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ADAPTING A GENERALIZED PLASTICITY MODEL TO REPRODUCE THE STRESS-STRAIN RESPONSE OF SILTY SOILS FORMING THE VENICE LAGOON BASIN

Simonetta Cola[(1)] and Laura Tonni[(2)]
[(1)] Dipartimento IMAGE, Università di Padova, simonetta.cola@unipd.it
[(2)] DISTART, Università di Bologna, laura.tonni@unibo.it

Summary

A Generalized Plasticity model, originally developed for the analysis of sandy soil behaviour, is modified in order to properly simulate the stress-strain response of a wide class of non-active natural soils, forming the upper profile of the Venice Lagoon basin. The main modification consists in introducing a state-dependent dilatancy, which allows proper modelling of granular soils over a wide range of pressures and densities, fulfilling at the same time basic premises of critical state soil mechanics. Moreover, according to recent developments on the isotropic compression of granular soils, few adjustments are introduced in the plastic modulus expression.
The approach is validated by comparing the model predictions with experimental data obtained from drained triaxial compression tests on natural and reconstituted samples of soils having different fine contents.

1. Introduction

Over the last twenty years the subsoil of the historical city of Venice and the surrounding lagoon has been extensively studied in order to formulate a reliable geotechnical model of the ground, useful for the design of the submersible barriers intended to protect the area against flooding.
Due to a very complex depositional history, the upper 100m of the Venetian basin profile are mainly composed of silts, always combined with sand or clay or both together in an endless, chaotic alternation of stratified sediments.
A comprehensive study carried out by Cola and Simonini (2000, 2002) showed that the mechanical behaviour of such soils is strongly dependent on the stress level and seems to be mostly controlled by inter-particle friction rather than electrochemical action.
Recently, Tonni *et al.* (2003) presented an attempt to model the experimental stress-strain response of Venetian soils through a Generalized Plasticity approach, using a constitutive model for sands developed by Pastor *et al*. (1990) within such versatile theoretical framework.
The preliminary calibration work, based on few drained triaxial tests, showed a rather satisfactory applicability of the model for predicting the behaviour of these soils, but suggested at the same time a more sensitive calibration study and few minor corrections to be introduced into the constitutive equations in order to get unified modelling over a wide range of densities and stress levels.
Therefore, following recent studies on the internal-state dependence of granular soil behaviour (Been & Jefferies, 1985; Muir Wood *et al*., 1994; Manzari & Dafalias, 1997; Li & Dafalias, 2000; Wang *et al.*, 2002) and according to the available experimental data, modifications in the plastic flow rule and in the plastic modulus expressions have been introduced.
In this paper, after a brief introduction to the original formulation of the constitutive model and a review of the basic properties of Venetian soils, results of the calibration studies are discussed, with particular reference to those material parameters appearing in the modified relationships of dilatancy and plastic modulus.
Finally, the approach is validated by comparing experimental data of drained triaxial compression tests with the model predictions.

2. Generalized Plasticity

2.1. Basic Theory

The basic idea of Generalized Plasticity (*GP*), introduced by Zienkiewicz and Mroz (1984) and later extended by Pastor & Zienkiewicz (1986), is to allow for plastic deformations irrespective of the direction of the stress increment $d\boldsymbol{\sigma}$, i.e. in both loading and unloading conditions. Moreover, plastic deformations are introduced without specifying any yield or plastic potential surfaces: the gradients to these surfaces are explicitly defined, instead of the functions themeselves.
The elastoplastic behaviour of the material is described by the general incremental relationship:

$$d\boldsymbol{\sigma} = \mathbf{D}_t : d\boldsymbol{\varepsilon} \tag{1}$$

in which the tangent elastoplastic stiffness tensor $\mathbf{D}_t$ depends on the current stress state, on the direction of the stress increment $d\boldsymbol{\sigma}$ and on a set of internal variables.
The dependence of $\mathbf{D}_t$ on the direction of $d\boldsymbol{\sigma}$ is expressed by simply distinguishing between two different loading classes, namely *loading* (L) and *unloading* (U). As shown in Pastor & Zienkiewicz (1986), loading/unloading conditions are determined by considering the sign of the dot product between the stress increment $d\boldsymbol{\sigma}$ and a normalized direction $\mathbf{n}$ defined in the stress space.
Correspondingly the tangent elastoplastic stiffness tensor $\mathbf{D}_t$, (represented with subscript L or U whether *loading* or unloading is occurring respectively), is expressed as follows:

$$\left(\mathbf{D}_{t,L/U}\right)^{-1} = \left(\mathbf{D}_t^e\right)^{-1} + \frac{1}{H_{L/U}}[\mathbf{m}_{L/U} \otimes \mathbf{n}] \tag{2}$$

where $\mathbf{m}_{L/U}$ is a direction of unit norm, $H_{L/U}$ stands for the plastic modulus and $\mathbf{D}_t^e$ is the tangent elastic stiffness tensor.
The limit case, called *neutral loading*, corresponds to stress increments for which the material behaviour is elastic and no plastic deformations occur.
Since the plastic modulus $H_{L/U}$ and the directions $\mathbf{n}$ and $\mathbf{m}_{L/U}$ are fully determined without reference to any yield surface or plastic potential function, different expressions can be selected for them depending on whether *loading* or *unloading* is occurring.
As usual, the strain rate $d\boldsymbol{\varepsilon}$ in eq. (1) can be decomposed into an elastic component $d\boldsymbol{\varepsilon}^e$ and a plastic component $d\boldsymbol{\varepsilon}^p$:

$$d\boldsymbol{\varepsilon} = d\boldsymbol{\varepsilon}^e + d\boldsymbol{\varepsilon}^p \tag{3}$$

with:

$$d\boldsymbol{\varepsilon}^e = (\mathbf{D}_t^e)^{-1} : d\boldsymbol{\sigma} \tag{4}$$

$$d\boldsymbol{\varepsilon}^p = d\boldsymbol{\varepsilon}_{L/U}^p = \frac{1}{H_{L/U}}[\mathbf{m}_{L/U} \otimes \mathbf{n}] : d\boldsymbol{\sigma} \tag{5}$$

Hence it follows that a constitutive model developed in the framework of *GP* is fully determined by specifying three directions (the loading direction $\mathbf{n}$ and the plastic flow directions $\mathbf{m}_L$ and $\mathbf{m}_U$), two scalars (the plastic moduli H_L and H_U) and the elastic stiffness tensor $\mathbf{D}_t^e$. The above "main ingredients" will be next expressed with reference to a specific constitutive model for granular soils - the "*PZ model*" (Pastor *et al*., 1990) - which is able to simulate various features of loose and dense sand behaviour under monotonic and cyclic loading, in drained as well as in undrained conditions.

