Correlation of Multiple Strategic Sources Decreases Their Age of Information Anarchy

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***Abstract*—We consider a sensor network, where multiple sources send status updates to a common receiver and, due to correlation in the content, transmissions from a given node can also be informative for others. The objective is to minimize the individual information freshness, quantified through age of information, at the receiver's side. We compare a centralized control with a disitributed minimization; in the former, the globally optimal data injection rate is chosen, whereas in the latter, sources behave like players of a non-cooperative game of complete information. We compute Nash equilibrium and quantify the price of anarchy. Even a moderate correlation among sources is shown to make a distributed approach more efficient. For a correlation of $1/3$ of the content, the anarchy is set to just 6–7% worse than the optimum.**

***Index Terms*—Age of Information; Queueing theory; Game theory; Remote sensing; Wireless sensor networks.**

I. Introduction

Remote sensing of real-time data generally follows the objective of getting up-to-date system information. Age of information (AoI) is used to quantify the freshness of status updates over time, and has gained popularity as an application independent and simple to compute performance indicator [1].

If a sensor sends status updates to a receiver at times in set $\mathcal{T} = \{\dots, \tau_1, \tau_2, \dots, \tau_N, \dots\}$, at time t the value of its AoI is $\delta(t) = t - \tau_{\ell(t)}$, where $\ell(t) = \arg\max_j\{\tau_j \leq t\}$. Many evaluations of AoI in the literature pertain to queueing systems with different disciplines [2]–[5].

In this brief, we focus on multiple sources, as in [4], [6], [7], but especially [8], which is the main source of inspiration for our analytical derivations. The key point of AoI investigations for multiple sources is to consider them as independent agents, which try to get their own updates ahead of the others in the queue. This can be tackled with *game theory* [9], which is popular to represent scenarios with multiple (competing) AoI values [10]–[13]. However, no previous game theoretic study focuses on queueing, and the interaction takes place among disconnected information sources, creating a *mors tua vita mea* situation [14], where updates sent by an agent are to detriment of the others, as they congest the queue at the receiver.

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TABLE I
Comparison of previous related work on multi-source AoI

contribution	distributed	correlation	game theory
Moltafet *et al.* [4]			
Tripathi & Modiano [6]		✓	
Abd-Elmagid & Dhillon [7]	✓		
Yates & Kaul [8]	✓		✓
Zhang *et al.* [10]			✓
Badia & Munari [11]	✓		✓
Saurav & Vaze [12]	✓		✓
Miguélez *et al.* [13]	✓		✓
Buratto *et al.* [15]		✓	
Kalør & Popovski [16]		✓	
Crosara *et al.* [17]	✓	✓	
Badia [19]	✓		✓
our contribution	✓	✓	✓

Yet, many sensing scenarios involve multiple sources from the same deployment, and whose data are correlated with one another [15]. This can be the case of cyberphysical systems used in smart agriculture, eHealth, or intelligent transportations: Multiple sensors can be placed at different locations to measure the same quantity, or physically located in the same spot but reporting on different, yet related, quantities (e.g., temperature and humidity, or multiple vital parameters) [16], [17]. This correlation can, and should, be taken into account, without any data exchange between the sensors, but relying on the presence of other sensors, as well as the statistical correlation in the information content [7]. The individual minimization of AoI calls for a game theoretic analysis, to assess whether the anarchical behavior of a distributed system can be efficient in terms of AoI [18], [19].

Thus, we bring the following contributions. First, we extend the analytical formulations of queueing theory to include *correlation* among sources, so that the updates sent by one node may be valid for each of the others with a certain probability. Moreover, we use *game theory* to evaluate the inefficiency of a distributed management, where each source decides its update injection rate, as opposed to a globally optimal choice. Even in the presence of correlation, nodes want to push their updates, surely beneficial for their AoI, over those of the others, which are so only probabilistically. This is quantified through the price of anarchy (PoA), i.e., the ratio between the cost at the Nash equilibrium (NE) over its optimal value [14]. To highlight that these aspects have never been investigated before in the same analysis, we summarize the related work in Table I. All these papers study multi-source AoI; yet, none considers a distributed optimization through game theory for correlated content, as we do.

