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## Granular flow impact forces on protection structures: MPM numerical simulations with different constitutive models

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### Abstract

Flows of granular materials are among the most destructive of all landslide phenomena. The assessment of the potential damage caused by the granular flow and the design of protection structures require the knowledge of the landslide-structure interaction. Numerical simulations of these phenomena are very complex because large displacements, soil-structure interaction and complex non-linear soil behavior have to be considered. A key issue in the simulation of these phenomena is the definition of a constitutive model able to describe the granular material response under a wide range of strain rates. This study examines and compares the results obtained by different constitutive models, namely the elastic perfectly plastic model with Mohr-Coulomb failure criterion, and the viscoplastic model with Drucker-Prager yielding condition. To this aim, we consider a finite volume of granular material instantaneously released from the top of an inclined channel; the material flows downslope and bumps a rigid wall on which the forces are measured.

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### 1. Introduction

Landslides are one of the major natural hazard worldwide, especially in mountainous and coastal region. To assess the potential damage caused by a landslide to existing structures, and to design protection measures, the knowledge

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of failure mechanisms, landslide patterns and impact forces are necessary. Impact forces are currently estimated with simplified approaches as function of velocity, density and thickness of the flow, disregarding the real mechanical response of the material. Simulating the landslide from the initiation to the impact would be useful for safe and cost-effective design of protection measures, but it is a very challenging task because the numerical approach has to be capable of dealing with large displacements, soil-structure interaction and a rather complex mechanical response of the material.

A key issue for the study of flow-like landslide is the implementation of a suitable constitutive model. The debate around the most appropriate constitutive equation for describing geophysical flows is still active [1,2]. Part of the scientific community uses soil-mechanics concepts (elastoplasticity) [1,3], while another part prefers viscoplastic models [4,5], and others provide phenomenological constitutive equations merging solid and fluid mechanics concepts [6]. The aim of this paper is to compare the results obtained with different constitutive models in simulating the impact of a dry granular flow on a rigid barrier. The elastic-perfectly plastic model with a Mohr-Coulomb failure criterion and the viscoplastic model for frictional fluids with Drucker-Prager yielding surface are applied. As already pointed out, the phenomenon is very complex and a multitude of aspects should be considered; however, this paper is limited to the analyses of the impact forces.

The experimental experience presented in [7] is considered. In the laboratory tests, loose Toyoura sand is instantly released from the top of a chute; it accelerates flowing down the slope and impacts a 30cm-high wall on which the impact force is measured. The inclination of the channel varies between 45° and 65°.

The Material Point Method (MPM) is used in this study to capture the large deformations that characterize the phenomenon. It is a continuum-based Lagrangian method, which simulates large displacements by material points (MPs) moving through a fix grid [8]. The MPM code used in this study (Anura3D) is being developed to solve 3D dynamic large deformation problems in geotechnical engineering [9], and it has been successfully applied in a number of applications such as slope collapse and soil penetration problems [10-13].

## 2. Constitutive models

This study considers an elastic perfectly plastic model with Mohr-Coulomb failure criterion and a viscoplastic model with Drucker-Prager yield criterion. Elastoplastic models showed to be able to capture the runout of dry granular flows [14,15] and were also applied to study the impact of snow avalanches on structures [16]. Viscoplastic constitutive laws have been often applied to simulate the propagation of saturated soil mixtures [17,18], but also of dry dense granular flows [5,7].

Both models assume that the material behaves elastically up to a yielding condition, which is a function of the stress state, and then it starts to flow accumulating irreversible plastic deformations. No volumetric deformations are induced by shearing, i.e. dilatancy is zero. In the elastoplastic description, the shear stress is independent of the shear rate; in contrast, the viscoplastic model assumes that the shear stress is a linear function of the shear rate.

With the viscoplastic approach, when the material yields, for each time step the stress tensor  $\boldsymbol{\sigma}$  is updated with

$$\boldsymbol{\sigma} = (p + K\Delta\varepsilon_{vol})\mathbf{1} + 2\mu_{eff}\mathbf{D} \quad (1)$$

where  $p$ =mean effective stress,  $K$ =bulk modulus,  $\Delta\varepsilon_{vol}$ =volumetric strain increment,  $\mathbf{D}$ = deviatoric strain rate tensor, and  $\mu_{eff}$ = effective viscosity which is defined as

$$\mu_{eff} = \begin{cases} \mu_d + \frac{\tau_y}{\|\mathbf{D}\|} & \text{for } \mu_{eff} < \mu_{max} \\ \mu_{max} & \text{for } \mu_{eff} \geq \mu_{max} \end{cases} \quad (2)$$

