

# Is the core mass-luminosity relation violated by the occurrence of the third dredge-up?

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**Abstract.** The core mass-luminosity ( $M_c - L$ ) relation of thermally pulsing asymptotic giant branch (TP-AGB) stars is a key ingredient in synthetic calculations of their evolution. Recently, Herwig et al. (1998) have presented full calculations of TP-AGB models with strong dredge-up occurring already during early thermal pulses. The resulting luminosity evolution differs appreciably from the simple linear  $M_c - L$  relation.

In this paper, we show that at least part of the luminosity evolution can be understood as being the result of two well-known effects: the gradual approach to the asymptotic behaviour that characterises the first thermal pulses, and the chemical composition changes of the envelope. Both effects are already implemented in the  $M_c - L$  relations used in synthetic models. Consequently, these models are able to reproduce the behaviour of full calculations. Whether additional effects, not yet taken into account, are present, can be decided only through additional calculations and data. We also comment on the validity of a linear  $M_c - L$  relation and its possible violation, as mentioned by Herwig et al.

**Key words:** stars: mass-loss – stars: evolution – stars: AGB and post-AGB

## 1. Introduction

The  $M_c - L$  relation for TP-AGB stars, first discovered by Paczyński (1970a), has since then been employed in many studies involving this evolutionary phase. It is a basic ingredient in synthetic calculations of TP-AGB evolution, and an important tool for the interpretation of observational data for AGB stars.

What the  $M_c - L$  relation means in the classical sense is simply that the quiescent luminosity of a TP-AGB star in the full-amplitude regime is mainly controlled by its core mass, without any dependence on the mass of its outer envelope.

Various theoretical analyses have been performed in the past to explain the existence of the  $M_c - L$  relation from first principles, using either homology relations (Refsdal & Weigert 1970; Havaelet & Barkat 1979; Kippenhahn 1981), or the equations

of stellar structure under specific physical conditions (see Eggleton 1967; Paczyński 1970b; Tuchman et al. 1983; Jeffery 1988).

A transparent discussion of the validity of the  $M_c - L$  relation was presented by Tuchman et al. (1983), to whom the reader should refer. There it is shown that an  $M_c - L$  relation *necessarily* holds when the star consists of:

- a *degenerate core* of mass  $M_c$  surrounded by
- a narrow *radiative burning shell* (or double shell) source providing most of the luminosity ( $L_H \sim L$ ), beyond which there must exist
- a *thin* (with a mass  $\Delta M \ll M_c$ ) and *inert* (the luminosity is constant) *transition region in radiative equilibrium*, extending up to the base of the convective envelope.

Then, because of the extreme steepness of the structural gradients across the radiative inert zone, it follows that the thermal evolution of the core is decoupled from that of the envelope. The relationship between the core mass and the luminosity, defined on the ground of this physical picture, is of linear nature, as confirmed by numerical results (e.g. Paczyński 1970a; Iben 1977; Wood & Zarro 1981; Boothroyd & Sackmann 1988a). Hereinafter, such linear relation will be referred to as *the classical  $M_c - L$  relation*.

However, such a simple  $M_c - L$  relation does not hold for all AGB stars. Blöcker & Schönberner (1991) have shown that the  $M_c - L$  relation can indeed break down in the most massive AGB stars ( $M \gtrsim 3.5\text{--}4.5M_\odot$  depending on the metallicity) experiencing envelope burning (or hot-bottom burning). In this context, substantial efforts have been made in order to accurately include this effect in synthetic TP-AGB calculations (Marigo et al. 1998; Marigo 1998; Wagenhuber & Groenewegen 1998). It must be emphasized that  $M_c - L$  relations in synthetic calculations are always technically motivated relations intended to fit results of full stellar evolution calculations. By no means they are just the classical, physically motivated linear relations mentioned above. In the following, such relations will be referred to as *technical  $M_c - L$  relations*.

Very recently, Herwig et al. (1998, hereinafter HSB98) have claimed that the classical  $M_c - L$  relation may also be violated in low-mass AGB stars, as a consequence of efficient third dredge-up. More specifically, they present evolutionary sequences with

a dredge-up efficiency close to  $\lambda = 1$  or even higher<sup>1</sup>, i.e. characterized by an almost constant or slightly decreasing core mass  $M_c$ . Despite of this fact, these sequences are found to evolve at increasing luminosity. This behavior is in apparent contradiction with the trend expected from the classical  $M_c - L$  relation, predicting lower luminosities at lower core masses.

In the present study we address the question whether these results present a *further* deviation from the classical  $M_c - L$  relation, as claimed by HSB98, or if they can be explained, at least partly, by the already known (and understood) deviations.

