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Design and Testing of a Double Acoustic Resonator

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A double acoustic resonator is a device which can cut down two resonance peaks of an acoustic cavity excited by sound sources in the 'low frequency' range. A straightforward design method is proposed; with this method a double resonator is designed in order to cut down two resonance peaks of an experimental cavity. Then the extensive series of tests which was performed to assess double resonator performance is described and the sensitivity of the system to resonator tuning errors is discussed.

Keywords: Acoustic vibrations; Cavity; Resonator

1. Introduction

Several acoustic cavities of machines are excited by sound sources in the 'low frequency' range, in which individual resonance peaks of cavity modes are distinguishable.

When a cavity mode is excited in resonance condition, noise emission strongly increases and machine operation is disturbed. The amplitude of cavity response can be reduced by coupling the cavity to a Helmholtz resonator tuned to the frequency of the mode excited in resonance condition [1], [2]. If sound sources, which are present in the machine, are able to excite in resonance condition two or more cavity modes at the same time, multiple acoustic resonators can be used. These devices are composed of a set of volumes connected by means of narrow ducts and have a finite number of natural frequencies which can be tuned to the natural frequencies of the lower order modes of the cavity. A double resonator was recently developed by the authors and several simulations showed that it was able to cut down two resonance peaks of a cavity [3].

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Since the design of a double resonator is not straightforward, in the first part of the paper the mathematical model of this device is presented and the parameters which affect resonator tuning and performance are pointed out; then a design method is proposed. The rest of the paper deals with the extensive series of tests which was performed to assess experimentally the effectiveness of the device; they were carried out both with the double resonator perfectly tuned to two modes of a rectangular cavity and in the presence of small tuning errors.

2. Design

The main problem in the design of a double resonator is the determination of resonator parameters which make its natural frequencies equal to those of the cavity. The parameters of a double resonator are represented in Fig. 1, which shows the resonator considered in this research. V_1 and V_2 are the volumes of the two chambers; A_1 and A_2, l_1 and l_2 are respectively the sections and the effective lengths of the ducts.

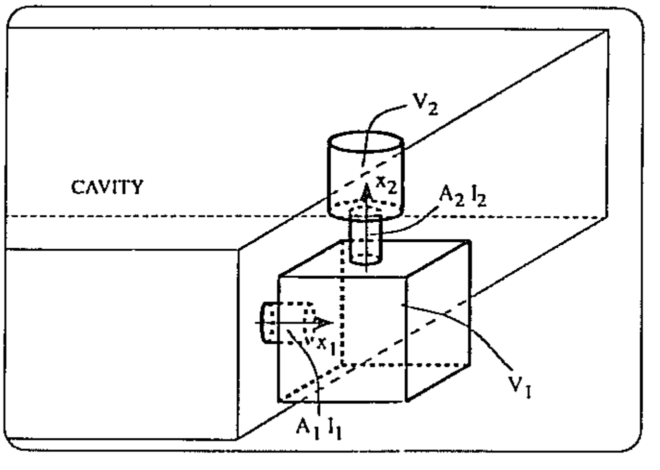

Fig. 1. Double resonator scheme.

In a frequency range where resonator parameters satisfy the following conditions:

$$V_1^{\frac{1}{3}} \ll \lambda;\; V_2^{\frac{1}{3}} \ll \lambda;\; A_1^{\frac{1}{2}} \ll \lambda;\; A_2^{\frac{1}{2}} \ll \lambda;\; l_1 \ll \lambda;\; l_2 \ll \lambda \quad (1)$$

(λ is sound wavelength), the resonator can be considered a lumped element device, whose generalized coordinates are the displacements x_1 and x_2 of the gas particles contained in the ducts. The equations of the undamped free vibrations of the resonator are calculated considering the equilibrium of the gas contained in the ducts under the action of the pressure caused by the compression of the gas inside the two chambers

$$\rho l_1 A_1 \ddot{x}_1 + \frac{\rho c^2 (A_1 x_1 - A_2 x_2)}{V_1} A_1 = 0 \quad (2)$$

$$\rho l_2 A_2 \ddot{x}_2 + \rho c^2 \frac{A_2 x_2}{V_2} A_2 - \frac{\rho c^2 (A_1 x_1 - A_2 x_2)}{V_1} A_2 = 0 \quad (3)$$

where ρ and c are the fluid density and the sound speed respectively.

