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Pricing Reliability Options under different electricity price regimes

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Abstract

Reliability Options are capacity remuneration mechanisms aimed at enhancing security of supply in electricity systems. They can be framed as call options on electricity sold by power producers to System Operators. This paper provides a comprehensive mathematical treatment of Reliability Options. Their value is first derived by means of closed-form pricing formulae, which are obtained under several assumptions about the dynamics of electricity prices and strike prices. Then, the value of the Reliability Option is simulated under a real-market calibration, using data of the Italian power market. We perform sensitivity analyses to highlight the role of the level and volatility of both power and strike price, of the mean reversion speeds and of the correlation coefficient on the Reliability Options' value. Finally, we calculate the parameter model risk to quantify the impact that a model misspecification has on the equilibrium value of the RO.

Keywords: Pricing; reliability option; option value; electricity markets.

1 Introduction

In several electricity markets worldwide there is an explicit remuneration of power through some Capacity Remuneration Mechanisms (CRMs). There exist different types of CRMs: *capacity payments* are explicit payments to power producers that are set administratively; *capacity auctions* are procurement auctions through which the System Operator (SO) remunerates a targeted amount of generation capacity; *capacity obligation* is the obligation for load serving entities to hold enough capacity to serve the load; *strategic reserves* is capacity that is withdrawn from the market and attributed to the SO in exchange for a predetermined remuneration. Interesting new CRMs that are gaining momentum are *reliability options* (ROs). Originally proposed by [41], [9], [31] and firstly implemented in Colombia [18], ROs are also adopted in ISO-New England [23], in Ireland [34, 35, 36] and in Italy [29, 39]. ROs are call options on power capacity, which are sold by power producers to the SO in exchange of a premium. By selling ROs, power producers commit to supply energy to the market and return to the SO the extra revenues that they would obtain when electricity prices rise above a predetermined level called *strike price*. The obligation of returning these extra revenues, which is termed *implicit penalty*, discourages any opportunistic behavior on the producers' side who might otherwise be tempted to withdraw capacity from the market in an attempt to benefit from price spikes. The aim of this paper is to propose a quantitative framework to evaluate ROs. We do so by following the financial approach, which requires identifying the

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stochastic property of the asset under evaluation and assuming that a continuous hedging between the financial derivative and the underlying asset is possible. At a first glance, this assumption seems quite hard to be met in the electricity sector, given that the underlying asset of the option is electricity, which is not a storable good[1]. However, there are cases of derivatives written on several underlying assets which are not liquidly traded, such as interest rates or temperatures. What is needed for the application of the risk-neutral pricing based on hedging is the existence of liquid assets that are traded and that correlate with the underlying of the derivative, such as forwards. This is our working assumption.

The RO is in its essence a contract for differences, in which the issuers give up some ex-post risky return in exchange for a known ex-ante premium. Therefore, a RO allows hedging price risk in the electricity market.[2] In markets where ROs are traded an explicit penalty for unavailability can also be introduced. For instance, in the Italian case the ROs issuers face an extra penalty that arises whenever they do not submit bids to the energy market [39]. The value of ROs should include also the expected value of such a penalty, calculated using some measure of expected power unavailability. We do not include such a value here and neglect the hedging against such an explicit penalty for two reasons: *i*) the latter is limited by the capacity derating that is calculated by the SO taking into account the average available capacity of a supplier, and we do not model here the quantity supplied by ROs issuers but just its value. *ii*) In general, the expected level of explicit penalty is rather low; for instance, in the Italian RO auctions, the expected amount of explicit penalty for power producers can be estimated as being not greater than 10% of the implicit penalty.[3]

We formulate different possible assumptions for the dynamics of the stochastic processes on which the RO depends, and estimate the relative RO value. ROs are complex options on power supply which can have different maturities and can be exercised several times at different, and possibly random, strike prices. Therefore, we provide a comprehensive mathematical treatment of all their aspects, and show how their fair value depends on the electricity price and the strike price definition and behavior.

Several authors have evaluated various exotic options on electricity.[4] However, to the best of our knowledge, our paper is the first one to evaluate ROs under different assumptions on the electricity price process. We choose a set of simple and significant models for electricity prices and present semi-explicit pricing formulae for ROs that have clear economic interpretations. We first start from the simplest possible assumption about electricity prices and strike prices, increasing then the level of complexity of the RO design, to allow for a mean reverting underlying, for stochastic strike prices and for possibly negative (but bounded from below) electricity prices. Furthermore, we simulate the RO value under different possible assumptions on the parameters. To provide a realistic example and gain further insights on their value, we calibrate the RO parameters against real electricity market data obtained from the Italian Power Exchange. The availability of long hourly price time series and the recent introduction of ROs in the Italian market both justify the choice.

We show how the ROs' value depends on the value of the parameters. We calculate the ROs' fair value before they are issued, as well as fine-tune their design with respect to the role and the impact that the strike price has on their value. Finally, we calculate the impact that a model misspecification has on the equilibrium value of the RO.

The paper is structured as follows. Section 2 places this paper in the relevant literature on the subject. Section 3 describes ROs and presents a general pricing formula under realistic assumptions. Section 4 provides semi-explicit solutions to the general pricing formula, for different electricity and

[1] At least as long as storage of electric energy by means of conversion into a different form of energy, such as kinetic energy of water in power dams or as chemical energy in batteries, is limited because of its cost or for technical reasons.

[2] This explains why in the RO literature [28], the implicit penalty is also termed *implicit covered penalty*, highlighting that ROs allow hedging against electricity price volatility.

[3] Source: own calculation based on Italian market data of year 2018.

[4] See the literature review section below for a discussion of these contributions.

strike price models. We start by defining the arbitrage-free boundaries of RO's evaluation. We then move from the simplistic model of geometric Brownian motion (GBM) with deterministic strike, to correlated GBMs with stochastic strike, and, by increasing realism on the model, to the case when both electricity and strike prices are seasonal and mean-reverting. For all these models, we present semi-explicit pricing formulae. Finally, we provide some insights for the case of negative prices. In Section 5, we showcase a simulation of the RO evaluation and perform a sensitivity analysis, using data of the Italian Power market for estimates and calibration. In Section 6 we present an analysis of parametric model risk, which allows quantifying how much a possible parameter misspecification affects the equilibrium RO price that we derive here. Section 7 draws conclusions, while all the proofs of the mathematical results are in the Appendix.

2 Literature review

ROs are a specific type of CRMs, aimed at enhancing security of supply. An introduction and an analysis of Capacity Remuneration Mechanisms can be found e.g. in [19, Chapters 22 and 23]. Among the different CRMs, ROs, were introduced by [9, 41] and further discussed by [3, 4, 18]. Interestingly enough, all these papers discuss ROs qualitatively and present examples showing how they can increase the security of supply. None of them present a quantitative analysis of ROs, as we do here.

The financial approach that we follow in this paper is the standard approach to price financial derivatives (see for instance [26]). It has been widely used in the literature to price derivatives on several underlying assets that are not liquidly traded, as it occurs in the present paper, such as interest rates or temperatures (see for instance [10, Chapter 15] for financial products and [20, 30, 38] for industrial applications). The assumption that there exists a liquid asset highly correlated with the price of electricity was questioned in the seminal paper [8], which considered the relationship between derivative (future) prices and spot prices in markets with limited liquidity and risk adverse agents. However, we believe that even if the assumption of limited liquidity was justified at the beginning of the liberalization process of the power market, this concern is less justified now, after several years of functioning of liberalized electricity markets. In fact, our approach is shared without concerns by other scholars who have evaluated other exotic options on electricity, such as spark-spread options (options on the differential between power prices and the heat content of the fuel, [20, 25]), Asian options (options written on average prices, [16]), and options which are implicit in demand response mechanisms, [38]. Our approach is also justified by utility-based arguments, see [14, Remark 3.6] for structured products in energy markets like that in Eq. (1) and the subsequent ones.

The nearest proxy of our analysis is [13], which evaluates, through a Monte Carlo approach, a contract composed of a portfolio of 4344 call options on hourly prices, all with the same strike price. This corresponds to a discrete-time version of the option we consider in Proposition 4.1. With respect to this paper, we provide a full mathematical treatment of ROs.

Finally, our work is also related to the analysis of arbitrage-free bounds of [20]. We start here by deriving model-free no-arbitrage bounds to the value of the ROs; similar no-arbitrage bounds have been derived by [20] for analogous contracts. However, the setting of that paper is different from ours: in [20], it is assumed that a continuum of forward contracts is traded, both for electricity and for the relevant fuel (whose spot price here would be regarded as equivalent to K), which deliver at any given date t. However, forward contracts on electricity guarantee the delivery of power over a period rather than on a single date t; therefore, the no-arbitrage bounds available in [20] cannot be directly applied.

3 Reliability options

We start by describing in general what ROs are. These contracts are sold in an auction, typically once a year, and they aim to deliver electricity with a given T_1-length period in advance (lead time), for a pre-defined period of delivery, with length $(T_2 - T_1)$. The rules of the RO specify that the capacity provider, the subject who sells the option, must commit to deliver a certain capacity to the subject buying the option, in general the SO. Such a commitment is made effective by prescribing that the seller must offer in the market an amount of electricity equal to the committed capacity, and return any positive difference between the reference market price and a previously set strike price K. Each RO contract scheme specifies what the reference market is. In a first approximation, the reference market can be a convex combination of different markets, such as the day-ahead and the balancing or real-time ones. In practice, different RO schemes can have different reference markets. For instance, in Ireland, the reference is exclusively the day-ahead market, while in NE-ISO it is the real-time one. If we call P the day-ahead market price and $P^{(b)}$ the price in the balancing market (or in the real-time market), we can define the reference market price R as the following convex combination

$$R = aP + (1 - a)P^{(b)},$$

where, $a \in [0, 1]$ depends on the country: $a = 0$ for ISO New England; $a = 1$ for Colombia and Ireland; $a \in [0, 1]$ in the case of Italy (see [29] for a description of the forthcoming Italian market).

The strike price is in general determined by taking into account the variable costs of the reference peak technology, that is, the dispatchable technology that would be included in the optimal generation mix with the lowest unitary investment cost. In actual RO markets, the rule for the strike price is communicated to potential sellers of ROs before the auction takes place. Thus, in some implementations it can be treated as a deterministic and constant parameter. However, it is also possible that the strike price changes over time during the life span of the RO. This is the case e.g. of the Italian scheme, where it is established that the rule linking the strike price to a reference marginal technology is set before the auction, but the marginal cost of such a technology is computed every given period during the life span of the RO.[5] This implies that the strike price can also be conceived as a stochastic process. We shall first derive the RO value starting with the simplest case, and then increase the level of complexity, to derive a general representation of the value of the RO.

3.1 A simple mathematical model for Reliability Options

The mathematical modeling of the general RO is quite complex, as many auctions and prices are involved. We simplify it by defining a mathematical model for the case when the reference price is simply the day-ahead price P, i.e. $a = 1$, as it is in the Colombian or the Irish CRM.[6] In this way, only one state variable is needed for the reference market price R, and it is indeed P.

We start by computing the fair price of a RO, written only on the reference price P and based on a generation capacity, i.e. for a power plant that is already in place. As said at the beginning of this section, the RO is sold in an auction at a certain time, but it becomes active in a subsequent time period. Let us denote by $t = 0$ the auction time and by $[T_1, T_2]$, with $T_1 > 0$, the time period when capacity has to be committed. It is assumed that the power plant will be productive at least until T_2. The idea of pricing the RO is to compute the expected operational profits at time $t = 0$ (auction time) of the power plant over the period $[T_1, T_2]$, both in the case when the capacity provider enters a RO scheme, and in the case it does not. The difference between these two operational profits will be the fair price of the RO.

[5] See [29] and [40].

[6] Moreover, we do not consider congestion in the transmission network, and therefore we implicitly assume that the market for ROs have the same size of the electricity market, namely, that there are no differences between the pricing zones of the electricity and the capacity markets.

We work on a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{Q})$ such that the probability measure $\mathbb{Q}$ is the risk-neutral pricing measure used by the market, and the day-ahead electricity price $P = (P_t)_{t\geq 0}$ is a $\mathbb{Q}$-semimartingale. We consider the simple case of a thermal plant, with total capacity $Q > 0$,[7] that converts a fuel, for example oil, gas or coal, into electricity. The cost $C = (C_t)_{t\geq 0}$ of running the thermal plant summarizes the fuel price, CO_2 price, operational and other costs. The power plant sells the electricity at time $t \geq 0$ when it wins the day-ahead auction, i.e. when its bid b_t is less than or equal to P_t. We adopt the usual simplifications, continuous time instead of hourly granularity and no ramping penalties/constraints. The plant can decide its bid process $b = (b_t)_{t\geq 0}$ to maximize its revenues.

