902. Dynamic analysis and design guidelines of mechanical oscillators for cutting soil through vibrating tools

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Abstract. The reduction of the soil cutting force through vibrating tools was the object of many studies during the second half of the last century. These studies initially focused on soil movement by bulldozers and then on soil tillage in agriculture. Over the past years, the field of tree-nursery mechanization has been employing this knowledge due to the use of equipment with oscillating tools for root-balling plants. Transportation and planting must be performed with the roots contained in a hemispherical ball of the original soil. This hemispherical root-ball is obtained by using a vibrating semicircular blade that cuts the soil underneath the plant. The blade oscillator is complex because the blade must oscillate and advance in the frame to cut the root-ball. For this reason, we correlated the oscillation and cutting movements with the oscillator features through a dynamic analysis using the Hong's formulae for Coulomb friction with a harmonic forcing torque. The resulting periodic motion has a substantial phase lag with respect to the forcing torque generated by the rotation of eccentric masses; instead, the amplitude predicted with the Coulomb friction is 15 % lower than the amplitude calculated without friction. Experiments were also conducted to verify the value of these amplitudes and to determine the correlations between the cutting torque of the blade in typical tree-nursery soil and the blade diameter. All the correlations proposed in this article, together with the performed literature survey, were useful for drafting new design guidelines for mechanical oscillators.

Keywords: mechanical oscillator, vibrating tools, soil cutting, root-balling machine.

Nomenclature

A_0 (mm)	Linear amplitude of oscillating blade (zero to peak)	M_{me} (Nm)	Torque of the eccentric masses (harmonic forcing torque)
B (m)	Blade width	M_R (Nm)	Friction torque
b_G (m)	Mass lever arm	M_t (Nm)	Cutting torque in soil
b_m (m)	Lever arm of the springs	n	Number of masses
D (m)	Root-ball diameter	r	Contact ratio
F_o (N)	Oscillating tool force	S_t (mm)	Total space available between the coils of the springs
F_{no} (N)	Non-oscillating tool force	t_t (s)	Root-ball cutting time
J (kgm^2)	Moment of inertia of oscillator+blade	v_a (m/s)	Tool feed velocity
k (Nm/rad)	Torsional spring rate	v_u (m/s)	Peak oscillating tool velocity
k_l (N/m)	Linear spring rate	v_u/v_a	Velocity ratio
m (kg)	Eccentric mass	y_G (m)	Eccentricity
Z	Number of springs	σ	Standard deviation
α_0 (rad)	Angular amplitude of oscillating blade	φ (rad)	Phase displacement
μ	External friction coefficient	ω (rad/s)	Angular velocity of the eccentric masses (angular frequency)
ρ (kg/m^3)	Soil density	ω_n (rad/s)	Natural angular frequency

Introduction

The first investigations on cutting soil through vibrating tools were made in the 1950s, initially aimed on soil movement by bulldozers and then on soil tillage in agriculture. In both cases the vibration reduces the cutting force on the soil.

Eggenmuller [1] discovered that the ratio (F_o/F_{no}) (the cutting force of an oscillating tool over the cutting force of a non-oscillating tool) is influenced by the amplitude and frequency of the oscillation and especially by the ratio of the oscillation peak velocity to the feed velocity (v_u/v_a) (tool peak velocity v_u during the oscillation over the feed velocity v_a). In particular, he demonstrated that the force ratio decreases with increasing velocity ratio, reaching a minimum of 0.4 for v_u/v_a equal to 6. Moreover, he showed that if the amplitude (zero to peak) of the blade motion is greater than 6 mm at all of the frequencies, then the decrease in the force ratio is small.

Other authors, [2] and [3], also reported that the force ratio considerably decreases for velocity ratios ranging from 1 to 3 and moderately for ratios ranging from 3 to 8, while the force ratio decrease is more modest with higher velocity ratios.

Narayanarao [4] investigated, both theoretically and experimentally, the effects of oscillation on a tool, such as a chisel, by verifying that an increase in the velocity ratio v_u/v_a up to 8 causes a reduction in the ratio of the required force (F_o/F_{no}) to reach the value of 0.4, confirming Eggenmuller's values.

Szabo [5] conducted experiments with an oscillating tool, pushing the velocity ratio beyond the limit of 8, concluding that a minimum force (F_o/F_{no} = 0.3) can be obtained by employing velocity ratios that are equal to or greater than 17.

