0.080	0.200	0.650	0.988	0.004	0.032	0.402	0.988	0.001	0.000	0.000	0.001
	No Correlation	0.004	0.002	0.000	0.000	0.132	0.336	0.823	0.997	0.013	0.078	0.648	0.997	0.001	0.001	0.001	0.001
P4	With Correlation	0.002	0.000	0.001	0.001	0.111	0.235	0.559	0.944	0.021	0.085	0.387	0.938	0.000	0.000	0.001	0.000
	No Correlation	0.003	0.000	0.000	0.000	0.168	0.357	0.712	0.974	0.046	0.173	0.562	0.981	0.001	0.001	0.000	0.000
P5	With Correlation	0.004	0.000	0.001	0.000	0.053	0.065	0.251	0.809	0.010	0.009	0.058	0.613	0.002	0.001	0.000	0.000
	No Correlation	0.010	0.001	0.000	0.000	0.078	0.129	0.480	0.943	0.020	0.013	0.142	0.820	0.003	0.001	0.000	0.000
P6	With Correlation	0.006	0.001	0.000	0.000	0.015	0.020	0.206	0.964	0.020	0.003	0.032	0.401	0.002	0.000	0.001	0.001
	No Correlation	0.006	0.001	0.001	0.000	0.023	0.035	0.463	0.997	0.026	0.005	0.069	0.653	0.002	0.000	0.000	0.000
P7	With Correlation	0.018	0.007	0.000	0.000	0.037	0.058	0.419	0.979	0.012	0.012	0.120	0.858	0.000	0.001	0.000	0.000
	No Correlation	0.020	0.007	0.000	0.000	0.059	0.147	0.674	0.995	0.014	0.020	0.313	0.958	0.001	0.001	0.000	0.000
	Average	0.006	0.002	0.000	0.000	0.076	0.201	0.606	0.970	0.020	0.064	0.371	0.869	0.001	0.001	0.000	0.000

**Notes:** This table reports the rejection frequencies of the Wald test for verifying the null hypothesis of zero coefficients in the off-diagonal terms of the parameter matrices A, B and C of the BEKK model  $H_{t+1} = C'C + B'H_tB + A'\varepsilon_t\varepsilon_t'A$  at the 1% significance level. P1-P7 refers to different cases of parametrization as given in Table 1.

**Table 5: Effect of a break in only one series on estimated volatility transmission using a BEKK model**

T	Test	Second series with break (x4)			
		1000	2000	4000	8000
P1	With Intercept	0.062	0.026	0.024	0.019
	No Intercept	0.053	0.041	0.040	0.054
P2	With Intercept	0.274	0.279	0.221	0.104
	No Intercept	0.144	0.116	0.125	0.086
P3	With Intercept	0.387	0.401	0.241	0.085
	No Intercept	0.180	0.177	0.126	0.066
P4	With Intercept	0.153	0.091	0.058	0.022
	No Intercept	0.101	0.050	0.044	0.054
P5	With Intercept	0.222	0.220	0.151	0.089
	No Intercept	0.197	0.152	0.088	0.062
P6	With Intercept	0.067	0.046	0.033	0.021
	No Intercept	0.080	0.062	0.043	0.030
P7	With Intercept	0.107	0.093	0.102	0.063
	No Intercept	0.100	0.065	0.088	0.068
Average		0.152	0.130	0.099	0.059

**Notes:** This table reports the rejection frequencies of the Wald test for verifying the null hypothesis of zero coefficients in the off-diagonal terms of the parameter matrices A, B and C (with and without intercept) of the BEKK model  $H_{t+1} = C'C + B'H_tB + A'\varepsilon_t\varepsilon_t'A$  at the 1% significance level. P1-P7 refers to different cases of parametrization as given in Table 1.

