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Reply to comment by Cao and Hu on "Long waves in erodible channels and morphodynamic influence"

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1. Introduction

[1] We thank *Cao and Hu* [2008] very much for their comment. We must honestly say that at first sight, we felt that their point, reproducing that raised by *Cao and Carling* [2003] to challenge the formulation of the 1-D governing equations proposed by *Lisle et al.* [2001], had already been proven to be erroneous by *Cui et al.* [2005]. However, after more careful examination of the matter, we have concluded that both the final forms of the continuity equation derived by *Cao and Hu* [2008] (hereinafter referred to as CH) and *Lanzoni et al.* [2006] (hereinafter denoted as LSFS) are not wholly correct. Indeed, while the main point raised by CH is definitely correct, they reach slightly incorrect conclusions, employing a framework which ignores the distinction between bed load and suspended load. On the other hand, the LSFS derivation is also incorrect in that it will be seen to ignore the defect of water flux due to the presence of particles transported as bed load, an effect which turns out to be of the same order of magnitude as that of the retained contributions. This effect modifies the final form of the continuity equation for the fluid phase which does indeed include a sediment transport correction, whose structure will be seen to be intermediate between the one derived by LSFS and that suggested by CH. While, as shown in section 4, this modification does not affect the treatment of one-dimensional long sediment waves presented by LSFS, the issue is of conceptual importance. We are therefore grateful to CH for motivating us to provide a hopefully more conclusive clarification of the matter. We are confident that the analysis presented below will be instructive for the reader, as it was for us.

2. Bed Load Versus Suspended Load

[2] Before we examine the main issue, it is preliminarily convenient to recall the physical distinction between suspended load and bed load, which plays an important role in the discussion below. We know from well-established field [*Drake et al.*, 1988] as well as laboratory [*Nelson et al.*, 1995] observations that the major difference between bed load and suspended load is the type of coherent turbulent wall structures able to entrain particles (''sweeps'' in the former case and ''bursts'' in the latter case). In other words, the above distinction is not conventional, rather it has a fundamental physical basis. The main distinguishing property of bed load is the fact that the dynamics of sediment particles is driven by, but distinct from, that of the fluid: a number of effects play a role, namely, fluid drag, shear- as well as rotation-induced lift, settling, and particle inertia. On the contrary, under the assumptions of sufficiently small particles and dilute suspensions, it is customary and rational to model transport in suspension by assuming that the particle velocity differs from the fluid velocity only because of the effect of particle settling: this leads to the well-known advection-diffusion equation for sediment suspensions.

3. Continuity Equation for the Fluid Phase and the Origins of the Distinct Incorrect Arguments of LSFS and CH

[3] We point out that (as also stated in LSFS) the present analysis is focused on describing only dilute suspensions of sediment particles transported both as bed load and as suspended load. Diluteness implies that $\hat{c}$, the instantaneous volume concentration of sediments, must be small. LSFS have taken advantage of the latter assumption by defining the local and instantaneous fluid velocity $\hat{\mathbf{u}}$ as a volume average that was performed by disregarding the presence of particles. This approximation, questioned by CH, is, in fact, neither necessary nor appropriate, though its influence on the analysis will be seen as crucial only when bed load is accounted for. Let us prove this statement by removing the above approximation. Hence, in the treatment presented below, the fluid velocity $\hat{\mathbf{u}}$ is taken to be a volume average performed accounting for the presence of particles. We will use the same notations as in Appendix A of LSFS. Mass conservation of the fluid phase requires that the following modified integral condition be satisfied:

$$\frac{\partial}{\partial t}\int_V (1-\hat{c})dV + \int_S (1-\hat{c})\hat{\mathbf{u}}\cdot\mathbf{n}dS = 0, \qquad (1)$$

where V is any finite control volume bounded by the surface S with outward normal $\mathbf{n}$.

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0043-1397/08/2007WR006188$09.00

[4] Applying the Ostrogradski transformation, the differential form of the continuity equation is found to read

$$-\frac{\partial \hat{c}}{\partial t} + \nabla \cdot [(1-\hat{c})\hat{\mathbf{u}}] = 0, \qquad (2)$$

and, averaging over turbulence and using letters without hat for the mean quantities,

$$-\frac{\partial c}{\partial t} + \nabla \cdot [(1-c)\mathbf{u}] - \nabla \cdot \langle \hat{c}' \hat{\mathbf{u}}' \rangle = 0 \qquad (3)$$

where angle brackets denote the average over turbulence and apex turbulent fluctuation. In equation (3) the effect of turbulent diffusion is also included. This contribution plays an important role in the context of 3-D modeling of suspensions, where vertical settling is dominantly balanced by the vertical component of turbulent diffusion. However, in the present context where we derive a depth-averaged formulation of the continuity equation, the only contribution which would be left in the depth-averaged equation is horizontal turbulent diffusion, a typically very small effect which will be neglected below for the sake of simplicity (as in CH). Let us then integrate (3) between the bed interface ($z = \eta(x, y, t)$) and the free surface ($z = h(x, y, t)$) to obtain

$$-\int_{\eta}^{h} \frac{\partial c}{\partial t} dz + \int_{\eta}^{h} \nabla \cdot (1-c)\mathbf{u} dz = 0. \qquad (4)$$