where  $\tau_y$  is the yield stress, which is a function of the friction angle and the stress state according to the Drucker-Prager condition,  $\|\mathbf{D}\|$  is the norm of the deviatoric strain rate tensor and  $\mu_{max} = 10^{10} Pa s$  is a bound for the apparent viscosity which prevents infinite values when the shear rate vanishes.  $\mu_d$  is the viscosity, which is assumed constant throughout the computation; in reality, in dense granular flows, it is dependent on shear rate and pressure [19], but

this will be considered in future developments of the research. Note that material compressibility is taken into account by the bulk modulus  $K$ .

### 3. Description of the numerical model

The geometry of the model follows the small-scale laboratory experiments by [7]. A box of sand is placed at the top of a 30cm-wide inclined channel whose slope varies between  $45^\circ$  and  $65^\circ$ . The material is instantly released and it hits a 30cm-high wall positioned 180cm downslope on which the impact forces are measured. The geometry and discretization of the numerical model are shown in Fig. 1. Since the problem is bi-dimensional, the width of the numerical model is only 2cm, which reduces the computational cost. The base of the chute is fully fixed, and roller boundary conditions are prescribed at the other surfaces; the wall is smooth. The computational mesh is refined where high stress and deformation gradients are expected, e.g. along the sliding plane and the face of the wall. 20 MPa are initially placed inside each sand element inside the box. The optimal discretization has been determined through preliminary analyses as a compromise between accuracy and computational cost.

Dry Toyoura sand is used in the experiments; the bulk density is  $1379\text{kg/m}^3$ . The constitutive parameters used for the two considered models are summarized in Table 1. They are chosen within a reasonable range of values for this material in such a way that the response is as similar as possible to the experiments for various slope inclinations.

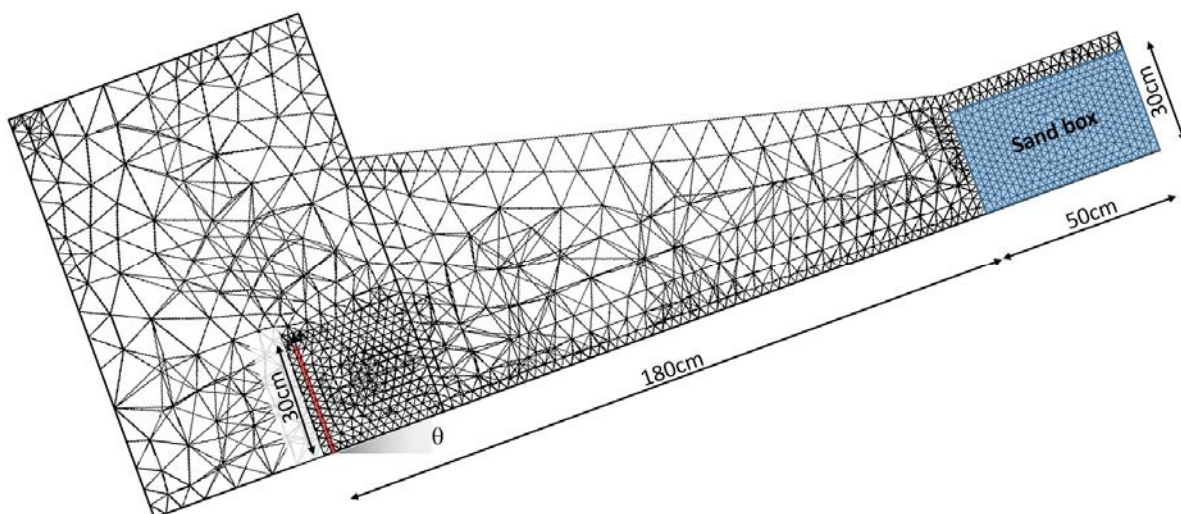


Fig. 1. Geometry and discretization of the numerical model

Table 1. Constitutive parameters

Elastoplastic		Viscoplastic	
Young modulus $E$	50kPa	Bulk modulus $K$	1000
Poisson's ratio $\nu$	0.2	Friction angle $\varphi$	$35^\circ$
Friction angle $\varphi$	$35^\circ$	Viscosity $\mu$	10 Pa s

### 4. Numerical results

Determining the input parameters of these constitutive models for a granular flow is difficult. Indeed, while the friction angle and the Young modulus are easy to define in quasi-static conditions, this is not true for granular flows, where their role in the dynamics of the landslide is still unclear. Similarly, while the importance of viscosity is well understood in Newtonian fluids, it is evident that soil is a completely different material. A parametric study is

preliminary performed to clarify the effect of the input parameters (Sec. 4.1). Despite the difficulties in calibrating the models, a comparison in terms of impact forces is presented in Section 4.2.