In order to seek the natural frequencies of the device, solutions $x_1 = x_{1_0}\, e^{i2\pi f t}$, $x_2 = x_{2_0}\, e^{i2\pi f t}$ are introduced and the following frequency equation is derived

$$\frac{c^4 A_1 A_2}{V_1 V_2} - \left(\frac{c^2 A_1 l_2}{V_1} + \frac{c^2 A_2 l_1}{V_2} + \frac{c^2 A_2 l_1}{V_1}\right)(2\pi f)^2 + l_1 l_2 (2\pi f)^4 = 0 \quad (4)$$

The two roots of the frequency equation are the natural frequencies of the double resonator, which are named f_1 and f_2 and are given by:

$$f_1 = \frac{1}{2\pi}\sqrt{\tfrac{1}{2}(T_a - T_b)}\, f_2 = \frac{1}{2\pi}\sqrt{\tfrac{1}{2}(T_a + T_b)} \quad (5)$$

where

$$T_a = \frac{A_1 c^2}{V_1 l_1} + \frac{A_2 c^2}{V_1 l_2} + \frac{A_2 c^2}{V_2 l_2}$$

$$T_b = \sqrt{\left(\frac{A_1 c^2}{V_1 l_1} - \frac{A_2 c^2}{V_2 l_2}\right)^2 + \frac{2 A_1 A_2 c^4}{V_1^2 l_1 l_2} + \frac{A_2^2 c^4}{V_1^2 l_2^2} + \frac{2 A_2^2 c^4}{V_1 V_2 l_2^2}} \quad (6)$$

They apparently depend on the six geometric parameters of the device, but it is easy to show that they actually depend only on three parameters. If the first duct and the first chamber of the double resonator were alone they would make a Helmholtz resonator with natural frequency

$$f_1^* = \frac{1}{2\pi}\sqrt{\frac{c^2 A_1}{l_1 V_1}} \quad (7)$$

If the second duct and the second chamber of the double resonator were alone they would make a Helmholtz resonator with natural frequency

$$f_2^* = \frac{1}{2\pi}\sqrt{\frac{c^2 A_2}{l_2 V_2}} \quad (8)$$

Dividing both sides of Eq. (4) by $(2\pi)^4 l_1 l_2$ it becomes

$$f_1^{*2} f_2^{*2} - (f_1^{*2} + f_2^{*2} + f_2^{*2}\alpha) f^2 + f^4 = 0 \quad (9)$$

where α is the ratio V_2/V_1.

This equation shows that the natural frequencies of the double resonator, and therefore the tuning of the resonator to the cavity, depend only on the three parameters f_1^*, f_2^* and α.

In order to reduce further the number of design parameters only double resonators composed of two single resonators having the same frequency $f_1^* = f_2^* = f^*$ are considered. With this assumption the frequency equation becomes

$$f^{*4} - (2+\alpha) f^{*2} f^2 + f^4 = 0 \quad (10)$$

By setting $f^2 = t$ the frequency equation becomes a standard second order algebraic equation:

$$f^{*4} - (2+\alpha) f^{*2} t + t^2 = 0 \quad (11)$$

The relationships among the coefficients and the roots (t_1 and t_2) of this equation are:

$$(2+\alpha) f^{*2} = t_1 + t_2$$
$$f^{*4} = t_1 t_2 \quad (12)$$

Therefore, if the two natural frequencies of the resonator $f_1 = \sqrt{t_1}$ and $f_2 = \sqrt{t_2}$ have to be equal to two natural frequencies of a cavity, Eq. (12) allows the calculation of the values of f^* and α which make the natural frequencies of the resonator equal to the assigned values.

After f^* and α have been selected by means of this procedure, it is necessary to fix three other parameters to define completely resonator geometry. In order to make easier the comparison among different resonators it is useful to define these parameters in a non-dimensional way with the ratios

$$\beta = \frac{l_2}{l_1},\; \sigma = \frac{V_1}{V},\; \tau = \frac{l_1}{V^{\frac{1}{3}}} \quad (13)$$

where V is cavity volume.

