

## Casimir forces and non-Newtonian gravitation

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**Abstract.** The search for non-relativistic deviations from Newtonian gravitation can lead to new phenomena signalling the unification of gravity with the other fundamental interactions. Various recent theoretical frameworks indicate a possible window for non-Newtonian forces with gravitational coupling strength in the micrometre range. The major expected background in the same range is attributable to the Casimir force or variants of it if dielectric materials, rather than conducting ones, are considered. Here we review the measurements of the Casimir force performed so far in the micrometre range and how they determine constraints on non-Newtonian gravitation, also discussing the dominant sources of false signals. We also propose a geometry-independent parameterization of all data in terms of the measurement of the constant  $\hbar c$ . Any Casimir force measurement should lead, once all corrections are taken into account, to a determination of the constant  $\hbar c$  which, in order to assess the accuracy of the measurement, can be compared with its more precise value known through microscopic measurements. Although the last decade of experiments has resulted in solid demonstrations of the Casimir force, the situation is not conclusive with respect to being able to discover new physics. Future experiments and novel phenomenological analysis will be necessary to discover non-Newtonian forces or to push the window for their possible existence into regions of the parameter space which theoretically appear unnatural.

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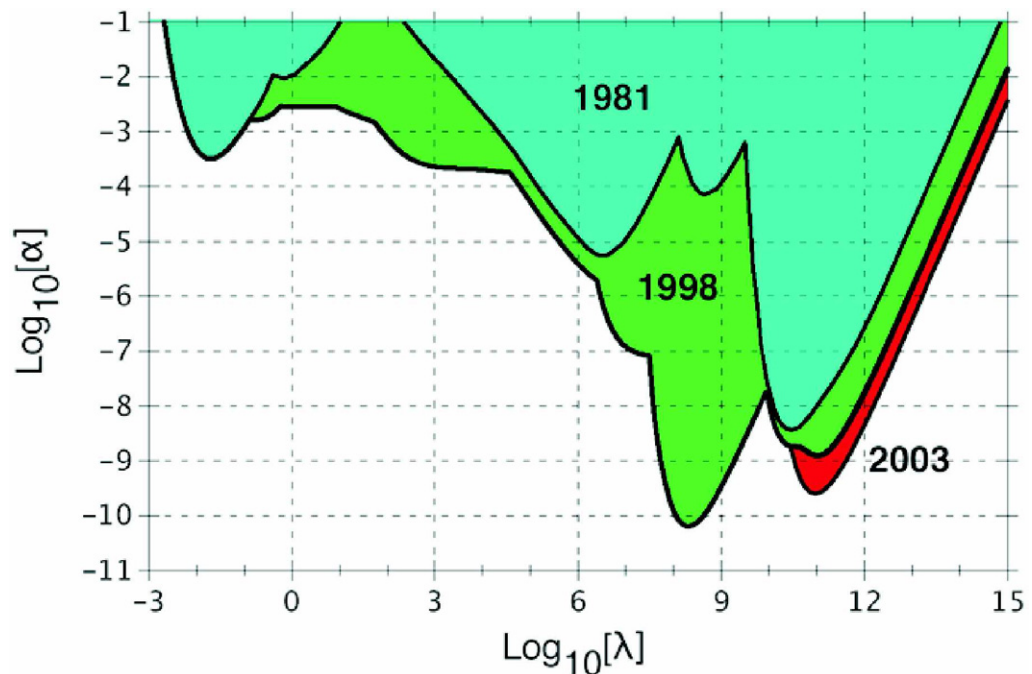
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**1. Introduction**

The search for deviations of non-relativistic nature from Newton's universal gravitation has been a recurrent issue since the initial proposals dating back to more than 30 years ago [1]–[6]. In the last two decades, further motivation has been added by the possibility to observe deviations from the equivalence principle due to a force with coupling of order of the gravitational one, acting on a short range, and coupled to the baryonic number [7]. Although the initial claim for such a force, also named *fifth force*, has not been confirmed by subsequent experiments, this has generated a revival of experiments in gravitation with a diversity of ingenious configurations. Thanks to these experiments, we now have a deeper knowledge of the gravitational field in the region from a few millimetres to planetary distances. The hypothetical forces superimposed on Newtonian gravity are typically parameterized by a Yukawa range  $\lambda$  and a coupling strength  $\alpha$  with respect to gravity, such that the total potential is  $V_{\text{tot}} = V_{\text{N}} + V_{\text{Y}}$ , where  $V_{\text{N}}$  is the Newtonian potential and  $V_{\text{Y}}$  the Yukawa potential. For a pointlike source of gravitational mass  $M$ , the two contributions respectively are:

$$V_{\text{N}}(r) = -G \frac{M}{r}, \quad V_{\text{Y}}(r) = -\alpha G \frac{M}{r} e^{-r/\lambda}, \quad (1)$$

where  $G$  is the Newtonian constant of gravitation. It is understandable that, due to the exponential suppression in the distance dependence due to a finite  $\lambda$ , even for a large  $\alpha$  the signal contribution due to the Yukawan term will be masked by the long-range Newtonian term unless experiments are performed on a scale comparable or smaller than  $\lambda$ . Such Yukawan forces are expected in many attempts to unify gravity to the other fundamental interactions. While variegated mechanisms have been proposed so far predicting gravitational-strength Yukawan forces [8], we would like to remark on the fact that even in minimal scenarios aimed at incorporating gravity into the standard model one should expect such forces. Indeed, by assuming that spontaneous symmetry breaking is the way to consider diverse interactions as different manifestations of a unified picture, the unification should give rise to the presence of new intermediate gauge bosons with characteristics in between the parent interactions. The successful electroweak unification is a prototypical example, as it predicts a new gauge boson, the  $Z^0$  particle, with features in between



**Figure 1.** Limits to the existence of new macroscopic forces parameterized by the coupling strength  $\alpha$  (normalized to Newtonian gravity) and the Yukawa range  $\lambda$  (in metres), coming from various experiments and astronomical observations. The shaded regions show the Yukawa couplings excluded by experiments performed before the corresponding year. The progress made in the knowledge of the gravitational field in the macroscopic range since the proposal for a ‘fifth force’ has been considerable. Limits are however rather weak at the two extreme lengthscales, in particular at the interesting submillimetre region for  $\lambda$  from the viewpoint of extra-dimensional models (courtesy of Coy, Fischbach, Hellings, Standish and Talmadge, 2003).

the purely electromagnetic sector (it is electrically neutral like photons) and the purely weak sector (it is massive similarly to the  $W^\pm$  bosons). By analogy, if we believe that spontaneous symmetry breaking is the universal path for unifying fundamental interactions (which will be scrutinized with the forthcoming experiments at the Large Hadron Collider), we should expect short-range gauge bosons with intermediate coupling between usual gravity and the other interactions. Unfortunately, without further assumptions, the mechanism of spontaneous symmetry breaking does not give quantitative hints on the couplings and range of the interactions. These parameters can be only determined from experimental inputs through new phenomena, which will be the counterpart of the discovery of the neutral weak currents (and the consequent measurement of the Weinberg angle) for the successful electroweak unification. This motivates the search for non-Newtonian forces in the broadest range of distances and couplings. The current limits in the  $\alpha$ - $\lambda$  plane, with  $\lambda$  spanning from 1 mm to  $10^{15}$  m, are shown in figure 1. It is recognizable that the constraints on extra-gravitational, short-range forces in the submillimetre range, and in the range above  $10^{15}$  m, do not match the more stringent limits in the intermediate region. On the other hand, various models based on supersymmetry predict the existence of forces

superimposed to gravity in the range between 1 and 100  $\mu\text{m}$  with intensity up to  $10^5$  times the strength of gravity in the same range [9]–[11]. The corresponding limits to their existence in this range are quite weak. Although in principle a measurement performed at any lengthscale (for instance a high-accuracy measurement of the gravitational force in the meter range with a Cavendish experiment) may constrain values of  $\lambda$  at very different lengthscales, the exponential sensitivity of  $\alpha$  to  $\lambda$  in the limit  $\lambda \rightarrow 0$  gives rise to very weak bounds for  $\lambda$  smaller than the lengthscale at which the experiment is performed. Therefore, in order to give significant limits in the micrometre range, one may opt for designing experiments intrinsically exploring small distances. In doing so a signal-to-noise issue is soon faced since, as first discussed in [12], the Newtonian gravitational force signal scales as the fourth power of the size of the experiment, with  $F_N \propto M^2/r^2 \simeq \rho^2 r^4$ , where  $M$  and  $\rho$  are mass and density of the test bodies and  $r$  their distance. This implies that background originating from a force having a slower scaling with distance is going to strongly affect our capability to obtain significant limits. In the range of  $\simeq 100 \mu\text{m}$ , limits obtained using Cavendish balances [13] have been recently improved by the group at the University of Washington using torsion pendula and rotating attractors [14, 15]. Decreasing the explored range of distances below the 100  $\mu\text{m}$  range is challenging due to the requirements necessary to ensure parallelism between the two surfaces during the rotation. An alternative technique to explore the range below 100  $\mu\text{m}$  consists in the use of microresonators and nanoresonators combined to dynamical detection techniques first introduced in [12, 16, 17]. Two groups, at Stanford and Boulder, have performed measurements based on this technique and limits in the  $\alpha$ – $\lambda$  plane have been reported [18, 19]. The presence of an electromagnetic shield between the source of the non-Newtonian field and the microresonator used for the detection of the force limits the minimum explorable distance.

