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Finite strains effects in 3D fully coupled geomechanical simulations
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Abstract:	Numerical modelling of geomechanical phenomena and geo-engineering problems often involves complex issues related to several variables and corresponding coupling effects. Under certain circumstances, both soil and rock may experience a non-linear material response, due to e.g. plastic, viscous or damage behavior, and even a non-linear geometric one, due to large deformations/displacements of the solid. Furthermore, the presence of one or more fluids (water, oil, gas, etc.) within the skeleton must be accounted for when evaluating the interaction between the different phases of the continuum body. A multiphase three-dimensional coupled model in finite strains, suitable for dealing with solid-displacements/fluid-diffusion problems, is here described and an elasto-plastic behavior for the solid phase is assumed. Particularly, a 3D mixed finite element is implemented to fulfill stability requirements of the adopted formulation, as well as a permeability tensor dependent on deformation is introduced. A consolidation scenario induced by filling of silos is investigated and the effects of the adoption of finite strains discussed.

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Finite strains effects in 3D fully coupled geomechanical simulations

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ABSTRACT

Numerical modelling of geomechanical phenomena and geo–engineering problems often involves complex issues related to several variables and corresponding coupling effects. Under certain circumstances, both soil and rock may experience a non-linear material response, due to e.g. plastic, viscous or damage behavior, and even a non-linear geometric one, due to large deformations/displacements of the solid. Furthermore, the presence of one or more fluids (water, oil, gas, etc.) within the skeleton must be accounted for when evaluating the interaction between the different phases of the continuum body. A multiphase three-dimensional coupled model in finite strains, suitable for dealing with solid-displacements/fluid-diffusion problems, is here described and an elasto-plastic behavior for the solid phase is assumed. Particularly, a 3D mixed finite element is implemented to fulfill stability requirements of the adopted formulation, as well as a permeability tensor dependent on deformation is introduced. A consolidation scenario induced by filling of silos is investigated and the effects of the adoption of finite strains discussed.

Keywords: Porous media, multiphase problems, finite strains, elasto-plasticity, Finite Element Method.

INTRODUCTION

Geomaterials such as soil, rock or concrete, are basic materials in the civil engineering field, with many different applications. The description of their mechanical behavior is a challenging task, requiring sophisticated numerical analyses. Particularly, such materials must be considered as multiphase porous media, composed by a solid skeleton and one (or even more) fluid within the pores. Hence, geomechanical problems are characterized by solid-fluid interaction, due to the presence of overlapping phases, and correspondingly a coupled analysis is required (Lewis and Schrefler 1998). Further, geomaterials can also experience material non-linearities of the solid skeleton, e.g. due to plasticity, creep or damage. Even in the elastic regime, the mechanical behavior of geomaterials is often non-linear.

Depending on the phenomenon to model, it may be necessary to take into account finite deformations, so introducing a source of geometric non-linearity in the formulation of the problem (Wang et al. 2009). Examples of geomechanical problems where finite deformations are involved are the inception of slopes (Lee et al. 2012; Zhu and Randolph 2009; Mohammadi and Taiebat 2014), the consolidation of heavy structures over soft soils (Bienen et al. 2015; Andresen et al. 2010), the excavation of tunnels (Meguid et al. 2002) and wellbores (Spiezia et al. 2016), the consolidation settlements around pile foundations (Osman and Randolph 2011; Zhang et al. 2015) and the consolidation of mine waste tailings (Caldwell et al. 1984), just to recall a few.

In the last two decades, theoretical and computational research has provided wide support for the solution of this kind of problems, where large deformations are encountered. Although several innovative methods have been proposed in literature, such as the Smoothed Particle Hydrodinamics (SPH) (Wang et al. 2016), the Material Point Method (MPM) (Abe et al. 2013), the Particle Finite Element Method (PFEM) (Carbonell et al. 2009) and the Meshless Local Petrov-Galerkin (MLPG) (Atluri and Zhu 1998), the Finite Element Method (FEM) is still probably the most widely used tool. It allows to solve the set of differential equations arising from the imposition of the balance equations to a continuum multiphase body, computing displacements, stress and strain fields, pressures etc. for the solid-fluid mixture. By using a FEM approach (Hughes 2012; Wriggers 2008),

finite strains can be rigorously taken into account as an extension of the infinitesimal framework, adopting an adequate formulation for both the balance laws and the constitutive model.

Even though the theory for coupled poromechanics in finite strains, solved within the framework of FEM, has been proposed in the late nineties by the pioneer works of Simo et al. (Simo and Meschke 1993), Borja et al. (Borja and Alarcón 1995; Borja et al. 1998) and Armero et al. (Armero 1999), this subject presents still some aspects that are worth being studied further.

In fact, even if in some recent works the effects of assuming finite strains when simulating geotechnical problems have been investigated (Nazem et al. 2006; Kardani et al. 2013; Zhang et al. 2018), only a few take into account the coupling between the different phases (Huang et al. 2014; Singh et al. 2016; Qi et al. 2017), which is an aspect of relevance when dealing with porous materials.

Hence in this work a coupled hygro–mechanical model in finite strains (Borja 2013) based on the modified mixture theory (Borja et al. 1998) is presented, following the lines of (Spiezia et al. 2016) for saturated porous media.

Particularly, the approach takes advantage of a constitutive model (Borja and Tamagnini 1998) which has demostrated to be particularly suitable in predicting different features of the granular materials as pressure sensitivity, hardening response with large plastic volumetric compaction, softening response with plastic dilation and coupled volumetric deviatoric plastic deformations. The model has been upgraded via the introduction of a permeability tensor variable with the deformation of the solid skeleton, as well as of a specific type of hexahedral elements, developed to guarantee solvability and stability of mixed formulations (Brezzi and Bathe 1990). The paper is organized as follows.

First, the balance equations, together with the constitutive laws for both the fluid and the solid phase, are briefly recalled (Borja and Alarcón 1995; Borja et al. 1998; Song and Borja 2014).

Section 2 presents the numerical implementation of the developed equations, describing in detail the formulation of the three-dimensional code and its novel features.

Section 3 presents the validation of the code, by comparing the results with the benchmark cases described in (Borja et al. 1998) and with the experimental results reported in (Callari et al. 1998;

[Al-Tabbaa 1987](#)).

Section 4 presents the numerical simulation of a consolidation process due to the filling of two tall structures over a fully saturated domain, evidencing the code capabilities in simulating soil–structure interaction.

