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Experiments on bar formation in a straight flume
2. Graded sediment

Stefano Lanzoni

Dipartimento di Ingegneria Idraulica, Marittima e Geotecnica, Università di Padova, Padua, Italy

Abstract. Laboratory experiments have been carried out in a large laboratory flume using a strongly bimodal sediment mixture in a range of flow conditions such that the initially flat bed of the flume became unstable enhancing the development of alternate bars. In order to elucidate the effect of grain sorting, the present experiments are compared with a previous set of experiments performed in the same flume under similar hydraulic conditions but using a nearly uniform sand with a mean geometric grain diameter equal to that characterizing present experiments. The comparison suggests that sediment heterogeneity may appreciably affect bed form characteristics. The development of small-scale (ripples) and mesoscale (dunes) sediment waves tended to be inhibited thus allowing a decrease in flow resistance. Owing to the bimodal character of the adopted mixture at low values of the bed shear stress a condition of partial transport was attained for which only the finer-grained portion of the mixture was observed to move, while the coarser-grained fractions remained essentially immobile throughout the experimental run. However, complete mobilization of all size fractions was observed to occur in runs carried out at higher slopes (i.e., at higher bed shear stress) in order to generate an alternate bar pattern. Selective transport of individual grain size fractions, coupled with the characteristic bar topography pattern led to an intense longitudinal sorting which accreted the coarser particles on bar crests. Furthermore, bar migration caused, through scour and fill, a significant vertical sorting. As a consequence of these sorting processes and in accordance with previous experimental observations, bar height turns out to be invariably damped with respect to uniform sediment experiments. The trend exhibited by the wavelengths is less clear and suggests that in the present experiments sorting effects do not enhance the clear shortening of bar wavelengths typically observed in other series of flume experiments carried out with weakly bimodal mixtures.

1. Introduction

Alternate bars are the most common type of bed forms which are observed both in sandy streams and in gravel bed rivers. They essentially consist of migrating alternating regions of scour and deposit with horizontal scales of the order of a few channel widths and vertical scales of the order of flow depth. Their vertical scale being so large, the problem of predicting their occurrence and their characteristics has a profound impact on several aspects of river engineering.

The latter argument has motivated a great interest in this subject both within the geomorphologic community (starting with the pioneering works of *Leopold and Wolman* [1957] and *Kinoshita* [1961]) and in the engineering literature (see the reviews in the *Ikeda and Parker* [1989] book. The question regarding the basic mechanism underlying the formation of alternate bars may be considered as fairly settled: These bed forms arise as a result of a three-dimensional instability of the bottom which gives rise to a stable periodic pattern. Both linear and weakly nonlinear theories are available [*Blondeaux and Seminara*, 1985; *Colombini et al*., 1987; *Struiksma and Crosato*, 1989; *Schielen et al*., 1993] for the case when the dominant form of transport is bed load. Linear analyses also exist which attempt to account for the effect of suspended load [*Fredsøe*, 1978; *Watanabe and Tubino*, 1992; *Repetto et al*., 1996].

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However, the existing literature mostly concentrates on the case of uniform sediment, while, on the contrary, sediment heterogeneity is a distinctive feature of natural rivers. This is partly motivated by the complexity and cost of laboratory experiments involving sediment mixtures, which, especially when dealing with large bed forms, require a large amount of sediment to be supplied. Moreover, a sufficient knowledge of the transport process for sediment mixtures has been achieved only recently (see the review papers of *Parker* [1991] and *Seminara* [1995]). In particular, the interaction between substrate, bottom surface, and bed load has been extensively investigated on the basis of both field and flume data [*Parker et al*., 1982; *Parker and Klingeman*, 1982; *Andrews and Parker*, 1987; *Iseya and Ikeda*, 1987; *Wilcock and Southard*, 1988; *Wilcock*, 1992; *Kuhnle*, 1992; *Wilcock and McArdell*, 1993]. Also, the mutual influence of sediment gradation on small-scale bed forms (such as ripples and dunes), on mean fractional transport rates, and on vertical sorting has been discussed [*Klaassen et al*., 1987; *Ribberink*, 1987; *Wilcock and Southard*, 1989; *Klaassen*, 1990].

In this paper the attention is focused on the influence of sediment nonuniformity on bar formation and on the equilibrium characteristics of bars. The effects of perturbations of bed elevation on flow field and the action of gravity on the direction of bed load transport have been shown to crucially affect bar instability when considering uniform sediment [*Blondeaux*

Table 1. Properties of the Various Sediment Mixtures[a]

Material	d_g^*	d_{50}^*	d_{10}^*	d_{30}^*	d_{70}^*	d_{90}^*	σ_g^*
FUNI	0.193	0.192	0.140	0.169	0.223	0.276	1.292
CUNI	2.072	2.078	1.081	1.440	3.075	3.968	1.680
FC70	0.494	0.262	0.157	0.199	1.280	3.210	3.305
MUNI	0.480	0.481	0.331	0.417	0.551	0.710	1.301

[a]The sediment mixture (FC70) has the same geometric mean diameter as the nearly uniform sand (MUNI) of previous experiments. The sediment was prepared by mixing two nearly unisize fine (FUNI) and coarse (CUNI) mixtures. Measurements are in millimeters.

and Seminara, 1985; *Colombini et al.*, 1987; *Struiksma and Crosato*, 1989; *Lanzoni*, this issue]. An attempt to incorporate the effects related to sediment heterogeneity within the classical framework of linear stability analysis of bar formation has been recently undertaken by *Lanzoni and Tubino* [1999]. Theoretical results and existing flume experiments [*Lisle et al.*, 1991; *Lanzoni et al.*, 1994] suggest that grain sorting associated with selective transport of graded sediment may induce a significant modification in the delicate balance between stabilizing and destabilizing effects. In particular, sediment heterogeneity was experimentally found to induce a somehow episodic bar development even under steady hydraulic conditions. Theoretical results, moreover, suggest that within a fairly wide range of the dimensionless control parameters (i.e., the average Shields stress Θ, the width ratio β, and the relative bed roughness d_s), various contributions related to sediment heterogeneity act simultaneously to produce an overall stabilizing effect on bottom development, which leads to an appreciable reduction of bar amplitude and a shortening of bar wavelengths.

The present contribution concerns the results of a series of experiments designed in order to get a better understanding of the effect of sediment heterogeneity on bar morphology and to give further insight on some basic physical mechanisms related to sediment nonuniformity such as longitudinal and vertical sorting. In order to elucidate, the effects of sorting experimental data obtained by recirculating a strongly bimodal mixture of fine and coarse sediment are here compared with the results of a former series of experiments presented in a companion paper [*Lanzoni*, this issue] performed under similar hydraulic conditions but using a nearly uniform sand. Here and in the following the latter experiments will be denoted as MUNI.

The paper is organized as follows. The experimental methods are described in section 2. Section 3 is devoted to the discussion of the experimental results. Finally, in section 4 some concluding remarks are presented.

