Physics Letters B 722 (2013) 330–335

Contents lists available at SciVerse ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

The effective chiral Lagrangian for a light dynamical "Higgs particle"

R. Alonso [a], M.B. Gavela [a,b], L. Merlo [a,b,*], S. Rigolin [c], J. Yepes [a]

[a] Departamento de Física Teórica and Instituto de Física Teórica, IFT-UAM/CSIC, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain
[b] CERN, Department of Physics, Theory Division, CH-1211 Geneva 23, Switzerland
[c] Dipartimento di Fisica "G. Galilei", Università di Padova and INFN, Sezione di Padova, Via Marzolo 8, I-35131 Padua, Italy

ARTICLE INFO

Article history:
Received 21 December 2012
Received in revised form 12 April 2013
Accepted 21 April 2013
Available online 22 April 2013
Editor: B. Grinstein

ABSTRACT

We generalize the basis of CP-even chiral effective operators describing a dynamical Higgs sector, to the case in which the Higgs-like particle is light. Gauge and gauge-Higgs operators are considered up to mass dimension five. This analysis completes the tool needed to explore at leading order the connection between linear realizations of the electroweak symmetry breaking mechanism — whose extreme case is the Standard Model — and non-linear realizations with a light Higgs-like particle present. It may also provide a model-independent guideline to explore which exotic gauge-Higgs couplings may be expected, and their relative strength to Higgsless observable amplitudes. With respect to fermions, the analysis is reduced by nature to the consideration of those flavor-conserving operators that can be written in terms of pure-gauge or gauge-Higgs ones via the equations of motion, but for the standard Yukawa-type couplings.

1. Introduction

A new resonance at the Electroweak (EW) scale has been established at LHC [1,2], consistent with the hypothesis of the SM scalar boson (so-called "Higgs boson" for short hereafter) [3–5] with mass around 125 GeV.

There are essentially two main frameworks that have been proposed to explain the EW symmetry breaking sector. The first possibility is that the Higgs is a fundamental particle, transforming linearly (as a doublet in the standard minimal picture) under the gauge symmetry group $SU(2)_L \times U(1)_Y$. Another possibility is, however, that the Higgs dynamics is not perturbative and the gauge symmetry in the scalar sector is non-linearly realized; this may be the case for instance if the Higgs resonance does not correspond to an elementary particle. In such a framework some strong dynamics should intervene at a scale Λ_s, and the characteristic scale of the associated Goldstone bosons f respects $\Lambda_s \leqslant 4\pi f$ [6]. In the original formulation [7–9] the physical Higgs particle is simply removed from the low-energy spectrum and only the three would-be-Goldstone bosons are retained, in order to give masses to the weak gauge bosons, with $f = v$, where $v = 246$ GeV denotes the electroweak scale defined via the W mass, $M_W = gv/2$. The smoking gun signature of this "technicolor" ansatz is the appearance of several vector and fermion resonances at the TeV scale.

However, several variants of the strong interacting ansatz exist, with some of them "predicting" the existence of a light Higgs resonance in the spectrum. In the best known of such scenarios, originally proposed in Refs. [10–15], the SM Higgs particle is substituted by a composite scalar degree of freedom that, being a quasi-Goldstone boson of a larger symmetry group, cannot acquire a large (i.e. $O(\text{TeV})$) mass.[1] Besides this light Higgs-like scalar particle, these models still present a strongly interacting sector at the TeV scale, while they may correct at lower energies the size of SM couplings. This path looks promising in the absence of new resonances in LHC data. For these sophisticated constructions, the characteristic scale f associated to the Goldstone bosons of the theory — which now include also the Higgs particle — does not need to coincide: i) neither with the scale of electroweak symmetry breaking, that will be denoted by $\langle h\rangle$, ii) nor with the electroweak scale v; while a constraint links together f, $\langle h\rangle$ and v. Indeed, in these hybrid schemes

$$\xi \equiv (v/f)^2 \tag{1}$$

parametrizes the degree of non-linearity of the Higgs dynamics. In the limit in which Λ_s and thus f go to infinity, the linear SM picture is recovered.

* Corresponding author at: Departamento de Física Teórica and Instituto de Física Teórica, IFT-UAM/CSIC, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain.

E-mail addresses: rodrigo.alonso@uam.es (R. Alonso), belen.gavela@uam.es (M.B. Gavela), luca.merlo@uam.es (L. Merlo), stefano.rigolin@pd.infn.it (S. Rigolin), ju.yepes@uam.es (J. Yepes).

[1] See for example Ref. [16] for a recent review on the subject.

0370-2693/$ – see front matter
http://dx.doi.org/10.1016/j.physletb.2013.04.037

Without referring to a specific model, one can attempt to describe NP effects by making use of an effective Lagrangian approach, with operators made out of SM fields. The transformation properties of the longitudinal degrees of freedom of the electroweak gauge bosons can always be described at low-energy[2] by a dimensionless unitary matrix transforming as a bi-doublet of the global symmetry group:

$$\mathbf{U}(x) = e^{i\sigma_a \pi^a(x)/v}, \qquad \mathbf{U}(x) \to L\mathbf{U}(x)R^\dagger,$$

with L, R denoting respectively the $SU(2)_{L,R}$ global transformations, spontaneously broken to the diagonal custodial symmetry $SU(2)_C$, and explicitly broken by the $U(1)_Y$ gauge interaction and by the (different) masses of fermions in each $SU(2)_L$ fermion doublet. The adimensionality of $\mathbf{U}(x)$ is the technical key to understand why the dimension of the leading low-energy operators describing the dynamics of the scalar sector differs for a non-linear Higgs sector [17–21] and a purely linear regime. In the former, non-renormalisable operators containing extra powers of a light h are weighted by powers of h/f [14], while the Goldstone boson contributions encoded in $\mathbf{U}(x)$ do not exhibit any scale suppression. In the linear regime, instead, the light h and the three SM GBs are encoded into the scalar doublet H, with mass dimension one: therefore any extra insertion of H is suppressed by a power of the cutoff.