2.2. A Generalized Plasticity model for sands

The *PZ* model assumes an isotropic material response in both elastic and plastic ranges, hence the constitutive equations can be written in terms of the three stress invariants p', q, ϑ and the work-conjugate strain invariants $d\varepsilon_v$ and $d\varepsilon_s$.
In what follows the model is presented only in the $q - p'$ formulation, since in this work the validation

of the constitutive equations is restricted to triaxial tests.
For *loading* stress increments, the plastic flow direction $\mathbf{m}_L^T = (m_{Lv}, m_{Ls})$ is given by:

$$m_{L,v} = \frac{d_g}{\sqrt{1+d_g^2}}\,; \qquad m_{L,s} = \frac{1}{\sqrt{1+d_g^2}} \qquad (6a, b)$$

where the soil dilatancy d_g is a linear function of the stress ratio $\eta = q/p'$:

$$d_g = (1+\alpha_g)\cdot(M_g - \eta) \qquad (7)$$

being M_g the slope of the critical state line in the *q-p'* plane and α_g a material parameter.
In unloading conditions irreversible strains are contractive and the m_v-component of the plastic flow direction $\mathbf{m}_U$ changes as follows:

$$m_{U,v} = -|m_{L,v}| \qquad (8)$$

The model assumes a non-associated flow rule, thus the loading direction $\mathbf{n}^T = (n_v, n_s)$ is different from $\mathbf{m}$, but with similar expressions for its components. These latter are given by:

$$n_v = \frac{d_f}{\sqrt{1+d_f^2}}\,; \qquad n_s = \frac{1}{\sqrt{1+d_f^2}} \qquad (9a, b)$$

where:

$$d_f = (1+\alpha_f)(M_f - \eta) \qquad (10)$$

with M_f and α_f material parameters.
In order to model the main features of sand behaviour (i.e. softening of dense sands, failure at the critical stress ratio M_g for all densities, liquefaction and cyclic mobility), Pastor & Zienkiewicz proposed for the plastic moduli H_L and H_U the following relationships:

$$H_L = H_0 \cdot p' \cdot H_f \cdot (H_v + H_s) \cdot H_{Dm} \qquad (11)$$

with:

$$H_f = \left(1 - \frac{\eta}{M_f}\frac{\alpha_f}{1+\alpha_f}\right)^4; \quad H_v = \left(1 - \frac{\eta}{M_g}\right); \quad H_s = \beta_0\beta_1 \exp(-\beta_0\xi) \quad H_{Dm} = \left(\frac{\zeta_{MAX}}{\zeta}\right)^{\gamma} \quad (12a, b, c, d)$$

and

$$H_U = H_{u0} \quad \text{for} \quad \left|\frac{M_g}{\eta_u}\right| \le 1\,; \qquad H_U = H_{u0}\left(\frac{M_g}{\eta_u}\right)^{\gamma_u} \quad \text{for} \quad \left|\frac{M_g}{\eta_u}\right| > 1 \qquad (13a, b)$$

Constants $H_0, \beta_0, \beta_1, \gamma, H_{u0}$ and γ_u appearing in eqs. (11)-(13) are constitutive parameters, ξ is the accumulated deviatoric plastic strain and ζ_{MAX} is the maximum value of the *mobilized stress function* ζ, accounting for the soil stress history. Finally, η_u stands for the stress ratio from which unloading takes place.
The material has a non-linear elastic response, according to the following relationships:

$$dp' = K_t \cdot d\varepsilon_v^e\,; \qquad dq = 3G_t \cdot d\varepsilon_s^e \qquad (14a, b)$$

The tangent bulk and shear moduli K_t and G_t are assumed to be dependent only on the hydrostatic part of the stress tensor:

$$K_t = K_o \cdot \frac{p'}{p'_0}\,; \qquad G_t = G_o \cdot \frac{p'}{p'_0} \qquad (15a, b)$$

being K_o and G_o the bulk and shear moduli at the reference mean effective stress p'_0, respectively.

As shown, the *PZ* model requires the definition of 12 material parameters, which can be calibrated from tests routinely performed in geotechnical laboratories, such as drained and undrained monotonic triaxial tests or undrained cyclic triaxial tests (Zienkiewicz *et al.*, 1999). Moreover, the number of material parameters that need calibration is dependent on the stress path under consideration: parameters such as γ, γ_u and H_{u0} were introduced in eqs. (12d)-(13a,b) for describing soil behaviour in unloading/reloading processes, hence they don't require any definition in case of monotonic loading.

3. The Venice Lagoon Soils

3.1. Basic properties

The Venice lagoon lies on 800m of Quaternary deposits, alternatively originated from continental and marine sedimentation, as a consequence of marine regressions and transgressions over the last 2 million years. The shallowest deposits, from ground level to 5-10 m below m.s.l., are due to the present lagunar cycle (Holocene).

Below Holocene sediments, down to a depth of 100 m, there is a chaotic interbedding of continental deposits, occurred in the last glacial Pleistocenic period (Würm).

The 95% of sediments can be grouped into 3 classes: medium-fine sands (SP-SM) with sub-angular grains, silts (ML) and very silty clays (CL). The remaining 5% may be classified as organic clay and peat.

Coarser sediments are mainly composed of silicate and carbonate minerals (dolomite, calcite, quartz and feldspars) while silts and silty clays, which originated from mechanical degradation of sands, have a higher content of clay minerals (illite with minor quantities of chlorite, kaolinite and smectite), though never exceeding 20% in weight. Generally, cohesive soils are slightly overconsolidated due to aging or exsiccation.

