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The Philosophy of Logical Practice

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While we now have an increasingly detailed understanding of the varied goals and methods which constitute the sciences and mathematics, our understanding of logic as a research area lags behind. A significant reason for this deficiency is that, unlike in the philosophy of science and mathematics, philosophers of logic have yet to embrace a *practice-based approach* to their field, re-orientating their attention towards logic as it is actually practiced by logicians. This paper makes the case for a new area of research, the *Philosophy of Logical Practice*, to sit alongside traditional philosophy of logic and systematically investigate logic's aims and methodologies using the practice-based approach.

Keywords: Practice-based approach · Logical Practice · Methodology of Logic · Epistemology of Logic · Philosophy of Logic

1 Introduction

Contemporary philosophy of science and mathematics are typified by their detailed analysis of the various facets of scientific and mathematical methodology, whether this be the norms of model building within a field of science (Braillard & Malaterre 2015), the notion of rigor that mathematicians adhere to when evaluating proofs (Hamami 2019), or the criteria scientists use when evaluating the utility of theoretical concepts (Brigandt 2010). In all of these cases, there is an acceptance that the philosophy of a research field should be primarily based upon the actual practices of experts within the field, rather than some idealised accounts of them.

The general rationale for this *practice-based approach* is clear, if often left explicitly unsaid (Soler et al. 2014). The sciences and mathematics, just like any field of enquiry, are social enterprises, albeit rational social enterprises, whose aims, values and techniques are the result of the collective actions and decisions of its practitioners.[1] Thus, any attempt to infer a picture of what scientific methodology *should look like* based upon our preconceptions of scientific rationality, or determine what mathematical evidence *must be* given certain metaphysical assumptions about the nature of mathematical objects, are likely to distort the realities of the research areas and provide us with an impoverished view of the fields. Ultimately, such idealised accounts will be but castles in the sky, no more insightful than an account of carpentry that fails to appreciate the intricacies of being a master carpenter. In comparison, if we build our accounts of these fields upon the ways in which they are actually practiced by the experts within them, we can produce more detailed and accurate reflections of these incredible human enterprises.

Yet, whilst one of the prominent features of contemporary philosophy of science and mathematics has been the successful implementation of this "practice-based turn", the practice-based approach has yet to be systemically embraced within the philosophy of logic, with only a handful of papers explicitly using the approach to elucidate

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[1] Note, this is not equivalent to proposing that a field's object of study, or the putative facts it discovers, are social constructions. There is no clear reason to think that the practice-based approach commits one to *social constructivism* about the domain (Soler et al. 2014). This is just as true for logic. While important, addressing this point will have to wait for another occasion.

features of logic's methodology.[2] The nuanced accounts of model building within the climate sciences (Steele & Werndl 2013), and the values that underpin economic evaluations within public health (Cenci & Hussain 2019), are in stark contrast to the lack of a detailed appreciation of the various methods, aims and values that constitute logic as a research area. It is still all too common to exclusively concentrate on what logical research *should look like* given our philosophical preconceptions about logic and its subject matter, rather than facing up to the reality of research in the field. Not only has this led to an impoverished picture of portions of logic's methodology, but a total neglect of others.

Our goal here is show that this predominant neglect of logical practice has been a mistake, by outlining the rationale and potential benefits for embracing a new practice-based approach to the philosophy of logic, by way of analogy with the benefits of the approach within the philosophy of science and mathematics. Our aim is not to argue that philosophy of logic as it is currently practiced should be wholly replaced; the field as it stands has provided us with fruitful debates in some areas, such as whether we have good reason to endorse multiple logics simultaneously (Cook 2010), and what role logic plays (if any) in reasoning (Steinberger 2020). Instead, our proposal is for a new field of research, the *Philosophy of Logical Practice* (hereafter, PLP), to sit alongside traditional philosophy of logic, which is both better able to answer certain established questions about logic than traditional approaches, and has the benefit of opening up new fruitful areas of research.

Our case for PLP proceeds as follows. Section 2 briefly outlines the general rationale for practice-based approaches, using established examples from the philosophy of science and mathematics. Section 3 then introduces PLP, detailing its motivation and aims. Section 4 and 5 then highlight the case for PLP, demonstrating how the positive considerations counting in favour of the approach in the philosophy of sciences and mathematics apply equally to the philosophy of logic. Section 4 outlines how PLP's methodology differs from that of traditional approaches to the philosophy of logic, and then shows how the former's methods are better suited to answering at least one prominent existent question in the philosophy of logic. In contrast, Section 5 details how PLP adjusts the philosophy of logic's scope of enquiry, leading to new fruitful areas of research which have been neglected by traditional philosophy of logic.

2 The Practice-Based Approach

Practice-based approaches to the philosophy of a research field are jointly characterised by their: (i) dissatisfaction with more traditional philosophical approaches to the field, and (ii) positive proposal for how to fix these shortcomings (Mancosu 2008; Soler et al. 2014). Prominent uses of the approach can be found in contemporary philosophy of science and mathematics, with each containing research programmes built around the approach, in the form of the *Philosophy of Scientific Practice* (PSP) and *Philosophy of Mathematical Practice* (PMP), respectively.

[2] To note some examples: Dutilh Novaes (2012) uses the approach to identify the theoretical values that underpin our use of formal languages, Martin (2019 & 2021) uses the approach to elucidate features of disagreements within logic and types of logical evidence, respectively, and Payette & Wyatt (2018) use it to provide an account of explanations in logic.