The total energy and consequently the total power required by an oscillating machine tool, which is the sum of the power of the thrust and the power of the oscillation, turned out to be equal or greater to that of a non-oscillating tool [6] and [7]. This result depends on the values of the ratio (v_u/v_a).

Using an inclined blade, [7] and [8] compared different kinds of periodic motion. They tested traditional harmonic motion, square-wave motion and saw-tooth motion; they did not find any difference regarding the required force.

To obtain the maximum efficacy in reducing the traction force, [9] and [10] have shown that the oscillations must occur lengthwise along the direction of motion. Lateral or vertical vibrations are not very useful.

This knowledge has been employed in the field of tree nursery, where the transportation and planting must be performed with the roots contained in a hemispherical ball of the original soil. This hemispherical root-ball is obtained by using a vibrating semicircular blade that cuts the soil underneath the plant. The vibrating tool is connected to a mechanical oscillator installed in a "root-balling machine" (Fig. 1).

Fig. 1. The semicircular blade and frame of a root-balling machine

As the blade must oscillate and advance in the frame to cut the root-ball, the blade oscillator is relatively complex. For this reason, the correlations between the dynamic features of the oscillator and the oscillation and cutting movements were investigated through theoretical analyses [11]. In this last work, the motion equation of the oscillator-blade system was integrated under the following hypothesis for the friction torque: the Coulomb formulation (square wave) was replaced by a viscous law (sinusoidal wave) in phase with the derivative of the blade angular position $d\alpha/dt$, hence a velocity, energetically equivalent as far as concerns the friction work.

The present work wants to overcome the approximation introduced in [11] and therefore aims at:

1) performing a better dynamic analysis by using the results of Hong´s mathematical model [12] to describe oscillating systems which keep in account the Coulomb friction;

2) making an experimental assessment of the presented model and, afterwards, giving new design guidelines concerning these mechanical oscillators.

Materials and methods

The oscillating system

The oscillator (Fig. 2) is composed of a train of five gear wheels: the power is supplied to the central wheel, while the two outer gear wheels transmit the motion to two eccentric masses[1]. When these eccentric masses are placed opposite each other (Fig. 2), the respective centrifugal forces are balanced, but when they are rotated by $\pm\pi/2$ from the initial position, they cause the gear housing to oscillate. This causes a forced oscillation of the gear housing that is transmitted, by the shaft, to the horizontal butterfly bush that is connected to the semicircular blade.

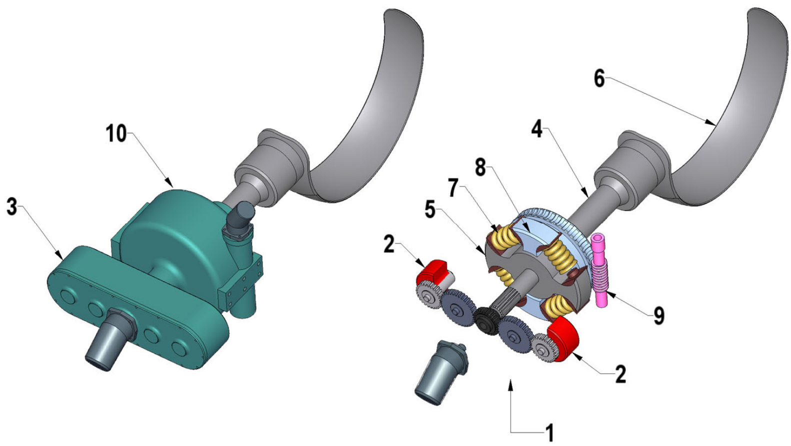

Fig. 2. Three-dimensional drawing of the oscillator-blade unit: 1) gear; 2) eccentric masses; 3) gear housing; 4) shaft; 5) horizontal butterfly bush; 6) semicircular blade; 7) springs; 8) vertical butterfly bush; 9) worm screw; 10) worm gear housing

This unit, composed of the gear housing, horizontal butterfly bush and blade, which is forced to oscillate, is then elastically connected, by four springs, as shown in Figure 2, to the vertical butterfly bush. This bush is not fixed but can rotate because it is connected to a worm gear. The

[1] The total number of masses can be greater than two, but it must be even (four, six, etc.).

worm screw is fed by a hydraulic motor, driven by an operator. In other words, the action of the worm screw rotates the worm wheel rigidly connected with the vertical butterfly bush. Thus, this bush rotates bringing with it, through the spring, the "gear housing-horizontal butterfly-blade" unit, to push the blade in the soil. As the gear housing oscillates and rotates, it is enclosed in a protective covering.

