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Coupled and multiphysics phenomena

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Editorial

The ALERT Doctoral School 2015 entitled *Coupled and multiphysics phenomena* is organized by Bernhard Schrefler, Lorenzo Sanavia (both University of Padova) and Frédéric Collin (University of Liège). The commitment of the school organizers and contributors of this printed volume must be highly appreciated!

Going through the contributions of their book, it becomes obvious how far we have proceeded from Terzaghi's theory of consolidation, which is perhaps one of the first works in this field. Nevertheless, looking at curricula of geotechnical courses at most universities, the impression can arise that time stands still and there is no need to teach more than Terzaghi's theory. The presented book demonstrates the opposite. It covers in a comprehensive manner various subjects of physical phenomena coupled together into powerful theories. It is fascinating to see excursions to environmental engineering, medicine or geology and to realize that we can describe those fields with a common language. I am convinced that the book will be useful not only to students attending the Doctoral School but to anybody with interests on modern geosciences. As usual, the pdf file of the book can be downloaded for free from the website of ALERT Geomaterials – `http://alertgeomaterials.eu`.

On behalf of the ALERT Board of Directors I wish all participants a successful ALERT Doctoral School 2015!

Ivo Herle
Director of ALERT Geomaterials
Technische Universität Dresden

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Coupled and multiphysics phenomena: Foreword

The contributions assembled in the present volume proceed from the lectures of the 2015 ALERT Geomaterials Doctoral School devoted to Coupled and Multiphysics Phenomena. The school has been organized and coordinated by Bernhard Schrefler (Università degli Studi di Padova), Lorenzo Sanavia (Università degli Studi di Padova) and Frédéric Collin (Université de Liège).

When dealing with the behaviour of multiphase porous systems, e.g. geomaterials, instances of complexity and interaction are numerous, mainly because of the coexistence of several constituents and phases, their physical and mechanical interactions, their reactivity and their often non-linear behaviour. The study of these coupled processes deals with a large number of applications, e.g. in geomechanics: underground structures (storage, tunnelling), surface structures (earth and concrete dams, embankments) as well as the exploitation of geo-resources (petroleum and gas extraction, mines and quarries).

This volume contains nine chapters in which emphasis is given to the presentation of the fundamental and new concepts that help understanding coupled and multiphysics phenomena in porous systems. The contributions cover experimental, theoretical, as well as numerical aspects. The school is divided into three main parts: the description of the couplings in multiphysics phenomena, including the experimental developments; the mathematical modelling of all these coupled processes, with an introduction to the constitutive modelling taking into account the dilatancy, which characterizes the mechanical behaviour of geomaterials; the numerical implementation of the mathematical models, comprising constitutive equations as well as balance equations and finally numerical modelling through advanced applications.

The experimental aspects of coupled and multiphysics phenomena are described in Chapter one. Pierre Delage introduced the different techniques to measure and control the environmental variables such as suction and temperature. The results obtained through advanced experimental techniques are presented, providing a global overview of the knowledge in this particular field.

Additional coupling phenomena are described by Jacques Huyghe in Chapters two and three, dealing with physical chemistry of mixtures and swelling, discussing mixtures of a liquid with a dissolved substance and considering electrolytic solutions and osmosis in biological tissues. For their mathematical description, theory of mixtures has been summarised. Moreover, Chapter three presents insights in finite deformation poromechanics with application to soil mechanics, poromechanics of the heart muscle and blood perfusion.

Chapter four by William Gray and Cass Miller deals with thermodynamically constrained averaging theory TCAT, which is the most advanced and most general method to develop governing equations of physical problems able to describe the complex system of couplings in the behaviour of multiphase porous systems. This theory is based on thermo-dynamic principles applied at microscale and averages all quantities from the microscale to the macroscale in a consistent and well-defined manner, so that the connection between microscale and macroscale quantities is explicitly known.

Manuel Pastor introduces in Chapter five the constitutive modelling of geomaterials based on Generalized Plasticity Theory. The underlying idea is to show how this general framework for the development of a constitutive law can be extended from a purely mechanical case to hydro-mechanical context. This helps in exhibiting the role of dilatancy in modelling of the most relevant phenomena in soils behaviour such as liquefaction, bonding and de-bonding due to chemical processes or changes in saturation conditions and influence of particle breakage.

The third part of this book is opened by Manuel Pastor with Chapter six, which introduces the numerical implementation of the mathematical models described in the previous chapters. The aim is to provide the reader with an overview both of the techniques and the difficulties encountered when modelling this type of problems. The analysis is restricted to the simplest case where only one fluid filling the pores is considered, as the main difficulties can be more easily explained and understood.

Then, with the last three chapters, the complexity of the problems increases step by step by considering variably saturated problems, thermo-hydro-mechanical problems and, finally, bio- chemo- thermo- hydro-mechanical problems. Hydraulic fracturing, the first numerical modelling of advanced applications, is presented by Bernhard Schrefler. Fluid-driven fracture propagating in porous media is a common problem in geomechanics. It is used, for example, to enhance the recovery of hydrocarbons from underground reservoirs.

Chapter eight by Lorenzo Sanavia presents a fully coupled and non-linear finite element model for the analysis of non-isothermal variably saturated soils in dynamics. Attention is given to the validation step when dealing with the development of numerical models.