3.2. Main features of soil behaviour

The mechanical behaviour of Venetian soils was recently analysed by Cola & Simonini (2002), who examined laboratory test results of a rather detailed investigation carried out at the Malamocco Test Site (MTS), located in one of the Lagoon inlets.

Such a study showed that, due to the high silty content and to the low activity of clay minerals, the mechanical behaviour of these soils is mainly controlled by inter-particle friction. Moreover, geotechnical parameters gradually vary as the grain size distribution changes from medium-fine sands (SP-SM) to silty clays (CL), with very few exceptions concerning organic samples.

According to these remarks, Cola and Simonini expressed a number of intrinsic parameters, such as the critical state parameters λ, e_{ref}, M or the maximum shear stiffness G_{max}, as functions of the grain size composition through the grain size index I_{GS}. Such index, defined as the ratio between the non-uniformity coefficient U and the mean particle diameter D_{50}, seemed to be particularly suitable to express the dependence of intrinsic parameters on the grain-size characteristics.

A number of relationships proposed by Cola and Simonini were successfully used in a preliminary attempt to model the Venetian soils behaviour (Tonni *et al.*, 2003) using a Generalized Plasticity approach.

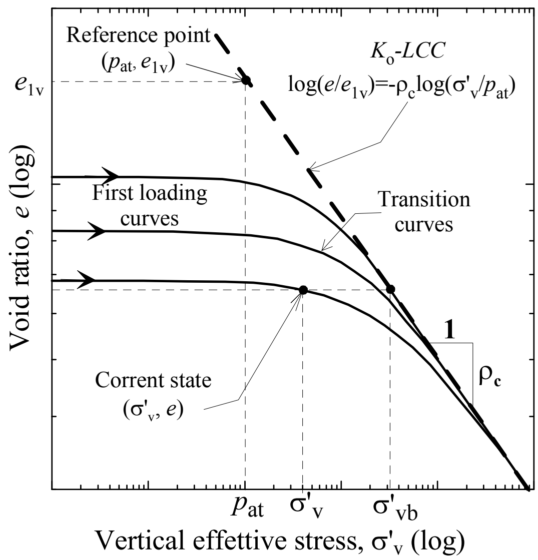

Figure 1. Compression curves for granular soils (adapted from Pestana & Whittle, 1995).

More recently, Biscontin *et al*. (2006) examined the non-linear compression behaviour of Venetian soils using the unified approach proposed by Pestana & Whittle (1995) for sandy and silty soils, which assumes that at very high pressures soils reach a unique Limiting Compression Curve (LCC), independently of the initial density.
In the log$e$-logσ'_v plane the LCC is well fitted by a line whose slope is intrinsically related to the soil mineralogy, being dependent only on the grain resistance to crushing.
Since in the LCC regime the coefficient of earth pressure K_0 can be considered constant, the isotropic and one-dimensional compression curves are parallel: hence, Biscontin *et al*. estimated the Pestana – Whittle model parameters using experimental results of a number of 1D compression tests on Venetian soils, pushed up to a maximum pressure of about 30 MPa.
Moreover, according to Mitchell theory on mixtures (1976), Biscontin *et al*. demonstrated that Venetian soils can be considered as mixtures of two fractions, namely a coarse-grained fraction composed of medium-fine sands and a fine-grained fraction composed of particles smaller than 5 μm.
The position of the LCC in the log$e$-logσ'_v plane – i.e. the void ratio e_1 at the reference pressure σ'_v=100 kPa – can be expressed as a function of the fine content FF, while the LCC slope is an intrinsic parameter which can be assumed as constant for all Venetian soils, except for few organic clays.

4. Calibration and modification of the original model

4.1. Experimental database

In Tonni *et al.* (2003), a preliminary attempt to apply the *PZ* model for analysing the mechanical behaviour of Venice soils was proposed, even though using a few tests on samples coming from the Malamocco Test Site (Cola & Simonini, 2002).
The triaxial testing programme on MTS soils, performed with a standard equipment, provided experimental evidence that sandy and silty samples tended to form shear bands as soon as the peak in the deviatoric stress was overcome, thus making it difficult to detect the critical state condition.
In order to dispose of a more reliable set of experimental data for the current calibration, four samples from MTS, 140 mm high and 70 mm diameter, were tested using an advanced triaxial cell available at the University of Padova, fully controlled through a personal computer and provided with local displacement transducers. According to suggestions of Rowe & Barden (1964) and Head (1982), the cell was outfitted with lubricated heads (San Vitale, 2004) in order to delay the development of shear bands. The main characteristics of the samples are listed in Table 1.
Table 1 also contains data of two more tests, SP-LD300 and SP-HD300, performed on 100 mm-diameter reconstituted samples of a medium-fine, highly uniform sand coming from MTS and prepared at different relative densities (Dalla Vecchia, 2002). Unlike the first group of tests, the latter was performed using a standard triaxial cell. The samples were prepared by dry pluviation in a mould placed on the triaxial base.
In Figure 2 deviatoric stress and volumetric strain of both sets of tests are plotted against the axial strain. Note that although lubricated ends were used, localization (characterized by a sudden drop in the deviatoric stress) was only delayed but not completely avoided.
Except for the sample tested in ML-200, containing few quantities of fines (FF = 10,5%), all the other samples show a dilatant behaviour which becomes less evident as the cell pressure increases (compare the volumetric response of SM-200 and SM-480 tests) or the density decreases (compare SP-HD300 and SP-LD300 tests). Such result confirms what observed by Cola and Simonini, i.e. that sandy and silty soils dilate at medium pressures and that dilatancy disappears when the mean stress goes over 1000 kPa. Contractive behaviour can be observed only when soils have a significant clay fraction.
In what follows, a calibration study of *PZ* model parameters, based on such new experimental data, will be presented. Since only drained triaxial tests in monotonic compression were considered, attention was merely focused on those features of the model accounting for monotonic loading conditions.

Table 1. List of tests and basic properties of the samples.

