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Firm Regulation and Profit Sharing: A Real Option Approach*

Michele Moretto and Paola Valbonesi

Abstract

To avoid the extremely high profit levels found in the recent experience of public utilities' regulation, some regulators have introduced a profit-sharing (PS) rule that revises prices to the benefit of consumers. However, in order to be successful, a PS rule should satisfy appropriate incentive conditions.

In this paper, we study the incentive properties of a second best PS mechanism designed by the regulator to induce a regulated monopolist to divert its "excessive" profits to the customers. In a real option model where a regulated monopolist manages a long-term franchise contract and the regulator has the option to revoke the contract if there is serious welfare loss, we first endogenously derive the welfare maximising PS rule under the verifiability of profits. We then explore the dynamic efficiency of this PS rule under the non-verifiability of profits and study the firm's incentive to comply with it in an infinite-horizon game. Finally, we derive the price adjustment path which follows the adoption of a PS rule in a price cap regulation.

We show that the riskiness of the distribution of the firm's future profits and the regulator's cost in revoking the franchise contract are key factors in determining the equilibrium properties of a dynamic PS rule.

KEYWORDS: regulation of the firm, profit-sharing, real option, price-cap regulation, public utilities

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1 Introduction

Recent European liberalization and US experience in the regulation of public utilities shows that price-cap regulation (PCR) allows prices to diverge greatly from actual costs and often generate "abnormal" profits for firms. This drawback of PCR as an incentive mechanism stems from its inability to set a contingent price that incorporates all uncertainties faced by the regulated firm in each period of the regulatory contract[1]. Regulators dislike high corporate profits under PCR because they reduce consumers' welfare and - favouring the firm - downgrade the regulator's own reputation to set the "right" price of the service. This is why over the last decade, regulators have modified PCRs with "profit-sharing" (PS) schemes in order to induce the regulated firms to rebate part of their profits to customers.

In the European experience of regulation, the textbook example for PS refers to price cuts implemented by the British electricity regulator between 1994 and 1995, well before the official price review due in 1999. Since the initial price control for the electricity companies turned out to be over-generous, the regulator intervened to reduce prices, thus directly returning some of the "excess" profits to consumers.[2] Sappington (2002), among others, shows that these PS practices are usually set in the US telecommunication industry by the regulator in the form of direct payment to customers or reduction in prices of key user services.[3]

A fundamental feature of these real-world PS mechanisms is therefore the discretion left to the regulator which entitles him to adjust the PCR adopted ex-ante, calling for a unilateral "renewal" of the regulatory contract. Thus, the implementation of PS scheme leaves room for dispute between the regulator and the regulated firm about both the profit (and price) level - i.e. the threshold value - that should trigger the PS rule, and the dynamic path that the regulated price should follow. This is because PS prescriptions would require audited cost information to calculate allowable profit levels, information which is often very difficult for the regulator to

[1] Actual regulation of public utility adopts mechanisms ranging between price-cap and cost of service. As underlined by regulation research, on the one hand, a price-cap rule provides effective incentives for managerial efficiency and cost minimization, but often cannot extract the benefits of the lower costs for consumers. On the other hand, cost of service contracts correctly align prices and costs, but the firm's cost could be excessive due to suboptimal managerial effort. Joskow (2006), considering the electric distribution and transmission, pointed out that "in presence of imperfect and asymmetric information the optimal regulatory mechanism will lie somewhere between these two extreme and will have a form similar to a profit sharing contract where the price that the regulated firm can charge is partially responsive or contingent on changes in realized costs and partially fixed ex ante".

[2] Similar often quoted examples of PS have since been made by other UK regulators such as British Gas Transco (gas transmission and distribution), National Grid Company (electricity transmission), Oftel (tlc sector) and Ofwat (water industry). See about Green, (1997), Armstrong et al.(1994), Ofwat (1997), among others.

[3] Specifically, it is there observed that PS in the form of reduction in prices has been widely used to regulate intrastate accounting rates affecting earnings of telecommunication providers.

collect. It has been often informally argued that this may make the adoption and enforcement of PS rules substantially more difficult.[4]

The aim of this paper is twofold: first of all, to derive the profit level which triggers the PS rule in a PCR setting; then, to show that under *non-verifiability of profits*, the PS rule can be an equilibrium strategy. Specifically, we analyzed a dynamic game in continuous time with two players: a regulated monopolist who manages a long-term franchise contract to provide a public utility service (such as water supply, waste management, gas or electricity distribution, highway tolls, etc.) and a welfare-maximising regulator who has the right, throughout the contractual relationship, to ask the regulated firm to reduce profits if he perceives they are "excessively" high, and to revoke the franchise contract if the firm does not comply with the PS rule. We modelled the regulator's "outside" option to revoke the contract as a perpetual Call option where the regulator - considering the firm's profit as an underlying asset - has to determine when to pay an exercise price to get the management of the utility back and re-determine its provision[5].

The game lasts a possibly infinite number of periods, and ends once the regulator exercises the option to revoke the franchise contract. Each period of this game is divided into four stages: in the first stage, nature chooses the realization of a random variable, determining the firm's profit; in the second stage, after having observed the firm's profit, the regulator decides whether or not to ask for profit reductions; in the third stage, the firm decides whether or not to comply with the regulator's prescription; finally, in the fourth stage - conditional on the firm's choice - the regulator may revoke the contract.

It turns out that the assumption on *profit verifiability* affects the game from the second stage: indeed, when the profit and the other regulatory variables are observable but non-verifiable, the regulator cannot force the firm to cut "excessive" profits as "no court or other third party will accept to arbitrate a claim based on the value taken by these variables".[6] This implies that - under the non-verifiability of profit - the firm can effectively choose whether to comply or not with the PS rule, retaining all the profits above the profit threshold that triggers the PS until the regulator revokes the contract. In contrast, when the verifiability of profit is assumed, the regulator's PS prescription reduces the game to a take-it-or-leave-it offer to the firm.

For the sake of clarity, our analysis is split into two parts. As a benchmark, we first investigated the simpler, though less realistic, case where the firm's profit is verifiable and - consequently - the PS scheme imposes contractual obligations contingent on realized profits. There, after having formally defined the PS scheme,

[4]See, among others: Green (1997) and Joskow (2006).

[5]It is easy to observe here the parallel with a financial Call option which "gives the holder the right to pay, for some specified amount of time, an exercise price and in return receive an asset (a share of stock) that has some value" (Dixit and Pindyck, 1995, p.9).

[6]Salanié (1997, p. 177).

we identified the profit threshold value that determines the regulator's introduction of the welfare-maximizing PS rule: at such a profit level, the regulator is indifferent between contract closure and imposing the PS.

We then moved to the more realistic case of profit non-verifiability. Here, the main difference compared to the previous case is that the regulated monopolist, who decides not to comply with the PS rule, can now retain all profits above the threshold, triggering PS until the regulator calls for contract closure. The regulator will now revoke the contract, say at period t, only if revocation at that period is effectively his best response. In other words, with no profit verifiability, an incentive constraint imposing dynamic optimality of the revocation policy must be satisfied in order to make the regulator's revocation threat credible. So, we formally show that for all the profits higher than the indifference threshold between PS and contract closure it is optimal for the regulator to revoke the contract, while revoking the contract for lower profit levels will never be optimal. Hence, the perfect equilibrium of the game is such that the firm complies with the PS rule chosen by the regulator in each period, as long as the revocation has not been carried out.[7] In equilibrium, the expectation of being able to induce profit sharing makes it rational for the regulator not to exercise its option to revoke, and this fact also makes it rational for the firm to continue to comply with the PS prescription. On the one hand, given that for the monopolist the loss from revocation of the contract is greater than the expected stream of profit cuts (prescribed by the PS rule), it will be efficient for the firm to continuously maintain profits at a level lower than the threshold that triggers the PS. On the other hand, the regulator will revoke the contract for any profit level higher than the this profit threshold.

This paper is related to two different strands of literature.

Concerning economic literature on the regulation of firms, our paper took stock of studies on drastic regulatory changes such as stochastic regulatory review (Bawa and Sibley, 1984) and expropriation by the regulator (Salant and Woroch,1992; Gilbert and Newbery, 1994). Bawa and Sibley (1984) showed that the firm's incentive to indulge in over-capitalization can be tempered by the fact that this raises profits and - consequently - makes it more likely that the regulator will cut prices. In contrast to their approach, which emphasizes the strategic firm behavior under both the regulator's probability of review function and price adjustment, we focussed on the regulator's decision to impose a regulatory review in the form of a PS rule and on the informational conditions which make it enforceable. Salant and Woroch (1992) and Gilbert and Newbery (1994) present models on expropriation by the regulator where the price regulation occurs endogenously as a self-enforcing and mutually beneficial cooperative equilibrium. In both these discrete-time repeated game frameworks, the regulatory lag does not affect the players' behavior. In contrast, in our continuous-time repeated game model, the explicit unilateral approach to contract

[7] Efficient sub-game perfect equilibria in infinite-horizon threat-games are investigated in Klein and O'Flaherty, 1993; Shavell and Spier, 2002.

renewal - i.e. the regulator sets the PS rule or calls for contract closure - allows us either to determine the regulatory lags endogenously or to study its determinants. Specifically, the endogeneity of the regulatory lag - which is in the essence of the most real-life adopted regulation mechanism - consistently belongs to both the level of the firm's profit and the regulator's revocation cost.[8]

On a formal level, our paper builds upon the real option techniques which have been widely used in the literature of irreversible investment and emphasize the option value of delaying investment decision, i.e. the value of waiting for better - although never complete - information on the stochastic evolution of a basic asset.[9] This paper adopts these techniques to investigate a regulatory problem. As far as we know, there have rarely been any relevant applications and, those that exist, mainly refer to a firm's investment decision in different regulated sectors (Hausman and Myers, 2002; Pindyck, 2004; Saphores et al., 2004), or under different compulsory regulatory mechanisms (Dobbs, 2004; Teisberg, 1993; Moretto et al., 2003). Unlike these contributions, our paper focuses on the endogenous determination of the profit level which triggers PS and on its dynamic sustainability. To investigate these issues, we have modelled the regulator's decision to introduce PS into the regulatory contract along with an option to revoke the contract. As far as we know, both these elements - the regulator's option and non-verifiability of the firm's profit - are absent from research on PS and regulator expropriation, as well as from option theory applications. Our approach allows us, first, to investigate the PS rule in an intertemporal regulatory setting and, second, to recognize that in presence of market uncertainty and non-verifiability of the firm's profit, the regulator's revocation cost (i.e. the regulator's credible threat) of contract closure is a crucial issue in dynamic PS enforcement.