On the other hand, the most promising theoretical predictions appear to be in the range of 1–10  $\mu\text{m}$ , and therefore it is important to design experiments which maximize the chance of detecting forces acting in this distance range. In order to minimize the background, and taking into account the huge difference in coupling strength between gravitation and electromagnetism, it seems compulsory to consider electrically neutral bodies. Then, the strongest expected background in the micrometre range for forces between electrically neutral bodies is due to the Casimir force [20]. This also means that any accurate measurement of Casimir forces gives limits on gravitational-like forces, as remarked in pioneering papers by Mostepanenko and Sokolov [21, 22] two decades ago. Since then, precision studies of Casimir forces in the 200 nm–2  $\mu\text{m}$  range have allowed us to grasp a better understanding of the background in which novel macroscopic forces of gravitational origin could be immersed.

Many excellent reviews on various aspects of Casimir physics have been available since two decades [23]–[33], and more recently reviews have also appeared on the experimental efforts to detect non-Newtonian gravitational forces above the range of 10–100  $\mu\text{m}$  [34]–[37]. Considering the abundance of existing review material on both Casimir forces and the search for non-Newtonian forces, we will only cover some selected topics lying between the two groups of reviews.<sup>1</sup> In particular, we will discuss limits to non-Newtonian forces in the micrometre range in which Casimir forces are expected to be the leading background. Obviously, this review should be considered as a progress report since the situation is evolving quickly due to the expected flow of new data and more stringent phenomenological and theoretical analysis. The spirit will be also to keep the formalism to a minimum and to put more emphasis on physical and

<sup>1</sup> Topics in between experimental gravitation and microscale physics have been also discussed in [38].

dimensional grounds, to allow as much as possible for a pedagogical, broad-audience approach. After a discussion of the Casimir force and its meaning and importance in current physics, we discuss its experimental evidence and comment on the consequent limits to gravitational forces. We then discuss the variety of leading systematic effects in the experiment, and propose a parameterization of the Casimir experiments which allows for a general comparison between various configurations. Some promising future research directions are then discussed, with more general remarks in the concluding section.

## 2. Casimir forces and quantum vacuum

The years 1947 and 1948 were milestones for the development of quantum field theories since almost simultaneously Lamb shifts in hydrogen were measured in high precision spectroscopy experiments, renormalization was proposed by Feynman, Schwinger and Tomonaga, and a macroscopic force originating from quantum vacuum fluctuations was predicted by Casimir. The following six decades of developments in microscopic physics may be considered as the refinement of these findings in quantum electrodynamics (QED) and its extension to other fundamental interactions.

With regard to QED in the macroscopic realm, one may ask if it leads to macroscopically detectable forces without classical counterpart. Evidently, if quantum vacuum fluctuations are isotropic, no force should be expected in the simplest case of an infinite mirror as the contributions due to the virtual photon radiation pressure will be equal on both sides. Such an isotropy is instead broken considering two indefinite and parallel plates. A simple stationary boundary condition for the electromagnetic fields on the surfaces, for instance the one typical of an ideal conductor, gives a drastic change in the density of states of the virtual photons inside the cavity with respect to that outside the cavity. When the net radiation pressure is evaluated, there is now a partial cancellation leading to the expression of the attractive<sup>2</sup> Casimir force per unit of surface area  $P_C$  [20]

$$P_C = \frac{K_c}{d^4}, \quad (2)$$

where  $d$  is the distance between the two plates and  $K_c$  is a constant. We want to stress some qualitative features of this very simple formula.

- (1) *The Casimir pressure depends on the product  $\hbar c$  and does not depend on the electric charge at the leading order.* The concrete expression for  $K_c$  contains only the two constants of nature allowed in free quantum field theory, i.e.  $\hbar$  and  $c$ , being  $K_c = \pi^2 \hbar c / 240$ . Indeed, all formulae for the Casimir force considered here have in common the dependence on the product  $\hbar c$ . In a classical, even if relativistic, setting (no quantum fluctuations, i.e.  $\hbar \rightarrow 0$ ) or in a non-relativistic, even if quantum, realm (by hypothetically imagining an infinite speed of light,  $c^{-1} \rightarrow 0$ , in which case however electromagnetism would be quite different) there are no Casimir forces. This is a distinctive feature with respect to the van der Waals–London forces in which relativity and the consequent finite speed of light propagation do not play any direct role. In fact, the Casimir force was initially originated from the

<sup>2</sup> Although Casimir forces are not expected to be always attractive, in the following we will consider only attractive forces and will express, in equations (2)–(5) and (7), the absolute value of the Casimir forces.

attempt to include the relativistic retardation into the London theory. We will exploit this feature of the Casimir force later on in discussing a comparison among different geometries. Also, although we are considering electromagnetic fluctuations, the electric charges which mediate the mechanical pressure between the virtual photons and the bulk material of the mirror do not appear explicitly. This apparently paradoxical property of Casimir pressure is also known as *saturation* [30, 39]. The combined effects of the value of the electric charge and the reflectivity of the mirrors are large enough to make the force asymptotically independent of the same electric properties producing the force. This saturation property is expected to fail if the reflectivity of the mirrors is not complete, which happens for any realistic material for electromagnetic frequencies above a finite cutoff frequency. In this case there is a next-to-leading correction to the Casimir force depending upon the electric charge and the finite conductivity properties of the material. Apart from the electromagnetic field, other fundamental interactions do not give significant contributions to the Casimir pressure. For gravitational quantum fluctuations, apart from the impossibility to identify a gravitational conductor, the force is too weak to play a role and cannot give a saturated contribution. Strong and weak fields mediated by gluons and  $W^\pm$ ,  $Z^0$  particles cannot play a role in the micrometre range since their influence is limited to the subatomic domain.

- (2) *The Casimir force may be interpreted as due to the interplay between quantum fluctuations and geometry.* The Casimir force has been interpreted in a very general way as emerging from the finite cancellation between the radiation pressure due to virtual photons present in quantum vacuum and acting on a macroscopic body. Seen from this perspective, the Casimir force is a strongly geometry-dependent force and is a macroscopic manifestation of the effect of boundary conditions on quantum fields, a manifestation of a sort of *quantum electrostatics*. In addition, it provides physical insights into the same concept of renormalization in quantum field theory. In principle, the finite cancellation of radiation pressure originates from the difference between diverging contributions, although in practice the high-frequency response of the conductors provides a physical regulator. Alternative views centred on the role of the sources of the electromagnetic field have been also proposed, based on the idea that the geometric, field theoretic interpretation of the Casimir result is more an issue of taste rather than an essential, substantial physical point [40]–[42] and that quantum fluctuations are actually originating from the sources of the fields. This is key in the calculation of the same effect by Lifshitz [43], using the fluctuations of the electric and magnetic fields of the sources in the materials. Therefore there are two distinct approaches to the same effect described by equation (2), the one adopted by Casimir, more idealistic, based on global, geometrical properties of quantum fluctuations, and that by Lifshitz, more materialistic, based on local, analytical properties of the sources. This dualism in the interpretation of the Casimir force still exists today, and the relative contributions of intrinsic field quantization and nonlinear features of source models have been discussed (see for instance [44]–[49]). The issue is currently important at the cosmological level, as the actual presence of quantum fluctuations at the macroscopic level gives rise to the outstanding problem of their expected large contribution to the cosmological constant [30], [50]–[52]. The presence of this contribution is instead an ill posed problem if one attributes no reality at all to intrinsic quantum field fluctuations, as discussed in [39].

We remark on the fact that a force proportional to  $\hbar c$  seems to naturally support a field theoretic interpretation in terms of irreducible quantum fluctuations. In fact, in the quantum

vacuum we pictorially expect a particle-antiparticle pair to appear and disappear on a timescale  $\tau \simeq \hbar/mc^2$  for massive particles (characterized by an energy-momentum dispersion law  $E^2 = p^2c^2 + m^2c^4$ ), and  $\tau \simeq 1/\omega$  for massless particles characterized by an energy-momentum dispersion relationship  $E = pc = \hbar\omega$ . The maximum distance the virtual particles may travel to preserve its virtuality cannot exceed the quantity  $c\tau$ . Then the product of the virtual energy transported in space times the travelled distance is limited to values smaller than  $Ect \simeq \hbar c$ , regardless of the particle mass. From this point of view, if we think of the Casimir force as originated from quantum vacuum fluctuations, it is not surprising that the only fundamental quantity appearing is then the product  $\hbar c$ , for any virtual particle.