Test	SP-200	SM-200	SM-480	ML-200	SP-HD300	SP-LD300
Soil type	Fine sand	Sandy silt	Sandy silt	Silt	Uniform fine sand	Uniform fine sand
Type of sample	Natural	Natural	Natural	Natural	Reconst.	Reconst.
Depth[(1)] (m below MSL)	40,5	33,1	33,1	18,5	40,8	40,8
D_{50} (mm)	0,170	0,045	0,045	0,034	0,170	0,170
U=D_{60}/D_{10}	1,98	2,68	2,68	7,08	1,64	1,64
I_{GS}	0,086	0,017	0,017	0,0043	0,104	0,104
LL, IP	-	-	-	29, 9	-	-
Fine Fraction, FF (d<5μm) (%)	0,0	6,0	6,0	10,5	0,0	0,0
Cell pressure, σ_c (kPa)	200	200	480	200	300	300
Void ratio at consolidation, e_c	0,676	0,787	0,748	0,726	0,712	0,820

[(1)] All the samples were collected from MSgM2 bore, located in MTS at a point with bottom sea at 10,2 m from MSL.

As a result, the validation work was restricted to the dilatancy relationship and to those components of the plastic modulus such as H_0, H_f, H_v *and* H_s. Components H_U and H_{Dm}, describing the unloading-reloading behaviour, were not examined in this work.

4.2. Critical state parameters

As mentioned before, Cola and Simonini found that for Venice soils the material constants characterizing the CSL in the e-ln$p$' and q-p' plane can be related to the grain size composition through the index I_{GS}. Such relationships are:

$$\phi'_c = 38.0 + 1.55 \cdot \log I_{GS} \quad (°) \qquad (16)$$

$$\lambda_c = 0.066 - 0.016 \cdot \log I_{GS} \qquad (17)$$

$$e_{ref} = 1.13 + 0.10 \cdot \log I_{GS} \qquad (18)$$

which apply in the range $8 \cdot 10^{-5} \le I_{GS} \le 0.12$.

In Tonni *et al.* (2003), only eq. (16) was used for calibrating the *PZ* model parameters. In this context, according to modifications introduced in the constitutive equations, the whole set of equations (16)-(17) and (18) had to be used in order to evaluate the state parameter ψ. Values of ϕ'_c, λ_c, e_{ref}, referring to the six available samples, are summarized in Table 2.

4.3. Elastic moduli

As shown in section 3.2, in the *PZ* model the elastic behaviour of the material is assumed non-linear and can be described by eqs. (14)-(15).

In this work, K_o and G_o in eqs. (14)-(15) were estimated using a method successfully adopted in Tonni *et al.* (2003). According to such procedure,

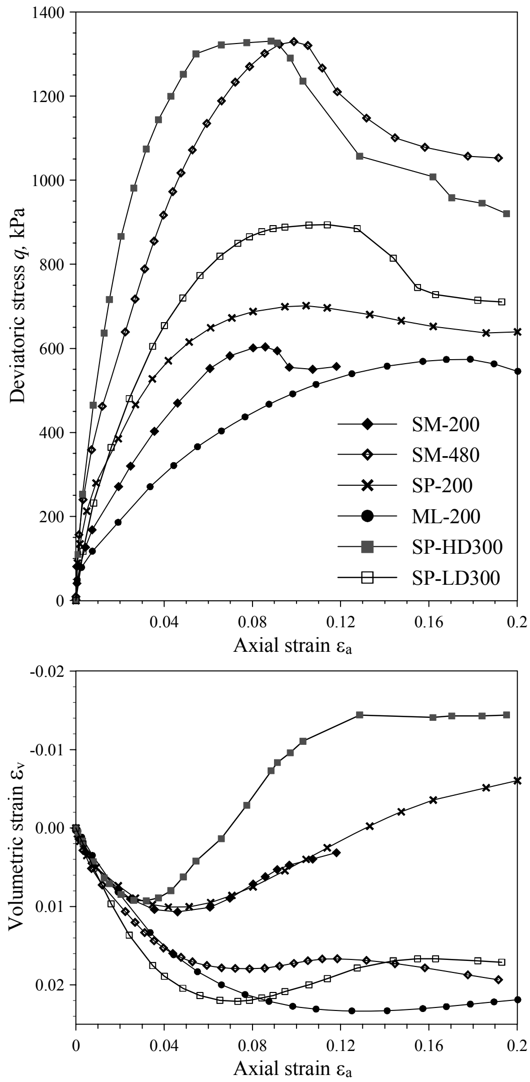

Figure 2. Deviatoric stress and volumetric strain vs. axial strain in tests used for calibration.

the initial shear modulus G_o can be estimated from G_{max}, which can be in turn determined by means of the Hardin & Drenevich (1972) relationship:

$$\frac{G_{\max}}{p'_{\text{ref}}} = D\frac{(2.97-e)^2}{(1+e)}\left(\frac{p'}{p'_{\text{ref}}}\right)^n \tag{19}$$

being D and n material constants and p'_{ref} a reference mean effective pressure assumed equal to 100 kPa. Parameters D and n were determined using the experimental fitting procedure recently proposed by Cola and Simonini for MTS soils. As a result, n can be assumed equal to 0.6 independently of the material, while D can be related to the grain size index as follows:

$$D = 470 + 60{,}4 \cdot \log I_{GS} \tag{20}$$

The modulus G_{max} in eq. (19) describes the response at very small strains (ε_s <0.001%) while G_0 refers to larger strain levels; hence G_0 values were calculated reducing G_{max} by a factor of 2.5, as also suggested by Gajo & Muir Wood (1999).
The bulk modulus K_0 was then calculated using the well-known relationship:

$$K_0 = \frac{2(1+\nu)}{3(1-2\nu)} \cdot G_0 . \tag{21}$$

in which ν is equal to 0.15.
Values of D, G_0 and K_0 are listed in Table 2.

