Replication Read Me for: Intergenerational Insurance

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This document provides guidance for replicating the results in *Intergenerational Insurance*.[1] The codes are generated using MATLAB (codes were last run with version MATLAB R2023b) and STATA (codes were last run with version StataMP 16). The MATLAB code can be readily replicated using any programming language that implements the pseudocode for numerical algorithms outlined in Part F of the Supplemental Material.

File Structure

The replication material contains the following six folders:

(1) *Three State Case* includes MATLAB scripts to generate outputs for policy functions corresponding to the three-state example outlined in Sections IV and VI of the article

(2) *Two State Case* includes MATLAB scripts to generate outputs for policy functions corresponding to the two-state example outlined in Section IX of the article

(3) *Supplemental Material* includes both STATA do-files to generate outputs outlined in Part A of the Supplemental Material, which includes descriptive statistics of cross countries comparison, and MATLAB scripts to generate outputs outlined in Parts C of the Supplemental Material, which includes comparatives statics on risk measures, and Part D of the Supplemental Material, which includes the impulse response function for a demographic shock

(4) *Matlab_functions* includes MATLAB subroutine functions called by MATLAB scripts located within folders (1), (2), and (3)

(5) *Stored_Figure* includes MetaPost input files utilized for plotting the figures in the style reported in the article

(6) *Stored_File* stores the output file generated by the MATLAB scripts described below

Detailed instructions for replication are reported below and the list of all files included in the replication material is reported in Table 1.

Description of the Matlab Codes (m.files)

[1] Lancia, Francesco, Alessia Russo, and Tim Worrall. 2024. "Intergenerational Insurance". *Journal of Political Economy*, forthcoming.

1. **MATLAB Codes for the Three-state Case**

   All codes for the numerical analysis with 3 endowment states are contained in the *Three State Case* folder. The codes generate outputs using the following parameters configuration: Logarithmic utility, no aggregate shocks and endowment shares for the young equal to $s(1) = 0.5$, $s(2) = 0.625$, and $s(3) = 0.8125$, whose realization probabilities are $\pi(1) = 0.5$ and $\pi(2) = \pi(3) = 0.25$. The discount factors are $\beta = \delta = \exp(-1/75)$.

   The m.files must be executed in the order described below:

   i. **Generate_policy_3s.m**: Generates the value function $V(s, \omega)$ and policy functions $f(s, \omega)$ and $g_r(s, \omega)$ (see, Lemmas 1-3 of the article), calling the following MATLAB functions: cheby_points.m, utility.m, Value_Iteration_3state.m

      - Note: The MATLAB function Value_Iteration_3state.m is based on Algorithm 2 described in Part F of Supplemental Material

      Output file: Policy_3state.mat

   ii. **Bond_policy_3s.m**: Generates debt policy functions $b_r(d)$ (see, Corollary 1 of the article), and fiscal reaction function $\tau(d)$ given by Equation (16) of the article, calling the MATLAB function utility.m and using the output file Policy_3state.mat from Point 1(i)

      Output file: Debt_policy_3state.mat

   iii. **Figure2.m**: Plots the sample path illustrated in Figure 2 of the article in MATLAB .fig format using the output file Policy_3state.mat from Point 1(i)

   iv. **Figure3.m**: Plots debt policy functions and the bond revenue illustrated in Figure 3 of the article in MATLAB .fig format using the output file Debt_policy_3state.mat from Point 1(ii)

2. **MATLAB Codes for the Two-state Examples**

   All codes for the numerical analysis with 2 endowment states are contained in the *Two State Case* folder. The codes generate outputs using the following parameters configuration: Logarithmic utility, no aggregate shocks and endowment shares for the young equal to $s(1) = \kappa - \epsilon$ and $s(2) = \kappa + \epsilon$ with $\kappa = 3/5$ and $\epsilon = 1/10$, whose realization probability is $\pi(1) = 0.5$. The discount factors are $\beta = \delta = \exp(-1/75)$.