Along these qualitative lines, we can in fact understand the Casimir formula by using dimensional arguments. In a parallel plane situation there is only a finite length available, i.e. the distance between the two plates  $d$ . A pressure is dimensionally equivalent to an energy density, and we can form an energy from  $\hbar c$  (energy times length) considering  $\hbar c/d$ . An energy density is obtained by further dividing by  $d^3$ , leading to

$$P_C \propto \frac{\text{energy} \times \text{length} / \text{distance}}{\text{volume}} \propto \frac{\hbar c / d}{d^3},$$

which gives the proper scaling with the fourth power of the distance. This elementary dimensional argument is of less unambiguous application in other geometries since more than one finite length plays a role, for instance the radius of the sphere and the distance between sphere and plane in the case of a sphere–plane geometry.

The study of the Casimir force is becoming an interdisciplinary subtopic of physics whose importance is growing in many fields. In cosmology and astrophysics, quantum vacuum is a candidate to support the *dark energy* hypothesis [52]–[58] which is considered responsible for the observed acceleration of the Universe [59]–[61]. In nanotechnology these forces may play a significant role in many artificial structures built on the nano-to-microscale [62]. Even in apparently unrelated fields, such as biophysics and chemistry, many of the relevant adhesion forces may call for a more complete framework than the one available through the van der Waals force, of which the Casimir/Casimir–Polder forces appear as the relativistic generalization [63]. This interdisciplinary flavour has grown since what was considered a scientific curiosity (a minute force between macroscopic and electrically neutral surfaces) has been increasingly explored in the laboratory. The laboratory findings have been complemented by theoretical insights then converted into further experimental investigations, in a close relationship between theory and experiment.

### 3. Experiments on Casimir forces

After the prediction of Casimir, a first generation of experiments was performed in the following decade. Since a detailed description of these experiments and a critical assessment of their limitations have been covered in a recent review [29], we will limit attention to brief considerations of the successful experiments with particular regard to some general methodological comments. It was soon recognized that the difficulty in the verification of the Casimir formula was not the expected force signal, which is relatively large at distances of order of a few micrometres for macroscopic (at least a few  $\text{mm}^2$ ) surface areas, but in the achievement and assessment of the parallelism, as well as in precisely measuring the distance between the

plates. An experiment in the original conducting parallel plate situation studied by Casimir was performed by Sparnaay [64], while other experiments were performed on configurations, different either in the geometry or in the conductivity properties. The sphere–plane geometry was investigated by van Blokland and Overbeek [65] to overcome the difficulty of maintaining the two plates parallel within the required accuracy to test the Casimir formula. The use of dielectric surfaces was proposed to have an accurate assessment of the gap distance, for instance by optical techniques. Obviously both these experimental shortcuts have associated issues with regard to the experiment–theory comparison. In the case of the sphere–plane geometry, an exact evaluation has only been obtained for a scalar field very recently [66], and all the experiments had to rely on an approximation used in classical electrostatics, known as proximity force approximation (PFA) [67]–[69]. This approximation gives reliable results if the relevant surfaces are separated by a distance much smaller than their typical local curvatures. This leads to an approximate expression for the Casimir force

$$F_C(d) = \frac{\pi^3 \hbar c R}{360 d^3}, \quad (3)$$

where  $R$  is the radius of the sphere and  $d$  is its distance from the plane, and its validity holds in the regime  $R \gg d$ . Since this approximation can be derived in classical electrostatics by relying on the additivity of the Coulomb force, care has to be taken in the case of forces of quantum nature which have a strong geometric (and non-additive) character. Likewise, in the case of dielectric materials the comparison with theory is complicated by the necessity to know the dispersive properties of the dielectric material. This is taken into account with a formula developed by Lifshitz and collaborators [43, 70]. The loss of *universality* intrinsic in the ideal Casimir formula makes the Lifshitz formula less appealing and complicates the theory–experiment comparison requiring a detailed knowledge of the dielectric response of the materials. Nevertheless, the problem was not felt to be important, as the experimental precision was limited in comparison to the more stringent tests of QED at the microscopic level with Lamb shifts and  $g - 2$  for electron and muon; no need for refined comparison with theory was then necessary.

The outcomes of the first generation of measurements can be summarized as follows. The Sparnaay experiment, with accuracy assessed at the 100% level, was considered as inconclusive in showing the expected scaling of the force with the distance, with also evidence for repulsive forces indicating a partial control over the electrostatic background. To use Sparnaay’s own words, the measurement ‘*did not contradict Casimir’s theoretical prediction.*’ The experiment by van Blokland and Overbeek was more successful from this viewpoint, obtaining agreement with the Casimir predictions at an estimated accuracy around 50%, and was thus the first uncontroversial verification of the Casimir force between metallic surfaces. Experiments with dielectric surfaces were performed using silica lenses [67, 71], crossed cylinders of muscovite mica [72, 73], thin films of liquid helium absorbed on surfaces of alkaline-earth fluoride crystals [74], flat surfaces of porosilicate glass [75]. The evidence for a crossover from the non-retarded component of the molecular force to the retarded component and an overall verification of the Lifshitz theory at the 20–40% accuracy level, apart from the experiment by Sabisky and Anderson reporting accuracy of order 1%, were the main results of these experiments.

After this burst of experimental activity on Casimir forces there was no further activity for many years. The spectacular success of QED and its unprecedented accurate verifications at the microscopic level could not be matched by measurements of (necessarily macroscopic) forces.



The attention at the macroscopic level was instead shifted on the atomic physics experiments, as the presence of a cavity with defined boundary conditions was found to affect the spontaneous emission properties of individual atoms. In this context, the microscopic counterpart of the Casimir force acting between an atom and a plane surface, also known as Casimir–Polder force [76], was measured by looking at the deflection induced on an atomic beam by two parallel plates [77] and comparing this with the theoretical predictions [78]. The new wave of Casimir force experiments was revamped after remarks by Sparnaay [79] concerning the possibility of a second generation of measurements at higher accuracy exploiting the emerging subfields of atomic force microscopy (AFM) [80] and of nanotechnology. Consequently, in partnership with Carugno at the INFN in Padova, we studied a scheme to measure the Casimir force in a parallel plate configuration, starting the first tests in the early summer 1993.<sup>3</sup> The apparatus capitalized on a variety of technological improvements not available at the time of the Sparnaay’s measurement. Most notably, the use of microresonators and of dynamical detection techniques based on the Fourier analysis of the tunnelling current of a single axis scanning tunnelling microscope were discussed and a first prototype tested inside a scanning electron microscope. Also, consideration was given to the capability of measuring the gravitational force in the same range [16]. Unfortunately the issues of parallelization, dust in the gap, and the large  $1/f$  noise present in electron tunnelling devices prevented a straightforward measurement of the Casimir force in the proposed configuration.

An attempt to measure the Casimir force using a torsional balance was initiated by Lamoreaux at the University of Washington in Seattle. The initial tests with flat plates, in 1994, also met difficulties in the alignment [81], until the experiment was reconfigured in the sphere–plane geometry by using a convex lens, and the Casimir force was then measured at distances up to  $6\ \mu\text{m}$  [82] with significant improvement in both range and accuracy with respect to the van Bockland and Overbeek measurement in the same configuration. These improvements were mainly due to the elimination of mechanical hysteresis in the torsion balance and the use of piezoelectric actuators for the positioning of the plates.<sup>4</sup>

Due to the large Volta potential present between the plates even after a nominal external short-circuit, even at the closest explored distance the Casimir force was evaluated to be about 20% of the total measured force, and required an ingenious subtraction technique to be evidenced. Theoretical discussions followed the appearance of the related paper, focusing on finite conductivity and temperature corrections. Given the large range investigated, this experiment with the accuracy initially quoted was in principle able to grasp both these corrections. A deeper analysis showed that the conductivity corrections were less trivial to manage due to the presence of a copper substrate deposited on the lens prior to the gold coating. Including a better assessment of the radius of curvature of the lens, found *a posteriori* to be aspheric [83], did not solve the conductivity issue. Further discussions of the experiment were carried out regarding the conductivity corrections [84, 85] and the thermal corrections [86]–[88]. While we suggest that the reader look at the related interesting exchange of comments, a likely assessment of

<sup>3</sup> The results of the calibrations and the proposal description were initially submitted to *Physical Review Letters* (manuscript number LN5333 Onofrio and Carugno) on January 24, 1994, and thereafter rejected on July 20, 1994.