Through our analysis, we find that correlation among sources, quantified through the probability of data injected by one node to be useful for the others, significantly decreases the loss of efficiency for distributed strategic management, which rapidly approaches 0 even without the nodes being strongly correlated. This implies that rational agents submitting mildly interrelated data can obtain an efficient injection rate even through distributed selection. This supports decentralized management of status updates from independent sources, whose loss of efficiency vanishes in the presence of correlation.

II. System Model

We consider a system as displayed in Fig. 1, where multiple sources of set $\mathcal{N} = \{1, 2, \ldots, N\}$ transmit their status updates to a common receiver, which enqueues all packets in an FCFS M/M/1 queue according to their order of arrival. Even though the status update packets share the same queue, each affects a different AoI value, related to its corresponding source.

This queueing model is well established and generally appropriate for multi-source wireless sensor networks [4], [20]. Thus, sources $1, 2, \ldots, N$ generate traffic according to a Poisson process with average arrival rate equal to $\lambda_1, \lambda_2, \ldots \lambda_N$, respectively. The global service rate of the queue is denoted as μ and packets from either source are identically served, with an exponentially distributed time of average $1/\mu$.

While a similar scenario was studied in [8], we assume that the system statuses under monitoring exhibit correlation, so that transmission from source i, while resetting AoI at this very source, can be also seen as a valid update for source $j \neq i$ with a certain probability that is a parameter $\alpha \in [0, 1]$.

In the literature on AoI, a common setup considers memoryless data injection systems treated as queues with different disciplines [3], [4], [21]. We will apply our reasonings to an M/M/1 FCFS queue, which is the most basic system also represents a good model for independent sensing nodes with adjustable injection rates. This choice is just made for simplicity and space limitations, as other more complex queueing systems could be used, without significantly changing the game theoretic rationale and the conclusions that can be obtained. On the other hand, this would unnecessarily complicate the analytical derivations and more in general obtain a more cumbersome mathematical analysis. The interested reader is referred, for example, to [17] for a comparison of memoryless vs deterministic data generation. The latter may be considered a more realistic description of sensing scenarios with periodic reporting, yet, it does not admit an elementary closed form solution. Nevertheless, it is shown that the same qualitative conclusions apply, the system with deterministic generation having, roughly speaking, AoI values that are just scaled down with respect to the memoryless generation and therefore similar conclusions in terms of PoA would be achieved.

For an M/M/1 queue with injection rate λ and service rate μ, the analytical expression of the average AoI Δ is [1]:

$$\Delta = \frac{1}{\mu}\left(1 + \frac{1}{\rho} + \frac{\rho^2}{1-\rho}\right), \tag{1}$$

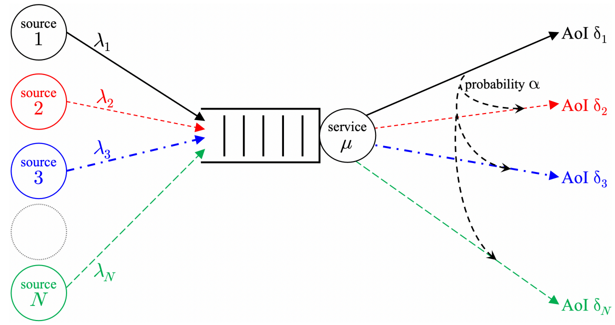

Fig. 1. Queueing system with multiple sources and correlated content

where $\rho = \lambda/\mu$ is the load factor. In the following, we consider a normalized service rate $\mu = 1$; this simplifies (1) by removing the coefficient $1/\mu$, and replacing ρ with λ, i.e.,

$$\Delta = 1 + \frac{1}{\lambda} + \frac{\lambda^2}{1-\lambda}, \tag{2}$$

which can be adjusted for a non-unitary service rate by simply re-scaling, and the same applies for all the equations introduced in the following. Also, for system stability, $\lambda \in [0, 1]$.