It is becoming customary to parametrize the Lagrangian describing a light dynamical Higgs particle h by means of the following ansatz [22,23]:

$$\begin{aligned}\mathscr{L}_h = {}& \frac{1}{2}(\partial_\mu h)(\partial^\mu h)\big(1 + c_H \xi \mathcal{F}_H(h)\big) - V(h) \\ & - \frac{v^2}{4}\mathrm{Tr}\big[\mathbf{V}^\mu \mathbf{V}_\mu\big]\mathcal{F}_C(h) + c_T \xi \frac{v^2}{4}\mathrm{Tr}\big[\mathbf{T}\mathbf{V}^\mu\big]\mathrm{Tr}[\mathbf{T}\mathbf{V}_\mu]\mathcal{F}_T(h) \\ & - \left(\frac{v}{2\sqrt{2}}\bar{Q}_L \mathbf{U}(x)\mathbf{Y} Q_R \mathcal{F}_Y(h) + \text{h.c.}\right) + \cdots, \end{aligned} \qquad (2)$$

where dots stand for higher order terms in the (linear) expansion in h/f, and $\mathbf{V}_\mu \equiv (\mathbf{D}_\mu \mathbf{U})\mathbf{U}^\dagger$ ($\mathbf{T} \equiv \mathbf{U}\sigma_3\mathbf{U}^\dagger$) is the vector (scalar) chiral field transforming in the adjoint of $SU(2)_L$. The covariant derivative reads

$$\mathbf{D}_\mu \mathbf{U}(x) \equiv \partial_\mu \mathbf{U}(x) + \frac{ig}{2} W_\mu^a(x)\sigma_a \mathbf{U}(x) - \frac{ig'}{2} B_\mu(x)\mathbf{U}(x)\sigma_3,$$

with W_μ^a (B_μ) denoting the $SU(2)_L$ ($U(1)_Y$) gauge bosons and g (g') the corresponding gauge coupling. In the equations above, $V(h)$ denotes the effective scalar potential describing the breaking of the electroweak symmetry. The first line in Eq. (2) includes the Higgs kinetic term, while the second line describes the W and Z masses and their interactions with h, as well as the usual custodial symmetry breaking term labeled by c_T. Finally, restricting our considerations to the quark sector, the third line in Eq. (2) accounts for the Yukawa-like interactions between h and the SM quarks, grouped in doublets of the $SU(2)_{L,R}$ global symmetry $Q_{L,R}$, and with $\mathbf{Y}$ being a 6×6 block diagonal matrix containing the usual Yukawa matrices Y_U and Y_D. The parameters c_H and c_T are model-dependent operator coefficients.

The functions $\mathcal{F}_H(h)$, $\mathcal{F}_C(h)$, $\mathcal{F}_T(h)$ and $\mathcal{F}_Y(h)$ above, as well as all $\mathcal{F}(h)$ functions to be used below, encode the generic dependence on $(\langle h\rangle + h)$ and are model-dependent. Each $\mathcal{F}(h)$ function can be expanded in powers of ξ, $\mathcal{F}(h) = g_0(h, v) + \xi g_1(h, v) + \xi^2 g_2(h, v) + \cdots$, where $g(h, v)$ are model-dependent functions of h and of v, once $\langle h\rangle$ is expressed in terms of ξ and v. For large ξ the whole series may need to be considered. In previous literature [22,23] the functional dependence of some of those functions has been expressed as a power series in h/v:

$$\mathcal{F}_C(h) = \left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + \ldots\right),$$
$$\mathcal{F}_Y(h) = \left(1 + c\frac{h}{v} + \ldots\right).$$

The constants a, b and c are model-dependent parameters and encode the dependence on ξ. The a and c_T parameters are constrained from electroweak precision tests: in particular $0.7 \lesssim a \lesssim 1.2$ [24] and $-1.7 \times 10^{-3} < c_T\xi < 1.9 \times 10^{-3}$ [25] at 95% CL.

The above Lagrangian can be very useful to describe an extended class of “Higgs” models, ranging from the SM scenario with a linear Higgs sector (for $\langle h\rangle = v$, $a = b = c = 1$ and neglecting the higher order terms in h), to the technicolor-like ansatz (for $f \sim v$ and omitting all terms in h) and intermediate situations with a light scalar h from composite/holographic Higgs models [9–15,26–28] (in general for $f \neq v$) up to dilaton-like scalar frameworks [29–35] (for $f \sim v$), where the dilaton participates to the electroweak symmetry breaking. Note that in concrete models electroweak corrections imply $\xi \lesssim 0.2$–0.4 [16], but we will leave the ξ parameter free here and account for the constraints on custodial symmetry through limits on the $d = 2$ and higher-dimensional chiral operator coefficients.

In this work we analyze the strong interacting scenario in the presence of a light Higgs particle and construct the tower of pure-gauge and gauge-h operators up to mass dimension 5, in the context of the effective chiral Lagrangian. We will assume a light h and a strong dynamics for the pseudo-Goldstone bosons which are the longitudinal degrees of freedom of the electroweak gauge bosons. This analysis enlarges and completes the operator basis previously considered in Refs. [17–23].

2. The effective Lagrangian

The parameter ξ, defined in Eq. (1), encodes the strength of the effects at the electroweak scale for theories which exhibit strong coupling at the new physics scale $\Lambda_s \leqslant 4\pi f$. Therefore, with a slight abuse of language ξ measures the degree of non-linearity of the low-energy effective theory: $\xi \to 0$ refers to the linear regime, and $\xi \to 1$ to the non-linear one.

