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Are Household Portfolios Efficient? An Analysis Conditional on Housing

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Abstract

Standard tests of portfolio efficiency neglect the existence of illiquid wealth. The most important illiquid asset in household portfolios is housing: if housing stock adjustments are infrequent, optimal portfolios in periods of no adjustment are affected by housing price risk through a hedge term and tests for portfolio efficiency of financial assets must be run conditionally upon housing wealth. We use Italian household portfolio data and time series on financial assets and housing stock returns to assess whether actual portfolios are efficient. We find that housing wealth plays a key role in determining whether portfolios chosen by homeowners are efficient.

I. Introduction

There has been an increased interest in recent years in household portfolio choice. A number of country studies look at the way that households allocate their financial wealth across different financial instruments and find that a decreasing but still sizeable proportion of households fail to invest in the stock exchange (Guiso, Haliassos, and Jappelli (2002)).

Households allocate their wealth into financial and real assets, but the portfolio allocation problem has typically been addressed empirically by focusing solely on financial assets. A few studies extend the analysis to cover other forms of household wealth, notably own business (Heaton and Lucas (2000)) and housing equity (Flavin and Yamashita (2002), Cocco (2005)). Both assets are illiquid; that is, subject to non-negligible trading costs. These trading costs are likely to be particularly high for the housing stock for homeowners. When consumption and

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investment needs differ and rental markets are imperfect (Henderson and Ioannides (1983)), short-run adjustments can be all but impossible. Flavin and Yamashita (2002) stress that in this sense "demand for housing is overdetermined," and investment considerations may be of secondary importance.

In this paper, we address the issue of efficiency of household portfolios when illiquid housing wealth is also considered. This issue has been investigated by Grossman and Laroque (1990) and more recently by Flavin and Nakagawa (2004). Grossman and Laroque show that the standard CAPM holds in a dynamic setting when households derive utility from just one good that is durable and illiquid (and, therefore, infrequently adjusted). In their model, there are risky financial assets and also a risk-free asset: given that the numeraire is the durable good, this implies that the nominal return on this asset has unit correlation with the housing return. Flavin and Nakagawa's (2004) paper extends Grossman and Laroque's (1990) model by allowing for the presence of two goods in the utility function. In their model, there is no correlation between housing returns and financial asset returns. Flavin and Nakagawa prove that over those periods where the housing stock is not adjusted, all households hold a single optimal portfolio of risky assets (the standard Markowitz optimal risky portfolio), the standard CAPM holds, and housing wealth affects portfolio allocations only through the relative risk aversion of individual investors. A number of recent papers produce micro evidence on the role of housing on portfolio allocations within this framework, in which housing wealth contributes to background risk (Flavin and Yamashita (2002), Kullmann and Siegel (2003), Yamashita (2003), Le Blanc and Lagarenne (2004), Cauley, Pavlov, and Schwartz (2005), and Cocco (2005)).

We extend the analysis to cover the case where returns are correlated, and show how efficient financial portfolios should be after allowance is made for the presence of a given housing stock. In these portfolios, housing wealth affects the optimal shares in two distinct ways: indirectly via risk aversion and directly via a hedge motive. In particular, we observe that all households will hold a single optimal portfolio of risky assets (the standard Markowitz optimal portfolio) and a hedge term covering house price risk.

On the basis of our theoretical analysis, we expect optimally chosen financial portfolios not to be mean-variance efficient in the standard sense when asset and housing returns are correlated. Also, if the housing stock is not frequently adjusted, we also expect the overall portfolios (that include financial assets and housing wealth) not to be mean-variance efficient. However, we show that optimal portfolios should be conditionally mean-variance efficient, that is mean-variance efficient when housing wealth is treated as given but stochastic. Our analysis provides the economic rationale for implementing the conditional test of mean-variance efficiency that treats housing wealth as predetermined suggested by Gourieroux and Jouneau (1999).

Our paper builds upon recent work by Flavin and Yamashita (2002), but differs from it in a number of important respects. Flavin and Yamashita characterize the efficiency frontier for homeowners, when the house cannot be changed in the short run and there are non-negativity constraints on all assets. But they consider the case where financial returns are not correlated with housing returns and, therefore, the main effect of housing is to change the background risk faced by investors. We instead allow for nonzero correlation, and show that even without imposing non-negativity constraints the optimal portfolio changes because investors who are homeowners hedge housing price risk. We also formally test for the efficiency of household portfolios, and are able to show that many financial portfolios that appear inefficient when housing is neglected are instead efficient, but many others that appear efficient when the hedge term is neglected are instead inefficient.

Our paper is also closely related to Cocco (2005), who numerically derives solutions of an intertemporal optimization problem that includes a risk-free asset, one risky asset, housing, and human capital under borrowing and short sale restrictions. That paper is ideally suited to address the issue of limited participation in the risky assets market, but does not investigate how short-term financial portfolio decisions should be made to hedge housing risk. Cocco not only limits the investment set to just one risky asset, but he also assumes zero correlation between housing and financial asset returns, thus ruling our hedging motives. Our paper complements Cocco's analysis by showing how financial portfolios should be chosen at a given point in time, when housing wealth is given, and by investigating whether household portfolios are optimally chosen in the presence of housing wealth risk.

To our knowledge, our paper is the first that formally tests for the efficiency of household portfolios and investigates the role played by housing wealth in making portfolios more or less efficient. In particular, our paper shows to what extent households use financial assets to hedge the risk posed by their housing position.

In our application, we use Italian household portfolio data from the Bank of Italy Survey on Household Income and Wealth (SHIW) for 1998 and time-series data on financial asset returns as well as housing stock returns to test the hypothesis that observed portfolios are efficient. We show in our data that there are significant partial correlations between financial and housing returns, and we argue that similar patterns can be found both in other European countries and the U.S.

The paper is organized as follows. Section II presents the theory, Section III discusses the test statistic and econometric issues, Section IV describes the data used, Sections V and VI report efficiency test results, Section VII presents results from robustness analysis, and Section VIII concludes.

II. Theory

In this section, we show how housing wealth can be introduced in the standard mean-variance one-period model—in the Appendix we provide conditions under which our analysis is valid in a multiperiod model where housing is not just an investment good but also provides consumption services.

We will now derive an equation for optimal financial assets holdings in the static mean-variance analysis framework if the existing housing stock is treated as an additional constraint to the optimization problem (see Mayers (1973) and Anderson and Danthine (1981) for the general case where an asset is constrained).

Let us consider a market with a riskless asset, n unconstrained, and one constrained risky asset. Denote the first two moments of asset returns as $\underline{m}+r_f$ (where

$\underline{m}^T = (\underline{\mu}^T \mu_H)$ and $\underline{\mu}$ is the expected excess return) and Ω. The variance-covariance matrix for excess returns can be decomposed into four blocks, corresponding to the n unconstrained risky assets and the constrained risky asset as follows:

$$\Omega = \begin{bmatrix} \Sigma & \Gamma_{b,P} \\ \Gamma_{b,P}^T & \sigma_P^2 \end{bmatrix}. \tag{1}$$

Consider an investor whose portfolio allocation in the risky asset is:

$$Z = \begin{pmatrix} \underline{x_0} \\ h_0 \end{pmatrix}, \tag{2}$$

where

$$\underline{x_0} \equiv \frac{\underline{X_0}}{W_0} \quad \text{and} \quad h_0 \equiv \frac{H_0 P_0}{W_0},$$

and $(1 - Z)^T \underline{1}$ in the riskless asset ($\underline{1}$ is an $n + 1$ vector of ones). Assume that this investor is constrained in his h_0 (that is, h_0 is given and equal to $\overline{h}_0$), but otherwise behaves according to the mean-variance model. The investor problem becomes:

$$\begin{cases} \min\limits_Z Z^T \Omega Z \\ \text{s.t.} \begin{cases} Z^T \underline{m} + r_f = m^* \\ h_0 = \overline{h}_0 \end{cases} \end{cases}, \tag{3}$$

where m^* is a given level of expected return.

The problem can be solved by defining the Lagrangian:

$$\Lambda = \left(\underline{x_0} \Sigma \underline{x_0}^T + h_0^2 \sigma_P^2 + 2h_0 \underline{x_0} \Gamma_{bP} \right) - 2\gamma \left[\underline{x_0}\, \underline{\mu} + h_0 \mu_H + r_f - m^* \right]. \tag{4}$$

The first-order conditions are:

$$\frac{\partial \Lambda}{\partial \underline{x_0}} = \left(2\Sigma \underline{x_0}^T + 2h_0 \Gamma_{bP} \right) - 2\gamma \left[\underline{\mu} \right] = 0, \tag{5}$$

$$\frac{\partial \Lambda}{\partial \gamma} = \underline{x_0}\, \underline{\mu} + h_0 \mu_H + r_f - m^* = 0. \tag{6}$$

The solution is:

$$\underline{x_0} = \gamma \Sigma^{-1} \underline{\mu} - h_0 \Sigma^{-1} \Gamma_{bP}, \tag{7}$$

where γ is the Lagrange multiplier of the constraint on the expected return, which has the standard relative risk aversion interpretation (Samuelson (1970)).

This result means that investors have to be efficient with respect to the risky financial assets and choose the efficient Markowitz portfolio according to their risk aversion (see Markowitz (1992)). However, they also use the risky financial assets to hedge their exposure on the constrained asset. If $\Gamma_{bP} = 0$ the hedge term vanishes and portfolio choice can be separated between financial and real assets.

III. Econometric Issues

In Section II, we saw that the notion of efficiency of household portfolios depends on the assumption that we make on the nature of housing investment. If housing investment is costless, then the efficient frontier should be computed using all financial asset returns as well as the return on housing.[1] If transaction costs affect housing investment, then the analysis differs according to the correlation between housing and financial returns. If this correlation is zero, household portfolios will be mean-variance efficient in the usual sense (i.e., with respect to the standard financial assets frontier). If this correlation is instead nonzero, household portfolios will be mean-variance efficient once we condition on the value of the housing stock as shown in equation (7).

In this section, we show how we can test for the efficiency of the observed household portfolios in all cases discussed above. To do this, we use time-series data on asset returns for a period prior to the survey to estimate the mean-variance frontier, taking into account the theoretical assumptions of rational expectations and normal return distributions. In particular, we use weighted sample means and covariances to estimate expected excess returns and risk (i.e., the first two unconditional moments). The weights are a declining function of the time distance from the end of the sample period.[2]

In the vast literature on efficient portfolios, only a few papers incorporate real estate as an asset. Goetzmann and Ibbotson (1990) and Goetzmann (1993) use regression estimates of real estate price appreciation, and Ross and Zisler (1991) calculate returns from real estate investment trust funds to characterize the risk and return to the real estate investment. Flavin and Yamashita (2002) use data from the 1968–1992 waves of the Panel Study of Income Dynamics that contain records on the owner's estimated value of the house and compute rates of return from regional real estate price data.