4.1. Parametric study

This section shows the effect of the input parameters of the considered constitutive models on the impact forces in case of a slope inclination  $\theta=55^\circ$ . Similar effects are observed with different slope inclinations.

Fig. 2 shows the effect of friction angle, Young modulus, and Poisson’s ratio of the elastoplastic model on the force evolution with time. Increasing the friction angle, the friction on the sliding surface increases, thus a larger amount of energy is dissipated and the impact velocity decreases, resulting in a lower peak force. When the granular flow meets the wall it is deviated upwards, part of the material passes beyond the wall while part stops in front of the wall forming a dead zone. Increasing the friction angle, less material overpasses the wall thus the post peak force, which depends on the mass of material deposited in front of the obstruction, increases. In the range of values of Young modulus and Poisson’s ratio, which is considered reasonable for loose sand at low stress, the peak force varies only slightly.

The effect of friction angle and viscosity of the viscoplastic model are shown in Fig. 3. In contrast with what is observed with the elastoplastic model, in this case the friction angle does not change significantly the impact force. More important is the viscosity, which affects the velocity, the thickness, and the front inclination of the flow at the impact. Increasing the viscosity, the velocity decreases, while the flow thickness and front inclination increase, these factors have opposite effects on the peak force, but in this case, the reduction of velocity prevails thus resulting in a decrease of impact forces with increasing viscosity. The bulk modulus does not influence significantly the forces.

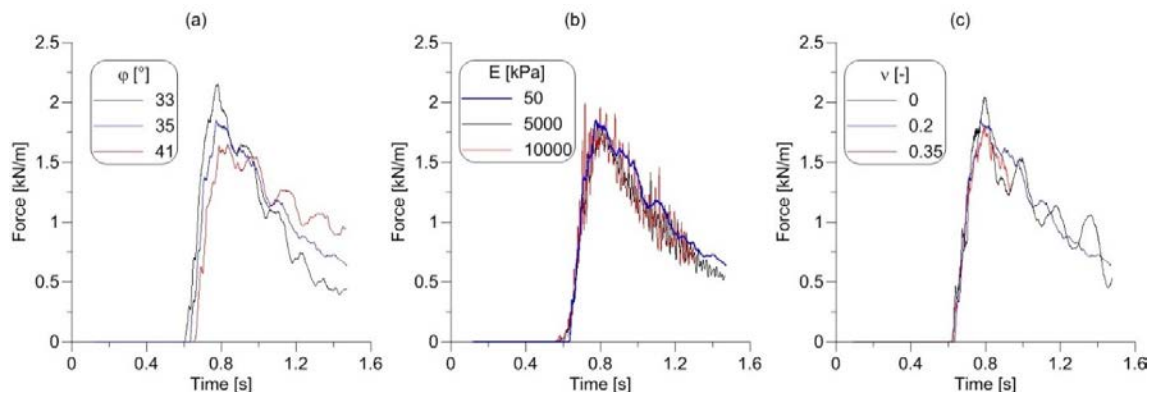


Fig. 2. Force evolution obtained with the elastoplastic model: (a) effect of friction angle, (b) effect of Young modulus, (c) effect of Poisson’s ratio.

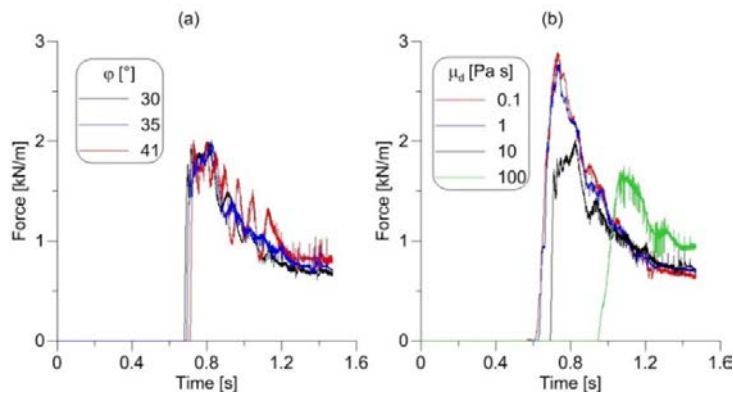


Fig. 3. Force evolution obtained with the viscoplastic model: (a) effect of friction angle, (b) effect of viscosity.

