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Theory and Methodology

Diffusion models in forecasting: A comparison with the Box–Jenkins approach

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Abstract: Diffusion models represent an interesting approach to the study of technical change and evolutionary processes. In the business area many models based on the diffusion approach have been developed particularly in forecasting, as in new product analysis. Some authors have also used these models extensively in technological forecasting applications. Despite the fact that at present diffusion models are a widely employed tool, their value compared to other forecasting techniques has not yet been well established. The aim of this paper is to carry out a comparative analysis on the descriptive and forecasting accuracy of the Box–Jenkins and diffusion models, on the basis of many different time series. The numerical procedures used in parameter calibration and the performance indexes employed in comparing the models' performance are explicitated. Some general conclusions regarding the conditions of diffusion models practical application are put forward.

Keywords: Forecasting; Model fitting; Diffusion models; Box–Jenkins method

1. Introduction

Diffusion models represent a powerful class of conceptual tools that have been used for various applications in a multitude of disciplines. Initially diffusion models were employed to describe the temporal spread of phenomena. In the economic perspective, for example, they are used to describe the spread of technological innovations. Some authors, e.g. Griliches (1957) and Mansfield (1961), have applied these models in an explanatory fashion, testing specific-based hypotheses.

Diffusion models have often been viewed as *normative* models. The normative approach is based on the assumption that there should be certain observed regularities in the data. The well known 'epidemic model' owes its notoriety to the fact that it represents the typical observed patterns by a simple S-shaped curve. In this view the epidemic model has given rise to a wide range of diffusion models. Normative models are used, for example, as a basis for planning commercial activities, and in considering the 'natural' growth shape of sales.

The most common application is perhaps in

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forecasting, particularly in the business area. In Marketing, e.g. new product analysis, many models based on the diffusion approach have been developed (Mahajan and Wind, 1986; Mahajan, Muller and Bass, 1990). In some cases these models have even shown themselves to be useful ex-ante (Easingwood, 1989) because they make it possible to take advantage of *analogies* with patterns of similar phenomena. Several authors, especially Martino (1983) have also utilized these models extensively for *technological forecasting* applications. Furthermore the diffusion models denote an interesting approach to the study of technical changes and evolutionary processes.

As is well known, there is at present a great need of new planning and forecasting tools in the business field (as in strategy formulation, policy making, purchasing, inventory control) particularly in the presence of turbulent environments. Despite the fact that diffusion models are today an important alternative approach to forecasting, their value compared to other forecasting techniques is not yet well established. In some cases the simplest diffusion models seem to work well (Bass, 1969; Coleman et al., 1966; Mansfield, 1961), in other cases the results are not as good (Bernhardt, 1970). Their success is normally tied to judicious choices of situation, type of innovation, population and time frame for analysing data. Note that in the literature these choices often have been made ex post, for successful innovations; but in operative contexts the choice of the forecasting model must be made ex-ante.

There is a virtual paucity of research on the validity of forecasts made using *diffusion models*. There are no systematic comparisons with best time-series techniques, particularly with the Box–Jenkins method (Mahajan and Peterson, 1985; Mahajan and Wind, 1986; Makridakis and Gardner, 1988).

The reason for this is the fact that the two classes of models have been developed within different research perspectives referring to different kinds of phenomena. Diffusion models originated from an industrial economics tradition, and only later they were applied in business economics; their development has been undertaken to try to improve the description of the temporal spread of new technologies. ARIMA models, instead, derive from mathematics and statistics tradition and can be applied to a great number of very different phenomena. The conditions in which they may be utilized are also different. Based on the literature one might expect that the ARIMA models to work well, as predictors, in the case of relatively stabilized phenomena, where there is a strong stocastic component, for which many data points are available (not less than 30, according to the basic work of Box and Jenkins, 1970). On the contrary one would expect the diffusion models to work well when dealing with the spread of a new product or technology. One would also expect a good fitting performance by ARIMA (Makridakis et al., 1984), but there are no sufficient comparative evaluations.

The aim of the paper is to compare, in an empirical way, the descriptive and the forecasting power of Box–Jenkins and diffusion models. The analysis is carried out on the basis of many time series, differentiated according to industry, country, technology and temporal horizon. The purpose is to contribute both in investigating the properties of these forecasting techniques and in formulating operative considerations on their performances and application fields.

2. Some conceptual problems in forecasting

Before going on to discuss the methods to be adopted in comparing the fitting and forecasting accuracy of Box–Jenkins and diffusion models, it is essential to recall some basic concepts on the use of forecasting algorithms. Any theory is based on several assumptions; if one of these assumptions is not realistic, the theory can predict events which are different from empirical observation. In the forecasting field there are often discrepancies between theoretical predictions and empirical results. "The major reason for such discrepancies is that some of the theoretical assumptions do not hold" (Makridakis et al., 1984).

Unfortunately there is no way to guarantee this stability. The habit some forecasters have of choosing the model with the largest correlation coefficient with respect to the available data as a basis for subsequent forecasts "is an extremely bad practice: forecasters are not concerned with how well a curve fits past data but, rather, with how well the curve will perform as a forecast" (Martino, 1983). Hence the most relevant aspect of forecasting "is to know the methods which can minimize the post-sample forecasting errors" (Makridakis et al., 1984).

Keeping this in mind, *empirical comparison* remains the fundamental methodology when different forecasting models are to be compared in an objective way. The empirical approach supplies a sound basis with which to investigate the accuracy of forecasting algorithms; furthermore it allows one to formulate various kinds of hypotheses, e.g. about the most appropriate methods for different classes of phenomena. From this point of view it is important to test the various forecasting models by means of many *different* time series, with respect to the length (number of data points) and the phenomenon from which they are obtained.

The choice of the time series which are to be analyzed is a delicate problem. Regarding the kind of phenomena and their measure unit, some authors suggest diffusion models should only be employed in the case of a single good unit–single adoption decision; but other authors do not agree. In reality diffusion models are largely employed in a broad meaning; the well known seminal work by Griliches on the hybrid corn did not consider single good units, but the new culture acreage percentage (a proxy of output amount) in different States as a measure of diffusion. In other recent works the diffusion of process innovation is evaluated through the output (e.g. in tons) made by new plants (see Poznansky, 1983; Meade, 1984) and there does not seem to be any major disagreement on this. Some authors even use diffusion models to study phenomena started long time before (e.g. electrical energy: Quaddus, 1986, Bodger and Tay, 1987; telephones: Young and Ord, 1989; tractors: Mar Molinero, 1980) and also there do not exist any objections.

Another critical aspect regards the length of the time series. It is well known that ARIMA models are totally unemployable, for theoretical reasons, in 'short' series, while diffusion models are more apt to represent the initial phases of the spread on the market of a new product. However, according to certain authors, ARIMA models provide acceptable results even when there are about 20 data points (see for example Lusk and Neves, 1984); on the other hand diffusion is often a very slow process which lasts many years (e.g. Rosemberg, 1976).

