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# Massive neutrinos and flavour violation

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**Abstract.** In spite of the large lepton flavour violation (LFV) observed in neutrino oscillations, within the Standard Model, we do *not* expect any visible LFV in the charged lepton sector ($\mu \to e, \gamma, \tau \to \mu, \gamma$, etc). On the contrary, the presence of new physics close to the electroweak scale can enhance the amplitudes of these processes. We discuss this in general and focus on a particularly interesting case: the marriage of low-energy supersymmetry (SUSY) and seesaw mechanism for neutrino masses (SUSY seesaw). Several ideas presented in this context are reviewed both in the bottom-up and top-down approaches. We show that there exist attractive models where the rate for LFV processes can attain values to be probed in pre-LHC experiments.


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## Contents



## 1. Introduction

Since the last couple of years of the previous century, tremendous progress has been made in our understanding of the nature of the most elusive Standard Model (SM) particles, the neutrinos. The main message of various experimental results has been that neutrinos have non-standard properties: they have masses and their flavour states mix and, indeed, with very large mixings. In the Standard Model, the phenomenon of flavour mixing is not surprising, as its presence has already been well established in the quark sector. Furthermore, the induced phenomenological implications such as $K^0$–$\bar{K}^0$ oscillations, $B_d$–$\bar{B}_d$ mixing and $b \to s, \gamma$ have been well understood as well as measured with high precision. Given this, and non-zero neutrino flavour mixing, one would expect that similar phenomenological and theoretical implications can now emanate from the leptonic sector.

The more obvious of the phenomenological implications of neutrino flavour mixing phenomena are processes like neutrino oscillations. On the other hand, the presence of this neutrino flavour mixing would also induce flavour mixing in its isodoublet partners, the charged leptons. This mixing, induced at the one-loop level through gauge bosons, is manifested by rare decay processes such as $\mu \to e, \gamma$, $\tau \to \mu, \gamma$, etc. If only the neutrinos carry this information on flavour mixing, as in the Standard Model with massive neutrinos, these processes are expected to be proportional to the ratio of masses of neutrinos over the masses of the W bosons, leading to extremely tiny branching ratios.

This situation can significantly change if there are some new additional particles carrying lepton flavour numbers, which have very large masses and simultaneously mix among themselves. The presence of such particles could lead to enhanced branching ratios for the above processes, perhaps bringing them into the realm of observability of the present and next generations of experiments. On the other hand, non-observability of these processes can lead to strong constraints on the nature of new physics, which is expected to be present just above the electroweak scale. Such constraints already exist from the hadronic sector, albeit riddled with typical uncertainties associated with them. The leptonic flavour changing processes, on the other hand, do not suffer from these uncertainties, and thus lead to stronger constraint on any new flavour violating physics. In fact, the present constraint on BR($\mu \to e, \gamma$) has long been considered to be the most stringent constraint on any new flavour physics. To get a feeling as to where we stand, we provide here a list of present and upcoming experimental limits:

*Present limits*

$$\begin{aligned}
&\mathrm{BR}(\mu \to e\gamma) \leqslant 1.2 \times 10^{-11}\ [1],\\
&\mathrm{BR}(\tau \to \mu\gamma) \leqslant 3.1 \times 10^{-7}\ [2],\\
&\mathrm{BR}(\tau \to e\gamma) \leqslant 3.7 \times 10^{-7}\ [3].
\end{aligned}$$

*Upcoming limits*

$$\begin{aligned}
&\mathrm{BR}(\mu \to e\gamma) \leqslant 10^{-13}\text{–}10^{-14}\ [4],\\
&\mathrm{BR}(\tau \to \mu\gamma) \leqslant 10^{-8}\ [3],\\
&\mathrm{BR}(\tau \to e\gamma) \leqslant 10^{-8}\ [3].
\end{aligned}$$

The impact of these limits could be felt in a wide class of new physics models setting in at a scale close to the electroweak scale. A particularly interesting class of models are the supersymmetric (SUSY) Standard Model(s). In these models, the supersymmetric partners of the leptons, namely the sleptons, carry the same flavour quantum numbers as the SM leptons. Since they are expected to be heavy, around the TeV scale, if flavour mixing is present in the (s)leptonic sector, large branching ratios are expected for the aforementioned rare decay processes. Interestingly enough, this naturally occurs if we marry the idea of low-energy supersymmetry to the mechanism of seesaw [5] giving rise to small neutrino masses (SUSY seesaw [6]). In the present review, we will mainly concentrate on this class of models.

## 2. Supersymmetric models and lepton flavour violation

The study of flavour violation in supersymmetric models is certainly quite complicated. Indeed, to the usual intricacies involved in the FCNC computations in the SM, we add several new SUSY contributions. The main source of this difficulty is the soft supersymmetry breaking Lagrangian, which can in general contain a large number of flavour violating couplings, leading to significant constraints on its parameters [7, 8]. An appealing way to cope with these tight constraints is to consider only a particular classes of soft Lagrangians, which result from models that break supersymmetry in a flavour blind manner, as in mSUGRA, anomaly-mediated supersymmetry breaking (AMSB) or its several variants, gauge-mediated supersymmetry breaking (GMSB), etc [9]. However, in general, even after choosing a particular model of supersymmetry breaking, flavour violation can still be present in the weak-scale Lagrangian. The various sources of flavour violation in such a case can be broadly classified as:[3]

(i) In models of supersymmetry breaking based on supergravity or superstring theories, although it is possible to achieve universality or even no-scale boundary conditions under some assumptions on the Kähler potential, non-universal soft terms are generically present in the high-scale effective Lagrangian [10].

(ii) In models with flavour symmetry imposed by a Froggatt–Nielsen mechanism, flavour violating corrections to the soft potential could be potentially large [11]. More so, if the flavon fields contain SUSY breaking F-VEVs [12, 13].

(iii) Finally, the existence of new particles at high scales with flavour violating couplings to the SM leptons (as right-handed neutrinos in a seesaw model [6]) or the presence of new Yukawa interactions (as in Grand Unified Theories where quark and leptonic fields sit in the same (super)multiplets [14]) can lead to flavour violation at the weak scale. In this case, the flavour violation is communicated to the low-energy fields through renormalization group equations (RGEs) [15].

Obviously, this list is not exhaustive. There can be other exotic sources which can be either additional heavy Higgs particles [16] or are related to the localization of the fermions in higher-dimensional space-time when MSSM/SUSY-GUT is embedded in a extra-dimensional model [17, 18].

In the present review, we will concentrate on the flavour violation solely due to a mechanism generating neutrino masses and mixings. To this effect, we would consider models where the *strong* universality is assumed at the high scale and thus point (i) would not be considered here. Similarly, we will not consider either effects generated by the imposition of a flavour symmetry as in (ii). Instead, the main aspect of this review will be to collect some salient features of the flavour violation induced by a neutrino mass model in a supersymmetric theory. As was mentioned in the introduction, the most natural and popular of them, the seesaw mechanism, will be our main focus. Notice that these RGE effects are always present in any SUSY seesaw model, independently of the presence of the other sources in (i) and (ii). In this sense, the effects considered here are independent of the particular mechanism of SUSY breaking and mediation. These model-dependent effects in (i) and (ii) can always be added to our results in the relevant cases.

Irrespective of the source, LFV at the weak scale can be parametrized in a model-independent manner in terms of a mass insertion (MI), $\Delta^l_{ij}$, the flavour violating off-diagonal entry appearing in the slepton mass matrix.[4] These MI are further subdivided into LL/LR/RL/RR types, labelled by the chirality of the corresponding SM fermions.[5] Depending on the model, one or several of these types of MI can simultaneously be present at the weak scale. In the presence of any of these parameters, one-loop diagrams mediated by gauginos, higgsinos (neutral and three charged fermionic partners of gauge and Higgs bosons) and sleptons lead to lepton flavour violating processes such as $\mu \to e + \gamma$, $\mu \to 3e$, $\mu \to e$ conversion in nuclei, etc (an example diagram is shown in figure 1). The strength of these processes crucially depends on the MI factor

[3] We assume R-parity conservation throughout the present work.
[4] In the basis where the charged lepton mass matrix is diagonal.
[5] $i$, $j$, $k$ denote generation indices throughout the work.