2. Materials, Experimental Apparatus, and Methods

2.1. Sediment

The sediment mixture adopted in the experiments (hereafter denoted as FC70) was chosen such that its geometric mean diameter was the same as the nearly uniform sand used in the MUNI experiments. It was prepared by suitably mixing two nearly unisize fine (FUNI) and coarse (CUNI) mixtures; in particular, FC70 was composed with 67% FUNI and 33% CUNI. The synthetic characteristics of the various mixtures are reported in Table 1 where d_i^* is the grain diameter such that ith percentage of the material is finer than d_i^*; d_g^* and σ_g^* are the geometric mean diameter and geometric standard deviation, respectively. All the sediments consisted almost entirely of quartz and nearly quartz density mineral, with a density $\rho_s =$ of 2.65 g/cm^3. The particle grain size distributions of FC70, FUNI, CUNI, and MUNI mixtures are shown in Figure 1. The poorly sorted, strongly bimodal character of FC70 sediment is quite clear. Indeed, the two modes of its grain size distribution are separated by a factor of approximately 10, thus yielding a value 5.7 of the index of bimodality formulated by *Wilcock* [1993]; that is well above the value 1.7 suggested as discriminating the bimodal character of a given mixture. The choice of such a mixture was first motivated by the fact that bimodal sediments are common in gravel bed rivers [*Shaw and Kellerhals*, 1982; *Parker*, 1991] as well as within the transition between gravel and sand beds [*Sambrook Smith et al.*, 1995]. Moreover, the theoretical interpretation of the dynamics of graded sediment is greatly simplified when the grain size distribution is bimodal; also, the influence of sorting effects is enhanced by increasing the geometric standard deviation of the mixture [*Lanzoni and Tubino*, 1999]. It must, however, be noted that poorly sorted, strongly bimodal mixtures exhibit a quite peculiar behavior of the various fractional transport rates [*Wilcock and McArdell*, 1993]. Depending on the ratio of the skin friction portion of the bed shear stress τ^{*I} to the reference shear stress τ_{ri}^* for incipient motion of ith size fraction, four transport regions can be defined. At $\tau^{*I}/\tau_{ri}^* < 1$, fractional transport rates are negligibly small. For $1 < \tau^{*I}/\tau_{ri}^* < 2.1$, fractional transport exists such that fractional transport rates of coarser particles are substantially smaller than those of smaller sizes, and at least some of the coarser particles are immobile. For $\tau^{*I}/\tau_{ri}^* > 2.1$, the transport rates of all fractions tend to be equal. Finally, as the friction velocity u_*

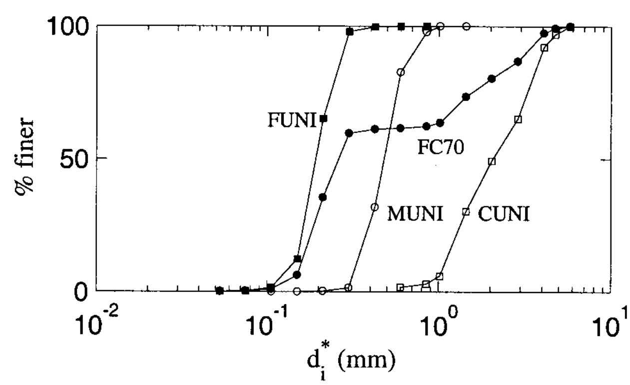

Figure 1. Particle size distribution of the sediment mixture (FC70) used in the present experiments. Such a mixture was obtained by mixing 67% of nearly unisize fine (FUNI) and 33% of coarse (CUNI) sand. Figure 1 also reports the grain size distribution of the nearly uniform sediment (MUNI) used in a previous series of experimental runs carried out under similar hydraulic conditions.

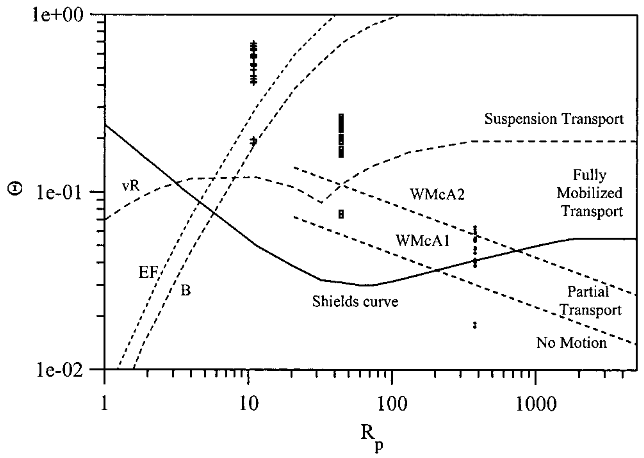

Figure 2. The critical values of the Shields parameter Θ for the beginning of motion, for partial and full mobilization, and for beginning of suspension are plotted against the particle number $R_p = (\Delta g d^{*3}/\nu^2)^{0.5}$, where ν is the kinematic viscosity coefficient, $\Delta = (\rho_s - \rho)/\rho$, and ρ and ρ_s are water and sediment densities, respectively. The Shields curve specifies the incipient motion threshold of unisize sediment. The curves denoted as WMcA1 and WMcA2 represent the initial-motion threshold and the threshold between partially and fully mobilized transport, respectively, obtained by *Wilcock and McArdell* [1993] in the case of a poorly sorted, strong bimodal mixture. The curves labeled as B, EF, and vR provide the estimates of the onset of suspension transport given by *Bagnold* [1966], *Engelund and Fredsøe* [1982], and *van Rijn* [1984b]. The points reported in the plot represent the experimental values of the Shields parameter evaluated by considering the geometric grain size (squares) and the grain sizes corresponding to the fine (plus signs) and the coarse (circles) mode of the FC70 mixture.

approaches the setting velocity w_i^* proper to the considered size fraction, a substantial proportion of the transport of this fraction will occur as suspension. The various transport regions are shown in Figure 2. Figure 2 also illustrates the Shields curve for incipient motion of uniform sediment together with various criteria for the onset of significant transport. The two curves delimiting the regions of no motion, of partial transport, and of fully mobilized transport have been obtained by estimating the reference shear stress of each size fraction composing the FC70 mixture (shown in Figure 1) through the relationship $\tau_{ri}^* = 0.68(d_i^*)^{0.55}$ suggested by *Wilcock and McArdell*'s [1993] experiments. Also, note that while the criteria suggested by *Bagnold* [1966] and *Engelund and Fredsøe* [1982] define the limit at which a concentration profile possibly starts to develop, the limiting curve of *van Rijn* [1984] defines an intermediate stage at which locally a certain amount of sediment begins to be brought into suspension by turbulent bursts (for a detailed discussion of particle-turbulence interactions see *Niño and Garcia* [1996]).

2.2. Apparatus

The present experiments, as well as the MUNI tests, were carried out in a rectangular channel, 50 m long, 1.5 m wide, and 1 m deep, with sidewalls consisting of a steel frame with glass windows. The main characteristics of this flume, the related water circuit, the various control devices, and the experimental procedures adopted to measure water discharge, sediment transport rate, and bottom profiles, are thoroughly described in a companion paper [*Lanzoni*, this issue]. Here only some additional information is given concerning the system adopted to supply the sediment and the procedure followed to measure the grain size distribution of the sediment.

Two options are available when working with graded sediment: One may either supply sediment with known particle size distribution or recirculate it. The question of whether sediment feeding or recirculation in flume experiments best approximates nature has been extensively discussed in the literature [*Parker et al.*, 1982; *Wilcock and Southard*, 1988; *Wilcock*, 1992]. While in the case of uniform sediment any difference is hardly detectable, for mixtures a difference is apparent in that the finer grains can filter below the pavement and thereby be removed from bed load in the recirculating case. The choice between these two options is not straightforward since natural gravel bed rivers are hybrid systems in themselves [*Milhous*, 1973; *Parker et al.*, 1982]. The recirculating option was adopted in present experiments because of the large amount of sediment transported during each run. The material settling on the bottom of the V-shaped sediment trap (10.45 m long and 3.90 m wide) located at the downstream end of the channel was then sucked out through a hydrocyclone pump and conveyed to the upstream end of the flume with a small amount of water. There, water and sediment were suitably separated through a second hydrocyclone, and the sediment was evenly distributed across the width of the flume via a diffuser located at the inlet section of the flume.

In some runs (P0109, P0609, P0709, and P1309) the transported sediment was sampled during the equilibrium phase by collecting into a bucket the sediment flowing over the diffuser. The samples were then saved for size analysis, and the sediment was not returned to the flume. Indeed, the dropping frequency (in terms of immersed weight, 10 kg of sediment every 2–3 min) was estimated to be high enough to prevent that sampling might affect sediment supply. The fractional transport rate q_{si}^* of the ith size fraction was then calculated as $q_s^* f_{ai}$, where f_{ai} was the proportion of the ith fraction in the transport and q_s^* was the measured total transport rate per unit width. Moreover, at the end of P2009 the bed material in the flume was sampled at several locations using the technique developed by *Ribberink* [1987]. After stopping the water flow and leaving approximately 20 cm of water depth in the flume, a thin cylinder with a diameter of 5 cm was gently pressed into the bed at fixed locations. Sediment layers with a thickness of 0.5 cm were then removed from the bed by siphoning and saved for size analysis.

