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# Fluid-Structure Interaction in the Localization of Saturated Porous Media

Dedicated to Professor FRANZ ZIEGLER in occasion of his 60th birthday

*In this paper, the length scales included both in rate-dependent single phase materials and in coupled problems such as saturated and partially saturated porous media where the viscous terms are introduced naturally by the fluid mass balance equations, are discussed. The viscous effects in the two problems are quite different and the internal length scales hence have different expressions. Also the wave speeds have different relationships with the wave number.*

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## 1. Introduction

Strain localization refers to the phenomenon by which deformations in solids localize into narrow bands of intense straining. For isothermal rate-independent solids, strain localization has been analysed as a material instability within a theoretical framework due to THOMAS [1], HILL [2], and RICE [3]. This (classical) continuum model does not involve an internal length scale parameter and the numerical analysis of localization problems show an excessive mesh dependence. This is due to the fact the that differential equations of motion cease to be hyperbolic in some part of the domain as soon as softening occurs. Various kinds of modifications and generalizations of standard continuum plasticity have been proposed in single phase materials to avoid these difficulties in localization simulation. As indicated in [4, 5], viscoplasticity (rate-dependent model) introduces a length-scale parameter into the dynamic initial value problem and provides a satisfactory framework for the analysis of strain localization in solids. Gradient models have also been proposed, introducing a Laplacian in the constitutive equations [e.g. 6]. A multiphase material model [7] as used here naturally contains a regularization due to the introduction of a gradient term through Darcy's law in the mass balance equation of the fluid [8]. Numerical simulations of dynamic strain localization in a fluid saturated geomaterial have evidenced that the maximum value of the equivalent plastic strain inside the shear band is mesh insensitive, while the shear band width remains weakly dependent on the mesh size [8, 9]. This behaviour has been explained in [10] in which a length scale for the one-dimensional problem in case of axial waves has been obtained. In ideal 1-D shear wave propagation multiphase material behaves like a single phase one in which no internal length scale exists [10].

In this paper standard linear stability analysis is employed to analyse the material instability in a one-dimensional rate-dependent single phase material and in a one-dimensional fully and partially saturated media. The Routh-Hurwitz criterion is used to determine whether the growth rate of the disturbance is positive or negative. The internal length scales obtained by the outlined method both for the visco-plastic single phase model and for the multiphase model are compared and the differences are pointed out. The second one in particular, is inversely proportional to the permeability of the medium which is linked to its microstructure.

## 2. Wave propagation, stability analysis, and internal length scale in rate-dependent single phase material

We investigate the wave propagation in a one-dimensional rate-dependent single phase strain softening bar. The dynamic governing equation for a rate-dependent model may be written as [5]

$$m\left(\frac{\varrho}{E}\,\frac{\partial^3 v}{\partial t^3}-\frac{\partial^3 v}{\partial x^2\,\partial t}\right)+(E+h)\,\frac{\varrho}{E}\,\frac{\partial^2 v}{\partial t^2}-h\,\frac{\partial^2 v}{\partial x^2}=0 \tag{1}$$

in which $m > 0$ is a rate-sensitivity parameter, $\varrho$ the density of the solid, $E$ and $h$ are the elastic and softening moduli, respectively. We assume $E + h > 0$. $v$ means the velocity of the material, $v = \dot{u}$.

To investigate the dispersive character of wave propagation in a rate-dependent, softening continuum, a general solution for a single harmonic wave with angular frequency $\omega$ and wave number $K$ is assumed to be of the following form:

$$v(x,\,t) = A\,\mathrm{e}^{iKx}\,\mathrm{e}^{\zeta t}\,, \qquad \zeta = -i\omega\,. \tag{2}$$

By substitution of (2) into (1), we obtain the dispersion relation

$$\zeta^3 + a\zeta^2 + b\zeta + c = 0 \quad \text{with} \quad a = (E+h)/m\,, \qquad b = EK^2/\varrho\,, \qquad c = hEK^2/\varrho m\,. \tag{3}$$

To study the stability of the problem, we use the Routh-Hurwitz criterion. A necessary and sufficient condition for stability is that all the roots of (3a) have negative real parts. This holds if and only if the coefficients of the characteristic polynomial (3a) satisfy

$$\Delta_1 = \frac{E+h}{m} > 0\,, \qquad \Delta_2 = \frac{E^2K^2}{\varrho m} > 0\,, \qquad \Delta_3 = h\,\frac{E^3K^4}{\varrho^2 m^2} > 0\,. \tag{4}$$

The determinant condition (4c) is violated when strain softening occurs; hence loss of stability may appear and a small perturbation can grow, for instance, into a shear band.