To conclude it can be said that in comparing ARIMA models with diffusion models, it seems important, in the interest of scientific analysis, to verify their performances empirically both with long and short time series describing different kinds of phenomena. Note also that the *ex ante* choice of only suitable data for each class of models could invalidate our analysis, the risk being that in this way one only ascertains what only initially intended to.

3. Diffusion models: A synthetical overview

Different types of forecasting methods are identified in literature. A first broad subdivision distinguishes between judgmental or *qualitative* methods (e.g. Delphi methods and so on) and more formal *quantitative* methods. Among the latter, another two classes are identifiable: *non-causal* and *causal* methods. The former includes time-series analysis techniques such as 'naive' methods, moving averages and so on; the latter encloses regression analysis, econometric models and other 'explanatory' techniques.

Although *diffusion models* in general could be classified as *causal models* (Lilien and Kotler, 1983; Martino, 1983) as they indeed attempt to take into account the process determinants, it is not correct to define their simplest releases (e.g. logistic models) as causal, in that they assume the *adopter imitative behavior* as the only force driving the diffusion process. So it is possible to define them as a 'deterministic' description, the generic S-shape of the time pattern being fixed. In the following pages the mathematical and descriptive features of some of the best known and most investigated S-shaped diffusion models are summarized (for further reading we suggest optimal and detailed overviews today available, for example Skiadas, 1985; here we refer however to the taxonomy suggested by Mahajan and Peterson, 1985).

The *basic or fundamental diffusion logistic model* can be expressed by the following differential equation:

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = g(t)\left[\bar{N} - N(t)\right] \tag{1}$$

where $N(t)$ is the cumulative number of adopters at time t; $\bar{N}$ is the total number of potential adopters, i.e. the ceiling of $N(t)$; $(\mathrm{d}N(t)/\mathrm{d}t)$ is the number of new adopters, i.e. the diffusion rate at t; $g(t)$ the *imitation or diffusion coefficient*.

The specific mathematical expression of $g(t)$ shapes the form of this 'deterministic' equation, settling, above all, its point of inflection, i.e. the maximum of the $\mathrm{d}N(t)/\mathrm{d}t$ function, and its symmetry, which is often considered in relation with the features of the imitative forces causing the diffusion process. If $g(t)$ is a constant, we obtain the so called *external-influence* diffusion model:

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = a\left[\overline{N} - N(t)\right]. \tag{2}$$

This model assumes that the information flowing from an external source is the diffusion agent. It was employed above all by Coleman et al. (1966) in studying new pharmaceutical products.

The *internal influence* model, so called because it considers the adoption as induced by interpersonal contacts, is based on the *epidemic paradigm*. Its mathematical form derives from the traditional biological studies on the spread of a disease through a population:

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = bN(t)\left[\overline{N} - N(t)\right], \tag{3}$$

and states that diffusion is a function of interactions between prior and potential adopters; it represents the *pure imitation* diffusion model. Integrating (3) we obtain a perfectly symmetric logistic curve.

The constant b, often inadequately named 'speed of diffusion', reflects the imitative force due to the communication channels operating within the reference population. Thanks to the seminal works of Griliches (1957) and Mansfield (1961), this is one of the best known and most applied diffusion models in the literature.

The *Gompertz function*, often employed in *technological forecasting* (see e.g. Martino, 1983) as a 'growth curve', may also be viewed as an internal diffusion model.

The *Bass mixed-influence diffusion model* (1969) is the 'simple' algebraic sum of the previous models:

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = \left[a + bN(t)\right]\left[\overline{N} - N(t)\right] \tag{4}$$

and takes into account both external and internal influence forces.

The shape of this generalised logistic curve is determined by both a and b whose ratio gives the relative weight of the two forces. Some forecasting applications of this model, especially with consumer durables, were studied by Bass (1969), Dodds (1973) and Tigert and Farivar (1981), giving quite satisfactory results.

Various important objections to the basic diffusion models can be raised: the number of potential adopters $\overline{N}$ is constant during the diffusion process; only one adoption is permitted; both coefficients, a and b, are constant in time so that the diffusion driving forces do not change (this in addition implies that the population is regarded as homogeneous) and consequently the innovation itself does not change during the diffusion process. In particular they are not 'flexible' in that their inflection point cannot occur at any moment of the diffusion process, they are either symmetric or non-symmetric because the diffusion speed is a constant; so their ability to represent many diffusion patterns is rather limited.

Hence, various models were developed to overcome this structural lack of flexibility. Based on some empirical diffusion curves, Floyd (1968), for example, suggested a model with a variable 'diffusion speed coefficient'.

Expressly from the need to overcome the time constancy of b, Easingwood, Mahajan and Muller (1981) proposed the NSRL (Nonsymmetric Responding Logistic) model:

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = B\left[N(t)/\overline{N}\right]^{\delta}\left[\overline{N} - N(t)\right]. \tag{5}$$

This curve may be made symmetric or nonsymmetric, with an inflection point varying between zero and one; in addition $b(t) = (B/\overline{N}^{\delta})N(t)^{\delta-1}$ can decrease or increase with time. Hence it is the most flexible model so far examined.

Another important reducing assumption of the fundamental model is the *time constancy of* $\overline{N}$. Hence other models, not analyzed here, attempt to make $\overline{N}$ 'dynamic' putting, for example, $\overline{N}(t) = f[S(t)]$ where $S(t)$ is a vector of variables affecting $\overline{N}(t)$. Some 'dynamic' models were developed by Mahajan and Peterson (1978), Sharif and Ramanathan (1981) and Kalish (1985).

In the literature more complex and advanced diffusion models which take into account other meaningful features of diffusion processes (e.g. repurchasing, multiple adoption and so on) have been developed: in this paper we have chosen to analyse only the models previously described because they are the best known, the easiest to employ, and "one of the more heavily used forecasting techniques in corporate environment" (Meade, 1984).

4. The Box–Jenkins approach

The widely known Box–Jenkins method is today the most general and perhaps the most powerful forecasting technique available. Indeed it can be employed to realize an appropriate model to analyse any set of data. This 'flexibility' makes it a reference point for comparing and analysing the performance of other forecasting techniques.

Referring to the basic work of Box and Jenkins (1970) for a more in-depth treatment of the theoretical foundations of this method, in the following pages the iterative approach which, starting from a real pattern of data, leads to the building of the actual forecasting model will be dwelt upon.

The generic form of an ARIMA (p, d, q), where p is the order of the AR part, q the order of the MA part, and d the level of differentiation that produces stationarity, is

$$a_t = \hat{w}_t - \Phi_1 \hat{w}_{t-1} - \dots - \Phi_p \hat{w}_{t-p} + \theta_1 a_{t-1} + \dots + \theta_q a_{t-q}, \qquad (6)$$

with $\hat{w}_t = w_t - \overline{w}$, $w_t = \nabla^{d*} z_t$, ∇ the derivative operator, $E[w_t] = \overline{w}$, and z_t represents the time series.

Usually the simplest processes, with a limited number of parameters, generally four, i.e. $0 \leq p \leq 2$, $2 \leq d \leq 2$, $0 \leq q \leq 2$, are adequate in a large number of situations.