## 2. Violations of the classical $M_c - L$ relation

A violation of the classical  $M_c - L$  relation implies that, for some reason, the configuration defined in Tuchman et al. (1983) is altered. For instance, the occurrence of hot-bottom burning (or envelope burning) in the most massive TP-AGB stars causes the inert radiative buffer to disappear, due to the deep penetration of the convective envelope into the H-burning shell. Another example refers to the first inter-pulse periods, when the luminosity of a TP-AGB star is found to be lower than predicted by the classical  $M_c - L$  relation for the same  $M_c$ . In these initial stages the condition  $L_H \sim L$  is not actually fulfilled, as the gravitational contraction of the core and the He-burning shell provide non-negligible contributions to the surface luminosity.

In the context of the recent results by HSB98, the first natural question is: does the third dredge-up in low mass TP-AGB stars lead to a real violation of the classical  $M_c - L$  relation? In other words, is any of the conditions listed above not fulfilled?

The answer is: no. In fact, the degenerate core and the H-burning shell still exist in the quiescent regime after the dredge-up has occurred, as does the radiative inert buffer, since HSB98 are only considering stars which do not experience hot-bottom burning.

The second question is: if the basic conditions for the existence of the classical  $M_c - L$  relation are still fulfilled, what causes the deviation from the linear  $M_c - L$  relation?

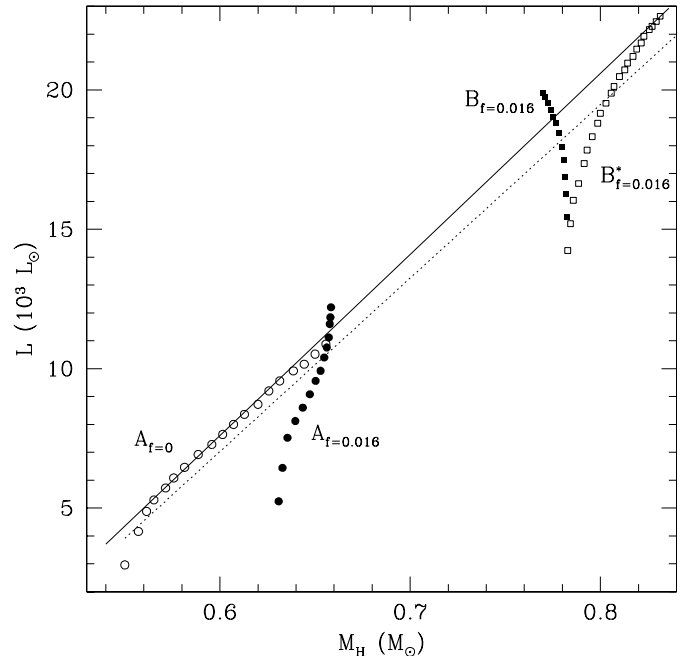
### 2.1. The first pulses

In order to answer the latter question, let us consider the previously known deviations from the classical  $M_c - L$  relation.

The first effect to consider is the initial luminosity evolution of TP-AGB stars. In complete calculations of AGB stars the first thermal pulses still take place during a phase of fast core contraction, at luminosities lower than given by the classical, linear  $M_c - L$  relation. During a few thermal pulses, the luminosity gradually approaches this relation, up to the so-called *full-amplitude regime*. During these first pulses unique relations between  $M_c$  and  $L$  are not expected to exist.

The sequences of models shown by HSB98 in their Figs. 2 and 3 refer to a relatively small number of pulse cycles, most of which have not yet attained the full-amplitude regime. More specifically, the two evolutionary sequences they computed with

<sup>1</sup>  $\lambda$  is defined as the ratio between the dredged-up mass and the core mass increase during each inter-pulse period.



**Fig. 1.** Evolution of the pre-flash quiescent luminosity for the HSB98 models. The symbols refer to the inter-pulse periods, and are taken from Fig. 2 of HSB98. The continuous line represents a linear  $M_c - L$  relation which is chosen here to describe the full-amplitude regime of the  $A_{f=0}$  and  $B_{f=0.016}^*$  sequences. The dotted line shows the  $M_c - L$  relation of Bloeker (1993).

overshooting, for  $3M_\odot$  (labelled  $A_{f=0.016}$  in HSB98) and  $4M_\odot$  ( $B_{f=0.016}$ ) models, present only 14 and 12 thermal pulses, respectively. In each of these sequences, at least 6 of the pulse cycles are clearly in the sub-luminous phase which characterizes the first pulses. This can be seen in Fig. 1.