2.3. Methods

The bed slope and water discharge ranges were similar to those investigated in the MUNI tests. A summary of the experimental conditions is reported in Table 2, where Q^* denotes water discharge, i_s is the water surface slope, D_0^* is the average flow depth, U_0^* is the average velocity, τ_0^{*I} is the skin friction portion of the total bed shear stress τ_0^*, and Q_s^* is the measured average volumetric solid discharge, including pores. Note that some runs were repeated more than once in order to test the reproducibility of the phenomena being investigated. Following *Wilcock* [1993], τ_0^{*I} was estimated using the drag-partition procedure of *Einstein* [1950] and assuming the mean geometric grain size of the CUNI mixture to be representative of grain roughness.

Before each test the bed material placed inside the flume

Table 2. Summary of Hydraulic Experimental Conditions[a]

Run	Duration, hours	Q^*, L/s	i_s, %	D_0^*, cm	U_0^*, m/s	τ_0^{*I}, Pa	τ_0^*, Pa	Q_s^*, L/h
P0206	93	30.0	0.176	6.4	0.31	0.586	1.049	···
P0807	73	30.0	0.416	4.3	0.46	1.306	1.690	199.7
P0606	44	35.0	0.179	7.2	0.32	0.617	1.209	···
P0507	62	35.0	0.409	4.5	0.52	1.523	1.730	240.4
P0609	24	35.0	0.511	4.5	0.52	1.603	2.191	345.1
P2006	22	40.0	0.263	5.3	0.50	1.291	1.291	···
P2906	21	40.0	0.322	5.2	0.51	1.405	1.561	···
P0307	45	40.0	0.393	4.9	0.54	1.640	1.781	257.3
P1207	46	40.0	0.393	4.6	0.58	1.677	1.677	276.9
P0109	51	40.0	0.513	4.7	0.57	1.829	2.285	378.5
P0509	19	40.0	0.491	4.8	0.56	1.802	2.187	388.0
P0809	31	45.0	0.505	5.1	0.59	1.961	2.400	···
P1309	29	45.0	0.525	5.0	0.60	2.004	2.474	471.0
P2009	3	45.0	0.526	5.0	0.60	1.998	2.484	391.7
P0709	7	50.0	0.501	5.3	0.63	2.139	2.478	502.4
P0806	3	55.0	0.209	6.8	0.54	1.352	1.284	···

[a]Definitions are as follows: Q^*, water discharge; i_s, water surface slope; D_0^*, average depth flow; U_0^*, average velocity; τ_0^{*I}, skin friction portion of total bed shear stress τ_0^*; and Q_s^*, measured average volumetric solid discharge, including pores.

was thoroughly mixed by hand in such a way that vertical sorting was prevented as much as possible. The bed was then leveled according to a selected slope in order to make the initial conditions for all runs as consistent as possible. During each run the tailgate located at the end of the sand trap was initially adjusted to obtain the required energy slope while the discharge was kept constant. The experiment was stopped when equilibrium conditions were obtained, that is, when the bed slope was equal to the water surface slope and the solid discharge and bar features were observed not to vary significantly in time.

3. Discussion of the Experimental Results

3.1. General Features

The characteristics of both small-scale (ripples and/or dunes) and large-scale (bars) bed forms growing within the flume were strongly affected by the behavior of the various fractional transport rates. The experimental values of the Shields parameter referred to the geometric grain size d_g^* of the adopted mixture and to the diameters corresponding to the two modes of the mixture; namely, $d_{g\mathrm{FUNI}}^*$ and $d_{g\mathrm{CUNI}}^*$, are plotted in Figure 2. It clearly appears that in two runs (i.e., P0206 and P0606), owing to the low value of the bed shear stress, a condition of partial transport was achieved for which only finer fractions were transported while coarser particles kept still, the coarse mode of the mixture having a value of the ratio τ_0^{*I}/τ_{ri}^* below the threshold for incipient motion. No bars were observed to form within the flume, and the bed was covered by ripples. Runs P0806, P2006, P2906, P0307, P0507, P0807, and P1207, though characterized by increasing values of the ratio τ_0^{*I}/τ_{ri}^*, fell in the region of partial transport ($1 < \tau_0^{*I}/\tau_{ri}^* < 2.1$) as well. In these runs, however, the proportion of grains that remained immobile over the duration of the experiments decreased appreciably as compared with P0206 and P0606. Generally, the bed remained nearly plane and very long, and almost steady bars formed as shown in Figures 3a and 3b; migrating bars with small heights were observed to grow episodically only in the downstream reach of the flume. Moreover, paved superficial layers (approximately 5–20 cm wide) typically formed near the sidewalls of the flume a few hours after the beginning of a given run. These layers, possibly triggered by the decrease of the velocity in the wall boundary layer, were characterized by a planimetric wavy pattern in the longitudinal direction which was strictly related to the slight flow deflection induced by the bottom topography associated to the steady bars. Increasing further the bed shear stress, a condition of fully mobilized transport was approached or attained (P0109, P0509, P0609, P0709, P0809, P1309, and P2009). In many of these runs, a few minutes after the beginning, longitudinal alternate streaks of coarse and fine material formed which, from time to time, joined together to form the oblique fronts of incipient small bars (see Figure 4). A regular sequence of well-formed alternate bars (Figures 3c and 3e) was observed only in early stages of a given run. These initially formed bars gradually migrated out of the flume, and later on only irregularly shaped alternate bars were observed to grow rather sporadically in the final reach of the flume (Figure 3d). The height of these latter bars tended to increase as the bars migrated downstream and was systematically lower than the height typical of the former, initially formed regular bars.

The observed bar characteristics are summarized in Table 3, where L_b^* denotes bar wavelength and H_b^* is the bar height defined as the difference between the maximum and the minimum bed elevation within a bar unit, while c_b^* is bar celerity estimated by comparing the plots of the longitudinal profiles after measurements taken at different times. It must be noticed that an unambiguous determination of bar features can be achieved only when considering the initially formed regular train of bars described above. Just a crude and somehow arbitrary estimation of bar height and wavelength can be attempted when bars grew episodically, showing highly irregular patterns. Therefore, when possible, two values of wavelength and height are reported in Table 3, the first referring to the initial regular bar sequence and the second representative of the final phase of a given run.

3.2. Flow Resistance and Sediment Transport

Before discussing in detail the effect induced by sediment heterogeneity on bar features, it may be worthwhile to briefly analyze how, in the experiments presented here, flow resistance characteristics are modified with respect to the MUNI