Building an ARIMA model requires the following three steps:
1. Identification (establishing p, d and q).
2. Estimation of Φ and Θ parameters.
3. Diagnostic checking.

The first stage is the most difficult and delicate one, as there is no existing deterministic approach with which to tackle it. Hence the basic task of this step is to focus on a class of models which will then be fitted and controlled, in other words to obtain some information on p, d and q.

The parameter estimation step, which is today fully computerized, may be divided into two stages. The first consists of a preliminary estimation of the parameters by a linear system where the previously estimated autocorrelations are the fixed terms. Then these preliminary values become the input of a 'least squares' procedure that computes the final identification.

The third step involves the adequacy control, regarding the specific data series under examination, of the selected model. In this stage a great number of statistical test can be employed, each of them takes into account a specific feature of the model. The statistical procedure proposed by Box and Jenkins is based on the comparison of the function of the residual autocorrelations with the chi-square distribution. If the diagnostic checking is not passed, the user will have to return to the first step and start the building procedure again.

5. Comparing Box–Jenkins with diffusion models: Design of the study

In this section, procedures and indexes used to compare the descriptive and forecasting performance of the previously described diffusion models with the Box–Jenkins method are briefly summarized.

Initially the procedures used to obtain the best estimation of the model parameters are taken up (a general survey on this problem, for diffusion models, is presented by Mahajan and Wind, 1986).

In the case of diffusion models a computational sub-routine from the IMSL Library (Digital E.C.) named ZXSSQ, has been utilized. ZXSSQ is based on a modification of the Levenberg–Marquardt algorithm which eliminates the need for explicit derivatives. For other references, in addition to IMSL Library, see Brown and Dennis (1972), Levenberg (1984) and Marquardt (1963). Other elements regarding this technique are given in the Appendix.

The procedure yields the curve fitting and computes the coefficient of correlations R^2 and the sum of square errors SSE:

$$SSE = \sum_{1}^{N} {}_i(S_i - Y_i)^2 \quad (7)$$

where i is the i-th of the N values considered, Y_i is the computed value and S_i is the actual value. They are the indicators chosen to evaluate the model's fitting performance.

As far as the calibration of the Box–Jenkins method parameters is concerned, the fully computerised routine we have employed is more complex.

Once an ARIMA (p, d, q) model has been selected, the NSPE (Non-Seasonal Preliminary Estimation) subroutine of the IMSL-Digital Library computes the prior estimates of parameters. Successively, employing the values previously yielded by NSPE as input, the NSLSE (Non-Seasonal Least Estimation) subroutine (from the IMLS Library too) computes the least squares estimates. When the identification and estimation phases are completed, the procedure goes on with the diagnostic control step, using the chi-square test. This step is followed by the fitting and the R^2 and SSE computation.

The procedure we have adopted in evaluating the *reliability* of forecasting methods, is similar to that used in other works (see e.g., Makridakis and Wheelwright, 1987). With a time series containing n observations, the identification of the model being studied is carried out on the first m observations, reserving the remaining $n - m$ to the comparison with the corresponding outputs of the model on different forecasting horizons. For $n - m = 1$ a forecast for the following period is obtained, for $n - m = 2$ another one-period forecast

Table 1
List of time series employed

Dynamic Ram 4-K (74–85)	S.G.S.-Thompson
Dynamic Ram 16-K (76–85)	S.G.S.-Thompson
Dynamic Ram 64-K (79–85)	S.G.S.-Thompson
Worldwide Dynamic RAM (74–85)	S.G.S.-Thompson
Steel produced by oxygen process FRANCE (60–80)	Poznanski (1983)
Steel produced by oxygen process JAPAN (58–80)	Poznanski (1983)
Steel produced by oxygen process SPAIN (63–80)	Poznanski (1983)
Steel produced by oxygen process USA (55–80)	Poznanski (1983)
Steel produced by oxygen process WEST G. (57–78)	Poznanski (1983)
Steel produced by oxygen process WORLD (60–78)	Poznanski (1983)
C.A.T. head scanner USA (72–78)	Easingwood et al. (1981)
C.A.T. body scanner USA (73–78)	Easingwood et al. (1981)
Optical scanner systems USA (74–79)	Tigert and Farivar (1981)
Worldwide photovoltaic modules (76–87)	Strategies Unlimited
Integrated circuits (70–85)	S.G.S.-Thompson
Peripheral equipment ITALY (76–86)	I.S.T.A.T.
Color television ITALY (74–86)	I.S.T.A.T.
Propylenical ITALY (67–86)	I.S.T.A.T.
Polyester ITALY (51–76)	I.S.T.A.T.
Car's production JAPAN (54–85)	I.S.T.A.T.
Shuttle-looms GREECE (74–81)	O.C.D.E.
Shuttle-looms UNITED KINGDOM (74–81)	O.C.D.E.
Shuttle-looms WEST GERMANY (74–81)	O.C.D.E.
Shuttle-looms ITALY (74–81)	O.C.D.E.
Telephone diffusion ITALY (30–42)	STET Group
Telephone diffusion ITALY (48–65)	STET Group
Telephone diffusion ITALY (65–80)	STET Group
Electric power SINGAPORE (61–82)	Quaddus (1986)
Electric power JAPAN (69–85)	I.S.T.A.T.
Telex SINGAPORE (66–82)	Quaddus (1986)
Acrylic fibers ITALY (72–86)	I.S.T.A.T.
Synthetic fibers ITALY (47–83)	I.S.T.A.T.
Float glass ITALY (53–86)	I.S.T.A.T.

and a two-period forecast are obtained and so forth.

Therefore from one series it is possible to obtain $n-p$ one-period forecasts, $n-p-1$ two-period forecasts and so forth, where p is the lowest number of samples used in the identification. This allows one to evaluate the accuracy of a large number of forecasts (for example 30 series yielded more than 350 one-period forecasts).

In the literature different performance measures of forecasting accuracy are used: the best known and most employed are the Average Squared Error (ASE) and the Mean Absolute Percentage Error (MAPE).

Following Makridakis (1984), our performance analysis will be carried out on the basis of the MAPE:

$$\text{MAPE}_i = \frac{1}{i} \sum_{k=1}^{i} {}_k \frac{|S_{n-k+1} - Y_{n-k+1}|}{S_{n-k+1}} *100,$$
$$i = 1, \dots, \text{MAX}, \quad (8)$$

where MAPE_i is the MAPE concerning the temporal horizon of length i, Y_{n-k+1} is the predicted value at $n-k+1$, S_{n-k+1} is the 'real' value at $n-k+1$ and MAX is the temporal horizon's greatest length.

By means of the procedure previously described a great number of forecasts (and MAPE-values) have been generated. The average MAPE computation and table presentation are carried out by a suitable computer subroutine.

The analysis was conducted on 33 time series collected from literature on diffusion, and therefore previously employed by other authors, and from national and international statistical sources (OCDE, ISTAT, etc.). For the reasons which were mentioned in Section 2, they were selected considering a set of different process and product technologies which are spreading into different geographical and social environments. Data belong to various industrial sectors (basis industry, manufacturing, chemical, electronics and biomedical industry, public services) and concern several phases of the diffusion process, from the initial stages to the complete spread (see Table 1). The series are also different as far as length and patterns are concerned.[1] Note that, compared to what is suggested in the literature, diffusion models are also fitted to many data (the longest series have more than 20 data points) and the ARIMA to few (less than 11 data points in the shortest series). This is done in the interest of scientific analysis as stated before.