Finally, we ought to mention a limit of this analysis. We do not consider the well-known trade-off generated by the introduction of a PS rule between lowering extreme profits and dulling the firm's incentive for cost reduction and investments.[10] In this respect, note that if our model's assumption of non-investment by the regulated firm - on the one hand - opens room for further extensions of the analysis - on the other hand - it results consistent with situations where the regulated firm's high profits are realized independent of the firm's strategic decision on investment as, for instance, when exogenous and unpredictable shocks affect positively market demand for the service supplied.

The paper is organized as follows. Section 2 presents the basic model of PCR

[8] As discussed in Laffont and Tirole (1994, p.15), endogeneity of the regulatory lag is especially important when the incentive properties of regulation are investigated.

[9] Dixit and Pindyck (1994) is the seminal text in this area.

[10] The literature on firm's regulation has mainly stressed that compulsory sharing of profit may: a) reduce the firm's incentive to minimize operating costs and increase revenue (Lyon, 1996; Crew and Kleindorfer, 1996); b) provide an incentive to undertake projects that are unduly risky (Blackmon, 1994); c) lead the utility to delay investment (Moretto et al., 2006).

in which a PS rule is introduced. Section 3 derives the regulator's value of the option to revoke the contract as well as the optimal profit threshold that triggers it (Proposition 1): when the firm's profits are verifiable, this threshold is the optimal level to introduce the PS rule (Proposition 2). Section 4 explores PS sustainability when the firm's profit is non-verifiable (Proposition 3). In these four sections, we have considered the PS rule as a general reduction of the firm's profit; in Section 5 - expressing explicitly the firm's profit function - we specifically investigated the adoption of a PS prescription in the form of price-cut: this allow us to study the price adjustment which follows in a PCR . Finally, Section 6 concludes with policy implications and extension of the model.

2 The basic set-up

We consider a risk-neutral profit-maximizing monopolist that manages a one-time sunk indivisible project for the provision of a public utility under a long-term franchise contract. For the sake of simplicity, we assume that the franchise term is sufficiently long to be approximated by infinity.

The firm's project produces a flow of profits π_t which develops over time according to a geometric Brownian motion, with instantaneous growth rate $\alpha > 0$ and instantaneous volatility $\sigma \geq 0$:

$$d\pi_t = \alpha\pi_t dt + \sigma\pi_t dW_t, \quad \pi_0 = \pi \tag{1}$$

where dW_t is the standard increase of a Wiener process, uncorrelated over time and satisfying the conditions that $E(dW_t) = 0$ and $E(dW_t^2) = dt$. Although equation (1) is an abstraction from real projects, we can think of π_t as the "reduced form" of a more complex model where the instantaneous cash flow $\pi_t = \pi(\mathbf{z}_t)$ depends on a vector of variables $\mathbf{z}_t$, which may include the market price, the quality of the service, the firm's investments and market shocks that account for some uncertainty in consumer demand and/or technological choice.

Furthermore, to emphasize the fact that our analysis is designed to cope with high profits and not with conduct that aims to conceal high profits, we assume that no new investments are undertaken during the contract period.[11] Therefore, under these assumptions, the value of an infinite project, $V(\pi)$, becomes (Harrison, 1985,

[11] A few contributions have investigated the relation between the adoption of a PS rule and the regulated firm's incentive to invest. Specifically, in a static setting and referring to technical efficiency, Lyon (1996) claims that a pure RPI-x regulation is preferable to a price-cap with PS. Moreover, Weisman (1993) shows that incorporating a PS rule in a price-cap regulation may be worsening relative to a pure cost regulation. Considering a dynamic setting with uncertainty on investment timing, Moretto et al. (2007) find that the inclusion of a PS rule in a PCR is less welfare detrimental than in a static framework, stressing that PS should be less stringent in sectors where there are bigger investment opportunities.

p.44):

$$V(\pi) = E_0 \left\{ \int_0^{\infty} \pi_t e^{-\rho t} dt \mid \pi_0 = \pi \right\} = \frac{\pi}{\rho - \alpha} \tag{2}$$

where $\rho > \alpha$ is the constant risk-free rate of interest,[12] and V is also driven by a geometric Brownian motion with the same parameters α and σ:

$$dV_t = \alpha V_t dt + \sigma V_t dW_t, \qquad \text{with } V_0 = V \tag{3}$$

When the monopolist makes "huge" profits, the regulator introduces a PS rule to divert these "excess" profits to consumers, or revokes the firm's contract to re-obtain responsibility to manage the utility and re-address the project's profitability. In what follows, we firstly model the PS rule and, in the next section, the contract closure.

Of the many ways of introducing PS, the simplest one is to set of an upper bound $\bar{\pi}$ on profits by the regulator, i.e. at $\bar{\pi}$ a "profit cut" stops π_t from going above $\bar{\pi}$.[13]

As from (2) choosing $\bar{\pi}$ is equivalent to choosing an upper limit to the value of the project $\bar{V}$, hereafter we take V_t as the primitive exogenous state variable for the regulatory process. Thus, if the monopolist starts with the initial project's value $V_0 < \bar{V}$, the PS rule works as follows:[14]

- for $V_t < \bar{V}$, the PS rule does not applies; V_t develops on its own and follows the geometric Brownian motion (3);

- for $V_t \geq \bar{V}$, the PS rule dr_t is introduced to stop V_t from going above $\bar{V}$. The new "regulated" process $V_t - r_t$ can be described by the following stochastic differential equation[15]:

  $$dV_t = \alpha V_t dt + \sigma V_t dW_t - dr_t, \quad V_0 = V, \text{ for } V_t \in (0, \bar{V}] \tag{4}$$

  where the increment dr_t represents the firm's profit reduction between time t and $t + dt$.

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[12] Alternatively, we could use a discount rate that includes an appropriate adjustment for risk and take the expectation with respect to a distribution for π that is adjusted for risk neutrality (see Cox and Ross, 1976; Harrison and Kreps, 1979).

[13] For qualitatively analogous rules and their discussion, see Sappington and Weisman (1996) and Sappington (2002).

[14] Really this is a "value-sharing" rule: we call it PS as there is a one-to-one relationship between the firm's value and profits. See Moretto and Valbonesi (2000) for the explicit model of a firm's production decision.

[15] Stochastic differential equations such as (4) are a notational convenience, since only their integral counterparts are well defined. The "impulse" dr must be interpreted as potentially taking finite values when a discrete jump occurs (Harrison and Taksar, 1983; Harrison, 1985).

To set up an appropriate mathematical model representing this PS rule we were guided by the theory of optimal barrier regulations (Harrison and Taksar, 1983; Harrison, 1985). The PS rule can be modelled by a process proportional to V_t, conditional to $\bar{V}$, right-continuous, non-decreasing and non-negative, defined as:

$$r_t = a(\bar{V})V_t \qquad \text{if } V_t \geq \bar{V}, \tag{5}$$

where $a(\bar{V}) \equiv [1 - \inf_{T^* \leq v \leq t}\left(\frac{\bar{V}}{V_v}\right)]$, $T^* = \inf(t \geq T^* \mid V_t - \bar{V} = 0^+)$ and $r_t = 0$ for all $t \leq T^*$(see Appendix 7.1).

As shown in Figure 1 below, the PS defined in (5) increases to keep V_t lower than $\bar{V}$ and is given by the cumulative amount of profit control exerted on the sample path of V up to t.

It is worth noting that this setting allows us to deal with PS mechanisms differently defined. Suppose, for example, that at $\bar{V}$ the regulator introduces a PS rule in the form of percentage-cut of the firm's profits: we can model this new PS rule adding to the above (5) a new stochastic differential equation for the profit cut.[16] Finally, note that this modelled PS rule can be easily related to the one-side sliding scale formula proposed by Joskow and Schmalensee (1986) to adjust prices under rate-of-return regulation.[17]

[16]Formally, this is:

$$dV_t = \alpha V_t dt + \sigma V_t dW_t - dr_t,$$

and

$$dM_t = s dr_t$$

where M is defined by giving up $1/s$ unit of V for each unit of M. Therefore, above $\bar{V}$, the PS rule can now be reformulated in terms of the new variable $Y_t = V_t/M_t$. When the existing combination of (V_t, M_t) places Y_t above s, the regulator intervenes immediately by cutting back on profits ($dr_t > 0$). The amount of profits cut is very small and is such as to push the firm's value along a sloped line $1/s$.