**Table 6: Impact of break on estimated hedge ratios using a BEKK model**

Model	Quantile	1000				2000				4000				8000			
		Full	Before	After	Ratio	Full	Before	After	Ratio	Full	Before	After	Ratio	Full	Before	After	Ratio
P1	0.025	0.479	0.464	0.504	1.088	0.519	0.500	0.562	1.124	0.567	0.536	0.616	1.150	0.598	0.565	0.655	1.160
	0.500	0.866	0.770	1.077	1.398	0.866	0.767	1.065	1.389	0.878	0.775	1.053	1.358	0.866	0.779	1.041	1.337
	0.975	1.309	1.222	1.903	1.558	1.299	1.205	1.869	1.551	1.307	1.204	1.825	1.516	1.297	1.200	1.789	1.491
P2	0.025	0.404	0.383	0.452	1.179	0.439	0.416	0.498	1.198	0.464	0.438	0.550	1.258	0.492	0.458	0.593	1.295
	0.500	0.785	0.713	1.006	1.412	0.786	0.705	1.015	1.439	0.784	0.702	1.018	1.450	0.793	0.698	1.024	1.466
	0.975	1.150	1.110	1.690	1.522	1.130	1.064	1.647	1.548	1.094	1.027	1.616	1.573	1.084	0.996	1.579	1.586
P3	0.025	0.960	0.926	1.167	1.260	1.026	0.987	1.280	1.296	1.073	1.026	1.358	1.324	1.117	1.061	1.406	1.324
	0.500	1.641	1.588	2.231	1.405	1.652	1.580	2.293	1.452	1.665	1.574	2.311	1.468	1.678	1.573	2.306	1.467
	0.975	2.480	2.373	3.575	1.506	2.393	2.288	3.522	1.539	2.329	2.229	3.511	1.575	2.320	2.180	3.451	1.583
P4	0.025	0.694	0.671	0.763	1.138	0.748	0.716	0.866	1.210	0.792	0.753	0.934	1.241	0.829	0.788	0.985	1.249
	0.500	1.211	1.106	1.567	1.417	1.209	1.095	1.582	1.445	1.218	1.093	1.574	1.440	1.206	1.096	1.557	1.421
	0.975	1.748	1.666	2.580	1.548	1.679	1.600	2.529	1.581	1.650	1.570	2.453	1.563	1.632	1.552	2.375	1.531
P5	0.025	0.690	0.674	0.817	1.213	0.721	0.695	0.890	1.282	0.754	0.725	0.947	1.306	0.785	0.753	0.985	1.307
	0.500	1.150	1.124	1.517	1.350	1.146	1.116	1.557	1.396	1.147	1.112	1.579	1.420	1.154	1.110	1.579	1.423
	0.975	1.741	1.670	2.370	1.419	1.684	1.624	2.354	1.450	1.642	1.583	2.347	1.482	1.623	1.552	2.317	1.492
P6	0.025	0.312	0.277	0.349	1.261	0.317	0.282	0.357	1.267	0.329	0.288	0.364	1.263	0.331	0.291	0.369	1.267
	0.500	0.531	0.482	0.663	1.377	0.525	0.475	0.666	1.403	0.526	0.477	0.661	1.386	0.521	0.476	0.662	1.390
	0.975	0.857	0.805	1.194	1.482	0.845	0.799	1.191	1.490	0.860	0.808	1.185	1.467	0.864	0.806	1.176	1.458
P7	0.025	0.311	0.295	0.346	1.173	0.320	0.303	0.367	1.212	0.329	0.307	0.376	1.222	0.346	0.315	0.390	1.236
	0.500	0.564	0.504	0.688	1.366	0.557	0.500	0.686	1.372	0.559	0.496	0.689	1.391	0.558	0.496	0.691	1.395
	0.975	0.808	0.775	1.124	1.450	0.791	0.755	1.095	1.451	0.780	0.742	1.086	1.463	0.773	0.731	1.073	1.467
Average		1.358				1.385				1.396				1.397			

**Notes:** The hedge ratio at time  $t$  is defined as the number of futures positions taken for each unit of spot position held at time  $t$ . The optimal (minimum variance) hedge ratio is given as  $\text{Covariance}(r_{s, t+1}, r_{f, t+1}) / \text{Variance}(r_{f, t+1})$ . BEKK model is given as  $H_{t+1} = C'C + B'H_tB + A'\varepsilon_t\varepsilon_t'A$ . P1-P7 refers to different cases of parametrization as given in Table 1. Ratio = After / Before.