HSB98 compare their sequences with the  $M_c - L$  relation from Bloeker (1993). The latter is shown as a dotted line in Fig. 1. This relation clearly predicts too faint luminosities if compared to the most luminous points in the sequences of models with no or little dredge-up,  $A_{f=0}$  and  $B_{f=0.016}^*$ . This inappropriateness of the Bloeker  $M_c - L$  relation to describe the present HSB98 models probably derive from the different input physics used in both sets of models.

We therefore prefer to define another linear  $M_c - L$  relation, more appropriate to describe the asymptotic behaviour of the HSB98 models without dredge-up. This is shown as the solid line in Fig. 1. This has been chosen to be the one which reasonably fits the 16 last inter-pulse periods (out of 19) in the  $A_{f=0}$  sequence, and the last 3 or 4 (out of 23) in the  $B_{f=0.016}^*$  one. Both evolutionary sequences asymptotically approach this linear relation.

In all the HSB98 evolutionary tracks, the first thermal pulses have luminosities which are lower than predicted by this asymptotic linear  $M_c - L$  relation we adopt (solid line). In the sequences with efficient dredge-up, only the very last quiescent models are more luminous than predicted by this relation for the same core masses. Specifically, only the last 3 inter-pulse periods of the  $A_{f=0.016}$  track, and the last 5 of the  $B_{f=0.016}$

one are above the  $M_c - L$  relation. The remaining points are all steadily increasing in luminosity, which is just the behavior expected for the first thermal pulses.

The luminosity increase in the initial phase of the TP-AGB evolution is partly due to the release of gravitational energy by the contracting core, and clearly constitutes a violation of the assumptions for the validity of the classical  $M_c - L$  relation. However, this effect is well known and already taken into account in the technical  $M_c - L$  relations in synthetic models (e.g. Groenewegen & de Jong 1993; Marigo et al. 1996, 1998, 1999; Marigo 1998; Wagenhuber & Groenewegen 1998). We note, however, that the behavior of  $L(M_c)$  for the two sequences with efficient dredge-up clearly deviates from those of the sub-luminous pulses without dredge-up, such that the presence of an additional effect resulting from the dredge-up is likely.

## 2.2. The composition dependence in the $M_c - L$ relation

Another important point is related to the change of the surface chemical composition produced by the third dredge-up, and thus to the composition dependence of the  $M_c - L$  relation.

Indeed, the fact that changes in the chemical composition of the envelope may affect – but do not violate – the classical  $M_c - L$  relation had already been pointed out long ago from theoretical arguments (e.g. Refsdal & Weigert 1970; Kippenhahn 1981; Tuchman et al. 1983). As clearly derived from Tuchman et al. (1983; see their Eqs. (1.17) and (1.29)) the  $M_c - L$  relation contains a non-negligible dependence on the composition of the envelope, essentially expressed by three parameters:

- a factor  $(1 + X)$  from the electron scattering opacity,
- a factor  $(5X + 3 - Z)$  from the mean molecular weight ( $\mu = 4/(5X + 3 - Z)$  for a fully ionized gas),
- a factor  $(X Z_{\text{CNO}})$  from the hydrogen burning rate.

Since then, various  $M_c - L$  relations, both classical linear and technical ones which include a composition dependence, have been presented by different authors (e.g. Lattanzio 1986; Boothroyd & Sackmann 1988a; Wagenhuber & Groenewegen 1998; Tuchman & Truran 1998). From these studies it turns out that at any given core mass, the quiescent luminosity of a TP-AGB star increases with increasing metallicity  $Z$ , helium content  $Y$ , (both leading to a higher mean molecular weight  $\mu$ ), and CNO abundances  $Z_{\text{CNO}}$ . For instance, based on calculations of full AGB models, Boothroyd & Sackmann (1988a) carefully analyzed the composition dependence, deriving a proportionality factor  $\sim Z_{\text{CNO}}^{1/25} \mu^3$  in their fitting formula of the  $M_c - L$  relation. They found that at given core mass, stars of solar composition ( $Z = 0.02$ ,  $\mu \sim 0.62$ ) are  $\sim 25\%$  more luminous than metal poor stars ( $Z = 0.001$ ,  $\mu \sim 0.598$ ).

It follows that the occurrence of recurrent dredge-up episodes in TP-AGB stars is expected to alter (not to break) even the classical  $M_c - L$  relation, as a consequence of the increase of the mean molecular weight in the envelope.

Of course, this effect has already been included in several synthetic calculations (e.g. Groenewegen & de Jong 1993; Marigo et al. 1996, 1998; Marigo 1998), where  $M_c - L$  rela-

tions with a metallicity dependence have been adopted. Marigo (1998) has already pointed out that a deviation from the  $M_c - L$  relation, corresponding to constant metallicity, can be caused by changes in the envelope composition.