We first evaluate the expected operational profits of the power plant over $[T_1, T_2]$ in the case when a RO scheme is not in place. This is the *value of the power plant* $V(T_1, T_2)$ at $t = 0$ and it depends on the power plant's income over $[T_1, T_2]$. It can be defined as

$$V(T_1, T_2) = \sup_{b\in\mathcal{B}} \mathbf{E}^{\mathbb{Q}} \left[\int_{T_1}^{T_2} e^{-rt} Q \, \mathbb{1}_{b_t \leq P_t} (P_t - C_t) \, dt \,\middle|\, \mathcal{F}_0 \right] , \tag{1}$$

where $\mathcal{B}$ is the set of adapted processes on $[T_1, T_2]$, r is the instantaneous risk-free rate of return and $\mathbf{E}^{\mathbb{Q}}$ is the expectation with respect to $\mathbb{Q}$.

Remark 3.1. In this setting, we assume that the investor is risk-neutral. Although here we are not evaluating financial assets, but rather incomes coming from industrial activity, this is in line with all the related literature, and it is justified by the following financial argument. The underlying assets P and C could be in principle not storable, or even not traded in some markets. However, even in such a situation, the risk-neutral evaluation in Eq. (1) can be applied as long as one can find hedging instruments that can be storable and liquidly traded, and that are correlated with P and C: for the mathematical derivation of such a result, see e.g. [10, Chapter 15] for vanilla products like call and put options (as we will end up to have), and [14, Remark 3.6] for structured products like that in Eq. (1) and the subsequent ones[8]. Here, we indeed have such suitable hedging instruments, i.e. forward contracts on power and fuel (for P and C, respectively), which are liquidly traded on financial markets, as they are basically equivalent to any other financial asset up to few days before physical delivery. When physical delivery approaches, in order to maintain the hedging position it is sufficient to liquidate the position on the maturing future(s) and open an equivalent new one on another future with a physical delivery further in time. This is a standard practice in energy markets, called *rolled-over portfolios*, see e.g. [1, 22] for two applications.

Going back to Eq. (1), it is optimal to choose b such that $\mathbb{1}_{b_t \leq P_t} = 1$ if and only if $P_t > C_t$, i.e. the optimal bidding process is $b_t = C_t \, \forall \, t \in [T_1, T_2]$. Thus, the final payoff for a thermal plant is

$$V(T_1, T_2) = \mathbf{E}^{\mathbb{Q}} \left[Q \int_{T_1}^{T_2} e^{-rt} (P_t - C_t)^+ \, dt \,\middle|\, \mathcal{F}_0 \right] .$$

We now consider the case when the thermal plant writes a RO with strike price $K = (K_t)_{t\geq 0}$. The plant must now pay back $(P_t - K_t)^+$. Therefore, the value $V_{ro}(T_1, T_2)$ of the thermal plant with a RO scheme in place is

$$V_{ro}(T_1, T_2) = \sup_{b\in\mathcal{B}} \mathbf{E}^{\mathbb{Q}} \left[\int_{T_1}^{T_2} e^{-rt} Q (\mathbb{1}_{b_t \leq P_t} (P_t - C_t) - (P_t - K_t)^+) \, dt \,\middle|\, \mathcal{F}_0 \right] .$$

[7] [41] have explicitly commented the problem for a RO issuer of not being able to produce the energy when called by the SO. We take this aspect into account here by interpreting Q as the available capacity, as described by [27] and [17]. This is coherent with the real-world application of RO, in which available capacity is computed by measuring the average availability of a power plant over a given time span (usually a year) and derating the nominal capacity accordingly (see [39] for the Italian scheme, and in [37] for Ireland). As an example, consider a 100MW plant with a maintenance period of one month per year. Its capacity factor is equal to 0.91; this figure can be used to de-rate the relevant capacity of the plant for the RO, which would amount to 91MW.

[8] This is exactly the same argument used to evaluate derivative assets written on non-tradable quantities like interest rates, temperature, etc.

The bidding strategy $b_t = C_t$ is again optimal for all $t \in [T_1, T_2]$. Thus,

$$V_{ro}(T_1, T_2) = V(T_1, T_2) - \mathbf{E}^{\mathbb{Q}} \left[\int_{T_1}^{T_2} e^{-rt} Q(P_t - K_t)^+ \, dt \,\middle|\, \mathcal{F}_0 \right] .$$

In a risk-neutral world, the value $RO(T_1, T_2)$ of a RO written on the time interval $[T_1, T_2]$ should make the investor indifferent between having the original plant without the RO, and having it with the RO written on it plus the price of the option, i.e. $V(T_1, T_2) = V_{ro}(T_1, T_2) + RO(T_1, T_2)$. Therefore, the final result is

$$\begin{aligned} RO(T_1, T_2) &= V(T_1, T_2) - V_{ro}(T_1, T_2) \\ &= \mathbf{E}^{\mathbb{Q}} \left[\int_{T_1}^{T_2} e^{-rt} Q(P_t - K_t)^+ \, dt \,\middle|\, \mathcal{F}_0 \right] \end{aligned} \tag{2}$$

Thus, the value of a reliability option issued by a thermal plant is equivalent to the price of an insurance contract against price peaks. Interestingly enough, notice that the operating strategy of the power plants does not change. In electricity markets, it is well known that perfectly competitive markets without CRMs, the so called energy only markets, provide enough incentives to investment, and the same is true for optimally designed CRMs, since the latter simply anticipate ex ante the supermarginal profits that investors would gain in energy only markets. In other words, the amount of remuneration of capacity accruing from perfectly competitive markets for CRMs equals the expected discounted value of the supermarginal profits gained in electricity markets; in a world without market failures, the two levels coincide (see [19, Chapter 22]). This is confirmed in our framework: without market power, the value of operating the plant is independent of the form of remuneration of power production, i.e. if revenues accrue ex-ante from the CRM or ex-post from selling electricity in the market.

4 Pricing of Reliability Options

4.1 Model-free no-arbitrage bounds

It is worth noticing that Equation (2) already allows us to produce model-free no-arbitrage bounds on the price of the RO. These model-free bounds do not require any assumption on the electricity price apart from P being bounded from below by a constant price floor $-P^*$, with $P^* \geq 0$. This is consistent with those electricity markets in which negative prices are allowed with a lower bound (as for instance in the German and French markets).

We start from the identity

$$(P_t - K_t)^+ = (K_t - P_t)^+ + P_t - K_t \, .$$

Since $0 \leq (K_t - P_t)^+ \leq K_t + P^*$, we have

$$P_t - K_t \leq (P_t - K_t)^+ \leq P_t + P^* \, .$$

By multiplying the inequalities by e^{-rt}, integrating and taking the expectation, we have that

$$Q\mathbf{E}^{\mathbb{Q}} \left[\int_{T_1}^{T_2} e^{-rt} (P_t - K_t) \, dt \,\middle|\, \mathcal{F}_0 \right] \leq RO(T_1, T_2) \leq Q\mathbf{E}^{\mathbb{Q}} \left[\int_{T_1}^{T_2} e^{-rt} (P_t + P^*) \, dt \,\middle|\, \mathcal{F}_0 \right] .$$

The right-hand side represents the forward price of delivering the quantity Q of electricity over the period $[T_1, T_2]$[9] with an additional constant $QP^* \frac{e^{-rT_1} - e^{-rT_2}}{r}$, depending on the price floor. We

[9] this is alternatively referred to as *flow forward* or *swap*, see e.g.[5].

label

$$F_P(0;T_1,T_2) := \mathbf{E}^{\mathbb{Q}}\left[\int_{T_1}^{T_2} e^{-rt}P_t\ dt \middle| \mathcal{F}_0\right]$$

the (unitary) forward price. Then, since $RO(T_1,T_2) \geq 0$, when $K_t \equiv K$, i.e. with fixed strike, we can rewrite the no-arbitrage relation above as

$$Q\left(F_P(0;T_1,T_2) - K\frac{e^{-rT_1}-e^{-rT_2}}{r}\right)^+ \leq RO(T_1,T_2) \leq QF_P(0;T_1,T_2)+QP^*\frac{e^{-rT_1}-e^{-rT_2}}{r}\,. \quad (3)$$

Thus, the value of a reliability option written on a total capacity Q over the period $[T_1,T_2]$ lies between the intrinsic value of Q call options on the forward $F_P(0;T_1,T_2)$ and the modified strike $K\frac{e^{-rT_1}-e^{-rT_2}}{r}$, and Q forwards $F_P(0;T_1,T_2)$ adjusted by an additional constant proportional to the price floor P^*.

Conversely, when K follows itself a stochastic process, we define

$$F_K(0;T_1,T_2) := \mathbf{E}^{\mathbb{Q}}\left[\int_{T_1}^{T_2} e^{-rt}K_t\ dt \middle| \mathcal{F}_0\right],$$

and obtain

$$Q\left(F_P(0;T_1,T_2) - F_K(0;T_1,T_2)\right)^+ \leq RO(T_1,T_2) \leq QF_P(0;T_1,T_2) + QP^*\frac{e^{-rT_1}-e^{-rT_2}}{r}\,. \quad (4)$$

Note that, even with a stochastic strike price K, the upper bound is unaffected. On the other hand, the lower bound is now the intrinsic value of Q exchange options on the forward $F_P(0;T_1,T_2)$ for the forward $F_K(0;T_1,T_2)$.

The no-arbitrage bounds above are model-free, in the sense that they hold for any no-arbitrage model that one can specify for the dynamics of P, and possibly of K, the only assumption needed being the existence of a price floor for P. However, to evaluate the RO as a financial contract, it is necessary to specify the stochastic process modeling electricity prices, as we shall do further below. Let us stress that electricity price have some peculiarities such as strong seasonality and mean-reversion; several processes have been adopted to reproduce its dynamics. For this reason, in the following sections we provide an overview on semi-explicit formulae to price a RO over $[T_1,T_2]$ under different price dynamics. We start with the simplest of the hypothesis, that serves us to build a reference model to better illustrate the key features of the pricing formula. We then increase the complexity of the dynamics in order to get a closer approximation to real price dynamics. In particular, notice that the price models generally used to evaluate options do not allow for negative prices, while we suggest a model of this kind in Section 4.6 below. In order to have a paper which is fully self-contained, we write in the Appendix all the proofs of the derivation of the pricing formulae.

4.2 Electricity spot price as a geometric Brownian motion

Let us start with the simplest assumption, i.e. that the price of electricity P evolves as a GBM, and that the option's strike price K is a fixed deterministic value. We stress that the former is an assumption that we regard as unreasonable, in the sense that it does not provide a realistic representation of the electricity price dynamics. However, it is the simplest possible assumption that is used to derive explicit pricing formulae for call options and we treat it as a first simplified approach to help us presenting the main features of the model. In this case, the price P, under the risk-neutral measure $\mathbb{Q}$, is assumed to be the solution of the following SDE:

$$dP_t = \mu P_t dt + \sigma P_t dB_t, \quad (5)$$

where B is a one-dimensional $\mathbb{Q}$-Brownian motion and μ is an appropriate yield, obtained by taking into account factors like risk-free rate of return and risk premium (which typically occurs in electricity markets, as shown, for instance by [8], given that P is typically a non-traded asset).

The price of a RO in this case is equivalent to the time integral over the interval $[T_1, T_2]$ of a European call option with strike price K and maturity ranging in $[T_1, T_2]$. In the following proposition, we provide a semi-explicit formula to price the RO, under the assumptions above.

Proposition 4.1. *Let the reference market price P follow the dynamics (5). The price of a reliability option over the time interval $[T_1, T_2]$ with fixed strike price $K \geq 0$ is given by the following formula:*

$$RO(T_1, T_2) = \int_{T_1}^{T_2} Q \left[P_0 e^{-(r-\mu)t} N(d_1(K, P_0, t)) - e^{-rt} K N(d_2(K, P_0, t)) \right] dt \,, \tag{6}$$

where N is the cumulative distribution function (CDF) of a standard Gaussian random variable and

$$\begin{aligned} d_1(K, P_0, t) \colon &= \frac{1}{\sigma\sqrt{t}} \left[\ln\left(\frac{P_0}{K}\right) + \left(\mu + \frac{\sigma^2}{2}\right) t \right] , \\ d_2(K, P_0, t) \colon &= d_1(K, P_0, t) - \sigma\sqrt{t} \,. \end{aligned}$$

Proposition 4.1 simply uses the Black and Scholes formula with dividends, since $RO(T_1, T_2)$ can be defined as the time integral of a family of call options with the same underlying and strike price, indexed by their maturity in $[T_1, T_2]$.[10] Thus, it provides a formula that can be applied to compute the value of the RO, once the parameters upon which the call depends on have been set; namely, the risk-free interest rate r, the starting price P_0 and the electricity price volatility σ.