In the last chapter, Frédéric Collin introduces the modelling of municipal waste considered as a bio-chemo-thermo-hydro-mechanical model. This latter material is a perfect example of porous media with coupled and multiphysics phenomena. As a consequence of the numerous physical processes, it is proposed to follow a step by step approach where each single aspect is introduced. The couplings are first presented through closed form solutions for simplified cases and then numerically modelled with all their complexity.

We believe that this volume may provide to postgraduate students, researchers and practitioners, a valuable introduction and a sound basis for further progress in the challenging fields of coupled and multiphysics phenomena in porous systems.

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Finite element analysis of non-isothermal multiphase porous media in dynamics

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This work presents a mathematical and a numerical model for the analysis of the thermo-hydro-mechanical (THM) behavior of multiphase deformable porous materials in dynamics. The fully coupled governing equations are developed within the Hybrid Mixture Theory. To analyze the THM behavior of soil structures in the low frequency domain, e.g. under earthquake excitation, the u-p-T formulation is advocated by neglecting the relative acceleration of the fluids and their convective terms. The standard Bubnov-Galerkin method is applied to the governing equations for the spatial discretization, whereas the generalized Newmark scheme is used for the time discretization. The final non-linear and coupled system of algebraic equations is solved by the Newton method within the monolithic approach. The formulation and the implemented solution procedure are validated through the comparison with other finite element solutions or analytical solutions.

1 Introduction

The analysis of the dynamic response of multiphase porous media has many applications in civil engineering. Onset of landslides due to earthquakes or rainfall and the seismic behavior of dams are examples where inertial forces cannot be neglected. Moreover, there are situations where it is important to consider also the effect of temperature variation. It is the case of catastrophic landslides, where the mechanical energy dissipated in heat inside the slip zone may lead to vaporization of the pore water creating a cushion of zero friction, which may accelerate the movement of the landslides [Var02]. Another interesting case is the seismic analysis of deep nuclear waste disposal.

Many authors have developed models for the analysis of the dynamic behavior of multiphase porous media in isothermal conditions. A state of art can be found in Zienkiewicz et al. [Zie99] and Schanz [Sch09]. Recently, Nenning and Schanz [Nen10] presented an infinite element for wave propagation problems; Heider et al. [Hei11] analyzed a numerical solution of dynamic wave propagation problems in infinite half spaces with incompressible constituents and Albers [Alb10] analyzed wave propagation problems in saturated and partially saturated porous media.

This work presents a formulation of a fully coupled model for deformable multiphase geomaterials in dynamics including thermal effects.
The model is derived introducing the u-p-T (displacements, pressures, temperature) formulation in the multiphase model developed in Lewis and Schrefler [Lew98], in which the relative acceleration of the fluids and their convective terms have been neglected following [Cha88], [Zie99]. This reduced model is valid for low frequency problems, as in earthquake engineering, [Cha88], [Zie99]. The standard Galerkin method is applied to the governing equations for the spatial discretization, while the generalized Newmark scheme is used for the time discretization. The final nonlinear set of equations is solved by the Newton method with a monolithic approach.
The model has been implemented in the finite element code COMES-GEO, [Gaw96], [Lew98], [San06], [San08], [San09], [Gaw09], [Gaw10], [San12] and has been validated through the comparison with analytical or finite element quasi-static or dynamic solutions.

2 Macroscopic balance equations

The full mathematical model necessary to simulate the thermo-hydro-mechanical behavior of partially saturated porous media in dynamics was developed within the Hybrid Mixture Theory (HMT) by Lewis and Schrefler [Lew98], using averaging theories according to Hassanizadeh and Gray [Has79a], [Has79b], [Has80], [Gra91]. This model can be derived from the more advanced averaging theory TCAT - Thermodynamically Constrained Averaging Theory (see the chapter of this book from Gray and Miller or [Gra14] and its references listing the journal papers on this topic).

The variably saturated porous medium is treated as a multiphase system composed of solid skeleton (s) with open pores filled with liquid water (w) and gas (g). The latter, is assumed to behave as an ideal mixture of dry air (non-condensable gas, ga) and water vapor (condensable gas, gw). At the macroscopic level the porous material is modeled by a substitute continuum of volume B with boundary ∂B that simultaneously fills the entire domain, instead of the real fluids and the solid which fill only a part of it. In this substitute continuum each constituent π has a reduced density which is obtained through the volume fraction $\eta^{\pi}(\boldsymbol{x},t) = dv^{\pi}(\boldsymbol{x},t) \,/\, dv(\boldsymbol{x},t)$, where dv is the volume of the average volume element (representative elementary volume, REV) of the porous medium and dv^{π} is the volume occupied by the constituent π in dv. $\boldsymbol{x}$ is the vector of the spatial coordinates and t the current time.

The solid is deformable and non-polar and the fluids, solid and thermal effects are coupled. All fluids are in contact with the solid phase. In the model, heat conduction and heat convection, vapor diffusion, (liquid) water flow due to pressure gradients or capillary effects and water phase change (evaporation and condensation) inside the pores are taken into account.
In the partially saturated zones the liquid water is separated from its vapor by a concave meniscus (capillary water). Due to the curvature of this meniscus, the sorption equilibrium equation [Gray91] gives the relationship $p^c=p^g-p^w$ between the capillary pressure $p^c(\boldsymbol{x},t)$ (also known as matrix suction), gas pressure $p^g(\boldsymbol{x},t)$ and liquid water pressure $p^w(\boldsymbol{x},t)$. This expression is approximated in dynamics; it is used here because of lack of experimental results. In the following, pore pressure is defined as compressive positive for the fluids, while stress is defined as tension positive for the solid phase.
The state of the medium is described by gas pressure p^g, capillary pressure p^c, temperature T and displacements of the solid matrix $\boldsymbol{u}$ [San06]. The balance equations are developed in geometrically linear framework and are written here at the macroscopic level.
For sake of completeness the equations of the model are only summarized in this chapter; the interested reader is refereed to [San15] for more details regarding the development of the mathematical model and its finite element implementation. Direct notation is adopted. Boldface letters denote vector or tensors and lightface italic letters are used for scalar quantities.