Interestingly, the AoI-optimal value of λ in (2) is found as $\lambda^* \approx 0.531$ [1]. This means that optimizing the average AoI implies adopting a non-aggressive data injection, where λ is somehow intermediate between 0, in which case the information would be stale, and 1, for which the queue will be unstable, which also causes the average AoI to soar.

In [8], the results of (2) are extended to the average AoI in the case of N independent sources. Without loss of generality, focus on a given source 1, whose expected AoI Δ_1 is found to be a function of vector $\boldsymbol{\lambda} = \{\lambda_i\}_{i \in \mathcal{N}}$ as [8]

$$\Delta_1(\boldsymbol{\lambda}) = \frac{\lambda_1^2(1 - \Lambda\Lambda_{-1})}{(1-\Lambda)(1-\Lambda_{-1})^3} + \frac{1}{1-\Lambda_{-1}} + \frac{1}{\lambda_1} \tag{3}$$

where $\Lambda = \sum_{i=1}^N \lambda_i$, $\Lambda_{-1} = \Lambda - \lambda_1 = \sum_{i=2}^N \lambda_i$. This derivation is immediate from [8], with just a few notational replacements. In particular, the original formulas consider two sources, but in an M/M/1 queue all data injected from sources others than 1, whose rate is Λ_{-1}, can be seen as a single memoryless source due to the superposition of Markov processes.

As an extension with respect to this literature, we consider that the packets transmitted by a source may contain data that correlate to the process tracked by another source. More precisely, a data packet injected by a source i can also enhance with probability α the instantaneous AoI value of another source j in the same way that all packets transmitted by source j do [17]. Due to the memoryless nature of the data injected, one can alternatively see α as the fraction of data sent by a source that is useful for another, instead of a probability. In common scenarios, all sources have identical interdependencies, and thus we consider α as a network parameter. The case of more complex dependencies is out of the scope of the present investigation, and left for future work. We remark that the present analysis still applies from a conservative standpoint by considering the worst-case correlation.

We can reformulate (3) by considering this correlation among the data, so that Λ is the same, but the rate of injected useful data for the AoI Δ_1 of source 1 increases to $\lambda_1+\alpha\Lambda_{-1}$, whereas the injection rate of data that do not enhance Δ_1 is $(1-\alpha)\Lambda_{-1}$. Thus, the average AoI of source 1 becomes [2]

$$\Delta_1(\boldsymbol{\lambda}) = \frac{(\lambda_1+\alpha\Lambda_{-1})^2\,[1-\Lambda(1-\alpha)\Lambda_{-1}]}{(1-\Lambda)[1-(1-\alpha)\Lambda_{-1}]^3} + \frac{1}{1-(1-\alpha)\Lambda_{-1}} + \frac{1}{\lambda_1+\alpha\Lambda_{-1}}\,. \quad (4)$$

Parameter α distinguishes a continuous range of scenarios. The case $\alpha = 0$ is that of multiple independent sources, as in [8], whose average AoI is given by (3). If $\alpha = 1$, all the sources behave as a unique flow with data injection rate Λ, falling back to [1] with a single source injecting $\lambda = \Lambda$, whose average AoI is given by (2). In the intermediate case $0 < \alpha < 1$, the status updates of the sources are correlated, so that some packets transmitted by one source can act as updates for another [2]; the average AoI is as per (4).

Also, the idea of AoI stems from time redundancy of data, implying that multiple close updates congest the processing at the end server without significantly enhancing the information freshness, and it may be convenient to evenly spread them over time. We are extending this concept to spatial redundancy, where some updates can be avoided if another source has already sent an update about a process that correlates with the one of interest or is even the same [22].

III. Game Theoretic Analysis

Game theory is the study of strategic interactions among multiple agents that follow individually different objectives. Some related papers [8], [11], [12] already applied such a methodology for scenarios with multiple sources, where each agent in the system is seen as minimizing a different AoI value. This leads to a formalization as a static game of complete information, for which the NE is computed, often in closed form, thanks to the aforementioned theoretical framework. This working point, which corresponds to the result of a distributed optimization by each individual source, is often suboptimal from a global system perspective and therefore can be compared with the best possible data injection strategy that results in lower AoI for all the nodes, i.e., a Pareto efficient solution. To quantify that the inefficiency of the NE, the PoA can be computed as the ratio between the system welfare of the optimal allocation and that achieved by distributed agents.