Linear regime

For $\xi \ll 1$ the hierarchy between $d \geqslant 4$ effective operators mimics the linear expansion, where the operators are written in terms of the Higgs doublets H: couplings with higher number of (physical) Higgs legs are suppressed compared to the SM renormalisable ones, due to higher powers of $1/f$ or, in other words, of ξ. The power of ξ keeps then track of the h-dependence of the higher-dimension operators.

In the extreme linear limit $\langle h\rangle = v$, and the Higgs sector enters the tower of operators through powers of the SM Higgs doublet H and its derivatives. It is illustrative to write H and its covariant derivative in terms of the Goldstone bosons matrix $\mathbf{U}$ (where from now on the variable x is left implicit) and the physical scalar h:

$$H = \frac{(v+h)}{\sqrt{2}}\mathbf{U}\begin{pmatrix}0\\1\end{pmatrix},$$
$$\mathbf{D}_\mu H = \frac{(v+h)}{\sqrt{2}}\mathbf{D}_\mu \mathbf{U}\begin{pmatrix}0\\1\end{pmatrix} + \frac{\partial_\mu h}{\sqrt{2}}\mathbf{U}\begin{pmatrix}0\\1\end{pmatrix}, \qquad (3)$$

with $\mathbf{D}_\mu \mathbf{U}$ being the covariant derivative previously defined. The Higgs kinetic energy term in the linear expansion reads then:

[2] Notice that in this low-energy expression for $\mathbf{U}(x)$, the scale associated to the eaten GBs is v and not f. Technically, the scale v appears through a redefinition of the GB fields so as to have canonically normalized kinetic terms.

$$\left(\mathbf{D}^\mu H\right)^\dagger\left(\mathbf{D}_\mu H\right)=\frac{1}{2}(\partial_\mu h)^2-\frac{v^2}{4}\left(1+\frac{h}{v}\right)^2\mathrm{Tr}\left[\mathbf{V}_\mu\mathbf{V}^\mu\right].$$

On the right-hand side of this equation one can recognize the phenomenological Lagrangian in Eq. (2) for $f\to\infty$, i.e. $\xi=0$, and $a=b=c=1$ (disregarding higher order terms in h/f), which corresponds to the SM case. A $(v+h)$ structure is clearly identified in the non-derivative term: the tower of $d>4$ operators would inherit generically an h-dependence in powers of $(v+h)/f=\xi^{1/2}(1+h/v)$, and of $\partial_\mu h/f^2$ [22,23,25]. A priori, the $\mathcal{F}(h)$ functions would also inherit that universal behavior in powers of $(1+h/v)$: for any operator weighted by ξ^n it could be expected a dependence $\mathcal{F}(h)=(1+h/v)^{2n}$. Nevertheless, the use of the equations of motion and integration by parts to construct the basis below will translate into combinations of operator coefficients, which lead to a generic h-dependence that, for instance at order ξ (i.e. for $d=6$ operators), reads

$$\mathcal{F}_i(h)=\left(1+2a_i\frac{h}{v}+b_i\frac{h^2}{v^2}\right),$$

where a_i and b_i are expected to be $\mathcal{O}(1)$. An obvious extrapolation applies to couplings weighted by higher powers of ξ (i.e. for $d>6$ operators). In the following, all the discussion will be carried out in terms of generic $\mathcal{F}(h)$ functions, though.

Non-linear regime

For $\xi\approx 1$, the ξ-dependence does not entail a suppression of operators compared to the renormalisable SM operators and the chiral expansion should instead be adopted, although it should be clarified at which level the effective expansion on h/f should stop. In fact, for any BSM theory in the non-linear regime the dependence on h will be a general function. For instance, in the $SO(5)/SO(4)$ strong interacting model with a composite light Higgs [26], the tower of higher-dimension operators is weighted by powers of $\sin((\langle h\rangle+h)/f)$, and in this case $\xi=\sin^2(\langle h\rangle/f)$. Below, the $\mathcal{F}(h)$ functions will be considered completely general polynomial of $\langle h\rangle$ and h (in particular not of derivatives of h) and, when using equations of motion and integration by parts to relate operators, they would be assumed to be redefined when convenient, much as one customarily redefines the constant operator coefficients.

Below, the $\mathcal{F}(h)$ functions encode the non-linear interactions of the light h and will be considered completely general polynomial of $\langle h\rangle$ and h (not including derivatives of h). Notice that, when using the equations of motion and integration by parts to relate operators, $\mathcal{F}(h)$ would be assumed to be redefined when convenient, much as one customarily redefines the constant operator coefficients.

2.1. Pure-gauge and gauge-h operator basis

All CP-even operators appropriate to the non-linear regime will be included in this work, up to mass dimension 5. In the absence of a light h, no pure-gauge or gauge-h $d=5$ operator exists, and it is thus a good guideline to start from the basis of $d=4$ pure-gauge chiral operators and complete it up to $d=5$ with suitable insertions of h. This will be implemented through generic $\mathcal{F}(h)$ functions, which will contain the dependence on h, ξ and/or $\langle h\rangle$, in the understanding that only those contributions which lead to an operator with total mass dimension 5 or lower will be retained. The connection to the linear regime will be made manifest exploiting the operator dependence on ξ. The Lagrangian can be decomposed as

$$\mathscr{L}^{d\leqslant 5}_{gauge\text{-}h}=\mathscr{L}^h_{\chi=0}+\mathscr{L}^h_{\chi=2}+\mathscr{L}^h_{\chi=3}+\mathscr{L}^h_{\chi=4},$$

where the subscript $\chi=n$ reminds the dimension of the non-linear parenthood of the operators. $\mathscr{L}^h_{\chi=0}$ contains only terms in h or its derivatives and corresponds to the first line of Eq. (2). The $\mathscr{L}^h_{\chi=2}$ accounts for the W and Z boson masses and their interactions with the h field, and is given in the second line of Eq. (2). The $\mathscr{L}^h_{\chi=3}$ is the Yukawa-type coupling and corresponds to the third line of Eq. (2). Finally, the $\mathscr{L}^h_{\chi=4}$ term can be written as:

$$\begin{aligned}\mathscr{L}^h_{\chi=4}=&-\frac{g_s^2}{4}G^a_{\mu\nu}G^{\mu\nu}_a\mathcal{F}_G(h)-\frac{g^2}{4}W^a_{\mu\nu}W^{\mu\nu}_a\mathcal{F}_W(h)\\&-\frac{g'^2}{4}B_{\mu\nu}B^{\mu\nu}\mathcal{F}_B(h)\\&+\xi\sum_{i=1}^{5}c_i\mathcal{P}_i(h)+\xi^2\sum_{i=6}^{20}c_i\mathcal{P}_i(h)\\&+\xi^3\sum_{i=21}^{23}c_i\mathcal{P}_i(h)+\xi^4c_{24}\mathcal{P}_{24}(h).\end{aligned}\tag{4}$$

The first two lines of Eq. (4) contain the kinetic terms for the gauge bosons, with $W_{\mu\nu}$, $B_{\mu\nu}$ and $G_{\mu\nu}$ denoting the $SU(2)_L$, $U(1)_Y$ and $SU(3)_C$ field strengths, respectively. The last two lines of Eq. (4) contain the following 24 CP-even operators, ordered by their ξ-dependence:

$$\begin{aligned}&\mathcal{P}_1(h)=gg'B_{\mu\nu}\,\mathrm{Tr}\left(\mathbf{T}W^{\mu\nu}\right)\mathcal{F}_1(h),\\&\mathcal{P}_2(h)=ig'B_{\mu\nu}\,\mathrm{Tr}\left(\mathbf{T}\left[\mathbf{V}^\mu,\mathbf{V}^\nu\right]\right)\mathcal{F}_2(h),\\&\mathcal{P}_3(h)=ig\,\mathrm{Tr}\left(W_{\mu\nu}\left[\mathbf{V}^\mu,\mathbf{V}^\nu\right]\right)\mathcal{F}_3(h),\\&\mathcal{P}_4(h)=ig'B_{\mu\nu}\,\mathrm{Tr}\left(\mathbf{T}\mathbf{V}^\mu\right)\partial^\nu\mathcal{F}_4(h),\\&\mathcal{P}_5(h)=ig\,\mathrm{Tr}\left(W_{\mu\nu}\mathbf{V}^\mu\right)\partial^\nu\mathcal{F}_5(h),\end{aligned}\tag{5}$$

$$\begin{aligned}&\mathcal{P}_6(h)=\left(\mathrm{Tr}\left(\mathbf{V}_\mu\mathbf{V}^\mu\right)\right)^2\mathcal{F}_6(h),\\&\mathcal{P}_7(h)=\left(\mathrm{Tr}\left(\mathbf{V}_\mu\mathbf{V}_\nu\right)\right)^2\mathcal{F}_7(h),\\&\mathcal{P}_8(h)=g^2\left(\mathrm{Tr}\left(\mathbf{T}W^{\mu\nu}\right)\right)^2\mathcal{F}_8(h),\\&\mathcal{P}_9(h)=ig\,\mathrm{Tr}(\mathbf{T}W_{\mu\nu})\,\mathrm{Tr}\left(\mathbf{T}\left[\mathbf{V}^\mu,\mathbf{V}^\nu\right]\right)\mathcal{F}_9(h),\\&\mathcal{P}_{10}(h)=g\epsilon^{\mu\nu\rho\lambda}\,\mathrm{Tr}(\mathbf{T}\mathbf{V}_\mu)\,\mathrm{Tr}(\mathbf{V}_\nu W_{\rho\lambda})\mathcal{F}_{10}(h),\\&\mathcal{P}_{11}(h)=\mathrm{Tr}\left(\left(\mathcal{D}_\mu\mathbf{V}^\mu\right)^2\right)\mathcal{F}_{11}(h),\\&\mathcal{P}_{12}(h)=\mathrm{Tr}\left(\mathbf{T}\mathcal{D}_\mu\mathbf{V}^\mu\right)\mathrm{Tr}\left(\mathbf{T}\mathcal{D}_\nu\mathbf{V}^\nu\right)\mathcal{F}_{12}(h),\\&\mathcal{P}_{13}(h)=\mathrm{Tr}\left([\mathbf{T},\mathbf{V}_\nu]\mathcal{D}_\mu\mathbf{V}^\mu\right)\mathrm{Tr}\left(\mathbf{T}\mathbf{V}^\nu\right)\mathcal{F}_{13}(h),\\&\mathcal{P}_{14}(h)=ig\,\mathrm{Tr}(\mathbf{T}W_{\mu\nu})\,\mathrm{Tr}\left(\mathbf{T}\mathbf{V}^\mu\right)\partial^\nu\mathcal{F}_{14}(h),\\&\mathcal{P}_{15}(h)=\mathrm{Tr}\left(\mathbf{T}[\mathbf{V}_\mu,\mathbf{V}_\nu]\right)\mathrm{Tr}\left(\mathbf{T}\mathbf{V}^\mu\right)\partial^\nu\mathcal{F}_{15}(h),\\&\mathcal{P}_{16}(h)=\mathrm{Tr}\left(\mathbf{V}_\nu\mathcal{D}_\mu\mathbf{V}^\mu\right)\partial^\nu\mathcal{F}_{16}(h),\\&\mathcal{P}_{17}(h)=\mathrm{Tr}\left(\mathbf{T}\mathcal{D}_\mu\mathbf{V}^\mu\right)\mathrm{Tr}(\mathbf{T}\mathbf{V}_\nu)\partial^\nu\mathcal{F}_{17}(h),\\&\mathcal{P}_{18}(h)=\mathrm{Tr}\left(\mathbf{V}_\mu\mathbf{V}^\mu\right)\partial_\nu\partial^\nu\mathcal{F}_{18}(h),\\&\mathcal{P}_{19}(h)=\mathrm{Tr}(\mathbf{V}_\mu\mathbf{V}_\nu)\partial^\mu\mathcal{F}_{19}(h)\partial^\nu\mathcal{F}'_{19}(h),\\&\mathcal{P}_{20}(h)=\mathrm{Tr}(\mathbf{T}\mathbf{V}_\mu)\,\mathrm{Tr}(\mathbf{T}\mathbf{V}_\nu)\partial^\mu\mathcal{F}_{20}(h)\partial^\nu\mathcal{F}'_{20}(h),\end{aligned}\tag{6}$$