3. Experimental Apparatus

The tests were performed in a rectangular cavity filled with air at room temperature. The natural frequencies and the acoustic mode shapes of a rectangular cavity are expressed by

$$f_i = \frac{1}{2} c \sqrt{\left[\left(\frac{i_x}{l_x}\right)^2 + \left(\frac{i_y}{l_y}\right)^2 + \left(\frac{i_z}{l_z}\right)^2\right]} \quad (14)$$

$$\psi_i(x,y,z) = \cos\left(\frac{\pi i_x x}{l_x}\right)\cos\left(\frac{\pi i_y y}{l_y}\right)\cos\left(\frac{\pi i_z z}{l_z}\right) \tag{15}$$

where l_x, l_y, l_z are the lengths of the cavity sides and i_x, i_y, i_z are the mode numbers. The lengths were chosen in order to make the natural frequencies of the first two modes equal to 200 Hz and 300 Hz respectively. Frequencies and mode numbers of the lower order modes are summarized in Table 1.

The cavity was excited by a loudspeaker located near a corner which is a pressure anti-node of both the first and the second cavity mode. The loudspeaker was driven by a wave-generator which could generate a constant amplitude harmonic signal or a constant amplitude signal with variable frequency (frequency sweep excitation).

Table 1.

Mode	Frequency [Hz]	i_x	i_y	i_z
1	200	1	0	0
2	300	0	1	0
3	360.5	1	1	0
4	400	2	0	0
5	500	2	1	0
6	600	0	2	0

The measurement equipment was composed of a sound level meter and a personal computer equipped with software for data acquisition and analysis. The microphone of the sound level meter was located in a corner of the cavity. The sound level meter gave directly sound pressure level inside the cavity and transmitted the microphone signal to the personal computer, which recorded the data. The personal computer could then display the time history of pressure inside the cavity and could perform the fast Fourier transform of the recorded data giving the response spectrum.

Figure 2 shows the response spectrum of the cavity alone; in order to speed up the test, the cavity was excited by a frequency sweep signal in the range 100–900 Hz and the fast Fourier transform of the recorded data was calculated. The frequencies of the peaks are in good agreement with the theoretical natural frequencies of the cavity (which are summarized in Table 1). The differences between theoretical and experimental frequencies (maximum difference about 5%) are caused by many phenomena: loudspeaker impedance may affect cavity behaviour; cavity walls are not perfectly stiff; sound spectrum is calculated exciting the cavity with a frequency sweep.

A double resonator tuned to the first two theoretical natural frequencies of the cavity (200 Hz and 300 Hz) was designed with the proposed method; the selected parameters of the resonator were: $f^* = 245$ Hz and $\alpha = 0.167$. The volume of the larger resonator chamber was set equal to a small fraction of cavity volume and

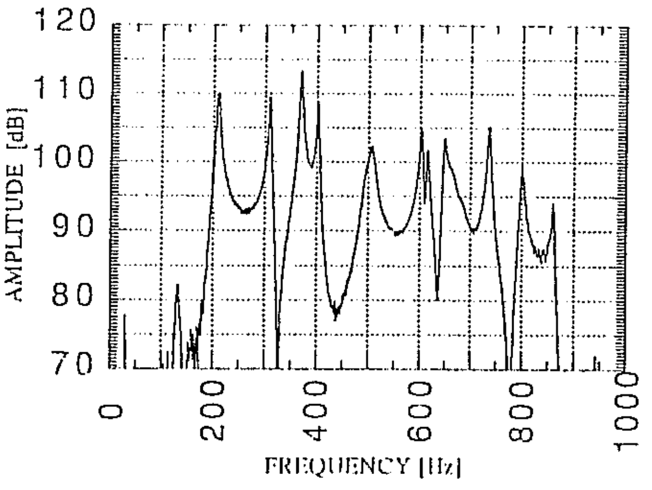

Fig. 2. Response spectrum of the cavity.

the duct effective lengths were chosen taking into account that the geometric lengths are smaller than the effective lengths owing to end-corrections [4]. The selected non-dimensional parameters of the resonator were: $\beta = 1.79$, $\sigma = 0.005$, $\tau = 0.09$.