However, in the studies following such an approach, the focus is on uncorrelated systems tracked by different nodes. This results in AoI values that are independent of each other, which leads to competition and stronger anarchy. Intuitively speaking, an allocation that improves AoI from a global standpoint is not seen as a NE by the players. Although not adversarial with each other, they see the service of data packets generated to some other source as uninteresting and irrelevant for them, and might prefer to push their own content.

We expect that the presence of correlation among the sources mitigates this competitive behavior, as in principle one source can also lower its own AoI by letting the others transmit. To formalize this with a quantitative reasoning, we define a static game of complete information $\mathcal{G} = (\mathcal{N}, \mathcal{A}, \mathcal{U})$, where the set of players is $\mathcal{N}$, i.e., it corresponds to the N sources, the action set $\mathcal{A} = [0,1]^N$ contains each $\lambda_j \in [0,1]$ as the action of the jth player, and the set of utilities $\mathcal{U}$ consists of $\{-\Delta_i\}_{i\in\mathcal{N}}$. The negative sign is because utilities denote a quantity to maximize for the ith player, and the average age of information Δ_i is a quantity to minimize instead [23].

The decisions are made individually and without coordination, i.e., the game is *static*. Also, the game theoretic condition of *complete information* holds; this refers to a preliminary common knowledge among all individual players of their objectives (as well as their existence in the scenario), not to any exchange of information content at runtime. Thus, the sensors are aware of each other and the fact that their contents are correlated, already from the deployment phase [17].

Albeit driven by selfish objectives, nodes anticipate the consequences of their actions, thus no player will monopolize the the queue with its traffic as it would lead to congestion and high AoI. The players are expected to also realize that they can be less aggressive and exploit correlation as a way to assist each other [15], since, depending on α, each individual agent may see that some other source updates its own information as partially beneficial for their own AoI as well.

Framing such a system in the context of game theory does not really cast it as a conflict between competing players, but more as a distributed system management, whose efficiency may be worth assessing [12], [13]. Thus, it is possible to find a *global optimum* vector $\boldsymbol{\lambda}^*$ of injection rates as

$$\boldsymbol{\lambda}^* = \arg\max_{\boldsymbol{\lambda}} \sum_{i=1}^{N} \Delta_i(\boldsymbol{\lambda})\,. \quad (5)$$

Symmetry considerations lead to the optimal vector $\boldsymbol{\lambda}^*$ being made of all identical elements, i.e., $\boldsymbol{\lambda}^* = (\lambda^*, \lambda^*, \ldots, \lambda^*)$ which implies that all sources must choose an identical λ^* value. In turn, this implies that they all experience the same average AoI value, and the optimality condition (5) becomes

$$\lambda^* = \arg\max_{\lambda} \sum_{i=1}^{N} \Delta_1(\lambda, \lambda, \ldots, \lambda)\,, \quad (6)$$

whose solution is easy to obtain in closed-form by finding the first-order derivative of the objective in λ, which can be computed through (4), and setting it as equal to 0 [18].

The NE is also found in a (generally different) symmetric point, computed from the selfish perspective of an individual source, say source 1, adopting the best response to any possible choice of Λ_{-1} of the other players as

$$\lambda_1^{(\mathrm{BR})}(\Lambda_{-1}) = \arg\max_{\lambda_1} \Delta_1(\lambda_1, \Lambda_{-1})\,. \quad (7)$$

Symmetry reasons imply that every other source will follow a similar approach, thus $\Lambda_{-1} = (N-1)\lambda_1$. The NE is ultimately achieved at the fixed point λ^{NE} of the best response, for which $\lambda^{\mathrm{NE}} = \lambda_1^{(\mathrm{BR})}\big((N-1)\lambda^{\mathrm{NE}}\big)$.