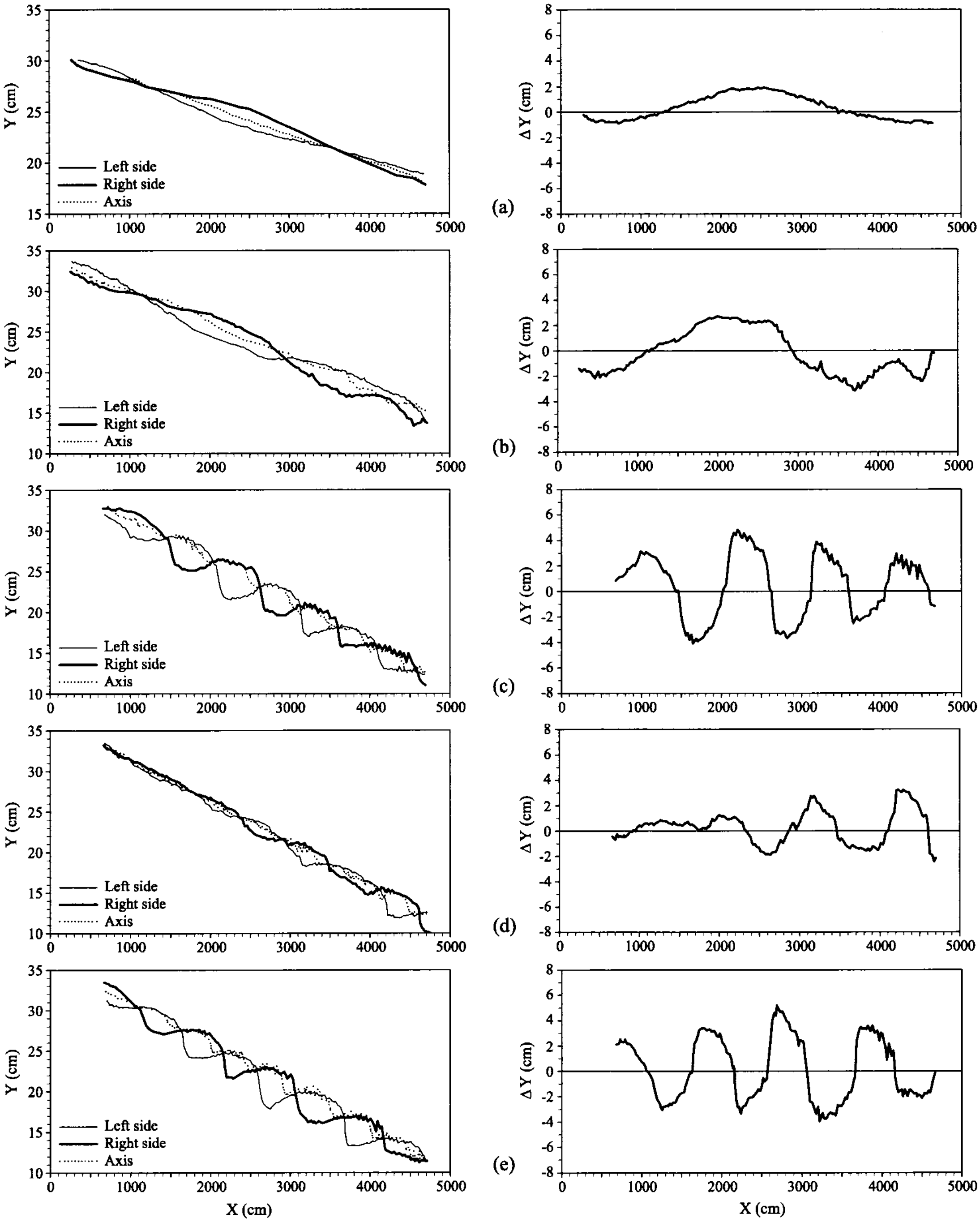

Figure 3. Examples of longitudinal bed profiles and difference between right side and left side bed elevation (ΔY). (a) Equilibrium phase of P2006. (b) Equilibrium phase of P0807. (c) Initial phase of P1309. (d) Equilibrium phase of P1309. (e) Initial phase of P2009. The data are plotted every 20 cm.

tests carried out with a uniform sediment. The comparison regarding the friction coefficient C_0 (corrected through *Einstein*'s [1950] procedure to account for sidewall effects) is shown in Figure 5a. It clearly appears that, on average, an appreciable decrease of the friction coefficient is attained in present experiments. This result is essentially due to the fact that sediment heterogeneity strongly inhibited the formation of small-scale bed forms (ripples and dunes) in all runs except for those carried out with smaller slopes (i.e., runs P0206 and P0606). As a consequence, unlike the MUNI tests, the bed was nearly plane, and the flow resistance was essentially dominated by bed friction, the relevant friction coefficient being of the form $C_0 = (6 + 2.5 \ln D_0^*/k_s^*)^{-2}$, with k_s^* denoting the equivalent grain roughness height. As shown in Figure 5b, a reasonable prediction of the actual flow resistance is provided by assuming k_s^* to be equal to the value of the coarse mode of the mixture (corresponding approximately to $d_{85FC70}^* = 2.89$ mm). This implies a remarkable increase of grain roughness with respect to the MUNI tests. Nevertheless, the form roughness induced by the ripple and/or dune covered–bed typical of

Figure 4. An example of the bed texture observed in run P0109 showing the coexistence of longitudinal alternate streaks of coarse and fine material and diagonal fronts of coarse particles associated to the incipient growth of alternate bars.

the MUNI experiments prevails, inducing the increased flow resistance shown in Figure 5a.

As far as the sediment transport is concerned, the sediment appeared to be transported mainly as bed load even though, especially at larger flow rates, a high concentration of fine material was observed microscopically in the lee side of coarser grains and macroscopically just downstream of the bar fronts. Furthermore, the occurrence of regular trains of two-dimensional water waves was observed occasionally. The accelerating flow arising in correspondence to the troughs of these waves caused a local increase of sediment transport in the form of bed load transport of patches of coarser particles and suspension of finer fractions. The accurate prediction of measured average solid discharge is quite a delicate task when considering a heterogeneous sediment. In the case of either bed load and suspended load a transport formula per size fractions should to be applied, and the different mobility of the various grain size fractions must be accounted for when introducing a suitable hiding function [*Egiazaroff*, 1965; *Ashida and Michiue*, 1972; *Profitt and Sutherland*, 1983; *Ribberink*, 1987; *Diplas*, 1987; *Parker*, 1990]. Nevertheless, Figures 6a and 6b suggest that even though a unique grain size (namely, the geometric mean grain diameter of the substrate, i.e., of the initial mixture) is considered to be representative of the overall mixture considered here, classical bed load predictors such as the *Meyer-Peter and Müller*'s [1948] formula (Figure 6a) and the *Parker*'s [1990] surface-based formula (Figure 6b) lead to reasonably good estimates of the sediment discharge in most of the runs. Accounting for size fractions does lead to relatively weak corrections (less than 2–3%) of the predicted transport rates. It must also be pointed out that *Parker*'s [1990] relationship yields rather satisfactory predictions of the solid discharge (the percentage error ranging from 5 to 30%) even though the adopted mixture exhibits a not negligible percentage of particles finer than 2 mm.

Table 3. Alternate Bar Characteristics Observed During the Initial Stages and the Final Equilibrium Phase of a Given Experimental Run

Run	L^*_b, m	H^*_b, cm	c^*_b, m/h	L^{*I}_b, m	H^{*I}_b, cm	Notes[b]
P0807	10.4	4.2	4.9	···	···	3
P0507	···	···	···	···	≈1	1
P0609	···	···	···	12.5	3.6	4
P0307	···	···	···	16–18	≈1	1
P1207	12.0	4.2	5.4	···	···	3
P0109	11.7	4.0	8.4	12.3	3.8	3
P0509	···	···	···	11.2	2.1	4
P0809	···	···	···	11.5	2.4	4
P1309	10.3	3.4	11.6	11.7	2.3	3
P2009	10.2	3.4	11.0	···	···	3

[a]Definitions are as follows: L^*_b, bar wavelength; H^*_b, bar height; and c^*_b, bar celerity. Final equilibrium phase is denoted by superscript I.

[b]Notes are as follows: 1, nearly plane bed with very long bars; paved sinuous streaks (about 20 cm wide) formed in the upstream 20–30 m of the flume. In the downstream reach very irregular migrating bars superposed episodically on the forced bars. 2: Bars were observed to form and grow irregularly only in the downstream reach on the flume 3: A very regular train of bars formed in the initial phase of the run. The final, equilibrium phase, on the contrary, was characterized by irregular bars growing only in the downstream reach of the flume. 4: Only the final equilibrium phase was monitored.

