Life-Cycle of Structures and Infrastructure Systems – Biondini & Frangopol (Eds)

3D collapse mechanisms of masonry bridges subjected to horizontal actions

L. Niero, P. Zampieri & C. Pellegrino
Department of Civil, Environmental and Architectural Engineering, University of Padova, Padova, Italy

ABSTRACT: A large number of existing masonry arch bridges are still in use in the Italian roadway and railway networks. Most of them were built more than one hundred years ago, designed considering only gravitational loads without any seismic analysis. To keep these works in service, it is necessary to have tools that can describe and predict the structural behavior of the bridge when subjected to extreme actions. For this reason, a three-dimensional rigid-block analysis for masonry arch bridges has been developed. In the model, the main structural masonry elements (vault, pier, abutments, spandrel wall) are discretized as an assemblage of rigid blocks, which interact via no-tension contact surface with Coulomb friction. This approach allows reproducing with good accuracy the tri-dimensional collapses mechanisms of some real masonry arch bridges recently collapsed due to extreme events.

1 INTRODUCTION

Masonry is one of the most widely used materials in ordinary structures and monumental buildings in Italy and, for this reason, there are several masonry arch bridges in the Italian road and rail network which are still in service today. Most of them were built more than a hundred years ago and were designed considering only gravity loads, without taking into account any seismic or extreme actions. For this reason, several existing bridges are characterized by structural and functional deficiencies: first, they were designed for less severe service conditions than those required today and, in addition, besides suffering progressive damage and natural deterioration, they can also be subjected to extreme actions such as earthquake, flooding and foundation settlements. In addition, flood-induced scour problems can occur on masonry piers because of the erosive effect and removal of bank material from bridge foundations located within the riverbed (Ragni, et al., 2019; Scozzese, et al., 2019).

To keep existing masonry arch bridges in service, it is necessary to have tools that can describe and predict their behavior. Currently, in the literature there are several structural models that enable the analysis of these structures up to the collapse condition (Zizi, et al., 2022; Galassi, 2023; Galassi & Zampieri, 2023) but, one of the most popular is rigid blocks analysis, which quickly provides an estimation of the carrying capacity of the structure and the associated failure mechanism. Kooharian (1952) and Heyman (1969) were among the first to apply plastic limit analysis theorems to masonry structures, assuming constituent blocks have infinite compressive strength, joints have zero tensile strength and sliding failures are not allowed. In the 1970s Livesley (1978) developed a formal linear programming (LP) procedure to compute the load factor of structures and plot their collapse mechanisms, considering also sliding failures. A similar approach was used by Melbourne & Gilbert (1995) to successfully model two-dimensional masonry arch bridges, assuming an associative friction model governed by Mohr-Coulomb law. Subsequently, to achieve more realistic behavior of structures, several authors such as Gilbert (2006), Ferris & Tin-Loi (2001), Orduña & Lourenço (2003) extended the formulation of the problem considering also a finite compressive strength of the material and a non-associated flow law. Some authors

DOI: 10.1201/9781003323020-193

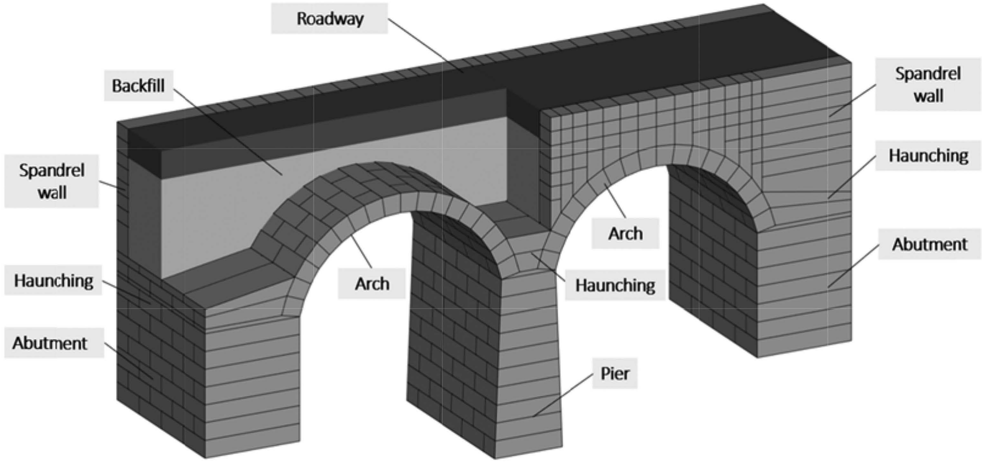

Figure 1. Three-dimensional rigid block model of a masonry arch bridge.

like Orduña & Lourenço (2005) and more recently Portioli et al. (2014) and Cascini et al. (2020) have proposed a three-dimensional formulation of the problem by applying it to out-of-plane loaded walls and arches.

