# A simple but effective approach to generate energy-efficient trajectories of a 2 degree-of-freedom planar manipulator

D. Dona', R. Minto, M. Bottin and G. Rosati

**Abstract** EU has the ambitious goal of reaching net-zero carbon emissions by 2050. To achieve this, energy savings in the industry can play a relevant role. Considering industrial robots, this paper presents a novel method based on adding a proper law to the base one, which both keeps the robot cycle time-invariant and is easy to implement. In particular, a 2 degree-of-freedom planar manipulator is taken as an example. The method consists of an optimization routine that takes into account the joint speed limit, reducing the energy needed for each movement.

## 1 Introduction

The reduction of energy expenditure (EE) in robotic systems is a well-known topic [6]. In a typical industrial scenario, motion planning is designed to minimize cycle time to maximize the throughput [3, 2]. At the same time, from both an environmental and an economic point of view, a reduction in EE can be desirable [5]. Various approaches have been proposed over the years to increase efficiency, such as using lighter parts [9, 8], adding elastic elements [11], or using efficient trajectories [4, 10]. The latter has the advantage that no hardware modifications are needed. In addition, efficient trajectories can be used in combination with other approaches, further reducing EE.

In this work, the optimization of a Point-to-Point (PTP) movement of a 2 degrees-of-freedom (DoF) manipulator is analyzed. The procedure is easily extendable to more complex open-chain manipulators and is easy to implement since it relies on simple trajectories and the well-known robot dynamic model [12]. The paper is organized as follows: Sect. 2 presents the mathematical model to be used in the optimization procedure explained in Sect. 3.1; Sect. 4 presents simulated examples; conclusions are drawn in Sect. 5.

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## 2 Mathematical model

Let us consider a 2 DoF planar manipulator with two revolute joints. Referring to Fig. 1, the inverse dynamic model of the manipulator is calculated starting from the Lagrange formulation.

$$\frac{d}{dt}\left(\frac{\delta \mathcal{L}}{\delta \underline{\dot{q}}}\right) - \frac{\delta \mathcal{L}}{\delta \underline{q}} = \underline{\tau} \quad , \quad \mathcal{L} = \mathcal{T} - \mathcal{U} \tag{1}$$

where $\mathcal{L}$ is the Lagrangian, $\underline{q}$ is the vector of the joint variables, $\underline{\dot{q}}$ is the vector of the joint speeds, $\underline{\tau}$ is the vector of the joint torques, $\mathcal{T}$ is the total kinetic energy, and $\mathcal{U}$ is the total potential energy. Substituting the expression of the kinetic and potential energy into (1) leads us to the following general form for the dynamics of a manipulator:

$$\boldsymbol{M}(\underline{q})\underline{\ddot{q}} + \boldsymbol{C}(\underline{q}, \underline{\dot{q}})\underline{\dot{q}} + \underline{g}(\underline{q}) = \underline{\tau} \tag{2}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$, and $\underline{g}$ are inertia, Coriolis matrix, and gravity vector, respectively, and $\underline{\ddot{q}}$ is the vector of joint accelerations.

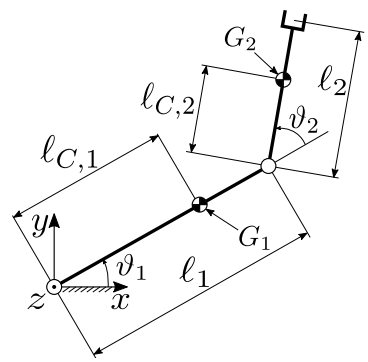


**Fig. 1:** Model of the 2 DoF manipulator

Let $I_{zz_i}$ be the inertia with respect to the CoM (center of mass) and $m_i$ be the mass of the $i-th$ link, also considering the mass of the motor stator $i+1$. Let $\ell_{C,i}$ be the distance of the CoM of the *i-th* link from the *i-th* joint and $\ell_i$ me the length of the *i-th* link. The matrix $\boldsymbol{M}$, for the manipulator studied, can be computed as follows:

$$\boldsymbol{M} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \tag{3}$$

Using the notation $\sin_i = \sin\theta_i$ and $\sin_{ij} = \sin(\theta_i + \theta_j)$ (and similarly for the cosine), we have:

$$m_{11} = I_{zz1} + m_1 \ell_{C,1}^2 + k_{r,1}^2 I_{m,1} + I_{zz2} + m_2 \left( \ell_1^2 + \ell_{C,2}^2 + 2\ell_1 \ell_{C,2} \cos_2 \right) + I_{m,2} \quad (4)$$

