# Numerical modelling of the initiation of landslides due to increase of water pressure with a multiphase material model

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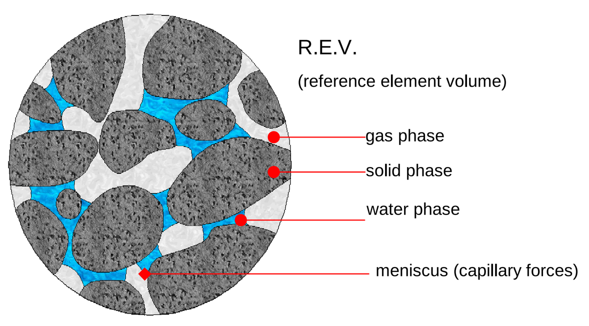
Aim of this work is the numerical analysis of the initiation of landslides due to increase of water pressure induced by rainfall. The slope is analysed as a multiphase elasto-plastic porous continuum where heat, water and gas flow are taken into account [1]. In particular, the gas phase is modelled as an ideal gas composed of dry air and water vapour, which are considered as two miscible species. Phase changes of water (evaporation-condensation, adsorption-desorption) and heat transfer through conduction and convection, as well as latent heat transfer are considered in the model. The modified effective stress state of the solid skeleton is limited by the Drucker-Prager yield surface for simplicity, with linear isotropic hardening and non associated plastic flow [2]. The macroscopic balance equations are discretised in space and time within the finite element method. The finite element code Comes-Geo [1] has been further developed for this work [2]. The independent variables are the solid displacements, the capillary and the gas pressure and the temperature. Small strains and quasi-static loading conditions are assumed.

The balance equations implemented in the code are:



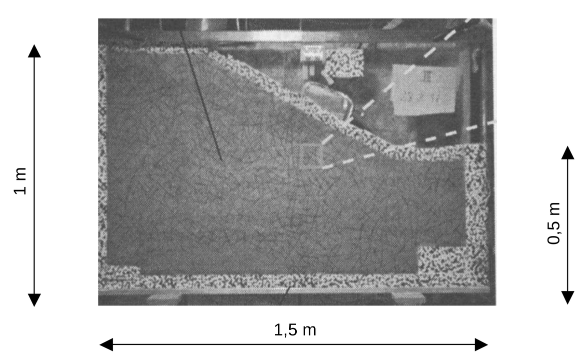

*Microscopic view of multiphase material.*

$$n(\rho^w-\rho^{gw})\left(\frac{\partial S_w}{\partial T}\frac{\partial T}{\partial t}+\frac{\partial S_w}{\partial p^c}\frac{\partial p^c}{\partial t}\right)+\left[\rho^w S_w+\rho^{gw}(1-S_w)\right]div\left(\frac{\partial \boldsymbol{u}}{\partial t}\right)$$
$$+(1-S_w)n\left(\frac{\partial \rho^{gw}}{\partial T}\frac{\partial T}{\partial t}+\frac{\partial \rho^{gw}}{\partial p^c}\frac{\partial p^c}{\partial t}\right)-div\left(\rho^g\frac{M_aM_w}{M_g^2}\boldsymbol{D}_g^{ga}grad\left(\frac{\partial p^{gw}}{\partial p^c}\right)\right)$$
$$+div\left(\rho^{gw}\frac{\boldsymbol{k}\,k^{rg}}{\mu^g}\left[-grad\left(p^g\right)+\rho^g\boldsymbol{g}\right]\right)-\beta_{swg}\frac{\partial T}{\partial t}$$
$$+div\left(\rho^w\frac{\boldsymbol{k}\,k^{rw}}{\mu^w}\left[-grad\left(p^g\right)+grad\left(p^c\right)+p^w\boldsymbol{g}\right]\right)=0$$

*Mass balance equation (solid + liquid + vapour).*

$$-n\rho^{ga}\left(\frac{\partial S_w}{\partial T}\frac{\partial T}{\partial t}+\frac{\partial S_w}{\partial p^c}\frac{\partial p^c}{\partial t}\right)-\beta_s\rho^{ga}(1-n)(1-S_w)\frac{\partial T}{\partial t}$$
$$+(1-S_w)\rho^{ga}div\left(\frac{\partial \boldsymbol{u}}{\partial t}\right)+(1-S_w)n\left(\frac{\partial \rho^{ga}}{\partial T}\frac{\partial T}{\partial t}+\frac{\partial \rho^{ga}}{\partial p^c}\frac{\partial p^c}{\partial t}+\frac{\partial \rho^{ga}}{\partial p^g}\frac{\partial p^g}{\partial t}\right)-$$
$$div\left(\rho^g\frac{M_aM_w}{M_g^2}\boldsymbol{D}_g^{ga}grad\left(\frac{p^{ga}}{p^g}\right)\right)+div\left(\rho^{ga}\frac{\boldsymbol{k}\,k^{rg}}{\mu^g}\left(-grad\left(p^g\right)+\rho^g\boldsymbol{g}\right)\right)=0$$