<sup>4</sup> It may be worth noting that this experiment started as an undergraduate project, cannibalizing parts from existing equipment in storage spaces, with an estimated \$300 of material construction [81], and without the delays usually related to external funding through peer reviewers/panels/committees. As is noticeable in the acknowledgment section of the papers reporting the recent six measurements on the Casimir force, minimal dedicated external support was received in all of them.

the situation can be summarized as follows: the initially quoted accuracy of 5% was probably reliable at the smallest explored distances, but it was worse at the largest distances. Lamoreaux himself pointed out the spirit of his measurements in one of the abovementioned replies [85]: ‘*I offer the caveat that my experiment was intended as a demonstration to show that, with modern experimental techniques, one could do a really accurate measurement of the Casimir force. As a demonstration, only minimal tests for possible systematic errors were performed: furthermore, I was satisfied with the agreement between my experimental result and my inaccurate calculation.*’ This remark by the pioneer of the modern generation of measurements on Casimir forces, as we will see in the following sections, is key for understanding the spirit with which the current generation of measurements on Casimir force has been carried out: they have to be considered more as demonstrations than experiments.<sup>5</sup>

The successful use of AFM techniques combined with the sphere–plane geometry was accomplished by Mohideen and Roy at Riverside in 1998 [89], after attempts started one year earlier. In their experiment, a metallized polystyrene sphere was mounted on the tip of the AFM cantilever, and the deflection of the cantilever measured as a function of the distance between the sphere and a metallized flat surface. The metal deposited on the sphere was initially aluminum but a second version of the experiment instead used gold [90] which was predicted to provide a cleaner situation [91]. In both cases, the experiment–theory comparison took a number of corrections into account, namely the finite conductivity, the roughness, and the finite temperature, unobserved in the Lamoreaux measurement. Due to the small range of distances investigated, down to 100 nm, and the smaller Volta potential, of order 30 mV, the Casimir force dominated the electrostatic contribution over a wide range of distances, with the latter contributing to the bare force only in an amount evaluated as less than 3%.

A third successful attempt was performed by Ederth [92]. This work is remarkable for a number of reasons. The force was measured in the novel geometry of crossed cylinders (previously only used by Tabor and Winterton [72] for dielectric surfaces), and in the very short range of 20–100 nm. The expected Casimir force in such a configuration can be written as:

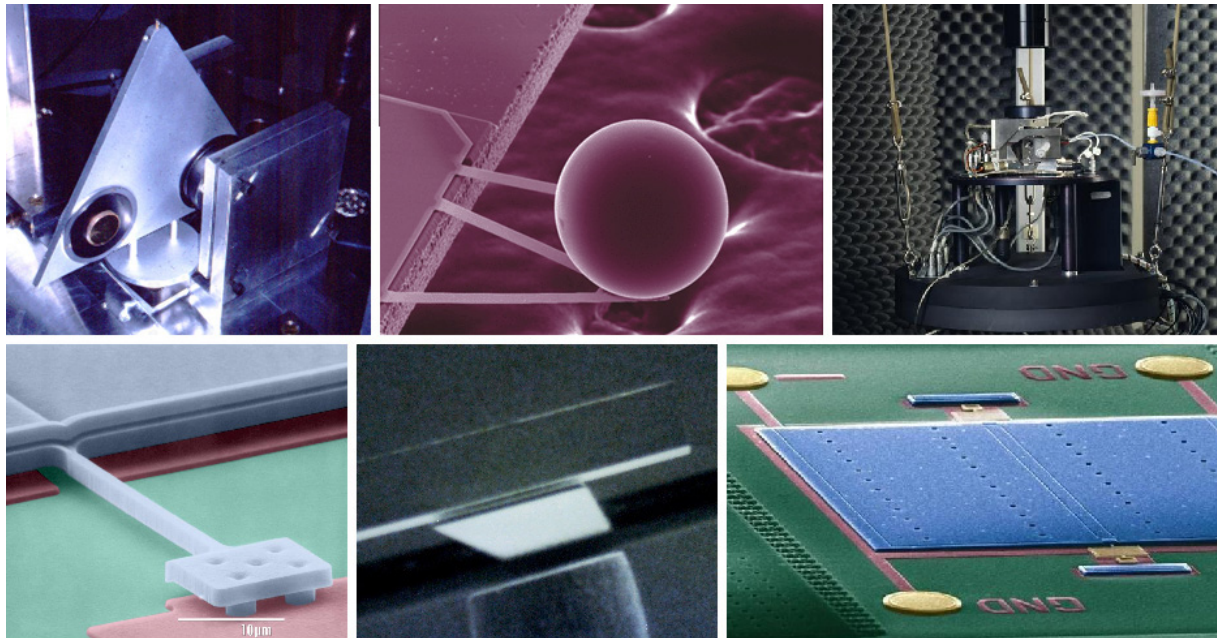
$$F_C(d) = \frac{\pi^2 \hbar c (R_1 R_2)^{1/2}}{360 d^3} \quad (4)$$

<sup>5</sup> Although the difference between demonstrations and experiments is known to scientists, it may be worth, for the more general audience reached by the *New Journal of Physics*, discussing this point in some detail. In a demonstration, scientists are aiming to validate an effect which is already expected, most likely with no surprises once the sought effect is found, apart from sources of discrepancy due to the approximations in schematizing the phenomenon. In fact, in a demonstration the theoretical comparison occurs by using the well-known theory describing the phenomenon. Its relevance is measured by the potential for enabling a new technology, opening a novel subfield of engineering, with the consequent social impact. In an experiment, scientists are looking for new, often unpredictable phenomena, not yet fully described by an existing, consolidated theory or model. Indeed, typically there is no theory at all, or there may be a large variety of different theories, all *a priori* possible descriptions of the phenomenon, which will be eventually confronted with the experimental outcome. Very rarely has an experiment immediate social impact as it is mainly driven by pure curiosity. Its relevance comes from ruling out many possible theories potentially describing the phenomenon, or by showing that an existing theory is falsified in a novel frontier, sometimes leading to a major scientific breakthrough. There is a natural crosstalk between demonstrations and experiments. The sensitivity of any apparatus used in experiments is assessed by means of a demonstration of a well established phenomenon, also called calibration. Likewise, experiments opening new frontiers allow for building models that will eventually lead to new technology. The complementary relationship between demonstrations and experiments reflects at the operative level the symbiotic relationship between technology and science.

where  $R_1$  and  $R_2$  are the radii of curvature of the two cylinders. Also, extreme care was put into polishing and chemically treating the surfaces, as well as assessing their roughness. Finally, a critical reanalysis of the precision was performed both with reference to the acquired data, and with respect to previous experiments in which the experimental precision was obtained with reference to theoretical predictions, in the spirit of a demonstration of an *a priori* known law. A further issue raised from Ederth was that the Casimir force is of such a nonlinear nature and spanning such a large range of values that it makes it difficult for an analysis with the rms error approach. He also noticed that an average, arbitrarily extended over a large region in which the force is small, will artificially decrease the rms error (see figure 6 in [92]). Moreover, the author discussed the effect of an unwanted oversight on the determination of the absolute distance between the two test bodies, concluding that if the separation between the surfaces is not measured directly with high precision, the induced uncertainty will make the relative error large (see figure 7 in [92]). While these may appear as details in the framework of demonstrating the Casimir force, they are actually critical if one wants to assess limits to new physics.

One variant of the sphere-plate measurement was performed at Bell Laboratories by a team led by Capasso and co-workers [93]. The force was measured between a sphere (similarly to the Mohideen set-up) but using a micro-electromechanical (MEM) resonator as the flat surface. The torque exerted by the sphere on the resonator unbalanced the capacitive bridge which used a second set of plates under the MEM. The rms deviation between theory and experiment was less than 0.5%, but the authors were more conservative and considered this quite accidental in the light of the presence of various factors not controllable at the same level of accuracy, most notably the spread in the optical properties of gold films. The main goal of this measurement was to show that Casimir forces are of relevance in micromechanics and (at least until they do not correspond to non-retarded components) in nanomechanics, and the interesting nonlinear dynamics predicted in [62] including Casimir switching was also demonstrated [94]. This experimental effort continues at Harvard, focusing on the influence on Casimir forces of materials with modulable reflectivity properties [95], torque induced by vacuum fluctuations [96], and effect of the finite thickness of the metallic deposit [97] (see also [98, 99] for reviews of these efforts). Previous claims of a possible role of the Casimir force in MEMs can also be found in [100, 101], but the related experiments found a discrepancy of a factor six with respect to the Casimir prediction. It has been suggested by the same authors that this discrepancy arose from source roughness or an adsorbed layer between the two surfaces, but regrettably no further investigations were carried out to solve this puzzle. An alternative and more conservative explanation was suggested in the conclusions of [94] as due to the electrostatic forces estimated to be  $10^6$  times larger than the expected Casimir force.