$$m_{12} = m_{21} = I_{zz2} + m_2 \left( \ell_{C,2}^2 + \ell_1 \ell_{C,2} \cos_2 \right) + k_{r,2} I_{m,2} \quad (5)$$

$$m_{22} = I_{zz2} + m_2 \ell_{C,2}^2 + k_{r,2}^2 I_{m,2} \quad (6)$$

Where $I_{m,i}$ is the inertia of the rotor of the *i-th* motor. Defining $h = -m_2 \ell_1 \ell_{C,2} \sin_2$ the Coriolis matrix takes the following form:

$$\boldsymbol{C} = \begin{bmatrix} h\dot{\theta}_2 & h\left(\dot{\theta}_1 + \dot{\theta}_2\right) \\ -h\dot{\theta}_1 & 0 \end{bmatrix} \quad (7)$$

The gravity vector $\mathbf{g} = [g_1 \; g_2]^T$ can be computed as:

$$g_1 = \left(m_1 \ell_{C,1} + m_2 \ell_1\right) g \cos_1 + m_2 \ell_{C,2} g \cos_{12} \quad (8)$$

$$g_2 = m_2 \ell_{C,2} g \cos_{12} \quad (9)$$

Once the joint torque is calculated with equation (2), it is possible to compute the torque required by the *i-th* motor by considering the efficiency $\eta_{r,i}$ and the reduction ratio $k_{r,i}$ of the *i-th* gearbox.

$$\tau_{mot,i} = \frac{\tau_i}{\eta_{r,i} \, k_{r,i}} \quad (10)$$

## 2.1 Electrical model

Since most manipulators are driven by DC motors, the electrical subsystem can be described by the electrical model of the DC motor.

The electrical energy $E$ absorbed by the motors from the power grid is the following:

$$E = \sum_{i=1}^{N} \int_{o}^{t_f} \mathcal{P}_i(t) dt \quad (11)$$

where $N$ is the number of motors and $\mathcal{P}_i$ is the power absorbed by the *i-th* motor. Since accumulator capacity is not given, regenerative braking power is dissipated in a braking resistor. From a mathematical point of view, this can be stated as follows.

$$\mathcal{P}_i(t) = \begin{cases} \mathcal{P}_i(t) & \text{if } \mathcal{P}_i(t) > 0 \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

The power delivered at every instant depends on the armature electrical quantities:

$$\mathcal{P}_i(t) = v_{a,i}(t) \; i_{a,i}(t) \quad (13)$$

Where $v_{a,i}$ and $i_{a,i}$ are the armature voltage and current of the *i-th* motor, respectively. The voltage armature can be obtained by solving the DC motor equivalent circuit model:

$$v_{a,i}(t) = R_{a,i} i_{a,i}(t) + L_{a,i} \frac{d\, i_{a,i}(t)}{dt} + k_{v,i} k_{r,i}\, \dot{q}_i \tag{14}$$

Where $R_{a,i}$, $L_{a,i}$, $k_{v,i}$ are the armature resistance, the armature inductance, and the Back EMF constant of the *i-th* motor, respectively. In the following, the inductive term is neglected under the assumption that the current transients are faster than those of the mechanical subsystem dynamics. Both (13) and (14) depend on the armature current:

$$i_{a,i}(t) = \frac{\tau_{mot,i}(t)}{k_{t,i}} \tag{15}$$

where $\tau_{mot,i}$ is the torque generated by the motor given by (10), and $k_{t,i}$ is the torque constant, numerically equal to $k_{v,i}$.

## 3 Motion planning algorithm

Hereafter, the subscript $i$ is removed for the sake of brevity; in the following, all the equations are considered to be valid for each joint. The movements of a manipulator are limited by the capability of the motors placed inside the joints. In other words, the movement time $T$ is the maximum of the times needed by all joints when each of them reaches the maximum speed ($\dot{q}_{lim}$) for a determined motion law. The trajectories of the joints that require less time are scaled to fit $T$, and thus the maximum joint velocities ($\dot{q}_{max}$) are reduced. Usually, it is one joint that reaches the limit [1]. This means that the other joints have some margin of velocity ($\Delta\dot{q} = |\dot{q}_{lim} - \dot{q}_{max}|$). As a result, we can exploit this difference to modify the base motion law.

Let us consider a PTP task; a common motion law is the symmetrical trapezoidal law of velocity, illustrated in Fig. 2. Given the end joint value and the acceleration time $T_a$, the law is fully defined.