We have estimated, from the data presented in Herwig et al. (1997; hereinafter HBSE97), Herwig (1998) and HSB98, the total change in the envelope composition due to the dredge-up events. For the  $3M_\odot$   $A_{f=0.016}$  track, the mean molecular weight  $\mu$  is estimated to increase from 0.6314 to 0.6394 during the TP-AGB evolution, whereas for the  $4M_\odot$   $B_{f=0.016}$  one, it increases from 0.6304 to 0.6376. This implies that in both cases  $\mu$  increases by 1.3% in total. Assuming  $L \propto \mu^3$  (following Boothroyd & Sackmann 1988a), this change in the envelope chemical composition would imply a change of 4% in the luminosity predicted by the linear  $M_c - L$  relation for constant metallicity. This already accounts for one-third of the luminosity increase above the  $M_c - L$  relation drawn in Fig. 1.

## 2.3. The presence of hot-bottom burning

HSB98 claim that their evolutionary sequences do not present hot-bottom burning, since their core masses are “lower than those associated to hot-bottom burning” (HBB). The highest core mass in their tracks is  $M_c = 0.83M_\odot$ , whereas they consider HBB to be present only at higher core masses.

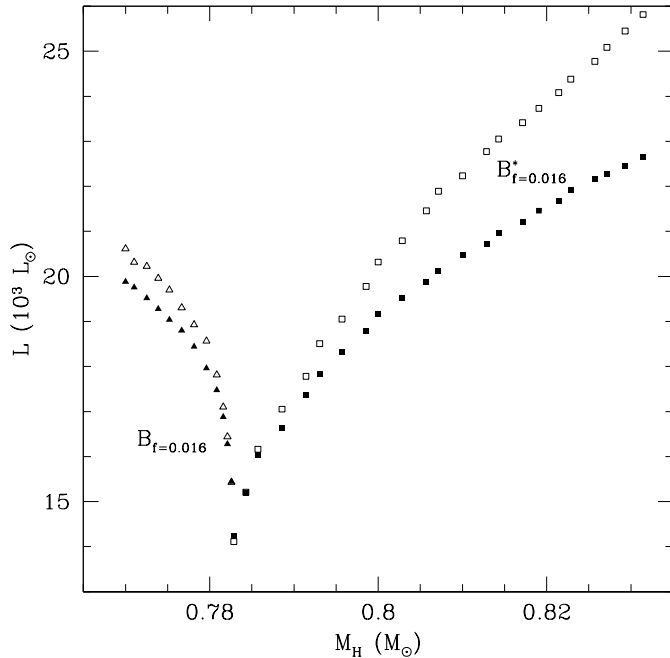
However, the knowledge of the core mass is not enough to diagnose the possible occurrence of HBB. Several authors (Boothroyd & Sackmann 1992; Vassiliadis & Wood 1993; D’Antona & Mazzitelli 1996; Marigo 1998) find that the presence of HBB, and its associated “over-luminosity”, are sensitive to other stellar parameters as well, as e.g. the envelope mass, metallicity, mixing-length parameter, and to the details of the convection theory.

The latter results have been obtained by means of stellar models that adopt canonical convection theories. It would be interesting to quantify whether the diffusive overshooting scheme applied by HSB98 to all convective boundaries, may also produce conditions favorable to HBB, i.e. higher temperatures at the bottom of the convective envelope at lower core masses. If this were the case the over-luminosity of the tracks may be partially ascribed to the occurrence of (a possibly mild) HBB. In this respect, however, no conclusion can be drawn without additional information about the HSB98 tracks.

## 3. The dependence on the core radius

As remarked above, most of the luminosity behavior of the HSB98 tracks can be understood by means of already known effects taking place during the TP-AGB evolution. HSB98, however, explicitly mention a violation of the classical  $M_c - L$  relation caused by dredge-up, and provide an explanation for the unusual behavior of their tracks based on the stellar core radius.

The authors find that the core radii,  $R_c$ , of their TP-AGB models follow quite different paths in the  $M_c - R_c$  plane, depending on whether the models experience dredge-up or not. Then, considering the apparent lack of a unique  $R_c - M_c$  rela-



**Fig. 2.** Evolution of the pre-flash quiescent luminosity (squares) for the HSB98  $4M_{\odot}$  models (filled dots). The open dots represent the luminosity predicted by means of Eq. (1).