The blade oscillating in soil

When the blade vibrates in the soil without cutting, hence without the hydraulic motor pushes on the blade through the worm screw, the worm wheel, the vertical butterfly bush and the springs (Fig. 2), the motion equation can be obtained from the equilibrium between the moment of inertia $J\dfrac{d^2\alpha}{dt^2}$, the torque of the elastic forces $k\alpha$, the torque of the eccentric masses $M_{me}\sin(\omega t)$ and the friction torque of the blade in the soil M_{Ra}.

Now, the differential equation of motion for the system with the blade only vibrating in the soil, can be written:

$$J\frac{d^2\alpha}{dt^2} + k\alpha + M_{Ra} = M_{me}\sin(\omega t) \tag{1}$$

where J is the moment of inertia of the gear housing and bush blade system (kg m^2); α is the angular position (rad), with zero as the static equilibrium of the system; k is the torsional spring rate (Nm/rad), which is correlated to the linear spring rate k_l (N/m) of the helicoidal springs (Fig. 2) by $k = z \cdot k_l \cdot b_m^2$; z is the number of springs (four in this case); b_m is the lever arm of the springs (m); M_{me} is the torque produced by the eccentric masses (Nm) and is $M_{me} = n \cdot m \cdot \omega^2 \cdot y_G \cdot b_G$; ω is the angular velocity of the eccentric masses and hence the angular frequency of the torque produced by the eccentric masses (rad/s); n is the number of eccentric masses (two in this case); m is the mass of an eccentric weight (kg); y_G is the mass eccentricity (m); b_G is the lever arm of the mass, which is the distance between the rotation axis and the blade oscillation axis (m); t is the time (s); and M_{Ra} is the friction torque.

According to Coulomb's law, M_{Ra} is constant with regards to the velocity $d\alpha/dt$, but changes its sign with the velocity (Fig. 3). Therefore, M_{Ra} is a square wave periodic function, and it is dephased by the angle ψ from the sinusoidal function of the forcing torque M_{me} because the velocity $\dfrac{d\alpha}{dt}$ is dephased by the same angle ψ.

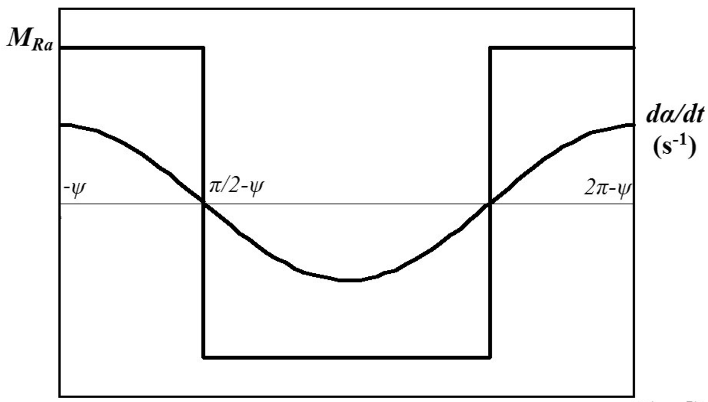

Fig. 3. Friction torque M_R and velocity $d\alpha/dt$ vs. angle ωt. M_R is in phase with the velocity $d\alpha/dt$

Thus, M_{Ra} is:

$$M_{Ra} = \begin{cases} M_R & \text{if } \dfrac{d\alpha}{dt} > 0 \\ -M_R & \text{if } \dfrac{d\alpha}{dt} < 0 \end{cases} \tag{2}$$

It is assumed that J, k and M_R are positive constants.

Using Coulomb's law, the amplitude of the friction torque M_R is: $M_R = \text{Friction force} \cdot \text{Lever} = 2\mu \cdot G \cdot \dfrac{D}{\pi}$, where D is the root-ball diameter (m) that, divided by π, represents the lever arm of the friction force arising from the friction interactions of the semicircular blade; μ is the external friction coefficient; and G is the weight on the blade (N), which passes from a null value when the blade meets the soil at the beginning and at the end of the cutting movement to a maximum value when the blade stands vertically at the maximum depth.