$$\begin{aligned}&\mathcal{P}_{21}(h)=\mathrm{Tr}\left(\mathbf{V}_\mu\mathbf{V}^\mu\right)\left(\mathrm{Tr}(\mathbf{T}\mathbf{V}_\nu)\right)^2\mathcal{F}_{21}(h),\\&\mathcal{P}_{22}(h)=\mathrm{Tr}(\mathbf{V}_\mu\mathbf{V}_\nu)\,\mathrm{Tr}\left(\mathbf{T}\mathbf{V}^\mu\right)\mathrm{Tr}\left(\mathbf{T}\mathbf{V}^\nu\right)\mathcal{F}_{22}(h),\\&\mathcal{P}_{23}(h)=\left(\mathrm{Tr}(\mathbf{T}\mathbf{V}_\mu)\right)^2\partial_\nu\partial^\nu\mathcal{F}_{23}(h),\end{aligned}\tag{7}$$

$$\mathcal{P}_{24}(h)=\left(\mathrm{Tr}(\mathbf{T}\mathbf{V}_\mu)\,\mathrm{Tr}(\mathbf{T}\mathbf{V}_\nu)\right)^2\mathcal{F}_{24}(h).\tag{8}$$

The 24 constant parameters c_i are model-dependent coefficients. The powers of ξ, factorized out in the last two lines of Eq. (4), do not reflect an expansion in ξ, but a reparametrisation that facilitates the tracking to the lowest dimension at which a "sibling" operator appears in the linear expansion. By sibling we mean an operator written in terms of the Higgs doublet H, that includes the pure-gauge part of the couplings $\mathcal{P}_{1-24}(h)$. It may happen that an operator listed in Eqs. (5)–(8) corresponds to a specific combination of siblings with different dimensions. This is the case, for instance, of $\mathcal{P}_{13}(h)$, whose siblings are of dimension 8 and 10.

For $\xi \ll 1$ the weight of the operators which are accompanied by powers of ξ is scale suppressed compared to that of SM renormalisable couplings. In this limit the Lagrangian above would encode a consistent linear expansion up to $d = 6$ operators, if only the terms of zero and first order in ξ are kept: indeed, operators $\mathcal{P}_6(h)$ to $\mathcal{P}_{24}(h)$ would correspond to $d = 8$ or higher-dimension siblings in the linear expansion. In contrast, in the non-linear regime, that is for $\xi \approx 1$, no such suppression appears and *all* operators in Eqs. (5)–(8) include $d \leqslant 5$ couplings and should be considered on equal footing. The leading terms of the linear and non-linear expansions do not match.

Operators in Eq. (2) and in the first two lines of Eq. (4), as well as $\mathcal{P}_{1-5}(h)$ had been already pointed out in the analysis of the linear–non-linear connection of the SILH framework [25]. Indeed, in the limit of small ξ, we can safely neglect all the terms proportional to $\geqslant 2$ powers of ξ and the resulting Lagrangian coincides with the SILH one. Nevertheless, to be complete the rest of the operators mentioned above should be included when fermions are taken into account and/or when dealing with theories in the non-linear regime. Equivalently in the linear regime, one should consider operators with $d > 6$: the complete basis of operators in this case accounts for operators of $d = 12$ at most, while all the higher order operators are redundant. This is consistent with the basis in the non-linear regime presented here, where the lowest dimensional sibling of $\mathcal{P}_{24}(h)$ has indeed dimension 12.

The different operators defined in Eqs. (5)–(8) correspond to three major categories: pure-gauge and gauge-h operators ($\mathcal{P}_{1-3}(h)$, $\mathcal{P}_{6-10}(h)$, $\mathcal{P}_{21-22}(h)$ and $\mathcal{P}_{24}(h)$) which result from a direct extension of the original Appelquist–Longhitano chiral Higgsless basis; operators containing the contraction $\mathcal{D}_\mu \mathbf{V}^\mu$ and no derivatives of $\mathcal{F}(h)$ ($\mathcal{P}_{11-13}(h)$); operators with one or two derivatives of $\mathcal{F}(h)$ ($\mathcal{P}_{4-5}(h)$, $\mathcal{P}_{14-20}(h)$ and $\mathcal{P}_{23}(h)$).

The extended Appelquist–Longhitano basis

$\mathcal{P}_{1-3}(h)$, $\mathcal{P}_{6-10}(h)$, $\mathcal{P}_{21-22}(h)$ and $\mathcal{P}_{24}(h)$ result from combining the basis of independent $d = 4$ chiral operators already considered in Refs. [17–21] with additional $\mathcal{F}(h)$ insertions. They appear in the Lagrangian with different powers of ξ: $\mathcal{P}_{1-3}(h)$ is linear in ξ, while $\mathcal{P}_{6-10}(h)$, $\mathcal{P}_{21-22}(h)$ and $\mathcal{P}_{24}(h)$ are proportional to ξ^2, ξ^3 and ξ^4, respectively.

This ensemble constitutes a complete basis of linearly independent pure-gauge and gauge-h $d \leqslant 5$ operators, when neither derivatives of h nor fermion masses or fermionic operators that cannot be related to pure-gauge or gauge-Higgs ones via the equations of motion are considered. It is worth noticing that, neglecting all terms in h (i.e. taking $\mathcal{F}(h)$ as a constant), the list of operators in Eqs. (5)–(8) reduces to the original Appelquist–Longhitano basis.