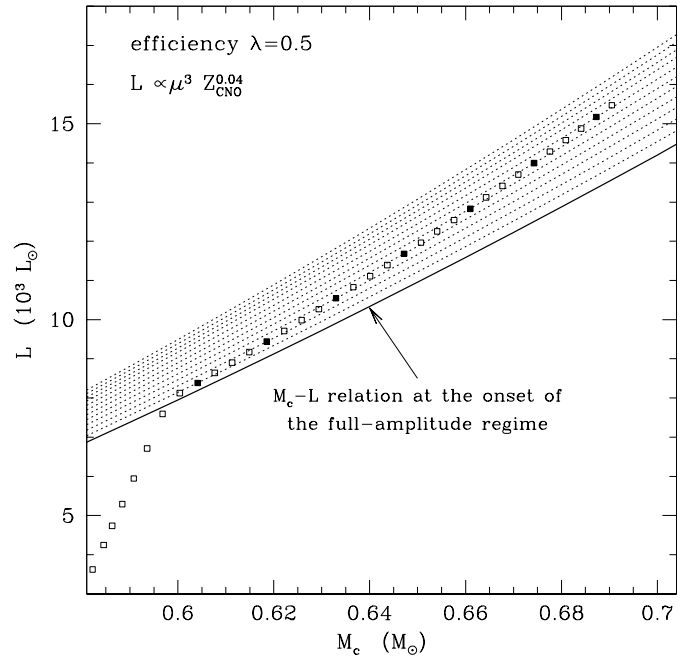
tionship and using the homology relation  $L(r/R_c) \propto M_c^2 R_c^{-1}$ , HSB98 conclude that the luminosity is not a function of  $M_c$  alone, but also of  $R_c$ . As a consequence, the  $M_c - L$  relation should rather be seen as a  $M_c - R_c - L$  relation.

Moreover, HSB98 claim that the over-luminosity above the  $M_c - L$  relation of their evolutionary sequences with  $M_c \sim 0.8M_{\odot}$  can be explained assuming

$$L = \text{constant } M_c^2 R_c^{-1}, \quad (1)$$

a fact we cannot confirm. In Fig. 2, we plot the luminosity evolution of the HSB98  $4M_{\odot}$  sequences, as derived both from their  $L$  and  $M_c$  values (from their Figs. 2 and 3; filled symbols). We then use the  $M_c$  and  $R_c$  values in order to obtain the equivalent luminosity from Eq. (1) above. This procedure however requires that we fix a value to the constant in this equation. The first point of the  $B_f=0.016$  sequence is used for this purpose, so that we obtain the relation  $L = 0.553 M_c^2 R_c^{-1}$  (where all quantities are in solar units). The open symbols in Fig. 2 then show the luminosities as obtained from this latter relation.

In this way, we have obtained the equivalent of Fig. 4 in HSB98. We find that the luminosities as derived from Eq. (1) are far from reproducing those given by the complete evolutionary sequences, although the general behaviour is similar. In contrast, HSB98 obtain quite a good match between the two sets of curves. Examining their Fig. 4, and comparing it with Fig. 2, we conclude that HSB98 adopt two different scales in their plot (i.e. for the true luminosity and for the luminosity as obtained from Eq. (1)), which are not related by a single multiplicative constant. Thus, their explanation of the luminosity evolution in terms of Eq. (1) is misleading, since obviously, the “constant”