[17]Formally, defining $V_t^r = V_t - r_t$ as the actual "regulated" process, simple algebra allow us to write it as (Moretto and Valbonesi, 2000):

$$\begin{aligned} V_t^r &= V_t + h_t\left(\bar{V} - V_t\right), \\ \text{with } h_t &= \begin{cases} 0 & , \quad \text{for } V_0 \leq V_t < \bar{V} \\ \frac{1 - \inf\limits_{T^* \leq v \leq t}(\bar{V}/V_v)}{1-(\bar{V}/V_t)}, & \quad \text{for } V_t \geq \bar{V} \end{cases} \end{aligned}$$

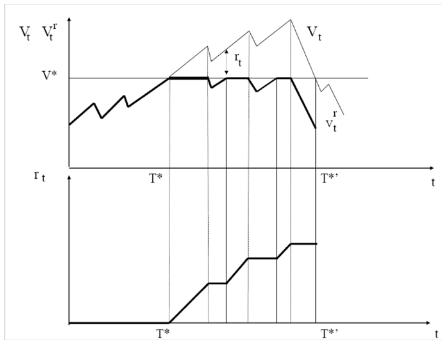

Figure 1: The value- (i.e., profit-) sharing dynamic

3 The optimal PS rule

In the previous section we modelled the PS rule (5) for a given exogenous upper bound value $\bar{V}$. In this section we define what firm's profit value triggers both the regulator's introduction of the PS and the regulator's adoption of the alternative strategy, i.e. the revocation of the contract. We develop the analysis under the simpler assumption of verifiability of the firm's profit. This assumption specifically implies that the regulator's threat of contract closure is binding when the firm's profit level is higher than the optimal trigger. Moreover, if the firm's profits are verifiable, the optimal value $\bar{V}$ that makes the regulator introduce the PS rule is the one, say V^*, that leads the regulator to revoke the contract.

We assume that the regulator decides whether or not to revoke the firm's contract solving the minimization of an intertemporal loss function; this loss function is an increasing function of the firm's profit. Indeed, an increase in the monopolist's profits reduces the monetary value of consumers' welfare. If the firm's profits become "too" high, i.e., when the social loss is "too" large, the regulator adopts one of the following alternative and equivalent strategies:

1. introduces a PS rule - defined as (5) - to divert profits from the firm to consumers;

2. revokes the contract to get the utility back and re-determine its provision, thus re-addressing the project's profitability.

Contract closure is then an "outside" option the regulator can always exercise. We model this option as a perpetual Call option, with the project's value V as the underlying asset.

3.1 Social loss and the option to revoke

The regulator minimizes an intertemporal loss function. Avoiding discounting for the sake of simplicity, we define this loss function as the difference in value of the social welfare between time 0 and the revocation time T, plus the revocation cost net of the firm's value. Hence if, according to the utilitarian criterion, we write the social welfare function[18] at time T as the sum of the net consumers surplus $K_T - (1+\lambda)V_T$, and the firm's value V_T, the loss function at T becomes:

$$\Delta K_T + \lambda(V_T - V) + (I - V_T) \tag{6}$$

where ΔK_T is the expected change in the consumers' willingness to pay for the service from time zero to T, $\lambda \in (0,1)$ is the opportunity cost of public funds to run the service by the regulator, $\lambda(V_T - V)$ is the expected rise in consumers' expenditure up to the revocation time T and the term $(I - V_T)$ is the regulator's net cost of revocation. Indeed, revocation is costly as contract closure determines that the management of the project returns to the regulator's hands and this - in turn - implies that the regulator should implement the new utility provision (i.e., through direct management, privatization or contracting out to another firm). Specifically, I is the regulator's cost in finding a new franchisee or - in the case of direct provision of the service - in training and hiring new personnel and/or adopting new technologies. The regulator's cost of revocation also includes legal expenditure if the firm decides to sue the regulator or, more generally, any cost from regulatory capture by the firm.[19]

Furthermore, we assumed $\Delta K_T = 0$. This is justified by the observation that short-run demand functions for necessary utilities such as water, electricity etc., are characterized by very low price and income elasticities and there is no close substitute for them. In other words, the consumers' reservation price for these utilities tends to be so high that the consumer surplus is not affected by small changes in the corresponding market price.[20]

Since minimizing (6) is equivalent to maximizing $V_T - I - \lambda(V_T - V)$, it is evident that rent extraction can be part of the regulator's objective in revoking the contract.[21] Thus, exercising the option to revoke requires payment of the sunk cost

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[18] As stressed by Laffont and Tirole (1994, p.55-56) in adopting a similar social welfare function, "the crucial feature of this kind of social welfare function is that the regulator dislikes leaving a rent to the firm". This could be justified by the fact that rent extraction is part of the regulator's goal.

[19] For a discussion on the different sources of regulatory capture from the firm, see Laffont and Tirole (1994, chapter 11).

[20] It is worth noting that our results hold even if $\Delta K_T \neq 0$. Specifically, if $\Delta K_T > 0$ is assumed, the effect of revocation results exacerbated.

[21] See Crew and Kleindorfer (1996, p. 218), for a discussion on rent extraction as included in the regulator's objective function.

I plus the social cost $\lambda(V_T - V)$. Due to the level of the sunk cost I, it is never optimal for the regulator to revoke when $V_T - I - \lambda(V_T - V)$ is equal to zero: it is better to wait until its value reaches a higher level.

Defining $F(V)$ as the value of the option at $t = 0$, we get:

$$F(V) = \max_T E_0 \left[((1-\lambda)V_T - \hat{I})e^{-\rho T} \mid V_0 = V \right] \quad (7)$$

where $T(\bar{V}) = \inf \left(t \geq 0 \mid V_t - \bar{V} = 0^+ \right)$ is the unknown future time when the option is exercised, V_T is the threshold value that triggers that action and $\hat{I} \equiv I - \lambda V$ is the exercise price. The optimization is subject to (3) and V[22].

Note that $F(V)$ is a perpetual Call option. By using standard results in the (Real) option valuation (Dixit and Pindyck, 1994), the solution of (7) is given by:

Proposition 1 *The value of the regulator's option to revoke at time $t \geq 0$ is given by:*

$$F(V) = \begin{cases} A(V^*)V^\beta & \text{for all } V < V^* \\ (1-\lambda)V - \hat{I} & \text{for all } V \geq V^* \end{cases} \quad (8)$$

where:

$$V_T \equiv V^* = \frac{\beta}{\beta - 1}\frac{1}{1-\lambda}\hat{I}, \quad \text{with} \quad \frac{\beta}{\beta-1} > 1^{23} \quad (9)$$

and:

$$A(V^*) = \frac{1-\lambda}{\beta}(V^*)^{1-\beta} > 0 \quad (10)$$

Proof. *see Appendix 7.2.* ■

Hence, the regulator's optimal revocation rule can be expressed as: "Revoke the contract as soon as the value of the project exceeds the adjusted break-even value V^*".

To interpret (8), let us rewrite it in the following form:

$$\begin{aligned} F(V) &= \left[(1-\lambda)V^* - \hat{I}\right]\left(\frac{V}{V^*}\right)^\beta \\ &= \left[(1-\lambda)V^* - \hat{I}\right] E_0\left[e^{-\rho T}\right] \end{aligned} \quad (11)$$

Maximizing (7) means maximizing the expected discounted value of the net benefit $(1-\lambda)V^* - \hat{I}$ when the utility is expropriated at time T, where $E_0\left[e^{-\rho T}\right] =$

[22] Moreover, we must also assume that $\hat{I} > 0$ and $V_T - \hat{I} > 0$.

[23] $\beta > 1$ is the positive root of the quadratic equation: $\Phi(\beta) = \frac{1}{2}\sigma^2\beta(\beta-1) + \alpha\beta - \rho = 0$

$\left(\frac{V}{V^*}\right)^\beta < 1$ is the expected discount factor. Then, maximizing (11) with respect to V^* gives the optimal revocation trigger as in (9).

Inspection of the opportunity cost λ in (9) reveals that:

- as $\lambda \to 0$, i.e., the regulator becomes socially "indifferent" between direct management of the utility and the franchising contract to a firm, V^* drops to $\frac{\beta}{\beta-1}I$ and the probability of revocation increases.
- as $\lambda \to 1$, i.e., the opportunity cost of direct management by the regulator rises, $V^* \to \infty$ and the regulator never revokes.

In other words, the regulator's direct cost of revocation $V^* - \hat{I}$ is weighed by the regulator's opportunity cost λ which, in turn, increases as fiscal distortion in raising public funds to run the service becomes larger. In real-world regulation this implies that the regulator's direct cost of revocation can be enhanced or reduced by the efficiency of the fiscal tools adopted in collecting funds.

3.2 Revocation vs Profit Sharing

Since for $V_t > V^*$ it is optimal for the regulator to revoke the contract, in this section we prove that, by setting $\bar{V} = V^*$, the regulator is indifferent to applying the PS rule and revoking the contract. Denoted by $(1-\lambda)R(V_T;\bar{V})$ the expected value of future cumulative profit reduction net of tax distortion due to (5)[24], the regulator's loss function at T results:

$$\lambda(V_T - V) + (I - V_T) + (1-\lambda)R(V_T;\bar{V}) \tag{12}$$

where $\bar{V}$ is a generic reflecting barrier. Using (12) as payoff, the value of the regulator's option to revoke becomes:

$$F^r(V) = \max_T E_0^r \left[(1-\lambda)V_T - \hat{I} - (1-\lambda)R(V_T;\bar{V}))e^{-\rho T} \mid V_0 = V \right] \tag{13}$$

where the superscripts indicate that the PS rule (5) is adopted. The solution of (13) shows that:

Proposition 2 *i) If $\bar{V} > V^*$, the regulator's optimal revocation trigger (once the PS is adopted) is still equal to (9), that is:*

$$V_T \equiv V^* = \frac{\beta}{\beta-1}\frac{1}{1-\lambda}\hat{I}$$

ii) If $\bar{V} = V^*$, the PS rule (5) keeps the regulator indifferent to revocation, i.e.:

$$F^r(V_t) = 0 \qquad \text{for } t \geq 0 \tag{14}$$

Proof. *see Appendix 7.3.* ■

[24] See about Appendix 7.3.

Proposition 2 states the optimality of the sharing rule (5) compared to the regulator's alternative equivalent strategy, that is, the regulator's optimal contract closure. This implies that if the monopolist keeps profits below V^*, revocation is never optimal.