Massive fermions: $\mathcal{D}_\mu \mathbf{V}^\mu \neq 0$

All operators $\mathcal{P}_{11-13}(h)$ contain the contraction $\mathcal{D}_\mu V^\mu$ and are physical only in the presence of massive fermions [20]. Indeed, considering the equations of motion for the field strengths,

$$\left(D^\mu W_{\mu\nu}\right)_j = i\frac{g}{4}v^2\,\mathrm{Tr}[\mathbf{V}_\nu\sigma_j] + \frac{g}{2}\bar{Q}_L\gamma_\nu\sigma_j Q_L,$$

$$\partial^\mu B_{\mu\nu} = -i\frac{g'}{4}v^2\,\mathrm{Tr}[\mathbf{T}\mathbf{V}_\nu] + \sum_{i=L,R} g'\bar{Q}_i\gamma_\nu \mathbf{h}_i Q_i,$$

with $\mathbf{h}_{L,R}$ the left and right hypercharges in the 2×2 matrix notation, and deriving these expressions, a connection is established between operators containing $\mathcal{D}_\mu \mathbf{V}^\mu$ and fermionic currents that preserve flavor but change chirality:

$$\frac{iv}{\sqrt{2}}\,\mathrm{Tr}\left(\sigma_j \mathcal{D}_\mu \mathbf{V}^\mu\right) = i\bar{Q}_L\sigma_j \mathbf{U}\mathbf{Y}Q_R + \text{h.c.},$$
$$\frac{iv}{\sqrt{2}}\,\mathrm{Tr}\left(\mathbf{T}\mathcal{D}_\mu \mathbf{V}^\mu\right) = i\bar{Q}_L\mathbf{T}\mathbf{U}\mathbf{Y}Q_R + \text{h.c.},$$

where the relation $\mathcal{D}_\mu\mathbf{T} = [\mathbf{V}_\mu, \mathbf{T}]$ and the Dirac equations

$$i\not{D}_L Q_L - \frac{v}{\sqrt{2}}\mathbf{U}\mathbf{Y}Q_R = 0,$$
$$i\not{D}_R Q_R - \frac{v}{\sqrt{2}}\mathbf{Y}\mathbf{U}^\dagger Q_L = 0,$$

with $\not{D}_{L,R}$ the usual covariant derivatives acting on the L, R doublet spinors, have been used.

In consequence, if fermion masses are not neglected the set of operators $\mathcal{P}_{11-13}(h)$ should be taken into account. Furthermore, these operators together with the pure-gauge and gauge-h ones in the class previously defined, constitute a complete basis of linearly independent $d \leqslant 5$ operators, upon disregarding: i) those resulting from combining chiral ones with derivatives of h; ii) flavor-changing fermionic operators [36,37] but the Yukawa coupling; iii) flavor-conserving fermionic operators that cannot be related to pure-gauge or gauge-Higgs ones via the equations of motion; iv) pure-h higher-dimension effective couplings.

Derivatives of h

Terms resulting from combining $\partial_\mu h$ or $\partial_\mu\partial^\mu h$ with $d = 2$ or $d = 4$ chiral couplings enlarge the basis by several operators: $\mathcal{P}_{4-5}(h)$, $\mathcal{P}_{14-20}(h)$ and $\mathcal{P}_{23}(h)$. Notice that $\mathcal{P}_{16}(h)$ and $\mathcal{P}_{17}(h)$ contain the contraction $\mathcal{D}_\mu \mathbf{V}^\mu$ and therefore the comments in the previous paragraph also apply for these two operators.

Among all the operators of this class, two of them have been already identified in Ref. [22,23,25]. We have provided the full set of 10 operators that need to be taken into account to complete the pure-gauge and gauge-h basis; they exhibit a ξ-dependence which starts at the linear, quadratic or cubic level.

Custodial symmetry nature

In the list in Eqs. (5)–(8), the operators $\mathcal{P}_1$, $\mathcal{P}_2$, $\mathcal{P}_4$, $\mathcal{P}_{8-15}$, $\mathcal{P}_{17}$, and $\mathcal{P}_{20-24}$ are custodial symmetry breaking. This can be understood either by the presence of the hypercharge coupling constant g' in front of the operators $\mathcal{P}_1$, $\mathcal{P}_2$, and $\mathcal{P}_4$, or by the connection to quark masses, as it is the case for $\mathcal{P}_{11-13}$, $\mathcal{P}_{16}$ and $\mathcal{P}_{17}$, or finally through the presence of the chiral scalar field $\mathbf{T}$ that explicitly violates the custodial symmetry.

2.2. Connection with other bases

It is easy to establish the correlation between the basis defined above and other possible gauge or gauge-h bases of operators with $d \leqslant 5$. To this aim, two equalities are useful:

$$\mathbf{V}_{\mu\nu} \equiv \mathcal{D}_\mu \mathbf{V}_\nu - \mathcal{D}_\nu \mathbf{V}_\mu = ig\mathbf{W}_{\mu\nu} - i\frac{g'}{2}B_{\mu\nu}\mathbf{T} + [\mathbf{V}_\mu, \mathbf{V}_\nu],$$
$$[\mathcal{D}_\mu, \mathcal{D}_\nu]\mathcal{O} = ig[W_{\mu\nu}, \mathcal{O}],$$

where $\mathcal{O}$ is a generic operator covariant under $SU(2)_L$ and invariant under $U(1)_Y$.