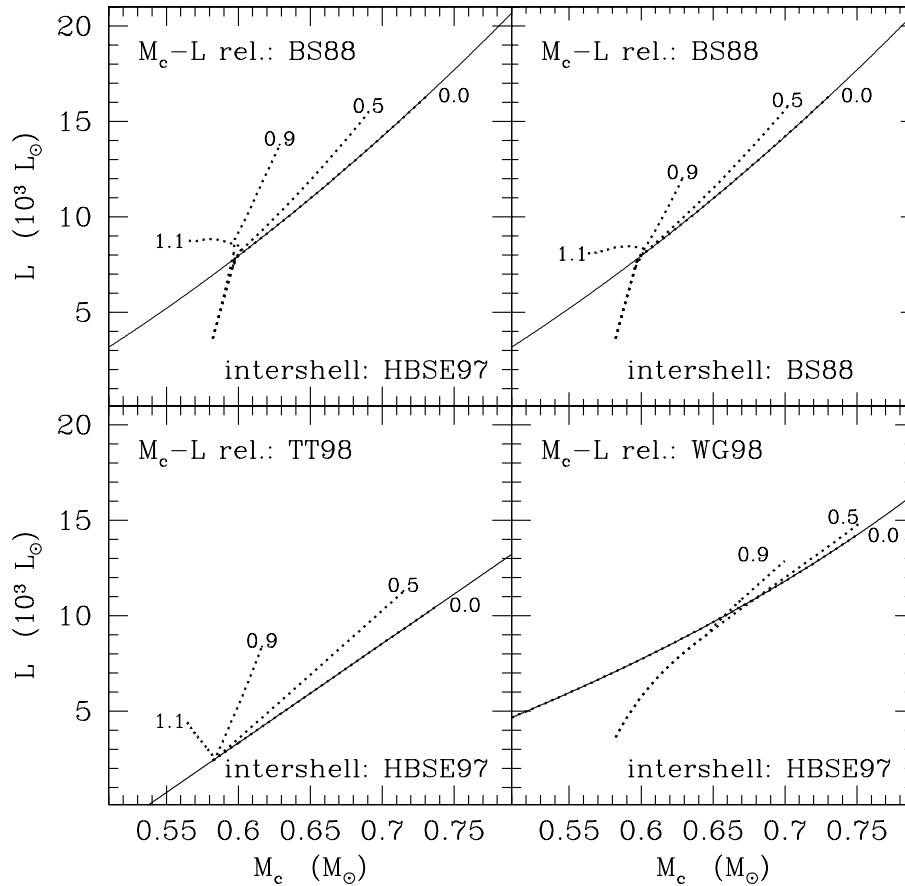


**Fig. 3.** Evolution of the pre-flash quiescent luminosity (squares) for the  $3M_{\odot}$  model, assuming  $\lambda = 0.5$ , the composition of the dredged-up material from Herwig et al. (1997), and the  $M_c - L$  relation from Boothroyd & Sackmann (1988a). The grid of dotted lines corresponds to  $M_c - L$  relations for various values of  $\mu$  and  $Z_{\text{CNO}}$  (both increasing with  $L$ ). The filled squares mark a few selected values of the quiescent luminosity, each determined, at given core mass, by that  $M_c - L$  relation of the grid which is consistent with the current surface chemical composition.

they adopt varies from model to model along the evolutionary sequences.

The possible dependence of the luminosity on core radius deserves the following remarks. This dependence would reflect the release of gravitational energy by the contracting core. During the TP-AGB evolution, the core contracts more rapidly during the first thermal pulses, until an almost constant and very low contraction rate is established in the full-amplitude regime (Herwig 1998). Another concurring effect comes from the decrease of the ratio  $\beta$  between the gas and the total pressure at increasing luminosities. The homology relation actually predicts  $L \propto M_c^{\sigma_1} R_c^{\sigma_2}$ , where the exponents  $\sigma_1$  and  $\sigma_2$  are given in Eq. 3 of HSB98. Since  $\sigma_2 \propto \beta$ , the radius dependence in Eq. (1) vanishes as we increase  $L$  and hence  $\beta \rightarrow 0$ .

For both reasons, an  $M_c - L$  relation independent of  $R_c$  should hold after a certain time. Unfortunately, the calculations by HSB98 have been stopped at the most important point, i.e. where the evolution of the core radius as a function of the core mass for the models with efficient dredge-up joins the standard  $M_c - R_c$  relation described by the models without dredge-up after the initial pulses (see their Fig. 3). If, from this point on, both relations follow the same path, then the entire effect presented by HSB98 is indeed related to peculiar behaviour of the first pulses, before the settling of the full-amplitude regime. In this case, there would be no real violation of the  $M_c - L$  relation.



**Fig. 4.** Quiescent luminosity (i.e. at the pre-flash maximum) as a function of the core mass for an evolving  $3M_{\odot}$  TP-AGB star (dotted lines). The luminosity evolution is derived adopting various  $M_c - L$  relations (i.e. Boothroyd & Sackmann 1988a, BS88; Tuchman & Truran 1998, TT98; Wagenhuber & Groenewegen 1998, WG98), assuming two different prescriptions for the chemical composition of the dredge-up material (BS88 or HBSE97; see the text for more details), and different choices for the efficiency parameter of dredge-up ( $\lambda = 0.0, 0.5, 0.9, 1.1$ ) as indicated nearby the corresponding curve. In each panel, the solid line refers to the  $M_c - L$  relation consistent with the envelope composition at the onset of the full-amplitude regime.

Otherwise, if there is a different dependence of the core radius upon the core mass, the  $M_c - L$  relation might be at least partly modified. If this were the case, it would be quite important to single out the physical effect produced by the convective dredge-up, which occurs at a thermal pulse, on the core radius during the subsequent quiescent evolution. In other words, why should the evolution of the core radius turn out different between models with and without dredge-up? This point is not clear in the HSB98 analysis.