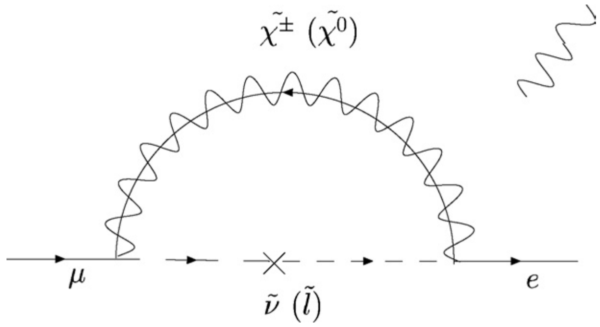


**Figure 1.** The diagrams contributing to $\mu \to e, \gamma$ decays.

$\delta^l_{ij} \equiv \Delta^l_{ij}/m^2_{\tilde{l}}$, where $m^2_{\tilde{l}}$ is the average slepton mass. For $|\delta| < 1$, which is expected to be the case for most models, one can always use the MI approximation [15, 19] to compute the amplitudes of the relevant processes. Such computations have been done long ago, considering the neutral gaugino diagrams [6, 7]. It has been realized later that, in addition to the flavour violating LL/RR MI, considering the Higgsinos/gaugino mixing, as well as the flavour diagonal left–right mixing in the slepton mass matrix, can significantly enhance the amplitudes of these processes at large $\tan\beta$ [20]. These computations have since then been updated by Hisano and Nomura [21] and Masina and Savoy [22], including this mixing as well as the charged gaugino/higgsino contribution.[6] Taking the $\tan\beta$ factor into account, the branching ratio of $l_j \to l_i, \gamma$ for the dominant LL MI is roughly given by

$$\mathrm{BR}(l_j \to l_i\gamma) \approx \frac{\alpha^3 |\delta^l_{ij}|^2}{G_F^2 m^4_{\mathrm{SUSY}}} \tan^2\beta, \qquad (1)$$

where $m_{\mathrm{SUSY}}$ represents the typical supersymmetry breaking mass such as the gaugino/slepton mass. For large $|\delta| \sim 1$ or for many $\delta$'s present simultaneously, it is instructive to diagonalize the slepton mass matrix and evaluate the precise amplitudes in the mass-eigenstate basis. A complete computation in this basis has been presented in [23] for several LFV processes such as $l_j \to l_i + \gamma$, $l_j \to 3l_i$, $\mu \to e$ conversion in nuclei. The processes discussed so far are the ones mediated by neutralino and chargino sector. However, Higgs bosons ($h^0$, $H^0$, $A^0$) are also sensitive to flavour violation and mediate processes such as $\mu \to e$ conversion [25], $\tau \to 3\mu$ [26] and $\tau \to \mu\eta$ [27]. The amplitudes of these processes are sensitive to a higher degree in $\tan\beta$ than the chargino/neutralino ones (the BRs grow as $(\tan\beta)^6$, although they are suppressed by additional Yukawa couplings) and thus could lead to large branching fractions at large $\tan\beta$. Detailed studies on the $\mu \to \tau$ sector have been presented in the literature [28].

In the rest of the review, we will consider the decay $l_j \to l_i, \gamma$ as the prototype signature of the lepton flavour violation. We will discuss several ideas put forward in the literature on the sensitivity of this decay process in determining the seesaw parameter space as well as the supersymmetric parameter space. Before this, we will briefly review the SUSY seesaw mechanism and the generation of flavour violation in this model.

[6] Another important feature is that the interference between various contributions could lead to suppressed amplitudes in some regions of the parameter space [21]–[23]. This typically occurs for RR type MI as long as universality in the gaugino masses is maintained at the high scale. Although in a completely generic situation without any universal boundary conditions, such cancellations can also occur for LL type MI [24].

## 3. Supersymmetric seesaw and leptonic flavour violation

The seesaw mechanism can be incorporated in the Minimal Supersymmetric Standard Model in a manner similar to what is done in the SM, by adding right-handed neutrino superfields to the MSSM superpotential:

$$W = h^u_{ij} Q_i u^c_j H_2 + h^d_{ii} Q_i d^c_i H_1 + h^e_{ii} L_i e^c_i H_1 + h^\nu_{ij} L_i \nu^c_j H_2 + M_{R_{ii}} \nu^c_i \nu^c_i + \mu H_1 H_2, \tag{2}$$

where we are in the basis of diagonal charged lepton, down quark and right-handed Majorana mass matrices. $M_R$ represents the (heavy) Majorana mass matrix for the right-handed neutrinos. Equation (2) leads to the standard seesaw formula for the (light) neutrino mass matrix

$$\mathcal{M}_\nu = -h^\nu M_R^{-1} h^{\nu\mathrm{T}} v_2^2, \tag{3}$$

where $v_2$ is the vacuum expectation value (VEV) of the up-type Higgs field, $H_2$. Under suitable conditions on $h^\nu$ and $M_R$, the correct mass splittings and mixing angles in $\mathcal{M}_\nu$ can be obtained. Detailed analyses deriving these conditions are already present in the literature [29].

Following the discussion in the previous section, we will assume that the mechanism that breaks supersymmetry and conveys it to the observable sector at the high scale $\sim M_\mathrm{P}$ is flavour blind, as in mSUGRA. However, this flavour blindness is not protected down to the weak scale [6].[7] The slepton mass matrices are no longer invariant under RG evolution from the super large scale where supersymmetry is mediated to the visible sector down to the seesaw scale, as the flavour violation present in the neutrino Dirac Yukawa couplings $h^\nu$ is now 'felt' by the slepton mass matrices in the presence of heavy right-handed neutrinos.

The weak-scale flavour violation so generated can be obtained by solving the RGEs for the slepton mass matrices from the high scale to the scale of the right-handed neutrinos. Below this scale, the running of the FV slepton mass terms is RG-invariant as the right-handed neutrinos decouple from the theory. For the purpose of illustration, a leading log estimate can easily be obtained for these equations.[8] Assuming the flavour blind mSUGRA specified by the high-scale parameters, $m_0$, the common scalar mass, $A_0$, the common trilinear coupling and $M_{1/2}$, the universal gaugino mass, the flavour violating entries in these mass matrices at the weak scale are given as

$$(\Delta^l_{ij})_{\mathrm{LL}} \approx -\frac{3m_0^2 + A_0^2}{8\pi^2} \sum_k (h^\nu_{ik} h^{\nu *}_{jk}) \ln \frac{M_X}{M_{R_k}}, \tag{4}$$

where $h^\nu$ are given in the basis of diagonal charged lepton masses and diagonal Majorana right-handed neutrino mass matrix $M_R$ and $M_X$ is the scale at which soft terms appear in the Lagrangian. Given this, the branching ratios for LFV rare decays $l_j \to l_i, \gamma$ can be roughly estimated using (1). From above it is obvious that the amount of lepton flavour violation generated by the SUSY seesaw at the weak scale crucially depends on the flavour structure of $h^\nu$ and $M_R$, shown in (2), the 'new' sources of flavour violation not present in the MSSM. If either the neutrino Yukawa couplings or the flavour mixings present in $h^\nu$ are very tiny, the strength of LFV

[7] This is always true in a gravity-mediated supersymmetry breaking model, but it also applies to other mechanisms under some specific conditions [30, 31].

[8] Within mSUGRA, the leading log approximation works very well for most of the parameter space, except for regions of large $M_{1/2}$ and low $m_0$. The discrepancy with the exact result increases with low $\tan\beta$ [32].

will be significantly reduced. Furthermore, if the right-handed neutrino masses were heavier than the supersymmetry breaking scale (as in GMSB models) they would decouple from the theory before the SUSY soft breaking matrices enter into play and hence these effects would vanish.