After neglecting the relative velocity and acceleration of the fluids in the governing equations of Lewis and Schrefler [Lew98], a set of balance equations for the whole multiphase medium is obtained as follows.

The linear momentum balance equations of the mixture in term of the generalized effective Cauchy's stress $\boldsymbol{\sigma}'(\boldsymbol{x},t)$ [Lew98], [Nut08] takes the form

$$div\left(\boldsymbol{\sigma}' - \left[p^g - S_w p^c\right]\mathbf{1}\right) + \rho\mathbf{g} = \rho\mathbf{a}^s \tag{1}$$

where $\rho = [1-n]\rho^s + nS_w\rho^w + nS_g\rho^g$ is the mass density of the overall medium, $S_w(\boldsymbol{x},t)$ is the degree of saturation of the liquid water $n(\boldsymbol{x},t)$ is the porosity and $S_g(\boldsymbol{x},t)$ is the degree of saturation of the gas, with $S_w + S_g=1$. $\rho^s(\mathbf{x},t)$ is the density of the solid grain, $\rho^w(\mathbf{x},t)$ is the density of liquid water and $\rho^g(\mathbf{x},t)$ is the density of the gas phase. $\mathbf{g}$ is the gravity acceleration vector, $\mathbf{1}$ is the second order identity tensor and $\mathbf{a}^s(\mathbf{x},t)$ the acceleration of the solid phase. The form of Eq. (1) assumes incompressible grains, which is common in soil mechanics. In order to consider compressible grains, the Biot coefficient should be set in front of the solid pressure (this becomes important when dealing with rock and concrete). The total stress of equation (1), using saturation as weighting functions for the partial pressures, was introduced in [Sch84] using volume averaging for the bulk materials and is thermodynamically consistent, e.g. [Gra91].

The mass balance equations for the dry air and the liquid water and its vapor are, respectively:

$$
\begin{aligned}
&\operatorname{div}\left(\rho^{ga}\frac{k^{rg}\boldsymbol{k}}{\mu^{g}}\left[-\operatorname{grad}p^{g}+\rho^{g}\boldsymbol{g}\right]\right)+\operatorname{div}\left(\rho^{g}\frac{M_{a}M_{w}}{M_{g}^{2}}\boldsymbol{D}_{g}^{ga}\operatorname{grad}\left(\frac{p^{gw}}{p^{g}}\right)\right)\\
&+\rho^{ga}S_{g}\operatorname{div}\boldsymbol{v}^{s}+nS_{g}\dot{\rho}^{ga}-\rho^{ga}n\dot{S}_{w}-\rho^{ga}\beta_{s}\left(1-n\right)S_{g}\dot{T}=0
\end{aligned}
\qquad (2)
$$

and

$$
\begin{aligned}
&\operatorname{div}\left(\rho^{w}\frac{k^{rw}\boldsymbol{k}}{\mu^{w}}\left(-\operatorname{grad}p^{w}+\rho^{w}\boldsymbol{g}\right)\right)+\operatorname{div}\left(\rho^{gw}\frac{k^{rg}\boldsymbol{k}}{\mu^{g}}\left(-\operatorname{grad}p^{gw}+\rho^{gw}\boldsymbol{g}\right)\right)\\
&-\operatorname{div}\left(\rho^{g}\frac{M_{a}M_{w}}{M_{g}^{2}}\boldsymbol{D}_{g}^{gw}\operatorname{grad}\left(\frac{p^{gw}}{p^{g}}\right)\right)+\left[\rho^{w}S_{w}+\rho^{gw}S_{g}\right]\operatorname{div}\boldsymbol{v}^{s}+\rho^{w}\frac{nS_{w}}{K_{w}}\left[\dot{p}^{g}-\dot{p}^{c}\right]\\
&-\left[\rho^{w}\beta_{sw}+\rho^{gw}\beta_{s}\left(1-n\right)S_{g}\right]\dot{T}+\left[n\rho^{w}-n\rho^{gw}\right]\dot{S}_{w}+nS_{g}\dot{\rho}^{gw}=0
\end{aligned}
\qquad (3)
$$

where $\mathbf{k}$ $(\boldsymbol{x},t)$ = $k(\boldsymbol{x},t)\mathbf{1}$ is the intrinsic permeability tensor of the porous matrix in water saturated condition [m^2], which is assumed to be isotropic, $k^{r\pi}(\boldsymbol{x},t)$ is the fluid relative permeability parameter and $\mu^{\pi}(\boldsymbol{x},t)$ is the dynamic viscosity of the fluid [Pa s], with $\pi = w, g$. K_{w} is the bulk modulus of the liquid water. β_{sw} = [1-n]$\beta_s[S_g\rho^{gw} + \rho^w S_w]$, with $\beta_s(\boldsymbol{x},t)$ the cubic thermal expansion coefficient of the solid. $\mathbf{D}_g^{gw}(\boldsymbol{x})$ is the effective diffusivity tensor of water vapor in the gas phase contained within the pore space, and M_a, M_w and $M_g(\boldsymbol{x},t)$ are the molar mass of dry air, liquid water and the gas mixture $M_g=\left[\frac{\rho^{gw}}{\rho^{g}}\frac{1}{M_w}+\frac{\rho^{ga}}{\rho^{g}}\frac{1}{M_a}\right]^{-1}$, respectively. These equations contain the mass balance equation of the solid phase, which has been introduced to eliminate the time derivative of the porosity.