Four operators with two derivatives acting on the generic functions $\mathcal{F}_i(h)$ can be written, in addition to $\mathcal{P}_{18-20}(h)$ and $\mathcal{P}_{23}(h)$. However, via integration by parts and pertinent redefinition of the generic functions $\mathcal{F}_i(h)$, one obtains that two of these new structures are given by:

$$\mathrm{Tr}(\mathbf{V}_\mu \mathbf{V}_\nu)\partial^\mu \partial^\nu \mathcal{F}(h) = \frac{1}{2}\mathcal{P}_4(h) - \mathcal{P}_5(h) - \mathcal{P}_{16}(h) + \frac{1}{2}\mathcal{P}_{18}(h), \tag{9}$$

$$\mathrm{Tr}(\mathbf{T}\mathbf{V}_\mu)\,\mathrm{Tr}(\mathbf{T}\mathbf{V}_\nu)\partial^\mu \partial^\nu \mathcal{F}(h) = \mathcal{P}_4(h) - \mathcal{P}_{14}(h) + \mathcal{P}_{15}(h) - \mathcal{P}_{17}(h) + \frac{1}{2}\mathcal{P}_{23}(h). \tag{10}$$

The remaining two, $(\mathrm{Tr}(\mathbf{T}\mathbf{V}_\mu))^2 \partial_\nu \mathcal{F}(h)\partial^\nu \mathcal{F}'(h)$ and $\mathrm{Tr}(\mathbf{V}_\mu \mathbf{V}^\mu) \times \partial_\nu \mathcal{F}(h)\partial^\nu \mathcal{F}'(h)$, can be reduced to $\mathcal{P}_{23}(h)$ and $\mathcal{P}_{18}(h)$, respectively, by the use of the h equation of motion. The latter operator and the one in Eq. (9) have been introduced in Ref [23].

Next, operators containing derivatives of the field strengths can be decomposed as

$$ig'\left(\partial_\mu B^{\mu\nu}\right)\mathrm{Tr}[\mathbf{T}\mathbf{V}_\nu]\mathcal{F}(h) = -\frac{g'^2}{2} B_{\mu\nu}B^{\mu\nu}\mathcal{F}(h) + \frac{1}{2}\mathcal{P}_1(h) + \frac{1}{2}\mathcal{P}_2(h) + \mathcal{P}_4(h),$$

$$ig\,\mathrm{Tr}\left[\left(\mathcal{D}_\mu W^{\mu\nu}\right)\mathbf{V}_\nu\right]\mathcal{F}(h) = \frac{g^2}{4} W^a_{\mu\nu}W_a^{\mu\nu}\mathcal{F}(h) - \frac{1}{4}\mathcal{P}_1(h) - \frac{1}{2}\mathcal{P}_3(h) + \mathcal{P}_5(h),$$

$$ig\,\mathrm{Tr}\left[\left(\mathcal{D}_\mu W^{\mu\nu}\right)\mathbf{T}\right]\mathrm{Tr}[\mathbf{T}\mathbf{V}_\nu]\mathcal{F}(h) = -\frac{1}{2}\mathcal{P}_1(h) - \mathcal{P}_3(h) + \frac{1}{2}\mathcal{P}_8(h) + \mathcal{P}_9(h) + \mathcal{P}_{14}(h).$$

Finally, disregarding the dependence on the h field, the three operators $\mathcal{P}_{14-16}$ containing the contraction $\mathcal{D}_\mu \mathbf{V}^\mu$ have already been considered in Ref. [20], although with a slightly different notation for the last two. The relation among $\mathcal{P}_{12}$ and $\mathcal{P}_{13}$ and the corresponding operators in Ref. [20] is the following:

$$\mathrm{Tr}\left(\mathbf{T}\mathcal{D}_\mu \mathcal{D}_\nu \mathbf{V}^\nu\right)\mathrm{Tr}\left(\mathbf{T}\mathbf{V}^\mu\right)\mathcal{F}(h) = -\mathcal{P}_{12}(h) + \mathcal{P}_{13}(h) - \mathcal{P}_{17}(h),$$

$$\begin{aligned}\mathrm{Tr}(\mathbf{T}\mathcal{D}_\mu \mathbf{V}_\nu)\,&\mathrm{Tr}\left(\mathbf{T}\mathcal{D}^\mu \mathbf{V}^\nu\right)\mathcal{F}(h) \\ &= -\frac{g'^2}{2}B_{\mu\nu}B^{\mu\nu}\mathcal{F}(h) + \mathcal{P}_1(h) + \mathcal{P}_4(h) - 2\mathcal{P}_6(h) + 2\mathcal{P}_7(h) \\ &\quad - \frac{1}{2}\mathcal{P}_8(h) + \mathcal{P}_{12}(h) - \mathcal{P}_{14}(h) + \mathcal{P}_{15}(h) + \mathcal{P}_{17}(h) \\ &\quad + 2\mathcal{P}_{21}(h) - 2\mathcal{P}_{22}(h) + \frac{1}{2}\mathcal{P}_{23}(h).\end{aligned}$$

3. Conclusions

In this Letter, we have considered the generic scenario in which a strong dynamics lies behind a light Higgs particle h, within an effective Lagrangian approach. The parameter describing the degree of non-linearity $\xi = (v/f)^2$ must lie in the range $0 < \xi < 1$. Small values lead to a low-energy theory undistinguishable from the SM, since all the effects of the strong interacting theory at the high scale become negligible. Larger values indicate a chiral regime for the dynamics of the Goldstone bosons, which in turn requires to use a chiral expansion to describe them, combined with appropriate insertions of the light h field.

This work generalizes the operator basis of Refs. [17–21] of chiral pure-gauge operators to include a light strong interacting h particle, up to $d = 5$ operators. The complete basis obtained includes several supplementary operators with respect to those previously identified in the literature [22,23,25,38], which need to be taken into account when approaching the non-linear regime. Furthermore, the results have been presented making explicit the leading dependence on ξ for each operator, which allows a direct identification of the equivalent leading operator of the linear regime. The consideration of $d = 6$, 8, 10 and 12 couplings of the linear expansion turns out to be required to establish the connection with the $d \leqslant 5$ set of operators of the non-linear one.