6. Main results

6.1. Model fitting

In the following pages the performances of the single diffusion models are compared; then the comparison with Box–Jenkins is carried out.

We attempted to put together the advantages of flexible and dynamic models (see Section 3) by eliminating the hypotheses of constancy of the population and of diffusion rate. In this way it is possible to develop a new model whose derivative expression is:

$$\frac{dN(t)}{dt} = BN(t)^{\beta+1}\left[\bar{N}^*N(t)^{\delta} - N(t)\right], \quad (9)$$

where the parameters to be estimated are four in number. Note that (9) becomes NSRL if $\delta = 0$. By means of (9), which is indicated in the following as SSDFM[2] the effects on the fitting and the forecasting power of an additional parameter can be evaluated.

Among diffusion models, NSRL and SSDFM, which in practice are often indistinguishable, appear to be the best fitting models in 19 cases out of a total of 33 time series. In the remaining 14 cases they give the second best results. A very good fitting ability is provided by the Bass (Mixed) model[3], which generates the lowest SSE value in 14 cases. In some series, Floyd and Gompertz models SSE values are comparable with the best one, but generally they are found to be at least one order higher. The epidemic model, here referred to as Dodd model, clearly gives the worst

[1] A copy of all series used in this paper is available from the authors on request. All but one series have one data point per year; the remaining one does not show seasonality.

[2] S-shaped Dynamic and Flexible Diffusion Model. Some analytical consideration on the characteristics of this model are in Gottardi (1988).

[3] The Bass model gives good results mainly in fitting the diffusion patterns in the public service sectors. This is probably due to the fact that the action of 'external agents' is taken into account in its mathematical formulation.

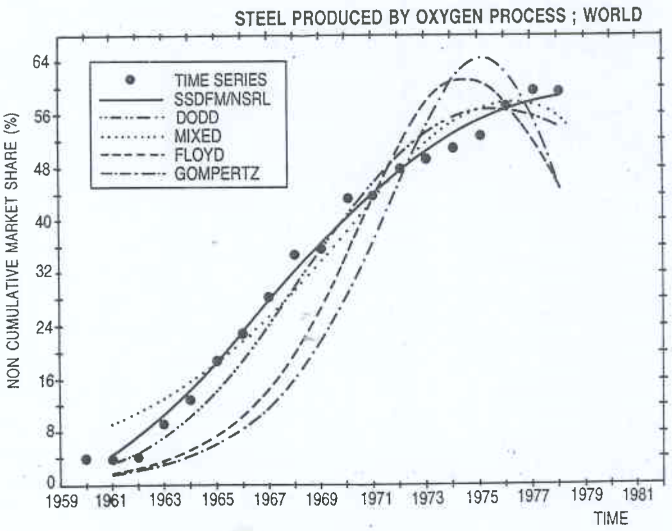

Figure 1

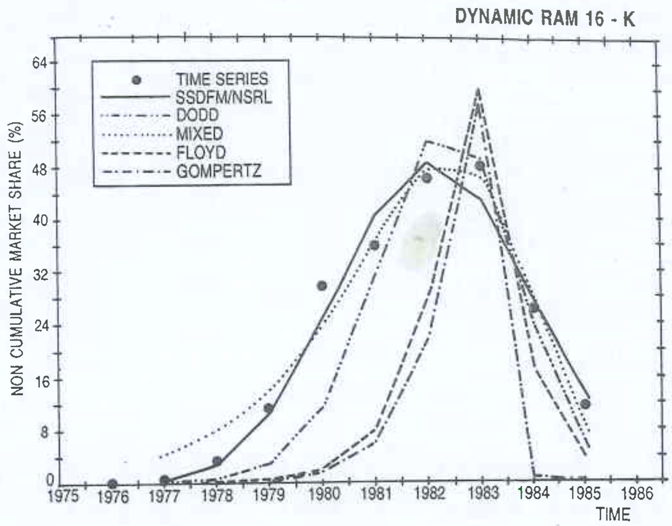

Figure 2

results, yielding the highest SSE and the lowest R^2 no less than 31 times (see Table 7 for the complete fitting comparison results).

On the basis of these results NSRL, SSDFM and Mixed models supply a *description* of the technological diffusion process which is better than other logistic models considered here. The descriptive ability of the Mixed model proves particularly good, especially in comparison with more flexible ones; also Gompertz shows a relatively good fitting performance. Figures 1 and 2 report some results on fitting comparation patterns.

Proceeding with the comparative evaluation of the Box–Jenkins method, our analysis displays its quite evident inferiority in descriptive power when referred to diffusion models. Only in 14 cases in fact it is equivalent to the latter; in the remaining cases it presents a much higher SSE-value.

According to the theory, this approach does not permit a good identification with length limited time series: this explains the improved fitting accuracy which can be observed when the number of data points increases (see Table 2, the worst models are omitted). Referring to fitting outcomes, this technique is however still employable when time series have a 15 data point length at least. This is certainly less than is usually indicated in the literature (as previously recalled, other authors' empirical results, e.g. Lusk and Neves, 1984, suggest about 20 data points). Figures 3 and 4 report some fitting comparisons between Box–Jenkins and the best working diffusion models.

6.2. Forecasting accuracy

To analyse the forecasting accuracy of the models only 30 time series have been employed, excluding the shortest ones for obvious reasons. The detailed results of the analysis are represented by the values of average MAPE which are

Table 2
Average values of R^2 for number of data points in different models

Model	Time series data points					
	$0 < n \leq 10$ (8) [a]	$10 < n \leq 15$ (7) [a]	$15 < n \leq 20$ (8) [a]	$20 < n \leq 25$ (4) [a]	$25 < n \leq 30$ (3) [a]	$n > 30$ (3) [a]
NSRL/SSDFM	0.966	0.947	0.965	0.993	0.985	0.971
Mixed	0.967	0.960	0.963	0.977	0.978	0.953
Gompertz	0.916	0.886	0.910	0.969	0.984	0.953
Box–Jenkins	0.706	0.871	0.950	0.985	0.973	0.974