The factor of 2 appears because there is a weight G on the upper side of the blade, along with an equal and opposite reaction to G that acts on the lower side of the blade.

The contribution of the epigean part of the plant is known [11], so it is easy to describe the weight G as twice that of the soil that is directly on the blade: $G \cong 2\rho \dfrac{1}{4}\dfrac{\pi}{6} D^3 g = \rho \dfrac{\pi}{12} D^3 g$, where ρ is the density of the soil (kg/m^3) and g is the acceleration due to gravity (m/s^2). Thus:

$$M_R \cong \mu \cdot \rho \frac{D^4}{6} g \tag{3}$$

Following the modelling approach of Hong [12], properly modified to the present case of a blade vibrating in the soil, the angular amplitude α_0 is therefore:

$$\alpha_0 = \sqrt{\alpha_{0wf}^2 - \left(\frac{B \sin \pi_1}{\Omega \left(1 - \cos \pi_1\right)} \right)^2} \tag{4}$$

where:

$$\alpha_{0wf} = \frac{M_{me}}{k - J\omega^2} \tag{5}$$

$$\omega_n = \sqrt{\frac{k}{J}} \tag{6}$$

$$\beta = \frac{M_{me}}{M_R} \tag{7}$$

$$\Omega = \frac{\omega}{\omega_n} \tag{8}$$

$$B = \frac{\left(1-\Omega^2\right)\alpha_{0wf}}{\beta} \tag{9}$$

in which:

- α_{0wf} (rad) is the angular amplitude of oscillating blade without friction; using $k \approx 20000$ Nm/rad, $J \approx 1$ kgm^2 and $\omega \approx 400$ rad/s (because the eccentric masses rotate at approximately 4000 rpm), we obtain negative values of α_{0wf} (eq. 5), meaning that α_{0wf} has a phase opposite to the forcing torque M_{me};
- ω_n (rad/s) is the natural angular frequency;
- β is the torque amplitude ratio;
- Ω is the angular frequency ratio;
- $\pi_1 = \frac{\pi}{\Omega}$;
- B (rad) may be viewed [12] as a virtual friction angular displacement at which the blade just sustains enough spring torque to overcome the friction bound on to start the sliding.

From the Hong's model [12] we can obtain the phase lag φ between the wave of angular position α and the sinusoidal wave of the forcing torque M_{me} when the blade is in soil and, hence, in presence of Coulomb friction:

$$\varphi = \arccos\frac{\alpha_0}{\alpha_{0wf}} \tag{10}$$

This angle ranges between $\frac{\pi}{2}$ and π because, based on typical values for the quantities of ω, k and J, previously indicated, $\omega > \sqrt{\frac{k}{J}} = \omega_n$ and hence α_{0wf} is negative.

The blade vibrating and cutting the soil

During the soil cutting, the vibrating blade is moved by the cutting torque M_t (generated by an hydraulic motor acting on the blade by the worm screw, the worm wheel, the vertical butterfly bush and the springs; Fig. 2). Therefore, the springs present a static angular deformation, equal to $\frac{M_t}{k}$, in addition to the angular displacement α due to oscillation. Therefore, the springs maximum deformation is the sum of $\frac{M_t}{k}$ and the angular amplitude α_0.

The cutting torque M_t was found experimentally by measuring the torque reaction of the support of the root-balling machine. These supports are the tracks; they were placed on four weight platforms linked to a data logger (accuracy class: 1).

The feed velocity of the blade, in its semicircular motion, was constant because the hydraulic pump and motor had constant volumetric capacity and operated at constant speed.

In the case that the blade meets an obstacle, such as a large stone or root, the cutting torque increases to such a value that the pressure of the oil reaches the relief pressure (17 MPa). The cutting torque M_t then stabilizes itself at a maximum value, and the blade, by its vibration, slowly breaks the rock or cuts the root and then continues its run at the constant speed determined by the gear ratio of the worm gear and the hydraulic motor speed.

The experiments were conducted on soil without asperities, such as stones or roots; for this reason, a constant velocity of the blade was verified.