The initial attempts started in Padova to perform a measurement in the parallel plane configuration became successful in the late Spring 2001 [102]. The key upgrades with respect to earlier efforts, as described in detail in [103, 104], were the use of a fibre optic interferometer as a displacement detector [105], the monitoring and cleaning of the two surfaces from dust with an *in situ* tool, the implementation of frequency-shift techniques. The frequency-shift technique was already well-known in the field of AFM [106] and its use in Casimir force experiments was first reported in [107]. The measurement was performed in the  $0.6\text{--}3\ \mu\text{m}$  range, with the lower limit approaching the expected Casimir stiction between the two surfaces considering the stiffness of the cantilever, and the upper limit due to the sensitivity of the fibre optic interferometer. The data were analysed using the plain Casimir formula without corrections, and the accuracy in the measurement of the Casimir coefficient (which can be directly translated into a measurement of



**Figure 2.** Pictures from the six Casimir force experiments of the second generation. In chronological order, from top-left to bottom-right, some views of the apparatus used in Seattle, Riverside, Stockholm, Murray Hill, Padova and Indianapolis are depicted.

$\hbar c$ ) was 15%. Major limitations to the accuracy were the precision in determining the absolute distance from the electrostatic calibration, the finite amount of parallelism, and the piezoelectric actuator with 10% deviation from the expected specifics, which was overcome with independent calibrations. Attempts to introduce the finite conductivity in the data fit led to better accuracy estimated to be around 5% [108]. However, in a conservative approach, no effort was made on a more sophisticated data analysis mainly since the better fitting involved points at the smallest distances which were taken close to the snapping critical point (estimated, given the stiffness of the resonator, to be around  $0.4 \mu\text{m}$ ) potentially inducing anharmonicity and instability in the resonator motion. Furthermore, the experiment was felt to be less accurate than previous sphere–plane experiments. Nevertheless, the result of this experiment represents a significant improvement over the pioneering but ambiguous result obtained by Sparnaay in the original plane–plane geometry studied by Casimir. Further motivations to study this geometry have been discussed in [109].

The most recent reports on Casimir force measurements are from Decca and collaborators at Indiana University. The Casimir force was initially measured in a configuration similar to the one studied at Bell Labs but using different metals [110, 111]. Apart from the use of different metals, gold and copper, the experiment makes use of a two-colour fibre optic interferometer to maintain the fibre–platform separation constant. The same dynamical frequency-shift technique previously used in [94, 102, 107] was implemented. The authors also claimed that the dynamical technique was equivalent to determining the Casimir pressure between two parallel plates, but this formal equivalency relies upon the validity of the PFA, similarly to all previous sphere–plane measurements, and therefore this is a way to restate the same physics. This group has made a

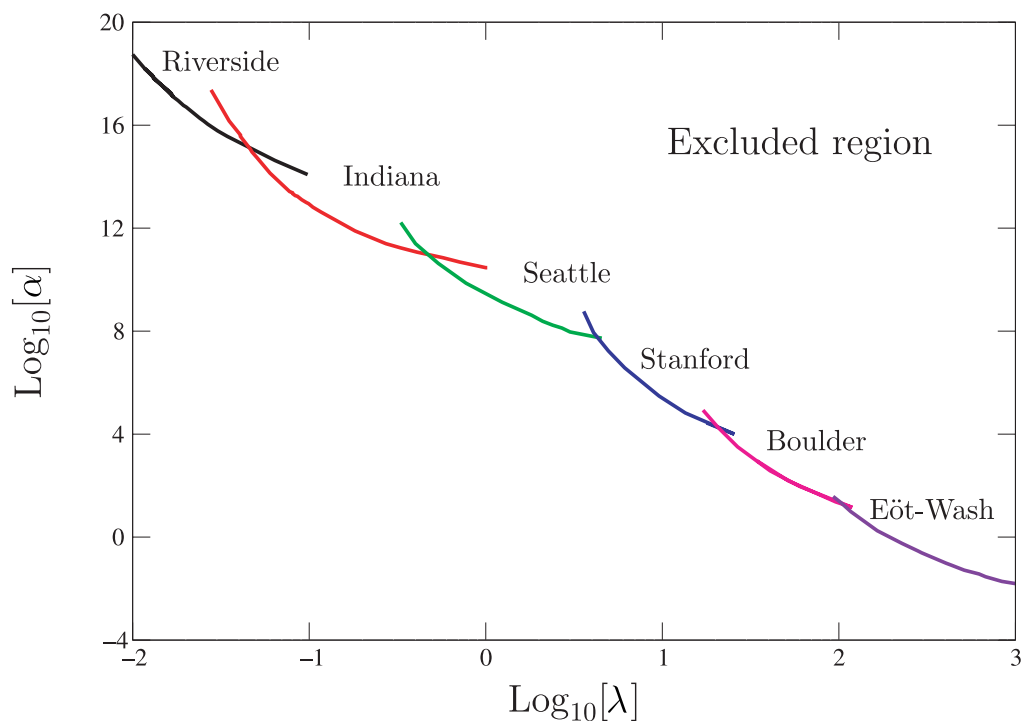
**Table 1.** Status of the experimental studies of Casimir forces between metallic surfaces. For each experiment, listed in chronological order, the investigated geometry, the explored range of distances, the claimed accuracy, and the first author and reference are reported. The claimed accuracy is the one quoted by each group, however it often corresponds to different definitions with different statistical meanings. In the experiment by Sparnaay repulsive forces were observed between aluminum surfaces, while the quoted accuracies for the van Blokland and the Lamoreaux experiments have to be considered as reliable only at the smallest distances. Whenever there was a sequel of experiments by the same group, the quoted reference is for the first reported data set.

Year	Geometry	Range ( $\mu\text{m}$ )	Accuracy (%)	Reference
1958	Plane–plane	$0.3 \div 2.5$	100	Sparnaay [64]
1978	Plane–sphere	$0.13 \div 0.67$	25	van Blokland and Overbeek [65]
1997	Plane–sphere	$0.6 \div 12.3$	5	Lamoreaux [82]
1998	Plane–sphere	$0.1 \div 0.9$	1	Mohideen and Roy [89]
2000	Crossed cylinders	$0.02 \div 0.1$	1	Ederth [92]
2001	Plane–sphere	$0.08 \div 1.0$	1	Chan <i>et al</i> [93]
2002	Plane–plane	$0.5 \div 3.0$	15	Bressi <i>et al</i> [102]
2003	Plane–sphere	$0.2 \div 2.0$	1	Decca <i>et al</i> [110]

well focused effort to constrain non-Newtonian forces using a variety of schemes, with particular regard to the cancellation of the Casimir force using the so-called isoelectronic technique [112]. In this approach, the differential force between a gold-coated sphere and flat surfaces of gold and germanium coated with a shared layer of gold was measured, leading to a fractional difference in the Casimir force estimated to be  $\simeq 10^{-6}$ , below the experimental sensitivity [113]. The presence of a force signal should be due to a non-Casimir like interaction. A nonzero force was evidenced, but it was attributed to the residual Casimir force due to the different height difference between the two flat surfaces, not controllable to better than  $\simeq 0.1$  nm. The experiment is planned to run using different isotopes of nickel to further reduce the fractional difference in the electronic properties. The more recent results have been questioned with regard to the claimed precision [114] in some of the parameters involved in the data analysis [115]. This issue seems still controversial and in the next section we will discuss some tests which could be possible *smoking guns* in assessing the sensitivity and the precision of all the Casimir force apparatuses.

We show in figure 2 pictures from the six recent experiments on Casimir forces, while in table 1 the current knowledge of Casimir forces between metallic surfaces is summarized. In figure 3 the corresponding reported limits in the  $\alpha$ – $\lambda$  plane are presented.

This succinct overview of the recent experiments on the Casimir forces needs to be complemented by the parallel developments in the measurement of the Casimir–Polder force, the long distance forces acting between atoms and macroscopic surfaces [76]. Casimir–Polder forces establish the link between the macroscopic Casimir forces and the microscopic atom–atom interactions (like the van der Waals forces). This force has been measured by looking at the deflection induced on an atomic beam of sodium atoms by the presence of a metallic surface [77]. The accuracy of this measurement was around 12% limited by the control of the distance between the atomic beam and the surface, but it was enough to confirm that the Casimir–Polder force, and



**Figure 3.** Experimental constraints and predictions for a hypothetical Yukawa force in the  $\alpha$ – $\lambda$  plane, with  $\lambda$  expressed in micrometres. The plot may be considered a prolongation of the  $\alpha$ – $\lambda$  plot of figure 1 on the upper left corner. The excluded region is delimited by curves compatible with no Yukawian-signal in the measured data within two standard deviations. From left to right, limits coming from three experiments on Casimir forces performed in the 100 nm–10  $\mu\text{m}$  range at room temperature [82, 90, 113], two experiments with microresonators operating in the 100  $\mu\text{m}$  distance range [18, 19], and one experiment using a torsion pendulum with rotating attractor [15], are shown.

not the van der Waals force, was acting in the configuration investigated. More detailed studies of Casimir–Polder force are necessary for a variety of reasons. It has been recently conjectured that their study at larger distance can more easily provide information on the thermal contribution to the force [116]. Interesting studies on non-equilibrium statistical mechanics are also possible [117], and their role in the formation of weakly bound macromolecular states of biological interest has yet to be explored. The technological interest in these studies is motivated by the manipulation of cold atoms on chips for the possible implementation of quantum information schemes or for characterization of surfaces. Quantum operations can be performed only by preserving quantum coherence, so it is important to evaluate the effect of surfaces in close proximity of cold atoms [118]–[123]. In a recent experiment, the Boulder group at JILA led by Cornell succeeded in measuring the Casimir–Polder force [124]. The dipole oscillation frequency of an elongated, cigar-shaped, condensate of rubidium atoms was determined versus the distance between the condensate and a conducting surface. This experiment could also soon lead to the observation of the thermal corrections to the Casimir–Polder force. In this case, the information could be

used to infer the thermal corrections to the purely Casimir force, especially if the investigations will be extended to dielectric surfaces. Another potentially fruitful technique is the use of atomic interferometry, with the first measurement of the van der Waals force reported in [125].