Mean-variance efficiency is usually assessed on the basis of a graphical comparison. However, Jobson and Korkie (1982), (1989) and Gibbons, Ross, and Shanken (1989) have proposed a test of the significance of the difference between the actual portfolio held by an investor and a corresponding efficient portfolio. This test is based on the difference between the slopes of arrays from the origin through the two portfolios in the expected return standard deviation space. If the actual portfolio is an efficient portfolio, the two slopes will be the same; if the actual portfolio is inefficient, the slope of the efficient portfolio will be significantly greater.

Gourieroux and Jouneau (1999) derive efficiency tests for the conditional or constrained case, i.e., for the case where a subset of asset holdings is potentially constrained (housing in our case). They define the Sharpe ratio of the unconstrained risky financial assets portfolio as:

$$S_1 = \underline{\mu}^T \Sigma^{-1} \underline{\mu}. \tag{8}$$

[1]Housing can be neglected if its return is spanned by financial assets.

[2]Another way is to consider the first two conditional moments from a time-series model of the returns data that allows for time-varying conditional heteroskedasticity as in Blake (1996). This modeling framework requires a long time series.

The Sharpe ratio for the observed (constrained) portfolio made of the first n (financial) assets is defined in this notation as:

$$S_1(Z) \quad = \quad \frac{\left[\underline{\mu}^T v_1\right]^2}{v_1^T \Sigma v_1}, \tag{9}$$

where $v_1^T = \underline{x}_0^T + h_0 \Sigma^{-1} \Gamma_{bP}$ (see equation (7)), that is the actual risky financial asset portfolio after eliminating the hedge term.

When all asset returns are normally distributed, Gourieroux and Jouneau (1999) show that the Wald statistic:

$$\xi_1 \quad = \quad T \frac{\hat{S}_1 - \hat{S}_1(Z)}{1 + \hat{S}_1(Z) \dfrac{Z^T \Omega Z}{v_1^T \Sigma v_1}}, \tag{10}$$

is distributed as a $\chi^2(n-1)$ under the null hypothesis that the risky financial assets portfolio (after eliminating the hedge term) lies on the financial efficient frontier.[3]

Gourieroux and Jouneau also show that a test for the efficiency of the whole portfolio can be derived as a special case by setting $v_1 = Z$. The test statistic becomes:

$$\xi_e \quad = \quad T \frac{\hat{S} - \hat{S}(Z)}{1 + \hat{S}(Z)}, \tag{11}$$

where:

$$\hat{S} \quad = \quad \underline{m}^T \Omega^{-1} \underline{m} \quad \text{and} \quad \hat{S}(Z) \quad = \quad \frac{[\underline{m}^T Z]^2}{Z^T \Omega Z}.$$

ξ_e is distributed as a $\chi^2(n)$ under the null hypothesis that the mean and standard deviation of the observed portfolio lie on the efficient frontier. In this special case, the test is asymptotically equivalent to the test derived by Jobson and Korkie (1982), (1989) and Gibbons, Ross, and Shanken (1989).

The intuition behind the conditional (constrained) test is the following. The standard test for portfolio efficiency is based on (the square of) the Sharpe ratio. The Sharpe ratio is in fact the same along the whole efficient frontier (with the exception of the intercept), that is, along the capital market line. This test breaks down when one asset is taken as given because the efficient frontier in the mean-variance space corresponding to all assets is no longer a line, but rather a curve. However, equation (7) implies that we can go back to the standard case when the analysis is conducted conditioning on a particular asset, once the hedge term component is subtracted from the observed portfolio. That is, a Sharpe ratio can be used to test for efficiency in the mean-variance space corresponding to the "unconstrained" assets after allowance has been made for the presence of the same hedge term in all efficient portfolios.

It is worth stressing that the test statistic is based on the square of the Sharpe ratio, thus portfolios with Sharpe ratios of the same magnitude but opposite sign

[3]For the sake of simplicity, we do not stress in our notation that the test statistic is defined as a function of sample estimates of the first two moments of the rates of return distribution and takes observed portfolio shares as given.

are treated in the same way. In our empirical application of the constrained case, we will treat as inefficient those portfolios that have a negative excess return. [4]

In our empirical analysis, we compute efficiency test statistics (either ξ_e or ξ_1) for each household in our sample. In particular, we compute the standard test (ξ_e) twice: once for the financial portfolio (as in standard practice), and once for the whole portfolio (inclusive of housing). In this latter case, we also compute the constrained test (ξ_1).

We use the computed test statistics in two different ways. First, we show which proportions of household portfolios fail the efficiency tests for a range of possible test sizes (from 0.10 to 0.01).[5] Second, we regress the computed test statistic (ξ_1) on household characteristics, income, and housing wealth as a way to investigate possible causes for inefficient portfolio allocations.

IV. The Data

To show the implications of our theoretical analysis, we use data on Italian asset returns and household portfolios. Italy provides a good test case to study the effect of housing on portfolios because home ownership is widespread and household stock market participation is relatively low, but has much increased in recent years. As we will see, in Italy housing returns unambiguously correlate with financial returns, thus providing the need for a hedge term in homeowners' portfolios. Finally, an attractive feature of Italy for our purposes is that pension wealth, whose amount is typically not recorded in survey data, is still almost entirely provided by the public pay-as-you-go social security system and, therefore, is both out of individual investors' control and not directly related to the financial markets performance.

Italian households traditionally have held poorly diversified financial portfolios (Guiso and Jappelli (2002)). In the 1980s and even more in the 1990s though, the stock exchange has grown considerably and mutual funds have become a commonly held financial instrument. Household financial accounts reveal that the aggregate financial portfolio share in stocks and funds amounted to 16.15% in 1985, 20.69% in 1995, and rose to an unprecedented 46.95% in 1998. This growth in the equity market paralleled the sharp decrease in the importance of bank accounts and short-term government debt in household portfolios. These aggregate statistics are uninformative on the participation issue, though. To this end, an analysis of survey data is required. The most widely used Italian survey data, the Bank of Italy-run SHIW, shows direct or indirect participation in equity markets (broadly defined to include life insurance, private pensions, and own business) to have increased from 26.43% in 1985 to 38.19% in 1995 and to 48.24% in 1998. For comparison, the percentage of homeowners in the same sample hovered around 63%–65% over the period. Finally, the share of financial to total wealth in SHIW was 11.7% in 1991 and rose to 14.59% in 1998—housing wealth accounted for

[4] We do this after checking that portfolios with the same standard deviation and excess returns just above zero are indeed counted as inefficient by the formal test.

[5] Throughout the paper, we use the term "test size" to denote the probability of type I error (probability of rejecting the null hypothesis when the null is true). This is sometimes known as significance level.

68.91% of total wealth in 1991 and fell slightly to 65.81% in 1998 (50.11% to 48.84% if we consider the principal residence only).

These summary statistics clearly show that household financial portfolios have changed a great deal over the years, and that real estate plays a key role in total household wealth. It makes sense to consider the interaction of housing and financial wealth holdings when assessing the efficiency of household portfolios. A financial portfolio may deviate from the mean-variance frontier for financial assets simply as a result of its covariance properties with the return on housing equity. A relevant issue is whether housing wealth is treated as a liquid or as an illiquid asset.

In our application, we use household portfolio data for 1998 and asset return data for the period 1989–1998. The 1998 SHIW wave contains detailed information on asset holdings of 7,115 households as of 31.12.1998, as well as self-assessed value of their housing stock (both principal residence and other real estate) and actual or imputed rent for each dwelling. For each household, we also know the region of residence and a number of demographic characteristics (that are used to characterize departures from efficiency). The survey does not oversample the very rich, and it therefore captures about a third of total household financial wealth. It does cover a relatively large number of assets, including individual pension funds: these are still remarkably unimportant in Italy, partly because of inadequate tax incentives. Occupation pension schemes are also relatively minor, even though recent reforms of the Italian Social Security system (particularly the Dini reform of 1995) imply that they should become widespread.[6]

Asset return data cover five major assets: short-term government bonds (three-month BOT), medium-term government bonds, long-term government bonds (BTP), corporate bonds, and equity (the MSCI Italy stock index).[7] We treat the short-term bond as risk free, and assume that this is the relevant return on bank deposits, once account is taken of non-pecuniary benefits. For medium-term, long-term, and corporate bonds, we derive the holding period returns by standard methods. In particular, for medium-term we use the RENDISTAT index (the index of the medium-term government bonds yields) and we determine the holding period return by assuming a duration of two years. For corporate bonds, we use the RENDIOBB index (the index of Italian corporate bond yields) and assume a duration of three years. For long-term bonds we use the estimated term structure of interest rates and determine the holding period returns of an equally weighted portfolio based on two assets with a duration of three years and five years. We checked the quality of this estimation by regressing our monthly returns determined with this procedure on those of the MSCI Italian bond index (that are only available since December 1993) and found that the fit is almost perfect (R^2 is equal to 99.62%).

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[6]Further information on the survey is provided in Guiso and Jappelli (2002) and D'Alessio and Faiella (2000). Information on the Italian pension system and its recent reforms is presented in Brugiavini and Fornero (2001).

[7]We take returns on the Italian stock exchange because, as we will see later, direct investment in foreign assets is rare in our data. We have checked that our results are robust to assuming that roughly half of indirect investment in stocks is held in the MSCI world market index, in line with aggregate statistics on Italian mutual funds portfolios in 1998.

We express all returns net of withholding tax on the assumption that for most investors other tax distortions are relatively minor (financial asset income is currently subject to a 12.5% withholding tax. Housing is taxed on the basis of its ratable value, while dividends on stocks directly held and actual rental income is taxed at the marginal income tax rate).

To evaluate the efficiency of the households' portfolio, we need to determine the expected return and the expected variance-covariance matrix of the assets. Given long, stationary series, we could simply compute the corresponding sample moments of the asset excess returns. However, this approach is unlikely to work in our case: our sample period is 1989–1998 (and cannot be extended because some assets did not exist prior to the mid-1980s), and in the decade we consider we observe a long convergence process of Italian interest rates to German interest rates that accelerated dramatically in the few years before the introduction of the Euro on 1st January 1999.