**Table 7: Descriptive statistics**

	S&P 500 (large cap)	S&P 600 (small cap)
Mean	0.0003	0.0004
Median	0.0006	0.0010
Maximum	0.1095	0.0811
Minimum	-0.0947	-0.1163
Std. Dev.	0.0116	0.0133
Skewness	-0.2706	-0.3163
Kurtosis	11.445	8.328
Jarque-Bera	17620.20 (0.00)	7083.207 (0.00)
Q(16)	98.95 (0.00)	54.43 (0.00)

**Notes:** The sample is comprised of daily index returns from January 2, 1995 to June 15, 2018. The number of observations is 5905. Q(16) is the Ljung-Box statistic for serial correlation. Jarque-Bera statistic is used to test whether or not the series resembles normal distribution. Actual probability values in parentheses. The correlation between the returns of two series is 0.873.

**Table 8: Structural breaks in volatility**

Series	Break Points	Time Period	Standard Deviation
S&P 500 (large cap)	8	Jan 2, 1995- Dec 14, 1995	0.0048
		Dec 15, 1995- July 28, 1998	0.0094
		July 29, 1998-June 13, 2002	0.0133
		June 14, 2002-Oct 16, 2002	0.0224
		Oct 17, 2002-April 1, 2003	0.0143
		April 2, 2003-Sept 30, 2003	0.0097
		Oct 1, 2003-July 27, 2006	0.0069
		July 28, 2006- Dec 19, 2011	0.0162
		Dec 20, 2011-June 15, 2018	0.0079
S&P 600 (small cap)	8	Jan 2, 1995-Oct 14, 1997	0.0065
		Oct 15, 1997-July 29, 1998	0.0105
		July 30, 1998-Dec 17, 2002	0.0145
		Dec 18, 2002-July 18, 2007	0.0101
		July 19, 2007-June 6, 2010	0.0225
		June 7, 2010-Sept 23, 2010	0.0178
		Sept 24, 2010-July 31, 2011	0.0112
		Aug 1, 2011-Nov 29, 2011	0.0292
		Nov 30, 2011-June 15, 2018	0.0100

**Notes:** Time periods detected by modified ICSS algorithm. Sample period is from January 2, 1995 to June 15, 2018.



**Table 9: Empirical estimates using BEKK model with and without structural breaks**

**Panel A: Model ignoring Structural Breaks**

(Log likelihood: 41202.50)

S&P 500 (large cap) conditional variance equation:

$$h_{1,t+1} = 1.71 \times 10^{-7} + 2.513h_{1,t} - 4.908h_{12,t} + 2.396h_{22,t} + 0.109\varepsilon_{1,t}^2 - 0.236\varepsilon_{1,t}\varepsilon_{2,t} + 0.127\varepsilon_{2,t}^2$$

(2256.32)    (73.38)    (-59.31)    (46.65)    (16.61)    (-18.47)    (13.72)

S&P 600 (small cap) conditional variance equation:

$$h_{22,t+1} = 1.98 \times 10^{-6} + 1.006h_{1,t} - 3.139h_{12,t} + 2.447h_{22,t} + 0.038\varepsilon_{1,t}^2 - 0.151\varepsilon_{1,t}\varepsilon_{2,t} + 0.149\varepsilon_{2,t}^2$$

(8015.35)    (35.06)    (-45.21)    (60.77)    (676.60)    (-20.70)    (10.41)

**Panel B: Model incorporating Structural Breaks**

(Log likelihood: 41529.92)

S&P 500 (large cap) conditional variance equation:

$$h_{1,t+1} = 6.98 \times 10^{-7} + 0.940h_{1,t} - 0.051h_{12,t} + 6.957 \times 10^{-4}h_{22,t} + 0.067\varepsilon_{1,t}^2 + 0.025\varepsilon_{1,t}\varepsilon_{2,t} + 0.002\varepsilon_{2,t}^2$$

(2.90)    (52.95)    (-2.93)    (1.50)    (4.32)    (2.38)    (0.97)

S&P 600 (small cap) conditional variance equation:

$$h_{22,t+1} = 1.560 \times 10^{-6} + 5.30 \times 10^{-5}h_{1,t} + 0.013h_{12,t} + 0.869h_{22,t} + 2.13 \times 10^{-4}\varepsilon_{1,t}^2 + 0.008\varepsilon_{1,t}\varepsilon_{2,t} + 0.088\varepsilon_{2,t}^2$$

(5.39)    (0.37)    (0.75)    (41.00)    (0.23)    (0.48)    (4.90)