[5] Using the Leibnitz rule, equation (4) becomes

$$-\frac{\partial}{\partial t}\int_{\eta}^{h} c dz + \nabla \cdot \int_{\eta}^{h} (1-c)\mathbf{u} dz + \left[c\frac{\partial h}{\partial t} + (1-c)\mathbf{u} \cdot \nabla \mathcal{F}_h\right]_h - \left[c\frac{\partial \eta}{\partial t} + (1-c)\mathbf{u} \cdot \nabla \mathcal{F}_\eta\right]_\eta = 0, \qquad (5)$$

where $\mathcal{F}_h = z - h$ and $\mathcal{F}_\eta = z - \eta$. Let us now use the boundary conditions at the free surface and at the bed interface. They read

$$[(1-c)\mathbf{u} \cdot \mathbf{n}]_{z=h} = u_{nh}\left(1 - c|_{z=h}\right) \qquad (6)$$

$$[(1-c)\mathbf{u} \cdot \mathbf{n}]_{z=\eta} = u_{n\eta}[(1 - c|_{z=\eta}) - (1 - c_M)], \qquad (7)$$

where u_{nh} and $u_{n\eta}$ denote the normal components of the speed of the free surface and bed interface, respectively. Moreover, c_M is the solid concentration of the bed. Observing that

$$\mathbf{n}|_{z=h} = \frac{\nabla \mathcal{F}_h}{|\nabla \mathcal{F}_h|}, \qquad u_{nh} = -\frac{\partial \mathcal{F}_h/\partial t}{|\nabla \mathcal{F}_h|}, \qquad (8)$$

$$\mathbf{n}|_{z=\eta} = -\frac{\nabla \mathcal{F}_\eta}{|\nabla \mathcal{F}_\eta|}, \qquad u_{n\eta} = \frac{\partial \mathcal{F}_\eta/\partial t}{|\nabla \mathcal{F}_\eta|}, \qquad (9)$$

with the help of (6)–(7), equation (5) takes the final form

$$\frac{\partial}{\partial t}[(1-C)D] + p\frac{\partial \eta}{\partial t} + \nabla \cdot [(1-C)\mathbf{q}] = 0, \qquad (10)$$

where p is the bed porosity $(1 - c_M)$, $\mathrm{q} = D\,(U_x, U_y)$ is the volumetric fluid flux and capital letters denote depth-averaged quantities. Note that in equation (10) we have neglected further dispersive terms arising when the depth average of products $(u_j\, c)$ $(j = 1, 2)$ is performed. Equation (10) is the modified version of the 2-D form of the continuity equation for the fluid phase arising when the approximation questioned by CH is removed.

[6] It is convenient at this stage to recall the depth-averaged form of the equation of mass conservation for the sediment, which reads

$$\frac{\partial}{\partial t}(CD) + (1-p)\frac{\partial \eta}{\partial t} + \nabla \cdot \mathbf{q}_s = 0, \qquad (11)$$

where $\mathbf{q}_s$ is the total volumetric flux of sediment given by the sum of bed load and suspended load. Substituting from (11) into the continuity equation (10) for the fluid phase, we find

$$\frac{\partial D}{\partial t} + \frac{\partial \eta}{\partial t} + \nabla \cdot [(1-C)\mathbf{q} + \mathbf{q}_s] = 0. \qquad (12)$$

[7] Equation (12) shows that the presence of sediment transport is felt in the continuity equation for the fluid phase through the term $\nabla \cdot (\mathbf{q}_s - C\mathbf{q})$, i. e., through the difference between the volumetric flux of sediment and the defect of water flux due to the presence of particles transported both as bed load and as suspended load.