### *3.1. Model-independent expectations for LFV?*

A crucial feature of the seesaw mechanism is that it has a larger number of parameters than those relevant for neutrino masses and mixings. This would inhibit us in computing model-independent expectations for, say, BR($\mu \to e, \gamma$), given that the supersymmetric seesaw mechanism works at the high scale. In fact, even after having a complete knowledge of the entire neutrino mass matrix elements, $\mathcal{M}_\nu$ as well as the heavy neutrino Majorana mass matrix eigenvalues $M_{R_k}$, it is still not sufficient to completely determine the rest of the seesaw parameters, namely the neutrino Dirac Yukawa coupling matrix $h^\nu$. This is best illustrated in the parametrization given by Casas and Ibarra [33]; starting from the seesaw formula (3), one can derive $h^\nu$ in terms of low-energy parameters as

$$h^\nu = U^\star_{PMNS}\mathcal{D}_{\sqrt{\mathcal{M}_\nu}}R^{\mathrm{T}}\mathcal{D}_{\sqrt{M_R}}\frac{1}{v_2}, \tag{5}$$

where $U_{PMNS}$ is the Pontecorvo–Maki–Nakagawa–Sakata leptonic mixing matrix, $\mathcal{D}_{\sqrt{\mathcal{M}_\nu}}$ is the square root of the diagonal matrix of light neutrino mass eigenvalues, $\mathcal{D}_{\sqrt{M_R}}$ is the square root of the diagonal matrix of heavy neutrino mass eigenvalues and $R$ is an arbitrary complex orthogonal matrix such that $RR^{\mathrm{T}} = 1$. The matrix $R$ parametrizes our ignorance of the neutrino Yukawas in spite of the complete knowledge of the neutrino mass matrix. Although it is difficult to give a physical definition to '$R$' it is important to note that it can have physical consequences for lepton flavour violation (as well as leptogenesis), even if the neutrino masses and mixings are already accounted for. For lepton flavour violation, the relevant part of the seesaw parameter space are the entries in the matrix $h^\nu h^{\nu\dagger}$ which are now given by

$$h^\nu h^{\nu\dagger} = U^\star_{PMNS}\mathcal{D}_{\sqrt{\mathcal{M}_\nu}}R^{\mathrm{T}}\mathcal{D}_{M_R}R^\star\mathcal{D}_{\sqrt{\mathcal{M}_\nu}}U^{\mathrm{T}}_{PMNS}\frac{1}{v_2^2}. \tag{6}$$

$R$ can be parametrized by three angles and three phases. Flavour violation is now determined in terms of the angles and phases in $R$, $U_{PMNS}$, $\mathcal{M}_\nu$ and $M_R$. For a given neutrino spectrum, $U_{PMNS}$ and neutrino mass eigenvalues are (approximately) known. However, the branching ratios can now be either enhanced or suppressed depending on the parameters in $R$. Choosing $M_R$ to be either completely hierarchical or degenerate leads to a reduction in the number of parameters affecting the branching ratios. It is further reduced if $R$ is chosen to be real. For example, if $M_R$ is completely hierarchical and $R$ is real, basically only one angle would affect the branching ratios. Similarly, when $M_R$ is degenerate and $R$ is real, the branching ratios are independent of $R$. Lepton flavour violating rates for various cases of interest have been analysed in [33] and special cases in [34, 35].

As mentioned earlier, flavour violation at the weak scale can be treated independently of the source in terms of the mass insertion parameters $\Delta^l_{ij}$. In the seesaw model, these $\Delta$ parameters are generated by RGE evolution and are thus proportional to the neutrino Yukawa couplings. In

**Table 1.** Limits on $\delta_{LL}$ parameters derived in the mSUGRA model for $\tan\beta = 10$. The average slepton mass is chosen to be around 450 GeV and $M_0 \sim 500$ GeV. Our results agree with those presented in [21, 22].

| $(\delta^l)_{LL}$ | Present limits | Upcoming limits |
|---|---|---|
| 12 | $2 \times 10^{-4}$ | $8.5 \times 10^{-6}$ |
| 13 | 0.09 | 0.02 |
| 23 | 0.09 | 0.02 |

mSUGRA, the relation between these two parameters is given as[9]

$$(\Delta^l_{ij})_{LL} \approx -\frac{3m_0^2 + A_0^2}{8\pi^2} C_{ij}, \tag{7}$$

where $C_{ij}$ is defined to contain all the information from neutrino Yukawa couplings, specifically, the left-mixing angles and the eigenvalues [36, 37] and is given by

$$C_{ij} = [h^\nu h^{\nu\dagger}]_{ij} \log \frac{M_X}{M_R}. \tag{8}$$

$C_{ij}$ is the part of the 'seesaw' parameter space that can be probed by LFV experiments. The various upper bounds on $\delta^l_{ij}$ from the experimental limits on $l_j \to l_i, \gamma$ decay rates and other LFV processes can now be converted to upper bounds on off-diagonal $C_{ij}$ parameters [36] for each point in the SUSY breaking parameter space. In table 1, we present limits on $(\delta^l_{ij})_{LL}$ from the present and upcoming limits on $l_j \to l_i + \gamma$ processes [38, 39].

The implied bounds on $C_{ij}$ can in turn be used to study constraints on neutrino Dirac Yukawa entries, for example, arising from a flavour model. The sensitivity of LFV experiments in probing the seesaw parameter space has been analysed by Ellis *et al* [37]. Choosing well-defined points in supersymmetric parameter space, such as the popular Benchmark/Snowmass points, scatter plots of $C_{ij}$ parameter space[10] probed by several LFV processes such as $\mu \to e, \gamma$ are presented.

### *3.2. CP violation in the lepton sector*

In addition to LFV, the SUSY seesaw can be responsible for several CP violating phenomena at both high and low energies in the leptonic sector. This can be facilitated by large amount of complex phases present in the seesaw couplings $h^\nu$ and/or $M_R$. We list some of these phenomena below:

(i) Leptogenesis [40].
(ii) CP violation in neutrino oscillations [41].
(iii) CP violation in $\Delta L = 2$ processes, such as neutrinoless double beta decay [42].
(iv) CP violation in $\Delta L = 1$ processes, such as CP violation in $\mu \to 3e$ etc [43, 44].

[9] Note that only $LL$ type $\Delta$s are generated in the SUSY seesaw mechanism as long as one sticks to the superpotential given in (2).
[10] $C_{ij}$ is denoted as $H_{ij}$ in [37].

(v) Leptonic EDMs [45]–[47].

(vi) Slepton oscillations and CP violation in the sleptonic sector [48].

Since the source of both LFV and some of the above phenomena, say, leptogenesis, is the same set of parameters in the Lagrangian, some amount of correlation can be expected between these phenomena for some regions of the entire seesaw parameter space. However, such a correlation is in general not guaranteed, even if the two phenomena exist simultaneously. Such a study was carried out, by random scanning of the parameter space, by Ellis and collaborators in the case of hierarchical heavy neutrinos in [49] and for degenerate heavy neutrinos in [50]. Implications of low-energy observables, specifically LFV, for leptogenesis and vice versa can also be studied by using the R-parametrization introduced above for $h^\nu$, or any of the other suitable parametrizations of a complex generic matrix, $h^\nu$. Such studies have been carried out in [51]–[54]; other related studies are [55, 56]. A detailed analysis of correlations between all the leptonic phenomena is clearly beyond the scope of the present work.

### *3.3. From LFV to seesaw parameters*

As we have seen so far, measuring the neutrino mass parameters $\Delta m^2_{\mathrm{atm}}$, $\Delta m^2_\odot$ and the mixing angles $\theta_{ij}$ more precisely would not lead us to any information on either the left- or the right-mixing angles present in $h^\nu$ or its eigenvalues. While it also holds true within the SM seesaw, the major advantage in the supersymmetric seesaw is the rich FCNC and CP violating phenomena associated with it. A natural question that follows is: Can we determine all the seesaw parameters by purely low-energy experiments? At this juncture, frankly, the question looks a bit ambitious given that we have still not yet completed the determination of the neutrino mixing matrix angles and phases as well as $\mathrm{sg}(\Delta m^2_{\mathrm{atm}})$. However, taking into consideration that (i) we have the possibility of low-energy supersymmetry being observable at the LHC, (ii) being probed in great detail at linear colliders, (iii) having improved sensitivity at the upcoming facilities such as MEGA and super-B/charm factories and (iv) improved determination of neutrino mass parameters at JPARC and long-base-line experiments, such an analysis may be required in the near future. As we will discuss later on, assuming a 'best case' scenario, the first hint of SUSY seesaw might in fact come from a $\mu \to e, \gamma$ decay, probably even before the advent of the LHC [57]. Later, more detailed evidence might pile up. Davidson and Ibarra [58] have presented a discussion on evaluating the seesaw parameters from low-energy observables. It has been shown that although there is a one-to-one correspondence between measurables at the weak- and the high-scale parameters, in practice, even in the simplest supersymmetric theories, such an analysis could be formidable to achieve. This is particularly true regarding the additional phases present in the sneutrino/slepton mass matrices. However, we continue to remain optimistic and let the future decide.