The cutting tests were performed using a root-balling machine with an oscillator having the features given in Table 1 and a blade 0.9 m in diameter. The cut, repeated five times, was made on a typical tree-nursery soil, that is a medium-textured soil with 20.2 % moisture, an external friction coefficient μ with the steel blade of 0.52 and a soil density of 1610 kg/m^3.

Table 1. Geometric and dynamic features of the oscillator

Linear spring rate	k_l (N/m)	572000
Number of springs	Z	4
Lever arm of the springs	b_m (m)	0.087
Torsional spring rate	k (Nm/rad)	17318
Moment of inertia of oscillator+blade	J (kg m^2)	1.4
Number of masses	N	4
Mass	m (kg)	1.27
Eccentricity	y_G (m)	0.0212
Mass lever arm	b_G (m)	0.163
Torque of the eccentric masses	M_{me} (Nm)	2775
Angular velocity of the eccentric masses (angular frequency)	ω (rad/s)	398
Root-ball diameter	D (m)	0.9
Blade width	B (m)	0.25

In addition, for each of the five repeated cutting tests, M_t was also measured when the blade was moving rearward in previously-cut soil. The torque M_t was then measured when the blade was moving forward in previously-cut soil.

Finally, cutting tests similar to the tests done with the 0.9 m diameter blade were performed also with blades of 0.6 m, 1.1 m and 1.2 m in diameter, to determine the influence of the root-ball diameter D on the cutting torque M_t.

Results and discussion

Table 2 shows the results of the cutting tests with the 0.9 m diameter blade. The Table 2 includes the cutting time of the root-ball t_t and the cutting torque M_t when (a) the blade was moving forward in un-cut soil, (b) the blade was moving rearward in previously-cut soil and (c) the blade was moving forward in previously-cut soil.

In Table 2, the values of the external friction coefficient μ are listed as well as the calculated friction torque using the equation (3).

Table 2. Results of the cutting tests with the root-balling machine with the 0.9 m diameter blade

Experimental case		Un-cut soil	Previously-cut soil (rearward)	Previously-cut soil (forward)
Cutting torque M_t (Nm)	Value	3297	2359	2251
	St. dev.	192	164	161
Cutting time t_t (s)	Value	11.3		
	St. dev.	0.5		
External friction coefficient μ		0.52		
Calculated friction torque M_R (Nm), eq. (3)		898		

In the comparison between the blade moving rearward and forward in previously-cut soil, the torque M_t shows a difference that is not statistically significant. However, the obtained values are higher than the friction torque M_R obtainable from (3), which could be explained because, when the blade moves in the already cut soil, it has to overcome friction but also has to strain the soil. It should be recalled that, after the first cut, the root-ball rests entirely on the hole.

Table 3 refers the 0.9 m diameter blade and compares the angular amplitude without friction α_{0wf}, calculated using equation (5), with the same quantity measured in the air, supposing negligible the viscous friction due to air. This evaluation was performed thanks to stroboscope. The experimental value was lower by about 4 %, probably due to the viscous friction of the lubricant within the worm gear housing (Fig. 2). Table 3 also shows the measured angular amplitude in the soil and those calculated using equation (4), with the first lower by about an acceptable 6 %, partly explained by the viscous friction of the lubricant and partly by the slight underestimation of the phenomena of friction between the blade and the soil. The same table also show the relative phase lag φ calculated using equation (10) through the calculated angular amplitudes ratio and those experimental. The difference between them is negligible (0.1 %). The phase lag φ, with a value of about 2.6 rad, confirms the discussion presented after equation (10).

Table 3. Calculated and measured amplitudes and phase lags for the 0.9 m diameter blade in the air and in soil

	Quantity		Calculated	Experimental
No friction (in air)	Angular amplitude	α_{0wf} (rad)	-0.0135 eq. (5)	-0.013*
	Phase lag	φ (rad)	3.14	
With friction (in soil)	Angular amplitude	α_0 (rad)	0.0117 eq. (4)	0.011*
	Phase lag	φ (rad)	2.583 eq. (10)	2.580 eq. (10)
	Linear oscillation amplitude of blade	$A_0 = \alpha_0 D \cdot 10^3/2$ (mm)	5.3	5
	Velocity ratio	$v_u/v_a = \alpha_0 \omega t_t/\pi$	16.6	15.8
	Maximum compression of the springs	$(M_t/k + \alpha_0)b_m 10^3$ (mm)	17.6	17.5

* using stroboscope

Table 3 also reports the linear amplitude of the oscillation of the blade, $A_0 = \alpha_0 D/2$ and the velocity ratio of the peak velocity of oscillation over the feed velocity of the blade, which is calculated by $v_u/v_a = \alpha_0 \omega \cdot t_t / \pi$.