[8] The contribution of suspended load to this term vanishes in the limit of dilute suspensions of very small particles (smaller than the size of Kolmogorov viscous eddies). In fact, in this limit, as mentioned above, the horizontal component of particle velocity contributing to the sediment flux is identical with the horizontal component of the local fluid velocity. Hence, using the index s to denote the suspended load contributions to the depth-averaged concentration (C_s) and to the sediment flux ($\mathbf{q}_{ss}$) and further denoting the unit vector in the horizontal direction and the particle velocity by $\boldsymbol{i}_h$ and $\mathbf{v}_p$, respectively, we may write

$$\nabla \cdot (\mathbf{q}_{ss} - C_s\mathbf{q}) = \nabla \cdot \int_{\eta}^{h} c_s\left(\mathbf{v}_p - \mathbf{u}\right) \cdot \mathbf{i}_h dz = 0, \qquad (13)$$

i.e., $C_s\mathbf{q} = \mathbf{q}_{ss}$. CH, however, do not account for the fact that bed load does not satisfy equation (13) and apply the latter argument to the whole sediment concentration C, concluding incorrectly that the continuity equation for the fluid phase in the presence of sediment transport reads

$$\frac{\partial D}{\partial t} + \frac{\partial \eta}{\partial t} + \nabla \cdot \mathbf{q} = 0. \qquad (14)$$

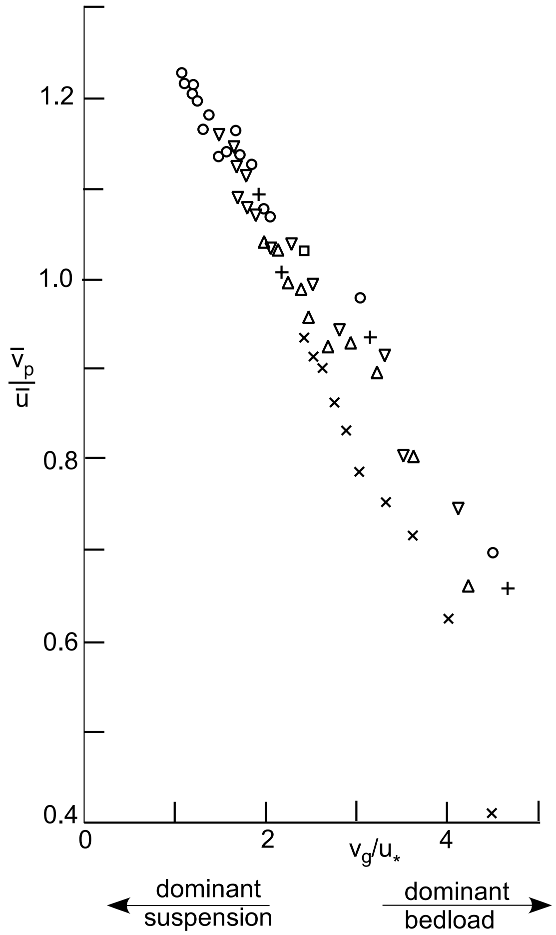

Figure 1. The ratio $\bar{v}_p/\bar{u}$ of the mean grain velocity, averaged over several trajectories, to the depth-averaged fluid velocity plotted versus the terminal settling velocity of grain in still water, v_g, scaled by the friction velocity u_* (adapted from *Abbott and Francis* [1977]). Dominant suspended load conditions tends to be achieved as $\bar{v}_p/\bar{u} \geq 1$.

[9] On the contrary, the contribution of bed load to the term $\nabla \cdot (\mathbf{q}_s - C\mathbf{q})$ does not vanish and may be written in the form

$$\nabla \cdot (\mathbf{q}_{sb} - C_b \mathbf{q}) = \nabla \cdot \int_\eta^h c_b (\mathbf{v}_p - \mathbf{u}) \cdot \mathbf{i}_h dz \neq 0, \qquad (15)$$

having used the index b to denote the bed load contribution to the depth-averaged concentration (C_b) and to the sediment flux ($\mathbf{q}_{sb}$). Note that the integral in equation (15) is negative as bed load particles experience a horizontal velocity smaller than the horizontal component of fluid velocity throughout their trajectory (see Figure 1).

[10] LSFS overestimate the latter contribution as they neglect the defect of water flux due to the presence of particles transported as bed load, namely, C_b $\mathbf{q}$: using the equation of mass conservation for the sediment transported as bed load (equation (10) with $C = C_b$ and $\mathbf{q}_s = \mathbf{q}_{sb}$) and neglecting the contribution $\partial(CD)/\partial t$, they express the quantity $\nabla \cdot \mathbf{q}_{sb}$ as $(p - 1)\partial\eta/\partial t$, concluding incorrectly that the continuity equation for the fluid phase in the presence of sediment transport reads

$$\frac{\partial D}{\partial t} + p\frac{\partial \eta}{\partial t} + \nabla \cdot \mathbf{q} = 0, \qquad (16)$$

namely, equation (A11) of LSFS.