#### 4. Precision and accuracy in Casimir force experiments

To gain more insight into the reliability of the limits on non-Newtonian forces, we briefly review some elementary notions of data analysis [126]. The *precision* of an experiment is commonly defined as the measure of how exactly the result is determined, regardless to what that result means in the framework of a specific model or theory. Any experimental set-up has a source of intrinsic uncertainties leading to different results for the repeated measurement of the same observable and the precision is a measure of how reproducible the result is in *itself*. If the random errors result from instrumental uncertainties, the precision is increased by using more precise instruments. If instead the random errors result from statistical fluctuations of counting finite sequences of events, the precision will be increased by counting more events. In an optimized experiment, both instrumental and statistical errors are minimized and made approximately equal to each other. The precision of the experiments, in the case of apparatuses aimed at measuring Casimir forces, should be expressed in terms of the spreading of the result in the observable directly measured, which can be the compensating voltage in the closed-loop actuation scheme for the Lamoreaux's torsion balance, the electric signal from the quadrant photodiode for the Mohideen configuration, the bridge unbalance voltage in the Bell Labs measurement, or the frequency shift in the Padova and Indiana measurement schemes. These primary observables are then indirectly translated into a measurement of force through the physics of the detection system and a proper calibration with a controllable force. At this point, although the force is always an indirectly measured quantity, it may be convenient, for the sake of standardization among different measurement techniques, to express the precision in terms of error bars for the force measurement.

The *accuracy* of an experiment is instead a measure of how close the result comes to the true value, and then is a measure of how much a test of a physical theory is stringent. The accuracy of an experiment is therefore dependent on how well the experimenter can skilfully control, compensate for or take into account in the data analysis for systematic errors. Improvement of the accuracy requires first of all high precision data, since a given targeted accuracy needs a precision at least as good. However, on the top of the precise data, one also needs to control sources of discrepancy with respect to a theory to be tested. Therefore a targeted accuracy relies on precise data plus auxiliary information about the modelization of the experiment in a given theoretical framework. In other words, while the precision of a measurement is a concept intrinsic to the apparatus and the protocol used to take data, the accuracy is a model-dependent concept and gets better and better as we refine the factors which take into account the sources for the so-called systematic errors. Once all known factors determining the accuracy are well assessed, a systematic error left over may be the manifestation of new physics.

It is evident that this logical procedure has to be taken with great care before making strong claims on new physics, and all possible conceivable sources of discrepancy must be scrutinized prior to a positive claim. All fields of physics (especially in recent times mostly due to sociological

reasons<sup>6</sup>) have examples of claims for evidence of new physics which were then withdrawn after closer scrutiny of the data and/or the running of independent experiments in similar conditions and under less hasty circumstances. These considerations, when applied to the experiments measuring the Casimir force, are translated into a long list of systematics which may mimic new effects. With respect to the bare Casimir formulae obtained in idealized geometries for perfect conductors electrically neutral at zero temperature and in ideal vacuum, there are indeed many possible systematic effects to be taken into account.

#### 4.1. Non-ideal geometries

There are several deviations from ideal geometries, such as ideal spheres and infinite parallel plates. In practice, all surfaces manifest a finite roughness, have finite size which always implies consideration of border effects, finite parallelism, and the geometrical parameters are determined with finite precision. Here is a list of considerations:

- (1) Roughness of the surface. The exact formula is derived, whenever a first principle derivation is available, assuming that there is ideal reflection on the walls of the cavity. Any realistic surface will have roughness and therefore the scattering of the photons will be more adequately schematized in terms of diffusion, rather than pure reflection.
- (2) Finite size of the test objects and border effects. Planar surfaces of finite size are always considered in realistic experiments. Therefore the predictions for infinite planes have to be corrected. This is particularly important in the parallel plane configuration as the *quantum field lines*, using an electrostatic analogue, are expected to manifest deviations from the symmetrical geometry. This effect may become relevant at large distances between the test bodies. Recently, Gies and collaborators have numerically evaluated border effects for a scalar field in the parallel plane configuration. In this case, one may introduce an effective surface area  $S_{\text{eff}}$  related to the geometrical area of the flat surfaces  $S$ , its perimeter  $C$ , as the distance between the plates  $d$  as  $S_{\text{eff}} = S + 0.36Cd$ , with a correction to the Casimir force linear, at leading order, in the distance  $d$  [128, 129].
- (3) Finite parallelism whenever applicable. The deviation from ideal parallelism, for instance in the case of a parallel plane geometry, must be kept under control to aim at high accuracy. The effect is relatively more important at small distances between the test bodies. The correction due to finite parallelism in the PFA has been discussed within the PFA scheme in [130].
- (4) Measurement of the size of the test objects, for instance the radius of the sphere, the size of the plates, and the distance between the two objects. The use of electron scanning microscopy allows one to obtain precise measurements of some geometric parameters, for instance the diameter of the spheres used in the AFM-MEMS experiments by the Riverside, Bell Labs and Indiana groups. One critical issue still awaiting a definitive solution is the precise determination of the distance between the two objects. Indeed, the *relative* distance between two different configurations corresponding to different distances can be accurately obtained by calibrating the displacement actuators, but the *absolute* distance is not, and

<sup>6</sup> The complex relationships among scientific communities, research funding agencies, peer-reviewed journals, and mass media in generating, propagating, and scrutinizing false claims of scientific discoveries have been recently the focus of many essays, papers, and books on sociology of science, see for instance [127].



this is important for forces strongly dependent on the distance. While in experiments using dielectric surfaces this problem was addressed by using optical interferometric techniques, in various experiments performed so far with metallic surfaces the absolute distance was determined as a fitting parameter. When using fibre optic interferometry, two-colour fibre optic displacement transducers like those of the Indiana group will be a standard solution for determining the absolute distance with an accuracy comparable to the determination of the two wavelengths. Other solutions could include the use of calibrated wires of small diameters (available down to 2–5  $\mu\text{m}$ ) as high precision spacers to assess for few experimental points the absolute distance. All these solutions have limitations. The more conceptual one is that, at the atomic level, the distance between two bodies is an ill-posed concept as surfaces are far from being accurately schematized as planes. It is then hard to assess the distance with a precision better than the surface roughness evaluated as the rms deviation over the considered surfaces. The same definition of distance may depend also on the probe used to assess the distance. In case of electrostatic calibrations, non locality in the electric properties of the surfaces may be a source of systematic errors. In the case of the use of fibre optic interferometry, light is not just reflected by the first atomic layer, and the procedure of obtaining the zero distance reference by touching the surfaces has to be taken with great care, due to potential uncertainties inherent to this somewhat subjective, not necessarily reproducible, procedure.

#### 4.2. *Electrical properties*

Ideally one needs materials which are perfect reflectors for photons at any wavelength, and which are electrically neutral. None of these conditions is obtained in practice:

- (1) There is no perfect conductor at all frequencies, so at frequencies large enough the photons will leak out from the cavity and will not contribute to the radiation pressure.
- (2) The presence of residual electric charges will spoil the neutrality and will give a direct Coulomb contribution. We can identify at least two distinct sources of residual electric charges, respectively due to the so-called patch effects, and the Volta potentials. The former are intrinsic to the same metal, while the latter refer to the differences in the mean electric potential between two connected metals.

The physical origin of the surface potential and charge on a metal is due to the presence of a finite surface dipole moment. This depends on the separation of the lattice planes parallel to the surface. If there are different crystallographic directions on the surface of a polycrystalline metal, there will be variations of the surface potential. The effect of such forces in the Casimir experiments has been discussed in detail, using analytical and numerical methods, in [131]. The mapping of the surface voltage can be obtained by using Kelvin probes (see [132] for a review), and measurements on various conductors, including gold, copper and aluminum, have been performed with a spatial and potential resolution of 1.5 mm and 1 mV, respectively [133]. The same authors have tried to contain the surface potentials by focusing on treatment of nonreactive materials like gold and graphite [134]. In particular, sublimation of fullerene films on metallic substrates has shown that there are no surface potential variations within the Kelvin probe resolution of 1 mV [135].