The enthalpy balance equation for the multiphase medium is:

$$
\begin{aligned}
&-\operatorname{div}\left(\rho^{w}\frac{k^{rw}\mathbf{k}}{\mu^{w}}\left[-\operatorname{grad}\left(p^{w}\right)+\rho^{w}\mathbf{g}\right]\right)\Delta H_{vap}-\operatorname{div}\left(\chi_{eff}\operatorname{grad}T\right)-\rho^{w}S_{w}\operatorname{div}\mathbf{v}^{s}\Delta H_{vap}\\
&+\left[C_{p}^{w}\rho^{w}\frac{k^{rw}\mathbf{k}}{\mu^{w}}\left[-\operatorname{grad}\left(p^{w}\right)+\rho^{w}\mathbf{g}\right]+C_{p}^{g}\rho^{g}\frac{k^{rg}\mathbf{k}}{\mu^{g}}\left[-\operatorname{grad}p^{g}+\rho^{g}\mathbf{g}\right]\right]\cdot\operatorname{grad}T\\
&+\left(\rho C_{p}\right)_{eff}\dot{T}-\rho^{w}\frac{nS_{w}}{K_{w}}\dot{p}^{w}\Delta H_{vap}+\beta_{sw}\dot{T}\Delta H_{vap}-n\left[\rho^{w}-\rho^{gw}\right]\dot{S}_{w}\Delta H_{vap}=0
\end{aligned}
\qquad (4)
$$

where $\left(\rho C_p\right)_{eff}(\boldsymbol{x},t)$ is the effective thermal capacity of the porous medium, $C_p^w(\boldsymbol{x},t)$ and $C_p^g(\boldsymbol{x},t)$ are the specific heat of water and gas, respectively, and $\chi_{eff}(\boldsymbol{x},t)$ is the effective thermal conductivity of the porous medium. The RHS term of Equation (4) considers the contribution of the evaporation and condensation. In equations (2)-(4) the advective fluxes have been described using Darcy's law for liquid water and gas, while the diffusion of vapor in the gas phase has been modeled with Fick's law. A recent development of a model which considers the air dissolved in the liquid water and its desorption at lower water pressures in quasi-statics loading conditions is presented in [Gaw09].

3 Constitutive relationships

For the gaseous mixture of dry air and water vapour, the ideal gas law is introduced. The equation of state of perfect gas (Clapeyron's equation) and Dalton's law are applied to dry air (ga), water vapor (gw) and moist air (g).

$$p^{ga} = \rho^{ga} TR / M_a\,, \quad p^{gw} = \rho^{gw} TR / M_w\,, \quad p^g = p^{ga} + p^{gw}\,, \quad \rho^g = \rho^{ga} + \rho^{gw} \qquad (5)$$

In the partially saturated zones, the equilibrium water vapor pressure $p^{gw}(\boldsymbol{x},t)$ can be obtained from the Kelvin-Laplace equation, where the water vapor saturation pressure, $p^{gws}(\boldsymbol{x},t)$, depending only upon the temperature, can be calculated from the Clausius-Clapeyron equation or from an empirical correlation. The saturation degree $S_w(\boldsymbol{x},t)$ and the relative permeability $k^{r\pi}(\boldsymbol{x},t)$ are experimentally determined functions dependent on capillary pressure and temperature (e.g. [Fra08] for S_w). The bulk density of liquid water that is dependent on the temperature is modeled using the relationship proposed by Furbish [Fur97]. The liquid water viscosity, dry air and water vapor viscosity, and the latent heat of evaporation are also temperature dependent relationships.

The solid skeleton is assumed elastic or elasto-plastic, homogeneous and isotropic in the numerical simulations described in Section 5. Its mechanical behavior is described within the classical rate-independent elasto-plasticity theory for geometrically linear problems. For the third numerical example, the yield function restricting the effective stress state $\boldsymbol{\sigma}'(\boldsymbol{x},t)$ is developed in the form of Drucker-Prager model for simplicity, with linear isotropic softening and non-associated plastic flow to take into account the post-peak and dilatant behavior of dense sands, respectively. The return mapping and the consistent tangent operator for the Jacobian matrix, equations (9), is developed in [San06], where the singular behavior of the Drucker-Prager yield surface in the zone of the apex is solved by using the multi-surface plasticity theory (following the formulation developed in [San02] for isotropic linear hardening/softening and volumetric-deviatoric non-associative plasticity in case of large strain elasto-plasticity).