Estimation error is of particular concern for first moments and calls for the use of prior information in estimation (see, for instance, Merton (1980) and Jorion (1985)). In our case, we should estimate the first moments by a Bayesian method that exploits prior information on the convergence of particularly long-term government bond rates to its German equivalent, and possibly a multivariate GARCH for the second moments. Unfortunately, we do not have enough data points to perform sophisticated estimation exercises. In fact, housing returns are available at a semiannual frequency, and we are therefore forced to use at most 21 data points. However, we can exploit prior information on convergence by using a simple weighted least squares procedure, where the raw return series data are down weighted more the farther away they are from December 1998. More precisely, we construct the weights to be a geometrically declining function of the lag operator multiplied by α (where α is set to 0.8). The weights are then multiplied by a constant so that the expected returns on long-term government bills are in line with the actual returns of the German Treasury bond in 1998–1999. The weighted series are used to compute sample first and second moments.[8]

Table 1 shows the first and second moments of the excess returns data we use. These are expressed as percentage semiannual rates of return net of the time-varying risk-free rate: for the risk-free rate, we report only the January 1999 six-month Italian Treasury bill interest rate.

We see that stocks have higher expected return and higher variance than all other risky financial assets. Correlation coefficients between bonds are quite high (they range between 0.84 and 0.97)—correlation coefficients of stocks and bonds are much lower (between 0.38 and 0.64). Most correlation coefficients are significantly different from zero at the 1% level.

This picture is, however, largely incomplete. We know that two households out of three own real estate, and we argue that this type of investment is highly illiquid. It is, therefore, of great interest for us to compute first and second moments of the housing stock. To this end, we use province level semiannual price data (source: Consulente Immobiliare[9]) covering the whole 1989–1998 period.

[8] A similar procedure for second-order moments is often used in the financial industry (see RiskMetrics (1999)) and can be shown to be equivalent to particular GARCH models (Phelan (1995)).

[9] Index constructed using repeat sales of houses.

TABLE 1

Sample First and Second Moments of Asset Excess Returns (1989–1998)

Panel A of Table 1 shows the descriptive statistics for the semiannual weighted excess returns of the four risky assets used in the analysis as well as the risk-free rate that refers to January 1, 1999. Panel B reports correlation coefficients of the same excess returns. ** indicates significant at the 1% level.

	BOT	BTP	MTG-Bonds	Corporate Bonds	Stocks
Panel A. Descriptive Statistics					
Expected return %	1.3169	0.8021	0.4270	0.4495	2.2932
Standard deviation %		1.2223	0.6469	0.7809	7.4875
Panel B. Correlation Coefficients					
BTP			0.965**	0.842**	0.379
MTG-bonds				0.871**	0.383
Corporate bonds					0.635**

We compute the return on housing by assuming that rent minus maintenance costs is a fixed proportion, κ, of the house price. We set $\kappa = 0.025$ (5% on an annual basis), as in Flavin and Yamashita (2002). It is worth stressing that the choice of κ is immaterial in the analysis of the constrained case as long as κ is a fixed number (see equation (10)), but is important in the case where housing is treated as unconstrained given that it affects its expected return directly.

Finally, we aggregate housing returns to the macro region level (provincial resident population numbers were used to generate weights). This way we generate average return data for the North West (NW), North East (NE), Centre (CE), and South (SO).[10] The first and second moments are then determined using the weighted least squares procedure described above.

Table 2 reveals that expected excess returns on housing are highest in the NE and in the SO and lowest in the CE (they range between 0.73% and 0.61% on a semiannual basis). They are close to returns on bonds, but are much lower than returns on stocks. Housing excess return standard deviations range between 0.46% and 0.78%, and are therefore much lower than on stocks, but comparable to government and corporate bonds. Of interest to us is the negative correlation between housing return and most financial asset returns. This is also found in the raw series, that is, in the series that are not weighted in the way described above.

The issue arises of whether these correlations are negligible. Some of the simple correlation coefficients are significantly different from zero (for the NW and CE regions). But simple correlation is not the relevant concept for our analysis: partial correlations are important in a multiple asset setting. The simplest way to assess the relevance of partial correlations is to estimate the coefficients of the hedge term in equation (17), that is, to estimate the beta hedge ratio $\Sigma^{-1}\Gamma_{bP}$. This can be done by running the regression of housing returns on financial asset returns as suggested by de Roon, Eichholtz, and Koedijk (2002). In our case, we use WLS instead of OLS for internal consistency, but stress that OLS point esti-

[10]This is a standard split. The North West includes the three large industrial cities of Milan, Turin, and Genoa; the North East includes many middle-sized cities and towns such as Bologna, Venice, Verona, and Trieste; the Centre includes the capital city, Rome, and many medium-sized town such as Florence, Perugia, and Ancona. In our classification, the South, which is largely rural despite the presence of important cities such as Naples and Bari, also includes the two large islands, Sicily and Sardinia.

TABLE 2

Expected Excess Returns and Correlation Matrix of Housing (1989–1998)

Panel A of Table 2 shows the descriptive statistics for the semiannual weighted excess returns of housing for the four macro regions. Panel B reports correlation coefficients of the same excess returns with financial asset excess returns. * indicates significant at the 5% level; ** indicates significant at the 1% level.

	NW	NE	CE	SO
Panel A. Descriptive Statistics				
Expected excess return %	0.6143	0.7108	0.6517	0.7303
Standard deviation %	0.7816	0.4607	0.5439	0.4986
Panel B. Correlation Coefficients				
BTP	0.018	−0.140	−0.237	−0.274
MTG-bonds	−0.0752	−0.246	−0.355	−0.142
Corporate bonds	−0.150	−0.086	−0.524*	−0.245
Stocks	−0.671**	−0.270	−0.675**	0.031

mates are similar.[11] Parameter estimates and their standard errors are summarized in Table 3.

TABLE 3

Regression of Excess Return on Housing on Financial Asset Excess Returns

Table 3 shows estimation results from the regression of each macro region housing excess return on financial asset returns. The p-value refers to the F-test of joint significance of all slope parameters. R^2 is the unadjusted coefficient of determination. Standard errors are in parentheses. Number of observations = 21.

Variable	NW	NE	CE	SO
Constant	−0.00141	−0.00030	−0.00041	−0.00037
	(0.00107)	(0.00095)	(0.00095)	(0.00106)
r_{BTP}	0.928275	0.559817	0.714788	−0.82673
	(0.28242)	(0.24867)	(0.25063)	(0.27920)
r_{MTG}	−2.38735	−1.88857	−1.29275	2.014983
	(0.60929)	(0.53646)	(0.54069)	(0.60233)
r_{BONDS}	1.080841	0.848318	−0.11126	−0.73687
	(0.3165)	(0.27867)	(0.28087)	(0.31289)
r_{STOCKS}	−0.12004	−0.04496	−0.04314	0.03540
	(0.01756)	(0.015464)	(0.01559)	(0.01736)
p-value	0.000034	0.014788	0.001414	0.02506
R^2	0.784422	0.518884	0.649422	0.482323

We see that in all regions there is at least one nonzero parameter at the 5% significance level and the slope coefficients are jointly significantly different from zero at the 5% level or lower (the p-value of the F-test is reported at the bottom of the table, together with the R^2). The region where this test is least significant is the SO (with a p-value of 2.5%).

On the basis of this evidence, we conclude that housing returns present significant correlations with financial asset returns in Italy, and that this provides the basis for introducing a hedge term in household portfolios of homeowners.[12]

[11] Significant coefficients retain their signs, but their magnitude and standard errors are inflated.

[12] In our analysis, we assume that the relevant stock return is purely domestic. However, according to Bank of Italy aggregate statistics, Italian equity mutual funds invested 52% in foreign stocks and 48% in the domestic stock exchange in the fourth quarter of 1998. Unfortunately, we do not know how household indirect equity holdings were split between domestic and foreign stocks or across countries, but we can run a robustness check. Given that direct stock holdings by SHIW households are mostly domestic, we assume a 50-50 domestic-foreign split in household portfolios, and take as

In Tables 4 and 5, we report the percentage participation for each asset and liability recorded in SHIW98 and the corresponding aggregate portfolio share. For instance, we see that almost 75% of the sampled households have a current bank (i.e., checking) account, and that 27.24% of all financial wealth is held in such accounts. We also show in the last column of Table 4, where each asset is classified, given that we use asset returns data at a much coarser aggregation level. So the first seven assets (cash, various deposits, and repos) are all classified as risk free. Of particular interest is the relatively low direct stock market participation (7.42% hold listed shares; 1.58% hold shares in unlisted companies).

TABLE 4

Participation Decision: Individual Financial and Real Assets

Table 4 shows the proportion of households reporting positive holdings of each asset recorded in SHIW98, as well as the way each asset is classified for the purpose of our efficiency analysis.

Asset	Participation	Broad Asset
Cash	100.00%	Risk-free
Bank Current Account Deposits	74.94%	Risk-free
Bank Savings Deposits (Registered)	19.31%	Risk-free
Bank Savings Deposits (Bearer)	10.90%	Risk-free
Certificates of Deposit	3.68%	Risk-free
Repos	0.94%	Risk-free
Post Office Current Accounts & Deposit Books	11.43%	Risk-free
Post Office Savings Certificates	6.55%	Long-term
BOT (Italian T-bills)	9.67%	Risk-free
CCT (Italian T-certificates)	4.74%	Risk-free
BTP (Italian T-bonds)	2.70%	Long-term
CTZ (Italian zero-coupon)	0.78%	Medium-term
Other Italian Government Debt (CTEs, CTOs, etc.)	0.31%	Medium-term
Corporate Bonds	5.55%	Bonds
Mutual Funds	10.86%	Bonds (1/2) Stocks (1/2)
Shares of Listed Companies	7.42%	Stocks
No. of Which are Privatized	4.30%	Stocks
Shares of Unlisted Companies	1.58%	Stocks
Shares of Limited Liability Companies	0.53%	Stocks
Shares of Partnerships	0.15%	Stocks
Managed Savings (by banks)	2.03%	Bonds (1/2) Stocks (1/2)
Managed Savings (by other financial intermediaries)	0.5%	Bonds (1/2) Stocks (1/2)
Managed Savings by Trust Companies	0.06%	Bonds (1/2) Stocks (1/2)
Foreign Bonds & Government Securities	0.52%	Bonds (1/2) Stocks (1/2)
Foreign Stocks and Shares	0.46%	Bonds (1/2) Stocks (1/2)
Other Foreign Assets	0.05%	Bonds (1/2) Stocks (1/2)
Loans to Cooperatives	1.67%	Stocks
House	69.76%	House
Mortgage	10.41%	Long-term (neg. position)
Debt	12.33%	Bonds (neg. position)

However, 10.86% of all households have mutual funds, and these holdings we classify partly as stocks and partly as bonds. Of great interest to us is the high proportion of households that own some housing stock (almost 70%) and the magnitude of this type of investment (which accounts for 85% of total wealth, see Table 5). Liabilities are relatively widespread (10.41% of the households report

stock return the average of the Italian stock exchange return and the MSCI world stock index return (in local currency). The simple correlation between the domestic return and this mixed stock return is 0.88; compared to the domestic return, the mixed return has a lower first moment (2.08% rather than 2.29%) and also a lower standard deviation (5.22% rather than 7.49%) resulting in a larger Sharpe ratio. The key regressions of housing returns on financial asset returns produce results quite similar to those shown in Table 3: the coefficients on the stock return lie in the $(-0.07, -0.14)$ interval for the NW, NE, and CE, and are all significant. For the SO, we find a significant, positive coefficient of 0.07. The rest of our analysis is largely unaffected. We thank the referee for suggesting this check.