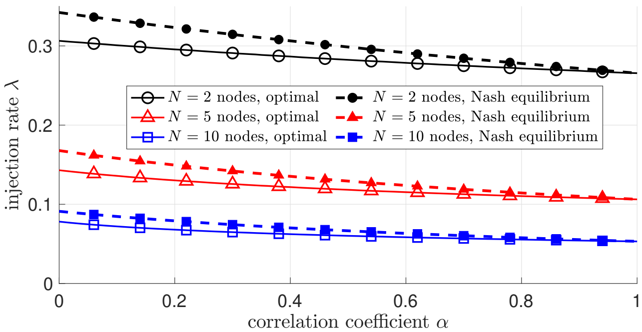

Fig. 2. Data injection rate λ at the NE (solid) and the global optimum (dashed), vs correlation parameter α.

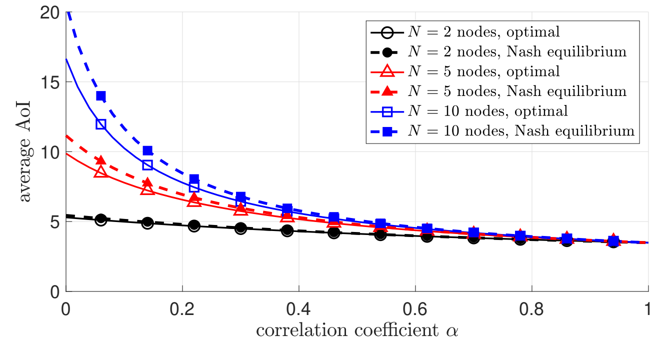

Fig. 3. Average AoI of one source at the NE (solid) and the global optimum (dashed), vs correlation parameter α.

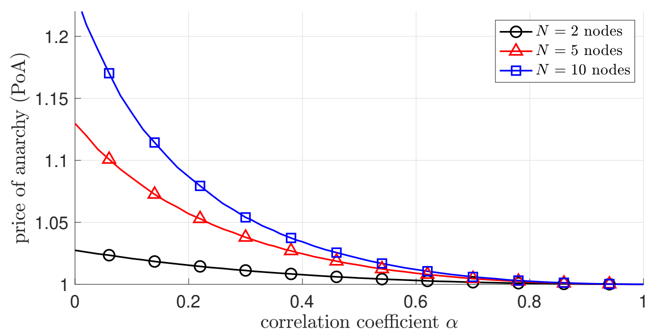

Fig. 4. Price of Anarchy vs correlation parameter α.

The difference in the two approaches is subtle but significant. The global optimum corresponds to setting λ^* for all sources, whereas the data injection rate at the NE, λ^{NE}, while analogously symmetric for all the sources, only focuses on a one-sided minimization of Δ_1 through λ_1, since in the distributed approach, every source does not care about minimizing AoI of others and cannot control their choices.

Since the service capacity is a shared resource, the NE differs from the optimal allocation. The latter corresponds to an efficient working point for the system as a whole, i.e., the server is shared so that the average AoI of all users is individually minimized. In contrast, the NE is reached when individual sources make strategic decisions to minimize their own AoI, without considering the collective welfare of the network. This individualistic approach can lead to suboptimal resource allocation, as users may prioritize their own interests over the overall network's efficiency, which is a classic situation known as the *tragedy of the commons* [14].

For our problem, the difference can be counterintuitive due to the symmetry of the sources. Indeed, both the optimal allocation and the NE correspond to all sources choosing the same data injection rate, λ^* and λ^{NE}, respectively, and achieving identical freshness, but the resulting AoI at the NE is higher because of source selfishness, and also $\lambda^{\mathrm{NE}} > \lambda^*$ according to the tragedy of the commons. The discrepancy arises because the optimal allocation requires centralized coordination and a global view of the network, whereas the NE is decentralized, with users making independent decisions based on their local information, which leads to overuse of the shared resource. Consequently, understanding the quantitative extent of this inefficiency is key, especially seeing whether correlation among sources plays a role in diminishing it.

IV. Numerical Results

We present evaluations comparing the optimization of data injection rate from either a global perspective or through the NE, i.e. selfish choice by the nodes. We consider a variable number of nodes N. All sources choose the same data injection rate for the individual source in both cases, and we denote it as λ, to be read as λ^{NE} or λ^* for the distributed or centralized approach, respectively. This can also be seen as a measure of the individual source throughput since μ is normalized, i.e., set to 1; also, both allocations are stable, i.e., $N\lambda < 1$.