#### 4.2. Comparisons

The force evolution obtained with the elastoplastic and the viscoplastic model, for the constitutive parameter of Table 1, are shown in Fig. 4. In order to compare the results with the experiments, the numerical values obtained for a model-width of 2cm are scaled to a width of 30cm. Both models predict a rapid increase of the force up to a peak, followed by a reduction to a quasi-static value. This response is qualitatively similar to the one observed in the experiments.

When using the elastoplastic model, for  $\theta=60^\circ$  and  $65^\circ$ , after the peak the force first decreases and then increases again; this phenomenon is due to the interaction of compressive waves, generated by the impact, with boundaries [20,21] that do not dissipate quickly with the elastoplastic model. The quasi-static force depends on the slope inclination; in contrast, it is nearly independent on  $\theta$  with the viscoplastic model, which agrees with the experimental observations. The peak force values obtained with the elastoplastic model and the viscoplastic model are similar, but they overestimates the experimental observations.

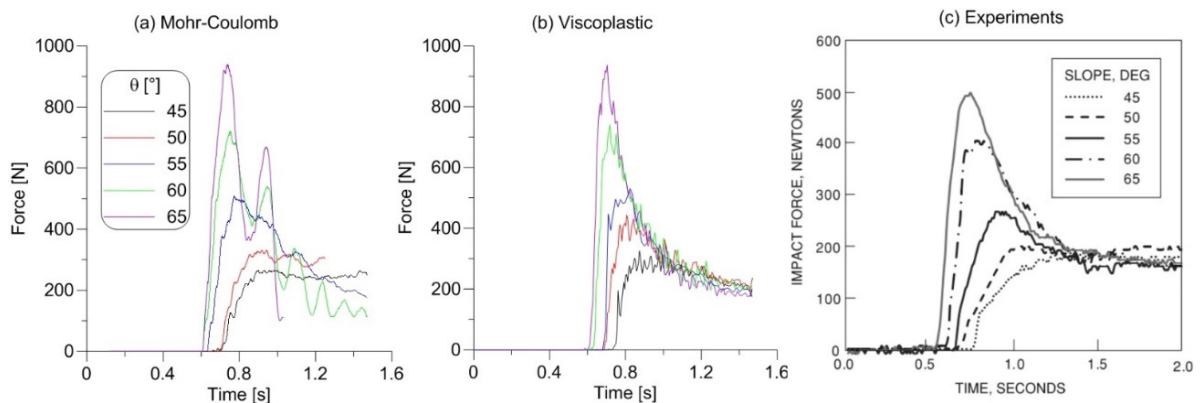


Fig. 4. Force evolution obtained with (a) elastoplastic model, (b) viscoplastic model, (c) experimental results by [7].

#### 5. Discussion and conclusions

This paper considers the propagation of a dry sand flow down a chute and its impact on a rigid wall. The impact forces obtained with two of the most common constitutive models for granular flows are compared, namely the elastic perfectly plastic model with Mohr-Coulomb failure criterion and the viscoplastic model with Drucker-Prager yield condition and constant viscosity.

The impact forces depend on a number of factors such as flow velocity, flow thickness, material response at the impact and so on. These factors are correlated and strongly influenced by the constitutive model. An exhaustive comparison, covering all these aspects, cannot be presented in this paper, which however gives an insight to the problem.

The friction angle governs the response of the elastoplastic model; in contrast, the viscosity is the most important parameter of the viscoplastic model. While the friction angle and the elastic parameters are simple to estimate in static conditions, for a granular flow they are very difficult to guess because they do not clearly represent measurable properties of the material and they depend on the state of motion. This renders the practical application of these models rather difficult.

Both models capture qualitatively well the force evolution: the force reaches a peak and then decreases to a static value. However, impact forces are overestimated compared to the experiments, meaning that none of the considered models can perfectly capture the behavior of this dense flow of granular material. Indeed, in dense granular flows, the grains interact with frictional contacts and collisions; the first mechanism prevails at low shear rates and it is reasonably well captured by elastoplastic models (quasi-static conditions). At higher shear rates, the collisional

contribute, which introduces a shear rate dependency of stresses (viscosity), gains more importance. Numerous attempts have been made to incorporate both frictional and collisional contributes in a constitutive model, see e.g. [19,22–24], but a satisfactory solution to the problem has not been found yet.

Future developments of the research will consider the implementation of more advanced constitutive models, in order to better capture the mechanical response of dense granular flows.

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