TABLE 5

Portfolio Share: Individual Financial and Real Assets

Table 5 shows the aggregate portfolio shares for each asset recorded in the SHIW98. They are defined relative to financial wealth or total wealth (the sum of financial wealth, housing wealth net of mortgage and debt).

Asset	Portfolio Share	
	Financial Wealth	Total Wealth
Cash	2.13%	0.31%
Bank Current Account Deposits	27.24%	2.86%
Bank Savings Deposits (Registered)	4.94%	1.00%
Bank Savings Deposits (Bearer)	2.75%	0.48%
Certificates of Deposit	2.52%	0.50%
Repos	1.19%	0.25%
Post Office Current Accounts & Deposit Books	2.54%	0.38%
Post Office Savings Certificates	2.00%	0.31%
BOT (Italian T-bills)	7.64%	1.22%
CCT (Italian T-certificates)	3.92%	0.58%
BTP (Italian T-bonds)	2.14%	0.37%
CTZ (Italian zero-coupon)	0.31%	0.06%
Other Italian Government Debt (CTEs, CTOs, etc.)	0.34%	0.04%
Corporate Bonds	4.92%	0.74%
Mutual Funds	13.99%	2.25%
Shares of Listed Companies	5.90%	0.99%
No. of Which are Privatized	1.86%	0.29%
Shares of Unlisted Companies	0.77%	0.12%
Shares of Limited Liability Companies	2.19%	0.26%
Shares of Partnerships	1.30%	0.16%
Managed Savings (by banks)	6.62%	1.23%
Managed Savings (by other financial intermediaries)	1.53%	0.31%
Managed Savings by Trust Companies	0.04%	0.01%
Foreign Bonds and Government Securities	0.25%	0.08%
Foreign Stocks and Shares	0.14%	0.03%
Other Foreign Assets	0.00%	0.00%
Loans to Cooperatives	0.80%	0.14%
House		85.05%
Mortgage		−2.07%
Debt		−0.54%

mortgages; 12.33% report other forms of consumer debt), but their quantitative importance is relatively minor.

In Table 6, we treat mortgages as negative holdings of long-term bonds (the only long-term bonds available are on government debt, BTP) and other debt as negative holdings of corporate bonds (other debt typically has medium-term maturity like corporate bonds). On this basis, we reclassify our households into four mutually exclusive groups. We then show how this classification changes according to the macro region. We see that the highest proportion of risk-free asset portfolios (30.1%) is found in the SO and the lowest in the CE (24.2%). The combination of risk-free and housing assets is highest in the SO (49.4%) and lowest in the NW (33.8%). The combination of risk-free and risky financial assets (included debts) is most common in the NE (5.6%), whereas the presence of all three assets is most common in the NE (36.2%) and least common in the SO (only 18.4%).

V. Estimation and Test Results: Standard Analysis

First, we show the mean-variance frontier for financial assets alone, using the returns information described in Section IV. We follow the literature and neglect both housing wealth and mortgages, and debts. Given that the latter are mostly

TABLE 6

Classification by Region

Table 6 shows the number and proportion of households holding various combinations of assets in each macro region and in the country as a whole (total). Mortgages and debt are treated as negative positions in risky assets.

	Total		NW		NE		CE		SO	
	N	%	N	%	N	%	N	%	N	%
Risk-Free Asset	1,567	26.47%	385	27.13%	217	20.43%	291	24.25%	674	30.10%
Risk-Free Asset + Housing	2,499	42.21%	479	33.76%	402	37.85%	511	42.58%	1,107	49.44%
Risk-Free + Risky Assets	223	3.77%	78	5.50%	59	5.56%	41	3.42%	45	2.01%
Risk-Free + Risky Assets + Housing	1,631	27.55%	477	33.62%	384	36.16%	357	29.75%	413	18.45%
Total Assets	5,920	100.00%	1,419	100.00%	1,062	100.00%	1,200	100.00%	2,239	100.00%

incurred to purchase housing stock, this is the most natural course of action when analyzing purely financial decisions.

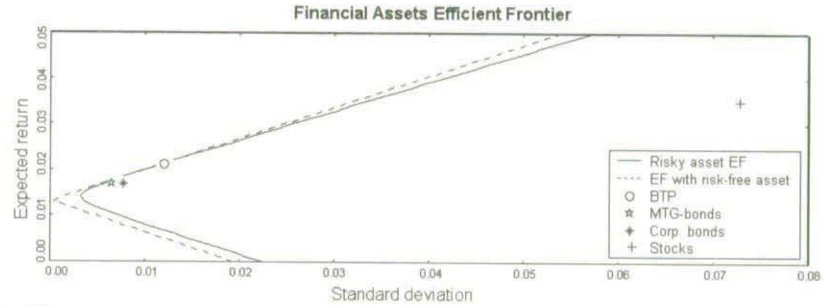
In Graph A of Figure 1, we show the risky financial assets efficient frontier and the efficient frontier with the risk-free asset (a broken line). Individual assets are also displayed: to the far right we have stocks (a + sign), to the extreme left of the risky frontier we find corporate bonds (an *). In Graph B, we show where individual portfolios lie. Notice that households which have a financial portfolio are 5,920 in total: 76.92% of these only have the risk-free asset while 23.08% also have risky assets.

FIGURE 1

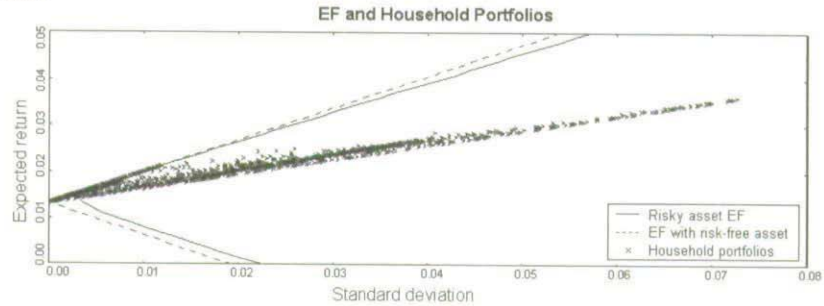
Financial Asset Efficient Frontier

Graph A of Figure 1 shows the four individual asset expected returns (y-axis) and standard deviations (x-axis) as well as the risky financial asset efficient frontier and the efficient frontier with the risk-free asset. Graph B shows, on the same scale, the efficient frontiers as above and the observed household portfolios.

Graph A



Graph B



The tangency of the upper portion of the broken line and the risky financial assets financial frontier define the market portfolio. In Table 7, first column, we report its weights: the mean-variance efficient portfolio is made of long positions in BTP (long-term government bonds), MTG bonds and stocks, and a short position in corporate bonds.[13]

TABLE 7

Tangency Portfolio Weights

Table 7 reports the tangency portfolio weights for the case when wealth is made of financial assets alone (column 1) and when wealth includes housing and debt (columns 2, 3, 4, and 5). Different tangency portfolios are computed for the four macro regions (NW, NE, CE, and SO).

		Financial Assets and Housing			
	Financial Assets	NW	NE	CE	SO
BTP	0.2923	−0.5462	−0.3324	−0.3829	1.4222
MTG-Bonds	0.8932	1.5346	1.21856	0.7724	−3.099
Corporate Bonds	−0.2030	−0.6615	−0.5252	0.0476	1.1269
Stocks	0.0175	0.0735	0.02842	0.0243	−0.052
House	—	0.5995	0.61071	0.5386	1.6027

We run the formal efficiency test (described in Section III) on observed household portfolios. The test statistic is computed for all valid observations (households whose wealth is not entirely in the risk-fee asset) and the percentages of non-rejections are computed at different values of the test size (from 10% to 1%). In this case, we can compute the test statistic for 1,366 households that have at least one risky asset. We find that 612 such households hold efficient portfolios when the test size is set at 10%. When the test size is set at 5% or 1%, all 1,366 portfolios are considered efficient.

It is perhaps surprising that all portfolios are considered efficient when the test is run at the 5% or 1% levels. This probably reflects three different facts: i) most households do not invest in stocks, in line with the tangency portfolio; ii) returns on bonds are highly correlated—optimization errors on their shares do not result in major efficiency losses; and iii) the efficient frontier is estimated using a small number of observations and is therefore estimated with limited precision. All this suggests that the test may have relatively low power, and the appropriate test size should be chosen at a conservative 10%.

Markedly different conclusions on the efficiency of household portfolios are reached if the investment set is extended to housing, and housing is treated as any other asset (that is, it is treated as unconstrained). We find that at any size of the test, there are very few efficient portfolios.