**Notes:**  $h_{11}$  is the conditional variance for the S&P 500 (large cap) return series and  $h_{22}$  is the conditional variance for the S&P 600 (small cap) return series. Conditional covariance series ( $h_{12}$ ) in each case is not reported since it does not capture the direct volatility transmission across the two series. Directly below the estimated coefficients (in parentheses) are the corresponding t-values. The mean equations included a constant term and a lagged return term (not reported for the sake of brevity). The coefficients for dummy variables for the BEKK model with breaks are not reported to conserve space as well. The estimated models satisfy the covariance stationarity condition put forward in Engle and Kroner (1995). Robust standard errors were calculated.

## Appendix

**Table A1: Effect of the number of simulations (1000 or 5000) on our results**

		(replica of Panel B of Table 3)							
		Both series with break (1x4 1x2)				Both series with break (1x4 1x2)			
		1000 simulations				5000 simulations			
T	Test	1000	2000	4000	8000	1000	2000	4000	8000
P1	With Intercept	0.602	0.591	0.565	0.699	0.619	0.538	0.561	0.659
	No Intercept	0.499	0.523	0.565	0.690	0.506	0.476	0.544	0.668
P2	With Intercept	0.587	0.514	0.361	0.286	0.575	0.472	0.376	0.289
	No Intercept	0.409	0.291	0.250	0.246	0.368	0.280	0.262	0.247
P3	With Intercept	0.737	0.716	0.642	0.548	0.711	0.700	0.650	0.524
	No Intercept	0.379	0.332	0.368	0.396	0.371	0.343	0.362	0.383
P4	With Intercept	0.683	0.550	0.499	0.511	0.657	0.553	0.491	0.515
	No Intercept	0.479	0.405	0.423	0.500	0.477	0.434	0.424	0.512
P5	With Intercept	0.680	0.609	0.576	0.502	0.670	0.622	0.570	0.508
	No Intercept	0.491	0.401	0.414	0.432	0.482	0.397	0.409	0.465
P6	With Intercept	0.542	0.411	0.347	0.403	0.512	0.392	0.360	0.403
	No Intercept	0.503	0.391	0.399	0.483	0.490	0.396	0.404	0.489
P7	With Intercept	0.613	0.550	0.498	0.536	0.612	0.524	0.506	0.532
	No Intercept	0.549	0.475	0.466	0.551	0.521	0.456	0.484	0.555
Average		0.554	0.483	0.455	0.485	0.541	0.470	0.457	0.482

**Notes:** This table shows that number of simulations (1000 or 5000) does not influence our overall results. This table reports the rejection frequencies of the Wald test for verifying the null hypothesis of zero coefficients in the off-diagonal terms of the parameter matrices A, B and C (with and without intercept) of the BEKK model  $H_{t+1} = C'C + B'H_tB + A'\varepsilon_t\varepsilon_t'A$  at the 1% significance level. P1-P7 refers to different cases of parametrization as given in Table 1.

## Appendix

**Table A2: Effect of the size of the break (4 or 2) on our results**

(replica of Panel B of Table 2)

T	Test	Both series with break (intercept x 4)				Both series with break (intercept x 2)			
		1000	2000	4000	8000	1000	2000	4000	8000
P1	With Intercept	0.739	0.672	0.604	0.651	0.641	0.543	0.436	0.407
	No Intercept	0.748	0.680	0.632	0.666	0.607	0.520	0.464	0.455
P2	With Intercept	0.633	0.503	0.459	0.390	0.638	0.545	0.397	0.386
	No Intercept	0.601	0.474	0.472	0.428	0.516	0.430	0.327	0.364
P3	With Intercept	0.739	0.672	0.588	0.618	0.736	0.661	0.569	0.528
	No Intercept	0.615	0.589	0.561	0.645	0.509	0.451	0.468	0.507
P4	With Intercept	0.758	0.616	0.560	0.503	0.732	0.644	0.547	0.485
	No Intercept	0.732	0.621	0.578	0.537	0.621	0.566	0.511	0.480
P5	With Intercept	0.710	0.559	0.456	0.378	0.750	0.594	0.428	0.376
	No Intercept	0.643	0.494	0.438	0.429	0.635	0.510	0.419	0.392
P6	With Intercept	0.437	0.263	0.128	0.105	0.419	0.256	0.147	0.108
	No Intercept	0.452	0.292	0.177	0.157	0.421	0.280	0.193	0.154
P7	With Intercept	0.651	0.570	0.491	0.400	0.675	0.562	0.455	0.424
	No Intercept	0.644	0.582	0.515	0.466	0.644	0.550	0.478	0.455
Average		0.650	0.542	0.476	0.455	0.610	0.508	0.417	0.394