Notations and symbols used throughout the paper are as follows: bold-face letters denote matrices and vectors; the symbol '·' denotes an inner product of two vectors (e.g. $\boldsymbol{a} \cdot \boldsymbol{b} = a_i b_i$) or a single contraction of adjacent indices of two tensors (e.g. $\boldsymbol{c} \cdot \boldsymbol{d} = c_{ij} d_{jk}$); the symbol ':' denotes an inner product of two second-order tensors (e.g. $\boldsymbol{c} : \boldsymbol{d} = c_{ij} d_{ij}$), or a double contraction of adjacent indices of tensor of rank two and higher (e.g. $\boldsymbol{C} : \boldsymbol{\epsilon}^e = C_{ijkl} \epsilon^e_{kl}$); the symbol '⊗' denotes a juxtaposition, e.g. $(\boldsymbol{a} \otimes \boldsymbol{b})_{ij} = a_i b_j$. For any symmetric second-order tensor $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ we have $(\boldsymbol{\alpha} \otimes \boldsymbol{\beta})_{ijkl} = \alpha_{ij} \beta_{kl}$, $(\boldsymbol{\alpha} \ominus \boldsymbol{\beta})_{ijkl} = \alpha_{il} \beta_{jk}$ and $(\boldsymbol{\alpha} \oplus \boldsymbol{\beta})_{ijkl} = \alpha_{jl} \beta_{ik}$. A positive stress is also assumed for tension according to the solid mechanics convention.

THEORETICAL FRAMEWORK

This section briefly recalls the coupled balance laws for a fully saturated porous media ([Borja and Alarcón 1995](#)), together with the constitutive model for both solid and fluid phase.

Balance laws

In agrement with the mixture theory and by assuming incompressibility of the two phases, the balance of linear momentum and the balance of mass for a fully saturated porous medium in the quasi static regime are

$$\rho \, \boldsymbol{g} + div \, \tilde{\boldsymbol{\sigma}} = 0 \, ; \tag{1}$$

$$div \, \boldsymbol{v} + div \left[\varphi(\boldsymbol{v}^F - \boldsymbol{v}) \right] = 0, \tag{2}$$

where $\rho_0 = J\rho$ is the reference mass density, $\boldsymbol{g}$ the vector of gravity acceleration, $\tilde{\boldsymbol{\sigma}}$ the total Cauchy stress tensor, related to the total Kirchhoff stress tensor $\tilde{\boldsymbol{\tau}}$ and the total first Piola-Kirchhoff tensor

$\tilde{\boldsymbol{P}}$ (Marsden and Hughes 1994) via the following expression

$$\tilde{\boldsymbol{P}} = \tilde{\boldsymbol{\tau}} \cdot \boldsymbol{F}^{-T} = J\tilde{\boldsymbol{\sigma}} \cdot \boldsymbol{F}^{-T}. \tag{3}$$

The term J is the Jacobian defined from the deformation gradient $\boldsymbol{F}$ of the motion $\boldsymbol{\phi}$

$$J = det(\boldsymbol{F}) ; \qquad \boldsymbol{F} = \frac{\partial \boldsymbol{\phi}}{\partial \boldsymbol{X}} ; \qquad \boldsymbol{\phi} = \boldsymbol{X} + \boldsymbol{u} . \tag{4}$$

In Eq. (2), $\boldsymbol{v}$ is the vector the solid phase velocity vector, $\boldsymbol{v}^F$ is the fluid phase intrinsic velocity and φ is the porosity of the soil skeleton defined through the Jabocobian as

$$\varphi = \frac{J dV - (1 - \varphi_0) dV}{J dV} = 1 - \frac{(1 - \varphi_0)}{J}. \tag{5}$$

The term $\varphi(\boldsymbol{v}^F - \boldsymbol{v})$ represents the Darcy velocity $\tilde{\boldsymbol{v}}$, defined as the relative volumetric rate of fluid per unit area through the deforming soil mass.

Constitutive models

Solid phase

For the description of the elasto-plastic mechanical behavior of the solid we employ the multiplicative decomposition of the deformation gradient and the product formula algorithm described by Simo (Simo 1992)

$$\boldsymbol{F} = \frac{\partial \boldsymbol{\phi}}{\partial \boldsymbol{x}^u} \cdot \frac{\partial \boldsymbol{x}^u}{\partial \boldsymbol{X}} \equiv \boldsymbol{F}^e \cdot \boldsymbol{F}^p ; \qquad \forall \boldsymbol{X} \in \mathscr{B} ; \qquad t \geq 0, \tag{6}$$

where $\boldsymbol{x}^u$ is the intermediate unloaded configuration. From the second law of thermodynamics we can define the following set of constitutive relations describing the elastoplastic process

$$\boldsymbol{\tau} = 2\frac{\partial \Psi}{\partial \boldsymbol{b}^e} \cdot \boldsymbol{b}^e ; \qquad -\frac{1}{2}\mathscr{L}_v \boldsymbol{b}^e = \dot{\gamma}\frac{\partial \mathscr{F}}{\partial \boldsymbol{\tau}} \cdot \boldsymbol{b}^e ; \qquad \dot{\xi} = \dot{\gamma}\frac{\partial \mathscr{F}}{\partial \chi}, \tag{7}$$

where $\boldsymbol{\Psi} = \boldsymbol{\Psi}(X, \boldsymbol{b}^e, \xi)$ is the stored energy function, $\boldsymbol{b}^e = \boldsymbol{F}^e \cdot \boldsymbol{F}^{e^T}$ the left elastic Cauchy-Green strain tensor, ξ a plastic variable and $\chi = \partial \boldsymbol{\Psi}/\partial \xi$ the hardening response of the solid matrix. The quantity $\mathscr{L}_v \boldsymbol{b}^e$ is the Lie derivative of $\boldsymbol{b}^e$, while $\mathscr{F}$ is the yield function and $\dot{\gamma}$ is a non-negative plastic multiplier satisfying the Kuhn-Tucker conditions: $\dot{\gamma} \geq 0$, $\mathscr{F}(\boldsymbol{\tau}, \chi) \leq 0$ and $\dot{\gamma}\mathscr{F}(\boldsymbol{\tau}, \chi) = 0$.