One of the purposes of this work is the evaluation of the effect of tuning errors on the behaviour of the system comprising the resonator and the cavity, therefore one or more geometric parameters of the resonator had to be easily adjustable in order to introduce tuning errors in the system. Since it is difficult to build a resonator duct with adjustable section or length, the regulation of resonator tuning was accomplished by varying the volumes of the chambers.

4. Experimental Results with Optimum Tuning

Analytical results showed that a double resonator tuned to two acoustic modes of a cavity cancels the two original resonance peaks and produces two pairs of new resonance peaks [3]. In each pair of peaks the frequency of the first peak is lower than the frequency of the original cavity peak, whereas the frequency of the second peak is higher than the frequency of the original cavity peak; the new peaks, owing to resonator damping, are lower than the original peaks.

The first experiments showed that the double resonator with nominal parameters was able to cut down the two resonance peaks of the cavity, but the heights of the four new peaks, owing to tuning errors, were rather different. Since the purpose of the double resonator is the control of noise in a large frequency band, the optimum performance is obtained when the four new peaks are lower than the original peaks and their heights are similar.

In order to achieve the optimum performance the volumes of the two chambers were slightly modified

Table 2. Modal damping ratios.

Cavity alone: first mode	$\zeta_{1c} = 0.016$
Cavity alone: second mode	$\zeta_{2c} = 0.005$
Cavity + resonator: first mode	$\zeta_1 = 0.021$
Cavity + resonator: second mode	$\zeta_2 = 0.021$
Cavity + resonator: third mode	$\zeta_3 = 0.011$
Cavity + resonator: fourth mode	$\zeta_4 = 0.017$

and after some attempts a good behaviour was obtained. The optimum value of the larger volume (V_{1opt}) was 92.5% of its nominal value; the optimum value of the smaller volume (V_{2opt}) was 93% of its nominal value. With these values of volumes the theoretical natural frequencies of the resonator of 208 Hz and 312 Hz resulted; they were higher than the natural frequencies of the two cavity modes. Fig. 3 shows the frequency response of the cavity coupled to the double resonator and for comparison the frequency response of the cavity alone. The introduction of the double resonator cuts down the two original peaks of the cavity with a noise reduction of more than 12 dB. The four new peaks are lower than the original peaks by at least 7 dB. The difference between the highest new peak (the second) and the lowest (the fourth) is about 6 dB.

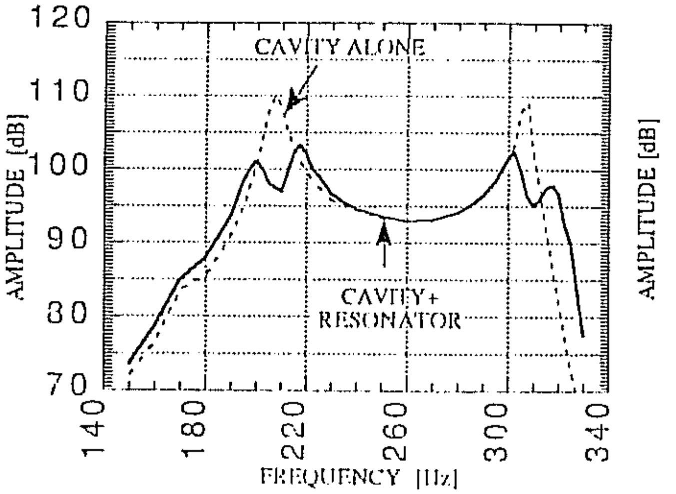

Fig. 3. Effect of a double resonator.

The frequency interval between the two peaks that substitute for the first cavity peak is ≈ 20 Hz, whereas the interval between the two peaks that substitute for the second cavity peak is ≈ 15 Hz. These intervals are in good agreement with those calculated analytically with the multi-mode model presented in [3], which resulted in 20 Hz and 17 Hz respectively.

The modal damping ratios of the two modes of the cavity alone and of the four modes of the coupled system were measured by calculating bandwidths at resonance. Results, which are summarized in Table 2, show that the new modes of the coupled system are more damped than the original modes of the cavity, the increment of damping ratio is relevant for the two modes that substitute for the second mode of the cavity.