The Drucker-Prager yield function with linear isotropic hardening/softening has been used in the form

$$F\left(p,\mathbf{s},\xi\right)=3\alpha_F\,p+\|\mathbf{s}\|-\beta_F\sqrt{\tfrac{2}{3}}\left[c_0+h\xi\right] \tag{6}$$

in which $p=\frac{1}{3}\left[\boldsymbol{\sigma}':\mathbf{1}\right]$ is the mean effective Cauchy pressure, $\|\mathbf{s}\|$ is the L_2 norm of the deviator effective Cauchy stress tensor $\boldsymbol{\sigma}'$, c_0 is the initial apparent cohesion, α_F and β_F are two material parameters related to the friction angle ϕ of the soil,

$$\alpha_F=2\frac{\sqrt{\frac{2}{3}}\sin\phi}{3-\sin\phi}\qquad \beta_F=\frac{6\cos\phi}{3-\sin\phi} \tag{7}$$

h the hardening/softening modulus and ξ the equivalent plastic strain.
To take into account the effect of capillary pressure and temperature on the evolution of the yield surface, the interested reader can refer, for example, to the chapter by Manzanal et. al of this book and [Fra08] for capillary dependent constitutive relationships in isothermal or non-isothermal conditions, respectively.

4 Spatial and time discretization

The finite element model is derived by applying the Galerkin procedure for the spatial integration and the generalized Newmark method for the time integration of the weak form of the balance equations (1)-(4) [Lew98], [Zie99], [Zie00].
In particular, after spatial discretization within the isoparametric formulation, the following non-symmetric, non-linear and coupled system of equations is obtained:

$$\begin{cases}\boldsymbol{C}_{gg}\dot{\bar{\boldsymbol{p}}}^{g}+\boldsymbol{C}_{gc}\dot{\bar{\boldsymbol{p}}}^{c}-\boldsymbol{C}_{gT}\dot{\bar{\boldsymbol{T}}}+\boldsymbol{C}_{gu}\dot{\bar{\boldsymbol{u}}}+\boldsymbol{K}_{gg}\bar{\boldsymbol{p}}^{g}-\boldsymbol{K}_{gc}\bar{\boldsymbol{p}}^{c}-\boldsymbol{K}_{gT}\bar{\boldsymbol{T}}=\boldsymbol{f}_{g}\\ \boldsymbol{C}_{cg}\dot{\bar{\boldsymbol{p}}}^{g}+\boldsymbol{C}_{cc}\dot{\bar{\boldsymbol{p}}}^{c}+\boldsymbol{C}_{cT}\dot{\bar{\boldsymbol{T}}}+\boldsymbol{C}_{cu}\dot{\bar{\boldsymbol{u}}}-\boldsymbol{K}_{cg}\bar{\boldsymbol{p}}^{g}+\boldsymbol{K}_{cc}\bar{\boldsymbol{p}}^{c}+\boldsymbol{K}_{cT}\bar{\boldsymbol{T}}=\boldsymbol{f}_{c}\\ -\boldsymbol{C}_{Tg}\dot{\bar{\boldsymbol{p}}}^{g}-\boldsymbol{C}_{Tc}\dot{\bar{\boldsymbol{p}}}^{c}+\boldsymbol{C}_{TT}\dot{\bar{\boldsymbol{T}}}-\boldsymbol{C}_{Tu}\dot{\bar{\boldsymbol{u}}}-\boldsymbol{K}_{Tg}\bar{\boldsymbol{p}}^{g}+\boldsymbol{K}_{Tc}\bar{\boldsymbol{p}}^{c}+\boldsymbol{K}_{TT}\bar{\boldsymbol{T}}=\boldsymbol{f}_{T}\\ \boldsymbol{M}_{uu}\ddot{\bar{\boldsymbol{u}}}+\int\boldsymbol{B}^{T}\boldsymbol{\sigma}'dW-\boldsymbol{K}_{ug}\bar{\boldsymbol{p}}^{g}+\boldsymbol{K}_{uc}\bar{\boldsymbol{p}}^{c}=\boldsymbol{f}_{u}\end{cases} \tag{8}$$

where the displacements of the solid skeleton $\mathbf{u}(\boldsymbol{x},t)$, the capillary pressure $p^{c}(\boldsymbol{x},t)$, the gas pressure $p^{g}(\boldsymbol{x},t)$ and the temperature $T(\boldsymbol{x},t)$ are expressed in the whole domain by global shape function matrices $\mathbf{N}_u(\boldsymbol{x})$, $\mathbf{N}_c(\boldsymbol{x})$, $\mathbf{N}_g(\boldsymbol{x})$, $\mathbf{N}_T(\boldsymbol{x})$ and the nodal value vectors $\bar{\boldsymbol{u}}(t),\bar{\boldsymbol{p}}^{c}(t),\bar{\boldsymbol{p}}^{g}(t),\bar{\boldsymbol{T}}(t)$.
Following the Generalized Newmark Method, equations (8) are rewritten at time $t_{(n+1)}$. The elements of the matrices $\boldsymbol{C}_{ij}$, $\boldsymbol{K}_{ij}$ and the vectors $\boldsymbol{f}_i$ are given in [San15].
In this study, the generalized Newmark time integration scheme [Zie00] is applied to the non-linear equation system (8) and a non-linear system of algebraic equations is

obtained, in which the unknowns are $\boldsymbol{x} = \left[\Delta\dot{\bar{p}}^g , \Delta\dot{\bar{p}}^c , \Delta\dot{\bar{T}} , \Delta\ddot{\bar{\boldsymbol{u}}} \right]$. The non-linear system is solved by Newton-Raphson method, thus obtaining the equation system that can be solved numerically (written below in a compact form) as:

$$\left. \frac{\partial \mathbf{G}}{\partial \mathbf{X}} \right|_{\mathbf{X}_{n+1}^{i}} \Delta \mathbf{X}_{n+1}^{i+1} \cong -\mathbf{G}\left(\mathbf{X}_{n+1}^{i} \right) \tag{9}$$

with the symbol $(\bullet)_{n+1}^{i+1}$ to indicate the current iteration (i+1) in the current time step (n+1) and where $\partial\mathbf{G}/\partial\mathbf{X}$ is the Jacobian matrix.
Owing to the strong coupling between the mechanical, thermal and the pore fluids fields, a monolithic solution of (9) is preferred.