4.3 Electricity price and strike price as correlated Geometric Brownian Motions

A first step to increase the level of complexity consists in modeling the strike price as a stochastic process. Recall that, in ROs, the strike price is the marginal cost of the marginal technology. Complex RO schemes can allow it to change over time, according to a predefined rule. For instance, it can be assumed that the strike price is given by the fuel cost of a predefined marginal technology, such as Combined Cycle Gas Turbines. In such a way, the strike price will be linked to a reference fuel price. Alternatively, it can be established that the reference price changes at fixed regular dates according to a given indexing formula.[11] Both cases imply that the strike price is a stochastic process (typically non-traded, in analogy with P). Thus, a first extension of the model defined in Section 4.2 is to model K and P as two (possibly correlated) geometric Brownian motions. This means that the prices $(K_t, P_t)_{t \geq 0}$ follow a risk-neutral dynamics of the following type:

$$\begin{cases} dK_t = \mu_k K_t dt + \sigma_k K_t dB_t^1 \,, \\ dP_t = \mu_p P_t dt + \sigma_p P_t dB_t^2 \,, \end{cases} \tag{7}$$

where (B^1, B^2) are correlated $\mathbb{Q}$-Brownian motions, with correlation $\rho \in [-1, 1]$. Notice that the correlation of the two stochastic processes depends on the rules defining the strike price and on the strike price nature. For instance, if the variable strike price is set to be equal to the marginal cost of the marginal technology, and if the electricity market is perfectly competitive, the system marginal price will be equal to the marginal cost of the marginal technology. Thus, the correlation coefficient would be equal to 1. If, on the contrary, the stochastic strike price equals some weighted average of different marginal costs at different hours, for instance at peak and off-peak hours, then

[10] Interestingly enough, this result solves also a problem firstly posed in [30], in the framework of firms' evaluations.
[11] As mentioned, this is the case of the Italian RO scheme.

the correlation coefficient would be positive but strictly less than 1, since the electricity price P would be more volatile than the strike price K. Finally, even if this possibility is rather unlikely, it may be that the strike price is negatively correlated with the electricity price, depending on how the strike price is defined and on what reference basket it is linked to.

The following proposition provides the value of the RO with two GBMs:

Proposition 4.2. *Let the reference market price P and the RO strike price K follow the dynamics (7). Then the price of a reliability option over the time interval $[T_1, T_2]$ is given by*

$$RO(T_1,T_2) = Q\int_{T_1}^{T_2}\left(P_0 e^{-(r-\mu_p)t}N(a_1(K_0,P_0,t)) - K_0 e^{-(r-\mu_k)t}N(a_2(K_0,P_0,t))\right)dt\,, \tag{8}$$

where N is the CDF of a standard normal random variable, and

$$\begin{aligned} a_1(K_0,P_0,t) :&= \frac{\ln\left(\frac{P_0}{K_0}\right) + (\mu_p-\mu_k)t}{\sigma\sqrt{t}} + \frac{1}{2}\sigma\sqrt{t}\,,\\ a_2(K_0,P_0,t) :&= a_1(K_0,P_0,t) - \sigma\sqrt{t}\,,\\ \sigma :&= \sqrt{\sigma_k^2+\sigma_p^2-2\rho\sigma_k\sigma_p} = \sqrt{(\sigma_k-\sigma_p)^2+2(1-\rho)\sigma_k\sigma_p}\,. \end{aligned}$$

In analogy to Proposition 4.1, in Proposition 4.2 we used the Margrabe formula with dividends (see, for instance, [15]). Here, the $RO(T_1,T_2)$ value is equal to the time integral of a family of options to exchange the (random) electricity price P with the (random) strike price K, again indexed by their maturity. As usual in the Margrabe formula, the relevant volatility is σ, that can be interpreted as the volatility of the ratio P/K (i.e. of the electricity price expressed in units of the strike price), which is decreasing with respect to the correlation ρ. In particular, for $\rho \to 1$ (i.e. when the strike price is highly correlated with the electricity price), we have $\sigma \to |\sigma_k - \sigma_p|$. In this case, when also $\sigma_k = \sigma_p$, the volatility vanishes, and the value of the option is determined just by its intrinsic value. Instead, for $\rho \to -1$ (i.e. when the strike price is highly negatively correlated with the electricity price), we have $\sigma \to \sigma_k + \sigma_p$, i.e. volatility is maximized. However, we stress that this latter case is rather unlikely for the case of RO, as typically a stochastic strike price K is defined in terms of quantities related to electricity generation (as e.g. the marginal price of the marginal technology, or some related market index), so that we should expect a positive correlation.

4.4 Mean-reverting electricity price with seasonality

As mentioned, a GBM does not capture typical stylized facts of electricity prices, namely seasonality and mean-reversion. A natural extension is thus to price the RO when the dynamics of the reference price reflects the aforementioned features. In particular, we model the log-spot price of electricity as a mean-reverting process encoding different types of seasonality by means of a time-dependent function, an approach that has been widely adopted in energy markets.[12] We first assume a deterministic strike price. In the next section, we shall remove this assumption.

We define the function describing seasonality trends for all $t \geq 0$ as

$$\mu(t) = \alpha + \sum_{i=1}^{12}\beta_i\, month_i(t) + \sum_{i=1}^{4}\delta_i\, day_i(t) + \sum_{i=1}^{24}\gamma_i\, hour_i(t)\,, \tag{9}$$

where $month_i(t)$, $day_i(t)$ and $hour_i(t)$ are dummies for month, day of week and hour, used to capture different types of seasonality. Specifically, we assume that *day* can take 4 values: 'Friday',

[12] For a presentation and critical discussion of various models for electricity prices proposed in the literature, see e.g. [6, 11, 16, 24, 25, 32, 42] and [5].

'Weekend', 'Monday', and 'other working day'. This captures the differences between working days and weekend as well as possible first- or end-of-the-working-week effect.

We then consider the day-ahead price P as

$$P_t = e^{\mu(t)} e^{X_t}\,, \tag{10}$$

where X_t, under the risk-neutral measure $\mathbb{Q}$, is the solution of the SDE

$$dX_t = -\lambda X_t dt + \sigma dW_t\,, \tag{11}$$

where W is a one dimensional $\mathbb{Q}$-Brownian motion, σ stands for the volatility and $\lambda > 0$ is the mean-reversion speed.

We have the following.

Proposition 4.3. *Let the reference market price P follow the dynamics (9)–(10)–(11). Then the price of a reliability option over the time interval $[T_1, T_2]$ with fixed strike price $K \geq 0$ is given by*

$$RO(T_1, T_2) = Q \int_{T_1}^{T_2} e^{-rt} \left[f(0,t) N(d_1(K, P_0, t)) - K N(d_2(K, P_0, t)) \right] dt\,, \tag{12}$$

where N is the CDF of a standard normal random variable, $P_0 = e^{\mu(0)+X_0}$ and

$$\begin{aligned}
f(0,t) :=& \mathbf{E}[P_t | \mathcal{F}_0] = \exp\left(\mu(t) + X_0 e^{-\lambda t} + \frac{1}{2} Var(t) \right) \\
=& P_0^{e^{-\lambda t}} \exp\left(\mu(t) - \mu(0) e^{-\lambda t} + \frac{1}{2} Var(t) \right), \\
Var(t) :=& \frac{\sigma^2}{2\lambda} (1 - e^{-2\lambda t})\,, \\
d_{1,2}(K, P_0, t) :=& \frac{1}{\sqrt{Var(t)}} \log \frac{f(0,t)}{K} \pm \frac{1}{2} \sqrt{Var(t)}\,,
\end{aligned}$$

where, by abuse of notation we mean that the definition of $d_1(K, P_0, t)$ involves the $+$ sign and the definition of $d_2(K, P_0, t)$ involves the $-$ sign.

Remark 4.1. Equation (12) is a generalization of Equation (6): in fact, if we let $\mu(t) := (\mu - \frac{1}{2}\sigma^2)t$ and $\lambda \to 0$, then we reobtain at the limit the model of the previous section. In fact, we have that $m_t \equiv X_0$, $Var(t) \to \sigma^2 t$,

$$e^{-rt} f(0,t) \to e^{\mu t + X_0}\,,$$

and

$$d_1(K, P_0, t) \to \frac{1}{\sigma\sqrt{t}} \left(X_0 + \mu t - \frac{1}{2}\sigma^2 t - \ln K \right) = \frac{1}{\sigma\sqrt{t}} \ln \frac{e^{X_0 + \mu t}}{K} - \frac{1}{2}\sigma\sqrt{t}\,.$$

Thus, the pricing formula in Equation (12) collapses into that of Equation (6).

4.5 Allowing for mean-reverting strike price with seasonality

As a natural extension of the model in Section 4.4, we now consider the case when the strike K is a mean-reverting process (with seasonality) as well. The dynamics of the state variables then becomes

$$\begin{cases} P_t = & e^{\mu(t)} e^{X_t}\,, \\ K_t = & e^{\nu(t)} e^{Y_t}\,. \end{cases} \tag{13}$$

Here, μ is given by (9) and ν is a seasonality function for K of the same form, while the processes X and Y are solution to

$$\begin{cases} dX_t = & -\lambda_x X_t dt + \sigma_x dW_t^1\,, \\ dY_t = & -\lambda_y Y_t dt + \sigma_y dW_t^2\,, \end{cases} \tag{14}$$

where (W^1, W^2) are correlated $\mathbb{Q}$-Brownian motions, with correlation $\rho \in [-1, 1]$.

Proposition 4.4. *Let the reference market price P and the RO strike price K follow the dynamics* (13)*; then the price of a reliability option over the time interval $[T_1, T_2]$ is given by*

$$RO(T_1,T_2) = Q\int_{T_1}^{T_2} e^{-rt}\left(f_P(0,t)N\left(d_2(K_0,P_0,t)\right) - f_K(0,t)N\left(d_1(K_0,P_0,t)\right)\right)dt\,, \tag{15}$$

where N is the CDF of a normal random variable, $P_0 = e^{\mu(0)+X_0}$, $K_0 = e^{\nu(0)+Y_0}$ and

$$\begin{aligned} f_P(0,t) &:= \mathbf{E}[P_t|\mathcal{F}_0] = \exp\left(\mu(t) + X_0 e^{-\lambda_x t} + \frac{\sigma_x^2}{2\lambda_x}(1-e^{-2\lambda_x t})\right) \qquad (16)\\ &= P_0^{e^{-\lambda_x t}} \exp\left(\mu(t) - \mu(0)e^{-\lambda_x t} + \frac{\sigma_x^2}{2\lambda_x}(1-e^{-2\lambda_x t})\right), \\ f_K(0,t) &:= \mathbf{E}[K_t|\mathcal{F}_0] = \exp\left(\nu(t) + Y_0 e^{-\lambda_y t} + \frac{\sigma_y^2}{2\lambda_y}(1-e^{-2\lambda_y t})\right) \qquad (17)\\ &= K_0^{e^{-\lambda_y t}} \exp\left(\nu(t) - \nu(0)e^{-\lambda_y t} + \frac{\sigma_y^2}{2\lambda_y}(1-e^{-2\lambda_y t})\right), \\ d_{1,2}(K_0,P_0,t) &:= \frac{1}{\sqrt{\overline{Var}(t)}}\log\frac{f_P(0,t)}{f_K(0,t)} \pm \frac{1}{2}\sqrt{\overline{Var}(t)}\,, \qquad (18)\\ \overline{Var}(t) &:= \sigma_x^2\frac{1-e^{-2\lambda_x t}}{2\lambda_x} + \sigma_y^2\frac{1-e^{-2\lambda_y t}}{2\lambda_y} - 2\rho\sigma_x\sigma_y\frac{1-e^{-(\lambda_x+\lambda_y)t}}{\lambda_x+\lambda_y}\,. \qquad (19) \end{aligned}$$

This result resembles that of Proposition 4.3 in the same sense as Proposition 4.2 is similar to Proposition 4.1: here $RO(T_1,T_2)$ can be again defined as the time integral of a family of options to exchange the electricity price P with the strike price K. Here too, the relevant volatility is $\overline{Var}(t)$, which can again be interpreted as the volatility of the ratio P/K (i.e., the electricity price expressed in units of the strike price: this is made explicit in the proof in the Appendix), which is again decreasing with respect to the correlation ρ. In particular, for $\rho \to 1$ (i.e. when the strike price is highly correlated with the electricity price), and $\lambda_x = \lambda_y =: \lambda$ (i.e. when the two mean-reversion speeds are the same), we have $\overline{Var}(t) \to \frac{1-e^{-2\lambda t}}{2\lambda}(\sigma_x - \sigma_y)^2$. In this case, when $\sigma_x = \sigma_y$, the volatility vanishes, and the value of the option is given just by its intrinsic value. Instead, in the unlikely case (see the discussion at the end of Section 4.3) when $\rho \to -1$ and $\lambda_x = \lambda_y =: \lambda$, we have $\overline{Var}(t) \to \frac{1-e^{-2\lambda t}}{2\lambda}(\sigma_x + \sigma_y)^2$, i.e., the volatility is maximized.