[a] Number of series

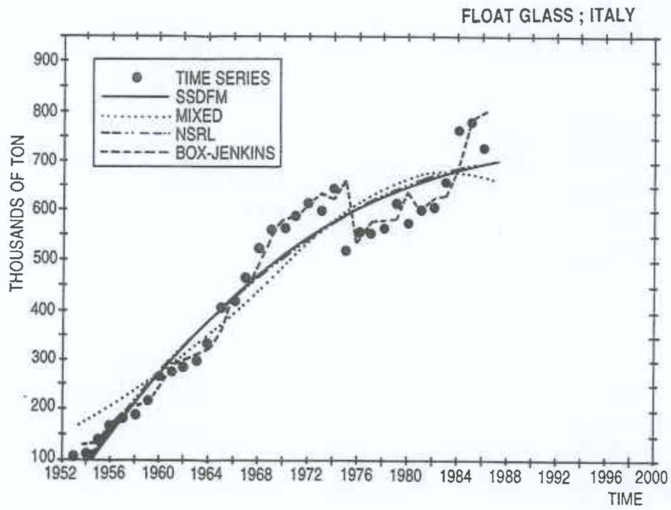

Figure 3

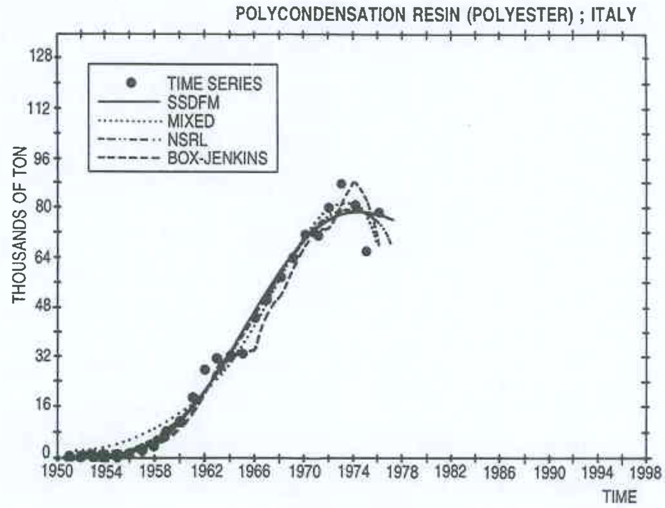

Figure 4

Table 3
Number of times in which each diffusion model turns out to be the best, according to MAPE, for different temporal horizons

	Temporal horizon							
	1 (30) [a]	2 (30) [a]	3 (26) [a]	4 (25) [a]	5 (25) [a]	6 (23) [a]	7 (21) [a]	8 (19) [a]
NSRL	17	14	12	12	16	13	11	8
Mixed	9	8	7	5	4	4	3	4
Gompertz	4	7	6	7	5	6	7	7
Floyd	–	1	1	1	–	–	–	–
Dodd	–	–	–	–	–	–	–	–

[a] Number of series analysed

Table 4
A comparison of relative MAPE-values (Box–Jenkins MAPE-values = 100) for time series with 13 data points or less

	Temporal horizon					
	1 (11) [a]	2 (11) [a]	3 (4) [a]	4 (4) [a]	5 (4) [a]	6 (4) [a]
NSRL	55.31	65.25	33.35	47.31	65.90	37.30
Mixed	66.96	77.97	53.04	77.42	107.28	62.34
Gompertz	74.57	82.57	49.23	61.96	90.28	52.73
Box–Jenkins	100.00	100.00	100.00	100.00	100.00	100.00

[a] Number of series analysed

Table 5
A comparison of relative MAPE-values (Box–Jenkins MAPE-values = 100) for time series with more than 13 data points

	Temporal horizon							
	1 (19) [a]	2 (19) [a]	3 (19) [a]	4 (19) [a]	5 (19) [a]	6 (19) [a]	7 (19) [a]	8 (19) [a]
NSRL	82.46	94.67	96.64	102.04	103.53	115.50	124.50	147.10
Mixed	102.83	119.70	135.03	152.22	160.09	178.45	198.30	215.58
Gompertz	133.46	140.66	155.58	154.60	158.44	164.03	176.99	198.65
Box–Jenkins	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

[a] Number of series analysed.

obtained from every model and time series for the different temporal horizons. These data are shown in Table 8. A preliminary analysis reveals that the MAPE-values obtained correspond completely to the typical size of the forecasting error (see Makridakis, 1986, p. 20).

To make comparison of the models and understanding of the results easier, some simple elaborations have been provided. Firstly the diffusion models are considered. Table 3 shows the number of times in which each model turns out to be the best, according to MAPE. Regarding this class of models, a substantial confirmation of the previously resulting performances can be observed. NSRL is relatively better; its performance are almost equal to SSDFM (see Table 9). Mixed and Gompertz give alternate results which are however on the whole good. As is clearly shown, unsatisfactory results were instead obtained from the Dodd and Floyd models.

Including Box–Jenkins in this comparative analysis, NSRL remains the best, except in the case of very long temporal horizons, while mixed and Gompertz supply results which are better than Box–Jenkins only in the case of short time series (13 data points or less, see Table 4). For a detailed analysis see Table 8.

As the temporal horizon increases, Box–Jenkins offers relatively superior results. A careful investigation on the basis of MAPE-values shows that Box–Jenkins degrades itself, enlarging the forecasting horizon, less quickly than diffusion models (see Table 10 for further details). Similar results in comparing Box–Jenkins with the simplest forecasting method were found by Makridakis et al. (1984).[4]

According to what previously recalled, the Box–Jenkins method requires a larger number of data points in carrying out a proper parameter calibration. The effect of length of series on the forecasting accuracy of all the best models can be verified in detail in Tables 4 and 5

[4] These model features seem to be confirmed by an analysis of the single time series. In fact diffusion models' forecasting accuracy results are more sensitive to the presence of 'exogenous noise' (i.e. non-predictable changes in the economic enviroment) which produce 'false inflection points' upon the basic S-shaped pattern.

7. Discussion of results and conclusions

The most common use of diffusion models is in forecasting, particularly in technology spread analysis and in business applications. Despite the fact they are widely used, their performance compared to other forecasting methods has not yet been well investigated. In this paper an empirical analysis on many time series, aimed at evaluating the performances of this class of models compared to the class of ARIMA and in particular Box–Jenkins models, has been carried out.

Our results seem to agree with the considerations of Makridakis and Gardner (1988) and regard in particular:

a) the opportunity of a distinction between model fitting and model forecasting performance;

b) the absence of a clear link between model complexity and forecasting accuracy (and therefore between its 'cost', in terms of time and scientific know-how, and its performances);

c) the fact that the models which may supply accurate forecasts in the short term do not necessarily do so in the long term.

Moreover the results allow to evaluate the effects of the length of series on the model descriptive power and on forecasting accuracy.

The results can be summarized in the following considerations. Regarding the performance of the main existing diffusion models, it can be said that from the classic epidemic model to the dynamic or flexible ones proposed in recent literature, there has been considerable improvement. The *descriptive power* of NSRL and Mixed is decidedly good, systematically better than that obtained using Box–Jenkins techniques. This is not stated explicitly in the literature. Employing a four-parameter diffusion model (SSDFM) one obtains R^2-values substantially identical to the ones given by NSRL, where there are three parameters. Furthermore it can be affirmed that, compared to Box–Jenkins, Gompertz also offers a relatively good fitting (remember that this model requests the estimation of only two parameters).