4.4. Dilatancy of Venetian soils: a state dependent relationship

As known, dilatancy plays a crucial role in the modelling of the mechanical behaviour of granular soils. In what follows we discuss some issues related to the dilatancy expression originally adopted in the *PZ* model. Then we will examine the effect on the stress-strain response induced by using two different flow rules.
Following the pioneristic work of Rowe and according to many later contributions on sand modelling (e.g. Nova & Wood, 1979), the *PZ* model assumes that dilatancy $d_g = d\varepsilon_v^p / d\varepsilon_s^p$ is a unique function of the stress ratio $\eta = q/p'$, irrespective of the material internal state.
One of the major shortcomings of considering dilatancy as uniquely related to η is that different sets of constitutive parameters are needed for a single sand with different initial densities, thus avoiding any chance of unified modelling of the mechanical response over a wide range of densities and stress levels. As pointed out by Li & Dafalias (2000), a sand model with its dilatancy following equation (7) works well only when the change in the internal state is minor.
In recent years attempts (Wan & Guo, 1998; Li *et al.*, 1999; Gajo & Wood, 1999; Li & Dafalias, 2000) have been made to treat dilatancy as a state-dependent quantity, with the concept of critical state as basis: in these contributions indeed dilatancy is expressed in terms of void ratio-dependent parameters which measure the deviation of the current state from the critical one.
According to such developments, in this work a modified expression of the plastic flow rule was introduced in the original constitutive equations, in order to address the deficiencies of the model in capturing the evolution of sand behaviour due to pressure and density changes.
In particular we focused our attention on two existing relationships, recently proposed by Li & Dafalias (2000) and Gajo & Muir Wood (1999) respectively:

$$d_g = \frac{d_o}{M_g}\left[M_g \exp(m_d \psi) - \eta\right] \tag{22}$$

$$d_g = A_d [M_g (1 + k_d \psi) - \eta] \tag{23}$$

in which d_0, m_d, A_d and k_d are material parameters while ψ is the well-known state parameter (Been & Jefferies, 1985) measuring the difference between the current and the critical void ratios at the same mean effective pressure *p'*:

$$\psi = e - e_{crit} \tag{24}$$

In this work eq. (22) was expressed in an equivalent form, in which the ratio d_0/M_g was replaced by the parameter D_0:

$$d_g = D_0\left[M_g \exp(m_d\psi) - \eta\right] \tag{25}$$

Eqs. (23) and (25), obtained from eq. (7) by introducing in it an exponential and a linear dependence on ψ respectively, still guarantee basic premises of critical state soil mechanics and embed at the same time both pycnotropy and barotropy through the state parameter ψ that changes during the deformation process.

The calibration of parameters appearing in the above equations can be performed fitting the experimental ε_v-ε_a curve, by a trial and error procedure, taking into account that constants m_d and k_d can be determined at the phase transformation state, where the soil behaviour changes from contractive to dilatative. In that state, being dilatancy equal to zero, parameter k_d of eq. (23) is given by:

$$k_d = \frac{1}{\psi_{PTP}} \cdot \left(\frac{\eta_{PTP}}{M_g} - 1\right) \tag{26}$$

while m_d of eq. (25) is given by:

$$m_d = \frac{1}{\psi_{PTP}} \cdot \ln\left(\frac{\eta_{PTP}}{M_g}\right) \tag{27}$$

being η_{PTP} and ψ_{PTP} the stress ratio and the state parameter at the phase transformation point, respectively.

According to these remarks, a preliminary estimate of m_d and k_d was determined for every single test, based on the phase transformation point; then, in order to avoid as much as possible multiple sets of parameters, mean values of m_d and k_d were considered. Particularly, a unique value was adopted for samples having the same I_{GS}. Final values of such parameters are listed in Table 2, together with the

Table 2. Parameters obtained from calibration.

Test	SP-200	SM-200	SM-480	ML-200	SP-HD300	SP-LD300
Soil type	Fine sand	Sandy silt	Sandy silt	Silt	Uniform fine sand	Uniform fine sand
ϕ'_c	36.3	35.3	35.3	34.3	36.5	36.5
λ	0.083	0.095	0.095	0.104	0.082	0.082
e_{ref}	1.023	0.953	0.953	0.893	1.032	1.032
M_g	1.48	1.43	1.43	1.39	1.48	1.48
M_f	0.70	0.81	0.81	0.79	1.10	0.88
K_0 (kPa)	84580	64300	115160	63360	103550	88310
G_0 (kPa)	77220	58710	105150	57850	94540	80630
α_f	0.45	0.45	0.45	0.45	0.45	0.45
m_d	0.05	1	1	0.2	0.05	0.05
D_0	1	1	1	1	1	1
$k_d^{(*)}$	-	1.1	1.1	-	-	-
$A_d^{(*)}$	-	1	1	-	-	-
β'_0	0.33	0.80	0.80	0.33	0.35	0.35
β'_1	10	10	10	10	10	10

(*) Parameters k_d and A_d refer to the dilatancy expression of eq. (23), which was only used for comparison purposes with eq. (25) in tests SM-200 and SM-400.

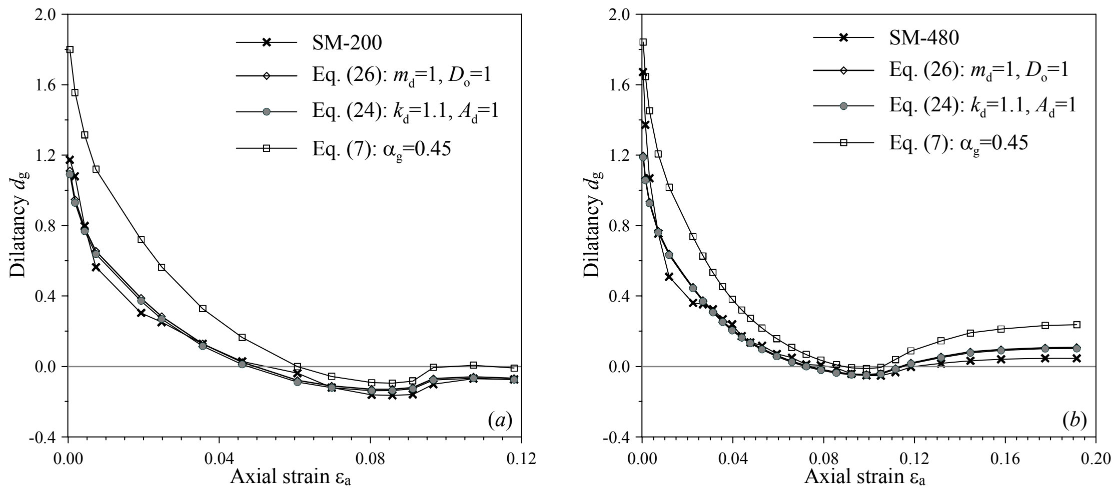

Figure 3. Comparison among experimental and predicted dilatancy in tests SM-200 (a) and SM-480 (b), using different flow rules

other material constants.

