

## PREFACE TO THE SPECIAL ISSUE DEDICATED TO JAMES MONTALDI



In August 2018 we held a conference in Guanajuato, Mexico, where several collaborators and students of James Montaldi had the privilege to homage his scientific contributions. James is best known as an expert in singularity and bifurcation theory, who is a world reference in equivariant Hamiltonian systems, and has an exceptional talent to turn symmetries into insightful theorems about the dynamics of mechanical systems. His unusual combination of mathematical depth, modesty, integrity and friendly personality, make him a greatly esteemed member of the extended geometric mechanics community.

The programme of this exciting meeting, which lasted one week, incorporated some of the many areas in which James has worked over the years. During the conference, we heard testimonials of James' mathematical sharpness, kindness and generosity as a supervisor and collaborator, accompanied with many nostalgic references of a meeting that he organised in Peyresq twenty years ago. This issue of the *Journal of Geometric Mechanics* is a continuation of our celebration of James' career and of our appreciation of having him as a teacher, colleague and friend.

Out of the many topics of the conference, the following two allow us to better understand his background and expertise, and to describe a noteworthy contribution to symmetric Hamiltonian systems that he made at an early stage of his career:

- *Singularity theory and geometry*: James is a direct descendant of the finest British geometry school of the twentieth century. He is an academic grandchild of William V. D. Hodge and Michael Atiyah, having obtained his PhD in Liverpool in the early 1980s under the supervision of Ian Porteous. In his thesis, James used singularity theory to study contact between submanifolds. Through the years James went on to become an expert on singularity and bifurcation theory. His publication list includes several excellent papers in these fields with outstanding applications to geometry. These expertises of James certainly distinguish him within the research community working on Hamiltonian systems and have been key to his contributions.
- *Nonlinear normal modes*: during the late 1980s, James spent some years at Warwick where he had a very fruitful collaboration with Mark Roberts and Ian Stewart working on the existence and stability of periodic orbits in the vicinity of equilibrium points of Hamiltonian systems. Their influential results clarify the role of symmetries and, in an equivariant setting, greatly improve the estimate on the number of families of periodic solutions predicted by previous works of Alan Weinstein and Jürgen Moser that extended the classic Lyapunov center theorem. Nowadays, the ideas of James' work constitute the core of any investigation of small oscillations in symmetric mechanical systems. The theory of James and collaborators has multiple applications. In particular, it is beautifully exemplified with the analysis of the normal modes of vibration of a liquid drop or bubble.

This volume of JGM contains 12 papers and 5 of them are specifically influenced by James' work or ideas. The subjects of these papers were also covered by the talks of the conference. These are:

- *Vortex dynamics on surfaces*: this is a topic which has captivated James since his 2001 paper with Chjam Lim and Mark Roberts. In his characteristic style, James' work on the subject systematically exploits the symmetries to classify and study bifurcations and stability of equilibrium and relative equilibrium solutions. Also in this field, we find an excellent example of James' insight and taste for beautiful mathematics in his proof of stability of Thompson's heptagon configuration, obtained with Tadashi Tokieda in 2013. The present volume contains an appraisal of James' contributions to vortex dynamics prepared by Jair Koiller, another world class vortician.
- *Hamiltonian relative equilibria*: James' world reputation as an expert in this field goes back to his influential 1997 paper where he extends a classical result of V. I. Arnold on the persistence and stability of relative equilibria for Hamiltonian systems. In this paper we can already appreciate James' expertise in topology and Lie theory put at the service of mechanics. Relative equilibria are ubiquitous in James' work, both as a topic of study in their own right and in different mechanical applications. Miguel Rodríguez-Olmos, who was a Marie Curie Fellow in Manchester working with James on this topic in 2008-2010, has prepared a special paper for this issue where he surveys the qualitative dynamics around singular relative equilibria of Hamiltonian systems, inspired by his collaboration with James. The last section of his paper describes in some detail some contributions of James to symmetric Hamiltonian systems.
- *Celestial mechanics*: James' reputation in this field greatly increased due to a very elegant 2013 paper on the classification of symmetry groups for choreographies of the planar  $n$ -body problem that he wrote in collaboration with his

former Phd student Katie Steckles. The results are illustrated with beautiful animations that have enchanted members of the community at several conferences and seminars, and which may be found on the link “Choreographies” on James’ website. Another contribution of James to the field, where we can appreciate the depth of his work, is his 2015 paper on the existence of symmetric central configurations. With an insightful symmetry analysis, in only a few pages, he recovers and extends several results that many other authors had obtained in an ad-hoc basis. Moreover, his analysis is general enough to apply to the study of the relative equilibria of the  $n$ -body problem in  $\mathbb{R}^d$  where  $d \geq 2$  is arbitrary. The present volume contains two papers on this latter topic, written by Holger Dullin & Jürgen Scheurle and Alain Albouy & Holger Dullin, that specifically consider the 3-body problem in  $\mathbb{R}^4$ . These papers were inspired by conversations held by the authors with James, Rick Moeckel and Alain Chenciner, at the Observatoire de Paris in 2015, and give interesting conclusions on the stability of the relative equilibria. The present issue also contains a short note written by Luis García-Naranjo on the (absence) of center of mass for the two body problem in spaces of constant curvature, a topic in which he’s had the fortune to have recently collaborated with James. The remarks in this note were inspired by interactions with James and are meant to symbolise his appreciation for his support, both professionally and personally, during the last 5 years.

The conference talks at the Montaldifest also covered other subjects in which James has contributed over the years, that we do not describe here in detail, such as molecular dynamics, symmetry breaking, Poisson geometry and momentum maps, nonholonomic mechanics, Lie theory and reduction and dynamical systems. The remaining 7 papers of this issue, that are not explicitly mentioned above, are in some way or another related with the subjects of James’ work. Indeed, their descriptive keywords contain at least one of either symmetry, Hamiltonian, Poisson, dynamics, mechanics or oscillator.

Any description of James’ career that does not mention his qualities as a teacher is necessarily incomplete. In fact, some of the best compliments about James come from his own students, who describe him as a kind and generous mentor, whose door is always open for mathematical discussion. At the publication time of this volume, James has completed the direction of 12 PhD theses and is working on two more. It is worth noticing his ability to mentor a great diversity of students, with some of his former students coming from extremely different geographical and cultural backgrounds. Also, gender inequality, which is unfortunately often found in mathematics, where women are greatly underrepresented, is not an issue among James students, with half of them being female.

To finish, we wish to add that the great depth and worth of James’ work is perhaps best explained in the words of his wife Margo, who sometimes refers to mathematics as *James’ secret lover* to which he passionately escapes for some hours every day. We hope that this volume serves the reader to appreciate James’ mathematical contributions and also the great pleasure of those of us who can count him as a teacher, colleague and friend, and who celebrated with him the Montaldifest.

Guest Editors:

Luis C. García-Naranjo

Departamento de Matemáticas y Mecánica

IIMAS-UNAM  
Apdo. Postal 20-126, Col. San Ángel  
Mexico City, 01000, Mexico  
luis.garcianaranjo@gmail.com

Manuel de León  
Instituto de Ciencias Matemáticas  
CSIC  
c/ Nicolás Cabrera 13-15  
28049 Madrid, Spain  
mdeleon@icmat.es

Juan Pablo Ortega  
Faculty of Mathematics and Statistics  
University of St Gallen  
Bodanstrasse 6, CH-9000 St. Gallen, Switzerland  
and Centre National de la Recherche Scientifique (CNRS), France  
Juan-Pablo.Ortega@unisg.ch