   The m.files must be executed in the order described below:

   i. **Generate_policy_2s.m**: Generates the value function $V(s, \omega)$ and policy functions $f(s, \omega)$ and $g_r(s, \omega)$ (see, Lemmas 1-3 of the article), calling the following MATLAB functions: utility.m, Autarky.m, Maxstat.m, Intconv.m, cheby_points.m, weight.m, Value_Iteration.m

      - Note: The MATLAB function Value_Iteration.m is based on Algorithm 2 described in Part F of the Supplemental Material

Output file: Canonical_policy.mat

ii. **Bond_policy_2s.m**: Generates debt policy functions $b_r(d)$ (see, Corollary 1 of the article), and fiscal reaction function $\tau(d)$ given by Equation (16) of the article, along with yields on the k-period bond $y^k(x)$ (see, Proposition 6), and long-run distribution ϕ and the Martin-Ross bound Υ (see, Proposition 8), calling the MATLAB function utility.m and using the output file Canonical_policy.mat from Point 2(i)

- Note: The invariant distribution is computed using Algorithm 3 described in Part F of the Supplemental Material

Output file: Debt_policy_2state.mat

iii. **Figure4.m**: Plots the invariant distribution and the multiplicative risk premium illustrated in Figure 4 of the article in MATLAB .fig format using the output file Debt_policy_2state.mat from Point 2(ii)

iv. **Shooting.m**: Generates the policy functions calling the MATLAB function saddle.m

- Note: The MATLAB function saddle.m is based on Algorithm 1 described in Part F of the Supplemental Material

3. **MATLAB Codes for Part C of the Supplemental Material**

All codes for the numerical analysis in Part C of the Supplemental Material are contained in the *Part C* subfolder inside the *Supplemental Material* folder. The m.files must be executed in the order described below:

i. **Comparative_betadelta.m**: Computes the comparative statics of various risk measures with respect to changes in the discount factor, calling the following MATLAB functions: utility.m, Autarky.m, Maxstat.m, Intconv.m, cheby_points.m, Value_Iteration.m, Matrices.m

Output file: Comparative_betadelta.mat

ii. **Comparative_epsilon.m**: Computes the comparative statics of various risk measures with respect to changes in the variability of endowment shares, calling the following MATLAB functions: utility.m, Autarky.m, Maxstat.m, cheby_points.m, Intconv.m, Value_Iteration.m, Matrices.m

Output file: Comparative_epsilon.mat

iii. **Comparative_kappa.m**: Computes the comparative statics of various risk measures with respect to changes in the mean of endowment shares, calling the following MATLAB functions: utility.m, Autarky.m, Maxstat.m, Intconv.m, cheby_points.m, Value_Iteration.m, Matrices.m

Output file: Comparative_kappa.mat

iv. **FigureC1.m**: Plots the comparative statics on various risk measures illustrated in Figure C.1 of the Supplemental Material in MATLAB .fig format using the output files Comparative_betadelta.mat from Point 3(i), Comparative_epsilon.mat from Point 3(ii), and Comparative_kappa.mat from Point 3(iii)

4. **MATLAB Codes for Part D of the Supplemental Material**

   All codes for the numerical analysis in Part D of the Supplemental Material are contained in the *Part D* subfolder inside the *Supplemental Material* folder. The m.files must be executed in the order described below:

   i. **IRF_demo.m**: Generates the impulse response functions following a temporary demographic shock, using the output file Canonical_policy.mat from Point 2(i) after calling the following MATLAB functions: utility.m, Autarky.m, Maxstat_n.m, cheby_points.m, Value_Iteration_demo.m

      Output file: IRF_demo.mat

   ii. **FigureD1.m**: Plots the impulse response functions for a demographic shock that increases the population size in both the first-best case and the case with limited enforcement, similar to the scenario illustrated in Figure D.1 of the Supplemental Material, in MATLAB .fig format, using the output file IRF_demo.mat from Point 4(i)

Approximate time needed to reproduce the analyses on a standard 2023 desktop machine is less than 2 minutes. The number of iterations typically ranges between 10 and 50 for each script.