4.6 Possible extension to negative day-ahead and strike prices

In principle, it is possible to allow for negative power prices, since we know this is a possibility in energy markets (see [21] and references therein). An analogous extension can be also envisaged for strike prices, especially when these are linked to power prices. A possible approach to model negative prices is to set negative values $-P^*$ and $-K^*$, for certain $P^*, K^* \geq 0$, as price floors for P and K, respectively, and to consider the following shifted dynamics

$$\begin{cases} P_t = & \left(e^{\mu(t)}e^{X_t} - P^*\right), \\ K_t = & \left(e^{\nu(t)}e^{Y_t} - K^*\right). \end{cases} \tag{20}$$

where μ and ν are again seasonality functions for P and K and the processes X and Y are solution of Equation (14), in analogy with the previous section.

By setting $C := P^* - K^*$, one can prove that the price of the reliability option is now given by the following expression:

$$RO(T_1,T_2) = Q\int_{T_1}^{T_2} e^{-rt}\mathbf{E}^{\mathbb{Q}}\left[\left(e^{\mu(t)}e^{X_t} - e^{\nu(t)}e^{Y_t} - C\right)^+ \,\middle|\, \mathcal{F}_0\right]dt\,. \tag{21}$$

The above formula is the time integral of a family of spread options with a fixed strike price C and indexed by their expiration date in $[T_1, T_2]$. Therefore, considering dynamics of type (20) relates the problem of pricing a Reliability Option to the problem of pricing a spread option (see [15] for a survey of classical frameworks and methods for spread options). Unfortunately, a general closed formula for the pricing of spread options is not available. However, since the RO is in principle a quite illiquid product, one can use a numerical method to price it in this general case, for example Monte Carlo.

5 Simulation and sensitivity analysis

In this section we simulate the value of the RO under realistic assumptions on the parameter values. To do so, we fit the parameters of the electricity price dynamics to a real market, using data of the Italian market. For simplicity, we consider day-ahead prices only, and use the weighted average of Italian zonal prices, called PUN (*Prezzo Unico Nazionale*), ranging from January 1 to December 31, 2016.

As previously explained, we used dummies to capture monthly, daily and hourly seasonality, as defined in Eq. (9). We chose 'January', 'Friday' and 'hour 1' as reference groups, against which the comparisons are made. Figure 1 shows the calibrated seasonality function, plotted against the historical PUN data. In line with the PUN mean price, which is equal to 42.77 €/MWh, when the strike price is supposed to be a constant K is arbitrarily chosen equal to 40 €/MWh. Furthermore, we considered an annual risk-free rate $r = 0.01$. According to the scheme to be implemented in Italy, the pricing of the RO starts 4 years from now, and the option has a maturity of 3 years ($T_1 = 4$, $T_2 = 7$).

The starting point X_0 is taken equal to 0. Table 1 reports the estimated parameters for each different model, while Table 2 shows the estimated seasonality parameters.

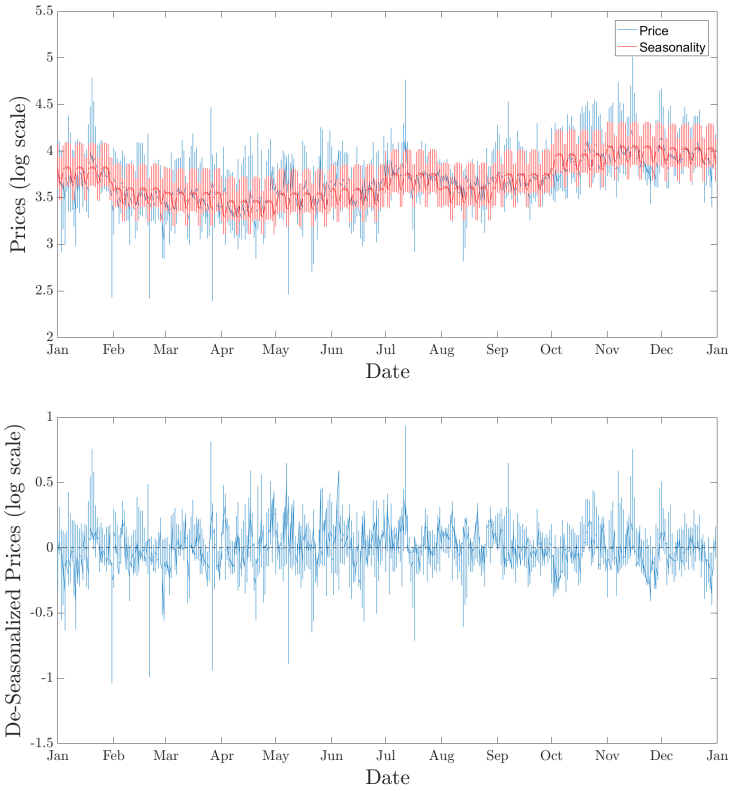

Figure 1: Seasonality function in (9) (solid red line, upper panel) calibrated on historical 2016 PUN electricity data (solid blue line, upper panel) and residuals (bottom panel).

	GBM	1-OU	2-OU
$\hat{\sigma}$	5.3033	6.8780	6.8780
	(0.041)	(0.056)	(0.056)
$\hat{\lambda}$	-	1302.89	1302.89
	(-)	(61.52)	(61.52)

Table 1: Estimated yearly parameters $\hat{\sigma}$ and $\hat{\lambda}$ for each pricing model (electricity price following a Geometric Brownian motion (GBM), electricity price following a mean-reverting Ornstein-Uhlenbeck process (1-OU), correlated electricity and strike prices following mean-reverting Ornstein-Uhlenbeck processes (2-OU)). Standard errors are in parentheses; all the estimated parameters are statistically significant, with P (largely) below 0.001 for all the parameters.

	Estimate	S.E.	pValue		Estimate	S.E.	pValue
Intercept	3.79	0.01	< 0.001	$hour_6$	-0.13	0.01	< 0.001
$month_2$	-0.22	0.01	< 0.001	$hour_7$	-0.01	0.01	0.497
$month_3$	-0.27	0.01	< 0.001	$hour_8$	0.1	0.01	< 0.001
$month_4$	-0.36	0.01	< 0.001	$hour_9$	0.18	0.01	< 0.001
$month_5$	-0.28	0.01	< 0.001	$hour_{10}$	0.16	0.01	< 0.001
$month_6$	-0.23	0.01	< 0.001	$hour_{11}$	0.12	0.01	< 0.001
$month_7$	-0.07	0.01	< 0.001	$hour_{12}$	0.07	0.01	< 0.001
$month_8$	-0.21	0.01	< 0.001	$hour_{13}$	0	0.01	0.800
$month_9$	-0.07	0.01	< 0.001	$hour_{14}$	-0.05	0.01	< 0.001
$month_{10}$	0.14	0.01	< 0.001	$hour_{15}$	-0.02	0.01	0.130
$month_{11}$	0.23	0.01	< 0.001	$hour_{16}$	0.04	0.01	0.002
$month_{12}$	0.21	0.01	< 0.001	$hour_{17}$	0.09	0.01	< 0.001
Monday	-0.01	0.01	0.045	$hour_{18}$	0.15	0.01	< 0.001
Weekend	-0.14	0.01	< 0.001	$hour_{19}$	0.22	0.01	< 0.001
Working_day	0.02	0.01	0.002	$hour_{20}$	0.28	0.01	< 0.001
$hour_2$	-0.08	0.01	< 0.001	$hour_{21}$	0.27	0.01	< 0.001
$hour_3$	-0.15	0.01	< 0.001	$hour_{22}$	0.2	0.01	< 0.001
$hour_4$	-0.18	0.01	< 0.001	$hour_{23}$	0.12	0.01	< 0.001
$hour_5$	-0.18	0.01	< 0.001	$hour_{24}$	0.03	0.01	0.013

Table 2: Linear regression estimates, standard errors and p-values obtained using the specification in (9). The base group categories for each dummy variable are $month_1$, $friday$ and $hour_1$.

As is evident from Table 1, where λ is statistically significant with $P << 0.001$, real electricity prices do not follow GBMs. Therefore, in the simulation, we restrict to the model defined in Section 4.4.

5.1 Mean reverting electricity price with seasonality, fixed strike

We simulate the value of the RO using the Monte Carlo methodology. Specifically, we compute the RO value using 10,000 simulations of the price path of the underlying.

Figure 3 shows the comparative statics for different ranges for the parameters σ and λ and strike price K. As expected, the higher the strike price, the lower the value of the reliability option for each value of σ (left panel). On the other hand, both the left and right panels show that, when σ increases, the RO value rises as well. Moreover, when λ is low, the relative increase in the RO value is high (right panel). This is consistent with the fact that a low λ allows fluctuations of the underlying that are far from the long term mean to be more persistent.

5.2 Electricity spot price and RO strike price as correlated OU with seasonality

We simulate now the value of the RO using the model described in Section 4.5, again by means of a Monte Carlo method (with 10,000 runs). For the reason mentioned above, consistently with the PUN mean price, K_0 is arbitrarily chosen equal to 40 €/MWh, so that, after de-seasonalizing (using the same estimated seasonality parameters of the PUN price), we obtain $Y_0 = -0.21$, while

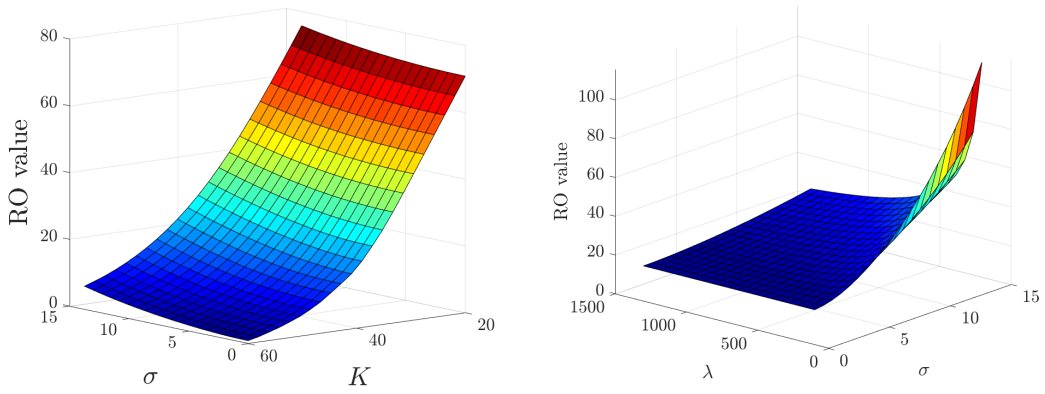

Figure 3: Sensitivity analysis of the results using a yearly σ in the range $(0; 2\hat{\sigma}]$ with a strike price K in the range $[20; 60]$ (left panel), and a yearly σ in the range $(0; 2\hat{\sigma}]$ with and a yearly λ in the range $(100; 2\hat{\lambda}]$ (right panel). The RO value is expressed in €/MWh.

$X_0 = 0$. If no otherwise stated, we consider a fixed correlation coefficient $\rho = 0.5$. In Figure 4 we plot the results of the simulations when the parameters of the strike price λ_K and σ_K are assumed to be equal to the ones estimated for the electricity price, while in Figure 5 the parameters of K and of P are decoupled.

As mentioned, Figure 4 shows the results of the simulations depending on σ_p, assuming the strike price process to have the same parameters estimated for the electricity price P. The upper left panel shows that the initial level of the strike price K_0 has no influence on the value of the reliability option, the reason for this lies in the magnitude of the estimated λ_P, and thus of λ_K: a mean reversion speed as high as that estimated makes the strike price process return to its mean level in an amount of time negligible with respect to the maturity. This implies that the starting point of the process has no relevant impact on the RO value.

The upper right panel of Figure 4 instead shows how sensitive the RO value is to changes in the electricity price parameters λ_P and σ_P (and thus in turn in λ_K and σ_K). Similarly to what we have observed before, the higher the volatility of the underlying (and, in this case, of the strike price), the higher the RO value. This relationship increases proportionally to the decrease of the speed of mean reversion, since it takes more time to return to the mean, and thus volatility has a higher impact.