The phase lag φ can be useful in determining the power P_o required by the oscillator. This value P_o can be obtained by using the expression: $P_o = (M_{me})_{eff} \cdot (d\alpha/dt)_{eff} \cdot \cos\psi$, where the effective values of the quantities, equivalent to the amplitudes multiplied by $\sqrt{2}/2$, are used; $d\alpha/dt = \alpha_0\omega$ and the phase displacement ψ is the phase lag between the forcing torque M_{me} and the velocity $d\alpha/dt$. Because this velocity is ahead by $\pi/2$ in comparison to the oscillation α, the oscillation lags behind by φ in comparison to M_{me}, resulting in $\psi = \varphi - \pi/2$.

This power required by the oscillator, $P_o = 0.5 \cdot M_{me} \cdot \alpha_0 \cdot \omega \cdot \cos\psi$, is supplied to the centre gear wheel (Fig. 2). It must be added to the cutting power supplied to the worm screw to obtain the total power.

The P_o term disappears at ω=0, i.e., when the blade does not vibrate. It is also not present when the blade vibrates in the air where the friction can be neglected; in this case the equation (5) is valid where the phase lag is $\varphi = \pi$ (in fact, $\psi = \varphi - \pi/2 = \pi/2$, which gives $\cos\psi = 0$).

On the contrary, when the blade vibrates in soil $\psi = \varphi - \pi/2 = 2.58 - \pi/2 = 1.01$ rad and P_o is the power required by the damped system (i.e. the blade in the soil), to overcome the Coulomb friction in the oscillating motion. The value of this term increases with the vibration frequency and thus with the velocity ratio: $\frac{v_u}{v_a} = \frac{\alpha_0 \cdot \omega \cdot t_t}{\pi}$.

If the velocity ratio increases, then the contact ratio $r = \frac{t_2 - t_1}{T}$, defined as the time for the oscillating cycle that the blade is in contact with the soil divided by the period T of oscillation, decreases. [8] showed theoretically that the extreme condition of $r = 0$ (then $v_u/v_a = \infty$) implies therefore that the total power $P_{tot} = \infty$. This total power is the sum of the power required by oscillator and by the cutting blade, where the latter is always a finite number. The results obtained by [8] are here confirmed. In fact, the power required by the oscillator is $P_o = 0.5 \cdot M_{me} \cdot \alpha_0 \cdot \omega \cdot \cos\psi = \infty$ because, with $v_u/v_a = \infty$, even $\omega = \infty$.

At the end of Table 3 appears the maximum compression of the springs, $\left(M_t / k + \alpha_0\right) b_m$, that is a very important quantity and must be compared to the total space S_t available between the coils. If the coils come into contact, they will transmit the vibration to the drive of the blade rotation and to the whole chassis of the root-balling machine. This situation must be avoided for ergonomic and machine-durability reasons.

The calculated and experimental values of springs maximum compression in Table 3 differ by a very small quantity (0.6 %), hence, negligible.

As a convergence of the results obtained in this work with the literature data (an oscillation amplitude of the blade of about 6 mm to provide a substantial reduction in the cutting torque [1] accompanied by a velocity ratio $v_u/v_a \geq 17$ [5]), some guidelines for designing the oscillators can be outlined as follows.

First, a maximum value of the angular frequency of the torque of the eccentric masses ω must be achieved, which requires the highest possible rotation speed that is compatible with the centrifugal forces and the structural strength of the rotating members (typically $\omega \cong 400$ rad/s).