The elastic left Cauchy-Green tensor $\boldsymbol{b}^e$ and the Kirchhoff effective stress tensor $\boldsymbol{\tau}$ can be expressed through the spectral decomposition

$$\boldsymbol{b}^e = \sum_{A=1}^{3} (\lambda_A^e)^2 \boldsymbol{m}^{(A)}; \qquad \boldsymbol{\tau} = \sum_{A=1}^{3} \beta_A \boldsymbol{m}^{(A)}, \qquad \boldsymbol{m}^{(A)} = \boldsymbol{n}^{(A)} \otimes \boldsymbol{n}^{(A)}, \tag{8}$$

where λ_A^e are the the elastic principal stretches, $\boldsymbol{\beta}_A$ the principal Kirchhoff effective stresses and $\boldsymbol{n}^{(A)}$ the principal direction for both strains and stresses thanks to isotropy assumption. Together with frame indifference assumption, the free energy can be written as a function of the elastic principal strains

$$\boldsymbol{\Psi}(X, \boldsymbol{b}^e) = \Psi(X, \varepsilon_1^e, \varepsilon_2^e, \varepsilon_3^e); \qquad \varepsilon_A^e = ln(\lambda_A^e); \qquad A = 1, 2, 3, \tag{9}$$

where ε_A^e are the principal elastic logarithmic strains, and from the first relation of Eq. (7) we obtain the following relation

$$\beta_A = \frac{\partial \Psi}{\partial \varepsilon_A^e}; \qquad A = 1, 2, 3. \tag{10}$$

In the present work, we adopt the following stored energy function (Borja and Tamagnini 1998) describing the elastic response of the soil in terms of volumetric ε_v^e and deviatoric ε_s^e elastic strain invariants

$$\Psi(\varepsilon_v^e, \varepsilon_s^e) = \tilde{\Psi}(\varepsilon_v^e) + \frac{3}{2}\mu^e \varepsilon_s^{e2}, \tag{11}$$

$$\tilde{\Psi}(\varepsilon_v^e) = -P_0 \hat{k} \, exp \, \Omega; \qquad \Omega = -\frac{\varepsilon_v^e - \varepsilon_{v0}^e}{\hat{k}}; \qquad \mu^e = \mu_0 + \frac{\alpha}{\hat{k}} \tilde{\Psi}, \tag{12}$$

where $\tilde{\Psi}$ is the contribution given by the isotropic part, $\hat{k}$ the elastic compressibility index, μ^e the elastic shear modulus, μ_0 a constant term, α a parameter coupling shear and volumetric parts and

finally P_0 the mean reference normal Kirchhoff stress invariant. The yield function is given by

$$\tilde{\mathscr{F}}(P, Q, P_c) = \frac{Q^2}{M^2} + P(P - P_c) = 0, \tag{13}$$

where P and Q are the effective Kirchhoff stress invariants, P_c the Kirchhoff preconsolidation pressure defining the size of the ellipsoid and M the slope of the critical state line, as shown in Fig. 1.

The model assumes a bi-logarithmic hardening law, as shown in Fig. 2, described by the following equation

$$ln\left(\frac{v}{v_0}\right) = -\hat{\lambda} ln\left(\frac{P_c}{P_{c0}}\right), \tag{14}$$

where $\hat{\lambda}$ is the virgin compression index, $v = V/V_S = 1 + e$ the specific volume of the soil and v_0 a reference value.

The hardening law governing the expansion/contraction of the ellipse through the parameter P_c is given by

$$\frac{\dot{P}_c}{P_c} = -\Theta \dot{\varepsilon}_v^p, \tag{15}$$

with $\Theta = 1/(\hat{\lambda} - \hat{\kappa})$.

Fluid phase

For the fluid phase, assuming laminar flow, the model adopts the generalized Darcy's law

$$\tilde{\boldsymbol{v}} = -\boldsymbol{k} \cdot grad\,\Pi, \tag{16}$$

where $\tilde{\boldsymbol{v}} = \varphi(\boldsymbol{v}^F - \boldsymbol{v})$ is the Darcy velocity and Π is the total fluid potential defined as

$$\Pi = \Pi^p + \Pi^e = \frac{p}{g\rho_F} - \Pi^e, \tag{17}$$

where Π^p is the pressure potential with p the the Cauchy pore pressure and ρ_F the mass density of the fluid; Π^e is the elevation potential. Finally, $\boldsymbol{k}$ is the second order permeability tensor,

which is assumed to be dependent on the deformation of the solid skeleton through the Jacobian J,

163 introducing therefore an additional source of non-linearity with respect to the original formulation

164 proposed in (Borja et al. 1998).

165 According to the Kozeny-Carman equation (Song and Borja 2014), the permeability reads

$$166 \qquad \boldsymbol{k}(J) = \frac{\rho_F g}{\mu} \frac{D^2}{180} \frac{\left[J - (1 - \varphi_0)\right]^3}{J(1 - \varphi_0)^2} \boldsymbol{1}, \tag{18}$$

167 where D is the effective diameter of the grains, μ the dynamic viscosity of water, φ_0 the initial

168 porosity of the solid and $\boldsymbol{1}$ the second order identity tensor.

NUMERICAL IMPLEMENTATION

Variational equations

171 For developing the variational counterpart of Eqs. (1) and (2), following the approach proposed

172 in (Borja and Alarcón 1995), we consider a fully saturated solid domain $\mathscr{B} \in R^{n_{sd}}$ and define the

173 motion of the solid phase ϕ, its first variation $\boldsymbol{\eta}$, the Cauchy pore pressure p and its first variation

174 ψ. The variational equation of the linear momentum G reads

$$175 \qquad G(\phi, p, \boldsymbol{\eta}) = \int_{\mathscr{B}} (grad\,\boldsymbol{\eta} : \tilde{\boldsymbol{\tau}} - \rho_0 \boldsymbol{\eta} \cdot \boldsymbol{g}) dV - \int_{\partial \mathscr{B}^{\mathrm{t}}} \boldsymbol{\eta} \cdot \boldsymbol{t}\, dA = 0, \tag{19}$$

176 and the variational equation of the mass balance equation H reads

$$177 \qquad H(\phi, p, \psi) = \int_{\phi_{\mathrm{t}}(\mathscr{B})} (\psi\, div\,\boldsymbol{v} - grad\,\psi \cdot \tilde{\boldsymbol{v}}) dv - \int_{\partial \phi_{\mathrm{t}}^{\mathrm{h}}(\mathscr{B})} \psi q\, da = 0. \tag{20}$$

178 These field equations G and H are expressed in the Eulerian form for developing an *updated*

179 *Lagrangian* formulation, allowing for obtaining the solution of the non-linear coupled problem.