3.3. Sorting Effects

We have already noted that only in the early stage of a given run can a regular train of migrating alternate bars form along the measuring reach of the flume. As suggested by both visual observations from the sidewalls and sieving analysis of bed samples taken at the end of run P2009, the process of scour and fill associated with the migration of the initially formed bars might cause an intense vertical sorting. The pool at the downstream face of a bar front, in fact, acts as a trap for the coarse fraction which is selectively subject to the downward pull due to gravity. As a consequence, a thick layer of coarse sediment forms above the average level of bar troughs. Moreover, in accordance with the experimental observations reported by *Lisle et al*. [1991], *Diplas and Parker* [1992], *Lisle and Madej* [1992], and *Ashworth et al*. [1992], the gentler riffles along the upstream face of a bar front are characterized by a longitudinal sorting which tends to accrete the coarser particles

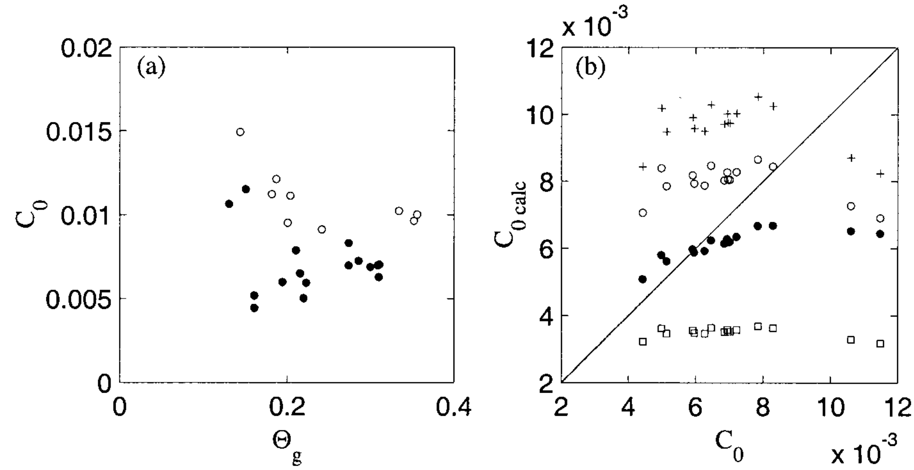

Figure 5. (a) Comparison between friction coefficients C_0 observed in present experiments (solid circles) and in MUNI's tests (open circles). (b) Comparison between theoretical ($C_{0\,\mathrm{calc}}$) resistance predictors and observed (C_0) friction coefficient: $k_s^* = 2d_{90}^*$ [*Parker*, 1991] (open circles); $k_s^* = 2d_{50}^*$ [*Engelund and Fredsøe*, 1978] (squares); $k_s^* = 3d_{90}^*$ [*van Rijn*, 1982] (plus signs); $k_s^* = d_{85FC70}^*$ (solid circles).

on the bar crests (e.g., Figure 7a). As the bar front migrates downstream, the coarse matrix formed at the bar trough is progressively covered by finer sand. Subsequently, a winnowing process initiates through which the coarse matrix is slowly filled with finer particles.

Owing to the highly three-dimensional character of both longitudinal and vertical sorting a quantitative description of the phenomenon is difficult. Nevertheless, a piece of information can be obtained from the sieving analysis of the bed samples collected at different locations along a unit bar. Figure 7b shows the results yielded by the samples taken during the initial phase of P2009 (i.e., about 1 hour after the beginning of the run), 20 cm from the left wall of the flume. In spite of a certain scatter in the data the vertical distributions of sediment particles seem to confirm the above description of the sorting process. Moreover, the grain size distribution obtained by averaging over all the samples suggests that the portion of bed analyzed tended to get only slightly coarser than the bulk mixture. This finding supports the idea that during the initial stage of a given run, the overall bed composition did not change appreciably.

As a given run went on, however, the bed was substantially reworked by bar migration, and the composition of the active layer was likely to differ significantly from the grain size composition of the substrate, as suggested by the samples of the transported sediment collected during the equilibrium phase of runs P0109, P0609, P0709, and P1309. The mean fractional transport rates q_{si}^* scaled by the proportion f_{si} of each fraction in the bulk mixture are plotted in Figure 8a as a function of the grain size d_i^*. Note that in this plot equal mobility would appear as the straight horizontal line $f_{ai}/f_{si} = 1$. The samples of transported sediment show a lack of both finer and coarser fractions which implies that equal mobility was never attained. Indeed, in these runs the bed shear stress was typical of the transition between the partial transport region and the fully mobilized region. This behavior is in accordance, at least qual-

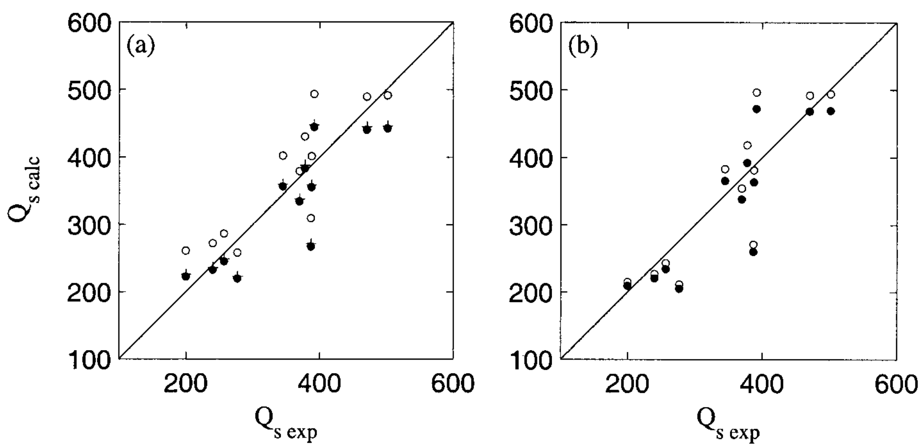

Figure 6. Comparison between theoretical bed load prediction and measured total load. (a) The *Meyer-Peter and Müller* [1948] bed load formula used per size fractions (plus signs) and assuming either d_g^* (solid circles) or d_{50}^* (open circles) as the grain size representative of the mixture. (b) The *Parker* [1990] bed load formula used per size fractions (solid circles) and assuming d_g^* (open circles) as the grain size representative of the mixture.

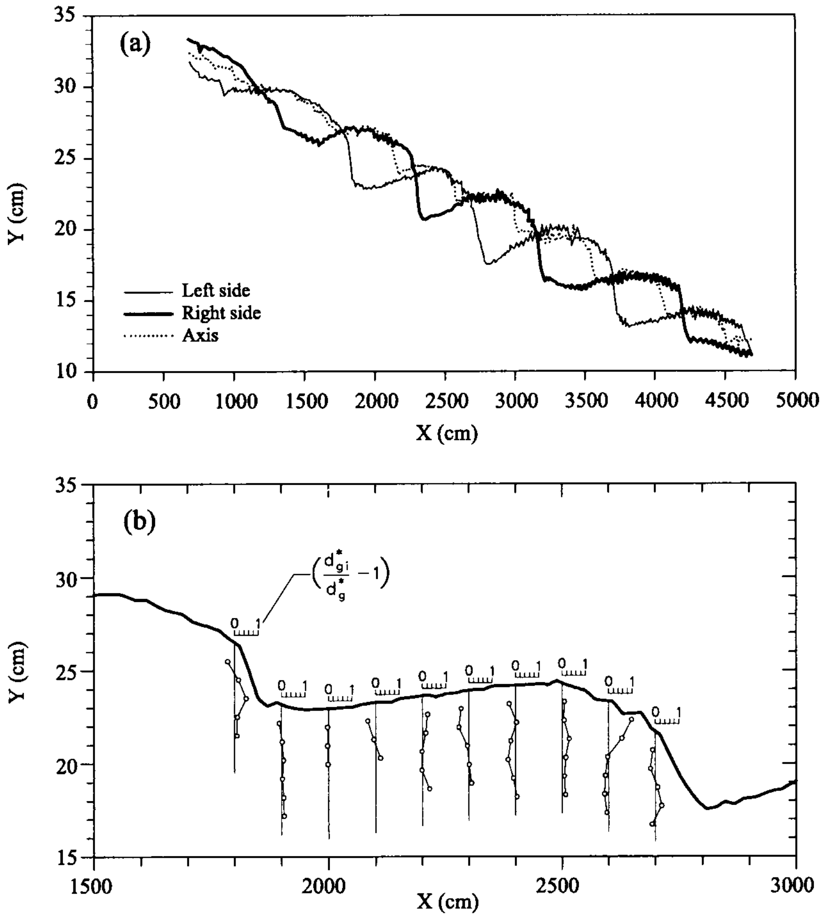

Figure 7. (a) Longitudinal bed profiles measured during the initial phase of P2009. The data, plotted every centimeter, clearly suggest a coarsening of the bed surface when approaching a bar front. (b) The vertical distribution of the quantity $d^*_{gi}/d^*_g - 1$ is plotted for each of the 10 different locations sampled along a bar unit, 20 cm from the left wall of the flume, during the initial phase of P2009. Here d^*_{gi} denotes the local geometrical mean grain size of the sampled sediment, while d^*_g is the geometrical mean grain size of the initially mixed sediment.