5 Finite element simulations

This section addresses the numerical validation of the model previously derived and presents an application studying a biaxial strain localization test.
Different tests have been simulated and presented in [San15], aiming to validate: a) the wave propagation in a solid material (equation (1) restricted to single phase solid material), b) the isothermal water saturated model (equations (1) and (3) with S_{w}=1), c) the isothermal variably saturated model (equations (1), (2) and (3)) and d) the non-isothermal water saturated model (equations (1), (3) and (4) with Sw=1). Analytical solutions are available in [Slu92] and [Boe93] for the first two tests respectively, while the numerical results from tests c) and d) have been compared with the numerical solution of the corresponding quasi-static models because of the lack of analytical solutions. Some representative results of tests c) and d) are illustrated here.

5.1 Drainage of liquid water from initially water saturated soil column

This numerical test is based on an experiment performed by Liakopoulos [Lia65] on a column 1 meter high (Figure 1) of Del Monte sand and instrumented to measure the moisture tension at several points along the column during its desaturation due to gravitational effects. Before the start of the experiment, water was continuously added from the top and was allowed to drain freely at the bottom through a filter, until uniform flow conditions were established. Then the water supply was ceased and the tensiometer readings were recorded. The finite element simulation is performed with the two-phase flow model in isothermal conditions. For the numerical calculation, a two-dimensional problem in plane strain conditions is solved; the spatial domain of the column is divided into 20 eight-node isoparametric finite elements of equal size. Furthermore, nine Gauss integration points were used. The

material parameters are listed in [Gaw96] or [San15], as well as the description of the boundary conditions and the equations for the saturation-capillary pressure and the relative permeability of water-capillary pressure relationships.
This problem has been solved considering single or two-phase flow mainly in quasi-static condition (e.g. [Gaw96]); a finite element solution in dynamics was presented in [Sch98]. The initial hydro-mechanical equilibrium state is obtained via a preliminary quasi-static solution.
The comparison between the dynamic and the quasi-static solution is plotted in Figures 2 to 4, where the profiles for liquid water pressure, liquid water saturation and vertical displacement along the column are plotted. Since the inertial loads are negligible in the experiment, the finite element solution in dynamics gives almost the same results of the quasi-static model [Gaw96], [Gaw09].

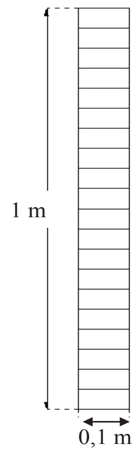

Figure 1: Geometry and finite element discretization of the sand column.

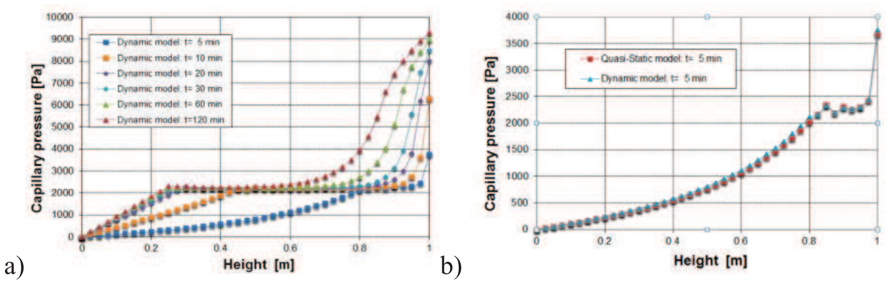

Figure 2: Profiles of capillary pressure versus height: a) dynamic solution; b) comparison between the quasi-static and the dynamic solution.

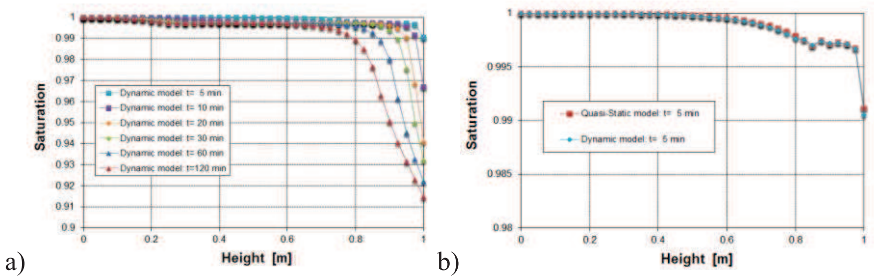

Figure 3: Profiles of liquid water saturation degree versus height: a) dynamic solution; b) comparison between the quasi-static and the dynamic solution.