In this case there are two kinds of solutions of equation (3a). The first one has one real positive root and two conjugate complex roots, the second one has three real roots in which at least one is positive. In this last case the dynamic governing equation becomes elliptic and hence the problem is ill-posed. The conditions for one case or the other to occur can be expressed as

$$Q > 0\,, \quad \text{one real and two complex conjugate roots}\,, \tag{5a}$$

$$Q \le 0\,, \quad \text{three real roots}\,, \tag{5b}$$

where

$$Q = (p/3)^3 + (q/2)^2\,, \qquad p = -a^2/3 + b\,, \qquad q = 2(a/3)^3 - ab/3 + c\,. \tag{6}$$

Upon substitution of (4a, b, c) and (6) into (5), the equality form of (5) will be obtained as function of $y = mK$ (the developments of the coefficients $w$, $r$, and $s$ are omitted for brevity):

$$Q(y) = wy^4 + ry^2 + s\,. \tag{7}$$

Indicating with $y_0$ the positive root of equation $Q(y) = 0$ the criterion (5) changes into

$$Q > 0 \Leftrightarrow K > y_0/m\,, \quad \text{one real and two complex conjugate roots}\,, \tag{8a}$$

$$Q \le 0 \Leftrightarrow K \le y_0/m\,, \quad \text{three real roots}\,, \tag{8b}$$

and the two regions identified by equations (5) or (8) are shown in Figure 1a.

The length scale value generated by the rate-dependent model will depend on the complex roots in equation (3). Substituting the complex root into (2), we have

$$v(x,\,t) = A\,\mathrm{e}^{iKx}\,\mathrm{e}^{-(\zeta_r + a/3)\,t - i\zeta_i t}\,. \tag{9}$$

The following relation between wave (phase) speed $c_w$ and wave number $K$ holds:

$$c_w = |\zeta_i|/K\,. \tag{10}$$

Then by means of $t = x/c_w$, the damping term $\mathrm{e}^{-(\zeta_r + a/3)\,t}$ in (9) changes into $\mathrm{e}^{-K((\zeta_r + a/3)/|\zeta_i|)\,x} = \mathrm{e}^{-\alpha x}$, where $\alpha$ is the damping coefficient. An internal length $l$ can be obtained as [5]

$$l = \alpha^{-1}\,, \qquad \alpha = ((\zeta_r + a/3)/|\zeta_i|)\,K\,. \tag{11}$$

In Figure 2a, the internal length $l$ is plotted as a function of wave number $K$ according to equation (11). The limit of $\alpha$ with respect to $K \to \infty$ sets an internal length scale of the rate dependent model given as

$$l_{\text{limit}} = 2mc_e/E\,, \qquad \alpha_{\text{limit}} = E/2mc_e\,, \qquad c_e = \sqrt{E/\varrho}\,. \tag{12}$$

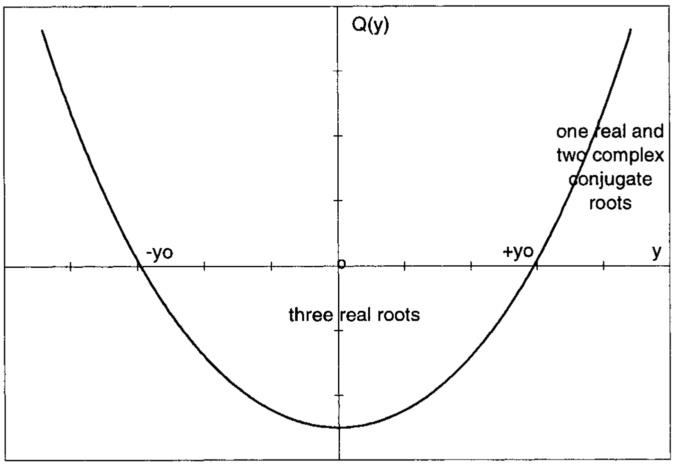


Fig. 1a. Distribution of the two regions identified by (8)