**Notes:** This table shows that size of the break does not much influence our overall results. Specifically, This table reports the rejection frequencies of the Wald test for verifying the null hypothesis of zero coefficients in the off-diagonal terms of the parameter matrices A, B and C (with and without intercept) of the BEKK model  $H_{t+1} = C'C + B'H_tB + A'\varepsilon_t\varepsilon_t'A$  at the 1% significance level. P1-P7 refers to different cases of parametrization as given in Table 1.

## Appendix

**Table A3: Effect of different timing of the break across series on our results**

T	Test	(replica of Panel B of Table 2)				Both series with break (intercept x 4)			
		Both series with break (intercept x 4)				Break after a 5 period time lag			
		1000	2000	4000	8000	1000	2000	4000	8000
P1	With Intercept	0.739	0.672	0.604	0.651	0.726	0.694	0.623	0.626
	No Intercept	0.748	0.680	0.632	0.666	0.734	0.711	0.659	0.667
P2	With Intercept	0.633	0.503	0.459	0.390	0.638	0.521	0.429	0.379
	No Intercept	0.601	0.474	0.472	0.428	0.603	0.500	0.429	0.424
P3	With Intercept	0.739	0.672	0.588	0.618	0.736	0.649	0.592	0.584
	No Intercept	0.615	0.589	0.561	0.645	0.628	0.557	0.565	0.618
P4	With Intercept	0.758	0.616	0.560	0.503	0.768	0.654	0.517	0.475
	No Intercept	0.732	0.621	0.578	0.537	0.754	0.653	0.538	0.501
P5	With Intercept	0.710	0.559	0.456	0.378	0.703	0.581	0.433	0.336
	No Intercept	0.643	0.494	0.438	0.429	0.644	0.524	0.417	0.379
P6	With Intercept	0.437	0.263	0.128	0.105	0.414	0.249	0.119	0.084
	No Intercept	0.452	0.292	0.177	0.157	0.430	0.275	0.170	0.131
P7	With Intercept	0.651	0.570	0.491	0.400	0.621	0.569	0.454	0.422
	No Intercept	0.644	0.582	0.515	0.466	0.634	0.574	0.493	0.479
Average		0.650	0.542	0.476	0.455	0.645	0.551	0.460	0.436

**Notes:** This table shows our results are not affected by the fact that the changes in the unconditional variance (intercept) are asynchronous across series. Specifically, this table reports the rejection frequencies of the Wald test for verifying the null hypothesis of zero coefficients in the off-diagonal terms of the parameter matrices A, B and C (with and without intercept) of the BEKK model  $H_{t+1} = C'C + B'H_tB + A'\varepsilon_t\varepsilon_t'A$  at the 1% significance level. P1-P7 refers to different cases of parametrization as given in Table 1.