The performance of the double resonator was then compared with that of a single Helmholtz resonator tuned to the first mode of the cavity. The duct of the single resonator was equal to the first duct of the double resonator, whereas the volume of the chamber was properly adjusted in order to achieve optimum tuning. The optimum volume resulting was 97% of its nominal value and the theoretical natural frequency was 203 Hz.

Figure 4 shows the frequency responses of the cavity coupled with the single resonator and of the cavity alone. The single resonator cuts down the first cavity peak with a noise reduction of about 15 dB. The two new peaks have the same height which is 10 dB lower than the height of the cavity peak. The damping ratios of the two new modes are $\xi_1 = 0.031$ and $\xi_2 = 0.025$. Therefore the single resonator does not affect the second resonance peak of the cavity, but produces a reduction of the first resonance peak of the cavity which is a bit larger than that produced by the double resonator.

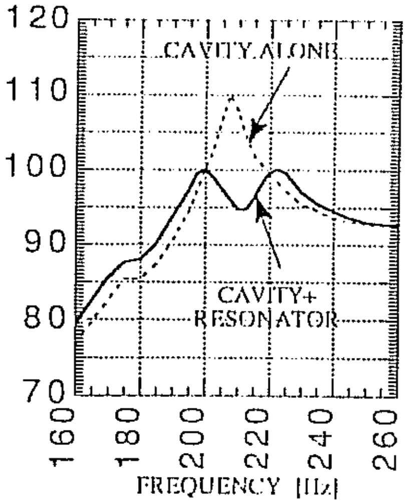

Fig. 4. Effect of a single resonator.

5. Experimental Results with Tuning Errors

The preceding section pointed out that an adjustment was necessary in order to achieve the optimum behaviour of the coupled system. In this section some experimental results are presented which show how much the behaviour of the coupled system differs from the optimum behaviour if tuning errors are present.

It is important to point out that small variations of resonator chamber volumes not only modify the natural frequency of the resonator (and therefore the tuning) but also the resonator/cavity volume ratio which is given by:

$$\frac{V_1 + V_2}{V} = \sigma(1 + \alpha) \qquad (16)$$

Some analytical results showed that a variation of the resonator/cavity volume ratio modifies the frequency interval between the two new resonance peaks that replace a resonance peak of the cavity [3]. In the case presented here the maximum variation of the resonator/cavity volume ratio is about 10% of the value in condition of optimum tuning, whereas the analytical results showed that larger variations are necessary to influence strongly the frequency intervals, therefore this effect is not very important.

In order to speed up the tests in the presence of tuning errors the cavity was excited by means of a constant amplitude signal with variable frequency (frequency sweep). The response of the cavity which is obtained with this method is a good approximation of the frequency response if the duration of the sweep is long enough.

Figure 5 shows the response spectra of the coupled system in the presence of tuning errors caused by modifications to the volume of the smaller chamber. It is important to highlight that even with large variations of the volume of the smaller chamber (≈ 25%), which causes large tuning errors (≈ 8%), the coupled system behaves slightly better than the cavity alone, because the new peaks are lower than the former peaks (which are 110 dB high), but in each pair of peaks one peak is much higher than the other. If the tuning errors become smaller the behaviour of the system improves and the heights of the peaks become comparable.

Figure 6 shows the response spectra of the coupled system in the presence of tuning errors caused by modifications to the volume of the larger chamber;

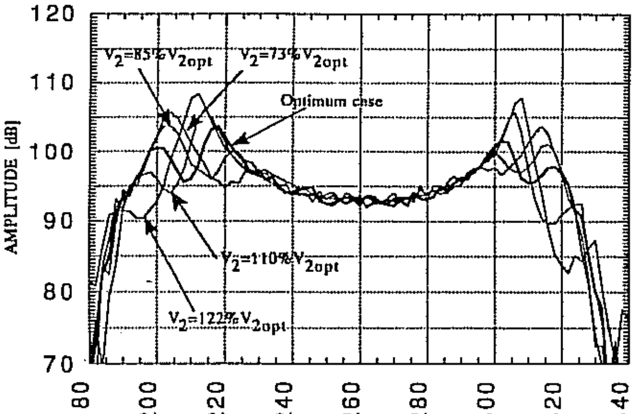

Fig. 5. Response spectra in the presence of tuning errors caused by modifications to the smaller volume.