## 4. What could be the neutrino Yukawa couplings?

In the above, we have seen that due to our lack of knowledge on the neutrino Yukawa couplings, our predictions for lepton flavour violation are not effective enough in a purely bottom-up approach. This can be considered as a limitation of the model we have been working with, namely, the MSSM with right-handed neutrinos. A possible way out then could be to enlarge the SM gauge group to a much larger group, such as a Grand Unified Theory. In these models, typically quark and leptons sit in the same multiplets, leading to relations between their Yukawa couplings. One would expect that similar correlations would occur when the seesaw mechanism is incorporated within a GUT. Under the simplest of the GUT groups, '$SU(5)$', however, the right-handed neutrinos remain singlets and the problem with LFV predictions still persists. One needs at least a group encompassing the Pati–Salam $SU(4)_c$ symmetry, like an $SO(10)$ model. Here we review one such example.

### *4.1. GUT models: an SO(10) example*

In the $SO(10)$ gauge theory, all the known fermions and the right-handed neutrinos are unified in a single representation of the gauge group, the **16**. The product of two **16** matter representations can only couple to **10**, **120** or **126** representations, which can be formed by either a single Higgs field representation or a non-renormalizable product of representations of several Higgs fields. In either case, the Yukawa matrices resulting from the couplings to **10** and **126** are complex-symmetric, whereas they are antisymmetric when the couplings are to the **120**. Thus, the most general $SO(10)$ superpotential relevant to fermion masses can be written as

$$W_{SO(10)} = h_{ij}^{10} 16_i 16_j 10 + h_{ij}^{126} 16_i 16_j 126 + h_{ij}^{120} 16_i 16_j 120, \tag{9}$$

where $i, j$ refer to the generation indices. In terms of the SM fields, the Yukawa couplings relevant for fermion masses are given by[11] [59]

$$\begin{aligned}
&16\,16\,10 \supset 5(uu^c + \nu\nu^c) + \bar{5}(dd^c + ee^c),\\
&16\,16\,126 \supset 1\nu^c\nu^c + 15\nu\nu + 5(uu^c - 3\nu\nu^c) + \overline{45}(dd^c - 3ee^c),\\
&16\,16\,120 \supset 5\nu\nu^c + 45uu^c + \bar{5}(dd^c + ee^c) + \overline{45}(dd^c - 3ee^c),
\end{aligned} \tag{10}$$

where we have specified the corresponding $SU(5)$ Higgs representations for each of the couplings and all the fermions are left-handed fields. The resulting mass matrices can be written as

$$M^u = M_{10}^5 + M_{126}^5 + M_{120}^{45}, \tag{11}$$

$$M_{LR}^\nu = M_{10}^5 - 3M_{126}^5 + M_{120}^5, \tag{12}$$

$$M^d = M_{10}^{\bar{5}} + M_{126}^{\overline{45}} + M_{120}^{\bar{5}} + M_{120}^{\overline{45}}, \tag{13}$$

$$M^e = M_{10}^{\bar{5}} - 3M_{126}^{\overline{45}} + M_{120}^{\bar{5}} - 3M_{120}^{\overline{45}}, \tag{14}$$

$$M_{LL}^\nu = M_{126}^{15}, \tag{15}$$

$$M_R^\nu = M_{126}^1. \tag{16}$$

[11] Recently, $SO(10)$ couplings have also been evaluated for various renormalizable and non-renormalizable couplings in [60].

A simple analysis of the above mass matrices leads us to the following result: *At least one of the Yukawa couplings in* $h^\nu = v_u^{-1} M_{LR}^\nu$ *has to be as large as the top Yukawa coupling* [61]. This result holds true in general, independent of the choice of the Higgses responsible for the masses in (11) and (12), provided that no accidental fine-tuned cancellations of the different contributions in equation (12) are present. If contributions from the **10**s solely dominate, $h^\nu$ and $h^u$ would be equal. If this occurs for the **126**s, then $h^\nu = -3h^u$ [62]. In case both of them have dominant entries, barring a rather precisely fine-tuned cancellation between $M_{10}^5$ and $M_{126}^5$ in equation (12), we expect at least one large entry to be present in $h^\nu$. A dominant antisymmetric contribution to top quark mass due to the **120** Higgs is phenomenologically excluded, since it would lead to at least a pair of heavy degenerate up-quarks.

Apart from sharing the property that at least one eigenvalue of both $M^u$ and $M_{LR}^\nu$ has to be large, for the rest it is clear from (11) and (12) that these two matrices are not aligned, in general, and hence we may expect different mixing angles appearing from their diagonalization. This freedom is removed if one sticks to particularly simple choices of the Higgses responsible for up-quark and neutrino masses. A couple of remarks are in order here. Firstly, note that in general there can be an additional contribution, equation (15), to the light neutrino mass matrix, independent of the canonical seesaw mechanism. Taking into consideration also this contribution leads to the so-called Type-II seesaw formula [63]. Secondly, the correlation between neutrino Dirac Yukawa coupling and the top Yukawa is in general independent of the type of seesaw mechanism, and thus holds true irrespective of the light-neutrino mass structure.

### *4.2. What could be the neutrino Yukawa mixing matrices?*

Within the $SO(10)$ example, we have seen above that the amount of mixing present in the neutrino Dirac Yukawa couplings, $h^\nu$ depends on the type and number of Higgs representations there are in the theory. Motivated by the flavour structure of SM fermions (including neutrino mixing angles), one can imagine two cases of mixings to be present in $h^\nu$. The first one corresponds to a case where the mixing present in $h^\nu$ is small and CKM-like. We will call this case 'the minimal case'. This is typical of models in which quarks and leptons have the same mixing angles at the high scale. The required large mixing angles in the light-neutrino sector is purely resultant of the seesaw mechanism. As a second case, we consider scenarios where the mixing in $h^\nu$ is no longer small, but large like the observed PMNS mixing. In this case, the heavy neutrino mass matrix only plays the role of a large scale due to which light neutrino masses are suppressed. We will call this case the 'the maximal case'.[12] These two cases serve as 'benchmark' scenarios for seesaw-induced lepton flavour violation in SUSY $SO(10)$. Similar studies of these two extreme cases have also been considered in [23, 37].

*4.2.1. Minimal case: a model for CKM mixings.* The minimal Higgs spectrum to obtain phenomenologically viable mass matrices includes two **10**-plets, one coupling to the up-sector and the other to the down-sector. In this way, it is possible to obtain the required CKM mixing [64] in the quark sector. The $SO(10)$ superpotential is now given by

$$W_{SO(10)} = \tfrac{1}{2} h_{ij}^{u,\nu} 16_i 16_j 10_u + \tfrac{1}{2} h_{ij}^{d,e} 16_i 16_j 10_d + \tfrac{1}{2} h_{ij}^{R} 16_i 16_j 126. \qquad (17)$$

[12] Note that these two cases are the two examples of how one can generate large mixing angles for light-neutrino mass matrices from the seesaw mechanism [29].