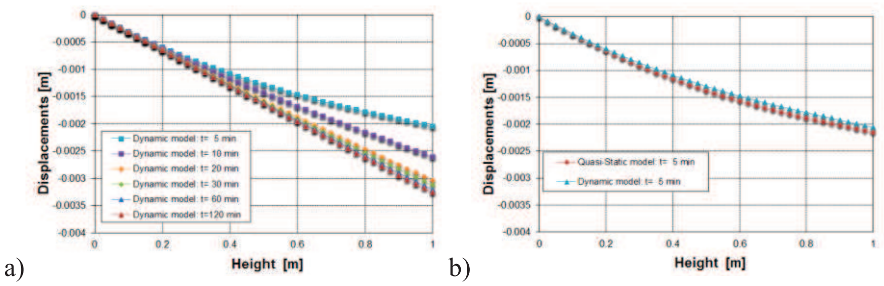

Figure 4: Profiles of vertical displacement versus height: a) dynamic solution; b) comparison between the quasi-static and the dynamic solution.

5.2 Numerical validation of the non-isothermal water saturated model

This problem deals with a water saturated thermo-elastic consolidation [Abo85], simulating a column, 7 m high and 2 m wide, of a linear elastic material subjected to an external surface load of 10 kPa and to a surface temperature jump of 50 K above the initial temperature of 293.15 K (Figure 5). The material parameters used in the computation are summarized in [San08]. The liquid water and the solid grain are assumed incompressible for the quasi-static analysis, whereas the compressibility of the liquid water is taken into account in the dynamic analysis. The initial and boundary conditions are described in [San08] and [San15]. Plane strain condition is assumed. The spatial domain is discretized with eight-node isoparametric elements; nine Gauss points are used.
The solution of the finite element model presented in this work is compared with the quasi-static solution [San08] and is plotted in Figures 6 and 7. The results show that the dynamic solution is faster than the quasi-static one at the beginning of the analysis, and that the dynamic solution reaches the quasi-static one at the steady-state.

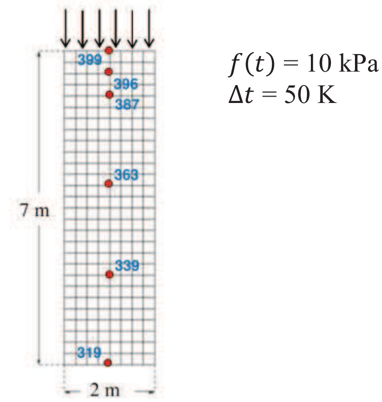

Figure 5: Description of the non-isothermal water saturated test.

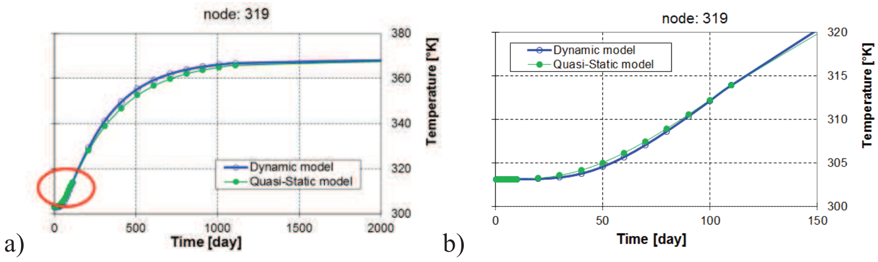

Figure 6: Temperature time history for node 319 up to the steady state solution (a) and in the first period (b) highlighted in a).

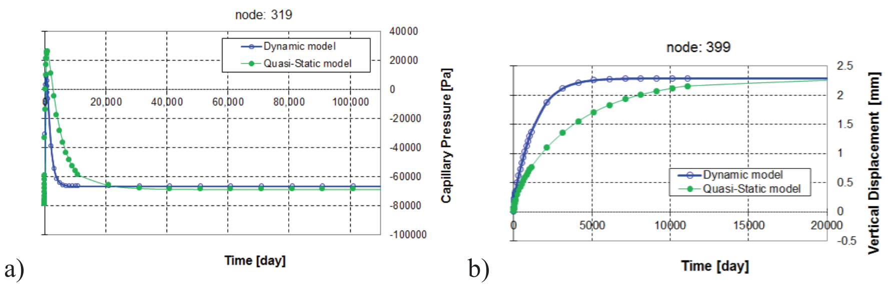

Figure 7: a) Capillary pressure time history for node 319 and b) vertical displacement time history for node 399.

5.3 Globally undrained biaxial compression test

A plane strain compression test of initially water saturated dense sand in globally undrained conditions is simulated here with the model developed in the previous sections. This case was solved in [San06] in quasi-static conditions and is inspired by the experimental work of Mokni and Desrues [Mok98], in which cavitation of the liquid water was experimentally observed at localization.

A sample of 34 cm height and 10 cm width is compressed with imposed vertical displacement applied to the top surface at a velocity of 3.6 mm/s (Figure 8). Vertical and horizontal displacements are constrained at the bottom surface; the boundary of the sample is impervious and adiabatic.

The mechanical behavior of the solid skeleton is simulated using the elasto-plastic Drucker-Prager constitutive model (with isotropic linear softening and non-associated plastic flow) summarized in Section 3. At time t= 0 seconds, the initial conditions for the domain are the hydrostatic water pressure, the gas pressure at atmospheric value and a temperature of 293.15 K. Gravity acceleration is taken into account; the initial stress state in equilibrium with the initial conditions and thermo-hydro boundary conditions is computed with the corresponding quasi-static model [San06]. The geomechanical characteristics of the dense sand are given in [San06].