The role of the correlation factor ρ is instead investigated in the bottom left panel, where we assess how different correlation factors in the range $[-1; 1]$ affect the price of the RO. When the two assets are perfectly correlated ($\rho = 1$), the RO value is zero for all levels of σ_P. In fact, as seen in Section 4.3, the volatility is minimized and the RO can be interpreted as an integral of calls, with maturity ranging in the interval $[T_1, T_2]$, being exactly at the money at the time of expiration, and thus having zero value. Instead, as shown, when the two processes are uncorrelated, the level of risk increases, and it reaches its maximum when they are perfectly negatively correlated. In this case, the volatilities of the two Brownian motions sum up, increasing the volatility of the option payoff and minimizing the risk of having the calls at the money. Finally, the bottom right panel shows that the RO price is negatively correlated with the risk free rate r: a higher r decreases the option value as it lowers the discounted cash flows.

In Figures 5 and 6 we show simulations' results when the parameters of the strike price's dynamics differ from the ones of the electricity price's . In the case of volatilities $\sigma_P = \sigma_K = \hat{\sigma}_P$, the left panel of Figure 5 reports the results for a variation in λ_K (in the range $(0; 2\hat{\lambda}_P]$ and shown in $\log_{10}$ scale) independent from the value of λ_P (which is instead fixed $\lambda_P = \hat{\lambda}_P$). The graph shows how K_0 hardly affects the RO value, as it has an impact only when both σ_K and λ_K are

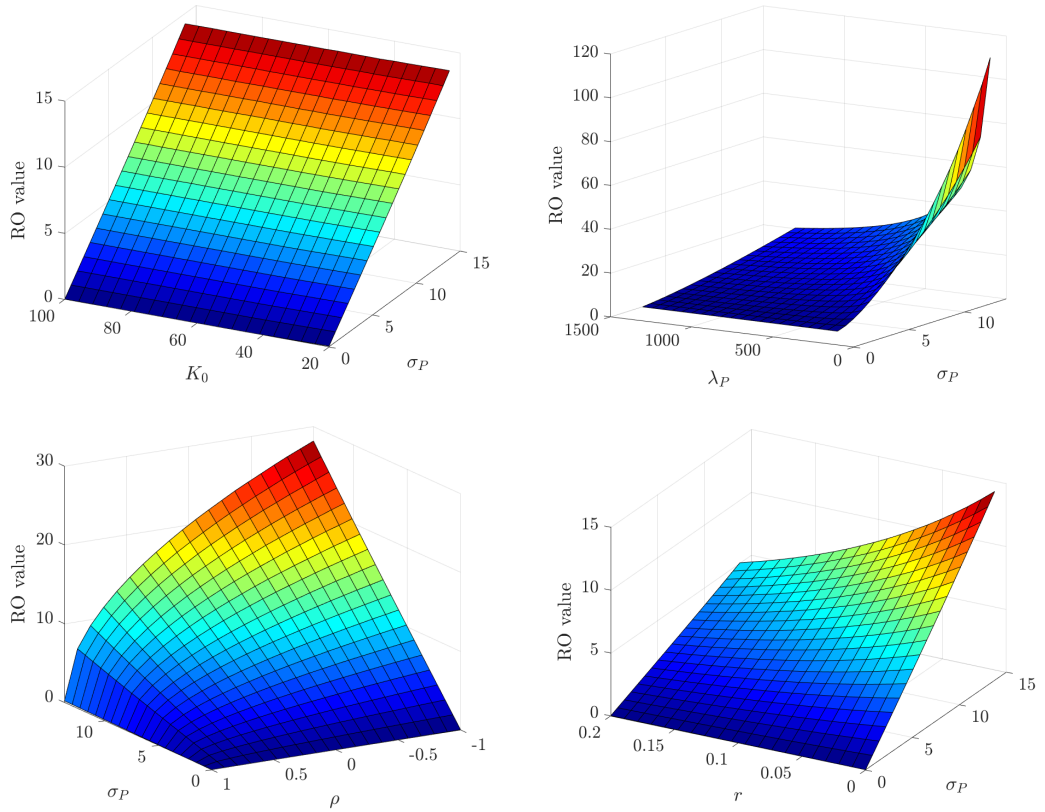

Figure 4: Sensitivity analysis of the results using a yearly σ_P in the range $(0; 2\hat{\sigma}_P]$ with an initial strike K_0 in the range $[20; 100]$ (upper left panel), with a yearly λ_P in the range $(100; 2\hat{\lambda}_P]$(upper right panel), with a correlation ρ in the range $[-1; 1]$ (left bottom panel) and with a yearly risk free rate r in the range $[0; 0.2]$ (right bottom panel).

sufficiently small. This confirms the result shown above that the initial value of the prices matters only when it takes a sufficient amount of time for them (i.e., for the strike price in this case) to return to their long term value. The right panel instead shows the sensitivity of the RO value to changes in the yearly λ_K (again in the range $(0; \hat{\lambda}_P]$) independent from the value of λ_P, and in the correlation factor ρ (in the range $[-1; 1]$). Here, the ρ value matters the most when both $\lambda_K = \lambda_P$ and $\sigma_K = \sigma_P$. In fact, ρ (negatively) affects the RO value only when it tends to -1 and λ_K is closer to the value of λ_P (note that, in the figure, $\lambda_K \in (0; \hat{\lambda}_P]$, where $\hat{\lambda}_P$ corresponds to the value of 2.47 in $\log_{10}$ scale). This confirms our intuition that, when the initial value of the electricity price and the strike price are close and the two random variables follow the same dynamics, the RO has a negligible value since it is likely that it will be always at-the-money. Conversely, if the two random variables are not perfectly correlated or the two variables follow different dynamics, it is unlikely that at every point in time P_t and K_t coincide, and this adds value to the RO.

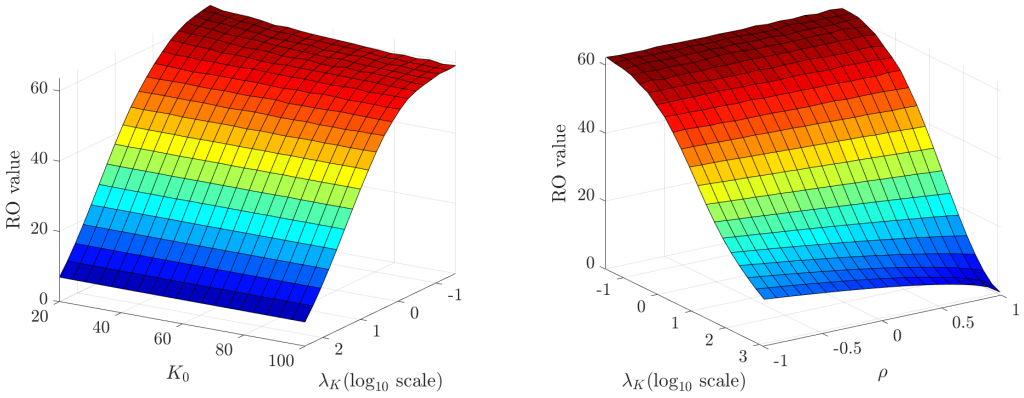

Figure 5: Sensitivity analysis of the results using a yearly λ_K in the range $(0; \hat{\lambda}_P]$ with an initial strike price K_0 in the range $[20; 100]$, both with a yearly σ_K equal to the yearly σ_P (upper left panel) and with and a scaled down yearly σ_K (upper right panel), and with a correlation ρ in the range $[-1; 1]$ (bottom panel) (here $\sigma_K = \sigma_P$). The RO value is expressed in €/MWh.

Finally, Figure 6 shows the effect of a disjoint variation in the two volatilities, with a yearly σ_P and σ_K in the range $(0; 2\hat{\sigma}_P]$, for different levels of ρ (in these graphs, $\lambda_K = \lambda_P = \hat{\lambda}_P$). As expected, when $\rho \leq 0$, the RO price is always increasing in both the electricity price volatility σ_P and the strike price's one σ_K, since volatility adds value to the call options. Instead, when $\rho > 0$, the fact that the two processes are somehow coupled can lower the aggregate risk, since the spread between the electricity price and the strike price reduces. This translates into a negative effect on the option value. The RO value is therefore minimized when $\sigma_P = \sigma_K$. In Figure 6, panel $\rho = 0.5$, we can see that the option value is still positive; in the panel $\rho = 1$, the RO value becomes null for $\sigma_P = \sigma_K$, since, as mentioned, if the two processes are perfectly positively correlated, the RO value coincides with its intrinsic value. Thus, there is a non-monotone effect of the volatility increase of one process, depending on the amount of volatility of the other process, and on the level of the correlation coefficient. The inflection is maximum when the two processes are perfectly positively correlated.

6 Parametric model risk

The fact that we have (semi-)closed formulae for the price of the RO allows us to investigate how the price depends on the parameters. This was partially done in the previous section, where a numerical price sensitivity with respect to various parameters was presented. A more accurate way to treat this involves parametric model risk, following the approach of [2], i.e. assessing which of the parameters would have the highest impact on the price in case of misspecification.

Indeed, the standard methodology to derive parameters in electricity markets (and that we followed in the previous section), lacking sufficiently liquid derivative products, is based on time series analysis. This allows estimating parameters and it makes use of their point estimates into the pricing formulae, but it completely disregards the information contained in the estimators' distribution (e.g. their biases and/or variances). This problem is known, among the various types of model risk, as *parameter risk*, i.e. the risk of picking the wrong parameter value(s) for the pricing formula.

The approach used in [2] to deal with parameter risk is the following: assume that we estimate a distribution R on the parameter space Θ, which expresses the trustworthiness that we give to the different parameters in Θ. Then, each parameter $\theta \in \Theta$ implies an expected derivative price $\mathbf{E}_\theta[X]$, where X is the payoff of our derivative contract (in our case, the RO). Since trustworthiness

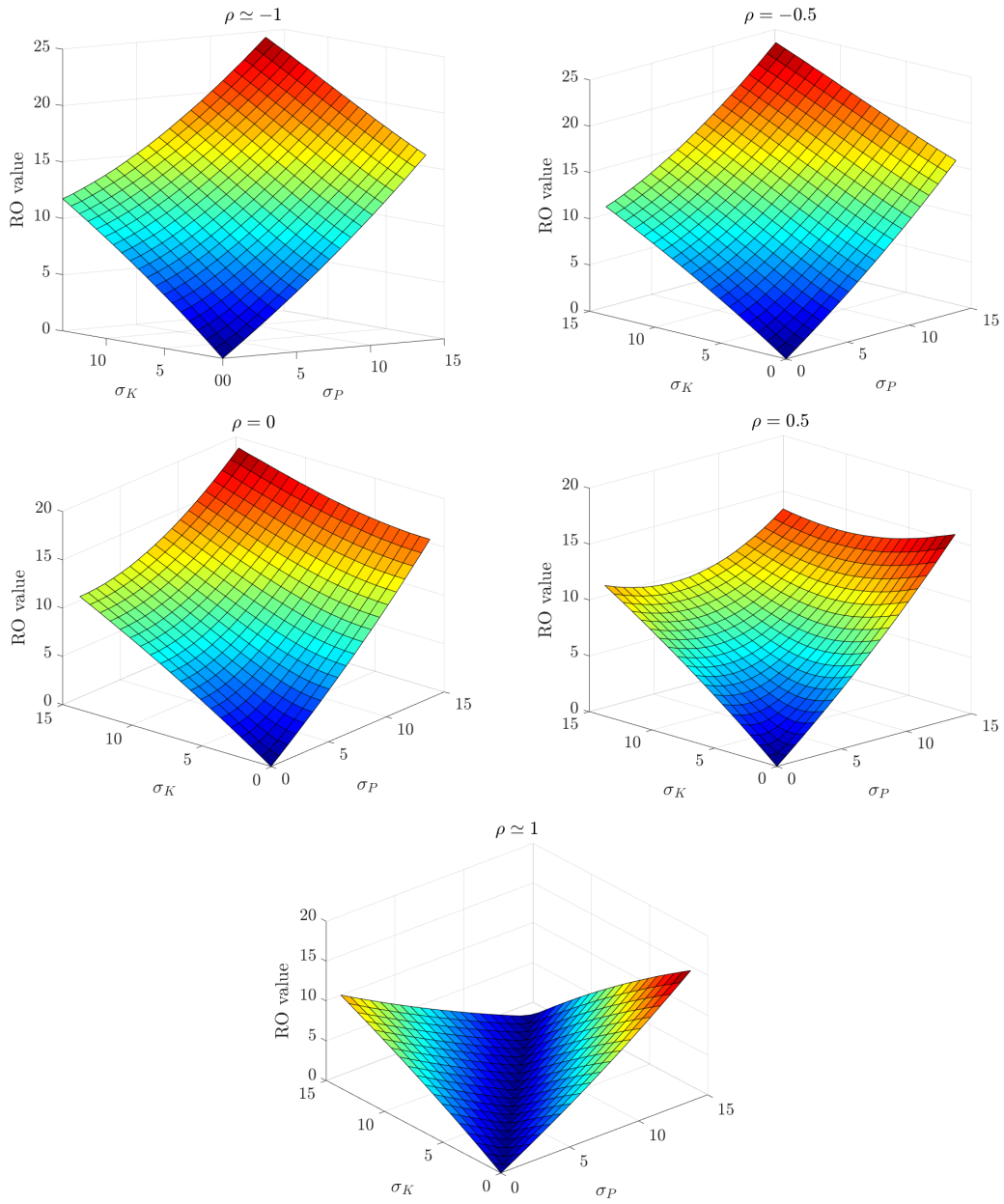

Figure 6: Sensitivity analysis of the RO value to a disjoint variation in the two volatilities, with a yearly σ_P and σ_K in the range $(0; 2\hat{\sigma}_P]$ (here $\lambda_K = \lambda_P$). In the different panels, we can see how a variation in the correlation coefficient ρ affects the RO value: when the two processes are independent or negatively correlated, higher σ_P and σ_K result in a higher option value. However, when the correlation is positive (middle right and bottom panels), the higher the correlation, and the more the two volatilities are similar, the lower the value of the option. The RO value is expressed in €/MWh.

of parameters is unknown, a convex risk measure ρ is used to average the pricing mechanism $\theta \to \mathbf{E}_\theta[X]$.