180 We rewrite the equations in the following way

$$181 \qquad G(\phi, p, \boldsymbol{\eta}) = \int_{\mathscr{B}} (grad\,\boldsymbol{\eta} : \boldsymbol{\tau} - Jp\, div\boldsymbol{\eta} - J\rho\, \boldsymbol{\eta} \cdot \boldsymbol{g}) dV - \int_{\partial \mathscr{B}^{\mathrm{t}}} \boldsymbol{\eta} \cdot \boldsymbol{t}\, dA\,; \tag{21}$$

$$H(\phi, p, \psi) = \int_{\mathscr{B}} \psi \dot{J} dV + \int_{\mathscr{B}} grad\,\psi \cdot \frac{J\boldsymbol{k}}{g\rho_F} \cdot (grad\,p - \rho_F \boldsymbol{g}) dV - \int_{\partial\mathscr{B}^h} \psi Q\,dA\,. \qquad (22)$$

In the first equation, the total Kirchhoff stress tensor has been decomposed into the effective part $\boldsymbol{\tau}$ and the pore pressure part Jp, through the classical Terzaghi's formula. The second equation is obtained from the definition of the Darcy's velocity, recalling that the time derivative of the Jacobian J is equal to $\dot{J} = J\,div\,\boldsymbol{v}$ (Marsden and Hughes 1994). Furthermore this integral is formulated with respect to the initial configuration $\mathscr{B}$, and hence $dv = JdV$ has been substituted. $\boldsymbol{t}$ is the prescribed stress vector on the boundary $\partial\mathscr{B}^t$ and Q is the prescribed rate of flux across the boundary $\partial\mathscr{B}^h$, assumed as positive when the fluid goes into the solid matrix. The condition $q = Q = 0$ means that no fluid flows through the boundary. The presence of $\dot{J}$ inside Eq. (22) requires the semi-discretization of the second variational equation

$$
\begin{aligned}
H_{\Delta t}(\phi, \theta, \psi) = &\int_{\mathscr{B}} \frac{\psi}{\Delta t}(J_{n+1} - J_n)dV \\
&- \int_{\mathscr{B}} \left[\beta(grad\,\psi \cdot J\tilde{\boldsymbol{v}})_{n+1} + (1-\beta)(grad\,\psi \cdot J\tilde{\boldsymbol{v}})_n\right]dV \\
&\int_{\partial\mathscr{B}} \psi\left[\beta Q_{n+1} + (1-\beta)Q_n\right]dA,
\end{aligned}
\qquad (23)
$$

where $\Delta t = t_{n+1} - t_n$ and β is the trapezoidal integration parameter defining the three time integration schemes: for $\beta = 0$ the Explicit Eulero, for $\beta = 1/2$ the Crank-Nicolson and for $\beta = 1$ the Implicit Eulero scheme, respectively.

The first variation of the variational equations $G(\phi, \theta, \boldsymbol{\eta})$ and $H(\phi, \theta, \psi)$, necessary for the solution of the problem through a Newton-Raphson scheme, is reported in Appendix A. By comparing the equations with those proposed in (Borja et al. 1998), a new contribution arises (see Eq. (45)), due to the permeability variation of the porous medium. The variation of the second order permeability tensor $\boldsymbol{k}$ with respect to the Jacobian of the gradient tensor J gives

$$\boldsymbol{k}'(J) = \frac{\partial\boldsymbol{k}}{\partial J} = \frac{\rho_F g}{\mu}\frac{D^2}{180}\frac{3[J - (1-\varphi_0)]^2 J - [J - (1-\varphi_0)]^3}{J^2(1-\varphi_0)^2}\boldsymbol{1}\,. \qquad (24)$$

Numerical integration

A three-dimensional mixed finite element has been implemented within the research code, which combines a three-quadratic 20–node displacement interpolation with a three-linear 8–node pore pressure interpolation (Fig. 3), so to fulfill the necessary stability requirements, satisfy the ellipticity requirement and the Ladyzhenskaya-Babuška-Brezzi (LBB) condition (Brezzi and Bathe 1990; Arnold 1990). The adopted element belongs to the Taylor-Hood family (Arnold et al. 1984; Guzmán and Sánchez 2015), in which the displacement interpolation is one-order higher than the pressure one. By assuming a quadratic order function for the displacement field, the continuity for the stress/strain field is also guaranteed.

Correspondingly, $N^\phi(x)$ and $N^p(x)$ indicate the shape function for the solid phase ϕ and the pore pressure field p. The displacements field $u^h(x) \in R^{n_{sd}}$, with $n_{sd} = 3$ becomes

$$u^h(x) = N^\phi(x)\{d + d_g\}, \tag{25}$$

where $d \in R^{NQ}$ and d_g are the unknown nodal displacements and the prescribed nodal displacements vector, respectively, $NQ = 20$. In the same way, the pore pressure field $p^h(x) \in R^1$ is expressed as

$$p^h(x) = N^p(x)\{p + p_r\}, \tag{26}$$

where $p \in R^{NP}$ is the unknown nodal pore pressures vector while p_r is the prescribed nodal pore pressures vector, with $NP = 8$. The weight functions η and ψ may be written as

$$\eta^h(x) = N^\phi(x)\tilde{\eta}; \qquad \psi^h(x) = N^p(x)\tilde{\psi}, \tag{27}$$

where $\tilde{\eta} \in R^{NQ}$ and $\tilde{\psi} \in R^{NP}$. The discretized form of Eq. (21) becomes

$$G^h(\phi, p, \tilde{\eta}) = \tilde{\eta}^T \left[N^S(d) + N^F(p) - F_{EXT} \right] = 0, \tag{28}$$

where:

$$N^S(d) = \int_{\mathcal{B}} B^T \hat{\tau} \, dV \, ; \tag{29a}$$

$$N^F(p) = -\int_{\mathcal{B}} b^T (N^p p + N_r^p p_r) J \, dV \, ; \tag{29b}$$

$$F_{EXT} = \int_{\mathcal{B}} \rho_0 N^{\phi T} G \, dV + \int_{\partial \mathcal{B}} N^{\phi T} t \, dA \, . \tag{29c}$$

The quantity $\hat{\tau} = \{\tau_{11}, \tau_{22}, \tau_{33}, \tau_{12}, \tau_{23}, \tau_{13}, \}^T$ is the vector containing the components of the symmetric Kirchhoff effective stress and $\rho_0 = J\rho$ is the reference mass density of the soil water-mixture. $B = [B_1, B_2, \ldots, B_{NQ}]$ is the classical strain-displacement matrix in spatial form, with B_A $(A = 1, \ldots, NQ)$

$$B_A = \begin{bmatrix} N_{A,1}^\phi & 0 & 0 \\ 0 & N_{A,2}^\phi & 0 \\ 0 & 0 & N_{A,3}^\phi \\ N_{A,2}^\phi & N_{A,1}^\phi & 0 \\ 0 & N_{A,3}^\phi & N_{A,2}^\phi \\ N_{A,3}^\phi & 0 & N_{A,1}^\phi \end{bmatrix} .$$

Matrix b is given by the product $b = m^T B$, where $\{m\} = \{1, 1, 1, 0, 0, 0\}^T$ for $n_{sd} = 3$, and $G \equiv g$ is the gravity acceleration vector.