Furthermore, it was considered [11] that the total space S_t between the coils must be larger than the maximum compression of the springs, that is:

$$S_t \geq \left(M_t/k + \alpha_0\right) b_m \tag{11}$$

Now, combining the equation (11), the oscillation amplitude of the blade $A_0 = \alpha_0 D/2$ that must be about equal to $6 \cdot 10^{-3}$ m, the linear spring rate k_l and the total space S_t between commercial spring coils (that are predetermined data) and the expression $k = z \cdot k_l \cdot b_m^2$ as derived after equation (1), the lever arm of the springs b_m was calculated [11] as:

$$b_m \geq \frac{D \cdot S_t}{4A_0} - \sqrt{\frac{D^2 S_t^2}{16A_0^2} - \frac{M_t D}{2A_0 z \cdot k_l}} \tag{12}$$

In this equation (12), the cutting torque M_t (Nm), which depends on the properties of the soil and the root-ball diameter and hence the blade diameter D (m), must be found. With the soil properties already discussed, the results of the influence of the diameter D are presented in Figure 4, which shows the cubic nature of the torque, well represented by the following equation ($R^2 = 0.99$):

$$M_t = 6385D^3 - 4272D^2 + 2478D \tag{13}$$

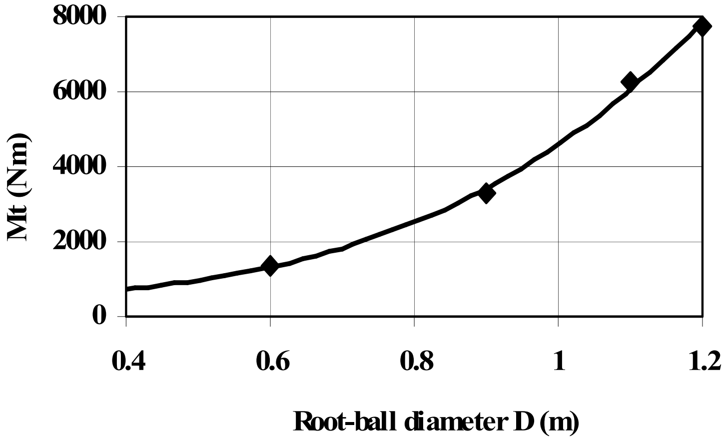

Fig. 4. Cutting torque M_t vs. root-ball diameter D

The moment of inertia J of the entire oscillating structure can be easily calculated; therefore, the torque of the eccentric masses M_{me} can be determined by appropriately choosing the number of masses n, their mass m, the eccentricity y_G and the lever arm of the masses b_G, satisfying the following equation, obtained from equations (4) and (5):

$$M_{me} = n \cdot m \cdot \omega^2 y_G \cdot b_G = \left| z \cdot k_l b_m^2 - J\omega^2 \right| \cdot \sqrt{\frac{4A_0^2}{D^2} + \left(\frac{B \sin \pi_1}{\Omega(1 - \cos \pi_1)} \right)^2} \tag{14}$$

Finally, the velocity ratio v_u/v_a is set to be equal to at least 17. By knowing the velocity $v_u = \alpha_0 \omega D/2$, it is possible to find the feed velocity of the blade v_a (therefore the hydraulic motor speed) and the gear ratio of the worm gear.

However, the springs lever arm b_m in eq. (12) is a function of the root-ball diameter D because the cutting torque M_t is a function of D according to eq. (13).

Moreover, the oscillators parameters defining the eccentric masses momentum M_{me} according to equation (14) are also function of the blade diameter D because square-rooted and because D has a great influence on the moment of inertia J, approximately according to a cubic power law. Therefore, from (12) and (14) we obtain that each different blade diameter D needs a different oscillator.

Several simulation of applying the calculation criterion represented by eq. (12) and (14) were executed to extend the criterion and couple a set of blades (typical diameter D: 0.6, 0.9, 1.2 m) with a single optimized oscillator.

We therefore concluded that the best choice is to optimize the oscillator for the average diameter D = 0.9 m (see Table 1), even if the oscillator could be further improved by raising the linear amplitude A_0 from 5 to about 6 mm. This can be achieved by increasing the linear spring rate k_l (commercial data) from 572 000 to 1 080 000 N/m and keeping constant the eccentric masses momentum M_{me}.

If such an oscillator is coupled with a blade having a lower diameter (D = 0.6 m), eq. (12) is surely valid and eq. (14) gives a linear amplitude A_0 that is 30 % greater than the suggested lower limit of 6 mm.