To this respect, an interesting point is discussed by Tuchman et al. (1983), in their analytical demonstration of the  $M_c - L$  relation from first principles. In brief, the authors shows that core radius of an AGB star is larger than the radius of an ideal zero-temperature white dwarf (for which a unique linear  $M_c - R_c$  relation exists) by a multiplicative factor,  $\alpha$ , which depends both on the core mass  $M_c$  and on the temperature  $T_c$  at the top of the H-burning shell (i.e. bottom of the overlying inert radiative buffer; see their Eqs. 1.18 and 1.19). This factor  $\alpha$ , being typically  $\lesssim 3$  for relevant burning shells, decreases with  $M_c$  and increases with  $T_c$ . Hence, in order to get a greater shrinkage of the core in the AGB models with efficient dredge-up, while the core mass is kept constant, a lower temperature  $T_c$  should be attained during the quiescent regime. This, in fact, would result in a smaller  $\alpha$ , and hence in a smaller  $R_c$ . The final result would be a certain excess of luminosity with respect to the reference  $M_c - L$  relation. Unfortunately, no information about  $T_c$  is given

in HSB98, but it would be worth investigating this point with the aid of full AGB calculations.

#### 4. Synthetic calculations with dredge-up

Here we present the results of synthetic calculations carried out with different technical  $M_c - L$  relations including a composition dependence and the first subluminescent pulses (see Figs. 3 and 4). The models are meant to be useful experiments, giving a first hint of how the quiescent luminosity of a TP-AGB star may behave when dredge-up events strongly alter the envelope composition.

Calculations are carried out over a limited number of interpulse periods for a  $3 M_{\odot}$  TP-AGB star with original solar composition (i.e.  $Z = 0.01886$ ,  $X = 0.708$ ). The chemical composition of the envelope at the first thermal pulse is characterised by ( $Z = 0.01899$ ,  $X = 0.68108$ ,  $\mu = 0.62633$ ,  $Z_{\text{CNO}} = 0.01357$ ). The third dredge-up is assumed to occur once the full amplitude regime is attained, i.e. after the first subluminescent pulses when the linear  $M_c - L$  relation is approached. We adopt various values of the dredge-up parameter ( $\lambda = 0., 0.5, 0.9, 1.1$ ) and two prescriptions for the composition of the dredged-up material. They are (in mass fraction):

$$- \text{}^4\text{He} = 0.76, \text{}^{12}\text{C} = 0.22, \text{}^{16}\text{O} = 0.02$$

(according to Boothroyd & Sackmann 1988b; BS88)

–  ${}^4\text{He}=0.25$ ,  ${}^{12}\text{C}=0.50$ ,  ${}^{16}\text{O}=0.25$   
(according to HBSE97)

Here we are not concerned to give a detailed description of dredge-up and its properties. For instance, we assume that dredge-up events take place at each thermal pulse in the full amplitude regime, whereas we expect that a significant increase of the envelope metallicity could, at a certain stage, even inhibit further occurrence of the process by decreasing the temperature at the base of the convective envelope during the post-flash luminosity maximum (see e.g. Boothroyd & Sackmann 1988c).

Since most available  $M_c - L$  formulae were obtained for relatively small ranges of metallicity, usually not super-solar, they may not give realistic results if the envelope metallicity increases to very high values. However, in this respect, the recent analysis developed by Tuchman & Truran (1998) is relevant. They have quantitatively investigated the composition influence upon the  $M_c - L$  relation, in order to better estimate the luminosity of classical novae, objects in which shell hydrogen burning is known to occur in extremely metal-rich material (e.g.  $Z = 0.25$ ). At such high values of the metallicity, the corresponding  $M_c - L$  relation is shifted to significantly higher luminosities than predicted for solar composition.

Fig. 3 shows the locus traced by the  $3M_\odot$  TP-AGB model experiencing dredge-up, adopting an efficiency  $\lambda = 0.5$  and the HBSE97 prescription for the chemical composition of the inter-shell. A grid of classical  $M_c - L$  relations (from Boothroyd & Sackmann 1988a) is also plotted for increasing values of the mean molecular weight, ranging from  $\mu = 0.62633$  to  $\mu = 0.65345$  in steps of about 0.0025. The envelope composition of the last calculated model (34<sup>th</sup> pulse) is characterised by ( $\mu = 0.64408$ ,  $Z_{\text{CNO}} = 0.04489$ ). For a core mass  $M_c = 0.6905M_\odot$  the quiescent luminosity is  $\log L/L_\odot = 4.1896$ , corresponding to an over-luminosity of about 14% with respect to the case in which the chemical composition were unchanged and equal to that of the first thermal pulse.

Other examples are presented in Fig. 4. All models show that the deviations from the  $M_c - L$  relation at constant metallicity are greater for increasing  $\lambda$  and/or for higher abundances of carbon and oxygen in the inter-shell. We can also note that models with  $\lambda = 1.1$  also evolve to luminosities above the reference  $M_c - L$  relation, despite the effective decrease of the core mass.