Both PSP and PMP are motivated by the perceived inadequacy of traditional philosophical approaches to the fields, on the basis that these traditional approaches produce accounts of the sciences and mathematics which are: (i) *too idealised*, in virtue of being based upon a priori reflections of what we *want* science and mathematics to look like given our preconceptions of the fields, rather than reflecting the realities of research in these fields; (ii) *over simplistic*, in failing to reflect the plurality of the fields' aims and methodologies; (iii) *too present-centred*, by falling foul of a tendency to produce Whig histories, presuming that the fields' histories are a story of smooth and unstoppable progress up to the present state of affairs; and, (iv) *too end-product focused*, by concentrating on the properties of final theories or proofs, and thereby neglecting the important processes which led to the discovery of these results, including communal processes (Carter 2019; Mancosu 2008).

An early, and particularly prominent example of these concerns within the philosophy of science is Kuhn's (1962) criticism of Popper's (1959) falsificationism. Popper's account of the scientific method was denounced for both idealising scientific methodology, by presenting a naïve picture of scientific progress as a continual chain of evermore informative theories that perpetually become falsified, and being too present-centred, by presuming the aims and norms for evaluation of past scientific theories were the same as those of contemporary science. Further, Kuhn (1962) highlighted how past accounts of scientific methodology had been deficient by neglecting important features of scientists' research activities, such as the designing and testing of experimental equipment, and their use in measuring constants. The failure of past theories of scientific methodology to recognise the various roles of experimentation within the sciences beyond the direct testing of hypotheses was further emphasised by Hacking (1983) and other 'New Experimentalists'. In both cases, past accounts of scientific methodology were criticised for paying too little attention to the rich variety of activities constituting part of the actual scientific method.

Similar concerns have been raised by advocates of PMP, with traditional approaches to the philosophy of mathematics being criticised for possessing too idealised a picture of mathematics, with mathematical knowledge conceived wholly in the form of theorems evidenced by formal proofs (Corfield 2003). Contrary to this traditional view, it's been argued that mathematical understanding progresses in many ways, including through the abundant use of informal proofs, whose positive epistemic features cannot be reduced to those of formal proofs (Larvor 2012; Tanswell 2015). Further, in virtue of being too concerned with philosophically foundational issues, such as the metaphysics of mathematical objects and the epistemological puzzles resulting from these metaphysical pictures, traditional philosophy of mathematics has produced an over simplistic picture of the mathematical enterprise, neglecting important features of contemporary mathematics, including the appraisal of definitions (Tappenden 2018), and the use of diagrams (Giardino 2017).

Conjoined with this negative component of the approach is the positive story of how the philosophy of these fields should then proceed in order to rectify the failures. The proposed solution requires both a re-evaluation of our *aims* when providing a philosophical account of a field, and a modification in the *methods* we ought to use in order to appropriately meet these aims.

Firstly, rather than attempting to build an account of the sciences and mathematics which conform to our philosophical assumptions about the norms of rationality, possible sources of evidence, and viable metaphysical theories, we should aim instead to produce accounts which: (i) reflect the reality of research in these fields; (ii)

recognise the plurality of aims and methodologies found across them; (iii) situate results in the field within their proper historical context; (iv) recognise the development of, and changes in, the methodological norms within the fields; and (v) give equal attention to the processes of discovery as the properties of the final products (Corfield 2003; Soler et al. 2014).

Secondly, meeting these aims will require embracing new methods. To ensure that their proposals reflect actual research in the fields, philosophers will need to spend less time deliberating over how science and mathematics *could* operate given established epistemological theories and certain metaphysical assumptions about the fields, and more time looking in detail at how scientists and mathematicians actually reach their results. Much of this work will take the form of case studies, whether this be an in-depth study of the activities of an individual researcher or research group, or a wider study of the norms within a particular sub-field (van Bendegem 2014). However, historiographic studies are also commonly used, in order to trace the development of a particular prominent concept, or track evolution of the methodological norms within a field (Krantz 2011), and studies from cognitive science are sometimes embraced to inform an account of how theories are evidenced or selected for (Giaquinto 2007).[3]

Through embracing these new aims and methods, both PSP and PMP have shown themselves to have two significant benefits over traditional approaches to the fields (Soler et al. 2014). Firstly, the approach is able to provide more insightful answers to established and prominent questions about scientific and mathematical methodology than traditional philosophical approaches. Clear examples of this benefit are found within the already noted debate over what constitutes a proof within mathematics, where we now have a much more detailed appreciation of the standards of proof and why different forms of proof are valued (De Toffoli 2021; Hamami 2019), and in the longstanding debate over the nature of explanations in the sciences (Braillard & Malaterre 2015). In the latter case, before the development of the practice-based approach, it had been assumed that there must be some essential characteristic shared by all instances of explanation, and thus a unified account of the phenomenon could be provided (see, for instance, Hempel 1965). Typically, these essentialist accounts struggled, in virtue of having to cope with numerous counterexamples, and declaring a significant number of scientific explanations illegitimate (Woody 2015). In comparison, since philosophers of science dropped the essentialist assumption, and began building accounts of scientific explanation from the "bottom-up" starting with instances of explanation from various sub-fields, research in the area has flourished with detailed theories of the wide variety of forms of explanations found across the life (Brigandt 2013), medicinal (Qiu 1989), and physical sciences (Fisher 2003) being produced.