It can be said that in providing a descriptive and explanatory analysis of growth and diffusion processes, the deterministic and causal framework characterizing diffusion models permits better results than the Box–Jenkins approach. These results are confirmed especially when data points are scarce, as clearly showed in Fig. 5. This could

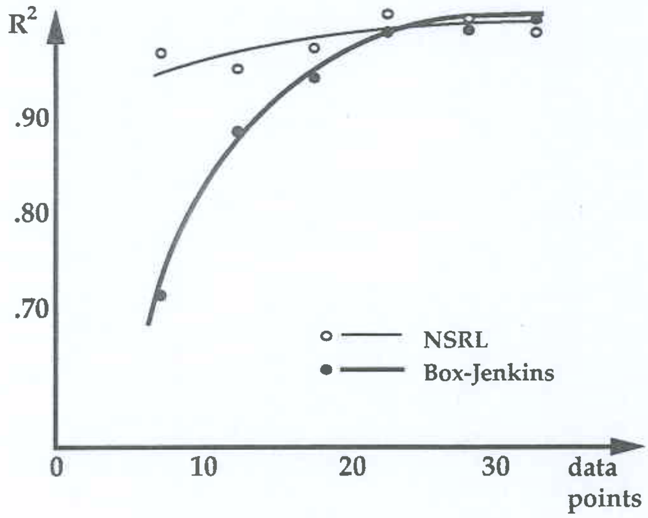

Figure 5

be explained remembering that diffusion models only require an estimation of the best fit parameters (as the function is predetermined) whereas the Box–Jenkins method requires a preliminary identification of the most appropriate model. Still, from a descriptive point of view, diffusion models also supply important information on the dynamic characteristics of spreading processes: because of their causal foundation, their parameters describe and measure particular features of diffusion, once they have been calibrated by means of ordinary numerical computation techniques.

As regards the *forecasting accuracy*, NSRL offers the best performance, Mixed and Gompertz give alternate results, and Dodds and Floyd appear unsatisfactory. NSRL remains the best also when Box–Jenkins is considered (Tables 4 and 5), except in the case of very long forecasting horizon, while Mixed and Gompertz are systematically better than Box–Jenkins in the time series with less than 14 data points (see Table 8).

An important aspect to underline is the model forecasting *reliability*, which we may define as the inverse of the number of times in which each model is found to be the worst. This is quite low for Box–Jenkins and very high for NSRL (Table 6). Compared to a four-parameter model (SSDFM), NSRL is the best or second best and at any rate offers satisfying results. Considering cost/effectiveness rate, we can affirm in conclusion that NSRL remains a very good tool.

The reliability of Mixed is comparable with that of Gompertz, and both are quite low. The performance of these latter models is however clearly better than that of Box–Jenkins *in short time series* (see Table 5).

An important consequence in comparing the methods in discussion derives from the fact that in cases of economic interest the data points available are scarce. This can cause serious problems in the use of statistically based techniques. In fact, even if Box–Jenkins is utilizable in practice with less than 30 data points (as also demon-

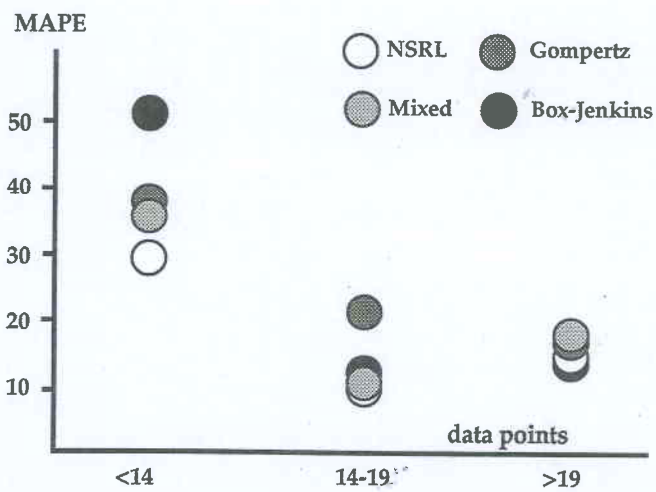

Figure 6. Average MAPE-values for different series lengths

Table 6
Number of times in which each model turns out to be the worst, according to MAPE, for different temporal horizons

	1 (30) [a]	2 (30) [a]	3 (26) [a]	4 (25) [a]	5 (25) [a]	6 (23) [a]	7 (21) [a]	8 (19) [a]
NSRL	1	1	1	–	–	1	1	1
Mixed	8	11	13	13	12	10	9	10
Gompertz	10	10	9	8	8	8	8	6
Box–Jenkins	11	8	3	4	5	4	3	2

[a] Number of series.

strated by other authors), the number of data points which are needed to calibrate the parameters remains however greater than that of the diffusion models; furthermore this approach requires pre-identification of the model. Hence, when time series are too short, Box–Jenkins becomes unreliable. This is confirmed by our results, which are summarized in Fig. 6: in the cases of short time series (less than 14 samples) the diffusion models give systematically better forecasting accuracy.

After all the results seen to be quite insensible to the kind of phenomenon (new consumer durables, innovative process technologies or services), the most meaningful factor being the length of the series. Keeping in mind that the practical utility of the forecasts is greater when there are few data points available and decreases as these increase, diffusion models could, from this point of view, prove to be irreplaceable instruments.

The last comment regards the temporal horizon of the forecast. The analysis which has been carried out confirms that minimizing the model fitting errors can guarantee fewer errors in forecasting only if structural changes in the data do not occur. In particular Box–Jenkins seems to supply a relatively better forecasting accuracy in the long term. The analysis of the results shows that its accuracy degrades less quickly than the diffusion models as temporal horizon increases. This effect could be explained remembering the deterministic nature of diffusion models: this makes them much more sensitive to structural modifications in the data, which are certainly more probable in the long run.

Appendix

Numerical procedures employed in parameters estimation of diffusion models.

The purpose of the ZZSSQ procedure is to minimize the sum of squares of M functions in N variables using a finite difference Levenberg–Marquardt algorithm, that is, solve a nonlinear least squares problem. The problem is stated as follows: given M nonlinear functions $f_1, f_2, \dots, F_M$ of a vector parameter X, minimise over X:

$$f_1(X)^2 + f_2(X)^2 + \dots + f_M(X)^2,$$

where $X = (X_1, X_2, \dots, X_N)$ is a vector of N unknown parameters to be estimated. When fitting a non-linear model to data, the function f_i should be defined as follows:

$$f_i(X) = y_i - g(X, V^i), \quad i = 1, 2, \dots, M,$$

where y_i is the i-th observation of the dependent variable, $V^i = (v_1, v_2, \dots, v_{NV})$ is a vector containing the i-th observation of the NV independent variables, and g is the function defining the non-linear model.

This subroutine utilizes three convergence criteria: the first is satisfied if on two successive iterations, the parameter estimates agree, component by component to K digits. The second is satisfied if, on two successive iterations, the residual sum of squares estimates have relative differences less than or equal to r; r may be set to zero. The third convergence criterion is satisfied if the norm of the approximate gradient is less than or equal to r'; r' may be set to zero.

The user has to choose and introduce proper convergence criteria values; moreover he has to define the maximum number of iterations permitted. The iteration is terminated and convergence is considered achieved if any one of the three conditions is satisfied or if the maximum iteration number is performed. Subroutine output is a vector with final estimates and some indexes revealing that convergence has been achieved and, if it is reached, which criterion was satisfied; otherwise, what kind of error there has been.