To exploit the velocity margin, it is introduced a law that has the propriety of having an overall zero displacement, which means that summing it to the base one makes no difference in the final position. In our work, a double trapezoidal law is used. It is composed of two trapezoidal velocity laws, one with positive velocity and the other with negative velocity, as illustrated in Fig. 3. The two periods are defined by the parameter $\alpha$, which is between 0 and $T$.

The maximum velocities in the two periods are $h_1$ and $h_2$, respectively. The ratio of acceleration times to the two periods is fixed and equal to $t_\alpha$.

The additional law is defined by two parameters: $h_1$ and $\alpha$; in fact, $h_2$ is defined by the condition that $\Delta q$ of the double trapezoidal law must be zero:

[1] In the case both reach the joint limit no optimization is possible.

$$\int_0^{\alpha} \dot{q}(t)dt = h_1\alpha(1-t_\alpha) = h_2(1-t_\alpha)(T-\alpha) = \int_{\alpha}^{T} \dot{q}(t)dt$$
$$h_2 = h_1\frac{\alpha}{T-\alpha} \tag{16}$$

Since joint speed limit $\dot{q}_{lim}$ must not be passed both in positive and negative values, $h_1$ is constrained as such, based on the maximum joint velocity for the base law $\dot{q}_{max}$:

$$h_1 \leq \dot{q}_{lim} - \dot{q}_{max} = h_{1,ub}$$
$$h_1 \geq -\dot{q}_{lim} - \dot{q}_{max} = h_{1,lb} \tag{17}$$

The expressions reported in (17) do not consider the possibility of a limit overtaking due to $h_2$. The constraints have to be updated through equation (16):

$$h^*_{1,ub}(\alpha) = \min\left[h_{1,ub}, h_{1,ub}\frac{\alpha}{T-\alpha}\right] \tag{18}$$
$$h^*_{1,lb}(\alpha) = \max\left[h_{1,lb}, h_{1,lb}\frac{\alpha}{T-\alpha}\right] \tag{19}$$

The relation between the two bounds and $\alpha$ is nonlinear and not concave.

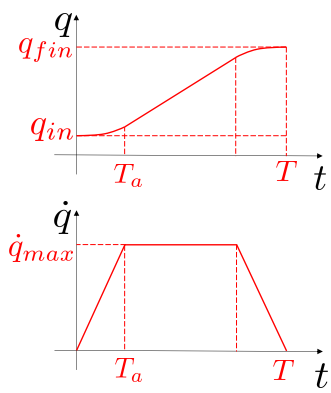


**Fig. 2:** Symmetrical trapezoidal law of velocity

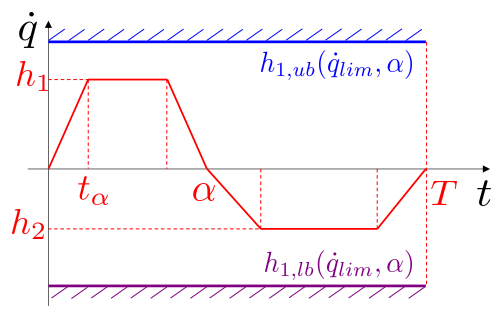


**Fig. 3:** Double trapezoidal law of velocity

### 3.1 Optimization procedure

Once the cycle time $T$ is calculated for the bottleneck joint, it is possible to choose $\alpha$ and $h_1$ for the other joint to minimize energy consumption. Calculating $\underline{\tau}_{mot}$, and then, by means of equations (14), (15) and (13) it is possible to calculate the

electrical energy absorbed by the motors $E$ (equation (11)). Formally, the following optimization problem must be solved:

$$\begin{aligned} \underset{\alpha, h_1}{\text{minimize}} \quad & E(\alpha, h_1) \\ \text{subject to} \quad & \begin{cases} 0 < \alpha < T \\ h_{1,lb} \leq h_1 \leq h_{1,ub} \end{cases} \end{aligned} \tag{20}$$

Since it is not convex, the global optimality is not ensured by the convergence of the routine [7]. To avoid this problem, several optimization routines with different random starting guesses can be performed.

## 4 Simulation

To validate our approach, we applied it to a 2 DoF planar manipulator performing a PTP motion; the inertial, electrical, and mechanical properties of the manipulator are listed in Tab. 1. The PTP motion, whose starting and ending positions are reported in Tab. 2, is studied with four different cases (Tab. 3), both with and without gravity and with both configurations (lefty/righty).