Proposition 2 - in both parts - provides further considerations. First, if $\bar{V} > V^*$, the optimal revocation trigger under PS is equal to the optimal revocation trigger without PS as in (9). This is just an application of the dynamic programming principle of optimality: if at $t = 0$ the regulator sets V^* as the optimal revocation trigger, this should be optimal for any $t > 0$, independent of any future policy after V^*.

Second, if the regulator sets $\bar{V} = V^*$ as a reflecting barrier, the value of its option to revoke is always equal to zero. The line of thought behind this result is a straightforward implication of the barrier control r_t applied to the process V_t. Indeed, the true cost of exercising the option for the regulator is not just equal to the strike price $\hat{I}$, but also includes the future profit cuts $R(V_t; V^*)$ and the value of the forgone option $F^r(V_t)$. Thus, the net expected present value of optimal exercise at time $t \geq 0$ is:

$$E_t^r \left\{ [(1-\lambda)V^* - \hat{I} - F^r(V^*)]e^{-\rho(t-T)} \right\} - (1-\lambda)R(V_t; V^*) = -F^r(V^*) \left(\frac{V_t}{V^*} \right)^\beta \tag{15}$$

where the last equality follows from $R(V_t; V^*) = [V^* - \frac{\hat{I}}{1-\lambda}] \left(\frac{V_t}{V^*}\right)^\beta$ (see about Appendix 7.3). Maximizing (15) with respect to V^* gives:

$$\beta \frac{F^r(V^*)}{V^*} - F^{r\prime}(V^*) = 0 \tag{16}$$

Since avoiding arbitrage at V^* the second term of (16) must be equal to zero, we get (14).[25] That is, from (15), the regulator is indifferent between introducing the PS rule and revoking the contract when the expected benefits from profit regulation exactly offset the expected social welfare loss due to the monopolist's excess profits.

Finally, although the PS rule (5) is simply proportional to the project's value, several new implications follow:

[25] The function $F^r(V_t)$ is defined as the expected value of the regulator's net benefit when the utility is expropriated at time T. As the net benefit is a continuous function of the primitive process V_t, also F^r is a continuous function except perhaps when $V_t = V^*$ and the profit-sharing rule r_t is applied. The behavior around $V_t = V^*$ is given by expanding $F^r(V_t)$ as:

$$F^r(V^*) = F^r(V^* - dr) = F^r(V^*) - F^{r\prime}(V^*)dr$$

which gives $F^{r\prime}(V^*) = 0$. This condition holds at any reflecting barrier without any optimization being involved (Dixit, 1993).

- r_t is dynamic and parametrized by the initial condition V^* which, in turn, depends on the revocation cost I and on the opportunity cost parameter λ.
- r_t is non-decreasing and is given by the cumulative amount of profit cuts exerted on the sample path of V_t up to t. Thus, r_t relates to past realizations of V_t, which makes the PS time-dependent.

Summing up, the PS rule r_t arises as the optimal response from the continuous interaction between the monopolist and the regulator: specifically, it is smaller as the regulator's revocation and opportunity costs, I and λ respectively, become larger; moreover, it determines a "regulated" process $V_t - r_t$ which is function solely of the starting state V_t.

4 Efficiency of the PS rule

An important and controversial fact of the PS rule implementation in the real world is its dynamic sustainability when the monopolist's profits are observable but non-verifiable. In this case, the regulator cannot force the firm to cut "excessive" profits as "no court or other third party will accept to arbitrate a claim based on the value taken by these variables".[26] Therefore, under profit non-verifiability, is the regulator's threat of contract closure still sufficient to induce the firm to comply with (5) as V_t intersects V^*?

In this section we formally demonstrate that the proposed PS rule sustains a perfect equilibrium for the repeated continuous time regulatory relationship that starts at $T^* = T(V^*)$. We have done this by showing that any firm's deviation from (5) makes contract closure worthwhile for the regulator. In addition, since V_t is a Markov process, it is easy to state that the equilibrium is also sub-game perfect.

The regulatory game we consider here lasts a possibly infinite number of periods, and ends once the regulator exercises the option to revoke the franchise contract. Each period is divided into four stages: in the first stage, nature chooses a parameter determining the profit of the regulated monopolist. In the second stage, after observing the firm's profit, the regulator decides whether or not to ask for PS: if the regulator perceives that the monopolist is making "excessively" high profits, he sets a profit ceiling, say V^*, according to (9), above which the PS rule (5) applies. The regulator accompanies its announcement with a threat to revoke the contract if the firm does not comply.[27] In the third stage, the monopolist decides whether or not to comply with the regulator's prescription (i.e., whether or not to share

[26] Salanié, 1997, p.177.

[27] By the Markov Property of (21), in our model it is not important when the regulator announces V^* as long as it is between zero and T^*.

profits, that is, to start with stream of payment $r_t \geq 0$).[28] In the fourth stage, the regulator, conditional on V_t, decides whether or not to revoke the contract. If the regulator does revoke, the monopolist suffers the loss V_t and the regulator obtains $V_t - I$ (i.e., the net gain from revocation). If the regulator does not revoke, the game goes ahead to the next period and it is repeated.

However, without a binding commitment by the regulator, any finite number of firm profit reductions will be inefficient. Indeed, the regulator's problem is that for any $t \geq T^*$ he has an incentive to carry out his threat, even if the monopolist reduces its profits. Since this means that the monopolist will not ward off the threat by reducing its profits, the monopolist will not reduce them. Thus, the unique sub-game perfect equilibrium is inefficient: revocation is carried out regardless of the monopolist's positive net present value. To avoid this inefficiency the monopolist must continuously "control" its profits; that is, for $t \geq T^*$, the monopolist should consider V^* as the ceiling not to be crossed, and reduce its expected profits just enough to keep $V_t < V^*$ to prevent contract closure.[29] To summarize:

Proposition 3 ***i)*** *Starting form the initial date $t < T^*$, the following strategy is a sub-game perfect equilibrium:*

- *As long as $V_t < V^*$ nothing is done*
- *As soon as V_t crosses V^* from below, the regulated firm reduces its profits by (5) and the regulator does not revoke.*

ii) *There always exists a time $T'^* > T^*$, defined as $T'^* = \inf(t \geq T^* \mid V_t - V^* = 0^-)$, where regulation stops.*

Proof. *see Appendix 7.4.* ■

As both players - the regulator and the monopolist - expect an infinite repetition of their relation, their choices in each period will depend on the previous moves. The players' strategy for each period $t \geq T^*$ can be described as follows: the regulated monopolist observes V_t and chooses to share - or not to share - its profit according to the rule r_t; the regulator "does not revoke" if the monopolist has adopted the rule r_t to keep $V_t < V^*$ for all $t' < t$.[30] On the contrary, the regulator "revokes" if the firm has deviated from r_t at any $t' < t$. Our stochastic-continuous time framework

[28] In our infinite-lived project without investment, the firm's dominant strategy is not to make the payment $r_t \geq 0$, that is not to comply.

[29] There are many efficient sub-game perfect equilibria where the threat of revocation induces an infinite flow of payments by the firm to prevent contract closure (see Shavell and Spier, 1996, Proposition 2).

[30] In our continuous time setting we assume, without any loss of generality, that when the regulator is indifferent between revoking the contract and not revoking, he does not exercise the option.

calls for an instantaneous reply by the regulator when the monopolist departs from the PS rule (5). That is, the regulator adopts the most severe punishment, i.e., revocation of the contract.[31]

The regulator believes that this mechanism, from the initial date and state (T^*, V^*), will be retained for the whole planning horizon and since the project is infinitely-lived, the present value of forgone profits if the contract is revoked will always ensure participation by the firm. On the other hand, the expectation of future profit regulations keeps the regulator from carrying out this threat.

Finally, although the project is infinite, profit regulation takes place within a finite (stochastic) time span. Intuitively, although the monopolist prefers to cut profits rather than terminate the contract (i.e., the loss from closure is greater than the expected profit cuts), it always prefers to stop payment if the revocation threat is not carried out. Then, since this threat relies on the fact that the regulator's option to revoke is always worth exercising at $V_t > V^*$, i.e. $F(V_t) > F(V^*)$, when V_t reaches, for the first time after T^*, the trigger V^* from above, if the firm sets $r_{T^{*\prime}} = 0$, the regulator will face a jump of his revocation option from zero to $F(V^*)$ but revocation is not carried out (see Figure 1 above).

In other words, owing to uncertainty, neither player - regulator or monopolist - can perfectly predict V_t each time. As V_t follows a continuous time random walk there is, for each time interval dt, a constant probability of moving up or down, i.e., of the game continuing one more period. The game ends in finite (stochastic) time with probability one, but everything is as if the horizon were infinite (Fudenberg and Tirole, 1991, p. 148).

5 PS and dynamic price adjustment in a PCR

Let us conclude by showing how the PS rule we have investigated in the previous sections can be implemented in a PCR setting. According to what seems the most adopted form of PS in the real world - i.e., price cutting - in this section we explicitly deal with a PS prescription by which the regulated monopolist should decrease the price once its profits are "too high". Thus, we introduce here a reduced form of the previous firm's profit function (1) that depends only on the price of the service and on a demand shock, i.e., $\mathbf{z}_t = (p_t, \theta_t)$. That is, we assume that:

1. The market demand at time t is a constant-elasticity function of the price p_t:

$$D(p_t) = \theta_t p_t^{-\varepsilon} \tag{17}$$

   with $\varepsilon \in (0, 1)$.

[31] In continuous repeated games there is no notion of *last time before t*, so induction cannot be applied. For examples on how to represent continuous time as a sequence of discrete-time grids that becomes infinitely negligible, we can refer to Simon and Stinchcombe (1989) and Bergin and MacLeod (1993).

2. The random parameter θ_t follows a trendless, geometric Brownian motion, with instantaneous volatility $\sigma > 0$, i.e.:

$$d\theta_t = \sigma\theta_t dW_t, \quad \text{with} \;\; \theta_0 = \theta \tag{18}$$

where dW_t is the standard increment of a Wiener process[32].