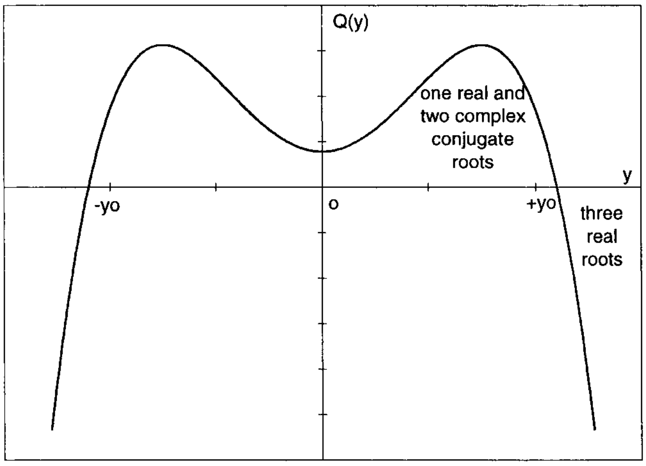


Fig. 1b. Distribution of the two regions identified by (15)

From Figure 2a appears that the damping effect will decrease with the increase of wave number and wave speed, and finally becomes a fixed value. This means that the high frequencies will be attenuated exponentially in the space domain to a limited extent determined by the length scale $l_{\text{limit}}$ of eq. (12).

## 3. Wave propagation, stability analysis, and internal length scale in multiphase materials

In this section, the problem of fully or partially saturated porous media is considered, following [10]. For these media, the following simplified equations hold, when assuming that the air phase remains at constant pressure in the partially saturated zone [7]:

$$\sigma'_{ij,j} - \alpha S_w p_{,i} - \varrho \ddot{u}_i = 0\,, \qquad Q^*(\alpha S_w \dot{u}_{i,i} + \dot{w}_{i,i}) + \dot{p} = 0\,, \tag{13}$$

where $\sigma'_{ij} = \sigma_{ij} + \alpha\delta_{ij} S_w p$ is the modified effective stress, $\sigma_{ij}$ the total stress, $u_i$ the displacement of solid skeleton, $S_w$ is the water saturation, $\dot{w} = -kp_{,i}$ the average water velocity relative to the solid phase, $p$ is the water pressure, $\alpha$ Biot's constant, $k$ the water permeability over the kinematic viscosity, function of $S_w$ [7] and $Q^*$ is indicated in [10]. By introducing in the above equations the total velocity of the fluid phase, $\dot{U}_i$, the $u - U$ form of the governing equations can be obtained [10] and the wave propagation problem can be analysed considering the set of equations for a one dimensional soil bar. The problem will be divided into two cases: compressive wave and ideal shear wave propagation, but only the first one will be reported here because it has been shown in [10] that the governing equation for ideal shear wave propagation can be obtained by setting $Q^* = 0$, and the behaviour coincides with that of a classical single phase material. The dispersion analysis for the one-dimensional soil bar considers the general solution for a single harmonic wave propagating through the soil bar with a displacement field of the form

$$\begin{Bmatrix} u \\ U \end{Bmatrix} = \begin{Bmatrix} A_u \\ A_U \end{Bmatrix} \mathrm{e}^{i(Kx - \omega t)} = A\,\mathrm{e}^{iKx + \zeta t}\,, \qquad \zeta = -i\omega\,. \tag{14}$$

We assume that the stress-strain relationship of the solid skeleton is established directly by the softening modulus $h$, i.e. $\sigma = h\varepsilon$ (no rate dependence), and that the condition $-\alpha^2 Sw^2 Q^* < h < 0$ always holds. When proceeding as for single phase materials, the dispersion relation has a similar form of eq. (3) and also eq. (7) is recovered, obviously with different coefficients and with $y = kK$ [10]. By indicating with $y_0$ the positive root of $Q(y) = 0$, criterion (5) then changes into

$$Q > 0 \Leftrightarrow K < y_0/k\,, \quad \text{one real and two complex conjugate roots}\,, \tag{15a}$$

$$Q \le 0 \Leftrightarrow K \ge y_0/k\,, \quad \text{three real roots}\,. \tag{15b}$$

Figure 1b shows the two regions identified by eq. (15). The differences with Figure 1a are clear. Further the wave speeds have different relationships (proportional and inversely proportional) to the wave number. Note that in Figure 1b the domains depend on the permeability of the medium.