Secondly, the approach has opened up new important research questions about the fields that were previously neglected using traditional philosophical methods. This has been achieved through a combination of the approach's methodology facilitating a more detailed consideration of the activities of practitioners, and further using these activities as a means to motivate new philosophical research questions, rather than simply imposing established philosophical questions upon the fields of enquiry. For instance, the approach has led to the investigation and a growing understanding of: (i) the various sources of evidence which mathematicians rely upon, such as visualisation (Giaquinto 2007) and computer-aided proofs (Avigad 2008); (ii) what constitutes

[3] For a more detailed discussion of the various methods of enquiry used within PMP see, Hamami & Morris (2020).

scientific understanding (de Regt 2017); (iii) the characteristics that mathematicians look for when devising and choosing formal notations (De Toffoli 2017); (iv) the role of (interdisciplinary) collaboration within the sciences (Andersen 2016); and, (v) the theoretical virtues mathematicians prize within a piece of mathematics (Rota 1997).

Later, in Sections 4 and 5, we'll argue that these same considerations which count in favour of PSP and PMP apply equally to PLP. Before that, however, we need to introduce the motivations behind a practice-based approach to the philosophy of logic, and how it differs from traditional approaches to the field.

3 PLP versus the Traditional Approach

3.1 The Motivation for PLP

In general terms, PLP aims to provide us with a new approach to the philosophy of logic, more capable of reflecting the reality of research within the field of logic.[4] As PSP and PMP before it, PLP is motivated by perceived deficiencies in the way that the philosophy of logic is traditionally practiced. In particular, we can pinpoint *five* (albeit connected) concerns with the traditional approach:

(i) It produces accounts of logic which *are far too idealised*.
Philosophers will often build an account of logic based upon what they *expect* its subject matter, sources of evidence, and methodology to be given certain philosophical presumptions about logic's properties, standards of rationality, and possible sources of evidence. What results is a picture of logic which is completely at odds with that practiced by logicians.[5]

(ii) It is *too focused on traditional philosophical concerns.*
Significant time is spent on established philosophical questions which are divorced from the realities of logical research. One prominent example is the preoccupation with the *metaphysics* of logic, whether this takes the form of providing an ontology of logic (Sider 2011), or assessing the existence of logical facts and their nature (McSweeney 2018). While of potential philosophical interest, there is little to no attempt in these debates to relate the proposals to the actual means through which logics are developed or chosen by practitioners. Another example is the attempt to provide detailed accounts of traditional philosophical properties that logic putatively possesses, such as the necessity of its truths (McFetridge 1990) and the topic-neutrality of its laws (Sher 1991). Again, while potentially of philosophical interest, rarely (if ever) do these accounts attempt to show that the formal systems which are regularly developed, analysed, and applied for various purposes in the field actually possess these putative properties, or how we would expect research in the field to be conducted in light of these properties.

[4] PLP should not be mistaken for the study of everyday reasoning; it is not an ethnography of reasoners. Rather, PLP is concerned with the theoretical field of logic and the activities of its practitioners. Of course, it may turn out that everyday reasoning is a topic of interest for logicians themselves, and this is an interesting question for the philosophy of logic. However, this doesn't mean that the subject matter of PLP itself is this everyday reasoning. The distinction here is analogous to that between linguistics and the philosophy of linguistics. While linguistics is (among other things) the study of the linguistic practices of speakers, the *philosophy of* linguistics is concerned with the aims and methodology of linguistics as a field. What should count as the *field of logic* is another question, which we'll come to below. Many thanks to an anonymous referee for pushing me on this point.
[5] We'll consider an example of this concern from the epistemology of logic in Section 4.

(iii) Its conclusions are often *too synchronically homogeneous*.

In virtue of being preoccupied with traditional philosophical concerns, or too focused on certain uses of logic deemed "philosophically important", a significant portion of the projects which logicians are engaged in, and the uses these logics are put to, are neglected. While dynamic logics are commonly used to model the semantics of complex linguistic phenomena (Keshet 2018), and Church's type theory is now being used to model ethical reasoning in AI (Benzmüller et al. 2020), when it comes to discussing the aims and methods of logic, such applications are either ignored completely or dismissed as not being the canonical, philosophically significant application of logic.[6] What results is a far poorer view of the field, with prominent applications of logic and substantial practices in the field being neglected.

(iv) Its proposals are often *ahistorical* (leading to implicit *diachronic homogeneity*).

In virtue of the propensity within contemporary philosophy of logic to build idealised accounts of logic based upon certain presumptions about rationality, knowledge, and logic's properties, what tends to result is an essentialist account of logic's aims and methods, which gives the impression that logic as a field is diachronically homogeneous. However, just like the empirical sciences and mathematics, logic is a social enterprise with continually changing priorities and techniques. Given the availability of excellent resources on the history of logic (Haaparanta 2009), such presumed diachronic homogeneity is perhaps surprising. However, as we'll see in Section 5, it is prominent in the literature.

(v) It is *too end-product focused*.