Description of the STATA Codes (do.files)

The codes for the descriptive analysis in Part A of the Supplemental Material are contained in the *Part A* subfolder inside the *Supplemental Material* folder. The LIS data is available upon registration to registered researchers only and and exclusively for non-commercial purposes. See www.lisdatacenter.org/data-access/lissy/eligibility/ for information on eligibility. The files are described below:

- **code_LIS.txt**: Contains the sequence of commands to remotely generate the dataset Data_FigureA1.dta based on LIS data[2]
- **FigureA1.do**: Plots the time series of the relative income measure for the young and the old illustrated in Figure A.1 of the Supplemental Material in STATA .gph format

  Data file: Data_FigureA1.dta

Source Code for Figures

All figures, in the format as they appear in the article, can be compiled by running the MetaPost file MS20211202R3.mp inside the *Stored_Figure* folder. The metapost file for the paper figures uses standard files graph.mp and hatching.mp

[2]Luxembourg Income Study (LIS) Database. 2024. Data retrieved for several countries, 13th January 2024. Luxembourg: LIS, https://www.lisdatacenter.org

- **Figure** 1**. Deterministic Policy Function**

  Output file: MS20211202R3-1.eps

- **Figure** 2. **Sample Path of the Young Consumption Share**

  Data file: SamplePath.d

  – Note: The source code for generating SamplePath.d is Figure2.m (see, Point 1(iii))

  Output file: MS20211202R3-2.eps

- **Figure** 3**. Debt Dynamics & Bond Revenue Function**

  – Note: The source code for generating the data points used in the MS20211202R3.mp is Figure3.m (see, Point 1(iv))

  Output file: MS20211202R3-3.eps

- **Figure** 4. **Invariant Distribution & Multiplicative Risk Premium**

  Data files: MRP.d and invidst.d

  – Note: The source code for generating MRP.d and invidst.d is Bond_policy_2s.m (see, Point 2(ii))

  Output file: MS20211202R3-4.eps

All figures, in the format as they appear in the Supplement Material, can be compiled by opening the *Stored_Figure* folder and running the MetaPost file SupMat.mp inside the *Supplemental Material* subfolder. The metapost file for the supplemental material uses graph.mp and latexmp.mp. It needs to be run twice with the switch -tex=latex

- **Figure A.**1**. Relative Income of Young and Old for six OECD Countries**

  Data files: Germany.d, Italy.d, Norway.d, Spain.d, UK.d and US.d

  – Note: The source code for generating the set of .d files used as input is Data_FigureA1.dta

  Output file: SupMat.1 (EPSF format)

- **Figure C.**1**. Comparative Statics**

  Data files: AfigC1.d, BfigC1.d, CfigC1.d, DfigC1.d, EfigC1.d, FfigC1.d, GfigC1.d, HfigC1.d and IfigC1.d

  – Note: The source codes for generating the set of .d files used as input are Comparative_betadelta.m, Comparative_epsilon.m and Comparative_kappa.m (see, Point 3(iv))

  Output file: SupMat.2 (EPSF format)

- **Figure D.**1**. Impulse Response Functions for a Demographic Shock**

  Data files: DemoShock.d and FigD1.d

  – Note: The source code for generating DemoShock.d and FigD1.d is IRF_demo.m (see, Point 4(ii))

  Output file: SupMat.3 (EPSF format)