Fig. 2 compares the optimal choice of the transmission rate from a global standpoint with the selfish perspective of the NE. The value of λ at NE always starts from a higher value than the optimal λ, as predicted by game theoretic reasonings, but, for increasing correlation, the two values become close, until the same point is reached when α=1. In that case, all sources send equivalent data, and even in the distributed case, they are aware of that. Thus, the sources choose an optimal data injection rate as $0.531/N$, so that their collective injection equals that of the optimal λ for one source only as per (2).

However, even different choices of λ may result in similar information freshness. For this reason, we plot in Fig. 3 the average AoI resulting from the injection rates chosen through either global or distributed optimization (i.e., the NE). The two curves start relatively different when α=0, i.e., for uncorrelated sources, but they get closer for all values of N already for moderate correlation. Even for the highest value of N=10, where a distributed optimization causes a more significant AoI increase, we do not require a very high value of α for them to be almost indistinguishable in terms of AoI.

To better quantify the difference, we report in Fig. 4 the PoA of the distributed allocation, which translates in the ratio of the achieved AoI between distributed and centralized optimization, respectively. This figure highlights that, while the scenario with 2 sources, which is the classic reference analyzed in [8], does not imply a very high PoA, this value becomes much

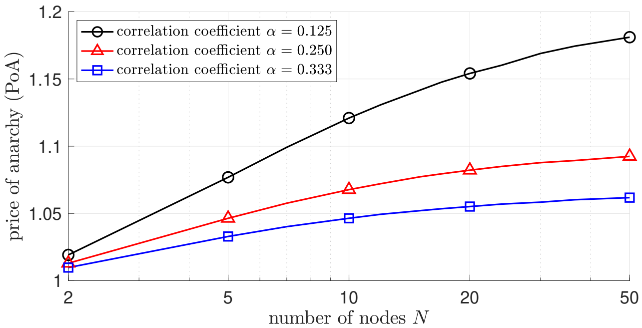

Fig. 5. Price of Anarchy vs number of nodes N, for different values of α.

worse when the number of sources increase. One can see that, in the absence of correlation, i.e., α=0, the loss of efficiency for N=10 is higher than 20%. At the same time, even a mild correlation brings the PoA significantly down. While the PoA can be expected to approach 1 for $\alpha \to 1$, it is strikingly around 1.06–1.07 for a correlation coefficient of around $1/3$.

A further result confirming this trend is shown in Fig. 5, where we consider the PoA vs the number of nodes N, which also allows us to evaluate what happens in larger networks. The plot shows that the PoA increases for larger N, a logical consequence of that, the higher the number of users, the higher the anarchy. But correlation does not only decrease the PoA but also causes it to saturate earlier, thereby confirming the same trend even in much larger networks.

An efficiency loss around $6-7\%$ can be certainly considered acceptable if compared to a centralized optimum injection that would require much more signaling exchange to establish coordination among the sources. Thus, our results do not only prove that decentralized IoT systems benefit from data redundancy, but also the correlation does not need to be very high for a distributed data injection approach to be effective from a practical standpoint [17].

V. Conclusions

We studied a remote sensing system, modeled as a multi-source M/M/1. We discussed how correlation of sources can make independent data injection more efficient. We modified existing analytical results for AoI [8] through the insertion of a parameter capturing the correlation among the sources [2]. We used game theory to find the NE, and correspondingly evaluate the PoA of a distributed management vs. optimal coordination.

We showed that, even in a non-collaborative game setup, competing behaviors among the sources are partially amended by the correlation of their content. This limits aggressive data injection by the individual nodes that lean towards collaboration, rather than conflict. These findings are definitely relevant criteria for practical implementations of cyber-physical systems involving correlated sources.

Future work may also involve a comprehensive study of different queueing systems [3], or more advanced game theoretic models, also involving the coexistence of multiple types of sources and/or security aspects [24].

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