Time integration of the mass balance equation (Eq. (23)) leads to

$$\Delta t \, H_{\Delta t}^h(\phi, p, \tilde{\psi}) = -\tilde{\psi}^T \left[J(d) + \Delta t \, \Phi(p) + \Delta t \, H_{EXT} \right] = 0 , \tag{30}$$

with

$$J(d) = -\int_{\mathscr{B}} N^{pT}(J_{n+1} - J_n)dV \,; \tag{31a}$$

$$\Phi(p) = \beta \int_{\mathscr{B}} E^T J_{n+1} \tilde{v}_{n+1} dV + (1 - \beta) \int_{\mathscr{B}} E_n^T J_n \tilde{v}_n dV \,; \tag{31b}$$

$$H_{EXT} = \int_{\partial\mathscr{B}} N^{pT} [\beta Q_{n+1} + (1 - \beta)Q_n] dA \,, \tag{31c}$$

where N^{pT} is the shape function matrix for the pressure field, $E = [E_1, E_2, \ldots, E_{NP}]$ the gradient-pressure transformation matrix, with E_A $(A = 1, \ldots, NP)$

$$E_A = grad\, N_A^p = \begin{bmatrix} N_{A,1}^p \\ N_{A,2}^p \\ N_{A,3}^p \end{bmatrix} .$$

By adopting the implicit Eulero scheme ($\beta = 1$), which is first order accurate and unconditionally stable, and referring to Darcy's velocity, $\tilde{v}$ (Eq (31b)) can be rewritten as

$$\Phi(p) = -\int_{\mathscr{B}} E^T \frac{k_{n+1}}{g\rho_F} [E\{p + p_r\}_{n+1} - \rho_F g] J_{n+1} dV \,, \tag{32}$$

with k non-linear permeability tensor and $\{p + p_r\}$ vector of prescribed and unknown nodal pore pressures. For sake of brevity the reader is referred to (Borja and Alarcón 1995; Borja et al. 1998) for the discretized expression of G^h and $H_{\Delta t}^h$. Particularly, the first variation of G is:

$$\delta G^h(\phi, p, \tilde{\eta}) = \tilde{\eta}^T [K_{\phi\phi}\delta d + K_{\phi p}\delta p] \,, \tag{33}$$

where

$$K_{\phi\phi} = \int_{\mathscr{B}} \left[\boldsymbol{Z}^T \boldsymbol{a} \boldsymbol{Z} + \boldsymbol{Z}^T (J \boldsymbol{I}_p) \boldsymbol{Z} - \rho_F J \boldsymbol{N}^{\phi T} \boldsymbol{G} \boldsymbol{b} \right] dV \; ; \tag{34a}$$

$$K_{\phi p} = - \int_{\mathscr{B}} \boldsymbol{b}^T \boldsymbol{N}^p J dV \; . \tag{34b}$$

$\boldsymbol{Z} = [\boldsymbol{Z}_1, \boldsymbol{Z}_2, \dots, \boldsymbol{Z}_{NQ}]$ is the full spatial gradient operator (see (de Souza Neto et al. 2011)), with components $\boldsymbol{Z}_A$ $(A = 1, \dots, NQ)$

$$\boldsymbol{Z}_A = \begin{bmatrix} N^{\phi}_{A,1} \boldsymbol{1} \\ N^{\phi}_{A,2} \boldsymbol{1} \\ N^{\phi}_{A,3} \boldsymbol{1} \end{bmatrix}$$

where $\boldsymbol{1}$ is the second order identity tensor, with 3×3 dimension for $n_{sd} = 3$. The quantity $\boldsymbol{a}$ is the total tangent operator defined by

$$\boldsymbol{a} = \boldsymbol{c} + (\boldsymbol{\tau} \oplus \boldsymbol{1}) = \boldsymbol{\alpha} - (\boldsymbol{\tau} \ominus \boldsymbol{1}) \, , \tag{35}$$

where $\boldsymbol{\alpha}$ is the algorithmic tangent operator (Borja 2013)

$$\boldsymbol{\alpha} = \sum_{A=1}^{3} \sum_{B=1}^{3} a^{ep}_{AB} \boldsymbol{m}^{(A)} \otimes \boldsymbol{m}^{(B)}$$
$$\sum_{A=1}^{3} \sum_{B \neq A} \left(\frac{\tau_B - \tau_A}{\lambda_B^{e\,tr} - \lambda_A^{e\,tr}} \right) \left(\lambda_B^{e\,tr} \boldsymbol{m}^{(AB)} \otimes \boldsymbol{m}^{(AB)} + \lambda_A^{e\,tr} \boldsymbol{m}^{(AB)} \otimes \boldsymbol{m}^{(BA)} \right) , \tag{36}$$

with $\boldsymbol{m}^{(A)} = \boldsymbol{n}^{(A)} \otimes \boldsymbol{n}^{(A)}$, and $\boldsymbol{m}^{(AB)} = \boldsymbol{n}^{(A)} \otimes \boldsymbol{n}^{(B)}$ with $\boldsymbol{n}$ (Eq. (8)). $\lambda_A^{e\,tr}$ is the trial elastic principal stretch and $a^{ep}_{AB} = \partial \tau_A / \partial \varepsilon_B$ (Borja and Tamagnini 1998) is the elastoplastic tangential modulus, obtained from the *return mapping* algorithm for determining the tensor $\boldsymbol{\tau}$.

$\boldsymbol{I}_p$ is provided by the second and third integrals of Eq. (44), and gives

$$\boldsymbol{I}_p = p^h (\boldsymbol{1} \ominus \boldsymbol{1} - \boldsymbol{1} \otimes \boldsymbol{1}) \, . \tag{37}$$

13

then Eq. (30) holds

$$\Delta t \, \Delta H_{\Delta t}^{h}(\phi, p, \tilde{\psi}) = -\tilde{\psi}^{T} \left[K_{p\phi} \delta d + K_{pp} \delta p \right],$$

(38)

where

$$K_{p\phi} = -\int_{\mathscr{B}} J N^{pT} b \, dV - \beta \Delta t \int_{\mathscr{B}} E^{T} \left(\frac{k + k'J}{\rho_{F} g} \right) (E\{p + p_{r}\} - \rho_{F} g) b J dV$$
$$+ \frac{\beta \Delta t}{\rho_{F} g} \int_{\mathscr{B}} E^{T} (A + W) Z J dV ;$$

(39a)

$$K_{pp} = -\frac{\beta \Delta t}{\rho_{F} g} \int_{\mathscr{B}} E^{T} k E J dV .$$

(39b)

k' operator is the first variation of the permeability tensor with respect to the Jacobian J of the deformation gradient F. A has 3×9 dimension

$$A = \left[\hat{v}_{1} \mathbf{1}, \hat{v}_{2} \mathbf{1}, \hat{v}_{3} \mathbf{1} \right],$$

(40)

where $\hat{v}_{i}$ are the components of the vector $\hat{v} = k \cdot (E\{p + p_{r}\} - \rho_{F} g)$. $W = \left[W_{1}, W_{2}, W_{3} \right]$ has 3×9 dimension as well

$$W_{1} = \begin{bmatrix} w_{111} & w_{121} & w_{131} \\ w_{211} & w_{221} & w_{231} \\ w_{311} & w_{321} & w_{331} \end{bmatrix}$$

$$W_{2} = \begin{bmatrix} w_{112} & w_{122} & w_{132} \\ w_{212} & w_{222} & w_{232} \\ w_{312} & w_{322} & w_{332} \end{bmatrix}$$

$$W_{3} = \begin{bmatrix} w_{113} & w_{123} & w_{133} \\ w_{213} & w_{223} & w_{233} \\ w_{313} & w_{323} & w_{333} \end{bmatrix}$$

where the components are obtained from $w_{ijk} = k_{ik} p_{,j}$.