As an example, in Figures (3a) and (3b) experimental data of $d\varepsilon_v / d\varepsilon_d \approx d\varepsilon_v^p / d\varepsilon_d^p = d_g$ are compared with dilatancy predictions obtained from eqs. (23), (25) and (7), the latter being the expression originally proposed in the model. Such curves refer to SM-200 and SM-480 tests performed on a sandy silt consolidated at two different stress levels.

It can be noted that both eqs. (23) and (25) can successfully simulate the experimental data; moreover, the best fit of experimental curves gives similar responses for both formulations.

Hence, the two expressions seem to be both suitable to describe dilatancy of Venetian soils and appear almost equivalent in their predictions. Moreover SM-200 and SM480 were reproduced without changing the material parameters.

4.5. A convenient plastic modulus expression for Venetian soils

4.5.1. Plastic modulus at constant stress ratio compression

There are a number of constitutive models assuming that sands, compressed at constant stress ratios, do not experience any plastic deformation before reaching the crushing regime. Such an assumption implies that in isotropic compression (which is a special case of constant stress ratio compression) the plastic modulus is infinite as long as the material remains within a “cap” limiting the elastic behaviour.

In the original *PZ* model, there is no need to introduce any cap in order to allow for plastic deformations in isotropic compression, since the volumetric plastic strain is inversely proportional to the material parameter H_0, as evident from eqs. (11)-(13) when η is assumed equal to 0.

According to the Cam-Clay model, plastic deformations in clay occur when the stress path moves along the normal compression line (NCL); hence, H_0 can be determined as:

$$H_0 = \frac{1+e}{\lambda - \kappa} \tag{28}$$

in which e is the initial void ratio while λ and κ are the slopes of the isotropic normal compression and swelling lines respectively. Using eq. (28), H_0 may vary in clay within 5-200, with the lowest values only for very soft clays: as an example, in Bangkok clay H_0 is equal to 6.6 (Pastor *et al.*, 1990).

As regards granular soils, Zienkiewicz *et al.* (1999) observed that such parameter can be determined by fitting the experimental curves p-ε_a or q-ε_a: estimations of H_0 reported in Pastor *et al.* (1990) vary between 350 and 16000, with higher values generally for dense sands.

In Tonni *et al.* (2003), the calibration of H_0, performed by fitting the q- ε_a curves of three drained triaxial tests, gave values equal to 800, 1000 and 2800 for CL, ML and SP-SM samples respectively. Although the estimated values seemed to be in good agreement with those reported in Pastor *et al.*, the procedure did not appear sufficiently reliable. In order to address this issue, alternative formulations for the isotropic compression of sands were examined, with particular reference to studies of Pestana & Whittle (1995) and Jefferies & Been (2000).

The unified approach proposed by Pestana & Whittle (1995) overcomes the limitations of some constitutive models assuming that soils exhibit irreversible volumetric strains for η-constant paths, also at low pressures: plastic strain gradually increases as long as the distance from the LCC decreases.

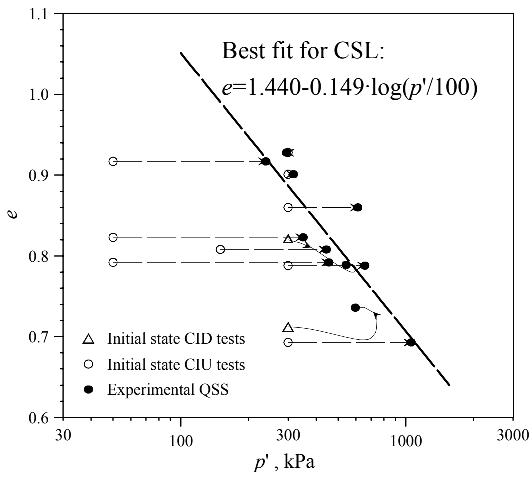

Figure 4. CSL for sand of tests SP-HD300 and SP-LD300 (from Dalla Vecchia, 2002).

As observed by Biscontin *et al.*, such a response characterizes also the 1D compression behaviour of Venetian soils, so that the Pestana-Whittle formulation can be successfully used to predict experimental data from normal compression tests on these soils. In such model, the volumetric plastic strain in isotropic compression is given by:

$$d\varepsilon_{\mathrm{v}}^{p} = \frac{e}{1+e}\rho_{\mathrm{c}}\left(1-\delta_{\mathrm{b}}^{\vartheta}\right)\frac{dp'}{p'} \tag{29}$$

being:

$$\delta_{\mathrm{b}} = 1 - p'/p'_{\mathrm{b}} \tag{30}$$

a normalized parameter varying in the range of [0-1], accounting for the distance from the limiting compression curve (LCC), and p'_b the octahedral stress on the LCC at the same void ratio. The exponent θ in eq. (29) governs the compression curve curvature when it comes close to the LCC: the higher θ, the higher the curvature.

Comparing eqs. (29)-(30) with eq. (11), it follows that H_0 can be expressed as:

$$H_0 = \frac{1+e}{e}\frac{1}{\rho_{\mathrm{c}}(1-\delta_{\mathrm{b}}^{\vartheta})} \tag{31}$$

Values of H_0, obtained from eq. (31) adopting data from Biscontin *et al.*, range from 109 to 640 (see Table 3).