Contemporary philosophy of logic concentrates primarily on the properties of *logics*, rather than on the processes which led to the formulation and evidencing of these logics, whether those of individual researchers or the community. This is troublesome not only because it can lead to us neglecting the many important techniques and methods which logicians use to develop, evidence, and apply their systems, but it can result in hasty conclusions about the field. For instance, following Haack (1978), there has been a tradition to understand (dis)agreement within logic in terms of ways in which logics as theories can (dis)agree (Stei 2020). Given that various such competing logics are still advocated in the literature, this has led some (Resnik 1999) to conclude that the persistent disagreement between candidate theories in the field is evidence that logic is more akin to ethics than the sciences. However, this neglects the significant ways in which *logicians* as participants in the debates agree, such as over: how logics should be evaluated (relative to a purpose), the relative strengths and weaknesses of the various competing theories, and what would constitute sufficient evidence for the resolution of the debate (Martin 2019). What results is a picture of theory-evaluation and debate more similar to the sciences than ethics (Martin & Hjortland 2021).[7]

[6] We'll revisit this topic in Section 5.

[7] Concerns over the lack of consensus and progress within logic also tend to be somewhat myopic, due to the ahistorical nature of contemporary philosophy of logic. Throughout the development of logic, there have been significant periods of consensus as well as recognition of progress (Haaparanta 2009). Very few contemporary logicians, for instance, deny that classical first-order logic is theoretically preferable to Aristotelian syllogistic logic as an account of mathematical reasoning, even if they admit it has its own weaknesses. This recognition should also go some way to addressing the concern that,

PLP's positive proposal is to address these shortcomings of traditional philosophy of logic through a combination of reorienting our theoretical aims back towards the practices of the field of logic, and suitability diverging from traditional approaches in terms of its methodology and scope of enquiry.

3.2 The Aims of PLP

In order to ensure we avoid producing idealised accounts of logic, putting too much focus on traditional philosophical concerns, and thereby neglecting important features of the field's actual aims and methods, PLP explicitly orientates its goals towards the concrete activities of the field, giving primacy to these activities over traditional philosophical concerns. In particular, PLP aims to:

(i) Provide the most detailed and accurate account of the aims, epistemology, and methods of logic possible, including recognising the diversity of these features of the field.

(ii) Explore the philosophical repercussions of the full diversity of logicians' activities, beyond what has traditionally been considered to be of philosophical importance.

(iii) Provide analysis of how the activities of logicians impact upon traditional philosophical topics, such as the nature of knowledge, rather than inversely how established answers to philosophical questions impact upon our view of logic (as has been standard in the philosophy of logic).[8]

Motivating each of these aims is the working assumption that a philosophy *of logic* should be concerned with logic as it exists as a living, breathing discipline, and that our philosophical theories about logic should be built upon these practices, not in spite of them. The purpose of the philosophy of logic, according to PLP, is not to use logic as an instructive case study to evidence one's choice philosophical theory, or to show how logical knowledge would be possible in light of one's favoured epistemology. This would, unjustifiably, suggest that logic as a field of research is somehow subservient to our wider philosophical theories and aims. Yet, just as with the empirical sciences and mathematics, logic is an important and diverse field of research that ought to be examined and

whereas PSP and PMP are effective because both the empirical sciences and mathematics are successful fields of enquiry, exhibiting progress and stable consensus on many matters, the same is not true of logic, and thus the case for PLP's effectiveness is weaker. We hope to discuss this matter in greater depth elsewhere.

[8] Here and elsewhere we've spoken of the *field of logic*, and allowing practices within the field to dictate our accounts of logic's aims, methodology and epistemology. Yet, one might plausibly be concerned that in order to identify and analyse these practices, one must have a prior conception of what constitutes the field of logic (and who is a member of it), which itself may be unduly influenced by one's philosophical views (unfortunately for PLP, given its motivations). Indeed, field specification is a problem for *all* practice-based approaches, given that they do not wish to excessively restrict the field's scope of enquiry due to preconceptions. Further, there is no straightforward answer. The delineation of research fields is a live research question within informetrics (Muñoz-Écija et al. 2019), and the various measures we currently have at our disposal are not guaranteed to deliver the same results. However, they do indicate that by beginning with certain paradigm cases of a logician and works within the field, we can then use citation, conference attendance, and publication venue data as reliable (if defeasible) evidence for membership in the field. While we'll provide some indication of how we foresee PLP expanding the philosophy of logic's scope of enquiry in Section 5, we hope to discuss in more detail the available means to delineate the field and the repercussions for PLP elsewhere.

analysed in its own right, and on its own field-specific terms. In other words, (logicians') practice has primacy, and it is these practices which should lead our philosophical analysis of logic.

Re-orientating the philosophy of logic's goals in this fashion requires both changes to its methodology and scope of enquiry, in comparison to traditional approaches. These changes, however, bring analogous benefits to those found with PSP and PMP. In the next section, we'll outline how PLP's methodology differs from traditional approaches, and the benefit this methodology brings in addressing established questions in the philosophy of logic. Then, in Section 5, we'll show how alterations to philosophy of logic's scope of enquiry lead to PLP opening up new fruitful areas of research, neglected by traditional approaches.