To understand why this happens, we show in Figures 2 and 3 the mean-variance frontier for financial assets and housing for two macro regions (given that we know where the households are and that house prices differ by region, we compute the relevant statistics for each macro region). We now treat outstanding

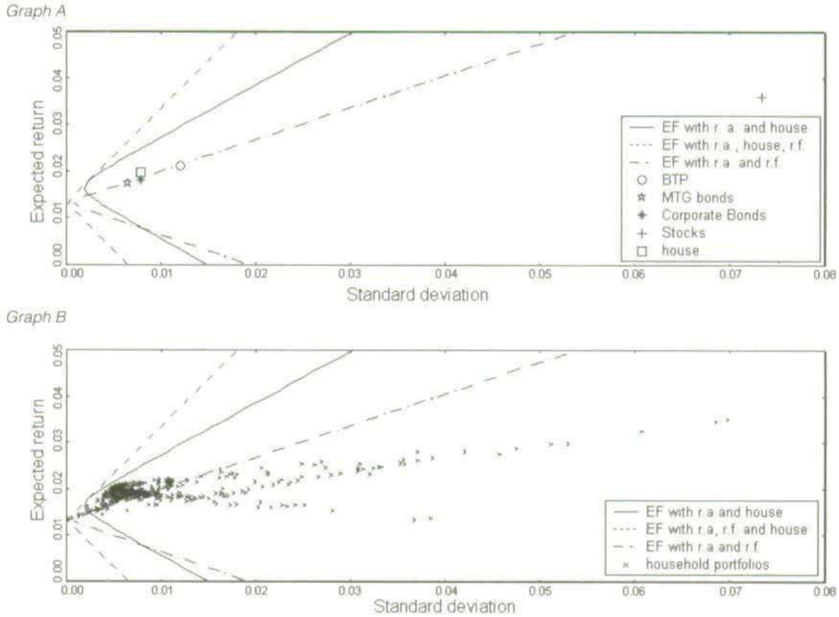
[13]In our conditional analysis, we treat mortgages as negative corporate bond positions—a negative equilibrium value of corporate bonds is thus possible. To avoid this result, one could aggregate MTG bonds and corporate bonds, given their similar duration and the high correlation of their returns, to obtain a tangency portfolio that has all positive weights (0.1747, 0.8032, 0.0221). Our key empirical results do not change much if we follow this route.

mortgages as negative holdings of long-term bonds (BTP) and debts as negative holdings of medium-term (corporate) bonds.

FIGURE 2

NW Efficient Frontiers with Housing

Graph A of Figure 2 shows the five individual asset expected returns (y-axis) and standard deviations (x-axis) as well as the risky asset efficient frontier and the efficient frontier with the risk-free asset. The corresponding efficiency frontiers without housing (see Figure 1) are also shown for comparison. Graph B shows, on the same scale, the efficient frontiers as above and the observed household portfolios. All these refer to the North West macro area (NW).



In Graph A of Figure 2, we show the risky asset efficient frontier and the efficient frontier with the risk-free asset (a broken line) for households living in the NW. Individual assets are also displayed: to the far right we still have stocks (+ sign), to the extreme left of the risky frontier we find MTG bond (a star) and corporate bonds (an *). Just above corporate bonds is housing (a square). Even though corporate bonds seem to be a dominated asset, we know from Tables 1 and 2 that its standard deviation is actually lower than the standard deviation on the house. Also, its highly positive correlation with MTG bonds, BTP, and stocks gives its short position some insurance value. This is borne out by the mean-variance efficient portfolio weights: as shown in the second column of Table 7, the optimal portfolio weight for housing in the NW region is 60% (and the BTP weight falls relative to the purely financial portfolio shown in the first column). This high wealth percentage in housing is of course largely explained by our assumption that the housing rental rate is as high as 5% in real terms.

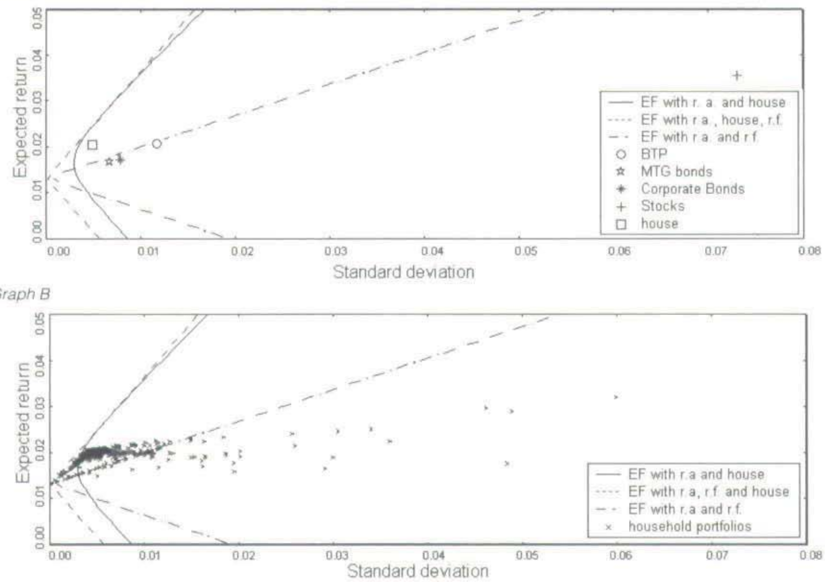
In Graph B of Figure 2, we show where individual portfolios lie. We see graphically that fewer portfolios lie close to the efficient frontier than we could see in Figure 1. We do not show graphs for the NE and CE because the tangency portfolios are similar to the NW (see the third and fourth column in Table 7) and the actual graphs are quite similar. For the SO, the picture is quite different (see Figure 3): the housing expected return is quite large and partial correlations are not in line with the rest of the country (see Table 3). As a result, the optimal portfolio has an extremely large weight on housing (160%). In this case, some observed portfolios appear to be close to the efficient frontier, so a formal test is required. When we run the formal efficiency test for the country as a whole, though, we find that only a handful of observations are efficient: 1 at the 10%, 11 at the 5%, and 17 at the 1% level.

FIGURE 3

SO Efficient Frontiers with Housing

Graph A of Figure 3 shows the five individual asset expected returns (y-axis) and standard deviations (x-axis) as well as the risky asset efficient frontier and the efficient frontier with the risk-free asset. The corresponding efficiency frontiers without housing (see Figure 1) are also shown for comparison. Graph B shows, on the same scale, the efficient frontiers as above and the observed household portfolios. All these refer to the South macro area.

Graph A



VI. Estimation and Test Results: Financial Assets Conditioning on Housing

Our results so far can be summarized as follows: i) when we consider only financial assets, household portfolios are mostly (76%) made of just the risk-free asset (of the diversified portfolios, at most 45% are mean-variance efficient at the 10% level); and ii) when we take a broader set of assets and liabilities (housing, mortgages, and debt) into consideration, many more households hold diversified portfolios (a common combination is the risk-free asset and housing), but only a tiny fraction of diversified household portfolios are now found to be efficient.

We have already argued (see Section II) that the illiquid nature of housing should be taken into account. If consumers hold a large fraction of their wealth in housing for reasons other than investment (because rental markets are imperfect, due to information asymmetry, as argued by Henderson and Ioannides (1983)), and do not trade frequently because of high pecuniary and non-pecuniary costs (Flavin and Nakagawa (2004)), then we should investigate their portfolio efficiency conditional on housing. It is, in fact, possible (and plausible) that their financial decisions are partly dictated by the need to hedge some of the risks connected with their illiquid housing investment.

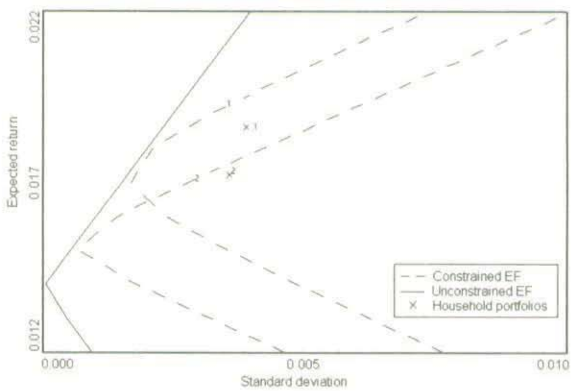
For each household that has nonzero housing wealth, we can compute a specific conditional efficiency frontier that treats housing, as constrained (for those without housing, the frontier displayed in Figure 1 still applies). It is worth stressing that in the constrained case the risk-free portfolio cannot be attained, except trivially (zero housing). This explains why the efficient frontiers we display in Figure 4 are not broken lines, contrary to what we have in Section V. We display the unconstrained and a few constrained frontiers, corresponding to a random subsample of homeowners whose actual portfolio is also shown (marked with a plus sign).

Figure 4 depicts the unconditional frontier with housing for the NW: the presence of a risk-free asset makes it a broken line. We also show two constrained frontiers for the same region, corresponding to two different shares of housing to total wealth (the frontier marked 1 has 18% of wealth into housing; the frontier marked 2 has 47% of total wealth into housing). They correspond to two observed portfolios, displayed as points x_1 and x_2. These frontiers lie entirely to the right of the unconstrained frontier (apart from a tangency point, corresponding to the case where the housing portfolio share is at its optimal value). They do not touch the vertical axis because a risk-free position cannot be achieved with positive housing wealth, given the correlations shown in Table 2.

FIGURE 4

NW Efficient Frontiers Conditional on Housing

Figure 4 shows the unconstrained efficient frontier for the NW where housing is included in the investment set (solid/broken line). It also shows two household portfolios expected returns (y-axis) and standard deviations (x-axis), and the corresponding constrained efficient frontiers (dashed curves).



We can now compute the test statistic for the conditional portfolios, ξ_1 (defined in equation (10), Section III), and calculate for how many portfolios the test fails to reject the null hypothesis of mean-variance efficiency. The test is not defined in the case of portfolios made entirely of risk-free assets (it is a ratio of zero to zero), and is identical to the standard test of Section V for portfolios consisting of just financial assets.

In the case of portfolios consisting of risk-free + housing, the test statistic takes the same value within the same region by construction. Also, in our application the expected return for all these portfolios (net of the hedge term) within regions is negative. Therefore, all portfolios with housing but no financial assets are inefficient.

When we consider housing as a constraint, we classify a much smaller number of households in the risk-free portfolios category: 1,567 instead of 4,554. In fact, of those without risky financial assets, homeowners without a mortgage are now classified in the risk-free + house category (2,499 households in all), and homeowners with a mortgage or debt (491) could be classified in the last category (risk free + house + financial assets) because the mortgage is treated as a negative position on long-term government bonds, but we keep them separate in our analysis because all such portfolios turn out to be inefficient for all test sizes. Therefore, we consider 1,140 diversified portfolios that are conditional on housing.

Table 8 reports efficiency results for the 1,363 households that have diversified portfolios[14] (223 have well-diversified financial portfolios, but no housing, and 1,140 with well-diversified financial portfolios and housing). We see that the test fails to reject efficiency in 261 cases (19%) at the 10% significance level, and this number rises to 595 (44%) at the 5% level and 901 (66%) at the 1% level. Not surprisingly, we find that in condition on housing many more portfolios are efficient than treating housing as an unconstrained asset (as stressed graphically in Flavin and Yamashita (2002)).