3. No operating costs are present but there is a fixed cost c per period[33]. Then, the monopolist's project gives a profit flow at each time t equal to[34]:

$$\pi(p_t, \theta_t) = v(p_t, \theta_t) - c \equiv p_t^{1-\varepsilon}\theta_t - c. \tag{19}$$

4. The monopolist is subject to price regulation. The price p is allowed to increase by the difference between expected inflation (the Retail Price Index, RPI) and an exogenously given expected increase in productivity over time (x):

$$dp_t = (RPI - x)p_t dt, \quad \text{with} \;\; p_0 = p \tag{20}$$

These assumptions enabled us to reduce the model to one dimension. Expanding $d\pi(p_t, \theta_t)$ and applying Itô's lemma, it is easy to show that $v(p_t, \theta_t)$ is driven by the following geometric Brownian motion:

$$dv_t = \alpha v_t dt + \sigma v_t dW_t \quad \text{with } v_0 = v, \tag{21}$$

with:

$$\alpha \equiv (1 - \varepsilon)(RPI - x)$$

The drift parameter of the process v_t is a linear combination of the corresponding parameters of the primitive process θ_t and of the price-cap rule (20). Finally, since the monopolist is risk-neutral, using the simplified expression for the profit function (21), the market value of the project becomes:

$$V = \frac{v}{\rho - \alpha} - \frac{c}{\rho} \tag{22}$$

As far as the price-cap revision is concerned, in the event of the monopolist's profits going beyond a "pre-determined" level, the PS rule requires the x factor to be automatically adjusted upward, making the price-cap adjustment rate $RPI - x$

[32] By the Markov Property of (18), the quality of all subsequent results does not change if we assume an increasing demand trend.

[33] The fixed costs we considered here are, as standard in the literature, flow fixed costs of production: that is, we assume that the firm begins the first period endowed with technology whose operation entails a flow cost c per unit of time.

[34] For the sake of simplicity, we avoided considering operating options such as reducing output or even shutting down which increase the value of the firm (MacDonald and Siegel, 1986; Dixit and Pindyck,1994, chs. 6 and 7).

more stringent (Sappington, 2002). According to this practice we can rewrite (4) as:

$$dV_t = (1-\varepsilon)(RPI - x')V_t dt + \sigma V_t dW_t, \quad V_0 = V, \text{ for } V \in (0, V^*] \tag{23}$$

where $x' = x - \frac{d \inf_{0\leq v\leq t}(V^*/V_v)/dt}{(1-\varepsilon)\inf_{0\leq v\leq t}(V^*/V_v)} > x$ is the new price decrease factor which stops the process V_t from going any higher than V^*. How the x' factor works seems intuitively appealing. As the numerical value for V^* is known, by (22) the optimal revocation trigger (9) can be written as $p_t^{1-\varepsilon}\theta_t = \frac{\beta}{\beta-1}\frac{1}{1-\lambda}\frac{\rho-\alpha}{\rho}(c+\rho\hat{I})$, from which the boundary value for θ^* is determined by:

$$\theta^*(p_t) = \frac{\beta}{\beta-1}\frac{1}{1-\lambda}\frac{\rho-\alpha}{\rho}\frac{c+\rho\hat{I}}{p_t^{1-\varepsilon}} \tag{24}$$

For any given value of the price p_t, random fluctuations of θ_t move the point (θ_t, p_t) horizontally left or right. If the point goes to the right of the boundary, then a price reduction is made immediately by shifting the point down to the boundary. If θ_t stays to the left of the boundary, no new price reduction is applied. Thus, price reduction proceeds gradually to maintain (24) equality. To illustrate this, suppose $RPI - x = 0$ so that $p_t = p_0$ for all t; by inverting (24) we obtain the optimal boundary function $p(\theta_t)$ which determines the optimal price regulation as a function of the sole state variable θ_t:

$$p_t = p_0\left(\frac{\theta^*}{\theta_t}\right)^{1/1-\varepsilon} \qquad \text{with} \quad \frac{dp_t}{d\theta_t} < 0 \tag{25}$$

Furthermore, higher costs shift the boundary (24) to the right, $\theta'^* > \theta^*$ and determine a smaller price reduction by the firm to comply with the PS rule.[35] The boundary function for this case is shown in Figure 2 below.

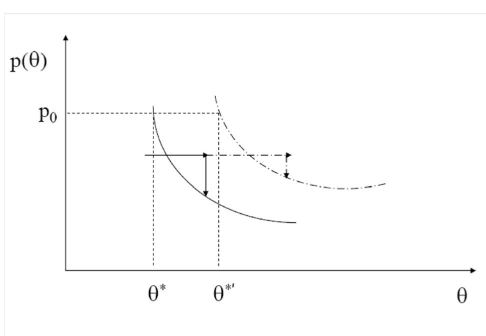

Figure 2: Price regulation

[35] Indeed, cost padding by the franchisee is another strategy that might be used to avoid the appearance of excess profit. It would be possible to model the franchisee as reporting costs and the regulator as employing auditors to determine the accuracy of cost reports, but this is not our aim here.

Thus, in our analysis of PS in a PCR setting, the ability to ratchet the expected increase in monopolist's productivity - that is, the x-factor - is directly related to the regulator's threat of revocation. The credibility of this threat determines the difference between contractual and real duration of a regulatory arrangement and, hence, the success of introducing a PS rule in the original PCR setting. The difference between contractual and real contract duration is another way of looking at the well-known "commitment problem" in regulation. Crew and Kleindorfer (1996) argue that a major issue in incentive regulation is commitment: ".. if a company is concerned that the regulator will penalize it at the end of or even during the price-cap period if it is successful, it may not pursue efficiency as strongly as implied by the apparent incentives in PCR. Thus, the notion that the regulator will not renege on the terms of PCR is very important for the efficiency to be achieves.. (p.218)". However, they subsequently admit that as the regulators' goal is rent extraction, it is easy to see that they have limited incentives to commit themselves, and that this problem is at the root of the emergence of regulatory contracts that incorporate sharing rules.

6 Final Remarks

When a welfare maximizing regulator delegates a monopolist to manage a long-run franchise contract for a public utility, we have employed a real options approach to determine: **a)** the threshold value that triggers profit-sharing (PS) and **b)** under what informational conditions a dynamic PS rule provides incentives to the firm to divert profits to customers. Specifically, we have assumed that if the monopolist's profits become "excessively" high - that is, if the social welfare loss is too large - the regulator can always adopt one of the following alternatives and equivalent strategies:

- introduce a PS rule to divert profits from the firm to consumers;
- revoke the contract to get the utility back and re-determine its provision, thus re-addressing the project's profitability.

We have modelled the regulator's option to revoke as a perpetual Call option which is a function of the firm's profits, the social welfare and the regulator's cost of revocation. The option value approach results particularly useful to the analysis of the PS at least for two reason.

First, option theory techniques are natural tools to analyze the intertemporal problem faced by a regulator who: (i) wishes to impose bounds on the profits of a public utility firm for welfare reasons; and (ii) cannot sign at the initial time a binding contract specifying the public utility market behaviour and profits in all possible contingencies (because of the non-verifiability of future states).

Second, the option value approach allowed us to consider uncertainty - from exogenous shocks, that typically affect the demand and cost function faced by the firm. This is relevant to fully understand the regulatory issue on PS, as sharing between the regulator and the firm is often called to face unpredictable events affecting the firm's demand or costs.

For the sake of clarity, our analysis is organized in two part. In the first one, we have shown *under the assumption of verifiability of profits* that the profit threshold that triggers revocation is the same as the one that triggers the PS (*Proposition 1*). That is, such a threshold keeps the regulator indifferent between revoking the contract and applying the PS rule (*Proposition 2*). Hence, as the regulator's contract closure can be very costly - i.e., it could include the revocation cost of finding a new franchisee, or legal expenditure if the firm sues the regulator - a considerable regulatory lag can occur before a PS rule is introduced. Moreover, if the regulatory process has been taken over by the monopolist, the PS - as unilaterally advocated by the regulator - is further delayed.

Then, *under the assumption of profit non-verifiability,* we investigated whether the PS rule is sustainable, that is, if it is dynamically efficient. We therefore showed that the regulator's credible threat to revoke the contract endogenously determines the rule by which the monopolist is induced to "control" its profits, and that this is a sub-game perfect equilibrium (*Proposition 3*). Given this result, the price adjustment which follows is endogenously and dynamically determined as the best response for the monopolist to the regulator's choice. Specifically, as shown in the last section of the paper, for a price-cap setting where a reduced form of the firm's profit function is adopted, we have provided how the price adjustment - via the firm's productivity adjustment (the so called X-factor) - is affected by PS enforcement.

These conclusions provide, on the one hand a reason for the firm's "voluntary" price cuts below the formally allowed price as, for example, was observed in the end of the 1990s for several water companies in England and Wales. Following the announced UK water regulator's prescription to rebate water prices in order to transfer savings to customers made by companies in reducing their managing costs' in the previous period, those firms agreed to comply with this PS rule. Furthermore, expecting substantial falls in their profit, offseted looking for new ventures and restructuring their budget sheet[36]. On the other hand, this conclusion also suggests a theoretical reason why the PS rule in its dynamic application tends to be unsuccessful in the real world. More specifically, as long as the regulator's threat to revoke the contract becomes non-credible, the regulated firm is no longer incentivized to comply with the adopted PS rule. These conjectures both stress the relationship between regulatory commitment and regulatory costs in a repeated relation such

[36] See: Andrew Taylor, Financial Times (London, England), April 17, 2000, "Water companies struggle in wake of regulator's price cuts"; Chris Godsmark, The Independent, (London), May 20, 1997, "Byatt questions regulation plans".

as the regulatory practice of public utility sectors. The sustainability of the PS mechanism crucially depends upon the magnitude of the regulator's revocation cost which, in turn, affect the credibility of the regulator's threat to revoke the contract. The revocation cost thus represents a form of capture of the regulator by the firm: the higher the revocation cost, the lower the profit shared and the less frequent the regulator's PS prescription.