4 Benefit One: Progress on Established Questions

4.1 The Methodology of PLP

PLP diverges from the methodology of traditional philosophy of logic in two noteworthy respects. Firstly, it has a distinct *methodological starting point*. As with its predecessors in the philosophy of science and mathematics, PLP operates a "bottom-up" approach, beginning with case studies of instances of practice within the field. From these initial case studies, tentative conclusions are drawn and hypotheses proposed, to then be tested against further case studies. The aim is to steadily build up a detailed theory of particular elements of the field through a process of testing proposals against an ever-increasing sample of case studies.[9]

In comparison, philosophy of logic traditionally has used a "top-down" approach, beginning with certain philosophical presumptions about the subject (Martin 2021). These could take the form of assumptions about logic itself, such as its privileged status in virtue of its laws being formal, wholly general and necessarily true, or wider philosophical assumptions, such as established accounts of what constitutes knowledge or rational standards of enquiry. From these postulates, philosophers of logic then attempt to *infer* viable theories about logic's aims, epistemology or methods, with the adequacy of any proposal ultimately tested against the background of these presumptions, and on the basis of the theory's ability to respect them. We'll consider a detailed example of this "top-down" approach from the epistemology of logic shortly.

Secondly, the approaches differ in the *evidential priority* they accord logicians' practices and philosophical background assumptions when the two clash. PLP treats the practices of experts within the field as the most reliable evidence we have to understand the field's aims, epistemology and methods. This means that, in most cases, if these practices conflict with philosophical presumptions, whether about logic itself, or established epistemological or metaphysical theories, then it is the latter which should go in this context.[10]

[9] Similarly to PSP and PMP, it's likely that other interdisciplinary methods, such results from cognitive science, will be useful to PLP when investigating certain questions, including the theoretical and aesthetic virtues logicians prize within systems. Discussion of the possibilities these further methods offer PLP are beyond the scope of the current paper; however, we hope to explore this topic elsewhere.

[10] Why do we say *in most contexts*? We must be open to the possibility that individual members of the community can make methodological mistakes, and thus not reflect the general methodological norms of the field. This potential concern can be addressed through considering a range of case studies (Martin 2021).

In comparison, it's common within traditional philosophy of logic to dismiss cases of logical practice as irrational or unviable if they clash with certain philosophical conclusions. Take, for example, the proposal that logical laws are *constitutive of rational thought* (Leech 2015). One consequence of this view is that if one fails to adhere to the (correct) logical laws, then one isn't reasoning rationally, which has consequences for the possibility of rational disagreement in logic. After all, if by definition a logician who does not adhere to the correct logical laws (whatever these are) cannot be reasoning rationally, their possibility of engaging in a *rational* disagreement with peers over the correct logic is precluded. However, this consequence runs contrary to what we find in the literature. Not only do we find advocates of competing logics debating with one another over the comparative strengths and weaknesses of their candidates, but debates over the validity of important logical laws, such as *modus ponens* (Martin & Hjortland 2021). While for advocates of a practice-based approach, this clash with logical practice spells a devastating blow to *constitutivism*, the constitutivist themselves seems content to admit that those who challenge the logical laws putatively constitutive of rational thought cannot be providing rational considerations (Martin 2019), *even if* these challenges are taken seriously by their peers.

One of the significant benefits of PLP's "bottom-up" method, in comparison to traditional "top-down" approaches, is that it offers the opportunity of greater progress on established questions within the philosophy of logic. As an exemplar of this proposed benefit, as well as an illustration of the marked difference between the methods of traditional philosophy of logic and PLP, we'll briefly consider a prominent debate from contemporary epistemology of logic over how we come to know claims about deductive validity.

4.2 An Example from the Epistemology of Logic

While there is wide disagreement within the philosophy of logic over what exactly constitutes deductive validity, whether it is facts about our natural languages, structural features of the world, or mathematical reasoning, there is a general consensus that one of the important philosophical applications which logical systems are put to is adequately capturing this deductive validity.[11] Given this, one important question for contemporary philosophy of logic is then, how do (philosophical) logicians come to evidence their candidate logics of validity? In other words, how do we come to know claims about deductive validity (Priest 2016; Williamson 2017).

Philosophical tradition has it that logical evidence about validity must be both non-inferential and *a priori* (Martin 2021). *Non-inferential*, for otherwise one would need to presume the validity of at least *some* rules of inference to establish the reliability of the inferences which partially constitute one's justification for the logical rules (Haack 1976). *A priori* because, firstly, no observable states of affairs directly demonstrate that a rule of inference is valid, and secondly, the possibility of *inferring evidence* for particular logical laws from empirical evidence is precluded by the non-inferentiality of logical evidence (Martin 2021). Consequently, if these two starting assumptions are correct, we must have *unmediated apriori* access to the truth of logical laws about validity. This would distinguish logic's epistemology from both the empirical sciences and mathematics, with the former relying significantly upon *a posteriori* evidence, and the latter involving inferences being made to establish results (Martin & Hjortland 2021).

[11] For the background to these debates over the metaphysics and epistemology of validity, see Martin & Hjortland (forthcoming).

Beginning with this pair of commitments, discussions of the epistemology of validity have traditionally embraced a "top-down" approach, by attempting to infer how knowledge of validity could be possible while respecting these traditional assumptions. Two accounts, in particular, have dominated the philosophical landscape: *logical rationalism* and *logical semanticism* (Martin 2021). Both positions agree that the justification for logical laws of validity must be non-inferential and *a priori*, but disagree on the source of this *apriority*. While according to rationalists, evidence of validity is constituted of intuitions facilitated by a quasi-perceptual intellectual faculty, with which one simply *sees* that a particular logical law is true or inference valid (Bealer 1998; BonJour 1998), semanticists deny the need to posit a novel cognitive faculty to accommodate logical knowledge. Instead, we gain evidence for logical laws directly through linguistic proficiency. In virtue of understanding the meaning of the constituent terms of a logical law or inference, we automatically become justified in assenting to its truth or validity (Ayer 1936). In other words, logical laws are epistemically analytic (Boghossian 1996).