*Dry air mass balance equation.*



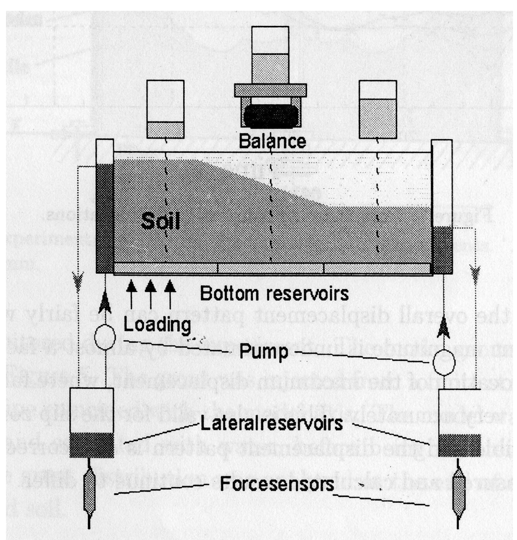

*Experimental slope [3].*

$$div\left(\sigma'-\boldsymbol{I}(S_gp^g-S_wp^c)\right)+\rho\boldsymbol{g}=0$$

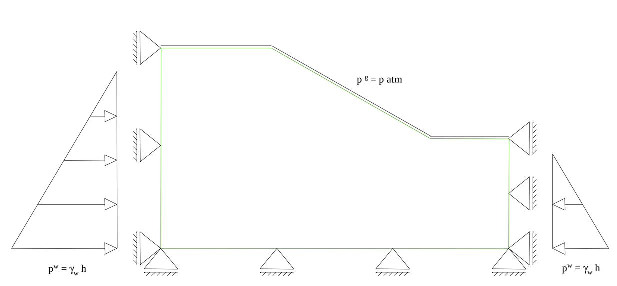
*Linear momentum balance equation of the mixture.*

$$(\rho C_p)_{eff}\frac{\partial T}{\partial t}+\rho^wC_p^w\left\{\frac{\boldsymbol{k}\,k^{rw}}{\mu^w}\left[-grad\left(p^g\right)+grad\left(p^c\right)+\rho^w\boldsymbol{g}\right]\right\}\cdot grad\left(T\right)$$
$$\rho^gC_p^g\left\{\frac{\boldsymbol{k}\,k^{rw}}{\mu^w}\left[-grad\left(p^c\right)+\rho^g\boldsymbol{g}\right]\right\}\cdot grad\left(T\right)-div\left(\chi_{eff}grad\left(T\right)\right)=$$
$$-\dot{m}_{vap}\Delta H_{vap}$$

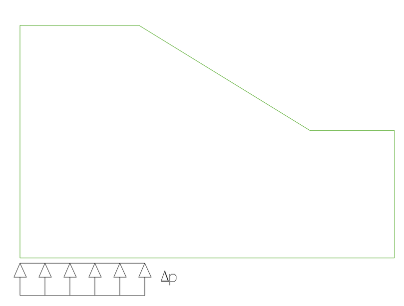
*Energy balance equation of the mixture.*




*Sketc of the experimental setup for the slope stability analysis [3].*



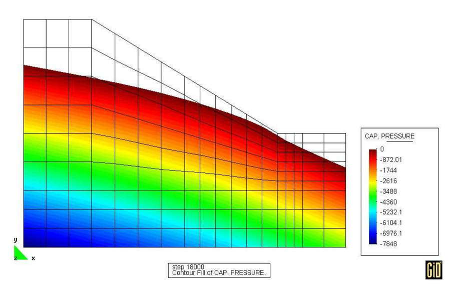
*Boundary conditions.*



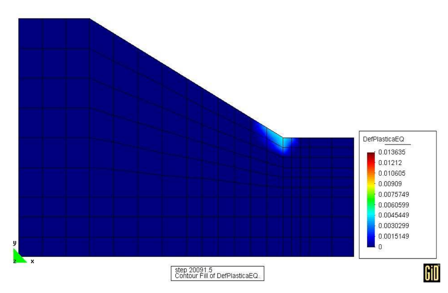
*Applied water pressure load.*

**Numerical analysis.** A 2D laboratory test [3] carried out at the LMS-EPFL laboratory is simulated numerically. The experiment reproduces a slope stability problem due to a pore water pressure of 1.6 kPa applied at the left third of the bottom surface. The slope is 1.5 m in length, 1 meter high and 0.25 m wide. A constant water table is imposed at the left and right hand sides of the slope at 0.2 and 0.15 m below the surface, respectively. The pore pressure load at the bottom is applied at time t=0. The first failure of the of the lower part of the slope occurred after about 80 s. The slope continued to fail by backward erosion and outflow appeared at the lower part shortly afterwards.