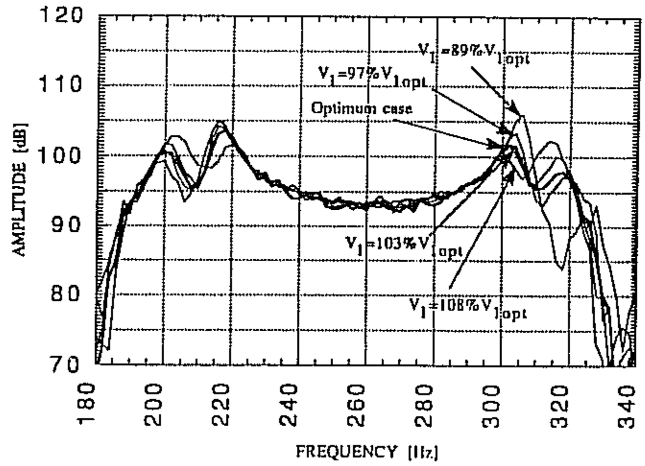

Fig. 6. Response spectra in the presence of tuning errors caused by modifications to the larger volume.

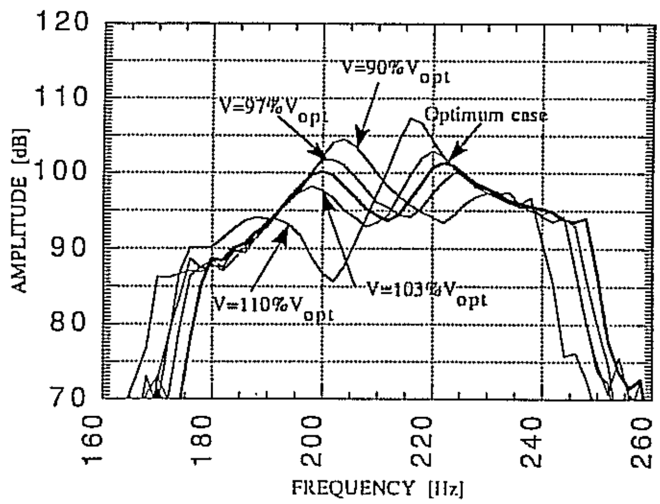

Fig. 7. Response spectra of a cavity coupled with a single resonator in the presence of tuning errors.

the results are similar to those obtained modifying the volume of the smaller chamber.

Tuning errors strongly influence the behaviour of a cavity coupled with a single resonator too. Fig. 7 shows the response spectra of the coupled system in the presence of tuning errors caused by modifications to resonator volume. With variations of resonator volume of about 10%, which causes tuning errors of about 5%, the difference between the two peaks is comparable with those measured when the double resonator was coupled to the cavity.

6. Conclusions

The design method presented in this paper is straightforward and allows the design of a double resonator tuned to a cavity with small tuning errors. With the subsequent adjustment it is possible to reach the optimum performance of the system comprising the double resonator and the cavity. The experimental tests showed that the double resonator strongly improves the acoustic behaviour of the cavity in proximity of two natural frequencies, the effect of the double resonator on the first resonance peak is comparable with that produced by a single Helmholtz resonator.

Small tuning errors do not endanger the behaviour of the system and the reduction of resonance peaks appears even if tuning errors of about 8% are present.

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Nomenclature

A_1	area of the first duct [m²]
A_2	area of the second duct [m²]
c	sound speed [m/s]
f_1	first natural frequency of the resonator [Hz]
f_2	second natural frequency of the resonator [Hz]
f_1^*	natural frequency of the resonator made by the first duct and the first volume [Hz]
f_2^*	natural frequency of the resonator made by the second duct and the second volume [Hz]
l_1	length of the first duct [m]
l_2	length of the second duct [m]
V_1	volume of the first chamber [m³]
V_2	volume of the second chamber [m³]
x_1	gas displacement in the first duct [m]
x_2	gas displacement in the second duct [m]
α	volume ratio of the two chambers
β	length ratio of the two ducts
ρ	gas density [Kg/m³]
λ	wavelength [m]
σ	volume ratio
τ	length ratio