Figures 9 and 10 show the contour plots at 13 seconds of the following thermo-hydro-mechanical variables: equivalent plastic strain, volumetric strain, capillary pressure, liquid water saturation and relative humidity. Positive volumetric strains are observed inside the dilatant shear bands (Figure 9b), inducing a liquid water pressure drop up to the development of capillary pressures (Figure 10a) desaturating the plastic zones (Figure 10b) because of the phase change of the liquid water into vapor due to cavitation (Figure 10c).

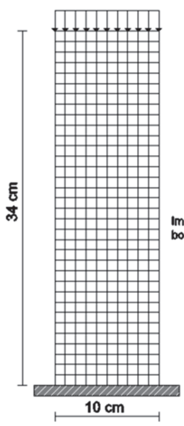

Figure 8: Finite element discretization and boundary conditions of the biaxial compression test.

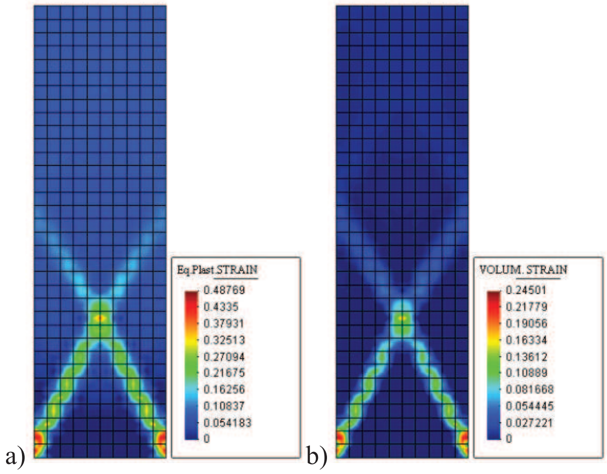

Figure 9: Numerical solution at 13 s: a) equivalent plastic strain, b) volumetric strain.

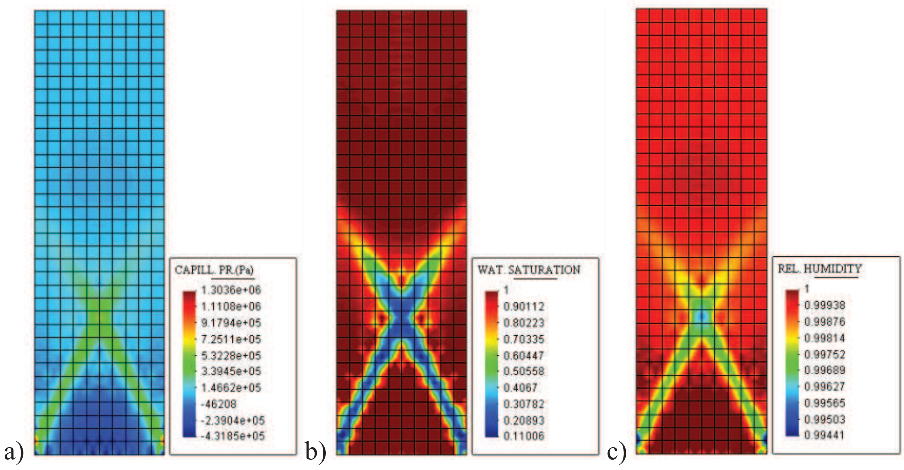

Figure 10: Numerical solution at 13 s: a) capillary pressure, b) liquid water saturation, c) relative humidity.

To study the independence of shear band width from the finite element size in dynamics, e.g. [Sch96], [Sch99], [Zha99] and [Sch06], test runs with meshes of 85, 340 and 1360 elements have been carried out. In this case, the analysis of the finite element results [Cao15] shows that the shear band width is reasonably mesh independent, while the peak value of the equivalent plastic strain and, as a consequence, of the volumetric strain, the capillary pressure, the water vapor pressure and the relative humidity are sensitive to mesh refinement and a regularization scheme would be needed as expected (e.g. [Zha99], [Sch99] and [Sch06]), because the internal length scale given by the liquid water motion [Zha99] is not sufficient to regularize the numerical solution.

6 Conclusions

A model for the analysis of the thermo-hydro-mechanical behavior of porous media in dynamics was developed. Starting from the generalized mathematical model developed in [Lew98] for deforming porous media in non-isothermal conditions, the u-p-T formulation was derived following [Zie99]. The validity of such an approximation is limited to low frequencies problems [Zie99], as in earthquake engineering. In this formulation, the relative accelerations of the fluids and the convective terms related to these accelerations are neglected.

The numerical model was derived within the finite element method: the standard Bubnov-Galerkin procedure [Zie00] was adopted for the discretization in space, while the implicit and unconditionally stable generalized Newmark procedure was applied for the discretization in time [Zie00].

The model was implemented in the finite element code Comes-Geo [Lew98], [Gaw96], [San06], [San08], [San09], [Gaw09], [Gaw10]. The formulation and the implemented solution procedure were validated through the comparison with literature benchmarks, finite element solutions or analytical solutions. In this work, comparison between the finite element solution in dynamics and the corresponding quasi-static solution is presented by studying the non-isothermal consolidation in a water saturated column and the drainage of liquid water in an initially water saturated soil column.

This work extends the model developed in [Sch98] to non-isothermal conditions and removes the passive air phase assumption of the multiphase porous media model in dynamics developed in [Zie99] and [Gaw98].

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