TABLE 8

Efficient Portfolios Conditional on Housing

Table 8 shows the numbers and proportions of efficient portfolios conditional on housing for three different test sizes. The sample is restricted to households that have at least one risky financial asset.

		Test Size					
		10%		5%		1%	
Portfolios	Tot. N	N	%	N	%	N	%
Risk-Free + Risky Fin. Ass.	223	104	46.64%	223	100.00%	223	100.00%
Risk-Free + Risky Fin. Ass. + Housing	1,140	157	13.78%	372	32.63%	678	59.40%
Total	1,363	261	19.15%	595	43.65%	901	66.10%

If we look at the group of households that have well-diversified financial portfolios (but no housing), we find that 46.64% of these portfolios are efficient when the test is conducted at the 10% significance level. It is worth stressing that there are just 223 households that fall into this category.

In the more interesting case, where the household holds both housing and risky financial assets (1,140 observations), we find that 157 cases are efficient at

[14]Compared to Section V, we have dropped three observations because of a missing value on the house.

the 10% significance level (14%). When we run the test at the 5% significance level, we find that 33% of these households hold efficient portfolios (372 in all, see Table 8). This number rises to 678 (59% of the group) at the 1% level.

The efficiency test results displayed in Table 8 suggest that a non-negligible proportion of homeowners hold portfolios that are not far from their conditional (or constrained) mean-variance frontier. This is in stark contrast to the case where housing is treated as a freely chosen asset (the unconditional test discussed in Section V).

We now consider the 1,140 fully diversified portfolios (risk-free, risky financial assets, and housing). In Table 9, we cross-tabulate diversified financial portfolios and total conditional portfolios according to the efficiency criterion (at the 10% level for both test statistics).

TABLE 9

How Diversified Portfolios Are Classified: A Comparison

Table 9 shows the numbers of portfolios with at least one risky financial asset or liability, according to the way they are classified by the efficiency tests run at the 10% level. Test size = 10%.

	Efficient (financial)	Inefficient (financial)	Total
Efficient (conditional)	71	86	157
Inefficient (conditional)	437	546	983
Total	508	632	1,140

We find that as many as 437 portfolios are classified as efficient when housing is neglected, but inefficient when it is considered. This suggests that hedging opportunities are not fully exploited. This is partly compensated by the presence of 86 portfolios for which the reverse holds. This could be evidence that these households use financial assets to hedge housing risk, but could also reveal that housing has diversification properties (for homeowners, financial risks are relatively small compared to total wealth). Given the high correlations (see Table 3) and the very large weight attached to housing wealth, the failure to exploit hedging opportunities outweighs the benefits from diversification, and the number of conditionally efficient portfolios (157) is smaller than the number of efficient financial portfolios (508). Similar conclusions can be drawn when the chosen test size for the conditional test is 5% (and kept at 10% for the financial test).

It is worth stressing that the estimated coefficients in Table 3 are the relevant indicators of the way hedging should be performed. For instance, in three regions out of four, more should be invested in MTG bonds compared to the mean-variance efficient portfolio weights displayed in Table 7 (the exception is the SO).

In Table 10, we display efficiency results by region (for two different test sizes: 10% in Panel A and 5% in Panel B). We see that the highest proportion of efficient portfolios obtains in the NW. This is particularly true for the conditional analysis—apparently NW households are the best at hedging housing risk. Purely financial portfolios are instead most often efficient in the SO.

The question rises of what makes a household more likely to hold an efficient portfolio. To address it, we run a regression of the test statistic (ξ_1) on observable household characteristics such as age, education, and employment position of the

TABLE 10

Efficient Portfolios Conditional on Housing by Region

Table 10 shows the numbers and proportions of efficient portfolios conditional on housing by macro region. Panel A reports results for a 10% test size, Panel B for a 5% test size. The sample is restricted to households that have at least one risky financial asset.

	NW		NE		CE		SO	
Portfolios	N	%	N	%	N	%	N	%
Panel A. Test Size = 10%								
Risk-Free + Risky Fin. Ass.	30	38.46%	23	38.98%	17	41.46%	34	75.56%
Risk-Free + Risky Fin. Ass. + House	112	29.71%	11	3.86%	15	5.86%	19	8.56%
Total	142	31.21%	34	9.88%	32	10.77%	53	19.85%
Panel B. Test Size = 5%								
Risk-Free + Risky Fin. Ass.	78	100.00%	59	100.00%	41	100.00%	45	100.00%
Risk-Free + Risky Fin. Ass. + House	169	44.83%	83	29.12%	73	28.52%	47	21.17%
Total	247	54.29%	142	41.28%	114	38.38%	92	34.46%

head, region, household income, and housing wealth (see Pelizzon and Weber (2006) for further details). Our key findings are that for high or low income levels inefficiency is lower and residence in the NW also has a negative effect on inefficiency, whereas a larger home value has a strong, positive impact.

VII. Discussion of Empirical Results and Extensions

An important issue that rises when housing is included in the asset mix is how to account for the liability every household has—to live somewhere. This issue is stressed in a number of papers, Sinai and Souleles (2005), Banks, Blundell, Oldfield, and Smith (2004), and Yao and Zhang (2005) who point out that housing is a hedge against increases in the price of housing services. It is clear that the risk posed by price increases of housing services is the more important the less easy it is to substitute out of housing into other goods and services. An extreme example where this substitution cannot take place at all is the case where housing consumption is already at its physical minimum.

In the framework we propose in this paper, we can account for housing needs in a relatively simple way. We define a minimum physical threshold for the main residence as $\overline{H}$ and estimate it in our data. We then take the observed price per squared meter as given and include in wealth only the difference between the current housing wealth and its minimum multiplied by that price. However, when the reported price is much higher than the local average, we replace it with a large, but more sensible value (on the assumption that the household could buy at that lower price if they moved into the smallest possible residence within the same area).

We define $(P_H H - \overline{P_H H})$ as net housing wealth, where $P_H H$ is the declared house value, $\overline{H}$ is the minimum house size for a given family size, and $\overline{P_H}$ is the relevant alternative house price within the current area of residence. We take the sample first percentile of squared meters for all possible family sizes as the minimum house size (this gives 20 square meters for a single, 35 for a couple, 40 for a couple with one or two children, and 46 for larger families). These values are in line with housing regulations (a single room must be at least nine square meters in Italy).[15] As for prices, for each household we take the observed price per square meter, except in those few cases where reported house prices are at the upper end of the distribution (top percentile), where we set them to 6m lire per square meter (roughly 3,000 euros).

The resulting net housing wealth variable has an average value of almost 143,000 euros, whereas the original home value is 216,000. Our procedure suggests that about a third of housing wealth should be disregarded when deciding the financial portfolio allocation because of the housing liability discussed so far.

When we compare the results of Table 11 to those in Table 8, we see that taking the housing liability into account makes some 6%–10% more fully diversified portfolios conditionally efficient, depending on the chosen size of the test. [16] Table 12, which compares directly to Table 9, shows that the fraction of portfolios that are efficient according to both financial and conditional tests rises substantially (from 6.2% to 9.9%). This is due to a reduction in the number of portfolios that are efficient financially, inefficient conditionally.

TABLE 11

Efficient Portfolios Conditional on Net Housing Wealth

Table 11 shows the numbers and proportions of efficient portfolios conditional on housing, for three different test sizes. The sample is restricted to households that have at least one risky financial asset. Housing wealth is defined net of exogenously determined housing needs.

		Test Size					
		10%		5%		1%	
Portfolios	Tot. N	N	%	N	%	N	%
Risk-Free + Risky Fin. Ass.	223	104	46.64%	223	100.00%	223	100.00%
Risk-Free + Risky Fin. Ass. + House	1,135	216	19.03%	479	42.20%	756	66.61%
Total	1,358	320	23.56%	702	51.69%	979	72.09%

TABLE 12

How Diversified Portfolios Are Classified: Net Housing Wealth

Table 12 shows the numbers of portfolios with at least one risky financial asset according to the way they are classified by the efficiency tests run at the 10% level. Housing wealth is defined net of exogenously determined housing needs. Test size = 10%.

	Efficient (financial)	Inefficient (financial)	Total
Efficient (conditional)	112	104	216
Inefficient (conditional)	393	526	919
Total	505	630	1,135

The definition of net housing wealth as the difference between the existing main residence and a minimal residence, which meets exogenously defined housing needs, is attractive, but fails to capture preference heterogeneity. Households with a strong preference for housing may consider a much higher minimum

[15]A square meter is roughly equivalent to 10 square feet.

[16]We lose 30 observations, five of which have fully diversified portfolios, because total wealth becomes negative. This explains why we have 1,135 households in Table 12 as opposed to 1,140 in Table 8.

threshold for housing services than households with a weaker preference for housing. A better approximation of housing needs may then be as a given proportion of housing services currently enjoyed.

We thus consider an alternative way to account for the notion that only a part of the main residence is perceived as wealth. We assume that households are not willing to reduce their housing consumption below a given fraction, x, of their existing consumption. This has an effect on the way they consider their main residence, but no effect on other real estate. We thus define net housing wealth, *nhw*, as $nhw = (1 - x) *$ main residence + other real estate − housing debt. We see that x times the main residence is the minimum threshold below which a household is not willing to go and for this reason is not counted as wealth.

Total wealth is the sum of financial wealth and net housing wealth. If $x = 0$, we are in the case considered in Section VI; if $x = 1$, total wealth is financial wealth (as in the first part of Section V) for those households that have neither other real estate nor housing-related debt. However, even if $x = 1$, total wealth does not coincide with financial wealth for households that have other real estate or housing-related debt.[17]

We then check to what extent conditional efficiency coincides with financial efficiency as a function of x. Figure 5 shows the results for the sample of 1,140 households that have both housing and risky financial assets. The lower curve represents the fraction of portfolios that are conditionally efficient out of all portfolios that are financially efficient (508 observations). The upper curve represents the fraction of portfolios that are conditionally inefficient out of all portfolios that are financially inefficient (632 observations). As expected, these two fractions increase in x, that is, conditional efficiency tends to coincide with financial efficiency the less the main residence is counted as wealth. As explained above, we apply this x correction to the main residence only—this explains why the proportions in Figure 10 do not reach unity even when we subtract 100% of the main residence value from housing wealth. In fact, 174 out of 508 (34%) households whose portfolios are financially efficient have other real estate, 244 out of 632 (39%) of households whose portfolios are financially inefficient have other real estate.