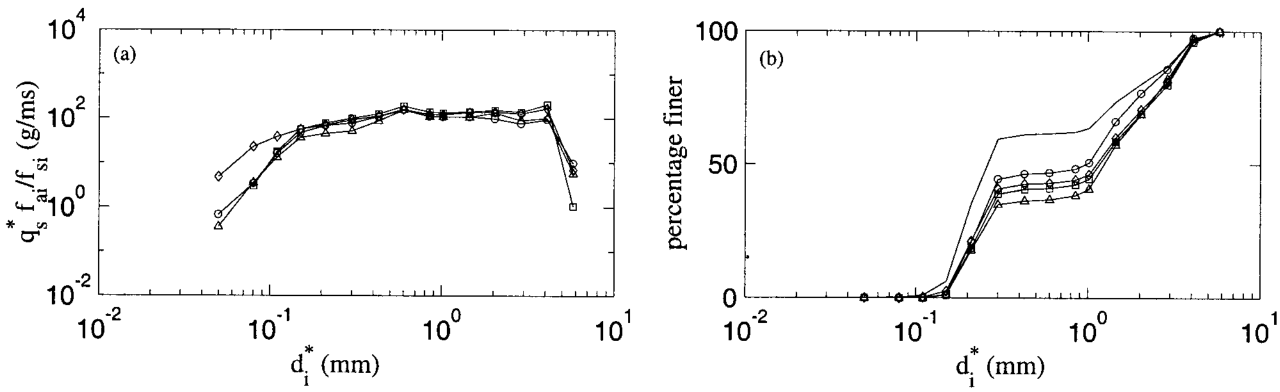

Figure 8. (a) Mean fractional transport rates $q^*_{si} = q^*_s f_{ai}$ scaled by the proportion of each fraction in the substrate f_{si} are plotted as a function of the grain size d^*_i. Size independence in fractional transport rates plots as a line parallel to the x axis. (b) Cumulative grain size distributions of transported sediment are plotted as a function of the grain size d^*_i. The symbols are as follows: open circles, P0109; triangles, P0609; squares, P0709; and diamonds, P1309. The solid line refers to the cumulative grain size distribution of the initially mixed bed.

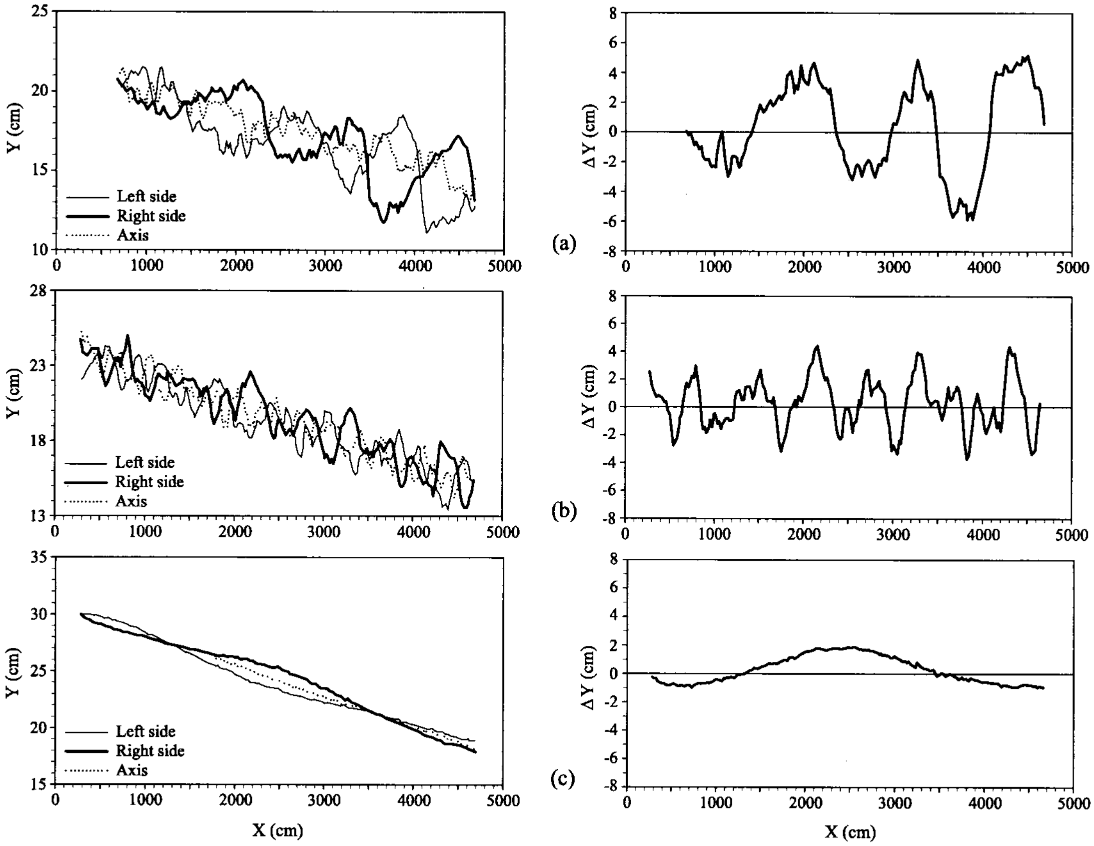

Figure 9. Comparison between longitudinal bed profiles and differences between right-side and left-side bed elevation (ΔY) measured under similar hydraulic conditions during experiments with uniform (MUNI) and mixed (FC70) sediments. (a) Equilibrium phase of P1801 (MUNI) with $Q = 30$ L/s and $i_s = 0.162\%$. (b) Equilibrium phase of P0404 (MUNI) with $Q = 40$ L/s and $i_s = 0.201\%$. (c) Equilibrium phase of P2006 (FC70) with $Q = 40$ L/s and $i_s = 0.263\%$. The data are plotted every 20 cm.

itatively, with the experimental observations made by *Wilcock* [1988, 1992] and *Wilcock and McArdell* [1993] with bimodal mixtures both in the case of plane and/or dune-covered bed. However, the lack of finer fractions appears quite high and induces a significant coarsening of the transported sediment with respect to the bulk mixture composition, as it clearly appears from Figure 8b showing the mean cumulative grain size distribution of the transport and of the bulk mixture. Though part of the detected deficiency of finer fractions can be related to the winnowing of finer grains into the bed substrate, such a result, possibly, indicates that in a recirculation feeding system the composition of transported sediment is significantly affected not only by dynamical armor but also by the intense longitudinal and vertical sorting due to bar formation. The resulting extremely irregular surface texture and vertical grain size distribution, in turn, caused the bars to grow episodically only in the final reach of the flume. Even though the irregular shapes of these bars make it quite difficult to specify their equilibrium characteristics, the comparison with bars formed in the initial stage of a given run (see Table 3, runs P1207, P0109, and P1309) shows that bar height clearly tended to be damped, while bar wavelength was slightly increased.

3.4. Comparison With Uniform Sediment Experiments

The analysis of bar characteristics observed in experiments carried out with different sediments is not straightforward. Theoretical studies [*Colombini et al.*, 1987; *Lanzoni and Tubino*, 1999; *Lanzoni*, this issue] suggest that bar features depend on the values attained by a few dimensionless parameters among which a fundamental role is played by the overall Shields parameter Θ (equal to $\tau^*/(\rho_s - \rho) g d_g^*$, with ρ water density), by the width ratio β (equal to B^*/D_0^*, with B^* half the channel width) and by the geometric grain roughness d_s (equal to d_g^*/D_0^*). A clear understanding of the effects of sediment heterogeneity on bar height and wavelength can therefore be attained only by comparing experiments exhibiting similar values of the above parameters. When considering present experiments, however, this requirement cannot be satisfied by simply comparing runs carried out under the same hydraulic conditions (i.e., the same water discharge and water slope). Small-scale bed forms, in fact, were not as well developed in the bimodal case, thus inducing the overall decrease in flow resistance previously discussed.