ZXSSQ needs a previous empirical identification of the parameters, because it could not converge if the starting values are too far from the resulting estimates. The lack of a general method to deal with the pre-identification problems (the simple algebraic estimation technique suggested by Mahajan and Sharma, 1986, does not work with all models and with all kinds of data) was solved allowing the operator to make various trials: if convergence is not achieved, he can modify either the pre-identification values or the stopping criteria.

Table 7
Values of R^2 and SSE (SSE in parentheses)

Time series	NSRL	SSDFM	Dodd	Mixed	Floyd	Gompertz	Box–Jenkins
Dynamic Ram	0.99567	0.99567	0.64977	0.96647	0.85536	0.96914	0.63747
4-K	(0.7164E+03)	(0.7164E+03)	(0.7973E+05)	(0.4999E+04)	(0.3198E+05)	(0.7353E+04)	(0.5773E+05)
Dynamic Ram	0.97076	0.97073	0.00146	0.97051	0.68951	0.89838	0.65209
16-K	(0.8929E+06)	(0.8929E+06)	(0.6586E+08)	(0.9559E+06)	(0.2011E+08)	(0.4978E+07)	(0.1250E+08)
Dynamic Ram	0.99980	0.99980	0.24340	0.99904	0.24340	0.99719	0.58557
64-K	(0.5425E+06)	(0.5425E+06)	(0.2838E+10)	(0.3718E+07)	(0.2838E+10)	(0.1123E+07)	(0.2176E+10)
Worldwide	0.99973	0.99973	0.89040	0.99977	0.99902	0.99173	0.95072
Dynamic Ram	(0.3020E+07)	(0.3020E+07)	(0.1122E+10)	(0.2308E+07)	(0.1356E+08)	(0.7178E+08)	(0.1795E+10)
Steel prod. by oxyg.	0.98631	0.98631	0.91416	0.98819	0.93524	0.98560	0.97747
France	(0.2485E+03)	(0.2485E+03)	(0.3140E+04)	(0.1970E+03)	(0.2252E+04)	(0.3643E+03)	(0.3801E+03)
Steel prod. by oxyg.	0.99440	0.99655	0.62350	0.94409	0.71624	0.91305	0.98474
Japan	(0.9493E+02)	(0.5564E+02)	(0.1876E+05)	(0.9464E+03)	(0.1268E+04)	(0.2894E+04)	(0.3925E+03)
Steel prod. by oxyg.	0.98429	0.98506	0.55707	0.97758	0.68276	0.91926	0.97063
Spain	(0.1136E+03)	(0.1144E+03)	(0.8560E+04)	(0.1308E+03)	(0.5892E+04)	(0.1457E+04)	(0.3984E+03)
Steel prod. by oxyg.	0.97330	0.97828	0.78306	0.95531	0.85828	0.97720	0.95705
USA	(0.4115E+03)	(0.3276E+03)	(0.4936E+04)	(0.7271E+03)	(0.2903E+04)	(0.3398E+03)	(0.6646E+03)
Steel prod. by oxyg.	0.99505	0.99710	0.77852	0.98226	0.84922	0.99101	0.99071
West G.	(0.9073E+02)	(0.5209E+02)	(0.7032E+04)	(0.3647E+03)	(0.4434E+04)	(0.2094E+03)	(0.1668E+03)
Steel prod. by oxyg.	0.99309	0.99486	0.85620	0.97230	0.88723	0.97760	0.98692
World	(0.5337E+02)	(0.3884E+02)	(0.2076E+04)	(0.2090E+03)	(0.1439E+04)	(0.1985E+03)	(0.1027E+03)
C.A.T. body scanner	0.98728	0.99185	0.9133	0.97180	0.96996	0.97277	0.41699
USA	(0.4824E+01)	(0.3243E+01)	(0.2786E+02)	(0.9415E+01)	(0.9061E+01)	(0.1347E+02)	(0.2595E+03)
C.A.T. head scanner	0.95703	0.95324	0.61752	0.91785	0.84392	0.95340	0.35437
USA	(0.1486E+02)	(0.1340E+02)	(0.1583E+03)	(0.2482E+02)	(0.5189E+02)	(0.1401E+02)	(0.2975E+03)
Opt. scanner systems	0.99327	0.99327	0.99350	0.99304	0.99331	0.98619	0.98835
USA	(0.1181E+04)	(0.1181E+04)	(0.1189E+04)	(0.1155E+04)	(0.1184E+04)	(0.2016E+04)	(0.2083E+04)
Worldwide	0.99202	0.99902	0.99204	0.99612	0.95145	0.99866	0.97218
photovoltaic modules	(0.50174+00)	(0.50174+00)	(0.6918E+01)	(0.2276E+01)	(0.4460E+02)	(0.8355E+00)	(0.5390E+00)
Integrated circuits	0.99240	0.99240	0.98716	0.98716	0.98337	0.95862	0.90064
World	(0.5763E+09)	(0.5763E+09)	(0.8329E+09)	(0.8330E+09)	(0.1074E+10)	(0.2696E+10)	(0.1037E+11)
Peripheral equipment	0.97350	0.97350	0.95137	0.97517	0.95301	0.96752	0.91593
Italy	(0.4362E+05)	(0.4363E+05)	(0.1326E+06)	(0.4023E+05)	(0.1221E+06)	(0.6147E+05)	(0.1636E+06)