We see that the proportion of portfolios that are efficient on both counts steadily increases: there are only 71 (14%) such portfolios when $x = 0$ (see the first main diagonal entry in Table 9), and as many as 389 (77%) when $x = 1$. This suggests that households that neglect real estate in their portfolio choice (while achieving financial efficiency) may do so for good reasons—because they do not consider most of their main residence disposable (part of wealth).

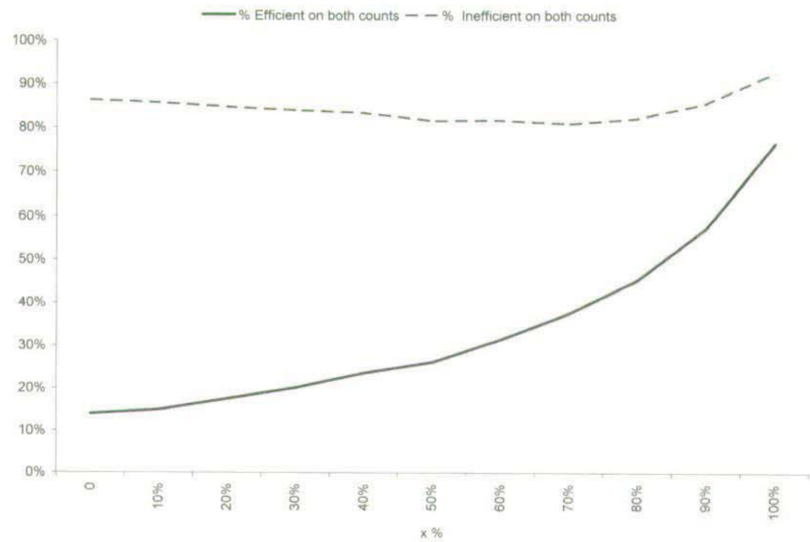
The picture for financially inefficient portfolios is different: the fraction of inefficient portfolios on both counts declines in x from 546 (86%) when $x = 0$ (see Table 9, second main diagonal entry) down to 515 (81%) when $x = 70\%$, then increases to 586 (93%). This suggests that some of these households may hedge housing risk, particularly if they consider about a third of the total main residence

[17]For all values of x, we make total wealth coincide with financial wealth also for those households where net housing wealth is negative. In fact, we replace *nhw* with zero: this replacement occurs in relatively few cases in our sample because housing debt is a relatively minor item (we set *nhw* to zero in less than 50 cases for $x < 90\%$, 65 cases when $x = 90\%$, and 119 cases when $x = 100\%$).

FIGURE 5

Sensitivity of Efficiency to Changes in Housing Wealth Definition

Figure 5 shows on the x-axis the percentage minimum threshold of the main residence below which a household is not willing to go. On the y-axis, it shows the percentage of portfolios that are conditionally efficient out of all portfolios that are financially efficient (solid line) and the percentage of portfolios that are conditionally inefficient out of all portfolios that are financially inefficient (dashed line).



value as wealth, but at least four out of five hold inefficient portfolios irrespective of the hedging motive.

Another issue worth considering is the effect of differential underreporting. We know from D'Alessio and Faiella (2000) that SHIW98 underestimates financial wealth by a wide margin (it accounts only for a third of aggregate household financial wealth), whereas housing wealth is in line with aggregate statistics. The reasons why financial wealth falls short of aggregate statistics can be non-response among the rich and underreporting among those who do respond. To assess whether the latter has an important impact on our test, we take the extreme case where differential non-response is not an issue, multiply all financial wealth holdings by a factor of three, and rerun the test.

Table 13 displays efficiency test results in this hypothetical case, where all households report the same fraction of their financial wealth. If we compare these results with those in Table 8, we see that more fully diversified portfolios are counted as efficient (for instance: 321 instead of 157 at the 10% level). This increase is in line with expectations (the hedge motive is relatively less important if housing wealth has a lower portfolio weight) and is quite sizeable (now 28.16% of fully diversified portfolios are efficient, rather than 13.78%).

One final issue is worth careful consideration. First and second moments of financial asset returns have been estimated using relatively accurate asset price data. Housing returns are instead based on averages of local house price data that are more likely to be affected by sampling variability. In Pelizzon and Weber (2006), we show how our analysis carries through to the case where housing returns are measured with error.

TABLE 13

Efficient Portfolios Conditional on Housing Corrected for Underreporting

Table 13 shows the numbers and proportions of efficient portfolios conditional on housing for three different test sizes. The sample is restricted to households that have at least one risky financial asset. Financial wealth is corrected for underreporting.

		Test Size					
		10%		5%		1%	
Portfolios	Tot. N	N	%	N	%	N	%
Risk-Free + Risky Fin. Ass.	223	104	46.64%	223	100.00%	223	100.00%
Risk-Free + Risky Fin. Ass. + House	1,140	321	28.16%	682	59.82%	895	78.51%
Total	1,363	425	31.18%	905	66.40%	1,118	82.02%

VIII. Conclusions

In this paper, we argue that standard tests of portfolio efficiency are biased because they neglect the existence of illiquid wealth. In the case of household portfolios, the most important illiquid asset is housing: if housing stock adjustments are costly and therefore infrequent, optimal portfolios in periods of no adjustment are affected by housing price risk.

We show that if financial assets' and housing returns are correlated the intertemporal expected utility model subject to transaction costs in housing investment implies that financial decisions are affected by the need to hedge some of the risks connected with the existing illiquid housing position. In particular, the investors' optimal strategy is to choose the standard Markowitz portfolio according to their risk aversion and use the risky financial assets to hedge their expositions on the constrained asset (this last decision is independent of their risk aversion). This hedging motive disappears in the case of zero correlation between housing return and financial returns, in which case housing price risk only affects the investor's degree of risk aversion (Flavin and Nakagawa (2004)).

We also show that the optimal investment in risky financial assets is equal to the one derived in a static mean-variance analysis framework if the existing housing stock is treated as an additional constraint to the optimization problem. Gourieroux and Jouneau (1999) have proposed an efficiency test for analyzing the performance of a portfolio of risky assets (in a mean-variance framework) when some constraints exist on a part of the portfolio. We are then able to claim that this test can be applied for a more general test of portfolio efficiency.

In our application, we use Italian household portfolio data and time-series data on financial asset and housing stock returns to assess whether actual portfolios are efficient. We first consider purely financial portfolios and portfolios that also treat the housing stock as another, unconstrained asset. We then consider the consequences of treating the housing stock as given and test for efficiency in this framework.

Our empirical results support the view that the presence of illiquid wealth plays an important role in determining whether portfolios chosen by homeowners are efficient. Our results can be summarized as follows. When we consider only financial assets, three portfolios out of four are made of just the risk-free asset. Of the diversified portfolios, a large fraction (45%) is mean-variance efficient. When we take a broader set of assets and liabilities (housing, mortgages, and debt) into consideration, many more households hold diversified portfolios (a common combination is the risk-free asset and housing). But very few diversified household portfolios are found to be efficient when housing is treated as unconstrained. When we calculate the efficiency test conditional on housing we find that one in seven of fully diversified portfolios (that include the risk-free asset, housing, and risky assets) are mean-variance efficient. We also find that these are largely not the same households whose financial portfolios were considered efficient.

An important issue that arises when housing is included in the asset mix is how to account for the liability that households have to live somewhere. In our robustness analysis, we propose two alternative ways to do this. First, we define net housing wealth as the difference between the home value and the value of the smallest property a household could move to in the same area. On average, our net housing wealth variable is worth around two-thirds of gross housing wealth. We show that taking the housing liability into account this way increases the number of conditionally efficient fully diversified portfolios by half. Second, we assume that housing needs are a given fraction of the housing services currently enjoyed. Net housing wealth is the difference between total housing wealth and the fraction required to meet housing needs. As this fraction approaches unity, we find that an increasing proportion of financially efficient portfolios are also conditionally efficient. Financially inefficient portfolios, instead, are more often conditionally efficient when this fraction rises to 70%, then more often inefficient.

Finally, compared to the efficiency results relating to portfolios consisting solely of financial assets such as stocks, bonds, and a risk-free asset, the introduction of housing and mortgage alters the risk and return trade-off in a direction that pushes very few household portfolios to be efficient. This is not the case once the illiquid nature of housing investment is taken into account, but there is strong evidence that hedging opportunities are not fully exploited even by those Italian households who hold well-diversified portfolios. This widespread failure to hedge house price risk has important implications for portfolio management.

Appendix. Derivation of Equation (7) in a Multiperiod Context

In this Appendix, we build on Flavin and Nakagawa's (2004) analysis of the dynamic optimization problem with housing, and use the same notation for comparison's sake. We show that the dynamic optimization problem produces the same asset allocation rule as a static problem that treats housing wealth as given.

Flavin and Nakagawa generalize Grossman and Laroque's (1990) model by making current utility a function of both a durable good, a house (H), and a non-durable good (C). The non-durable good is infinitely divisible and costlessly adjustable. As in Grossman and Laroque, the durable good is instead subject to an adjustment cost proportional to its value and is therefore adjusted infrequently. This generalization is of great relevance for the analysis of portfolio choice because it allows us to consider explicitly the relation between the real rate of return on housing investment and the real rates of return on financial assets.

The household maximizes expected lifetime utility:

$$U = E_0 \int_0^{\infty} e^{-\delta t} u(H_t, C_t) dt. \tag{A1}$$

For analytical simplicity, the house is not subject to physical depreciation.[18] Using the non-durable good as numeraire, define:

(A2) $P_t \quad = \quad$ house price (per square meter) in the household's market.[19]

Assume that wealth is held only in the form of financial assets and housing. The household can invest in a riskless asset and in any of n risky financial assets. Holdings of the financial assets can be adjusted with zero transaction cost.