Nevertheless, a survey of the values attained by Θ, β, and d_s in the MUNI and FC70 experiments clearly indicates that two groups of experimental runs exist which exhibit similar ranges of the above parameters. The first group includes the uniform sediment tests P1801, P0404, and P2403 and the bimodal sediment runs P0606, P2006, and P2906. The second group consists of runs P1605, P1505, P2709, P2809, and P2909 (MUNI) and P0807, P0109, P1309, and P2009 (FC70). The comparison among similar runs is pursued in Figures 9 and 10. It clearly

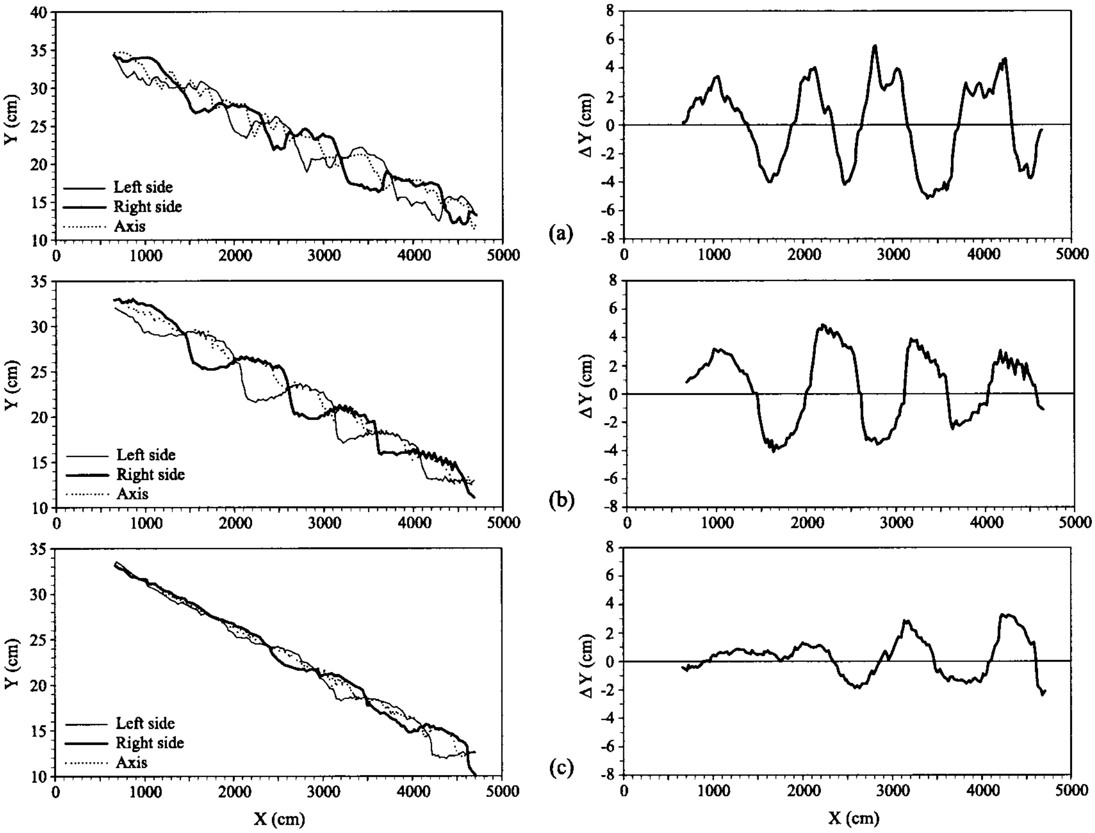

Figure 10. Comparison between longitudinal bed profiles and differences between right-side and left-side bed elevation (ΔY) measured under similar hydraulic conditions during experiments with uniform (MUNI) and mixed (FC70) sediments. (a) Equilibrium phase of P2709 (MUNI) with $Q = 45$ L/s and $i_s = 0.514\%$. (b) Initial phase of P1309 (FC70) with $Q = 45$ L/s and $i_s = 0.525\%$. (c) Equilibrium phase of P1309 (FC70) with $Q = 45$ L/s and $i_s = 0.525\%$. The data are plotted every 20 cm.

appears that at low values of the shear stress, partial mobility related to the strong bimodal character of the adopted mixture crucially affected bar features. The growth of an alternating migrating bar was substantially prevented, and almost steady bars formed within the flume (Figure 9) with a wavelength similar to that of the forced bars observed in the experiments with uniform sand (e.g., P2102). A possible explanation of the formation of these bars is that they were triggered by spatially growing disturbances originating from nonuniform initial conditions in the lateral distribution of grain sizes. The extremely low mobility of coarser particles, in fact, made it quite difficult to ensure a perfectly symmetrical feeding of the recirculated sediment. The formation of asymmetrical patches of coarse particles near the walls of the flume then induced a slight but permanent deflection of the flow which, in turn, enhanced the development of the observed long steady bars.

A reduction of bar height with respect to the case of the MUNI experiments is also evident in runs carried out at higher values of the bed shear stress, as shown in Figure 10. The decrease in bar height was further enhanced as a given experimental run approached its equilibrium stage (Figure 10c), that is, after the bed has been significantly reworked by bar migration. These findings agree with the experimental observations of *Lisle et al*. [1991], *Lisle and Madej* [1991], and *Lanzoni et al*. [1994] and with the theoretical picture outlined by *Lanzoni and Tubino* [1999]. In particular, the nonnegligible reduction of the depths of scour and deposition associated with bar formation can be explained as follows. The decrease of bottom shear stress over the shoaling bar head enhances the selective deposition of coarse particles and leads to the formation of an armored surface layer. This persistent deposition of coarse particles, in turn, tends to deflect flow and sediment transport around the bar and, consequently, inhibits bar-head erosion and bed load transport over bars. The effectiveness of this mechanism, obviously, tends to decrease when equal mobility is approached.

An overall view of the results in terms of the Shields parameter Θ is given in Figure 11 which also includes the experimental data of *Lanzoni et al*. [1994]. The latter have been obtained from experiments carried out in a rectangular recirculating flume, 18 m long and 0.36 m wide, adopting either a uniform mixture of glass spheres with diameter of 1.5 mm or a weakly bimodal mixture made up by glass spheres with diameters of 1 mm and 2 mm in proportion 1:1 (i.e., a mixture with a mean diameter of 1.5 mm and a standard deviation of about 0.5 mm). In this case the comparison between uniform and heterogeneous sediments is simplified by the fact that the bed remained nearly plane in both series of experimental runs, thus ensuring that if similar hydraulic conditions are considered, the same values of the parameters Θ, β, and d_s are recovered. Figure 11a confirms the damping of bar height induced by sediment heterogeneity discussed above. Note, in fact, that the experimental points falling on the x axis refer to nonuniform sediment experiments in which migrating bars did not form. It also clearly appears that sorting effects invariably tend to weaken as the Shields stress increases, that is, as a condition of equal mobility is approached. The above picture is not altered if the results are analyzed by plotting the dimensionless bar height in terms of the parameter β instead of Θ.

The trend exhibited by the wavelength is less clear. In *Lanzoni et al.*'s [1994] experiments, sediment heterogeneity always induced a reduction of bar wavelength. Present data, on the contrary, exhibit a much more involved trend suggesting that bar wavelength does not change too much with respect to MUNI tests.