The presence of contacts between different metals allows also for different mean voltages. For instance, in the Lamoreaux experiment the two plates were externally shorted through a loop of 40 separate electrical connections, leading to an overall Volta potential of 430 mV [82]. This results in an electrostatic force which overwhelmed the Casimir force. Even more important, the combination of this electrostatic attraction between the two plates and the finite retaining force of all experimental configurations results in stiction of the surfaces at relatively large distances, thus preventing the achievement of the smaller gaps at which the Casimir force is expected to prevail over the electrostatic contribution. Provided that the drift in time of the effective Volta potential is minimal, the solution adopted consists in the application of a *counterbias* voltage cancelling, at least at the leading order, the Volta potential contribution. This allows for the achievement of smaller gaps to the point where the stiction will result from the Casimir force itself. The residual voltage difference due to the incomplete cancellation can be taken into account in the off-line data analysis through a simultaneous fitting with the expected Casimir and Coulomb forces.

#### 4.3. Environmental properties

The measurement of the Casimir force is also affected by environmental factors. The leading ones are due to the real photons present in the blackbody radiation at finite temperature, the presence of residual gas in the gap, and the acoustic noise in the laboratory. So far, all experiments have been performed in conditions such that magnetic fields should have no effect, although at the microscopic level this requires a control over magnetic properties of the test masses or magnetic impurities present in them which eventually could be relevant for accurate measurements.

- (1) Finite temperature. An obvious effect of finite temperature, at the instrumental level, is the presence of thermal drifts in the apparatus, which may limit the effectiveness of taking long integration times. The use of Peltier coolers with active feedback and materials with low expansion coefficients mitigates this effect. Moreover, at any finite temperature one also expects a blackbody background of photons. Most of these photons are blocked by the cavity as the condition of stationarity for the electromagnetic field implies that no electromagnetic waves with wavelength exceeding twice the length of the cavity can be sustained. The thermal correction for a given temperature is particularly important at large distances. This effect is included in the realistic approach pioneered by Lifshitz, but complications arise due to the necessary inclusion of the reflectivity coefficients. In the Lifshitz formula for the force between two parallel surfaces [43]:

$$F_C = \frac{S}{\pi\beta d^3} \sum_{m=0}^{\prime} \int_{m\gamma}^{\infty} dy y^2 \left[ \frac{r_{\text{TM}}^{-2} e^{-2y}}{1 - r_{\text{TM}}^{-2} e^{-2y}} + \frac{r_{\text{TE}}^{-2} e^{-2y}}{1 - r_{\text{TE}}^{-2} e^{-2y}} \right] \quad (5)$$

both finite temperature (with  $\beta = 1/k_B T$  the inverse temperature, and  $\gamma = 2\pi d/\beta \hbar c$ ), and finite conductivity effects (expressed through the reflection coefficients  $r_{\text{TE}}$  and  $r_{\text{TM}}$  for the two independent polarizations, the transverse electric (TE) and transverse magnetic (TM) modes, computed at imaginary frequencies  $\omega_m = i\xi_m$ , where  $\xi_m = 2\pi m/\beta \hbar$  are the Matsubara frequencies) are taken into account. The  $m = 0$  term is counted with half weight, as denoted by the prime on the summation sign. The reflection coefficients are expressed in

terms of the dielectric permittivity  $\epsilon(\omega)$  as

$$r_{\text{TM}}^{-2} = \left[ \frac{\epsilon(i\xi_m) p_m + s_m}{\epsilon(i\xi_m) p_m - s_m} \right]^2, \quad r_{\text{TE}}^{-2} = \left[ \frac{s_m + p_m}{s_m - p_m} \right]^2, \quad (6)$$

where  $p_m = y/m\gamma$  and  $s_m = \sqrt{\epsilon(i\xi_m) - 1 + p_m^2}$ . Using tabulated optical data for different metals in the bulk, it is possible to compute the dielectric permittivity along the imaginary frequency axis. To calculate the  $m = 0$  contribution, the available data have to be extrapolated in the static limit. There is no unique prescription to do this. Different choices have led to controversial predictions for the Casimir force between parallel plates (see for instance [115], [136]–[147] and the in-depth discussion in [32]). The definitive control over the thermal fluctuations will be required to give limits to other forces. Detailed data analysis such as the one proposed in [111] cannot be used to simultaneously constrain two physically different effects as the thermal contribution and the presence of gravitational forces of Yukawan origin in absence of other experimental inputs.

- (2) Residual gases can influence the measurement both by giving additional random forces acting on the test bodies, by changing in time the properties of the surfaces, and by favouring the migration of contaminants, oxide layers, dust particles, and residual charges.
- (3) Acoustic and seismic noise. The experiments should be shielded from mechanical noise originating either by natural sources, as seismic noise, for instance using air tables and damping systems, and from artificial sources including vacuum pumps. In some cases, this can limit the measurement time as the vacuum chamber must be disconnected from the vacuum line, as described in [114]. In the Padova experiment, the demand for vacuum also comes from the use of a scanning electron microscope evacuated through a diffusion pump, which was in turn periodically fed by a (quite noisy) roughing pump. In the case of the extremely sensitive torsional balance set-up, Lamoreaux reported that his own presence in the laboratory during the measurement, and the consequent tilting of the floor due to the additional weight in the room, was a factor initially affecting the data [81].

Furthermore, even for idealized physical and geometrical conditions, one often has to deal with the PFA formula in the case of sphere–plane or cylinder–plane geometries, and with data analysis where often unambiguous choices cannot be made. The use of the PFA has generated a wide debate, since it is used in the sphere–plane configuration for which an exact formula does not exist and, due to this reason, it is then hard to assess its validity. Under idealized conditions for all the other possible sources of systematics, it is generally believed that the PFA is valid within 0.1% for large radii of curvature [29]. Recently, progress has been made by various groups towards understanding the limitations of the PFA and its interplay with some of the abovementioned effects. The Paris group led by Lambrecht and Reynaud has discussed the interplay between roughness corrections, assuming the plasma model for conductivity corrections, and the PFA, showing that the roughness correction is underestimated in the PFA framework [148]. The Casimir energy between a cylinder and a plate has been evaluated exactly [149, 150], allowing for a comparison with the corresponding PFA formula previously evaluated in [151]. The worldline numerics approach pioneered by Gies *et al* [152] has been applied to both sphere–plate and cylinder–plate geometries for a scalar field with Dirichlet boundary conditions, showing significant deviations from PFA if the ratio between the gap and the radius of curvature of the surfaces is too large [153].

The systematic factors listed above do not contribute with equal weights in the entire range of explored distances. Finite roughness, finite parallelism, finite conductivity, vacuum pressure, and Volta potentials more strongly affect the measurement at small distances. The finite size and border effects, the finite temperature contribution, and the PFA (whenever applied) are corrections expected to play a major role at larger distances.

Another issue to be experimentally assessed is the sensitivity to the minimum detectable force using physical measurements. For larger scale experiments (like torsional balances) this is better assessed for instance in terms of the maximum distance at which the gravitational force is still detectable. For smaller scale experiments (like AFM-based ones) the sensitivity should be naturally expressed in terms of the minimum detectable electric field. There are also other physical effects to be exploited to assess the precision and the sensitivity of the measurements in the short range. In particular, the conducting properties of the substrate can be used, for instance using semiconducting surfaces with modulated conductivity. The sensitivity could then be expressed in terms of the minimum charged carrier modulation detectable through the Casimir force measurement. Even simpler, the Casimir force is expected to depend on the thickness of the substrate due to the finite skin depth, as evidenced in [97]. The claimed accuracy of the measurement could be checked with a *blind* test, by providing two surfaces having a thickness unknown to the same Casimir force experimenter. If the claimed accuracy is real, he/she should be able to infer, from the Casimir force measurements, the difference between the thickness with a corresponding confidence level, to be compared with actual measurements of the thickness of the substrates performed by a third party. This unbiased benchmark could be the ultimate test of the reliability of the accuracy claimed in each experiment. Similar benchmarks are also available by detecting the effect of external fields, like the presence of a magnetic field [154], in a controllable way. In principle, also effects like the so-called *lateral* Casimir force [155] could help to assess the sensitivity, however the control over the related theoretical predictions based upon the PFA seems to be an issue [156].