[11] It is clear at this stage that the correct form of the continuity equation for the fluid phase in the presence of sediment transport has the intermediate form

$$\frac{\partial D}{\partial t} + \Pi\frac{\partial \eta}{\partial t} + \nabla \cdot \mathbf{q} = 0, \qquad (17)$$

with $p < \Pi < 1$. The coefficient Π is not constant but depends on the Shields parameter and particle Reynolds number and tends to 1 for high values of the Shields stress. Determining its form would require accurate calculations of

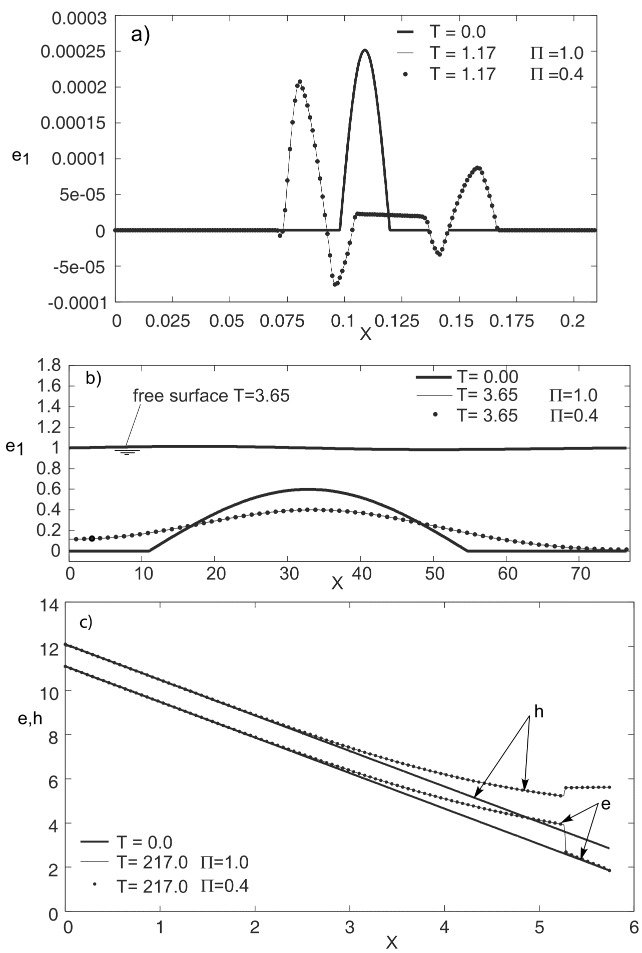

Figure 2. The numerical results reported in LSFS Figures 5, 6, and 7 (i.e., the nonlinear morphodynamic responses to an initial small perturbation in the form of either (a) a short or (b) a long hump and (c) the response to the propagation of a hydraulic jump), obtained for $\Pi = p$ (=0.4), are here compared with the calculations carried out setting $\Pi = 1$, as prescribed by CH equation.

the integral expression of equation (15), an endeavour which is outside the scope of the present discussion.

4. Actual Relevance of the Issue Raised by CH

[12] Having hopefully clarified the issue raised by CH, let us check its actual relevance on the numerical treatment of one-dimensional waves reported in section 2.2 of LSFS. Figure 2 shows the results of three simulations carried out by setting either $\Pi = p$ (=0.4) or $\Pi = 1$, reproducing the numerical examples of LSFS' Figures 5, 6, and 7. It clearly appears that the value attained by the coefficient Π does not affect the numerical simulations in any appreciable way. This result is not surprising. Indeed, the analysis carried out by *Lyn and Altinakar* [2002] already indicated that neglecting the terms $\Pi\partial\eta/\partial t$ and $\partial(CD)/\partial t$ in the mass balance equations for the fluid phase and the sediment, respectively, does not significantly affect the celerities at which disturbances of the water surface and the bed are propagated even close to critical flow conditions (i.e., when the Froude number approaches unity). On the other hand, the numerical investigations carried out by *Correia et al.* [1992] and *Cao et al.* [2002] suggest that the bed evolution term appearing in the continuity equation for the fluid phase may play a nonnegligible role only when considering long-term simulations of sedimentation and aggradation processes due to overloading. The latter issue, however, would require a careful analysis of the numerical treatments employed to reach the above conclusions, an endeavour which is outside the scope of the present note.

[13] Finally, we observe that in the case of intense sediment transport a number of further effects arise, e.g., hydrodynamic and possibly collisional interactions between sediment particles, which modify the structure of fluid turbulence and the relative motion between the two phases, thus invalidating the whole model. While a number of attempts have been proposed in the literature to describe the dynamics of highly concentrated sediment mixtures (see the vast literature concerning debris and mud flows), it is fair to state that this is as yet an unsolved problem which can hardly be treated on the basis of the approach discussed above.

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