On the other hand, the alternative approach on the isotropic compression of sands, recently proposed by Jefferies & Been (2000), combines premises of CSSM (as the uniqueness of critical state line) with the idea that there is a single, unique LCC only when grain crushing becomes prevalent.

Based on several isotropic compression tests on Erksak sand, the authors found that the plastic bulk modulus K^p could be expressed as follows:

$$K^{p} = 0{,}3\cdot K^{e}\cdot\exp(-6{,}5\psi)\left(1+1{,}3\frac{p'}{\sigma_{\chi}}\right) \tag{32}$$

where ψ is the well-known state parameter and σ_χ is the apparent grain crushing pressure in shear, corresponding to a discontinuity of the CSL slope in the *e*-ln*p*' plane (Verdugo, 1992). Indeed, the critical void ratio is proportional to the logarithm of *p'* when $p' < \sigma_\chi$ and becomes proportional to the mean pressure for $p' > \sigma_\chi$.

As K^p coincides with H_L, H_0 can be estimated as:

$$H_0 = \frac{K^{p}}{p'} \tag{33}$$

To estimate the apparent grain crushing pressure in shear σ_χ of Venetian soils, required in eq. (32), two

Table 3. Values of H_0 from Pestana-Whittle and Jefferies-Been formulations.

Test	SP-200	SM-200	SM-480	ML-200	SP-HD300	SP-LD300
Pestana-Whittle	640	207	109	220	332	169
Jefferies-Been	1017	226	159	214	611	258

experimental observations were taken into account.
The first one is that sandy and silty samples did not exhibit any dilatancy only when triaxial tests were performed at mean confining pressures higher than 1-1.2 MPa. Moreover, results (Dalla Vecchia, 2002) from drained and undrained triaxial tests performed on the same soil tested in SP- DH300 and SP-LD300, showed that in a semilogarithmic plot the critical state line is linear up to 1 MPa (Figure 4).
Secondly, sandy and silty fractions in Venetian soils are a mixture of carbonates and silicates with very fine grains, the grain size distribution varying from uniform to well graded.
Since McDowell & Bolton (1998) remark that fine well-graded soils crush at higher pressures than coarse-grained soils, it follows that the apparent grain crushing pressure in shear for Venetian soils should be higher than the value (700 kPa) adopted for Erksak sand, being the latter a coarse, uniform soil. All considered, a final value of 1,2 MPa was adopted for σ_χ.
For comparison purposes, value of H_0 obtained through both Pestana-Whittle and Jefferies-Been methods, at the same initial confining pressures, are listed in Table 3: note that the two procedures seem to be alternative as they give values that are similar or, in some cases, within the same order of magnitude. Such a result confirms the reliability of the Been & Jefferies approach.
Eq. (32) was obtained by fitting experimental tests on Erksak sand samples, thus a more sensitive calibration work would be necessary in order to properly apply the relationship to other granular soils. However, the formulation proposed by Jefferies & Been has the advantage of describing the isotropic compression behaviour of sands in terms of the same parameter ψ which was introduced in the modified dilatancy equation, thus allowing a unified modelling within the state parameter framework. As a result, in this work H_0 was determined using eq. (32), according to Been & Jefferies formulation.

4.5.2. Other components of plastic modulus

The components of the plastic modulus defined in eqs. (12a,c) account for the decrease in the plastic modulus H_L as the shear plastic deformation increases and the critical state is reached. In what follows we briefly discuss some issues related to the calibration of parameters appearing in the expression of H_L. Since no particular tests had been performed for determining α_f and M_f, reliable estimations of such parameters were obtained according to suggestions of Zienkiewicz *et al.* (1999). Hence a constant value equal to 0.45 was adopted for α_f while a preliminary estimation of M_f was obtained by considering the following relationship:

$$M_f = M_g \cdot D_R \tag{34}$$

where the relative density D_r was assumed equal to 50%, being this a reliable value for Venetian

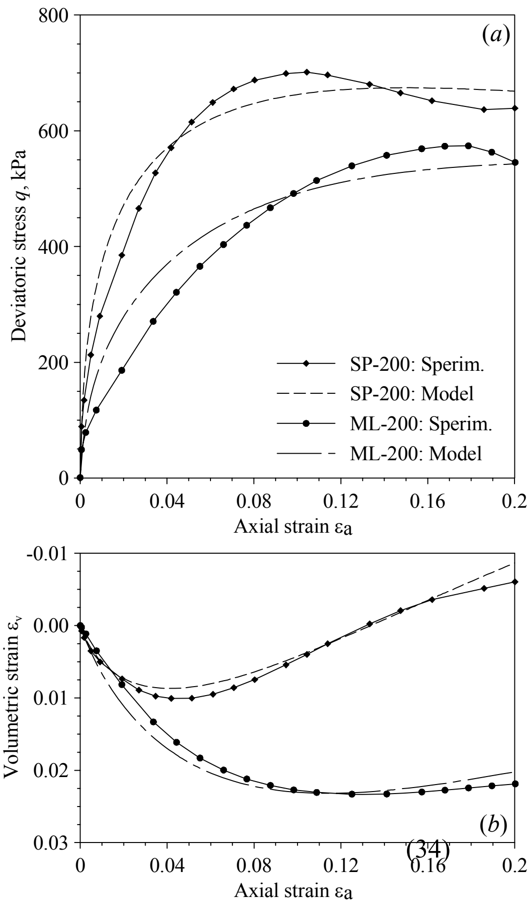

Figure 5. Experimental and predicted curves for tests SM-200 and SM-480 on sandy silt.

soils. M_f was then adjusted in order to get a better fit of the stress ratio at which the soil behaviour changes from contractive to dilatative.
The expression of H_s was slightly modified as follows:

$$H_s = \beta'_0 \exp(-\beta'_1 \xi) \quad (35)$$

being β_0' e β_1' two material parameters corresponding to $\beta_0 \cdot \beta_1$ and β_0 of eq. (12c) respectively. The modification was introduced in order to make the calibration of such parameters easier: indeed, keeping fixed all the other material parameters, experimental values of H_s can be determined from test results and fitted in the H_s – logξ plot. From calibration, parameter β_1' turned out equal to 10 for all the tests, while β_1' varied from 0.3 to 0.8, as summarized in Table 2.