## Appendix

**Table A4: Estimated parameter estimates of volatility transmission using a BEKK model in case of no break**

T	P1				P2				P3				P4				P5				P6				P7			
	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000
C11	0.36	0.61	0.87	0.98	0.30	0.54	0.86	0.99	0.11	0.23	0.49	0.82	0.19	0.33	0.71	0.94	0.12	0.24	0.50	0.86	0.52	0.88	0.99	1.00	0.36	0.60	0.86	0.98
C12	0.25	0.10	0.03	0.01	0.24	0.13	0.02	0.01	0.46	0.33	0.13	0.03	0.37	0.24	0.06	0.01	0.33	0.25	0.11	0.02	0.15	0.10	0.03	0.00	0.17	0.10	0.02	0.01
C22	0.31	0.59	0.82	0.96	0.29	0.65	0.93	1.00	0.11	0.35	0.66	0.92	0.20	0.47	0.85	0.99	0.08	0.33	0.70	0.96	0.10	0.33	0.71	0.96	0.14	0.41	0.81	0.98
A11	0.86	0.97	1.00	1.00	0.58	0.81	0.96	1.00	0.44	0.78	0.96	1.00	0.66	0.88	0.98	1.00	0.39	0.74	0.95	1.00	0.89	0.98	1.00	1.00	0.63	0.85	0.96	0.99
A12	0.04	0.03	0.01	0.01	0.05	0.03	0.01	0.02	0.02	0.03	0.02	0.02	0.05	0.04	0.01	0.01	0.06	0.05	0.03	0.02	0.06	0.03	0.01	0.01	0.07	0.04	0.02	0.01
A21	0.05	0.03	0.01	0.01	0.05	0.02	0.01	0.01	0.04	0.03	0.01	0.01	0.06	0.04	0.02	0.01	0.07	0.04	0.02	0.01	0.06	0.04	0.03	0.02	0.07	0.03	0.03	0.01
A22	0.61	0.78	0.90	0.99	0.89	0.98	1.00	1.00	0.93	0.99	1.00	1.00	0.77	0.92	0.99	1.00	0.77	0.94	0.99	1.00	0.46	0.70	0.92	0.99	0.64	0.83	0.97	1.00
B11	0.97	1.00	1.00	1.00	0.96	0.98	1.00	1.00	0.96	0.99	1.00	1.00	0.97	0.99	1.00	1.00	0.97	0.99	1.00	1.00	0.99	1.00	1.00	1.00	0.95	0.99	1.00	1.00
B12	0.17	0.10	0.04	0.02	0.11	0.06	0.04	0.03	0.08	0.09	0.07	0.04	0.12	0.11	0.04	0.02	0.14	0.12	0.09	0.04	0.17	0.11	0.03	0.00	0.18	0.09	0.03	0.01
B21	0.08	0.04	0.01	0.01	0.12	0.07	0.02	0.01	0.09	0.08	0.04	0.02	0.10	0.06	0.02	0.01	0.13	0.07	0.03	0.01	0.16	0.10	0.06	0.03	0.15	0.09	0.03	0.01
B22	0.69	0.75	0.80	0.90	0.95	0.99	1.00	1.00	0.98	1.00	1.00	1.00	0.77	0.85	0.93	0.99	0.97	0.99	1.00	1.00	0.96	0.98	1.00	1.00	0.96	0.99	1.00	1.00

**Notes:** This table reports the frequency of the 1% statistically significant individual parameters of the BEKK model given as  $h_{1,t+1} = c_{11}^2 + a_{11}^2 \varepsilon_{1,t}^2 + 2a_{11}a_{12}\varepsilon_{1,t}\varepsilon_{2,t} + a_{21}^2 \varepsilon_{2,t}^2 + b_{11}^2 h_{1,t} + 2b_{11}b_{12}h_{12,t} + b_{21}^2 h_{22,t}$  and  $h_{22,t+1} = c_{12}^2 + c_{22}^2 + a_{12}^2 \varepsilon_{1,t}^2 + 2a_{12}a_{22}\varepsilon_{1,t}\varepsilon_{2,t} + a_{22}^2 \varepsilon_{2,t}^2 + b_{12}^2 h_{11,t} + 2b_{12}b_{22}h_{12,t} + b_{22}^2 h_{22,t}$ . P1-P7 refers to different cases of parametrization as given in Table 1. We see that frequency of 1% statistically significant coefficient go to 0.01 for increasing size of the sample in the case of off-diagonal coefficients (12 and 21).