For the rate-dependent single phase material the following non-dimensional parameters have been used: $E = 2.0E4$, $\varrho = 2.0E\text{-}8$, $h = -5.0E3$, $m = 0.2$, and $y_0 = 5.8998E\text{-}3$ is obtained. For the multiphase media has been used: $Q^* = 2000$, $h = -500$, $\varrho = 0.2$, $\alpha = 1.0$, $S_w = 1.0$. The value of $y_0$ is equal to 0.110 09.

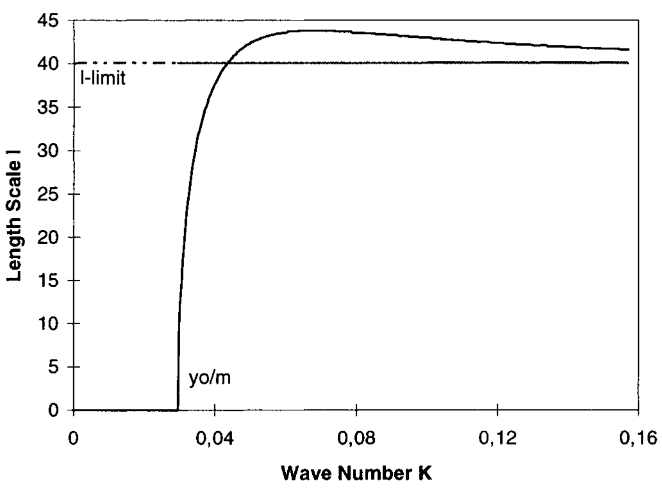


Fig. 2a. Length scale value as a function of wave number $K$ for a rate-dependent single phase material. For the selected material parameters the value of $l_{\text{limit}}$ is equal to 40

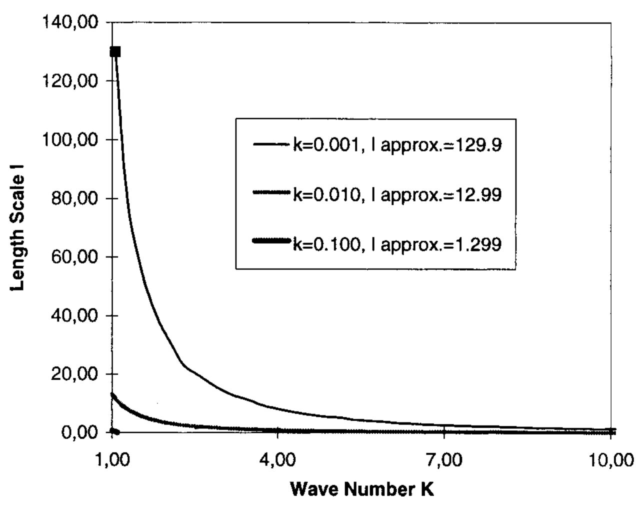


Fig. 2b. Internal length as a function of wave number for permeabilities $k = 0.001$, 0.01, 0.1. The value of $l_{\text{approx}}$ based on above material parameters is equal to $0.1299/k$

An internal length is again expressed as (11) but the relationship is rather complicated. Since the seepage permeability is generally small, we can obtain the approximate length scale with respect to small permeability [10]:

$$\alpha_{\text{approx}} = \frac{kK_0^2 \alpha^2 S_w^2 Q^*}{2c_m m\eta}\,, \qquad l_{\text{approx}} = \frac{6c_m \eta}{kK_0^2 \alpha^2 S_w^2 Q^*}\,. \tag{16}$$

In Figure 2b the internal length $l_{\text{approx}}$ is drawn as a function of wave number $K$ for the following permeability values: $k = 0.001$, 0.01, 0.1 ($K_0 = 1.0$ is adopted here).

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