Thus, wealth is given by:

$$W_t \quad = \quad P_t H_t + B_t + \underline{X}_t \underline{\ell}, \tag{A3}$$

where $\underline{X}_t = (1 \times n)$ vector of amounts (expressed in terms of the non-durable good) held of the risky assets and $\underline{\ell} = (n \times 1)$ vector of ones. B_t is the amount held in the form of the riskless asset. All financial assets, including the riskless asset, may be held in positive or negative amounts.[20]

Assuming that dividends or interest payments are reinvested so that all returns are received in the form of appreciation of the value of the asset, let b_{it} = the value (per share) of the i-th risky asset, and assume that asset prices follow an n-dimensional Brownian motion process:

$$db_{it} \quad = \quad b_{it}\left((\mu_i + r_f)dt + d\omega_{it}\right). \tag{A4}$$

The vector $\underline{\omega}_{Ft} \equiv (\omega_{1t}, \omega_{2t}, \ldots, \omega_{nt})$ follows an n-dimensional Brownian motion with zero drift and with instantaneous covariance matrix Σ, the corresponding vector $(n \times 1)$ of expected excess returns on risky financial assets is $\underline{\mu} \equiv (\mu_1, \mu_2, \ldots, \mu_n)$, and r_f is the riskless rate. The i-th element of $\underline{X}_t$ in equation (3) is given by $X_{it} \equiv N_{it} b_{it}$, where N_{it} is the number of shares held of asset i. Since asset prices, b_{it}, are taken as exogenous, the household determines X_{it} by its choice of N_{it}.[21]

House prices also follow a Brownian motion:

$$dP_t \quad = \quad P_t\left((\mu_H + r_f)dt + d\omega_{Ht}\right), \tag{A5}$$

where ω_{Ht} is a Brownian motion with zero drift and instantaneous variance σ_P^2.

Stacking equations (4) and (5), define the $((n+1) \times 1)$ vector $d\underline{\omega}_t$:

$$d\underline{\omega}_t \quad = \quad \begin{bmatrix} d\omega_{1t} \\ \vdots \\ \\ d\omega_{nt} \\ d\omega_{Ht} \end{bmatrix}, \tag{A6}$$

which has instantaneous $((n+1) \times (n+1))$ covariance matrix Ω:

$$\Omega \quad = \quad \begin{bmatrix} \Sigma & \Gamma_{b,P} \\ \Gamma_{b,P}^T & \sigma_P^2 \end{bmatrix}, \tag{A7}$$

---

[18] Damgaard, Fuglsbjerg, and Munk (2003) have developed a model similar to ours by deriving the numerical solution for the case with nonzero depreciation. Depreciation implies that the target value for housing is above the midpoint of the (s, S) interval. We prefer to subtract maintenance costs from the return on housing, assuming that maintenance restores the housing stock to its previous state.

[19] Unlike Flavin and Nakagawa (2004), we do not consider a separate price process for the next house to be bought.

[20] This model does not deal with labor income or borrowing restrictions that are instead considered in the model developed by Cocco (2005).

[21] We follow Flavin and use X_{it} rather than N_{it} as the choice variable representing the portfolio decision.

where:

$$\text{(A8)} \qquad \Gamma_{bP} = \begin{bmatrix} \sigma_{b1P} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \sigma_{bnP} \end{bmatrix}.$$

Note that we depart from Flavin and Nakagawa here in that we do not assume the covariance matrix Ω to be block diagonal. This is the substantial difference between our models that generates qualitatively different results.

We shall show that, under the assumptions listed in this Appendix, the optimal holding of risky financial assets is given by:

$$\text{(A9)} \qquad \underline{X}_0^T = \left[\frac{-\frac{\partial V}{\partial W}}{\frac{\partial^2 V}{\partial W^2}}\right] \Sigma^{-1}\underline{\mu} - P_0 H_0 \Sigma^{-1}\Gamma_{bP}.$$

In equation (A9), the expression in square brackets is the reciprocal of the coefficient of absolute risk aversion:

$$\text{(A10)} \qquad ARA \equiv -\frac{\frac{\partial^2 V}{\partial W_t^2}}{\frac{\partial V}{\partial W_t}} > 0.$$

It is worth pointing out that risk aversion affects the first term on the RHS of equation (A9) but not the second term, which bears the interpretation of a hedge portfolio.[22] In Flavin and Nakagawa's analysis, this second term disappears because they assume $\Gamma_{bp} = 0$, and therefore can prove that the traditional CAPM holds.

Suppose that at time $t = 0$, the household decides that it is not optimal to change the housing stock immediately. During a time interval $(0, s)$ when the possibility of such change is negligible, wealth evolves according to:

$$\text{(A11)} \quad dW_t = \left[P_t H_0(\mu_H + r_f) + \underline{X}_t(\underline{\mu} + r_f) + r_f B_t - C_t\right] dt + \underline{X}_t\, d\underline{\omega}_{Ft} + P_t H_0 d\omega_{Ht},$$

or rewriting in order to eliminate the term representing risk-free bonds,

$$\text{(A12)} \qquad dW_t = \left[r_f W_t + P_t H_0 \mu_H + \underline{X}_t \underline{\mu} - C_t\right] dt + \underline{X}_t\, d\underline{\omega}_{Ft} + P_t H_0 d\omega_{Ht}.$$

Let $V(H, W, P)$ denote the supremum of household expected utility, be twice continuously differentiable, conditional on the current values of the state variables (H, W, P). Bellman's principle of optimality can be stated as:

$$\text{(A13)} \quad V(H_0, W_0, P_0) = \sup_{\{\underline{X}_t\},\{C_t\}} E\left[\int_0^s e^{-\delta t} u(H_0, C_t)\, dt + e^{-\delta s} V(H_0, W_s, P_s)\right],$$

subject to the budget constraint (A12) and the process for house prices (A5). The term inside the brackets intuitively represents the sum of the rewards on the interval $(0, s)$ and

[22] This term is different from the classical Merton hedge term that accounts for shifts in the investment opportunity set.

the maximized expected value by proceeding optimally on the interval (s, ∞) with the system started at time s in state (H_0, W_s, P_s).[23]

Subtracting $V(H_0, W_0, P_0)$, dividing by s, and taking the limit as $s \to 0$ gives:

$$\text{(A14)} \qquad 0 = \lim_{s\to 0} \sup_{\{\underline{X}_t\}\{C_t\}} E\left[\frac{1}{s}\int_0^s e^{-\delta t} u(H_0, C_t)\,dt + \frac{1}{s}\left(e^{-\delta s} V(H_0, W_s, P_s) - V(H_0, W_0, P_0)\right)\right].$$

Evaluating the integral and using Ito's lemma, equation (A14) can be rewritten as:

$$\text{(A15)} \qquad 0 = \sup_{\underline{X}_0, C_0}\Bigg\{u(H_0, C_0) - \delta V(H_0, W_0, P_0, P_0') + \frac{\partial V}{\partial W}(r_f W_0 + P_0 H_0 \mu_H + \underline{X}_0 \underline{\mu} - C_0) + \frac{\partial V}{\partial P} P_0 \mu_H + \frac{1}{2}\frac{\partial^2 V}{\partial W^2}\left(\underline{X}_0 \Sigma \underline{X}_0^T + P_0^2 H_0^2 \sigma_P^2 + 2P_0 H_0 \underline{X}_0 \Gamma_{bP}\right) + \frac{1}{2}\frac{\partial^2 V}{\partial P^2} P_0^2 \sigma_P^2 + \frac{\partial^2 V}{\partial W \partial P}\left(P_0^2 H_0 \sigma_P^2 + P_0 \underline{X}_0 \Gamma_{bP}\right)\Bigg\}.$$

That is:

$$\text{(A16)} \qquad 0 = \sup_{C_0}\left\{u(H_0, C_0) - C_0 \frac{\partial V}{\partial W}\right\} - \delta V(H_0, W_0, P_0, P_0') + \frac{\partial V}{\partial W}(r_f W_0 + P_0 H_0 \mu_H) + \frac{\partial V}{\partial P} P_0 \mu_H + \frac{1}{2}\frac{\partial^2 V}{\partial P^2} P_0^2 \sigma_P^2 + \frac{\partial^2 V}{\partial W \partial P}\left(P_0^2 H_0 \sigma_P^2 + P_0 \underline{X}_0 \Gamma_{bP}\right) + \frac{1}{2}\frac{\partial^2 V}{\partial W^2} P_0^2 H_0^2 \sigma_P^2 + \sup_{\underline{X}_0}\left\{\frac{\partial V}{\partial W}\underline{X}_0 \underline{\mu} + \frac{1}{2}\frac{\partial^2 V}{\partial W^2}\left(\underline{X}_0 \Sigma \underline{X}_0^T + 2P_0 H_0 \underline{X}_0 \Gamma_{bP}\right)\right\}.$$

Non-durable consumption satisfies the standard first-order condition:

$$\text{(A17)} \qquad \frac{\partial u}{\partial C} = \frac{\partial V}{\partial W}.$$

The vector of holdings of risky financial assets, $\underline{X}_0$, is chosen according to:

$$\text{(A18)} \qquad 0 = \text{constant} + \frac{\partial V}{\partial W}(r_f W_0 + P_0 H_0 \mu_H - C_0) + \frac{1}{2}\frac{\partial^2 V}{\partial W^2} P_0^2 H_0^2 \sigma_P^2 + \sup_{\underline{X}_0}\left\{\frac{\partial V}{\partial W}\underline{X}_0 \underline{\mu} + \frac{1}{2}\frac{\partial^2 V}{\partial W^2}\left(\underline{X}_0 \Sigma \underline{X}_0^T + 2P_0 H_0 \underline{X}_0 \Gamma_{bP}\right) + \frac{\partial^2 V}{\partial W \partial P}(P_0 \underline{X}_0 \Gamma_{bP})\right\}.$$

Assuming that $(\partial^2 V)/(\partial W \partial P) = 0$ we can derive the optimal holding of risky financial assets as:

$$\text{(A19)} \qquad \underline{X}_0^T = \left[\frac{-\dfrac{\partial V}{\partial W}}{\dfrac{\partial^2 V}{\partial W^2}}\right] \Sigma^{-1} \underline{\mu} - P_0 H_0 \Sigma^{-1} \Gamma_{bP}$$

[23]We assume that the transversality condition holds such that $V(H_0, W_s, P_s)$ is bounded.

and the amount held of the riskless asset is:

$$B_0 = W_0 - P_0 H_0 - \underline{X}_0 \underline{\ell}. \quad \text{(A20)}$$

Equation (A19) is the same as equation (7) if both members are divided by total initial wealth, W_0.

The assumption that $(\partial^2 V)/(\partial W \partial P) = 0$ is justified under two sets of circumstances: a) if the utility function does not depend on housing, as pointed out by Damgaard, Fuglsbjerg and Munk (2003); and b) if the utility function is additive in housing and non-durable consumption. While condition a) rules out a consumption role for housing, condition b) provides a useful benchmark for the analysis.

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