14

The final discretized coupled system of equations can be written as

$$\left\{ \begin{matrix} r_\phi(d, p) \\ r_p(d, p) \end{matrix} \right\} = \left\{ \begin{matrix} N^S(d) \\ J(d) \end{matrix} \right\} + \left\{ \begin{matrix} N^F(p) \\ \beta_0 \Delta t\, \Phi(p) \end{matrix} \right\} + \left\{ \begin{matrix} -F_{EXT} \\ \beta_0 \Delta t\, H_{EXT} \end{matrix} \right\} = \left\{ \begin{matrix} 0 \\ 0 \end{matrix} \right\}, \qquad (41)$$

and the Newton-Raphson incremental solution is calculated from

$$\begin{bmatrix} K_{\phi\phi} & K_{\phi p} \\ K_{p\phi} & K_{pp} \end{bmatrix}_k \left\{ \begin{matrix} \delta d \\ \delta p \end{matrix} \right\}_{k+1} = \left\{ \begin{matrix} r_\phi \\ r_p \end{matrix} \right\}_k . \qquad (42)$$

The tangent operator K in Eq. (42) is in general non-symmetric and indefinite.

VALIDATION OF THE NUMERICAL CODE

The numerical analyses are performed by using *GeoMatFEM*, a Matlab research code for three-dimensional coupled geomechanical simulations.

The code has been validated against two numerical examples (Borja et al. 1998), namely the uniform consolidation of a soil column and the consolidation of a strip foundation. Both numerical simulations assume fully saturated hyperelastic-plastic porous media.

Additionally, two experimental tests have been considered for validating the constitutive model.

Hyperelastic-plastic consolidation of a column

Let's consider a column of fully saturated soil with square base of 1 m side and 5 m height, as shown in Fig. 4. The mesh is composed by 10 three-dimensional finite elements.

The material parameters are in shown in Table 1.

The initial stress configuration is not stress-free, but it balances the gravity load (Borja et al. 1998; Borja and Tamagnini 1998); it is obtained by an uncoupled small strain analysis, applying self-weight in three steps and then determining the internal stresses. We recall that nonzero initial stresses are required to activate the procedure, so a small initial value p_0 and preconsolidation pressure p_{c0} have been assumed at all Gauss points for the first run of the model.

The displacements are subsequently reinitialized to zero, and the consolidation analysis is carried

out by applying a vertical downward load $\Delta w = 0.09$ MPa at the top of the soil column in three equal time steps at a constant rate of 0.03 MPa/day, while time steps are increased $t_{n+1} = 1.5\Delta t_n$. The results of the consolidation analysis are shown in Figs. 5(a) and 5(b) in terms of total fluid potential and average degree of consolidation versus time, respectively. The total fluid potential $\Pi = \Pi^p + \Pi^e = p/(\rho_w g) + x_z$ is calculated at the Gauss point A , with $X_{PGA} = [\,0.106\,\text{m}\ 0.106\,\text{m}\ 0.106\,\text{m}\,]$ close to the column base. The average degree of consolidation $\bar{U}_{ave}$ and the time factor T are computed as

$$\bar{U}_{ave} = \frac{\bar{u}_z(t)}{\bar{u}_z(\infty)} \qquad T = \frac{c_v t}{H_0^2} = \frac{\mu_0\,k}{\rho_w g}\frac{t}{H_0^2}. \tag{43}$$

Fig. 6 reports the isochrones of Cauchy pore pressure predicted by the small strains and finite strains approaches; the obtained results are superimposed to those reported in (Borja et al. 1998), so proving the correctness of the implemented procedure.

The convergence velocity (Fig. 7) exhibits a quadratic profile, typical of Newton-Raphson schemes, so configuring on this side the correct implementation of the tangent operator.

Hyperelastic-plastic footstrip consolidation

The consolidation of an half-space of clay subjected to a flexible footing strip (Fig. 8(a)) has been additionally considered following (Borja et al. 1998).

A constant value of total potential equal to $\Pi = 20.0$ m is applied, together with an initial hydrostatic Cauchy pore pressure distribution as shown in Fig. 8(b).

The material data of the clay are the same as in the previous example but the porosity and permeability parameters change with the deformation of the porous matrix. The material properties are reported in Table 2, assuming an hyperelastic behavior for sand and and hyperelastic-plastic one for clays.

Three preloading stages (case #1, case #2 and case #3) have been considered, leading to different initial stress states, similarly to what performed in the previous example. For case #1 an initial stress state due to the self–weight has been considered, while for case #2 and #3 both self-weight and two preloading conditions have been assumed, equal to 0.015 MPa and 0.030 MPa, producing

two different over-consolidation states. Fig. 9(a) shows the evolution of vertical displacements for a node on top surface located on the symmetry plane. Fig. 9(b) depicts the evolution of Cauchy pore pressure $p = \theta/J$ at Gauss point B with $X_{PGB} = [\, 0.106\,\text{m}\;\; 0.106\,\text{m}\;\; 16.211\,\text{m}\,]$.

Again, as observed in the previous example, the results are superimposed to the benchmark ones and the convergence velocity shows a quadratic profile (Fig. 10).

Experimental tests

The constitutive model has been further validated against an isotropic compression test and a standard drained triaxial test (Callari et al. 1998; Al-Tabbaa 1987); the material parameters are listed in Table 3.

Isotropic compression test

A normally consolidated sample with initial isotropic pressure $p_0 = p_{c0} = 0.1\,\text{MPa}$ has been reconstructed and a set of loading-unloading cycles has been applied. Figure 11 depicts the evolution of the specific volume v with Kirchhoff isotropic pressure P, evidencing the agreement between numerical results and experimental data, both in the loading and in the unloading stages.