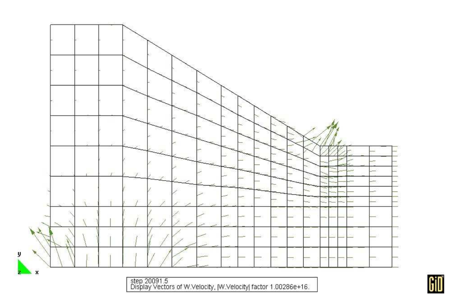
Numerical results performed with the finite element code Comes-Geo show that the first failure of the lower part of the slope is captured and that outflow appears shortly afterward, as experimentally observed. The computation was made with a uniform initial condition of almost dry material (water saturation of 42%). Then the hydrostatic water load was applied at the left and right side, and finally, the water pressure load was applied at the bottom.



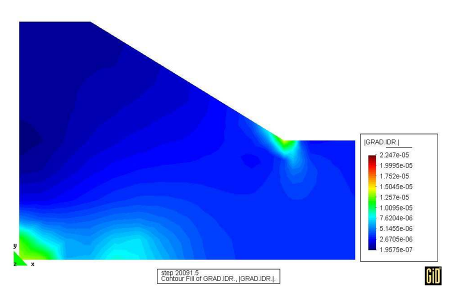
*Phreatic line ($p^c$=0) due to the boundary conditions.*



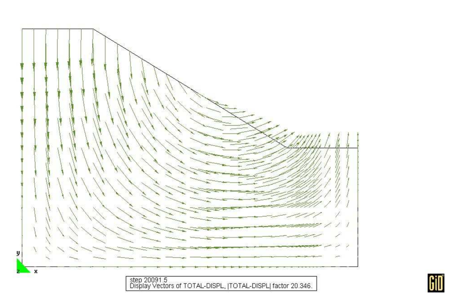
*Equivalent plastic strain.*



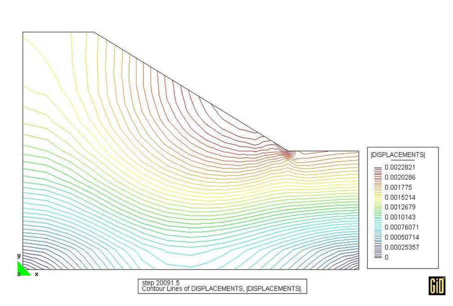
*Water flow rate.*



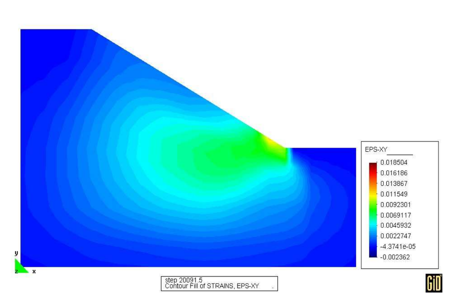
*Hydraulic gradient.*



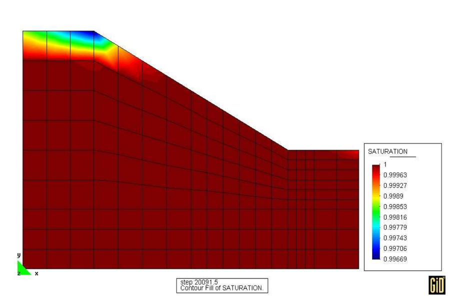
*Total displacements.*



*Displacements due to load.*



*Shear strains.*



*Water saturation.*

[1] R.W. Lewis and B. A. Schrefler, *The Finite Element Method in the Static and Dynamic Deformation and Consolidation of Porous Media*. Wiley, 1998.

[2] L. Sanavia, F. Pesavento, B.A. Schrefler, *Strain localisation simulation in non-isothermal multiphase geomaterials*, Proc. ISSMGE International Conference From Experimental Evidence towards Numerical Modelling of Unsaturated Soils, Bauhaus University of Weimar, Germany, September 18-19, 2003 (Keynote lecture), in print.

[3] G. Klubertanz, F. Bouchelaghem, L. Laloui, L. Vulliet, *Miscible and immiscible multiphase flow in deformable porous media*, Mathematical and Computer Modelling, 37, pp. 571-582, 2003.