We further assume that the **126**-dimensional Higgs field gives Majorana mass *only* to the right-handed neutrinos. An additional feature of the above mass matrices is that all of them are *symmetric*. From here, it is clear that the following mass relations hold between the quark and leptonic mass matrices at the GUT scale:[13]

$$h^u = h^\nu, \qquad h^d = h^e. \tag{18}$$

In the basis where charged lepton masses are diagonal, we have

$$h^\nu = V_{\rm CKM}^{\rm T} h^u_{diag} V_{\rm CKM}. \tag{19}$$

The large couplings in $h^\nu \sim \mathcal{O}(h_t)$ induce significant off-diagonal entries in $m^2_{\tilde{L}}$ through the RG evolution between $M_{\rm GUT}$ and the scale of the right-handed Majorana neutrinos,[14] $M_{R_i}$. The induced off-diagonal entries relevant to $l_j \to l_i, \gamma$ are of the order of

$$(m^2_{\tilde{L}})_{21} \approx -\frac{3m_0^2 + A_0^2}{8\pi^2} h_t^2 V_{td} V_{ts} \ln \frac{M_{\rm GUT}}{M_{R_3}} + \mathcal{O}(h_c^2), \tag{20}$$

$$(m^2_{\tilde{L}})_{32} \approx -\frac{3m_0^2 + A_0^2}{8\pi^2} h_t^2 V_{tb} V_{ts} \ln \frac{M_{\rm GUT}}{M_{R_3}} + \mathcal{O}(h_c^2), \tag{21}$$

$$(m^2_{\tilde{L}})_{31} \approx -\frac{3m_0^2 + A_0^2}{8\pi^2} h_t^2 V_{tb} V_{td} \ln \frac{M_{\rm GUT}}{M_{R_3}} + \mathcal{O}(h_c^2). \tag{22}$$

In these expressions, the CKM angles are small but one would expect the presence of the large top Yukawa coupling to compensate such a suppression. The required right-handed neutrino Majorana mass matrix, consistent with both the observed low-energy neutrino masses and mixings as well as with CKM-like mixings in $h^\nu$ is easily determined from the seesaw formula defined at the scale of right-handed neutrinos[15]

$$M_R = V_{\rm CKM} h^u_{diag} V_{\rm CKM}^{\rm T} m_\nu^{-1} V_{\rm CKM} h^u_{diag} V_{\rm CKM}^{\rm T}, \tag{23}$$

where we have used equation (14) for $h^\nu$. For hierarchical neutrino mass spectrum, $m_{\nu_3} \approx \sqrt{\Delta m^2_{\rm atm}}$, $m_{\nu_2} \approx \sqrt{\Delta m^2_\odot}$ and $m_{\nu_1} \ll \sqrt{\Delta m^2_\odot}$ and for a nearly bi-maximal $U_{PMNS}$, it is straightforward to see that the right-handed neutrino mass eigenvalues are given by

$$M_{R_3} \approx \frac{m_t^2}{4m_{\nu_1}}, \qquad M_{R_2} \approx \frac{m_c^2}{4m_{\nu_1}}, \qquad M_{R_1} \approx \frac{m_u^2}{2m_{\nu_1}}. \tag{24}$$

[13] Clearly, this relation cannot hold for the first two generations of down-quarks and charged leptons. One expects small corrections due to non-renormalizable operators or suppressed renormalizable operators [65] to be invoked.
[14] Typically, one has different mass scales associated with different right-handed neutrino masses.
[15] The neutrino masses and mixings here are defined at $M_R$. Radiative corrections can significantly modify the neutrino spectrum from that of the weak scale [66]. This is more true for the degenerate spectrum of neutrino masses [67] and for some specific forms of $h^\nu$ [68]. For our present discussion, with hierarchical neutrino masses and up-quark like neutrino Yukawa matrices, we expect these effects not to play a very significant role.

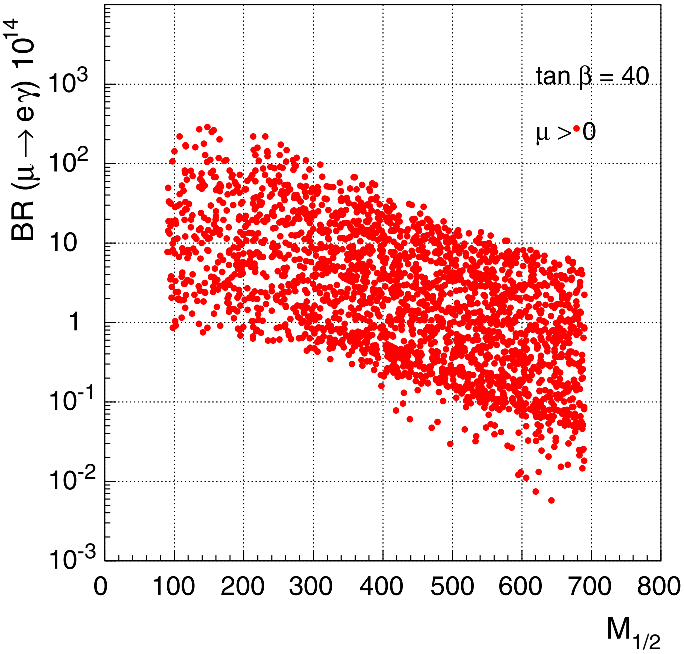


**Figure 2.** The scatter plots of branching ratios of $\mu \to e, \gamma$ decays as a function of $M_{1/2}$ are shown for the (minimal) CKM case for $\tan\beta = 40$. Results do not alter significantly with the change of sign($\mu$).

The Br($\mu \to e, \gamma$) is now predictable in this case. Considering mSUGRA boundary conditions, we compute these branching ratios numerically. In figure 2 we show the scatter plots (in mSUGRA parameter space $m_0$, $A_0$, $M_{1/2}$) for BR($\mu \to e, \gamma$) for the CKM case and $\tan\beta = 40$. We see that reaching a sensitivity of $10^{-14}$ for BR($\mu \to e\gamma$) would allow us to probe the SUSY spectrum completely up to $M_{1/2} = 300$ GeV (notice that this corresponds to gluino and squark masses of order 750 GeV) and would still probe large regions of the parameter space up to $M_{1/2} = 700$ GeV. Thus, in summary, although the present limits on BR($\mu \to e, \gamma$) would not induce any significant constraints on the supersymmetry-breaking parameter space, an improvement in the limit to $\sim\mathcal{O}(10^{-14})$, as foreseen, would start imposing non-trivial constraints especially for the large $\tan\beta$ region.

A further comment on the 'minimal' mixing case is in order. Strictly speaking, this is not the 'minimalest' mixing possible in the Dirac neutrino Yukawa couplings. It has been shown in models where the right-handed neutrinos attain their masses through Yukawa couplings, one can essentially set the Dirac neutrino Yukawa mixing to be zero and the entire mixing comes from the right-handed neutrino sector [69]. Such a situation can be realized within left–right symmetric models, with or without an $SO(10)$ embedding. The renormalization group flow is different in this case and the LFV is now related to Yukawa couplings of the right-handed neutrino mass matrix in an indirect manner [69, 70].

*4.2.2. Maximal case: a method for PMNS mixings.* The minimal $SO(10)$ model presented in the previous subsection would inevitably lead to small mixing in $h^{\nu}$. In fact, with two Higgs fields in symmetric representations, giving masses to the up-sector and the down-sector separately, it would be difficult to avoid the small CKM-like mixing in $h^{\nu}$. To generate mixing angles larger than CKM angles, asymmetric mass matrices have to be considered. In general, it is sufficient to introduce asymmetric textures either in the up-sector or in the down-sector. In the present case, we assume that the down-sector couples to a combination of Higgs representations (symmetric and antisymmetric)[16] $\Phi$, leading to an asymmetric mass matrix in the basis where the up-sector is diagonal. As we will see below, this would also require that the right-handed Majorana mass matrix be diagonal in this basis. We have

$$W_{SO(10)} = \tfrac{1}{2}h_{ii}^{u,\nu}16_i 16_i 10^u + \tfrac{1}{2}h_{ij}^{d,e}16_i 16_j \Phi + \tfrac{1}{2}h_{ii}^{R}16_i 16_i 126, \tag{25}$$

where the **126**, as before, generates only the right-handed neutrino mass matrix. To study the consequences of these assumptions, we see that at the level of $SU(5)$, we have

$$W_{SU(5)} = \tfrac{1}{2}h_{ii}^{u}10_i 10_i 5_u + h_{ii}^{\nu}\bar{5}_i 1_i 5_u + h_{ij}^{d}10_i \bar{5}_j \bar{5}_d + \tfrac{1}{2}M_{ii}^{R}1_i 1_i, \tag{26}$$

where we have decomposed the 16 into $10 + \bar{5} + 1$ and $5_u$ and $\bar{5}_d$ are components of $10_u$ and $\Phi$, respectively. To have large mixing $\sim U_{PMNS}$ in $h^\nu$ we see that the asymmetric matrix $h^d$ should now be able to generate both the CKM as well as PMNS mixing. This is possible if

$$V_{\mathrm{CKM}}^{\mathrm{T}} h^d U_{PMNS}^{\mathrm{T}} = h_{diag}^{d}. \tag{27}$$