As in [2], as convex risk measure we choose the *Average-Value-at-Risk* (AVaR), defined at

significance level $\alpha \in (0,1)$ as:

$$\mathrm{AVaR}_\alpha(X) := \frac{1}{\alpha}\int_0^\alpha q_{1-\beta}(X)\; d\beta\,,$$

where $q_\beta(X)$ is the (lower) β-quantile of the random payoff X. The standard interpretation is that $\mathrm{AVaR}_\alpha(X)$ is the average of all quantiles of X above the chosen confidence level $1-\alpha$.

This approach can be implement in our case by identifying that our multidimensional parameter is

$$\theta = (\alpha, \beta_1, \ldots, \beta_{12}, \delta_1, \ldots, \delta_4, \gamma_1, \ldots, \gamma_{24}, \lambda, \sigma)$$

in the case of the mean-reverting model of Section 4.4, and

$$\begin{aligned}\theta \;=\; & (\alpha^P, \beta_1^P, \ldots, \beta_{12}^P, \delta_1^P, \ldots, \delta_4^P, \gamma_1^P, \ldots, \gamma_{24}^P, \lambda_x, \sigma_x,\\ & \alpha^K, \beta_1^K, \ldots, \beta_{12}^K, \delta_1^K, \ldots, \delta_4^K, \gamma_1^K, \ldots, \gamma_{24}^K, \lambda_y, \sigma_y, \rho)\end{aligned}$$

in the case of the two-dimensional mean-reverting model of Section 4.5. However the whole parameter distribution is very complex and difficult to obtain, see [12]. Therefore, in analogy with [2], we reduce the problem by considering the distributions of treatable subsets of parameters separately, disregarding the remaining parameter risk by considering the rest of parameters as fixed and known. In what follows we sketch the procedure.

1. Each individual parameter, or a group of parameters, θ_j is estimated by a maximum likelihood estimator (MLE) $\hat\theta_j(P_1, \ldots, P_n)$, where n is the length of the time series used for the estimation.

2. Since $\hat\theta_j$ is a MLE, its distribution R is asymptotically Gaussian, i.e. $\sqrt{n}(\hat\theta_j(P_1, \ldots, P_n) - \theta_0) \to N(0, \Sigma)$, where θ_0 is the true parameter value and Σ is the parameter's covariance matrix and the distribution R can be approximated by $N(\theta_0, \frac{1}{n}\Sigma)$.

3. The AVaR can be computed explicitly in the case of a Gaussian distribution, therefore, with the approximation of the previous step, we obtain

$$\mathrm{AVaR}_\alpha(X) \simeq \mathbf{E}_\theta[X] + \frac{\phi(N^{-1}(1-\alpha))}{\alpha\sqrt{n}}\sqrt{(\nabla\mathbf{E}_\theta[X])'\Sigma(\nabla\mathbf{E}_\theta[X])}\Big|_{\theta=\hat\theta_j(P_1,\ldots,P_n)}\,, \tag{22}$$

   where ϕ is the density of a standard normal random variable and ∇ is the gradient with respect to θ_j.

This procedure is well-suited to a situation in which the parameters are estimated from time series of market data (as it is for our case). In Section 5 we estimated the seasonality parameters for the power price P, using time series from the Italian market, together with the mean-reversion and volatility parameters for the stochastic factor. Instead, parameters for the strike price K, even when this was assumed stochastic, were assumed to be equivalent to those of P. This would occur, for instance, when power price depends on an underling fuel cost, and the strike price is defined as an indexed formula of such a fuel cost (this is what happens in the Italian RO [39]). In any case, the lack of a proper time series of the strike price does not allow us to estimate its distribution and compare it with the estimate distribution of P. For this reason an accurate model risk procedure currently makes sense only for the single-factor model, allowing us to provide realistic results. Notice however that the procedure is general enough to be applied also to the two-factor model, provided that a time series exists for K, making its estimation possible.

The closed-form expression for the normal AVaR in (22) allows computing risk-captured prices efficiently with the model in Section 4.4. The second term in the right-hand side of (22) is the risk adjustment value, i.e. when subtracted or added to the RO value $\mathbf{E}_\theta[X]$ it provides the ask or bid

prices for the RO, respectively. As a measure of model risk, we can thus define the relative width of the bid-ask spread as

$$\Delta = \frac{\text{bidPrice} - \text{askPrice}}{\text{midPrice}} . \tag{23}$$

Table 3 shows the relative bid-ask spread as defined in equation (23) for different significance levels of the underlying risk measure, and for different treatable subsets of parameters, i.e. for different model risk sources. The table shows that the major source of risk lies in the seasonality parameters distribution, which has the larger impact on the RO price, while volatility also has a significant impact on the total risk (though five times less than the seasonality). Notice however that seasonality contains 37 distinct parameters while volatility is a single one; thus, the latter is the parameter that has the largest impact on model risk. On the contrary, the analysis shows a negligible model risk due to the speed of mean reversion. Since these results depend on the estimated variances of the estimators the reason why the model risk due to a misspecification of λ is negligible with respect to the other ones, originates in the low variance of th estimator of λ. Finally, the model risk is quite small also because of a high n (equal to $24 \times 366 = 8784$), which is the same for all group of parameters[13].

		$\alpha = 0.001$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Δ	Total	5.196%	4.113%	3.183%	2.708%
	Seasonality	5.099%	4.035%	3.123%	2.657%
	Speed of mean reversion	0.161%	0.128%	0.099%	0.084%
	Volatility	1.068%	0.845%	0.654%	0.556%

Table 3: Relative width of the bid-ask spread (Δ, in percentage) as a measure of model risk for different significance levels α and for different risk sources.

7 Conclusions

In this paper, we have studied the value of the RO from a financial perspective. The financial approach to option pricing relies on the assumption that the market prices financial products by risk-neutral measure exists. This is not a problem for pricing options on electricity prices, as long as they can be written on electricity futures that can be rolled over the delivery period of the RO. Nevertheless, it must be kept into account that such an approach does require that RO markets are competitive and that forward markets are liquid. therefore, our analysis provides a benchmark value for the RO under the assumption that the market for the derivative is liquid enough to bring about competition.[14] In this framework, the simplified mathematical model that we proposed can be seen as a starting point in the analysis of ROs. We obtain semi-explicit formulae for the value of the RO, under a set of different assumptions with increasing realism and complexity. We move from simple integrals of call options written on GBMs to correlated mean reverting processes that capture the behavior of realistic electricity price time series, on the one hand, and complex rules for RO, on the other. Moreover, we simulate the RO value through a real-market calibration of the parameters.

Our results are important from two different point of views. From a theoretical perspective, we provide a mathematical treatment that allows to show how the value of the RO depends on the values of its parameters. The results are consistent with expectations from option theory: a rise in the strike price lowers the RO value, which depends positively on the volatility of the electricity

[13] If K was traded on a time-basis with a smaller frequency (for instance on a daily basis) the n for this time series would be reduced by a factor 24. This would be the case if K was a gas-linked index. As a consequence, also the impact of n in equation (22) would be smaller by a factor $\sqrt{24} \simeq 5$. This effect was also noticed in [2], where some parameters had to be estimated on a much smaller dataset and thus carried the highest significant parameter risk

[14] Note that, according to [9, 17], ROs are instruments that enhance competition in the electricity market.

price, as well as on the volatility of the strike price itself. The mean reversion speed of the process reduces the impact of the starting point, which was another expected result. However, when both the strike price and the electricity price are assumed to be stochastic processes, the value of the RO depends crucially on their correlation coefficient ρ. In particular, a positive correlation reduces the value of the RO. Moreover, there is a non-monotone impact of the volatility of one process, depending on the level of volatility of the other process and on a positive correlation. We also provide a parametric model risk analysis, which revealed that the most important risk source is the seasonality, while the single parameter carrying the most risk is the volatility. The parametric model risk we use here allows us to quantifies the magnitude of these combined effects. To show it, we calculate them with regards to the value of the RO we estimate using Italian market data.

Our results are also relevant to support a proper design of ROs, in particular avoiding undesired outcomes. For instance, our analysis shows that ROs might not contribute to deliver security of supply, providing very little remuneration to capacity. This is the outcome predicted by our study when the strike price of the ROs is defined as a mark-up on the marginal cost of power production, as it is for the Italian case. In this case, the electricity price and the strike price of the RO covariate positively and this implies that ROs have a low value, for every possible starting value of the state variables P and K. The limited revenue raised by ROs might even hinder security of supply, which is the opposite of what CRMs are designed for.

More in general, our results show that a careful estimate of the parameters is needed to calculate the value of the ROs. *Ceteris paribus*, the RO value will be lower as the volatility of the electricity price decreases, the strike price increases, the speed of mean reversion increases, the correlation of the electricity price with the strike price increases (if the strike price is allowed to change over time), and the two volatilities are closer. These are all factors that need to be taken into account when designing the market for ROs and calculating the equilibrium value.

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Appendix

A.1 Proofs of pricing formulae

Proof of Proposition 4.1. The quantity $f(s,\omega) := e^{-rs}Q(P_s(\omega)-K)^+$ in Equation (2) is non-negative.

Then, if we set

$$A(K,P_0,s) := e^{-rs}\mathbf{E}^{\mathbb{Q}}\left[(P_s-K)^+\middle|\mathcal{F}_0\right], \tag{A.1}$$

by Tonelli's theorem, we get

$$RO(T_1,T_2) = Q\int_{T_1}^{T_2} A(K,P_0,s)ds\,. \tag{A.2}$$

$A(K, P_0, s)$ is clearly the price of a European call option with strike price K and maturity s, thus Equation (6) is simply obtained with the Black and Scholes formula. □

Proof of Proposition 4.2. As in the proof of Proposition 4.1, if we write

$$A(K_0, P_0, s) \colon = e^{-rs}\mathbf{E}^{\mathbb{Q}}\left[(P_s - K_s)^+ \middle| \mathcal{F}_0 \right], \tag{A.3}$$

then, by Tonelli's theorem, we have

$$RO(T_1, T_2) = Q \int_{T_1}^{T_2} A(K_0, P_0, s) ds\,.$$

Here, $A(K, P_0, s)$ is the price of an exchange option between the electricity price P and the strike price K, with maturity s, thus Equation (8) is simply obtained with the Margrabe formula with dividends (see [15]).