Drained triaxial test

A normally consolidate sample is now subjected to an initial isotropic pressure $p_0 = p_{c0} = 0.3\,\text{MPa}$. The soil has been loaded in order to reach a deviatoric stress $Q = 0.12\,\text{MPa}$ and then unloaded. Again, by considering Fig. 12, the real material response appears to be correctly caught by the numerical model.

THREE-DIMENSIONAL ANALYSIS

A consolidation process due to the filling of two tall silos over a fully saturated clay domain is considered.

Most foundation failures in clayey soils occur when a silo is quickly loaded for the first time. The rapid filling process leads to a possibly hazardous increase in pore water pressure, so reproducing a typical undrained condition associated to a decrease in effective stress, with eventual large

343 irreversible strains and possible mechanical failure (Dogangun et al. 2009) (Fig. 13). Water over-

344 pressure hinders soil compaction and causes dangerous shear deformations that could compromise

345 the structural stability.

346 The investigated example has been inspired by the case study presented in (Puzrin et al. 2010). The

347 example describes the soil behavior underneath two adjacent silos built in the Red River Valley,

348 Canada, which did not have strength enough to resist to the applied loads. The two silos were too

349 close, and therefore pressure bulbs under the foundations overlapped. This caused large stresses

350 and, in turn, large settlements under the parts of the ring foundations. The final result was tilting

351 and touching.

352 This proposed example, although simple and straightforward, is particularly suitable to investigate

353 the potentiality of the approach, and specifically to evaluate the effects of a finite strains assumption

354 on the modeled scenario. In fact, this geomechanical problem is of interest: a three-dimensional

355 simulation is required, furthermore both material and geometric non-linearities must be taken into

356 account, along with the interaction between solid and fluid phases. Additionally, as reported below,

357 even the so called $P - \Delta$ effect can be caught thanks only to the introduced geometric non linearity.

358 Two cylindrical silos with 10 m diameter, 40 m height and placed at a distance of 2 m m one

359 to the other (Fig. 14) have been reconstructed. The silos are built on a normally consolidated and

360 fully saturated clay layer of 30 m, resting on a rigid rock base. In order to reduce the number of

361 elements, only half of the model has been realized, to take advantage of the symmetry with respect

362 to the X–Z plane. Lateral surfaces are assumed to be horizontally restrained with the bottom surface

363 fixed; free flux can occur on top and bottom of the clay layer.

364 The soil discretization is composed by 3542 D20P8 mixed finite elements, with a total of 3622

365 elements, with 16644 nodes for the displacement field, 4356 of which also for the pore pressure

366 field; the total degrees of freedom are 54288.

367 As previously done, an initial stage accounting for the self-weight, plus a surface pressure p_i =

368 0.2 MPa, has been considered.

369 By assuming that the silos are used for the storage of cereals, a load of $0.8 \, \text{t/m}^3$ is added

to the consolidation analysis. The silos are modeled as rigid elastic elements (Young's modulus $E = 1 \times 10^4$ MPa and Poisson ratio $\nu = 0$). Permeability k is calculated via the Kozeny-Carman equation (Eq. 18), whereas for the small strain analysis the value indicated in Table 4 applies.

The two silos are simultaneously filled during four constant time steps of 12 h, then the weight is maintained during the consolidation stage. The time steps are increased according to the equation: $\Delta t_{n+1} = 1.5 \Delta t_n$. Fig. 15 shows the time evolution of the vertical displacements for the central point of the silos base (points C and D of Fig. 14), evidencing pretty similar results when small or finite strains are considered: particularly, a difference of 0.9 % in the final settlements have been obtained; when horizontal displacements of top of silos are taken into account (Fig. 16, points A and B), the difference reaches 68 % and the silos rotation predicted by the finite strains analysis is about three times higher (1.56°) than that reported by the small strains one. Such a difference evidences the $P - \Delta$ contribution, essential in the correct description of the ultimate scenario. Correspondingly, the represented situation is particularly hazardous, implying that the silos under such a rotation can come into contact (at approximately 90 days when the analysis is then stopped).

Fig. 17 shows the evolution of Cauchy pore pressure at points E and F (Fig. 14), i.e. 3 m below the top of clay layer. After the fast filling of the two silos, the pore pressure reaches the maximum peak value of 0.26 MPa for both small and finite strains analysis (typical undrained condition). Since initially the load is sustained by the pore pressure, the skeleton does not deform, and therefore the two models give a very similar result in terms of pressure peak. Further, with the evolution of pore pressure in time (drained condition), the results show a slightly lower rate of pore pressure if finite strains are accounted for, due to the change in permeability with soil deformation. The final value of pore pressure is hence of 0.031 MPa, slightly higher than the initial hydrostatic pressure of 0.030 MPa. This is the consequence of imposing a constant hydraulic potential at the bottom of the clay layer, and represents a local artesian condition due to the deformation of the soil. Finally, an overview of pore pressure evolution in finite strains regime is visible in Fig. 18, which shows the contours of Cauchy pore pressure for some time steps. The typical consolidation bulbs is evidenced under the two silos, which slowly dissipates until the initial hydrostatic condition is

reached (Fig. 18.f).

The results in terms of plastic deformations are plotted in Figs. 19 and 20. Both analyses give similar values but those from finite strains model are higher, plus a wider zone of soil plasticization between the two silos, producing in turn a differential settlement at silos' foundation $\Delta u_z = 26.4\,\text{cm}$ ($\Delta u_z = 8.5\,\text{cm}$ for the small strain model). By considering Fig. 20, it is interesting to observe two conical zones characterized by high deviatoric plastic deformations, resembling hence typical 3D shear bands of strain localization.

Fig. 21 depicts the plastic deformations plotted along vertical Z-Z axis (see Fig. 14), evidencing the relevance of a finite strains approach. Both volumetric and deviatoric strains show their peak in proximity of the silos' foundation, with a deformation mechanism essentially of deviatoric nature, as reported below. Finally, Fig. 22 plots the P–Q stress path for Gauss points G (see Fig. 14), showing no appreciable differences in terms of stress for small and finite strains analyses. Figs. 23 evidences that, even if the predicted volumetric stress is always higher than the deviatoric one, the deformation mechanism is mainly driven by deviatoric strains in the undrained stage (I; higher deviatoric strains and higher deviatoric strain rate) and by volumetric strains in the drained stage (II; consolidation, higher volumetric strain rate). Anyway, overall larger deviatoric strains (as reported by Fig. 21) show a typical soil behaviour more sensitive to shear straining, a mechanism appreciable via a finite strains approach only.

CONCLUSIONS

In this work a fully coupled hydro-mechanical model has been described and validated against available literature and experimental results. Particularly, the model has been developed within a 3D Finite Element research code by assuming material and geometric non-linearities, also introducing a dependence of permeability on deformation as well as a specific type of mixed finite element. The former allows for correctly reproducing fully saturated scenarios in finite strains, the latter for solving stability issues of the adopted formulation.