Finally, it is interesting to note that the main effects of sediment nonuniformity emerging from the experimental data shown in Figure 11 appear to be adequately reproduced by theoretical models developed within the classical framework of linear stability analysis. The theoretical values of maximum growth rate of alternate bars and of bar wavelength are reported in Figure 12. In particular, the points referring to heterogeneous sediment tests have been calculated on the basis of the dispersion relationship obtained by *Lanzoni and Tubino* [1999] for bimodal mixtures. The computations pertaining to uniform sediments have been carried out by using the dispersion relationship given by *Lanzoni* [this issue] in which, for homogeneity with the bimodal sediment case, the coefficients Λ_1, Λ_2, and Λ_3 arising from effects due to the secondary helical flow and to the longitudinal bed slope have been set to zero. In both cases of uniform and bimodal sediments the surface-based bed load transport formula of *Parker* [1990] has been employed; moreover, the *Richardson and Simons* [1967] roughness predictor has been used in the MUNI tests. Figure 12a clearly shows that sorting effects invariably induce a significant damping of the maximum growth rate of alternate bar perturbations. This result indirectly suggests that at equilibrium, a reduction of the depths of scour and deposition associated with bar formation is likely to be attained. Note that, unlike observational evidence, theoretical results would predict the growth of alternate bars also for the bimodal runs P0606, P2006, and P2906; nevertheless, in these tests the experimental value of β turns out to be very close to the critical width ratio β_c theoretically predicted, below which bars are not expected to develop. Figure 12b, moreover, appears to adequately reproduce the behavior of experimental bar wavelengths shown in Figure 11b. Indeed, while sediment heterogeneity leads to a shortening of bar wavelength in the case of the *Lanzoni et al.* [1994] experiments, the theoretical values of bar wavelength pertaining to the present experimental runs do not change appreciably when considering either bimodal or uniform sediments.

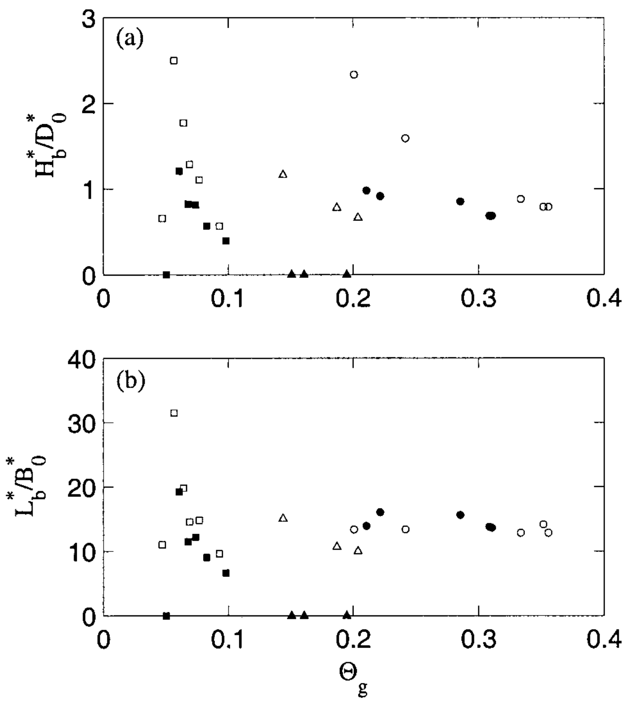

Figure 11. Dimensionless (a) bar wavelength and (b) bar height observed in MUNI (open circles and triangles) and FC70 (solid circles and triangles) experimental runs are plotted against the Shields parameter based on the geometric grain diameter. In particular, the triangles refer to runs P1801, P0404, and P2403 (MUNI) and to runs P0606, P2006, and P2906 (FC70); the circles refer to runs P1605, P1505, P2709, P2809, and P2909 (MUNI) and to runs P0807, P0109, P1309, and P2009 (FC70). Experimental data from *Lanzoni et al.* [1994] are also reported in Figure 11. The experiments were carried out in a laboratory flume 0.36 m wide and 18 m long by using, alternatively, a uniform (open squares) sediment made up of glass spheres with diameter 1.5 mm and a weakly bimodal (solid squares) mixture composed by glass spheres with diameters of 1 mm and 2 mm in proportion 1:1.

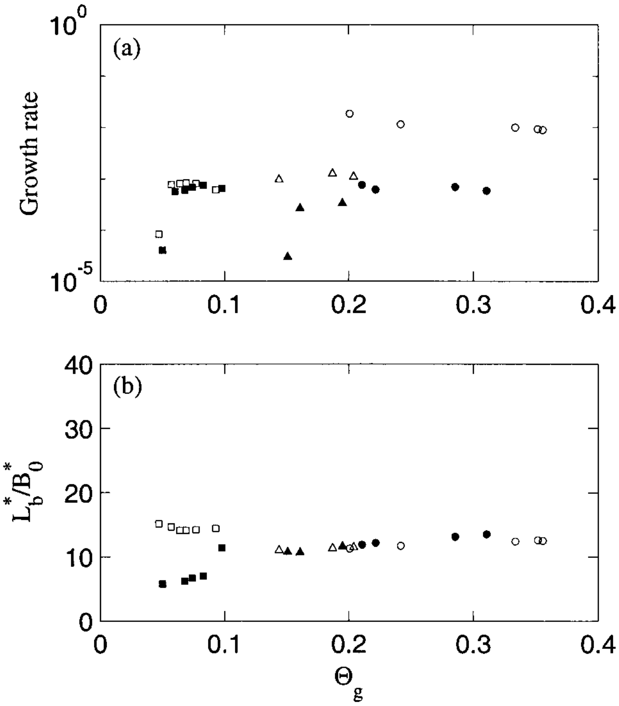

Figure 12. Theoretical values (a) of the maximum growth rate of alternate bars and (b) of the dimensionless bar wavelength corresponding to the experimental data reported in Figure 11. Open and solid symbols refer to uniform and bimodal sediments, respectively. The calculations have been carried out by adopting either the dispersion relationship given by *Lanzoni* [this issue] for a uniform sediment or the dispersion relationship obtained by *Lanzoni and Tubino* [1999] for bimodal mixtures. In the latter case a hiding exponent of 0.2 has been adopted.

4. Conclusions

The results of a series of experiments carried out in a quite large straight flume by recirculating a strongly bimodal mixture of sediments have been presented. Hydraulic conditions were chosen such that the initially flat bed of the flume became unstable, giving rise to the development of alternate bars.

Owing to the strong bimodal character of the mixture, in runs carried out at lower values of the bottom shear stress a condition of partial transport occurred such that fractional transport rates of coarser particles were substantially smaller than the ones corresponding to the finer-graded portion of the mixture. Increasing the bed shear stress led to full mobilization of all grain sizes. However, as shown by sediment samples collected during some typical runs, equal mobility was never attained.

The typical topographic pattern associated with alternate bars also causes, through the unequal response of grain to bottom stress, a selective deposition of coarser particles toward bar head. This tendency is obviously counteracted by the action of gravity which tends to pull coarser particles downward selectively (i.e., toward bar pools). In the bar case the former mechanism appears to prevail because of the relatively low spatial variations typical of bottom elevation. The resulting longitudinal sorting influences the delicate balance between stabilizing and destabilizing actions which govern bar growth. In particular, bar front coarsening not only prevents bar-head erosion but also deflects flow and sediment transport around the bar, thus reducing bed load transport along the rising side of the bar profile.

In experimental runs carried out at lower shear stress, these processes, combined with partial transport induced by sediment bimodality, are strong enough to prevent bar migration and lead to the formation of quite small, long stationary alternate bars. These steady bars can possibly be regarded as spatially growing disturbances originating from nonuniform initial conditions in the lateral grain size distribution of the bed surface. The decrease in alternate bar height with respect to MUNI tests is evident also at higher shear stresses and appears to be further enhanced as, owing to bar migration, the bed is consistently reworked. Also, the differences progressively reduce as the Shields parameter increases, that is, as equal mobility tends to be approached.

While the results concerning the damping of bar height are in accordance with existing flume experiments, the effect of sediment heterogeneity on bar wavelength is not univocal. Present data suggest that bar wavelength does not change appreciably compared with the uniform sediment case. On the contrary, a previous series of experimental tests, carried out with a weakly bimodal mixture, indicate that sorting effects may also lead to a reduction of bar wavelength.

These findings appear to be adequately reproduced by theoretical models developed within the classical framework of linear stability analysis. However, predicting the actual bar topographical pattern in the presence of heterogeneous sediment would require not only the removal of the assumption of small perturbations but, also, the development of a suitable model for vertical sorting.

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S. Lanzoni, Dipartimento di Ingegneria Idraulica, Marittima e Geotecnica, Università di Padova, via Loredan 20, I-35131 Padua, Italy. (lanzo@idra.unipd.it)

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