## Appendix

**Table A5: Estimated parameter estimates of volatility transmission using a BEKK model in break in both series**

T	P1				P2				P3				P4				P5				P6				P7			
	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000
C11	0.28	0.42	0.62	0.69	0.18	0.20	0.21	0.25	0.09	0.07	0.07	0.07	0.15	0.26	0.41	0.56	0.10	0.11	0.11	0.18	0.34	0.68	0.94	1.00	0.21	0.30	0.43	0.53
C12	0.15	0.17	0.19	0.24	0.24	0.20	0.17	0.12	0.36	0.38	0.32	0.26	0.18	0.17	0.18	0.17	0.20	0.22	0.23	0.13	0.10	0.06	0.01	0.00	0.14	0.17	0.13	0.09
C22	0.02	0.01	0.04	0.05	0.05	0.14	0.27	0.46	0.03	0.10	0.23	0.39	0.01	0.01	0.03	0.04	0.01	0.07	0.18	0.41	0.02	0.09	0.25	0.57	0.02	0.07	0.15	0.26
A11	0.77	0.91	0.98	0.99	0.75	0.93	0.97	0.99	0.50	0.81	0.97	1.00	0.62	0.84	0.93	0.96	0.48	0.80	0.96	0.99	0.91	0.98	0.99	1.00	0.76	0.94	0.98	0.96
A12	0.20	0.22	0.22	0.32	0.13	0.14	0.14	0.10	0.10	0.10	0.11	0.13	0.16	0.14	0.13	0.19	0.21	0.22	0.19	0.22	0.24	0.21	0.11	0.04	0.24	0.31	0.32	0.30
A21	0.28	0.28	0.30	0.40	0.24	0.22	0.21	0.12	0.20	0.19	0.14	0.13	0.26	0.24	0.27	0.27	0.26	0.24	0.21	0.18	0.20	0.16	0.14	0.16	0.26	0.29	0.32	0.29
A22	0.78	0.92	0.98	0.99	0.87	0.93	0.96	0.98	0.92	0.96	0.99	1.00	0.75	0.85	0.93	0.96	0.85	0.95	0.97	0.99	0.79	0.98	1.00	1.00	0.79	0.93	0.97	0.95
B11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
B12	0.58	0.55	0.49	0.50	0.44	0.40	0.45	0.47	0.45	0.50	0.57	0.69	0.51	0.49	0.49	0.43	0.47	0.45	0.42	0.44	0.29	0.20	0.07	0.01	0.49	0.49	0.44	0.38
B21	0.58	0.58	0.59	0.64	0.40	0.30	0.22	0.11	0.22	0.17	0.16	0.17	0.51	0.47	0.46	0.46	0.31	0.28	0.22	0.16	0.35	0.26	0.22	0.24	0.48	0.49	0.44	0.38
B22	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

**Notes:** This table reports the frequency of the 1% statistically significant individual parameters of the BEKK model given as  $h_{11,t+1} = c_{11}^2 + a_{11}^2 \varepsilon_{1,t}^2 + 2a_{11}a_{12} \varepsilon_{1,t} \varepsilon_{2,t} + a_{21}^2 \varepsilon_{2,t}^2 + b_{11}^2 h_{11,t} + 2b_{11}b_{12} h_{12,t} + b_{21}^2 h_{22,t}$  and  $h_{22,t+1} = c_{12}^2 + c_{22}^2 + a_{12}^2 \varepsilon_{1,t}^2 + 2a_{12}a_{22} \varepsilon_{1,t} \varepsilon_{2,t} + a_{22}^2 \varepsilon_{2,t}^2 + b_{12}^2 h_{11,t} + 2b_{12}b_{22} h_{12,t} + b_{22}^2 h_{22,t}$ . P1-P7 refers to different cases of parametrization as given in Table 1. Break size was shift in unconditional variance of 4. We see clear evidence of spurious spillover in intercept, ARCH and GARCH coefficients (stronger effect on GARCH coefficients in all cases).

## Appendix

**Table A6: Estimated parameter estimates of volatility transmission using a BEKK model in break in both series with dummy**