A consolidation case study has evidenced the potentialities of the code and the relevance of a finite strains approach, particularly when $P - \Delta$ effects must be accounted for in realistically reproducing

hazardous scenarios of soil-structure interaction. Together with the capability of reaching such an ultimate state of silos tilting, the upgraded code has even demonstrated to better describe the evolution of the deformation state for the foundation soil, experiencing a transition from higher deviatoric strains and higher deviatoric strain rates to higher volumetric strain rates when passing from an undrained stage to a drained one.

ACKNOWLEDGMENTS

APPENDIX I. FIRST VARIATION OF THE LINEAR MOMENTUM AND MASS BALANCE

434 **EQUATIONS**

435   The variation of the linear momentum, Eq. (21), in the spatial form is written as:

$$
\begin{aligned}
\delta G = \int_{\mathscr{B}} grad\,\boldsymbol{\eta} : (\boldsymbol{c} + \boldsymbol{\tau} \oplus \mathbf{1}) : grad\,\delta\boldsymbol{u}\,dV \\
- \int_{\mathscr{B}} grad\,\boldsymbol{\eta} : (Jp\mathbf{1} \otimes \mathbf{1}) : grad\,\delta\boldsymbol{u}\,dV + \int_{\mathscr{B}} Jp\,grad^{T}\boldsymbol{\eta} : grad\,\delta\boldsymbol{u}dV \\
- \int_{\mathscr{B}} J\delta p\,div\,\boldsymbol{\eta}\,dV - \int_{\mathscr{B}} \rho_{F}J\,div(\delta\boldsymbol{u})\boldsymbol{\eta} \cdot \boldsymbol{g}\,dV - \int_{\partial\mathscr{B}} \boldsymbol{\eta} \cdot \delta\boldsymbol{t}\,dA,
\end{aligned}
\tag{44}
$$

436   where $\boldsymbol{c}$ is the fourth order spatial tangent tensor (Borja and Alarcón 1995), $\boldsymbol{\tau} \oplus \mathbf{1}$ is a fourth order

437   tensor representing the initial stress term and $Jp\mathbf{1} \otimes \mathbf{1}$ is a fourth order tensor representing the

438   pore pressure term. The quantities $\delta\boldsymbol{u}$, δp and $\delta\boldsymbol{t}$ are the variation of the displacement vector, the

439   Cauchy pore pressure and stress vector, respectively.

440   The first variation of mass balance, Eq. (23), integrated over a fixed Δt in the spatial configuration

441   is:

$$
\begin{aligned}
\delta H_{\Delta t} = \int_{\mathscr{B}} \frac{\psi}{\Delta t}J\,div\,\delta\boldsymbol{u}\,dV + \beta \int_{\mathscr{B}} grad\,\psi \cdot \frac{J\,\boldsymbol{k}}{g\,\rho_{F}} \cdot grad\,\delta\theta\,dV \\
+ \beta \int_{\mathscr{B}} grad\,\psi \cdot \left(\frac{\boldsymbol{k}}{g\,\rho_{F}} + \frac{J}{g\,\rho_{F}}\frac{\partial\boldsymbol{k}}{\partial J} \right) \cdot [grad\,p - \rho_{F}\boldsymbol{g}]\,J\,dV \\
- \beta \int_{\mathscr{B}} grad\,\psi \cdot grad\,\delta\boldsymbol{u} \cdot \frac{\boldsymbol{k}}{g\,\rho_{F}} \cdot [grad\,p - \rho_{F}\boldsymbol{g}]\,J\,dV \\
- \beta \int_{\mathscr{B}} grad\,\psi \cdot \frac{J\,\boldsymbol{k}}{g\,\rho_{F}} \cdot grad^{t}\,\delta\boldsymbol{u} \cdot grad\,p\,dV - \beta \int_{\partial\mathscr{B}} \psi\,\delta Q\,dA,
\end{aligned}
\tag{45}
$$

442   where δQ is the variation of the flux Q through the surface dA.

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TABLE 1

Parameter	U.M.	Small strains	Finite strains
μ_0	MPa	0.2	0.2
α		0.0	0.0
$\tilde{k}$		0.0476	-
$\tilde{\lambda}$		0.1667	-
$\hat{k}$		-	0.05
$\hat{\lambda}$		-	0.2
M		1.00	1.00
p_0	MPa	-0.01	-0.01
p_{c0}	MPa	-0.01	-0.01
ϵ_{v0}^e		0.00	0.00
ρ_S	t/mm^3	2.7×10^{-9}	2.7×10^{-9}
ρ_W	t/mm^3	1.0×10^{-9}	1.0×10^{-9}
φ		0.7024	0.7024
k	mm/s	1.0×10^{-5}	1.0×10^{-5}

TABLE 2

Sand layer			
Parameter	U.M.	Small strains	Finite strains
ν		0.0	0.0
φ_0		-	0.4118
ρ_S	t/mm^3	-	2.7×10^{-9}
ρ_w	t/mm^3	-	1.0×10^{-9}
ρ	t/mm^3	$2{,}00\times10^{-9}$	-
Clay layer			
Parameter	U.M.	Small strains	Finite strains
φ		0.5441	0.5441
k	mm/s	1.0×10^{-6}	1.0×10^{-6}

TABLE 3

Parameter	U.M.	Values
α		90.0
$\hat{k}$		0.013
$\hat{\lambda}$		0.93
M		0.80
ϵ_{v0}^{e}		0.00
v_0		2.37

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TABLE 4

Parameters	U.M.	Small strain	Finite strain
μ_0	MPa	5.0	5.0
α		0.0	0.0
$\tilde{k}$		0.0196	-
$\tilde{\lambda}$		0.0385	-
$\hat{k}$		-	0.02
$\hat{\lambda}$		-	0.04
M		1.00	1.00
p_0	MPa	-0.050	-0.050
p_{c0}	MPa	-0.050	-0.050
ϵ_{v0}^e		0.00	0.00
ρ_S	t/mm^3	2.7×10^{-9}	2.7×10^{-9}
ρ_W	t/mm^3	1.0×10^{-9}	1.0×10^{-9}
φ		0.36	0.36
k	mm/s	0.6328×10^{-5}	-
D	mm	-	1.0×10^{-3}
μ	MPa·s	-	1.0×10^{-9}

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(b)

Fig. 5

(a)   (b)

Fig. 6

(a)

(b)

Fig. 7

(a)   (b)

Fig. 8

(a)

(b)

Fig. 9

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(a)

(b)

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Fig. 20

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Fig. 21

Fig. 22

(a)

(b)

Fig. 23

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