Finally, we remark that our synthetic results shown in Fig. 4 reproduce the behavior of the luminosity as found by HSB98. For instance, the cases with  $\lambda = 1.1$  and  $\lambda = 0.5$  clearly resemble the sequences  $B_{f=0.016}$  and  $B_{f=0.016}^*$  in their Fig. 2, respectively. However, it must be specified that in our calculations with extremely efficient dredge-up ( $\lambda \sim 1$ ), a considerable over-luminosity above the  $M_c - L$  relation shows up after a much larger number of dredge-up episodes ( $\sim 10^2$ ) if compared to the results by HSB98 ( $\sim 10$ ). This difference can be partly ascribed to the fact that in our case the onset of third dredge-up occurs only when the full-amplitude regime is attained, whereas in HSB98 dredge-up takes place from the first thermal pulses

on, when other effects (in addition to the chemical composition, see Sects. 2.1 and 3) are likely to play a role.

## 5. Concluding remarks

In this paper we claim that the results from HSB98 may partially be understood by means of the already known deviations from the classical linear  $M_c - L$  relation.

An important effect certainly present in their evolutionary calculations is the increase in luminosity associated with the initial core contraction that occurs during the first thermal pulse cycles of any TP-AGB star. This phase of rapid luminosity evolution represents a substantial fraction of the tracks presented by HSB98. In order to determine if dredge-up really leads to a violation of the classical  $M_c - L$  relation, which is expected to hold for the later evolution of AGB stars, the HSB98 evolutionary sequences should be extended in order to include a much larger number of thermal pulses.

In fact, the most evident effects of the efficient dredge-up in HSB98 evolutionary sequences are:

1. The small or negative changes in the core mass from pulse to pulse, which cause the tracks to evolve almost vertically in the  $M_c - L$  diagram, instead of along a line of increasing core mass and luminosity.
2. The changes in the surface chemical composition which make their quiescent luminosity deviate from that predicted by an  $M_c - L$  relation obtained for a constant value of metallicity.

None of these effects, however, implies a violation of the classical  $M_c - L$  relation. The structural conditions for the existence of a  $M_c - L$  relation are expected to hold only after the tracks enter in the full-amplitude regime, as remarked above.

In this regard, we remark that the evolutionary tracks should be compared with the  $M_c - L$  relation obtained from the current chemical composition of the envelope, and not with those obtained from tracks of constant metallicity. Also, the possible presence of hot-bottom burning should be completely ruled out before we can tell about deviations from the  $M_c - L$  relation. It would be of particular interest, for instance, to investigate the evolution of low-mass stars ( $M \lesssim 2M_\odot$ ) computed with a similar algorithm for convection as in HSB98.

It turns out that the correct interpretation of HSB98 results requires the analysis of additional quantities along their evolutionary tracks, other than the core mass, luminosity, and core radius. These quantities are: the fraction of the stellar luminosity provided by the release of gravitational energy (necessary to identify if the full-amplitude regime has been reached), the surface chemical composition of the models (necessary to better quantify the deviation from the initial  $M_c - L$  relation due to composition changes); the luminosity provided by nuclear burning in the convective envelope (necessary to rule out the presence of hot-bottom burning); and the temperature  $T_c$  at the top of the H-burning shell (useful to investigate its effect on the factor  $\alpha$ , defined in Sect. 3, and hence on the core radius  $R_c$ ). Unfortunately, this information is not provided by HSB98.

We stress once more that synthetic TP-AGB models have already been adopting *technical non-linear*  $M_c - L$  relations, i.e. including significant deviations from linearity due to the sub-luminous first thermal pulses and changes in the surface chemical composition produced by dredge-up (e.g. Groenewegen & de Jong 1993; Marigo et al. 1996; Marigo 1998). Moreover, the real breakdown of the  $M_c - L$  relation caused by hot-bottom burning in the most massive AGB stars have been accurately taken into account (Marigo et al. 1998; Marigo 1998; Wagenhuber & Groenewegen 1998) in these models. Finally, we recall that the  $M_c - L$  relation applies only to the quiescent inter-pulse periods, but not to the luminosity variations driven by thermal pulses. Even the effect of the post-flash low-luminosity dip is usually included in synthetic TP-AGB calculations.

Therefore, synthetic AGB evolution calculations already include all known effects affecting the  $L(M_c)$ -relation and do not rely on the assumption that the classical, linear  $M_c - L$  relation is valid. A corresponding comment in HSB98 turns out to be inappropriate. As such, any *new* effect, as possibly indicated by the HSB98 calculations can easily be incorporated after sufficient data from full calculations are available.

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