Moreover, the regulator's revocation cost determines - via the profit threshold that triggers the PS rule - the (expected) time interval between each pair of regulatory reviews (i.e., the regulatory lags). So, revocation cost measures the inefficiencies that the regulator incurs in the direct provision of the service or in finding a new franchisee, costs which can vary greatly in different regulatory contexts. In the regulation of local utilities, for instance, the inexperience of the municipal authority as regulator may determine proportionally higher revocation costs than for a nationally supplied utility (Clark and Mondello, 2000).

Note that the PS rule we have investigated here – as PS rules observed in the real world – is a relatively simple and inflexible long-term contract. As observed by Bolton and Dewatripoint (2005, p.483) - "enforcement costs are likely to escalate significantly with contractual complexity" as disagreement and litigation are more likely for more complex contracts. The advantage of our simple PS rule is that it relies on self-enforcement which is often more efficient than legal enforcement. This strengthens in the case of non-verifiability of profit. Moreover, the self-enforcement of the PS rule is based on the credibility of the threat of contract revocation by the regulator. As such a threat is triggered by the regulator maximizing of an intertemporal social welfare function, it is never optimal for the regulator to ex-post renegotiate the PS rule.

To close, we would like to briefly suggest an extension of our model which looks at the main simplifying assumption we adopted and which warrants further investigation. The economic literature on firm's regulation generally holds that PS rules reduce a firm's incentive to invest (Lyon, 1996; Crew and Kleindorfer, 1996; Sappington, 2002). However, our analysis - which specifically addresses the definition of threshold value triggering the PS rule and on its self–enforcement - is carried out under the assumption of no investment. Specifically, our analysis realistically applies to cases where the regulated firm realizes high profits independent of its strategic decision on investment. This frequently occurs in utility markets, for instance, when exogenous and unpredictable shocks positively affect the firm's demand (or decreases the firm's costs). Thus, a natural extension of our model would include the firm's intertemporal investment choice to reduce costs and increase demand as well as assess the effects of the threat of contract closure and/or the PS rule by the regulator on this strategic firm's decision.

7 Appendix

7.1 The regulation process

We define the regulation which follows the introduction of the PS rule as the reduction dV_t needed to keep V_t at $\bar{V}$. This is a one-sided non-decreasing adapted control process (as in Harrison, 1985) on the state variable V which is right-continuous and non-negative. So, the control policy consists of a process $Z = \{Z_t, t \geq 0\}$ and a regulated process $V^r = \{V_t^r, t \geq 0\}$ such that:

$$V_t^r \equiv V_t Z_t, \quad \text{for} \quad V_t^r \in (0, \bar{V}], \tag{26}$$

where:

- $i)$ V_t is a geometric Brownian motion, with stochastic differential as in (3);
- $ii)$ Z_t is a decreasing and continuous process with respect to V_t ;
- $iii)$ $Z_0 = 1$ if $V_0 \leq \bar{V}$, and $Z_0 = \bar{V}/V_0$ if $V_0 > \bar{V}$ so that $V_0^r = \bar{V}$;
- $iv)$ Z_t decreases only when $V_t^r = \bar{V}$.

Applying Ito's lemma to (26), we get:

$$d\,V_t^r = \alpha\, V_t^r dt + \sigma V_t^r dW_t + V_t^r \frac{dZ_t}{Z_t}, \qquad V_0^r \in (0, \bar{V}]$$

where $V_t^r \frac{dZ_t}{Z_t} \equiv V_t dZ_t = -dr_t$ is the infinitesimal level of value forgone by the firm. In terms of the regulated process V_t^r, we can write:

$$r_t \equiv r(V_t) = V_t - V_t^r \equiv (1 - Z_t)V_t, \tag{27}$$

Although the process Z_t may have a jump at time $t = 0$, it is continuous and keeps V_t below the barrier $\bar{V}$ exercising the minimum amount of control: in fact, control is exercised only when V_t crosses $\bar{V}$ from below with a probability one in the absence of regulation. Therefore, in the case of $V_0 < \bar{V}$, we get $V_t^r \equiv V_t$, with the initial condition $V_0^r \equiv V_0 = V$, and $Z_t = 1$.

At $T^* \equiv T(\bar{V}) = \inf(t \geq 0 \mid V_t - \bar{V} = 0^+)$ control starts so as to maintain $V_t^r = \bar{V}$.

The firm's values are adjusted downward by the amount $r_t = V_t - V_t^r \geq 0$ every time $\bar{V}$ is hit. The same conditions $(i) - (iv)$ uniquely determine Z_t with the representation form (Harrison,1985; Proposition 3, p. 19-20):[37]

$$Z_t \equiv \begin{cases} \min(1, \bar{V}/V_0) & \text{for } t = 0 \\ \inf\limits_{0 \leq v \leq t} (\bar{V}/V_v) & \text{for } t \geq 0 \end{cases} \tag{28}$$

[37] This is an application of a well-known result of Levy (1948), for which the process:

$$\ln V_t^r \equiv \ln V_t + \ln Z_t \equiv \ln V_t - \inf_{0 \leq v \leq t} (\ln V_v - \ln \bar{V})$$

has the same distribution as the "reflected Brownian process" $\mid \ln V_t - \ln \bar{V} \mid$.

7.2 Proof of Proposition 1

The function $F(V_t)$ is defined as the expected value at time t of the regulator's net benefit when the utility is expropriated at time T. As the net benefit is a continuous function of the primitive process V_t , also F is a continuous function of V_t. Then, by a short arbitrage argument (Cox and Ross, 1976; Harrison and Kreps, 1979), applying Ito's lemma to F, the value of the regulator's option to revoke becomes the solution of the following differential equation (Dixit and Pindyck, 1994, p. 147-152):

$$\frac{1}{2}\sigma^2 V_t^2 F''(V_t) + \alpha\, V_t F'(V_t) - \rho F(V_t) = 0, \qquad \text{for } V_t \in (0, V^*], \tag{29}$$

where $F(V_t)$ must satisfy the following boundary conditions:

$$\lim_{V_t \to 0} F(V_t) = 0 \tag{30}$$

$$F(V^*) = (1-\lambda)V^* - \hat{I} \tag{31}$$

$$F'(V^*) = 1 - \lambda \tag{32}$$

If the value of the utility tends to zero, so does the option. Conditions (31) and (32) imply respectively that, at the trigger level V^*, the value of the option is equal to its liabilities where $\hat{I}$ indicates the sunk cost of revocation (matching value condition) and the sub-optimal exercise of the option is ruled out (smooth pasting condition). By the linearity of (29) and using (30), the general solution is of the form:

$$F(V_t) = AV_t^{\beta}, \tag{33}$$

where A is a constant to be determined and $\beta > 1$ is the positive root of the quadratic equation:

$$\Phi(\beta) = \frac{1}{2}\sigma^2\beta(\beta-1) + \alpha\beta - \rho = 0 \tag{34}$$

As (33) is the option value of optimally revoking the contract, the constant A must be positive and the solution is valid over the range of V_t for which it is optimal for the regulator to keep the option alive $(0, V^*]$. By substituting (33) for (31) and (32) we get:

$$V^* = \frac{\beta}{\beta-1}\frac{1}{1-\lambda}\hat{I}\ , \tag{35}$$

and:

$$A(V^*) \equiv \left[(1-\lambda)V^* - \hat{I}\right](V^*)^{-\beta} \equiv \frac{1-\lambda}{\beta}(V^*)^{1-\beta} > 0. \tag{36}$$

This concludes the proof.

7.3 Proof of Proposition 2

We prove that when the regulator uses (27), its option to revoke is always equal to zero. We organize the proof in two parts as follows.

7.3.1 Cost of regulation

Let us denote with $R(V_t^r;\bar{V})$ the expected value of future cumulative profit cuts. At $t=0$ this is given by:

$$\begin{aligned} R(V_0;\bar{V}) &\equiv E_0^r\left\{\int_0^\infty e^{-\rho t}dr(V_t) \mid V_0^r \equiv V_0 \in (0,\bar{V}]\right\} \qquad (37)\\ &= -E_0^r\left\{\int_0^\infty e^{-\rho t}V_t dZ_t] \mid V_0^r \equiv V_0 \in (0,\bar{V}]\right\} \end{aligned}$$

where $\bar{V}$ is the (generic) upper reflecting barrier defined in (26). Since V_t^r is a Markov process in levels (Harrison, 1985, Proposition 7, p. 80-81), we know that the foregoing conditional expectation is in fact a function of the starting state alone.[38] Keeping the dependence of R on V_t^r active and assuming that it is twice continuously differentiable, by Itô's lemma we get:

$$\begin{aligned} dR &= R'dV_t^r + \frac{1}{2}R''(dV_t^r)^2 \qquad (38)\\ &= R'(Z_t dV_t + V_t dZ_t) + \frac{1}{2}R''Z_t^2(dV_t)^2\\ &= R'(\alpha\, V_t^r dt + \sigma V_t^r dW_t + V_t^r\frac{dZ_t}{Z_t}) + \frac{1}{2}R''Z_t^2\sigma^2 dt\\ &= \frac{1}{2}R''\sigma^2 V_t^{r2}dt + R'\alpha\, V_t^r dt + R'\sigma V_t^r dW_t + R'V_t^r\frac{dZ_t}{Z_t} \end{aligned}$$

where it has been taken into account that for a finite-variation process like Z_t, $(dZ_t)^2=0$. As $dZ_t=0$ except when $V_t^r=\bar{V}$ we can rewrite (38) as:

$$dR(V_t^r;\bar{V}) = [\frac{1}{2}\sigma^2 V_t^{r2}R''(V_t^r;\bar{V}) + \alpha\, V_t^r R'(V_t^r;\bar{V})]dt \qquad (39)$$

$$+\sigma V_t^r R'(V_t^r;\bar{V})\, dW_t - R'(\bar{V};\bar{V})dr(V_t)$$

[38] For $V_0 = V > \bar{V}$ optimal control would require Z to have a jump at zero so as to ensure $V_0^r = \bar{V}$. In this case the integral on the right of (37) is defined to include the control cost r_0 incurred at $t=0$, that is (see Harrison 1985, p. 102-103):

$$\int_0^\infty e^{-\rho t}dr_t \equiv r_0 + \int_{(0,\infty)} e^{-\rho t}dr_t$$

where $r_0 = V - V_0^r$.