Table 1: File List

Folder	File Name	File Description	Application
Matlab_function/	Autarky.m	Matlab function to generate the autarky allocation	MATLAB
	cheby_points.m	Matlab function to generate Chebychev nodes	
	Intconv.m	Matlab function to generate the first-best allocation	
	Maxstat.m	Matlab function to generate the maximum feasible promise utilities	
	Matrices.m	Matlab function to generate the transition probability matrix with $s = 1, 2, 3$	
	Maxstat_n.m	Matlab function to generate the maximum feasible promise utilities with $n > 1$	
	saddle.m	Matlab function to compute the saddle path of consumption with $s = 1, 2$	
	utility.m	Matlab function to define the type of utility function	
	Value_Iteration.m	Matlab function to approximate policies with $s = 1, 2$	
	Value_Iteration_demo.m	Matlab function to approximate policies with $s = 1, 2$ in the case of a demographic shock	
	Value_Iteration_3state.m	Matlab function to approximate policies with $s = 1, 2, 3$	
	weight.m	Matlab function to give weights to grid points	
Store_File/	Canonical_policy.mat	Output file of Generate_policy_2s.m	MATLAB
	Comparative_betadelta.mat	Output file of Comparative_betadelta.m	
	Comparative_epsilon.mat	Output file of Comparative_epsilon.m	
	Comparative_kappa.mat	Output file of Comparative_kappa.m	
	Debt_policy_2state.mat	Output file of Bond_policy_2s.m	
	Debt_policy_3state.mat	Output file of Bond_policy_3s.m	
	IRF_demo.mat	Output file of IRF_demo.m	
	Policy_3state.mat	Output file of Generate_policy_3s.m	
Two State Case/	Bond_policy_2s.m	Matlab script to generate policies $b_r(d)$, $\tau(d)$, $y^k(x)$, ϕ, and Υ with $s = 1, 2$	MATLAB
	Figure4.m	Matlab script to plot Figure 4	
	Generate_policy_2s.m	Matlab script to generate the value $V(s, \omega)$ and policies $f(s, \omega)$ and $g_r(s, \omega)$ with $s = 1, 2$	
	Shooting.m	Matlab script to replicate Algorithm 1 described in Part F of the Supplemental Material	
Three State Case/	Bond_policy_3s.m	Matlab script to generate policies policies $b_r(d)$ and $\tau(d)$ with $s = 1, 2, 3$	MATLAB
	Figure2.m	Matlab script to plot Figure 2	
	Figure3.m	Matlab script to plot Figure 3	
	Generate_policy_3s.m	Matlab script to generate the value $V(s, \omega)$ and policies $f(s, \omega)$ and $g_r(s, \omega)$ with $s = 1, 2, 3$	

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Table 1: Continued from previous page

Folder	File Name	File Description	Application
Supplemental Material/Part C	Comparative_betadelta.m	Matlab scripts to compute the comparative statics with respect to δ or β	MATLAB
	Comparative_epsilon.m	Matlab script to compute the comparative statics with respect to ϵ	
	Comparative_kappa.m	Matlab script to compute the comparative statics with respect to κ	
	FigureC1.m	Matlab script to plot Figure C.1 in the Supplemental Material	
Supplemental Material/Part D	IRF_demo.m	Matlab script to compute impulse response function for demographic shock	MATLAB
	FigureD1.m	Matlab script to plot Figure D.1 in the Supplemental Material	
Supplemental Material/Part A/	code_LIS.txt	Sequence of commands to remotely generate the dataset based on LIS data	Any Text Editor
	Data_FigureA1.dta	STATA data-files generated by remotely access to LIS data	STATA
	FigureA1.do	STATA do-file to plot Figure A.1 in the Supplemental Material	
Stored_Figure/	MS20211202R3.mp	MetaPost file for figures in the format as in the article	MetaPost
	MRP.d	Data files for Figure 4	Any Text Editor
	invdist.d		
	SamplePath.d	Data files for Figure 2	
	MS20211202R3-1.eps	EPS file of Figure 1	Any EPS Viewer
	MS20211202R3-2.eps	EPS file of Figure 2	
	MS20211202R3-3.eps	EPS file of Figure 3	
	MS20211202R3-4.eps	EPS file of Figure 4	
Stored_Figure/ Supplemental Material/	Germany.d	Data file for Figure A.1	Any Text Editor
	Italy.d		
	Norway.d		
	Spain.d		
	UK.d		
	US.d		
	DemoShock.d	Data files for Figure D.1	
	FigD1.d		

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Table 1: Continued from previous page

Folder	File Name	File Description	Application
Stored_Figure/Supplemental Material	AfigC1.d	Data files for Figure C.1	Any Text Editor
	BfigC1.d		
	CfigC1.d		
	DfigC1.d		
	EfigC1.d		
	FfigC1.d		
	GfigC1.d		
	HfigC1.d		
	IfigC1.d		
	SupMat.mp	MetaPost file for figures in the format as in the Supplemental Material	MetaPost
	SupMat.1	EPSF file of Figure A.1 in the Supplemental Material	Postscript Viewer
	SupMat.2	EPSF file of Figure C.1 in the Supplemental Material	
	SupMat.3	EPSF file of Figure D.1 in the Supplemental Material	