On the contrary, with a bigger blade (D = 1.2 m) the important condition expressed by eq. (12) is not yet satisfied: vibrations will be dangerously transmitted to the chassis and to the operator. Therefore, we suggest the adoption of a second set of four linear springs with a spring rate k_l = 1 080 000 N/m and a leverage b_{m2} longer than that of the first set ($b_m = b_{m1}$ = 0.087 m). In this case the oscillator should operate compressing only the springs of the first set even for D = 0.9 m and 0.6 m. By doing so, eqs. (12) and (14) are still valid, but it is necessary to calculate the effective compression S'_1 of the first set of springs using the following equation, analogous to (11):

$$S'_1 = \left(\frac{M_{t(0.9)}}{k_1} + \alpha_{0(0.9)} \right) b_{m1} \tag{15}$$

This equations gives 10 mm if $k_1 = z \cdot k_l \cdot b_{m1}^2 = 4 \cdot 1080000 \cdot 0.087^2 = 32698$ Nm/rad, $b_{m1} = b_m$ and M_t are set at the values of Tables 1 and 2, α_0 = 0.0127 rad as resulting from eqs. (4)-(9).

Therefore, for a compression lower than S'_1, the only compressed springs belong to the first set. With 0.9 m and 0.6 m diameter blades, the intervention of the second set of springs is undesirable as this latter set is in parallel with the first one. The natural frequency ω_n of the system would rise causing an excessive increase of the angular amplitude α_0 according to eqs. (6), (5) and (4). In such a situation there is the serious risk to approach to the resonance condition for the system, especially for the smaller blade.

For compressions greater than the value of S'_1 resulting from (15), the second set of springs has to begin operating. Only in this way it will be possible to overcome the high cutting torques occurring with the bigger blade diameter (D = 1.2 m) without the coils of the first spring series come into contact.

Being the residual compression of the springs of the first set equal to $S''_1 = S_t - S'_1$ = 8 mm (e.g. for the spring with k_l = 1 080 000 Nm/rad, S_t has a commercial value of 18 mm), it is possible to solve the following equation (dynamic balance) for the only unknown quantity concerning the second set, i.e. their leverage b_{m2}. It is necessary to notice that, in this second set of four springs, only the two in compression are involved ($k_2 = 2 \cdot k_l \cdot b_{m2}^2$):

$$b_{m2} \geq \sqrt{\left(\frac{M_{t(1.2)} b_{m1} - k_1 S'_1}{S''_1 - \frac{2 A_0 b_{m1}}{D_{(1.2)}}} - k_1 \right) \frac{1}{2k_l}} \tag{16}$$

This equation (16), to be satisfied together with (12) and (14), completes the new calculation method for an oscillator suitable for operating with 3 different blade diameters. E.g., using the same numerical values used for the quantities appearing in (16), this latter would be valid if adopting, for the second series of springs, a leverage $b_{m2} \geq 0.1$ m.

Conclusions

We performed a dynamic analysis of a system formed by a mechanical oscillator and a soil-cutting blade, typically adopted in the modern root-balling machines used in the field of tree-nursery.

Differently from [11], Hong's model [12] for Coulomb friction under harmonic forcing torque was used, thus allowing a faster calculation of amplitudes and phase lag. The obtained results are however very close to that obtained in the approximate modelling in [11].

The amplitude predicted with the Coulomb friction is 15 % lower than the amplitude calculated without friction. We also found an important phase lag φ (2.6 rad) of the blade periodic motion with respect to the forcing torque produced by a rotation of eccentric masses of oscillator. The experimental survey of these amplitudes confirmed the goodness of the adopted approach and the mean relative error was about 5 %.

Another important experimental validation concerns the correlation between the blade cutting torque in a typical tree-nursery soil and the blade diameter, obtaining a 3rd degree polynomial law.

Starting from the knowledge of the cutting torque of the blade, it is possible to calculate the deformation of the springs. This quantity, added to the oscillation amplitude, allows predicting if the spring coils will come in contact, a situation to be absolutely avoided not to transmit the vibrations to the whole root-balling machine. This evaluation is essential to complete the knowledge and propose new design guidelines of mechanical oscillators for root-balling machines capable to operate with different blade diameters.

These guidelines can be summarized by the solution of the system of equations (12), (14) and (16), providing the optimal characteristics of the springs, eccentric masses and their speeds varying the diameter of the root-ball and the inertia of the oscillating system.

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