This is a stochastic differential equation in R. Integrating by part the process Re^{-rt} we get (Harrison, 1985, p. 73):

$$e^{-\rho t}R(V_t^r;\bar{V}) = R(V_0;\bar{V})+ \tag{40}$$

$$+\int_0^t e^{-\rho s}\left[\frac{1}{2}\sigma^2 V_s^{r2}R''(V_s^r;\bar{V}) + \alpha V_s^r R'(V_s^r;\bar{V}) - \rho R(V_s^r;\bar{V})\right]ds$$

$$+\sigma\int_0^t e^{-\rho s}V_s^r R'(V_s^r;\bar{V})\,dW_s - R'(\bar{V};\bar{V})\int_0^t e^{-\rho s}dr(V_s)$$

Taking the expectation of (40) and letting $t \to \infty$, if the following conditions apply:

(a) $\lim_{l\to 0}\Pr[T(l) < T(\bar{V}) \mid V_0 \in (0,\bar{V}]] = 0$ for $l \leq V_t^r < \bar{V} < \infty$, where $T(l) = \inf(t \geq 0 \mid V_t^r = l)$ and $T(\bar{V}) = \inf(t \geq 0 \mid V_t^r = \bar{V})$;

(b) $R(V_t^r;\bar{V}))$ is bounded within $(0,\bar{V}]$;

(c) $e^{-\rho t}V_t^r R'(V_t^r;\bar{V})$ is bounded within $(0,\bar{V}]$;

(d) $R'(\bar{V};\bar{V}) = 1$;

(e) $\frac{1}{2}\sigma^2 V_t^{r2}R''(V_t^r;\bar{V}) + \alpha V_t^r R'(V_t^r;\bar{V}) - \rho R(V_t^r;\bar{V}) = 0$,

we obtain $R(V^r;\bar{V})$ as indicated in (37). Condition (a) says that the probability of the regulated process V_t^r reaching zero before reaching another point within the set $(0,\bar{V}]$ is zero. As V_t^r is a geometric type of process this condition is, in general, always satisfied (Karlin and Taylor, 1981, p. 228-230). Furthermore, if condition (a) holds and $R(V^r;\bar{V})$ is bounded, then conditions (b) and (c) also hold. According to the linearity of (e) and using (d), the general solution has the form:

$$R(V_0;\bar{V}) = B(\bar{V})(V_0)^{\beta}, \tag{41}$$

with:

$$B(\bar{V}) = \frac{1}{\beta}(\bar{V})^{1-\beta} > 0 \tag{42}$$

As for $V_0 \leq \bar{V}$, $Z_0 = 1$ and $V_0^r \equiv V_0 = V$, then $R(V_0;\bar{V}) = R(V;\bar{V})$. On the other hand, if $V_0 > \bar{V}$, we get $Z_0 = \bar{V}/V_0$, so that $V_0^r = \bar{V}$ and $R(V_0^r;\bar{V}) = R(\bar{V};\bar{V})$.

7.3.2 The value of revocation under profit control

Indicating by $F^r(V)$ the regulator's value of the option under profits control, this can be expressed, at time zero, by:

$$F^r(V) = \max_T E_0^r \left[((1-\lambda)V_T - \hat{I} - (1-\lambda)B(\bar{V})V_T{}^{\beta})e^{-\rho T} \mid V_0 = V \right] \qquad (43)$$

As in (33) this takes the form:

$$F^r(V) = AV^{\beta}$$

If the regulator decides for revocation, the optimal threshold, say V^*, must satisfy the two familiar conditions:

$$A(V^*)^{\beta} = (1-\lambda)V^* - \hat{I} - (1-\lambda)B(\bar{V})(V^*)^{\beta} \qquad (44)$$

$$\beta A(V^*)^{\beta-1} = (1-\lambda) - (1-\lambda)\beta B(\bar{V})(V^{**})^{\beta-1} \qquad (45)$$

These two equations can be solved for the trigger V^* and for the constant A. Simple algebra shows that V^* is independent of B, (i.e. of the barrier $\bar{V}$), and it is equal to (35):

$$V^* = \frac{\beta}{\beta-1}\frac{1}{1-\lambda}\hat{I}$$

Yet, the constant A is equal to:

$$A = -(1-\lambda)B(\bar{V}) + \frac{(1-\lambda)}{\beta}(V^*)^{1-\beta} = (1-\lambda)\left[\frac{1}{\beta}(\bar{V})^{1-\beta} - \frac{1}{\beta}(V^*)^{1-\beta}\right]$$

Therefore setting $\bar{V} = V^*$ the constant A drops to zero as well as the value of the option to revoke. Finally, as r_t depends only on the primitive exogenous process V_t, the "regulated" process $V_t - r_t$ is also a Markov process in levels (Harrison, 1985, Proposition 7, p. 80-81). Thus, any option value beginning at a point t at which revocation has not taken place has the same solution. This concludes the proof of Proposition 2.

7.4 Proof of Proposition 3

We prove that the regulatory scheme proposed in Proposition 2 is also optimal when the regulator cannot force the firm to adopt it. We proceed in the following way. First, since by Proposition 2 the sharing rule r_t makes the regulator's option to revoke the contract always equal to zero, it is also a good candidate for supporting a long-run equilibrium of the threat-game. Next, we prove that this is indeed the case by applying a sort of one-stage-deviation principle and showing that any deviation from r_t makes revocation worthwhile (the non-decreasing property of r_t is crucial to this result). Finally, the Markov property of the "regulated" process $V_t - r_r$ makes the equilibrium sub-game perfect.

7.4.1 Revocation strategy and perfect equilibrium

It is well known that infinitely repeated games may be equivalent to repeated games that terminate in finite time. At each period there is a probability that the game continues one more period. The key is that the conditional probability of continuing must be positive (Fudenberg and Tirole, 1991, p. 148). This is indeed our case, neither player can perfectly predict V_t at any date and the sharing rule (27) with form (28) is viewed by both agents as a stationary strategy for evaluating all future profit reductions.[39] In the strategy space of the regulator it appears as:

$$\phi(V_t, r_t) = \begin{cases} \text{Do not revoke at } t = T^* \text{ if the firm} \\ \text{plays the rule } r_t = (1 - Z_t)V_t \text{ for } t' < t \\ \\ \text{Revoke if the firm deviates from} \\ r_t = (1 - Z_t)V_t \quad \text{at any } t' < t \end{cases}$$

where $\phi(V_t, r_t)$ is the strategy at t with history (V_t, Z_t). The regulator's "threat" strategy is chosen if the firm deviates by adjusting V_t less than r_t or by abandoning $r_t = (1 - Z_t)V_t$ as a rule to evaluate future adjustments. The regulator must believe that the regulation, from the initial date and state (T^*, V^*), will be kept in use for the whole (stochastic) planning horizon. If the firm deviates, the regulator believes that the firm intends to switch to a different rule in the future and knows for sure that the regulator will revoke immediately thereafter. The regulator does not revoke at t if $r_{t'} \geq V_{t'} - V_{t'}^r$ for all $t' \leq t$, because profit cuts are expected to continue with the same rule and $F^r(V) = 0$ for all $t \geq T^*$. If $r_{t'} < V_{t'} - V_{t'}^r$ for some $t' < t$ the regulator expects a different rule and carries out the threat, switching from $F^r(V_t) = 0$ to $F(V_t) \geq V^* - I$. The game is over. To prove this, we first need to prove the following Lemma:

[39] Integrating the differential form (3), the geometric Brownian motion can be expressed as:

$$V_{t+dt} = V_t e^{dY_t}$$

where $dY_t = \mu dt + \sigma dW_t$ and $\mu = \alpha - \frac{1}{2}\sigma^2$. The differential dY_t is derived as the continuous limit of a discrete-time random walk, where in each small time interval of length Δt the variable y either moves up or down by Δh with probabilities (Cox and Miller, 1965, p. 205-206):

$$\Pr(\Delta Y = +\Delta h) = \frac{1}{2}\left(1 + \frac{\mu\sqrt{\Delta t}}{\sigma}\right), \quad \Pr(\Delta Y = -\Delta h) = \frac{1}{2}\left(1 - \frac{\mu\sqrt{\Delta t}}{\sigma}\right)$$

or defining $\Delta h = \sigma\sqrt{\Delta t}$:

$$\Pr(\Delta Y = +\Delta h) = \frac{1}{2}\left(1 + \frac{\mu\Delta h}{\sigma^2}\right), \quad \Pr(\Delta Y = +\Delta h) = \frac{1}{2}\left(1 - \frac{\mu\Delta h}{\sigma^2}\right)$$

That is, for small Δt, Δh is of order of magnitude $O(\sqrt{\Delta t})$ and both probabilities become $\frac{1}{2} + O(\sqrt{\Delta t})$, i.e. not very different from $\frac{1}{2}$. Furthermore, considering again the discrete-time approximation of the process Y_t, starting at $V^* e^{+\Delta h}$, the conditional probability of reaching V^* is given by (Cox and Miller, 1965, ch.2):

$$\Pr(Y_t = 0 \mid Y_t = 0 + \Delta h) = \begin{cases} 1 & \text{if } \mu \leq 0 \\ e^{-2\mu\Delta h/\sigma^2} & \text{if } \mu > 0 \end{cases}$$

which converges to one as Δh tends to zero.