**Table 1:** Model data

| | Link 1 | Link 2 |
|---|---|---|
| $\ell$ [mm] | 300 | 200 |
| $I_{zz}$ [kg·m$^2$] | 0.1 | 0.1 |
| $m$ [kg] | 10 | 10 |
| $\ell_C$ [mm] | 150 | 100 |
| $I_m$ [kg·m$^2$] | $6 \cdot 10^{-7}$ | $6 \cdot 10^{-7}$ |
| $k_r$ [/] | 120 | 120 |
| $\dot{q}_{lim}$ [rpm] | 4000 | 4000 |
| $k_t$ [Nm/A] | 1.5 | 1.5 |
| $R_a$ [Ω] | 1.5 | 1.5 |

**Table 2:** Initial and final points of the task

| Coordinate | Initial P. | Final P. |
|---|---|---|
| x [mm] | 300 | 0 |
| y [mm] | 0 | 400 |

**Table 3:** Cases studied

| Case | Configuration | Gravity |
|---|---|---|
| (a) | righty | ✓ |
| (b) | righty | ✗ |
| (c) | lefty | ✓ |
| (d) | lefty | ✗ |

Using the Matlab function `fmincon` and random (but feasible) starting points, the results summarized in Tab. 4 are obtained. The comparison between the reference trajectory and the optimized trajectory is reported in Fig. 4.

**Table 4:** Optimization results, the energies are expressed in [mJ]

| | EE ref. | EE opt. | $\Delta$[%] | $h_1$ [rad/s] | $\alpha$ [s] |
|---|---|---|---|---|---|
| Case (a) | 618.4 | 502.1 | 19 | 2.40 | 0.496 |
| Case (b) | 406.8 | 336.9 | 17 | 2.23 | 0.346 |
| Case (c) | 74.3 | 66.5 | 11 | -4.10 | 0.478 |
| Case (d) | 103.5 | 100.7 | 3 | -0.79 | 0.245 |

In all cases, the introduction of the double trapezoidal law reduces the EE. The savings are from 3% to 19% with a mean of 12.5%, whilst the computational time to calculate the optimal motion law is 89.2s on a Windows PC equipped with an AMD Ryzen 4500U processor and 8 GB of RAM. The cost can be reduced using lower initial guesses (in this case 64 different starting points are used). It is clear that in a practical context, a trade-off between optimality and computational cost has to be evaluated in the design phase. However, it should be noted that, by changing the motion law, the trajectory of the end-effector changes, so collision checks must be performed to ensure the feasibility of the trajectory [1]. The reference and optimal trajectories for each case are shown in Fig. 4.

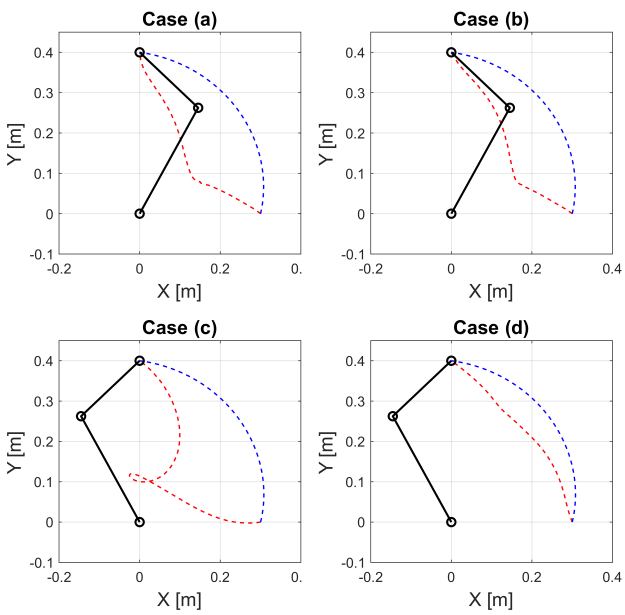


**Fig. 4:** Comparison of optimal (in red) and reference (in blue) trajectories in the different cases studied. Same starting and final points are used. The left side cases are with gravity (manipulator in the vertical plane) while the right ones are without gravity. The above are in the righty configuration and the below in the lefty one.

## 5 Conclusions

In this article, a novel optimization method has been presented to achieve energy efficiency in open-chain manipulators, with a particular focus on a 2 DoF planar manipulator. The method, which is based on the electromechanical dynamic model of the manipulator, is based on a modification of a standard motion law to exploit full joint capabilities. In this sense, the movement time is not affected, but it is possible to reduce Energy Expenditure.

Furthermore, this method can be used in more complex robotics systems, since more DoF are added more joints are potential candidates to be optimized.

The method has been implemented in Matlab and four different use cases have been tested. The results show that Energy Expenditure can be reduced by more than 10% with ease.

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