□

Proof of Proposition 4.3. As in the previous proofs, we write $A(K, P_0, s) \colon = e^{-rs}\mathbf{E}^{\mathbb{Q}}\left[(P_s - K)^+ \middle| \mathcal{F}_0 \right]$ and apply Tonelli's theorem to obtain

$$RO(T_1, T_2) = Q \int_{T_1}^{T_2} A(K, P_0, s) ds\,.$$

We now notice that

$$A(K, P_0, s) = e^{-rs}\mathbf{E}^{\mathbb{Q}}\left[(f(s,s) - K)^+ \middle| \mathcal{F}_0 \right]$$

where $f(t, s)$, $t \in [0, s]$, has the dynamics

$$df(t, s) = f(t, s)\sigma e^{-\lambda(s-t)}\, dW_t$$

The result then follows from the Black-Scholes formula with time-dependent (deterministic) volatility, which enters into the formula via the integral of its square, here equal to

$$\int_0^s \left(\sigma e^{-\lambda(s-t)} \right)^2 \, dt = \frac{\sigma^2}{2\lambda}(1 - e^{-2\lambda s}) = Var(s)$$

Equation (12) follows. □

Proof of Proposition 4.4. As before, we write $A(P_0, K_0, s) \colon = e^{-rs}\mathbf{E}^{\mathbb{Q}}\left[(P_s - K_s)^+ \mid \mathcal{F}_0 \right]$, we use Tonelli's theorem and obtain

$$RO(T_1, T_2) = Q \int_{T_1}^{T_2} A(P_0, K_0, s) ds\,.$$

Now, as in the proof of Proposition 3.3, we now notice that

$$A(P_0, K_0, s) = e^{-rs}\mathbf{E}^{\mathbb{Q}}\left[\left(f_P(s,s) - f_K(s,s)\right)^+ \middle| \mathcal{F}_0 \right]$$

where $f_i(t, s)$, $t \in [0, s]$, $I = P, K$, have the dynamics

$$\begin{aligned} df_P(t, s) &= f_P(t, s)\sigma_x e^{-\lambda_x(s-t)}\, dW_t^1, \\ df_K(t, s) &= f_K(t, s)\sigma_y e^{-\lambda_y(s-t)}\, dW_t^2, \end{aligned}$$

The result then follows from the Margrabe formula with time-dependent (deterministic) volatilities, which now enters into the formula via the integral of the squared volatility of $f_p(\cdot, s)/f_K(\cdot, s)$ (see e.g. [20]), here equal to

$$\int_0^s \left(\sigma_x^2 e^{-2\lambda_x(s-t)} + \sigma_y^2 e^{-2\lambda_y(s-t)} - 2\rho\sigma_x\sigma_y e^{-(\lambda_x+\lambda_y)(s-t)} \right) \, dt = \overline{Var}(s)$$

Equation (15) follows.

□

References

[1] Alexander. C. *Hedging the risk of an energy futures portfolio.* In: Risk management in commodity markets – from shipping to agriculturals and energy. H. Geman (ed.). Wiley Finance series. John Wiley & Sons, 2008

[2] Bannör, K., Kiesel, R., Nazarova, A. and Scherer, M. (2016), Parametric model risk and power plant valuation, *Energy Economics* **59**, 423–434

[3] Batlle, C., Mastropietro, P., Rodilla, P. and Perez-Arriaga, I.J. (2015) The system adequacy problem: lessons learned from the American continent. In: Chapter of the Book Capacity Mechanisms in the EU Energy Markets: Law, Policy, and Economics. Oxford University Press.

[4] Bhagwat, P.C. and Meeus, L. (2019) Reliability Options: Can they deliver on their promises? EUI Working Papers, RSCAS 2019/21, Robert Schuman Centre for Advanced Studies, Florence School of Regulation.

[5] Benth, F. E., Benth, J. Š. and Koekebakker, S. (2008). *Stochastic Modelling of Electricity and Related Markets.* Advanced Series on Statistical Science & Applied Probability, 11, World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2008.

[6] Benth, F. E. and Koekebakker, S. (2008), Stochastic modeling of financial electricity contracts, *Energy Economics* **30(3)**, 1116–1157.

[7] Benth, F. E., Piccirilli, M. and Vargiolu, T. (2019). Mean-reverting additive energy forward curves in a Heath-Jarrow-Morton framework. Math. Finan. Econ., to appear. DOI:10.1007/s11579-019-00237-x

[8] Bessembinder, H. and Lemmon, M. L. (2002), Equilibrium pricing and optimal hedging in electricity forward markets, *The Journal of Finance* **57 (3)**, 1347–1382.

[9] Bidwell, M. (2005), Reliability options: A market-oriented approach to long-term adequacy, *The Electricity Journal* **18(5)**, 11–25.

[10] Bjork, T. (1998). *Arbitrage Theory in Continuous Time.* Oxford University Press 1998.

[11] Borovkova, S. and Schmeck, M. D. (2015), Electricity price modeling with stochastic time change. Available at SSRN: `https://ssrn.com/abstract=2685045` or `http://dx.doi.org/10.2139/ssrn.2685045`

[12] Bunn, D., Andresen, A., Chen, D. and Westgaard, S. (2015), Analysis and forecasting of electricity price risks with quantile factor models. *Energy Journal* **37 (1)** 169–190.

[13] Burger, M. Klar, B. Mueller, A. Schindlmayr, G. (2004), A spot market model for pricing derivatives in electricity markets, *Quantitative Finance* **4(1)**, 109–122.

[14] Callegaro, G., Campi, L. Giusto, V. and Vargiolu, T. (2017). Utility indifference pricing and hedging for structured contracts in energy markets. *Mathematical Methods of Operations Research* **85 (2)**, 265–303.

[15] Carmona, R. and Durrleman, V. (2003). Pricing and Hedging Spread Options. *SIAM Review* **45 (4)**, 627–685.

[16] Clewlow, L. and Strickland, C. (1999), Valuing energy options in a one factor model fitted to forward prices, *SSRN Electronic Journal.*

[17] Cramton, P., Ockenfels A. and Stoft, S. (2013). Capacity Market Fundamentals. *Economics of Energy & Environmental Policy,* **2 (2)**, 28-48.

[18] Cramton, P. and Stoft, S. (2007). *Colombia Firm Energy Market*. Proceedings of the Hawaii International Conference.

[19] Creti, A. and Fontini, F. (2019). *Economics of Electricity*. Cambridge University Press, Cambridge, UK.

[20] Deng, S. Johnson, B. and Sogomonian, A. (2001), Exotic electricity options and the valuation of electricity generation and transmission assets, *Decision Support Systems* **30 (3)**, 383–392.

[21] Edoli, E. Gallana, M. and Vargiolu, T. (2017). Optimal intra-day power trading with a Gaussian additive process. *Journal of Energy Markets* **10 (4)**, 23–42.

[22] Edoli, E., Tasinato, D. and Vargiolu, T. (2013). Calibration of a multifactor model for the forward markets of several commodities. *Optimization,* **62 (11)**, 1553–1574.

[23] Federal Energy Regulatory Commission (2014). *Order on Tariff Filing and Instituting Section 206 Proceeding*. Docket No. ER14-1050-000, issued on May 30.

[24] Geman, H. and Roncoroni, A. (2006). Understanding the fine structure of electricity prices, *The Journal of Business* **79(3)**, 1225–1261.

[25] Hikspoors, S. and Jaimungal, S. (2007), Energy spot price models and spread options pricing, *International Journal of Theoretical and Applied Finance* **10 (7)**, 1111–1135.

[26] Hull, J. C. (2005), *Options, Futures and other Derivatives*, Prentice Hall, 6th edition.

[27] Joskow, P. and Tirole, J. (2007), Reliability and competitive electricity markets, *The RAND Journal of Economics,* **38(1)**, 60–84.

[28] Mastropietro, P., Herrero, I. Rodilla, P. and Batlle, C. (2016). A model-based analysis on the impact of explicit penalty schemes in capacity mechanisms. *Applied Energy,* **168**, 406–417.

[29] Mastropietro, P., Fontini, F. Rodilla, P. and Batlle, C. (2018). The Italian Capacity Remuneration Mechanism: Description and Open Questions. *Energy Policy,* **123**, 659–669.

[30] McDonald, R. and Siegel, D. (1985). Investment and the valuation of firms when there is an option to shut down. *International Economic Review* **26 (2)**, 331–349.

[31] Oren, S.S. (2005) Generation adequacy via call options obligations: Safe passage to the promised land. *The Electricity Journal* **18**, 28–42.

[32] Paraschiv, F., Fleten, S.-E. and Schürle, M. (2015), A spot-forward model for electricity prices with regime shifts, *Energy Economics* **47**, 142–153.

[33] Rodilla, P. and Batlle, C. (2013). *Security of generation supply in electricity markets*. In: Regulation of the Power Sector, 2013, Springer, Perez-Arriaga, Ignacio J. (Ed.).

[34] Single Electricity Market Committee, SEM (2015). *Integrated Single Electricity Market (I-SEM) - Capacity Remuneration Mechanism Detailed Design – Decision Paper 1.* Decision Paper SEM-15-103, released on December 16.

[35] Single Electricity Market Committee, SEM (2016a). *Integrated Single Electricity Market (I-SEM) - Capacity Remuneration Mechanism Detailed Design – Decision Paper 2.* Decision Paper SEM-16-022, released on May 10.

[36] Single Electricity Market Committee, SEM (2016b). *Integrated Single Electricity Market (I-SEM) - Capacity Remuneration Mechanism Detailed Design – Decision Paper 3*. Decision Paper SEM-16-039, released on July 8.

[37] Single Electricity Market Committee, SEM (2016c) Capacity Requirement and De-Rating Factor Methodology Detailed Design – Decision Paper. Decision Paper SEM-16-082, released on December 8.

[38] Sezgen, O., Goldman C.A. and Krishnarao P. (2007). Option value of Electricity demand response, *Energy* **32 (2)**, 108–119.

[39] TERNA (2019) Disciplina del sistema di remunerazione della disponibilità di capacità di energia elettrica - Fase di prima attuazione. Approvata con D.M. del 28 giugno 2019.

[40] TERNA (2017). *Metodologia per il calcolo del prezzo di esercizio*. Annex to Terna (2017a), released in January 2017. Available at http://download.terna.it/terna/0000/0842/29.PDF.

[41] Vázquez, C., Rivier, M. and Perez-Arriaga, I.J. (2002). A Market Approach to Long-Term Security of Supply. *IEEE Transactions on Power Systems* **17 (2)**, 349–357.

[42] Vehvilainen, I. (2002), Basics of electricity derivative pricing in competitive markets, *Applied Mathematical Finance* **9(1)**, 45–60.

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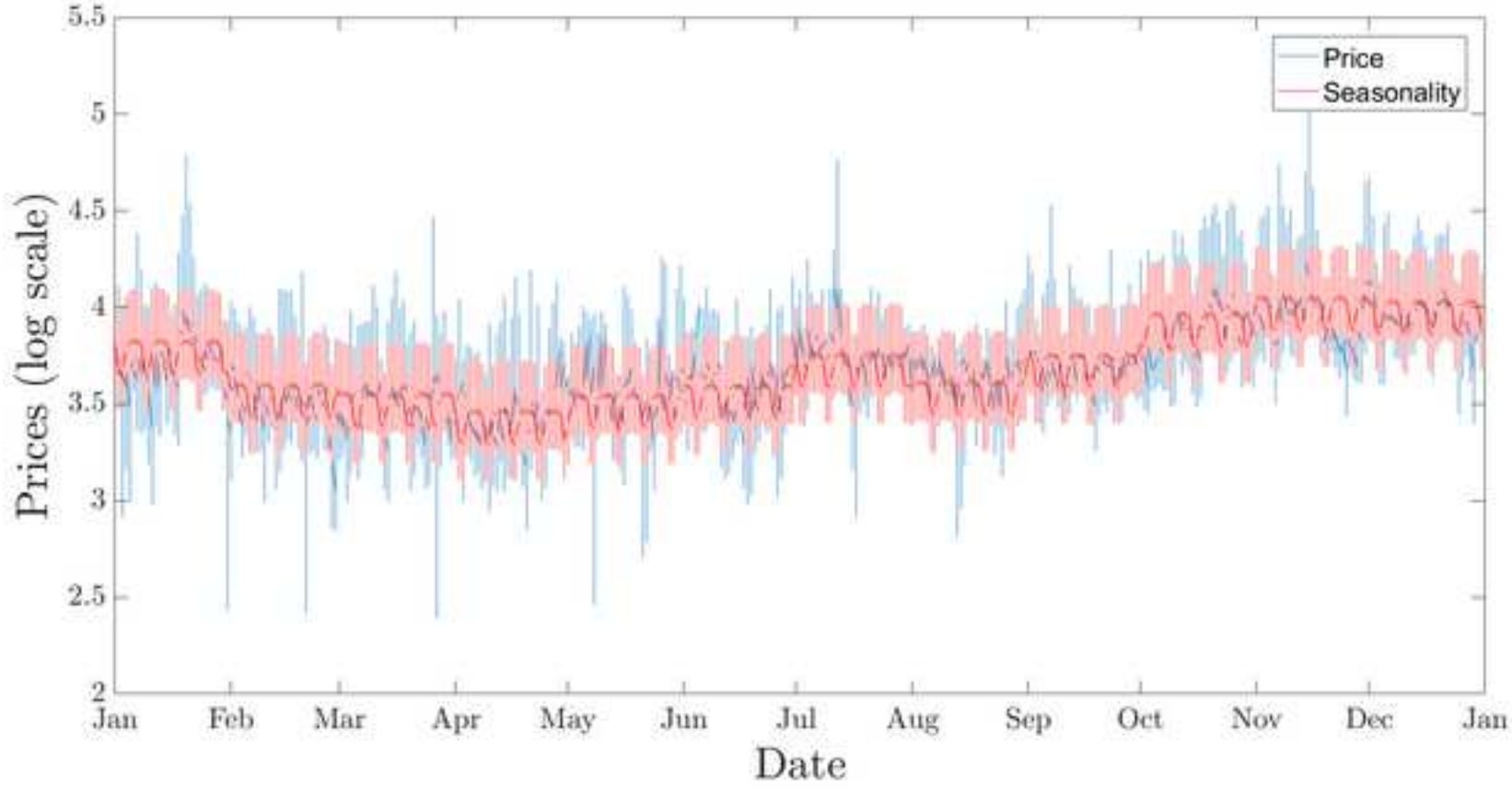