Lemma 4 *For each* $t' > T^*$ *we get:*

$$R(V_{t'}; V^*) = (\rho - \alpha) E^r_{t'} \int_{t'}^{\infty} e^{-\rho(s-t')} r_s ds \tag{46}$$

Proof. Let's consider R as in (37). For each $t' > T^*$, integration by parts gives:

$$\int_{t'}^{t} e^{-\rho(s-t')} V_s dZ_s = \tag{47}$$

$$e^{-\rho(t-t')} V_t Z_t - V_{t'} Z_{t'} + \rho \int_{t'}^{t} e^{-\rho(s-t')} V_s Z_s ds - \int_{t'}^{t} e^{-\rho(s-t')} Z_s dV_s$$

Taking the expectations of both sides and using the zero-expectation property of the Brownian motion (Harrison, 1985, p. 62-63), we have:

$$E^r_{t'} \int_{t'}^{t} e^{-\rho(s-t')} V_s dZ_s = E^r_{t'}[V_t Z_t e^{-\rho(t-t')}] - V_{t'} Z_{t'} + (\rho - \alpha) E^r_{t'} \int_{t'}^{t} e^{-\rho(s-t')} V_s Z_s ds \tag{48}$$

By the Strong Markov property of V^r_t[40] it follows that $E^r_{t'}[V_t Z_t e^{-\rho(t-t')}] = E^r_{t'}[V_t Z_t] E^r_{t'}[e^{-\rho(t-t')}] = V^* E^r_{t'}[e^{-\rho(t-t')}] \to 0$ almost as surely as $t \to \infty$, so that:

$$E^r_{t'} \int_{t'}^{\infty} e^{-\rho(s-t')} V_s dZ_s = -V_{t'} Z_{t'} + (\rho - \alpha) E^r_{t'} \int_{t'}^{\infty} e^{-\rho(s-t')} (V_s - r_s) ds$$

Since $-V_{t'} Z_{t'} + (\rho - \alpha) E^r_{t'} \int_{t'}^{\infty} e^{-\rho(s-t')} V_s ds = 0$, substituting (37) and rearranging we get:

$$R(V_{t'}; V^*) = (\rho - \alpha) E^r_{t'} \int_{t'}^{\infty} e^{-\rho(s-t')} r_s ds$$

■

We now prove that r_t is sub-game perfect by showing that the firm cannot gain by deviating from r_t in an arbitrarily short interval and conforming to r_t thereafter. In particular, let's assume (t', t) an interval in which r_s is kept flat at $r_{t'}$ so that $V^r_s \leq V^*$, and t is the first time that $dZ_t > 0$. Considering the decomposition (48) we can write (46) as:

$$\begin{aligned} R(V_{t'}; V^*) &= (\rho - \alpha) \left\{ E^r_{t'} \int_{t'}^{t} e^{-\rho(s-t')} r_s ds + E^r_{t'} \left\{ \int_{t}^{\infty} e^{-\rho(s-t')} r_s ds \right\} \right\} \\ &= (\rho - \alpha) \left\{ E^r_{t'} \int_{t'}^{t} e^{-\rho(s-t')} r_s ds + E^r_{t'} \left\{ e^{-\rho(t-t')} \int_{t'}^{\infty} e^{-\rho(s-t')} r^*_s ds \right\} \right\} \end{aligned}$$

[40] The Strong Markov Property of regulated Brownian motion processes stresses the fact that the stochastic first passage time t and the stochastic process V^r_t are independent (Harrison, 1985, Proposition 7, p. 80-81).

where we have defined $V_s^{r*} = V_{t+s}^r$ and $r_s^* = r_{t+s} - r_t$ for $t' \leq t$. Applying, again, the Strong Markov Property of V_t^r we get:

$$\begin{aligned} R(V_{t'};V^*) &= E_{t'}^r \int_{t'}^t e^{-\rho(s-t')} r_s ds + E_{t'}^r \left\{ e^{-\rho(t-t')} E_{t'}^r \int_{t'}^\infty e^{-\rho(s-t')} r_s^* ds \right\} \\ &= (\rho-\alpha) E_{t'}^r \int_{t'}^t e^{-\rho(s-t')} r_s ds + E_{t'}^r \left\{ e^{-\rho(t-t')} R(V_{t'};V^*) \right\} \\ &= (\rho-\alpha) E_{t'}^r \int_{t'}^t e^{-\rho(s-t')} r_s ds + R(V_{t'};V^*) E_{t'}^r \left\{ e^{-\rho(t-t')} \right\} \end{aligned}$$

where the second equality follows from the assumption that $r_s = r_{t'} \equiv V_{t'} - V_{t'}^r$ for all $s \in (t', t)$. Finally, noting that $e^{-\rho(t-t')} \simeq 1 - \rho(t - t')$ and $\int_{t'}^t e^{-\rho(s-t')} ds \simeq (t - t')$ we can simplify the above expression as:

$$R(V_{t'};V^*) \simeq \frac{(\rho-\alpha)}{\rho} r_{t'} \equiv \frac{(\rho-\alpha)}{\rho} (V_{t'} - V_{t'}^r) \tag{49}$$

From (49), any application of controls $r_{t'} < V_{t'} - V_{t'}^r$ leads to a reduction of (46) for all $t \geq t'$ and, then, by Proposition 2, to $F^r(V_t;V^*) > 0$ which triggers revocation by the regulator.

Furthermore, the firm does not adjust by more than r_t since doing so would not increase the probability of delaying revocation. It does not pay less, since $r_t < V_t - V_t^r$ induces closure making it worse off, i.e. $0 < V_t$.

Finally, as $V_t^r \equiv V_t - r_r$ is a Markov process in levels, it is clear from (46) that any sub-game that begins at a point at which revocation has not taken place is equivalent to the whole game. The strategy ϕ is efficient for any sub-game starting at an intermediate date and state (t, V_t). This concludes the first part of the Proposition.

7.4.2 Non-decreasing path of r_t within $[T^*, T'^*)$.

So far we have implicitly assumed that, once started at T^*, the profit sharing goes on forever. Earlier interruptions are not feasible as long as the threat of revocation is credible. Credibility relies on the fact that the agency's option to revoke if the firm deviates from r_t is always worth exercising at $V_t > V^*$. However, in a Brownian path there is a positive probability of the primitive process V_t crossing V^* again starting at an interior point of the range (V^*, ∞). In this case, the firm may be willing to stop cutting profits. That is, the firm "regulates" profits until $V_t \geq V^*$according to r_t, but when V_t reaches, for the first time after T^*, a predetermined level, say $V' \leq V^*$, it ceases regulation. The regulator will then face a jump from zero to $F(V') \leq F(V^*)$ making the threat of revocation no longer credible.

To see how this happens let's assume that the firm stops adjusting profits at time T' with $T^* < T' < \infty$, and $T' = \inf(t \geq T^* \mid V_t \geq V'$ and $V' \leq V^*)$, i.e. T' is the first time that the primitive process V_t reaches $V' \leq V^*$ with profit regulation under way. Then the value of the revocation option starting at any $t \in [T^*, \infty)$ with $t < T'$ can be expressed as:

$$\tilde{F}(V_t, V_t^r; V') = P(V'; V_t) E_t^r [F(V_{T'}) e^{-r(t-T')}] + \tag{50}$$

$$(1 - P(V'; V_t))E^r_t[F^r(V_{T'})e^{-r(t-T)}]$$

where $P(V'; V_t)$ is the probability of the primitive process V_t reaching $V' \leq V^*$ starting at an interior point of the range (V^*, ∞), which is equal to (Cox and Miller, 1965, p. 232-234):

$$\Pr(T' < \infty \mid V_t) \equiv P(V'; V_t) = \left(\frac{V_t}{V'}\right)^{-2\mu/\sigma^2}$$

with $\mu = (\alpha - \frac{1}{2}\sigma^2)$[41]. As the starting point is now any $t \in [T^*, \infty)$, we can immediately see in (50) the dependence on both V^r_t and V_t.

Since the option value under profit regulation is zero, if V' is never reached we get $F^r(V_{T'}) = 0$. On the contrary, if V' is reached and the contract is revoked, it is simply $F(V_{T'}) = F(V')$, and:

$$\tilde{F}(V_t; V') = P(V'; V_t)E_t[F(V')e^{-r(T'-t)}]$$

According to the Strong Markov Property of V_t equation (50) becomes:

$$\tilde{F}(V_t; V') = P(V'; V_t)F(V')\left(\frac{V_t}{V'}\right)^{\gamma} \tag{51}$$

where $\gamma < 0$ is the negative root of (34). Since at t the primitive process V_t is greater than V' and $P(V'; V_t)\left(\frac{V_t}{V'}\right)^{\gamma} = \left(\frac{V_t}{V'}\right)^{\gamma-2\mu/\sigma^2} \leq 1$, we obtain $\tilde{F}(V_t; V') \leq F(V')$ for all $t \in [T^*, T')$, which implies that the following inequality holds:

$$\tilde{F}(V_t; V') = F(V^*)\left(\frac{V'}{V^*}\right)^{\beta}\left(\frac{V_t}{V'}\right)^{\gamma-2\mu/\sigma^2} \leq F(V^*) \tag{52}$$

Since $\tilde{F}(V_t; V') \leq F(V^*)$ for all $t \in [T^*, T')$, the regulator's optimal strategy is to revoke immediately at T^*. To prevent revocation the profit adjustment must continue until time $T'^* \equiv T'(V^*) = \inf(t \geq T^* \mid V_t - V^* = 0^-)$ when the trigger value $V' = V^*$ is reached for the first time after T^*. The game ends and can then be started afresh. This concludes the second part of the Proposition.

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