T	P1				P2				P3				P4				P5				P6				P7						
	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000
C11	0.49	0.70	0.91	0.98	0.51	0.76	0.95	1.00	0.29	0.55	0.83	0.98	0.32	0.53	0.80	0.96	0.30	0.57	0.87	0.99	0.64	0.91	0.99	1.00	0.54	0.74	0.94	0.99			
C12	0.13	0.07	0.03	0.01	0.09	0.04	0.01	0.00	0.22	0.10	0.03	0.02	0.18	0.09	0.02	0.01	0.17	0.06	0.02	0.01	0.07	0.04	0.01	0.00	0.07	0.03	0.01	0.01			
C22	0.46	0.66	0.85	0.97	0.49	0.79	0.96	1.00	0.40	0.70	0.93	1.00	0.39	0.70	0.92	0.99	0.34	0.66	0.93	1.00	0.34	0.68	0.91	0.99	0.34	0.65	0.92	1.00			
C11	0.51	0.72	0.91	0.98	0.55	0.78	0.96	1.00	0.42	0.63	0.85	0.98	0.35	0.56	0.80	0.97	0.43	0.66	0.89	0.99	0.72	0.93	0.99	1.00	0.57	0.76	0.95	0.99			
C12	0.13	0.06	0.03	0.01	0.07	0.04	0.01	0.00	0.14	0.07	0.02	0.02	0.16	0.09	0.02	0.01	0.10	0.06	0.02	0.01	0.05	0.04	0.01	0.00	0.06	0.03	0.01	0.01			
C22	0.48	0.68	0.86	0.97	0.55	0.81	0.97	1.00	0.55	0.80	0.95	1.00	0.44	0.72	0.92	0.99	0.49	0.70	0.94	1.00	0.49	0.74	0.92	0.99	0.43	0.69	0.92	1.00			
A11	0.85	0.97	1.00	1.00	0.51	0.77	0.96	1.00	0.31	0.64	0.94	1.00	0.61	0.89	0.99	1.00	0.34	0.66	0.96	1.00	0.87	0.99	1.00	1.00	0.59	0.83	0.98	1.00			
A12	0.03	0.03	0.02	0.01	0.04	0.03	0.01	0.01	0.04	0.03	0.02	0.01	0.05	0.03	0.02	0.02	0.07	0.05	0.02	0.01	0.05	0.03	0.01	0.01	0.06	0.03	0.02	0.02			
A21	0.05	0.02	0.02	0.01	0.04	0.03	0.01	0.01	0.03	0.02	0.02	0.01	0.07	0.03	0.02	0.01	0.07	0.05	0.02	0.01	0.06	0.03	0.02	0.01	0.08	0.03	0.02	0.01			
A22	0.56	0.74	0.91	0.99	0.86	0.98	1.00	1.00	0.93	0.99	1.00	1.00	0.74	0.92	1.00	1.00	0.70	0.92	1.00	1.00	0.34	0.67	0.92	1.00	0.59	0.85	0.97	1.00			
B11	0.97	0.99	1.00	1.00	0.93	0.97	1.00	1.00	0.99	1.00	1.00	1.00	0.97	0.99	1.00	1.00	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	0.93	0.98	1.00	1.00			
B12	0.17	0.10	0.03	0.02	0.09	0.07	0.04	0.02	0.10	0.09	0.05	0.03	0.13	0.07	0.03	0.02	0.15	0.10	0.05	0.03	0.14	0.07	0.02	0.00	0.14	0.07	0.03	0.01			
B21	0.08	0.03	0.02	0.00	0.11	0.06	0.01	0.00	0.07	0.05	0.02	0.00	0.10	0.05	0.01	0.01	0.12	0.05	0.02	0.00	0.13	0.07	0.04	0.02	0.14	0.08	0.03	0.01			
B22	0.67	0.71	0.81	0.92	0.94	0.99	1.00	1.00	0.97	0.99	1.00	1.00	0.75	0.82	0.93	0.99	0.95	0.99	1.00	1.00	0.97	0.98	1.00	1.00	0.95	0.98	1.00	1.00			

**Notes:** This table reports the frequency of the 1% statistically significant individual parameters of the BEKK model given as

$$h_{11,t+1} = c_{11}^2 + a_{11}^2 \varepsilon_{1,t}^2 + 2a_{11}a_{12} \varepsilon_{1,t} \varepsilon_{2,t} + a_{21}^2 \varepsilon_{2,t}^2 + b_{11}^2 h_{11,t} + 2b_{11}b_{12} h_{12,t} + b_{21}^2 h_{22,t} \text{ and } h_{22,t+1} = c_{12}^2 + c_{22}^2 + a_{12}^2 \varepsilon_{1,t}^2 + 2a_{12}a_{22} \varepsilon_{1,t} \varepsilon_{2,t} + a_{22}^2 \varepsilon_{2,t}^2 + b_{12}^2 h_{11,t} + 2b_{12}b_{22} h_{12,t} + b_{22}^2 h_{22,t}$$

P1-P7 refers to different cases of parametrization as given in Table 1. Break size was shift in unconditional variance of 4. We see including a dummy variable takes care of the spurious volatility transmission as the frequency of 1% statistically significant coefficient go to 0.01 for increasing size of the sample in the case of off-diagonal coefficients (12 and 21).