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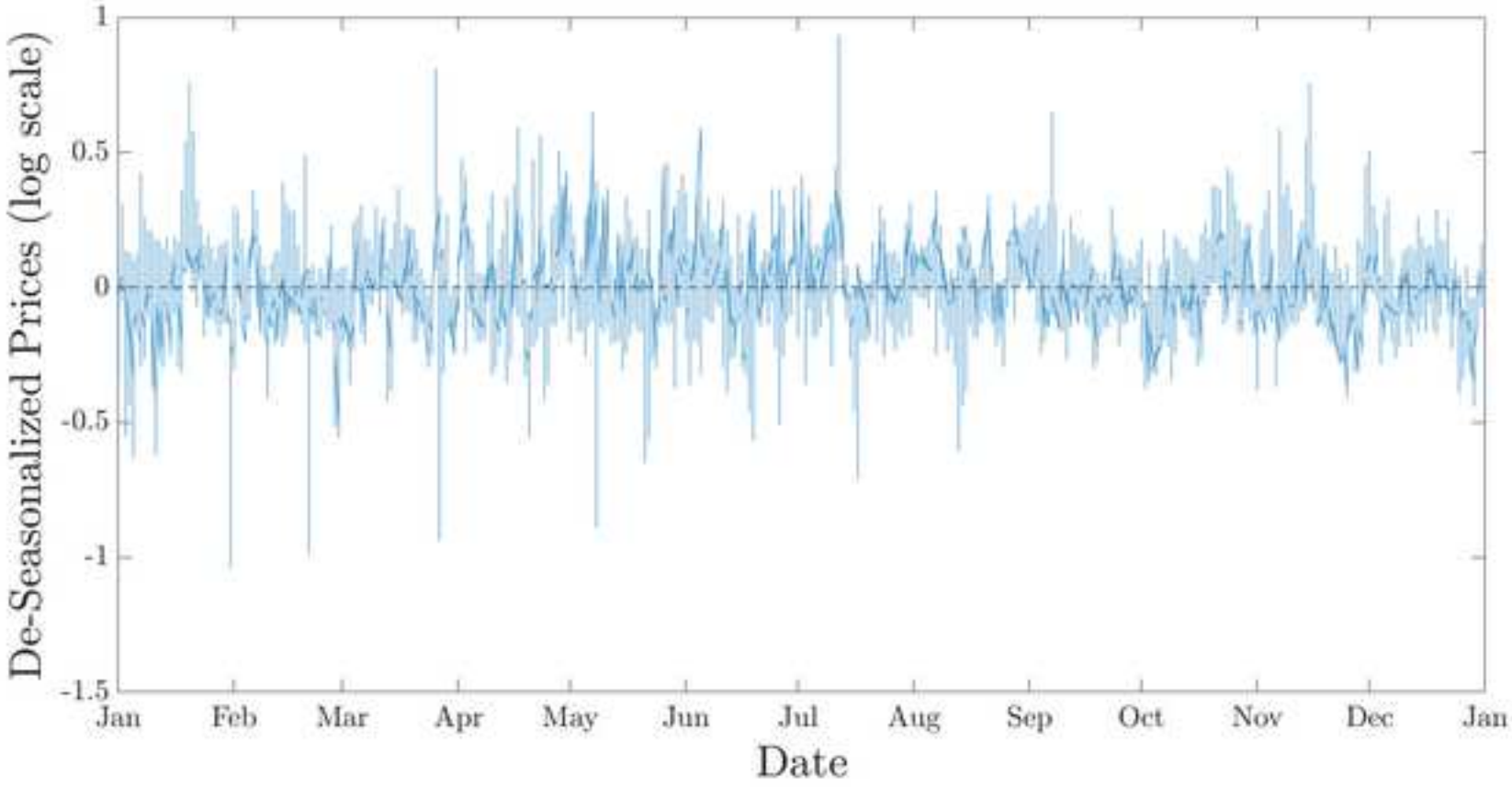

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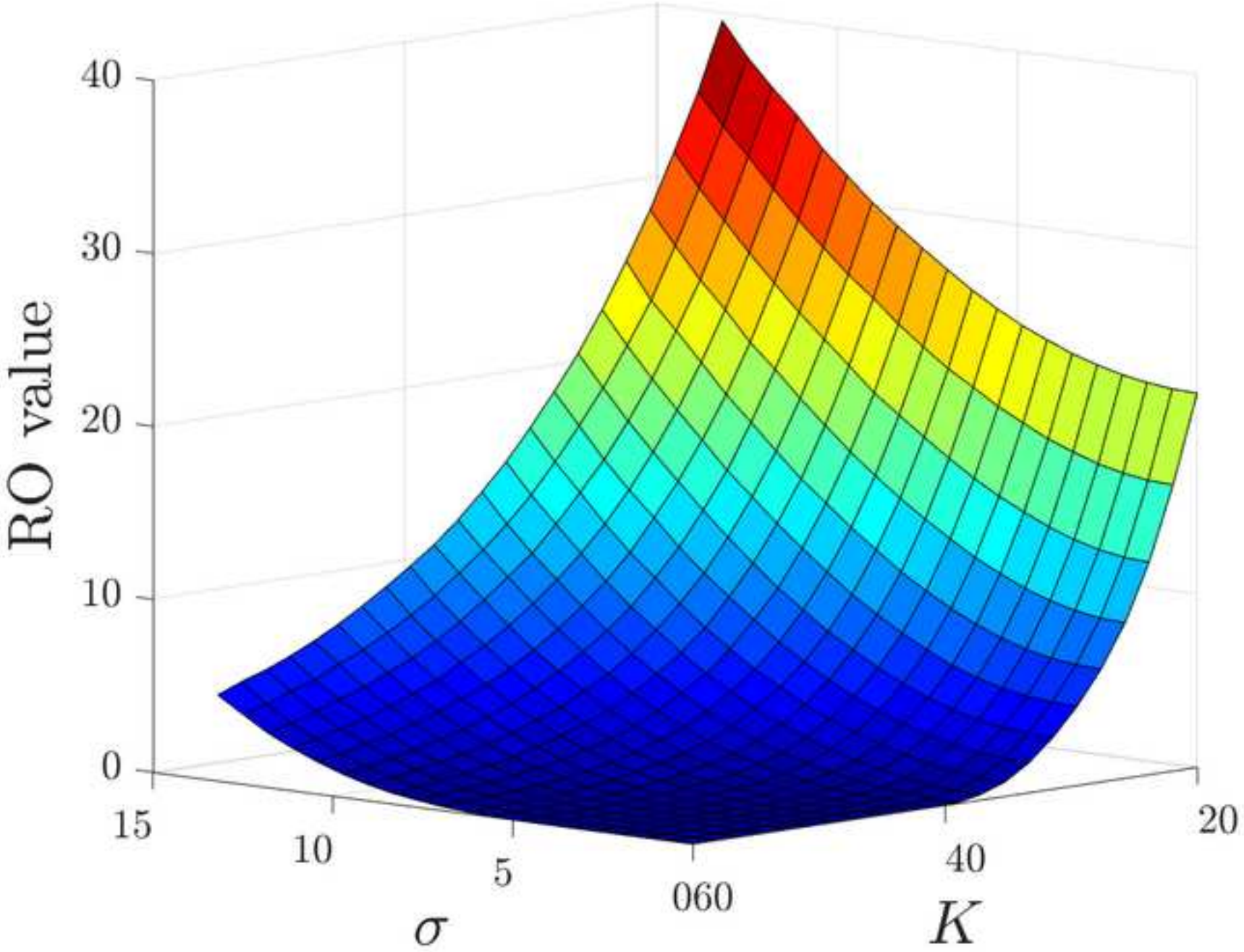
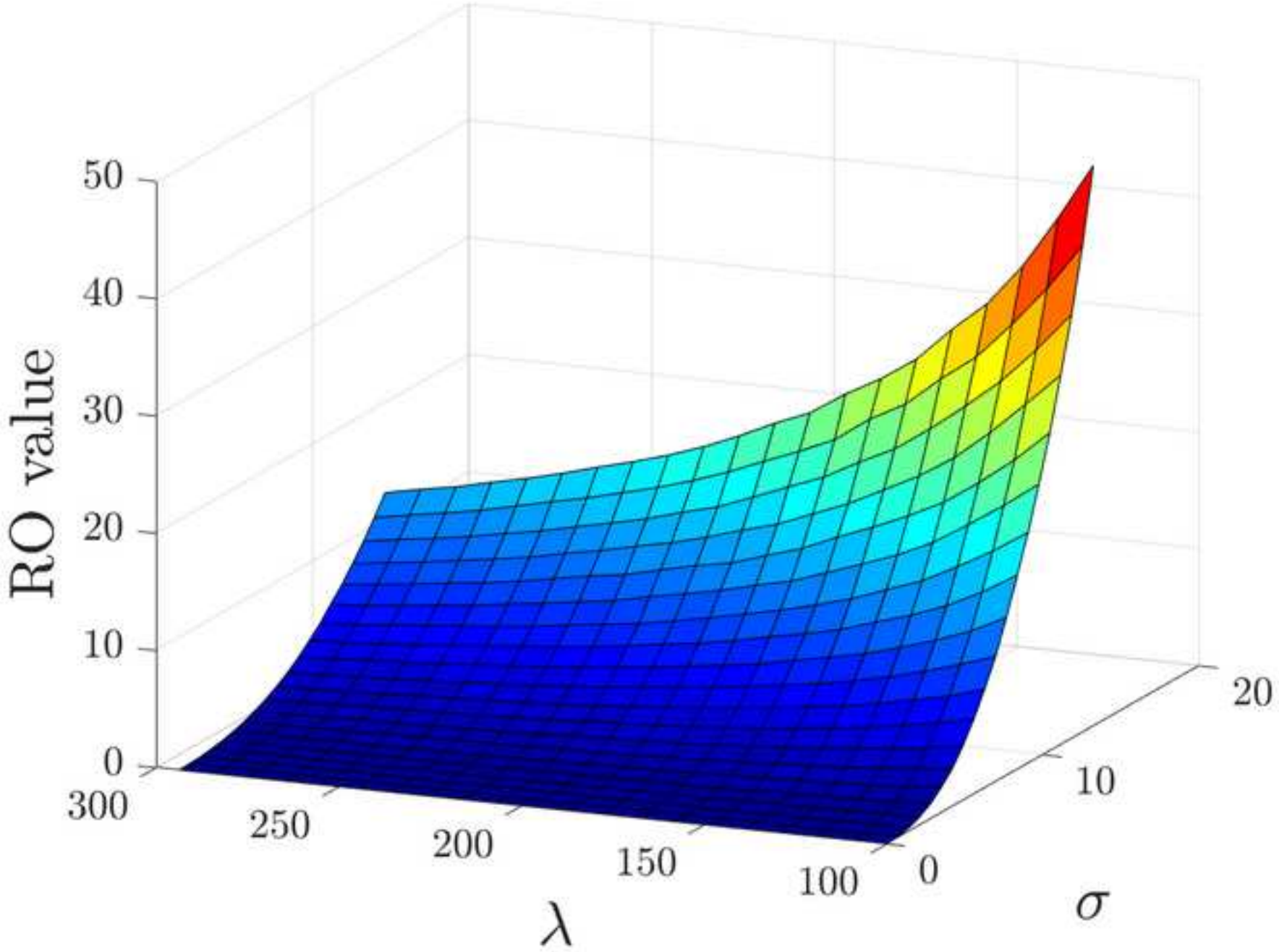
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