These results may also provide a model-independent guideline to explore which exotic gauge-Higgs couplings may be expected, and their relative strength to Higgsless observable amplitudes. Complementary information could come from the flavor sector [36,37] and hopefully will be able to shed light on the origin of the electroweak symmetry breaking mechanism.

Acknowledgements

We thank C. Delaunay, C. Grojean, J. Perez and W. Skiba for interesting questions and comments on the first version of the Letter. We acknowledge partial support by European Union FP7 ITN INVISIBLES (Marie Curie Actions, PITN-GA-2011-289442), CiCYT through the project FPA2009-09017, CAM through the project HEPHACOS P-ESP-00346, European Union FP7 ITN UNILHC (Marie Curie Actions, PITN-GA-2009-237920), MICINN through the grant BES-2010-037869 and the Juan de la Cierva programme (JCI-2011-09244), Italian Ministero dell'Università e della Ricerca Scientifica through the COFIN program (PRIN 2008) and the contracts MRTN-CT-2006-035505 and PITN-GA-2009-237920 (UNILHC). We thank the Galileo Galilei Institute for Theoretical Physics for the hospitality and the INFN for partial support during the completion of this work. R.A. acknowledges the Harvard Physics department for hospitality during the completion phase of this work. S.R. and J.Y. acknowledge CERN TH department for hospitality during the completion phase of the work.

References

[1] ATLAS Collaboration, G. Aad, et al., Phys. Lett. B 716 (2012) 1, arXiv:1207.7214.
[2] CMS Collaboration, S. Chatrchyan, et al., Phys. Lett. B 716 (2012) 30, arXiv:1207.7235.
[3] F. Englert, R. Brout, Phys. Rev. Lett. 13 (1964) 321.
[4] P.W. Higgs, Phys. Lett. 12 (1964) 132.
[5] P.W. Higgs, Phys. Rev. Lett. 13 (1964) 508.
[6] A. Manohar, H. Georgi, Nucl. Phys. B 234 (1984) 189.
[7] L. Susskind, Phys. Rev. D 20 (1979) 2619.
[8] S. Dimopoulos, L. Susskind, Nucl. Phys. B 155 (1979) 237.
[9] S. Dimopoulos, J. Preskill, Nucl. Phys. B 199 (1982) 206.
[10] D.B. Kaplan, H. Georgi, Phys. Lett. B 136 (1984) 183.
[11] D.B. Kaplan, H. Georgi, S. Dimopoulos, Phys. Lett. B 136 (1984) 187.
[12] T. Banks, Nucl. Phys. B 243 (1984) 125.
[13] H. Georgi, D.B. Kaplan, P. Galison, Phys. Lett. B 143 (1984) 152.
[14] H. Georgi, D.B. Kaplan, Phys. Lett. B 145 (1984) 216.
[15] M.J. Dugan, H. Georgi, D.B. Kaplan, Nucl. Phys. B 254 (1985) 299.
[16] R. Contino, arXiv:1005.4269.
[17] T. Appelquist, C.W. Bernard, Phys. Rev. D 22 (1980) 200.
[18] A.C. Longhitano, Phys. Rev. D 22 (1980) 1166.
[19] A.C. Longhitano, Nucl. Phys. B 188 (1981) 118.
[20] F. Feruglio, Int. J. Mod. Phys. A 8 (1993) 4937, arXiv:hep-ph/9301281.
[21] T. Appelquist, G.-H. Wu, Phys. Rev. D 48 (1993) 3235, arXiv:hep-ph/9304240.
[22] R. Contino, C. Grojean, M. Moretti, F. Piccinini, R. Rattazzi, JHEP 1005 (2010) 089, arXiv:1002.1011.
[23] A. Azatov, R. Contino, J. Galloway, JHEP 1204 (2012) 127, arXiv:1202.3415.
[24] A. Azatov, J. Galloway, Int. J. Mod. Phys. A 28 (2013) 1330004, arXiv:1212.1380.
[25] G.F. Giudice, C. Grojean, A. Pomarol, R. Rattazzi, JHEP 0706 (2007) 045, arXiv:hep-ph/0703164.
[26] K. Agashe, R. Contino, A. Pomarol, Nucl. Phys. B 719 (2005) 165, arXiv:hep-ph/0412089.
[27] R. Contino, L. Da Rold, A. Pomarol, Phys. Rev. D 75 (2007) 055014, arXiv:hep-ph/0612048.
[28] B. Gripaios, A. Pomarol, F. Riva, J. Serra, JHEP 0904 (2009) 070, arXiv:0902.1483.

[29] E. Halyo, Mod. Phys. Lett. A 8 (1993) 275.
[30] W.D. Goldberger, B. Grinstein, W. Skiba, Phys. Rev. Lett. 100 (2008) 111802, arXiv:0708.1463.
[31] L. Vecchi, Phys. Rev. D 82 (2010) 076009, arXiv:1002.1721.
[32] B.A. Campbell, J. Ellis, K.A. Olive, JHEP 1203 (2012) 026, arXiv:1111.4495.
[33] S. Matsuzaki, K. Yamawaki, arXiv:1207.5911.
[34] Z. Chacko, R. Franceschini, R.K. Mishra, arXiv:1209.3259.
[35] B. Bellazzini, C. Csaki, J. Hubisz, J. Serra, J. Terning, arXiv:1209.3299.
[36] R. Alonso, M. Gavela, L. Merlo, S. Rigolin, J. Yepes, JHEP 1206 (2012) 076, arXiv:1201.1511.
[37] R. Alonso, M. Gavela, L. Merlo, S. Rigolin, J. Yepes, arXiv:1212.3307.
[38] I. Low, R. Rattazzi, A. Vichi, JHEP 1004 (2010) 126, arXiv:0907.5413.