5. Validation of the model and final remarks

Table 2 summarizes the parameter values obtained through the calibration work, which was performed according to the procedures described in previous sections.
Correspondingly, in Figures 5, 6 and 7 model predictions of soil response, plotted in terms of deviatoric stress and volumetric strain versus axial strain, are compared with experimental data.
Such predictions, obtained using the Li-Dafalias flow rule, show that as long as the strains do not localize, the volumetric behaviour is successfully modelled in all the tests.
As regards parameter m_d appearing in dilatancy equation (25), it must be noted that different values had to be considered with varying the tests. Nevertheless, a unique value could be adopted when considering two soils having the same fine content. Moreover, the small number of available tests did not allow to explain m_d oscillations: difficulties in determining a reliable value of such parameter could

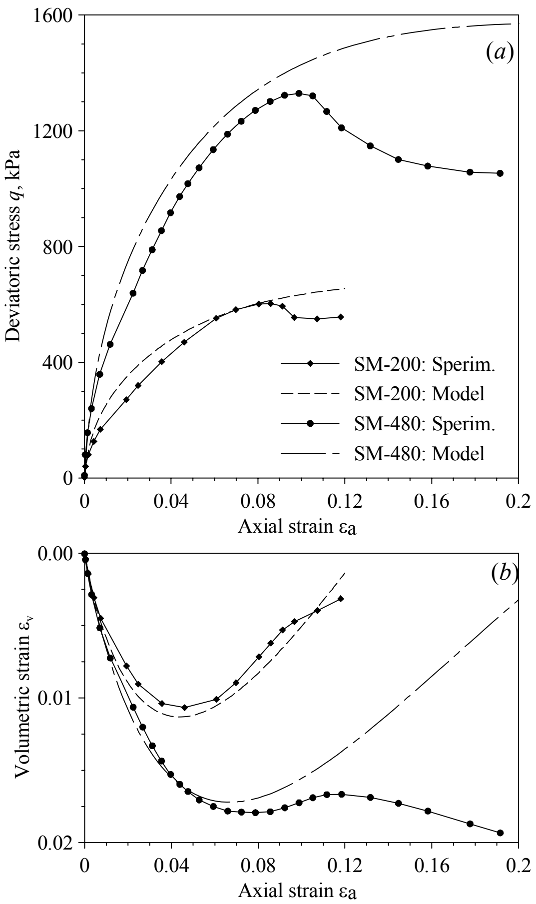

Figure 6. Experimental and predicted curves for tests SP-200 and ML-200.

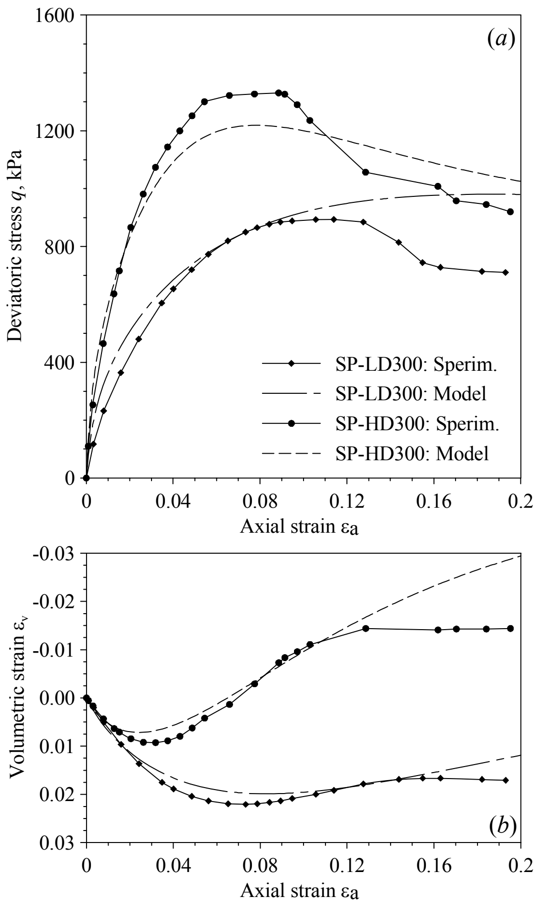

Figure 7. Experimental and predicted curves for tests SP-HD300 and SP-LD300.

be maybe related to its dependence on the state parameter ψ, which was in turn calculated from empirical relationships having some degree of uncertainty.

Observing curves of deviatoric stress vs. axial strain, it can be noted that there is a good agreement between model predictions and experimental data, particularly at small and medium strain levels.

It's worth remarking that the model predictions match fairly well the experimental stress-strain curves for pre-peak deformations, while in post-peak regime they show a lower rate of softening.

Nevertheless it is well known that, due to localization phenomena, an inhomogeneous distribution of stresses and strains occurs in the specimens, leading to potentially unreliable data. As a result, the steepness of the post-peak load-displacement curve can be significantly overestimated in laboratory tests and the softening, as experimentally observed, cannot be regarded as a material property. According to this remarks and considering that the model in use cannot describe localization of shear bands, the calibration of the parameters was performed so as to achieve the best fit of the experimental curves before reaching the post-peak regime, although numerical analyses were in general pushed up to an axial strain of about 20%. It's obvious that the comparison is no longer meaningful.

Finally, although high heterogeneity makes difficult proper modelling of Venetian soils, it must be noted that the proposed calibration procedure provides a rather simple tool for evaluating with a reasonable accuracy the model parameters, for all the Venetian soil classes.

Moreover, the introduction of a state dependent dilatancy into the original *PZ* model resulted in reliable predictions of the volumetric response using a unique set of constitutive parameters over a wide range of pressures.

On the other hand, the new expression of the plastic modulus component H_0 allowed embedding recent developments on the isotropic compression behaviour of sands within a Generalized Plasticity approach.

Further improvements should be introduced in the constitutive equations, in order to get a completely unified modelling of such natural soils over a full range of densities and stress levels. Modifications should concern those components of the plastic modulus different from H_0, by introducing in them a further dependence on the state parameter ψ.

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