This would mean that the 10 that contains the left-handed down-quarks would be rotated by the CKM matrix, whereas the $\bar{5}$ that contains the left-handed charged leptons would be rotated by the $U_{PMNS}$ matrix to go into their respective mass bases [71, 72]. Thus we have, in analogy with the previous subsection, the following relations to hold true in the basis where charged leptons and down-quarks are diagonal:

$$h^u = V_{\mathrm{CKM}} h_{diag}^{u} V_{\mathrm{CKM}}^{\mathrm{T}}, \tag{28}$$

$$h^\nu = U_{PMNS} h_{diag}^{u}. \tag{29}$$

Using the seesaw formula of equations (3) and (29), we have

$$M_R = \mathrm{diag}\left\{\frac{m_u^2}{m_{\nu_1}}, \frac{m_c^2}{m_{\nu_2}}, \frac{m_t^2}{m_{\nu_3}}\right\}. \tag{30}$$

We now turn our attention to lepton flavour violation in this case. The branching ratio, $\mathrm{BR}(\mu \to e, \gamma)$ would now be dependent on

$$[h^\nu h^{\nu\mathrm{T}}]_{21} = h_t^2 U_{\mu 3} U_{e3} + h_c^2 U_{\mu 2} U_{e2} + \mathcal{O}(h_u^2). \tag{31}$$

It is clear from the above that in contrast to the CKM case, the dominant contribution to the off-diagonal entries depends on the unknown magnitude of the element $U_{e3}$ [73]. If $U_{e3}$ is very close to its present limit $\sim$0.2 [74], the first term on the rhs of the equation (31) would dominate.

[16] The couplings of $\Phi$ in the superpotential can be either renormalizable or non-renormalizable. See [71] for a non-renormalizable example.

Moreover, this would lead to large contributions to the off-diagonal entries in the slepton masses with $U_{\mu 3}$ of $\mathcal{O}(1)$. We have

$$(m_{\tilde{L}}^2)_{21} \approx -\frac{3m_0^2 + A_0^2}{8\pi^2} h_t^2 U_{e3} U_{\mu 3} \ln \frac{M_{\rm GUT}}{M_{R_3}} + \mathcal{O}(h_c^2). \tag{32}$$

The above contribution is larger than the CKM case by a factor of $(U_{\mu 3}U_{e3})/(V_{td}V_{ts}) \sim 140$ compared with the CKM case. From equation (1) we see that it would mean about a factor $10^4$ times larger than the CKM case in BR($\mu \to e, \gamma$). In case $U_{e3}$ is very small, i.e. either zero or $\lesssim (h_c^2/h_t^2)U_{e2} \sim 4 \times 10^{-5}$, the second term $\propto h_c^2$ in equation (31) would dominate. However the off-diagonal contribution in slepton masses, now being proportional to charm Yukawa, could be much smaller, in fact even smaller than the CKM contribution by a factor

$$\frac{h_c^2 U_{\mu 2} U_{e2}}{h_t^2 V_{td} V_{ts}} \sim 7 \times 10^{-2}. \tag{33}$$

If $U_{e3}$ is close to its present limit, the current bound on BR($\mu \to e, \gamma$) would already be sufficient to produce stringent limits on the SUSY mass spectrum. Similar $U_{e3}$ dependence can be expected in the $\tau \to e$ transitions where the off-diagonal entries are given by

$$(m_{\tilde{L}}^2)_{31} \approx -\frac{3m_0^2 + A_0^2}{8\pi^2} h_t^2 U_{e3} U_{\tau 3} \ln \frac{M_{\rm GUT}}{M_{R_3}} + \mathcal{O}(h_c^2). \tag{34}$$

The $\tau \to \mu$ transitions are instead $U_{e3}$-independent probes of SUSY, whose importance was first pointed out in [75]. As in the rest of the cases, the off-diagonal entry in this case is given by

$$(m_{\tilde{L}}^2)_{32} \approx -\frac{3m_0^2 + A_0^2}{8\pi^2} h_t^2 U_{\mu 3} U_{\tau 3} \ln \frac{M_{\rm GUT}}{M_{R_3}} + \mathcal{O}(h_c^2). \tag{35}$$

In the PMNS scenario, figure 3 shows the plot for BR($\mu \to e, \gamma$) for $\tan\beta = 40$. In this plot, the value of $U_{e3}$ chosen is very close to the present experimental upper limit [74]. As long as $U_{e3} \gtrsim 4 \times 10^{-5}$, the plots scale as $U_{e3}^2$, while for $U_{e3} \lesssim 4 \times 10^{-5}$ the term proportional to $m_c^2$ in equation (32) starts dominating; the result is then insensitive to the choice of $U_{e3}$. For instance, a value of $U_{e3} = 0.01$ would reduce the BR by a factor of 225 and still a significant amount of the parameter space for $\tan\beta = 40$ would be excluded. We further find that with the present limit on BR($\mu \to e, \gamma$), all the parameter space would be completely excluded up to $M_{1/2} = 300$ GeV for $U_{e3} = 0.15$, for any value of $\tan\beta$ (not shown in the figure).

In the $\tau \to \mu\gamma$ decay the situation is similarly constrained. For $\tan\beta = 2$, the present bound of $3 \times 10^{-7}$ starts probing the parameter space up to $M_{1/2} \leqslant 150$ GeV. The main difference is that this does not depend on the value of $U_{e3}$, and therefore it is already a very important constraint on the parameter space of the model. In fact, for large $\tan\beta = 40$, as shown in figure 4, reaching the expected limit of $1 \times 10^{-8}$ would be able to rule out completely this scenario up to gaugino masses of 400 GeV, and only a small portion of the parameter space with heavier gauginos would survive. In the limit $U_{e3} = 0$, this decay mode would provide a constraint on the model stronger than $\mu \to e, \gamma$, which would now be suppressed as it would contain only contributions proportional to $h_c^2$, as shown in equation (32).

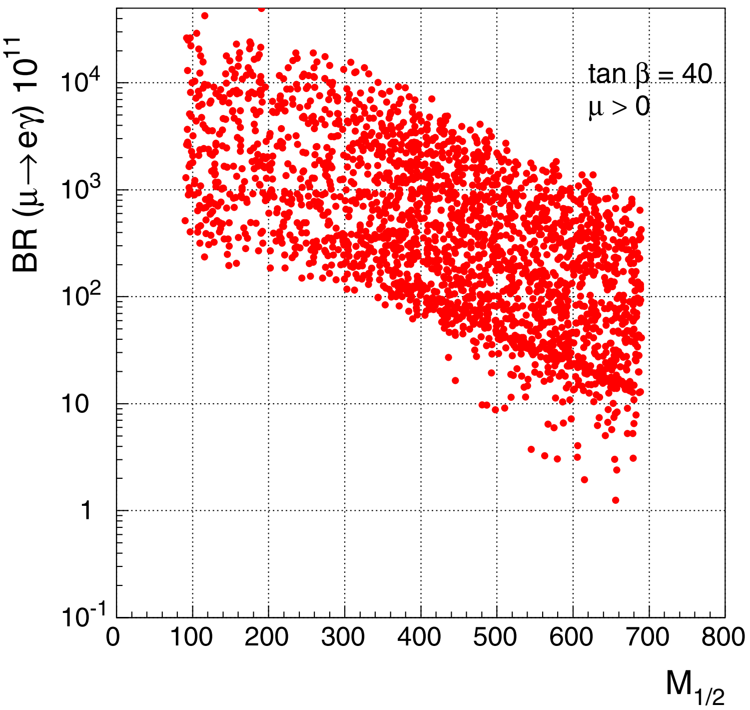


**Figure 3.** The scatter plots of branching ratios of $\mu \to e, \gamma$ decays as a function of $M_{1/2}$ are shown for the (maximal) PMNS case for $\tan\beta = 40$. The results do not alter significantly with the change of sign($\mu$).