Colour television	0.91359	0.91559	0.67732	0.87354	0.69074	0.80488	0.75514
Italy	(0.1825E+06)	(0.1781E+06)	(0.1781E+07)	(0.2663E+06)	(0.1316E+07)	(0.5707E+06)	(0.6738E+06)
Propylenical	0.84380	0.84380	0.69039	0.82835	0.68643	0.78069	0.83582
Italy	(0.3144E+05)	(0.3144E+05)	(0.1208E+06)	(0.3462+05)	(0.1045E+06)	(0.5296E+05)	(0.3451E+05)
Polyester	0.98505	0.98291	0.79811	0.98179	0.86136	0.97595	0.96686
Italy	(0.3768E+03)	(0.4299E+03)	(0.8754E+04)	(0.4951E+03)	(0.5686E+04)	(0.5589E+03)	(0.8854E+03)
Car's production	0.98713	0.98907	0.81378	0.97716	0.85885	0.98306	0.98459
Japan	(0.3159E+07)	(0.2626E+07)	(0.7056E+08)	(0.6000E+07)	(0.4971E+08)	(0.4168E+07)	(0.3717E+07)
Shuttle-looms	0.97576	0.97579	0.58227	0.98900	0.89714	0.94758	0.95379
Greece	(0.3655E−01)	(0.3657E−01)	(0.3151E+01)	(0.1640E−01)	(0.4160E+00)	(0.1373E+00)	(0.1339E+00)
Shuttle-looms	0.90484	0.90985	0.58934	0.87598	0.63584	0.66605	0.81856
United Kingdom	(0.8922E+00)	(0.8448E+00)	(0.8909E+02)	(0.1165E+01)	(0.9342E+01)	(0.5406E+01)	(0.4056E+01)
Shuttle-looms	0.98312	0.98315	0.88460	0.98663	0.97556	0.98170	0.93184
West Germany	(0.1250E+01)	(0.1251E+01)	(0.2037E+02)	(0.9811E+00)	(0.2964E+00)	(0.1502E+00)	(0.5836E+01)
Shuttle-looms	0.94299	0.94299	0.89797	0.96825	0.87841	0.91454	0.96939
Italy	(0.3387E+01)	(0.3387E+01)	(0.2992E+02)	(0.1819E+01)	(0.1203E+02)	(0.6434E+01)	(0.3206E+01)
Telephone diffusion	0.86400	0.86402	0.82924	0.97413	0.62430	0.67928	0.98487
Italy (30–42)	(0.1573E−01)	(0.1573E−01)	(0.4206E+00)	(0.2934E−02)	(0.1623E+00)	(0.8770E−01)	(0.2767E−02)
Telephone diffusion	0.99461	0.99461	0.88602	0.99942	0.96323	0.98490	0.99280
Italy (48–65)	(0.1525E+00)	(0.1525E+00)	(0.1198E+02)	(0.1335E−01)	(0.2491E+01)	(0.7203E+00)	(0.6606E+01)
Telephone diffusion	0.97340	0.97340	0.84012	0.99851	0.86909	0.91289	0.99879
Italy (65–80)	(0.3273E+01)	(0.3273E+01)	(0.1919E+03)	(0.1677E+00)	(0.5293E+02)	(0.2247E+02)	(0.5671E+00)
Electric power	0.99623	0.99623	0.97670	0.99759	0.97901	0.99412	0.99657
Singapore	(0.4820E+06)	(0.4820E+06)	(0.7552E+07)	(0.2852E+06)	(0.6212E+07)	(0.1231E+07)	(0.1168E+07)
Electric power	0.92959	0.92912	0.68478	0.94151	0.68248	0.74737	0.91914
Japan	(0.9101E+04)	(0.9159E+04)	(0.3881E+07)	(0.7556E+04)	(0.2897E+06)	(0.1312E+06)	(0.1281E+05)
Telex	0.99955	0.99955	0.99785	0.99962	0.99796	0.99949	0.99650
Singapore	(0.8180E+05)	(0.8180E+05)	(0.6322E+06)	(0.6071E+05)	(0.5891E+06)	(0.1044E+06)	(0.2294E+07)
Acrylic fibers	0.87931	0.87931	0.77808	0.93163	0.76026	0.80050	0.87895
Italy	(0.6561E+04)	(0.6561E+04)	(0.3806E+05)	(0.3611E+04)	(0.3405E+05)	(0.1830E+05)	(0.6429E+04)
Synthetic fibers	0.98167	0.98429	0.80986	0.97316	0.85507	0.98246	0.97352
Italy	(0.2330E+05)	(0.1961E+05)	(0.3356E+06)	(0.3622E+05)	(0.2364E+06)	(0.2175E+05)	(0.3442E+05)
Float glass	0.93823	0.94007	0.70898	0.90852	0.75444	0.89267	0.96458
Italy	(0.8161E+05)	(0.7926E+05)	(0.1024E+07)	(0.1227E+06)	(0.7201E+06)	(0.2130E+06)	(0.4905E+05)

Table 8
Average MAPE-values of various models for different number of data points and time horizons

Number of data points considered	Model	Temporal horizon							
		1	2	3	4	5	6	7	8
≥ 20 (11 series)	NSRL	11.012	13.221	15.813	17.670	19.830	23.612	26.446	32.788
	SSDFM	9.395	12.011	14.425	15.970	17.573	20.438	23.060	26.870
	Dodd	45.707	54.191	62.245	68.169	73.182	77.329	80.079	82.571
	Mixed	14.198	18.831	23.538	27.652	32.395	38.347	44.590	50.705
	Gompertz	13.780	16.133	18.060	20.020	21.772	22.624	26.885	29.688
	Floyd	34.393	39.934	45.069	49.742	50.038	58.541	61.812	65.001
	Box–Jenkins	11.804	13.542	14.476	14.735	15.196	15.547	15.980	15.414
< 20 and ≥ 14 (8 series)	NSRL	7.318	10.863	11.702	13.302	14.260	16.660	19.870	20.446
	SSDFM	7.741	8.452	9.946	11.345	13.629	15.188	18.516	20.024
	Dodd	30.172	48.111	60.355	63.551	89.562	98.598	112.905	131.866
	Mixed	8.484	10.829	14.366	18.064	19.669	23.170	28.262	26.318
	Gompertz	17.401	20.970	29.008	29.437	33.685	38.659	42.979	47.675
	Floyd	31.051	38.183	45.427	51.266	58.080	61.192	66.099	71.064
	Box–Jenkins	11.005	12.058	14.702	16.585	19.216	21.155	23.197	23.354
< 14 and ≥ 12 (4 series)	NSRL	26.993	32.144	34.555	41.939	37.581	37.072		
	SSDFM	27.279	34.170	40.648	47.510	43.702	41.868		
	Dodd	101.471	139.627	219.134	445.134	627.405	808.405		
	Mixed	37.050	45.838	54.960	65.970	61.178	61.294		
	Gompertz	38.712	45.207	51.012	54.923	51.486	51.853		
	Floyd	41.617	48.184	54.158	59.743	66.319	68.654		
	Box–Jenkins	53.071	65.709	103.617	88.643	57.026	98.334		
≤ 11 (7 series)	NSRL	22.898	32.653						
	SSDFM	22.339	35.995						
	Dodd	80.838	108.920						
	Mixed	25.333	34.939						
	Gompertz	29.669	38.906						
	Floyd	46.290	58.351						
	Box–Jenkins	39.127	40.851						

Table 9
Number of times in which each diffusion model turns out to be the best, according to MAPE, for different temporal horizons

	Temporal horizon							
	1 (30) [a]	2 (30) [a]	3 (26) [a]	4 (25) [a]	5 (25) [a]	6 (23) [a]	7 (21) [a]	8 (19) [a]
NSRL [b]	9	9	7	8	11	8	8	5
SSDFM [b]	12	11	9	8	12	12	10	9
Mixed	7	7	6	5	4	4	3	4
Gompertz	4	5	5	6	4	5	6	6
Floyd	–	1	1	1	–	–	–	–
Dodd	–	–	–	–	–	–	–	–

[a] Number of series analysed.
[b] In some cases NSRL and SSDFM are tied first.

Table 10
Number of times in which each model turns out to be the best, according to MAPE, for different temporal horizons (Floyd and Dodd models are omitted)

	Temporal horizon							
	1 (30) [a]	2 (30) [a]	3 (26) [a]	4 (25) [a]	5 (25) [a]	6 (23) [a]	7 (21) [a]	8 (19) [a]
NSRL	12	11	9	7	9	6	6	3
Mixed	7	6	5	4	2	3	2	3
Gompertz	3	4	4	2	2	1	1	1
Box–Jenkins	8	9	8	12	12	13	12	12

[a] Number of series.

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