A further potential concern for the assessment of the limits to non-Newtonian forces also arises from the use of electrostatic calibrations at larger distances. For instance, in all the experiments determining the absolute distance through a fitting to the purely electrostatic data, one cannot logically expect strong and meaningful limits to non-Newtonian forces. In doing this, one first insulates a region of parameters, at a large distance where a purely electrostatic contribution is expected. In this regime, all groups do not consider any of the usual corrections included in the evaluation of the Casimir force, something which may limit the accuracy of the calibrations in the first place. For some corrections (like for instance roughness) this is partially justified since the electrostatic calibrations are performed at such large distances that they are negligible. From this analysis in a partial region one extract some parameters, for instance the absolute distance and the Volta potential, and then this determination is used in a region where the Casimir force is expected to give a significant contribution. This works fine for the latter force due to the sharp dependence on distance, but the expected Yukawan contribution may fall off slowly at large distance depending on the value of the Yukawa range  $\lambda$ , so to first neglect this term in the fitting of the electrostatic calibrations and then to look at it in the residuals of the Casimir force has a somewhat tautological flavour. In particular, a Yukawan force of relatively large intensity and long range could be hidden in the electrostatic calibration. Its presence would be evidenced by a less accurate fit of the data in the long distance with the Coulomb contribution alone. However if the fitting parameters are then used for obtaining information on the Casimir force it will be very hard to infer from the residuals the presence of this Yukawan force. To

avoid such loopholes, a combination of improvements should be implemented, first of all a precise determination of the distance between the test objects independent on the electrostatic calibrations, and thereafter a fitting procedure in the widest range of distances with the Casimir force, the residual electrostatic force, and the hypothetical Yukawan force all considered at the same time.

As should be clear from previous discussions, it is very important to analyse the data for extracting information about the windows of opportunity for non-Newtonian forces. A possible approach to data analysis which should help comparison of various experiments and assessments on non-Newtonian limits is based on the following argument. As remarked earlier, Casimir physics is intrinsically ruled, at the leading order, by the presence of the two fundamental constants present in relativistic quantum field theory, namely  $\hbar$  and  $c$ . In all formulae for the Casimir effects the two constants appear in a multiplicative way. All the experiments on Casimir forces can then be rephrased as macroscopic determinations of the product  $\hbar c$ . This has the advantage that it allows for a direct comparison of experiments performed in a variety of geometries. Moreover, since high precision measurements of  $\hbar$  ( $c$  being now exactly determined) are independently available, deviations of  $\hbar c$  obtained from Casimir experiments with respect to the more precisely known value of  $\hbar c$  allow for a unified discussion of the accuracy of each experiment. The accuracy of the measurement can be expressed in terms of the relative difference between the central value for  $\hbar c$  obtained from the data fitting of the Casimir measurements and its *true* value (i.e. with much smaller error) as available from CODATA,  $\hbar c = (3.16152636 \pm 0.00000054) \times 10^{-26}$  J m. Deviations of statistical significance from the CODATA value obtained from the data analysis of the experiments on Casimir force should signal either a not well controlled background or new physics in the micrometre range. The latter hypothesis could become stronger if confirmed by similar pattern of deviations in independent and quite different experimental set-ups.

## 5. Future experimental directions

In this section we summarize some directions currently pursued to understand the long-distance behaviour of the Casimir force which are relevant for limits to non-Newtonian forces. The current status and future directions of short-distance Casimir physics, for instance the interplay of quantum fluctuations with surface collective excitations [157, 158], will be more adequately discussed in other review papers of this Focus issue.

Among the directions to be explored, there is the possibility of improving the AFM sensitivity to detect the Casimir force in the 1–2  $\mu\text{m}$  range or to use differential techniques to measure the force at smaller distances at different temperatures [159], as proposed by Mohideen and collaborators. At large distances, the  $1/d^3$  scaling of the sphere–plane Casimir force is more favourable than the  $1/d^4$  scaling of the force between parallel plates, and the expected thermal corrections are more pronounced [151]. In particular, high precision AFM apparatuses can be developed to reach sensitivity in the  $10^{-18}$  N/ $\sqrt{\text{Hz}}$  range by using cryogenic techniques. Apart from the broader technological benefits of developing high precision AFM apparatuses, this direction can lead to better limits on non-Newtonian forces without the cross-talk with the still not completely understood thermal correction as lower temperatures correspond to longer thermal effective lengths  $\propto \hbar c/k_B T$ . Pursuing this direction will require a well focused effort with the need to adapt a Casimir force apparatus in a cryogenic environment.



**Figure 4.** Image of the experimental set-up for the study of the cylinder-plane configuration, with (from top to bottom) the optical fibre for the displacement measurement, the silicon rectangular cantilever, and the cylindrical lens. Cantilever and cylindrical lens, coated with  $(144 \pm 20)$  nm of Au, have respectively width and radius of curvature equal to 1 cm. Notice the mirror image of the optical fibre on the cantilever, which allows to estimate the fibre to cantilever distance. With proper illumination and the use of an optical microscope by looking at the mirror image of the cylinder on the cantilever, it is likewise possible to estimate the cylinder–cantilever distance at large (above  $\simeq 10 \mu\text{m}$ ) gaps, which allows for consistency checks with the electrostatic calibrations.

A second direction to be pursued is an improved version of the experiment using a torsional balance. This direction is under development both at Los Alamos for the sphere–plane geometry [160, 161] and in Grenoble for the parallel plate geometry [162]. Lamoreaux has discussed a series of improvements to his original set-up. The continuation of his efforts seems quite natural and this could clarify open issues related to the original measurements reported in 1997. Meanwhile, a torsion balance for a measurement of the Casimir force between large surface parallel plates has been built in at ILL in Grenoble and electrostatic calibrations are currently being performed. The experiment will also make use of already developed inclinometer technology used to measure the quantum states of neutrons in the gravitational field [163] to deal with issues of parallelization and absolute measurement of the gap distance. Even in this case an improvement of the sensitivity is expected, at the price of a longer development stage, using cryogenic techniques [164].

A third direction actively pursued is the study of the Casimir force in a cylinder-plane geometry. This configuration has advantages and disadvantages in between the sphere–plane and the parallel plates geometries. The Casimir force in the cylinder–plane configuration, evaluated in the PFA, is [151]

$$F_C(d) = \frac{\pi^3 \hbar c}{384\sqrt{2}} \frac{La^{1/2}}{d^{7/2}}, \quad (7)$$

where  $L$  is the length of the cylinder, and  $a$  its radius of curvature. This configuration looks particularly promising to detect the thermal effect and allows for an easy procedure to obtain accurate parallelization. An exact solution for the Casimir force has been found recently [149],

and thus this configuration is also a candidate to test the validity of the PFA in case the cylinder is made of small (around few  $\mu\text{m}$  diameter) wires. An experiment in this configuration is now under development at Dartmouth with electrostatic calibrations having shown that the measurement should be feasible [165]. As of this writing the first data acquisition with an upgraded apparatus aimed at measuring the Casimir force in the 2–5  $\mu\text{m}$  range is underway (see figure 4).

## 6. Conclusions

We have reviewed past experiments which have successfully measured the Casimir force, and from which limits on the existence of non-Newtonian forces have been derived. We have also discussed some of the future directions which are being explored with the aim of better mastering the systematics of the corrections to the Casimir force, in particular the urgent issue of the finite temperature correction. Without a solid understanding of this effect, limits to non-Newtonian forces appear very weak and prone to criticism. This in turn demands reliable assessments of the precision and sensitivity of the experimental set-up, for instance also by means of blind tests as proposed in section 4. It will be also necessary, in order to avoid unintentional bias in the data analysis and to collect diverse feedbacks, to allow for a broad *data sharing* among various experimental groups and phenomenologists interested in independent analysis. This is possible at zero cost through internet resources and simply adopting what is already implemented in other subfields of physics, most notably in astronomy and high-energy physics, with data archives available to all interested astronomers and physicists.

All the demonstrations of the Casimir force performed so far have not provided evidence, within their claimed accuracy, for any residual signalling non-Newtonian physics. We have briefly discussed a proposal to parameterize the data collected in the six Casimir force experiments, or future experiments, in terms of a measurement of the constant  $\hbar c$ . Due to the long list of systematic effects, it is also clear that an extremely careful and conservative analysis has to be carried out prior to any claim of new physics. This could be evidenced as a significant deviation between the central value of the  $\hbar c$  value determined from the Casimir data and the  $\hbar c$  CODATA value. The repetition of experiments in dissimilar conditions and aimed at observing the same signal will help disentangle a possible positive claim from false signals. We believe however that a definitive confirmation of a claimed effect will most likely happen by collecting complementary evidences based upon radically different classes of experiments from the one described here and operating at different lengthscales. For instance, relevant informations could also be acquired by exploiting techniques close to the ones developed for detecting light neutral particles coupled to photons by Zavattini *et al* [166], by accurate measurements of neutron's states in the Earth's gravitational field [167], by observing surface excitations in superfluid helium [168, 169], by measuring the dynamics of intrinsically nanotechnological structures like nanotubes [170], or significantly improving the sensitivity of existing dynamical detection schemes for rotating [171] or vibrating masses [172, 173].

The message of this review in a nutshell is that, in spite of the recent progress which brought solid demonstrations of the Casimir force, the challenge of discovering new gravitational-like phenomena in the micrometre range with table-top experiments is still an issue more open than ever. A third generation of Casimir force experiments combining creativity, hard work, metrological accuracy, and substantial support both in intensity and duration will be necessary to further pursue non-Newtonian gravitation at the microscale.

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