In summary, in the PMNS/maximal mixing case, even the present limits from BR($\mu \to e, \gamma$) can rule out large portions of the supersymmetry-breaking parameter space, if $U_{e3}$ is either close to its present limit or within an order of magnitude of it (as the planned experiments might find out soon [76]). These are more severe for the large $\tan\beta$ case. In the extreme situation of $U_{e3}$ being zero or very small $\sim\mathcal{O}(10^{-4}\text{–}10^{-5})$, BR($\tau \to \mu, \gamma$) will start playing an important role, with its present constraints already disallowing large regions of the parameter space at large $\tan\beta$. While the above example concentrated on the hierarchical light neutrinos, similar 'benchmark' mixing scenarios have been explored in great detail, for degenerate spectra of light neutrinos, by Illana and Masip [77], taking also in to consideration running between the Planck scale and the GUT scale.

### 4.3. Textures and other examples

While the bottom-up approach studies the possibility of 'measuring' the seesaw parameters through low-energy experiments, the top-down approach gives the opportunity to study several theoretically well-motivated models encompassing the seesaw mechanism. For example, a flavour symmetry based on either abelian or non-abelian family symmetries could be at work at the scales giving rise to specific patterns in all the Yukawa coupling matrices in the Lagrangian, including that of neutrino Dirac Yukawa couplings. Low-energy flavour violation is then dependent on these patterns of the Yukawa matrices which are predictable. Several analyses of this kind have been presented in the literature mostly within a supersymmetric GUT [16, 36], [78]–[80]. A recent review of several textures presented in the literature can be found in [81].

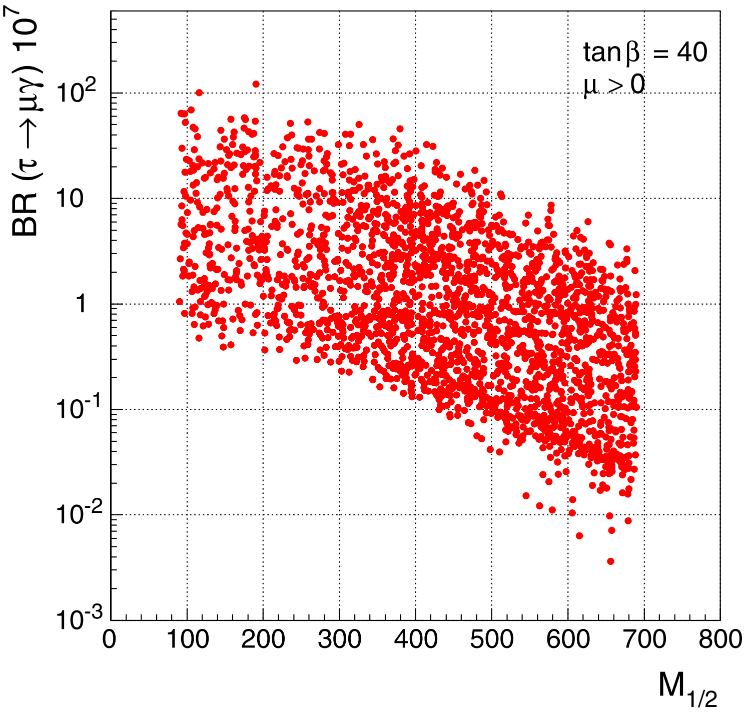


**Figure 4.** The scatter plots of branching ratios of $\tau \to \mu, \gamma$ decays as a function of $M_{1/2}$ are shown for the (maximal) PMNS case for $\tan\beta = 40$. The results do not alter significantly with the change of sign($\mu$).

Textures that tend to lead to large left-mixing in $h^\nu$ are typically prone to constraints from the present limits on $\mu \to e + \gamma$. A class of textures which goes by the name of 'lop-sided' generically predict large branching ratios [79] within the reach of experiments at MEGA. Finally, in addition to the $SO(10)$ example presented here, there have been several other examples both within the context of $SO(10)$ and otherwise that have been explored in the literature [82].[17]

## 5. Seesaw induced LFV and associated phenomenology

So far we have looked at LFV generated by a seesaw mechanism both through a bottom-up as well as top-down perspectives. In the following, we will discuss the impact the generated LFV can have on the associated phenomenology of the SUSY model. We will only concentrate on a few issues, leaving out the leptonic CP violation which we have already commented on previously. Most of these correlated effects are only valid within a class of SUSY-GUT models; as such correlations cannot be constructed from a purely bottom-up approach.

### *5.1. LFV and experimental sensitivity to $U_{e3}$*

In the maximal mixing situation, which we have discussed in the previous subsection, we have seen that the BR($\mu \to e, \gamma$) would depend crucially on the neutrino mixing matrix element $U_{e3}$.

[17] For a study of an $SO(10)$ texture leading to interesting correlations between various flavour and CP phenomena, see [83]. We thank the author for sending us the paper before publication.

**Table 2.** SPS points for mSUGRA.

| | $m_0$ | $M_{1/2}$ | $A_0$ | $\tan\beta$ | sg($\mu$) |
|---|---|---|---|---|---|
| SPS1a | 100 | 250 | $-100$ | 10 | $>0$ |
| SPS1b | 200 | 400 | 0 | 30 | $>0$ |
| SPS2 | 1450 | 300 | 0 | 10 | $>0$ |
| SPS3 | 90 | 400 | 0 | 10 | $>0$ |
| SPS4 | 400 | 300 | 0 | 50 | $>0$ |
| SPS5 | 150 | 300 | $-1000$ | 5 | $>0$ |

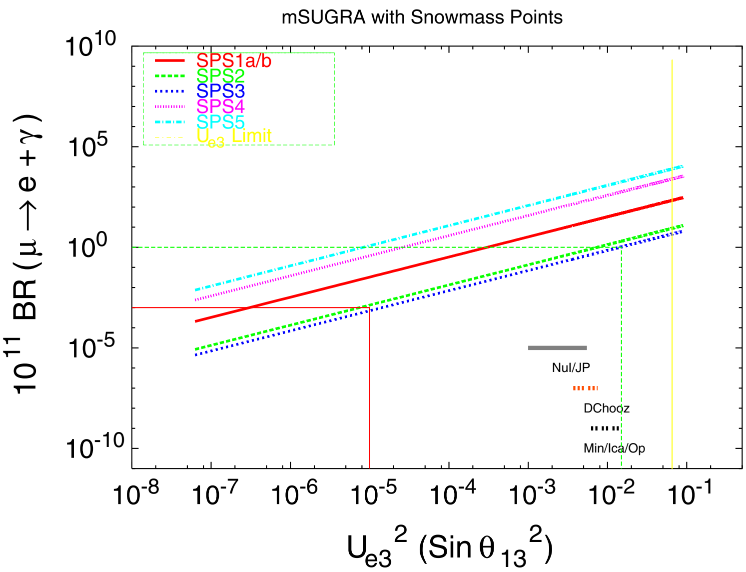


**Figure 5.** The variation of the BR($\mu \to e + \gamma$) with respect to $U_{e3}^2$ in the maximal mixing, MNS case. Each (diagonal) line corresponds to an SPS point in mSUGRA as denoted in the legend of the figure. The sensitivity of future LFV experiments $\mathcal{O}(10^{-11}$–$10^{-14})$ is projected on the $U_{e3}^2$ axis. The ranges probed by future long-base-line experiments are shown as horizontal lines. NuI/JP represents the projected sensitivity of Nu-MI/J-PARC, DChooz, reactor-based experiments like double CHOOZ, Min/Ica/Op represents the experiments MINOS, ICARUS and OPERA [82].

To illustrate the correlations between $\mu \to e + \gamma$ and the neutrino mixing angle $U_{e3}$, we chose specific points in the supersymmetric parameter space as given by the Snowmass collaboration [85], and are presented in table 2.

In figure 5, we plot BR($\mu \to e + \gamma$) with respect to $U_{e3}^2$ or $\sin^2\theta_{13}$. We also present the sensitivity of various future experiments probing $U_{e3}$ and well as the expected improvements on the limits on BR($\mu \to e + \gamma$). It is seen that $\mu \to e\gamma$ can have a stronger sensitivity on $U_{e3}$ if both SUSY seesaw and maximal mixing case are realized in nature [86]. A detailed study of the impact of $U_{e3}$ on observability of SUSY at the LHC can be found in [57].