Importantly, neither rationalists nor semanticists are motivated in their accounts by the types of evidence logicians actually appeal to when justifying logics of validity. Instead, with the starting assumptions of logic's epistemic foundationalism and apriority, it's presumed that either rationalism or semanticism must be correct if we are to avoid the unfortunate sceptical conclusion that we don't possess knowledge of validity (Boghossian 2000). Further, the answer of which of these two candidates one should then favour is not made on the basis of which provides us with a more realistic answer to logical knowledge but rather, firstly, which is more able to avoid undesirable sceptical conclusions and, secondly, the compatibility of these accounts with further philosophical commitments their advocates embrace. For example, logical semanticists such as the logical positivists have traditionally been motivated to accept logic's analyticity on the basis of their scepticism of the existence of a special cognitive faculty providing direct rational insight into the truth of logical claims and, further, the desire to accommodate the putative necessary truth of logical laws without relying on a dubious notion of metaphysical necessity (Carnap 1963: 46). In other words, semanticists are commonly motivated by both metaphysical and epistemological naturalism (Warren 2020: Ch. 1). By embracing analyticity, the semanticist can aim to account for the necessary truth of logic's laws in terms of linguistic conventions, rather than in terms of ways the world must be. In comparison, rationalists desire to uphold the objectivity of logic which they believe the semanticist throws away by demoting logic to the status of conventions (BonJour 1998), and attempt to achieve this by rejecting naturalism and admitting both abstract non-spatiotemporal facts, and a special faculty *rational intuition* in order to access them (Katz 1998).

What has resulted is a longstanding debate between rationalists and semanticists, in which the goal has been to undermine the viability of the competitor while staving off sceptical conclusions. On the semanticist side, this has comprised both appeals to naturalism (Warren 2020: Ch. 1) and the opacity of rational intuition as a faculty (Boghossian 2000: 231) in order to undermine rationalism, and attempts to show that the permissibility of rule circularity can allow the semanticist to avoid sceptical conclusions (Boghossian 2000). In comparison, rationalists have attempted to undermine semanticism by appealing to classical Quinean concerns over the viability of analyticity and the dangers of committing ourselves to conventionalism (BonJour 1998: Ch. 2)

However, neither position has been able to make significant progress and succeed, even by its own lights. While we still lack any case for the reliability of rational intuitions as a source of knowledge (De Cruz 2014), past advocates of the semanticist approach now question the viability of the rule circularity justification for logical knowledge (Boghossian 2014). When it comes to top-down approaches to logic's epistemology, therefore, we have reached something of an impasse. What is even more concerning, however, is that both positions problematize the actual debates logicians have over the validity of rules of inference, and the forms of evidence these logicians appeal to. As has been shown through case studies, logicians appeal to a vast array of forms of evidence when arguing for their theories of validity, including the ability of their logics to: (i) solve pressing theoretical puzzles, such as the logico-semantic paradoxes; (ii) facilitate important mathematical results; and, (iii) explain why mathematicians are warranted in making certain inferential moves within informal proofs (Martin 2021; Martin & Hjortland 2021). That it's reasonable for logicians to appeal to these forms of evidence, as far as both rationalism and semanticism are concerned, is a mystery. In fact, both are committed to saying that the debates logicians are engaged in are epistemically inappropriate, in virtue of not being based on the forms of evidence sanctioned by their respective accounts (Martin 2021). Thus, the "top-down" approach has not only produced an impasse, but accounts which problematize actual logical debates.

In comparison, through recent attempts to build an account of the epistemology of validity from the "bottom up", beginning with the reasons logicians actually give to support their theories of validity, the practice-based approach has shown itself to have three advantages over the "top-down" approach. Firstly, in virtue of taking the forms of evidence that logicians actually appeal to as a given, and building an account of logic's epistemology up from these practices, the resulting theory should not problematize actual logical debates. Secondly, there's good reason to think that the approach will not lead to an impasse. The practice-based approach provides clear criteria for the success of an account of the epistemology of validity, in terms of the ability of the proposal to make sense of the forms of evidence logicians actually appeal to and the types of debates they engage in. Of course, having this clear set of criteria does not preclude that for a period of time we may have several candidate proposals that are equally viable. However, it does mean that we can expect over time for a particular proposal to gain greater traction than its competitors, in virtue of its expectations regarding logicians' practice to be met to a greater degree as more cases are considered.

Thirdly, while relatively young in comparison to "top-down" approaches, attempts to produce an account of the epistemology of validity using the practice-based approach have already delivered more detailed accounts of the mechanisms by which we come to be justified in believing logical theories than "top-down" approaches. For instance, Martin & Hjortland (2021) have shown how we can make sense of many of the forms of evidence logicians appeal to within a *predictivist* model of logical justification. According to this model, logical theories of validity are evidenced by their ability to: (i) produce successful predictions about which inferences mathematicians will deem acceptable within informal proofs; (ii) explain why these particular inferences are acceptable in virtue of their form; and, (iii) establish their compatibility with other well-established commitments, such as mathematical results, through the logico-semantic paradoxes. Whilst the *predictivist* model may well, ultimately, be found to be flawed in some regard (as the practice-based approach would expect), the detailed

account of logic's epistemology which the approach has been able to produce already (albeit tentatively) evidences its fruitfulness.

Combined, these considerations give us good reason to think that, over time, the "bottom-up" practice-based approach will be able to provide us with a more detailed and nuanced account of the epistemology of validity than "top-down" approaches. This is just one prominent example of the benefit PLP provides over traditional approaches, in offering progress to established debates within the philosophy of logic. The benefits with PLP do not stop here, however. In the next section, we'll highlight how PLP also opens up new fruitful areas of enquiry in virtue of expanding the philosophy of logic's scope of enquiry.

5 Benefit Two: New Fruitful Areas

As we made note of in Section 3, traditional philosophy of logic typically concentrates on established philosophical questions and how they relate to logic, be it the rationality *of logic*, the metaphysics *of logic*, or the epistemology *of logic*. Further, attention is paid almost exclusively to logics as objects, whether discussing how logics can provide normative guidance to reasoning, or in what sense different logics can disagree with one another. In addition, when focus *is* put on the aims and methodology of logic, consideration is restricted to those particular aims of logic which philosophers are interested in, and how logics might be selected for in accordance with these purposes, thereby neglecting multiple other applications of logic. In this section, we outline three regards in which PLP widens philosophy of logic's scope of enquiry, and highlight three corresponding new and fruitful research questions PLP raises in virtue of doing so.

5.1 Pure Logic

Firstly, PLP recognises that logic as a field is concerned with both developing formal systems and studying these systems for their own sake (sometimes called, *pure* or *mathematical* logic), and further applying these formal systems to various phenomena for different purposes (often known as, *applied* logic). While contemporary philosophy of logic is well aware of this distinction (Priest 2006), little to no attention is ever paid to pure logic, in terms of its aims, content and methods. This is a mistake, not only because pure logic constitutes a significant portion of the field of logic as a whole, but because by considering its activities we should gain a better understanding of how innovations within pure logic lead to breakthroughs in applied logic, just as technical innovations tend to lead to breakthroughs in the sciences, and the relationship between methodological norms within pure logic and (other) fields of mathematics. Answering both of these questions are important for philosophers of logic, given the recent interest in the extent to which logic is methodologically distinct from both mathematics (Sagi 2021) and the empirical sciences (Martin & Hjortland 2021).

Let's focus momentarily on one notable research question that arises from widening philosophy of logic's scope to include consideration of pure logic: which properties of formal systems are logicians interested in establishing, and why?

Just a cursory look at recent work in the field highlights the multitude of results which logicians are interested in establishing. Firstly, logicians are concerned to establish that a given system has certain desirable

properties. This can take the form of establishing a widely-regarded beneficial property of logic, such as their decidability (Payne 2015; Wintein & Muskens 2016), but it also often takes the form of solving a non-generic open problem for a particular logic (Badia 2018; Uckelman et al. 2014). For instance, Slaney & Walker (2014) set about establishing that the pure implication fragment of Anderson & Belnap's (1975) logic **T** has infinitely many pairwise non-equivalent formulae in one propositional variable; a particular instance of the problem set by Meyer (1970) for all substructural logics. Such cases raise interesting questions for the philosopher of logic. While a logic's decidability has clear practical implications when it comes to applying the logic in multiple contexts, the matter is less clear for some of these logic-specific open problems. Is there any shared characteristic behind the open problems logicians choose to address, rather than those they deem less important? Further, are these open problems motivated by community-wide desiderata for a formal system, or research-programme specific?

The import of this latter question is further raised by logicians' interest in demonstrating the limitations of a given system. While these undesirable characteristics can have clear connections to widely shared desiderata of a system, as is the case with Kosterec's (2020) demonstration that the definition of substitution in Transparent Intensional Logic leads to a contradiction in the system, and Randall Holmes' (2018) proof that monadic third-order logic cannot provide a general representation of functions, they need not. For instance, Yang (2013) demonstrates that the (putatively) relevant logic **R** fails to satisfy Anderson & Belnap's (1975) own relevance principle. Yet, of course, adherence to a relevance principle is a not a desideratum shared by the whole logic community. It is, therefore, not only an interesting question which characteristics are considered (un)desired within a formal system, and the relation of these desiderata to potential applications of logics, but whether these desiderata are shared community wide, or peculiar to particular research programmes in the field.[12] The benefit of PLP is not only that it provides us with a means to generate these interesting research questions, but provides us with the clear and concrete means to go about addressing them.

5.2 Non-Philosophical Applications

Secondly, rather than restricting the philosophy of logic to those applications of logics which are deemed philosophically interesting (for whatever reason), PLP considers the full array of applications of logic, whether this be the use of fuzzy logics to model national incomes (Ferrer-Comalat et al. 2020), or possible-world semantics to model belief revision (Grove 1988). Considering the full diversity of these applications of logic is important, for it is only under these conditions that we can appropriately judge the fruitfulness of logic as a field, the full extent and diversity of its methodological norms, and whether similar standards of evaluation persist across all of logic's applications. Further, again, these questions are clearly important to philosophers, as the extent to which logic as a field possesses certain methodological features which align it with the sciences is a prominent debate in the literature (Martin & Hjortland 2021; Williamson 2017).