### 5.2. Correlations with other SUSY search strategies

In addition to the improvements in LFV experiments, this is also going to be the decade in which we should be able to establish whether low-energy supersymmetry exists or not through direct searches at the LHC [87]. On the other hand, improved astrophysical observations from experiments like WMAP [88] and Planck are going to determine the relic density of supersymmetric LSP at unprecedented accuracy. Within mSUGRA, correlations between these two search strategies have been studied [89]. Incorporating the seesaw mechanism in the model via $SO(10)$, would generate another discovery strategy through the lepton flavour violation channel. This is especially true when the LFV entries in the slepton mass matrices are maximized, as in the PMNS case.

We see that the following three main regions in the mSUGRA parameter space would survive after imposing all the present phenomenological and astrophysical (dark matter) constraints [57].[18] (a) The stau coannihilation regions, where the lightest stau is quasi-degenerate with the neutralino LSP and efficient stau–stau as well as stau–neutralino (co)annihilations suppress the relic density. (b) The A-pole funnel region, where the neutralino(bino)–neutralino annihilation process is greatly enhanced through a resonant s-channel exchange of the heavy neutral Higgs A and H. (c) Focus point or hyperbolic branch regions, where a non-negligible higgsino fraction in the lightest neutralino is produced. In each of these regions the LFV rates emanating from the seesaw mechanism can be computed and contrasted with the sensitivity of direct searches at the LHC. Assuming the maximal mixing PMNS case, we find [57]

- *Coannihilation regions*: In these regions, which are mostly accessible at the LHC, an improvement of two orders of magnitude in the branching ratio sensitivity from the present limit, would make $\mu \to e\gamma$ visible for most of the parameter space, as long as $U_{e3} \gtrsim 0.02$, even for the low $\tan\beta$ region. For large $\tan\beta$, independent of $U_{e3}$, $\tau \to \mu\gamma$ will start probing this region provided a sensitivity of $\mathcal{O}(10^{-8})$ is reached.
- *A-pole funnel regions*: In these regions the LHC reach is not complete and LFV may be competitive. If $U_{e3} \gtrsim 10^{-2}$, the future $\mu \to e\gamma$ experiments, with limit of $\mathcal{O}(10^{-14})$ will probe most of the parameter space. As before, $\tau \to \mu\gamma$ will probe this region once the BR sensitivity reaches $\mathcal{O}(10^{-8})$.
- *Focus point regions*: Since the LHC reach in this region is rather limited by the large $m_0$ and $M_{1/2}$ values, LFV could constitute a privileged road towards SUSY discovery. This would require improvements of at least a couple of orders of magnitude (or more, depending on the value of $U_{e3}$) of improvement on the present limit of $\mathrm{BR}(\mu \to e, \gamma)$. Dark matter (DM) searches will also have partial access to this region in future, leading to a new complementarity between LFV and the quest for the cold dark matter constituent of the Universe.

### 5.3. Seesaw-induced Hadronic FCNC and CPV

So far we have seen that the SUSY version of the seesaw mechanism can lead to potentially large leptonic flavour violations, so large that they could compete even with direct searches at the LHC. If one combines these ideas of supersymmetric seesaw with those of quark–lepton unification, as in a supersymmetric GUT, one would expect that the seesaw resultant flavour effects would now

[18] For a bottom-up analyses, see [90].

also be felt in the hadronic sector, and vice versa [14, 15]. In fact, this is what happens in a SUSY $SU(5)$ with seesaw mechanism [72], where the seesaw-induced RGE effects generate flavour violating terms in the right-handed squark multiplets. However, as is the case with the MSSM + seesaw mechanism, within the $SU(5)$ model also, information from the neutrino masses is not sufficient to fix all the seesaw parameters; a large neutrino Yukawa coupling has to be *assumed* to have the relevant phenomenological consequences in hadronic physics, such as CP violation in $B \to \Phi K_s$, etc.

As we have already seen within the $SO(10)$ model, a large neutrino Yukawa, of the order of that of the top quark, is almost inevitable. Using this, it has been pointed in [71], that the observed large atmospheric $\nu_\mu$–$\nu_\tau$ transitions imply a potentially large $b \to s$ transitions in SUSY $SO(10)$. In the presence of CP violating phases, this can lead to enhanced CP asymmetries in $B_s$ and $B_d$ decays. In particular, the still controversial discrepancy between the SM prediction and the observed $A_{\mathrm{CP}}(B_d \to \Phi K_s)$ [91] can be attributed to these effects. Interestingly, despite the severe constraints on the $b \to s$ transitions from $B \to X_s, \gamma$ [92, 93], subsequent detailed analyses [94, 95] proved that there is still enough room for sizeable deviations from the SM expectations for CP violation in the $B$ systems. The reader interested in various correlations in $b \to s$ transitions with all possible FV off-diagonal squark mass entries can find an exhaustive answer in [95].

Finally, let us make a short comment about possible correlations between the hadronic and leptonic FV effects in a SUSY GUT. If the FV soft breaking terms appear at a scale larger than that of the grand unification, they must be related by the GUT symmetry. This puts constraints on the boundary conditions for the running of the FV soft parameters. From this consideration, one might intuitively expect that some correlation between various leptonic and hadronic FCNC processes [38] can occur at the weak scale. If in the evolution of the sparticle masses from the grand unification scale down to the electroweak scale, one encounters seesaw physics, then the quark–lepton correlations involving the left-handed sleptons, although modified, lead to even stronger constraints on hadronic physics [38, 39]; for some other related works see [96].

## 6. Alternatives to canonical seesaw and LFV

In the final section before conclusions, we briefly mention lepton flavour violating studies conducted in mechanisms other than the canonical seesaw. As earlier, our list is not exhaustive or detailed.

(i) *Type II seesaw*: In the $SO(10)$ example, as we have shown, one can consider a situation where the non-seesaw contribution (15) dominates over the seesaw one. Flavour violation in this case has been studied in specific models [97].

(ii) *Triplet seesaw*: The seesaw mechanism itself can be implemented with fermionic triplets instead of singlet fermions. In the supersymmetric context, the flavour violation is then proportional to the triplet couplings and the neutrino mixing angles [98].

(iii) *3 × 2 seesaw*: The seesaw parameter space drastically reduces when one of the neutrinos is assumed to be too heavy and decouples from the model. The predictive power of the model is now enhanced for various low- and high-energy observables [99]–[101].

(iv) *X-dimensional models*: In models with warped X-dimensional scenarios, large LFV is expected with either Dirac- [102] or Majorana-type neutrinos [103].

Flavour violation has also been computed in various other models as R-parity violation [104], seesaw models with additional leptons (or inverse seesaw) [105], left–right symmetric models [106] and SM seesaw with a large number of Higgs bosons [107].

## 7. Conclusions

Since the discovery of neutrino masses in atmospheric neutrino oscillations, there has been a number of activities trying to understand the seesaw mechanism and its low-energy implications. On the one hand, we use a bottom-up approach quantifying our ignorance on the seesaw parameter space to compute the LFV, while on the other, in top-down approach, various models and textures are constrained by LFV. Associated implications to $B$-physics GUT models, DM abundances in SUGRA theories and implications for LHC searches have been and are being done. Although we have not discussed it here, LFV in $Z$-decays [108] and other collider processes [109] is also being investigated.

Undoubtedly, the seesaw mechanism represents (one of ) the best proposal to generate small neutrino masses naturally. But how can we make sure that this is indeed nature's choice? Even establishing the Majorana nature of the neutrinos through a positive evidence of neutrinoless double beta decay, it will be difficult to assess that such Majorana masses come from a seesaw. Indeed, as we said at the beginning, in the SM seesaw, we expect very tiny charged LFV effects, probably without any chance of ever observing them. When moving to SUSY seesaw we add an important handle to our effort to establish the presence of a seesaw. In fact, as we tried to show in this work, SUSY extensions of the SM with a seesaw have a general 'tendency' to enhance (or even strongly enhance) rare LFV processes. Hence, the combination of the observation of neutrinoless double beta decay and of some charged LFV phenomenon would constitute an important clue for the assessment of SUSY seesaw in nature.

There is no doubt that after the discovery of the neutrino masses, among the indirect tests of SUSY through FCNC and CP violating phenomena, LFV processes have acquired a position of utmost relevance. It would be spectacular if, by the time the LHC observes the first SUSY particle, we could see also a muon decaying to an electron and a photon! After 30 years, we could have the simultaneous confirmation of two of the most challenging physics ideas: seesaw and low-energy SUSY.

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