[12] We raise here two further questions worthy of future study. Firstly, we've mentioned that logicians are keen to establish a given system's decidability. This is understandable. However, they are often interested in providing *multiple* sound and complete derivation systems for the logic (cf. Kamide 2018), without any clear rationale. What is the motivation here? Secondly, logicians are often interested in developing the resources to study a range of logics simultaneously, even if most of the (comparative) properties of these logics are already known (cf. Kamide 2021). Does this suggest that the development of new powerful tools is considered a good in and of itself, potentially for promising future use?

In addition, considering the array of logic's applications should help us address and evaluate certain prominent presumptions within the philosophy of logic, such as the longstanding assumption that there is some *primary* or *canonical* purpose to logic. While this assumption goes back at least as far as the terminist tradition in the 12th century (Lu Adler 2018: Ch. 2), when nominalists and realists argued over whether logic's purpose was to capture the relationship between the *meaning* of terms, or rather the *nature of objects* terms denote, it still has its advocates now (Priest 2006). This is a little surprising, given that with the development of mathematical logic came the recognition that logical systems can be applied to distinct phenomena with a myriad of purposes. Kripke frames, for instance, are used to model multiple agents' mental attitudes within multi-agent systems in artificial intelligence (Wooldridge 2009), and linear logic has been fruitfully applied to model meaning composition in formal semantics (Dalrymple 2001). Yet, even given the acknowledgment of this fact, we find contemporary philosophers proposing that logical systems have a *canonical application*, whether this be the "analysis of reasoning" (Priest 2006: 196) or the "codification of logical consequence in natural language" (Cook 2010: 495).

Unfortunately, no detailed defences accompany these proposals. In the background seems to be a presumed *essentialism* about the aims of logic, based upon the perceived purposes of logic according to founding figures of the field, such as Aristotle and Tarski (Cook 2010: 495). Yet, even if historically accurate, any such appeal to historical precedent is bound to fall foul of the *embryonic fallacy*: the presumption that an activity has the same aims and purposes as when it was initially developed. Astronomy, for example, was originally developed with the purpose of facilitating astrological predictions (Campion 2008), yet that is far from the field's purpose now. Just as the techniques at disposal within a field change over time, so can its primary aims.

A strength of the practice-based approach is that, by explicitly recognising logic as a living breathing social activity constituted by the decisions and actions of its practitioners, it frees us of the need to assume that there is some primary or canonical aim which *defines* logic. The aims and purposes of logic are those given to it by its practitioners, and each deserve to be fully explored. By becoming over preoccupied with one possible purpose of logic, it is likely that our view of logic's methods and epistemology will also be too narrow. After all, we should expect that practitioners will use methods and sources and evidence suitable to achieving their theoretical aims (Woody 2015). Consequently, the practice-based approach not only facilitates us exploring the full range of purposes to which logicians put their theories, but in doing so opens up the possibility that logic's methodology is not homogeneous. A research question that itself the practice-based approach is well-suited to investigate.

5.3 Theory Development

Lastly, in virtue of being an exemplar of the practice-based approach, PLP is not primarily interested in logics as objects of study, but rather in logic as a field of research, including the many activities and norms that constitute the field. This includes the development of formal systems, the development of techniques in order to prove some

desired results, and the communal processes that lead to the ultimate acceptance or rejection of a logic as (un)interesting or (un)successful for some given purpose.[13]

An excellent example of one of these processes worthy of study is the development and assessment of *concepts*. It is well recognised within the philosophy of science that scientists are engaged not only in theory choice but also concept development and choice, evaluating the relative fruitfulness of theoretical concepts (Brigandt 2010). Recent work has also been carried out on the criteria mathematicians use in assessing concepts (Tappenden 2008). In comparison, due to the neglect of a practice-based approach, we have lacked the resources to consider instances of concept development within logic, and identify the means through which logical concepts are evaluated and chosen. Yet, it's quite clear that just as in the empirical sciences and mathematics, concepts are developed in order to solve theoretical problems in the field. For instance, Henkin's (1949) introduction of the concept MAXIMALLY CONSISTENT SET in order to construct his completeness proof for first-order logic, and Kripke's (1963) development of the novel concepts of RELATIONAL FRAME and ACCESSIBILITY RELATION forming part of his relational semantics, providing a much-needed semantics for modal and intuitionistic logics. The question of how these concepts are developed, and their success assessed, is of interest not only to those working in the philosophy of language on the topic of conceptual engineering (Cappelen 2018), but again for the live question of the extent to which logic's methodology mirrors that of the empirical sciences.

6 Conclusion

Unlike the philosophy of science and mathematics, the philosophy of logic has yet to recognise the importance of building its understanding of the field upon the actual practice of its researchers. Our goal here has been to provide some initial motivation for embracing a practice-based approach within the philosophy of logic, showing that those considerations which justified a practice-based turn within the philosophy of science and mathematics apply equally to the philosophy of logic. While it should be recognised that more needs to be done in terms of vindicating PLP's promise through further successful applications of its method, these considerations should provide philosophers of logic with sufficient reason to take the approach seriously and explore its fruitfulness.

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[13] The question of which practices or activities of a field, precisely, we should concentrate on when using a practice-based approach is a difficult question not only or PLP, but for PSP and PMP. Some within PMP (van Kerkhove & van Bendegem 2004) have attempted to provide frameworks in which to think about the relevant mathematical practices worthy of study. While it may in the future be viable (and fruitful) to provide such a framework in which to understand the scope of logical *practices*, we trust such a framework isn't necessary to either defend PLP or engage in its activities.

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