This paper aims to develop a three-dimensional rigid block analysis for masonry arch bridges, capable of capturing the out-of-plane behavior of the structures subjected to extreme actions. The main structural masonry elements, such as vaults, pier, abutments and spandrel wall are discretized as an assembly of rigid blocks, interacting through contact surfaces with no-tensile strength and Mohr-Coulomb friction. The developed code, based on rigid-block analysis, defines the collapse multiplier and the associated failure mechanism without taking load history into account, using a linear optimization procedure and an associated flow law.

2 THE RIGID BLOCK MODEL

To analyze the three-dimensional behavior of the masonry arch bridge in Figure 1, its structural elements (arches, central pier, side abutments and spandrel wall) are modeled using a set of rigid blocks to study the interaction between the different elements. Instead, the backfill and road pavement are considered as external forces acting on the structural masonry elements, proportional to their load area.

Rigid blocks typically have larger geometric dimensions than physical blocks to also account the mortar joint, which is not modeled explicitly. This approach is particularly appropriate for analyzing existing masonry structures characterized by poor quality mortar (Baggio & Trovalusci, 1993). In fact, some experimental tests (Trovalusci, 1992) have validated the use of discrete, rather than homogeneous and isotropic, models and have shown how the global behavior of the structure is strongly influenced by the size, arrangement and orientation of the different elements rather than by their mechanical properties. For this reason, in the rigid-block analysis the collapse multiplier is obtained from local equilibrium conditions, assuming the hypothesis of: i) absence of tensile stresses at the interface and ii) the impossibility of crushing failure. In fact, masonry bridges are old structures subjected to cyclic loading, fatigue phenomena and freeze-thaw cycles for a long time so it is realistic to assume zero tensile strength in mortar joints. Finally, the attritive resistance at the interface is considered using the Mohr-Coulomb law and an associated flow law. The presented model is therefore capable of predicting the three-dimensional behavior of a masonry arch bridge subject to both vertical loads and hazard events such as earthquakes. In this paper, the constraint and loading conditions shown in Figure 2 are applied to the discrete model of a masonry arch bridge.

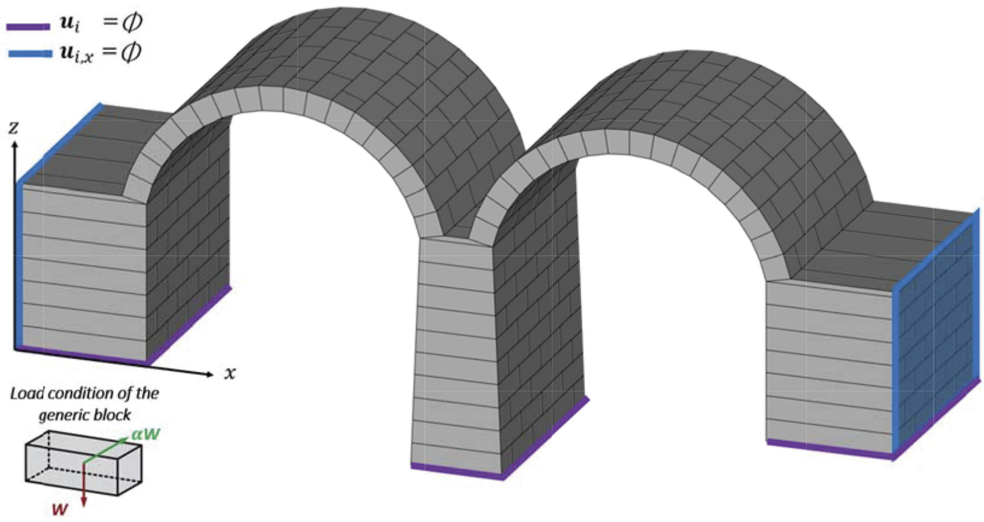

Figure 2. Loading and constraint condition of the masonry arch bridge.

3 LINEAR PROGRAMMING FOR RIGID-BLOCK ANALYSIS

The numerical model of a masonry arch bridge is composed of n rigid blocks and m contact points, located at the vertexes of interfaces (Figure 3). External loads and the displacement rates applied to the centroid of each 3D rigid block are respectively collected in vector $\boldsymbol{f} \in \mathbb{R}^{6n}$ and $u \in \mathbb{R}^{6n}$:

$$\boldsymbol{f} = \left[f_{x1},\ f_{y1},\ f_{z1},\ m_{x1},\ m_{y1},\ m_{z1}, \ldots \ldots, f_{xn},\ f_{yn},\ f_{zn},\ m_{xn},\ m_{yn},\ m_{zn}\right]^T \tag{1}$$

$$\boldsymbol{u} = \left[u_{x1},\ u_{y1},\ u_{z1},\ u_{\theta x1},\ u_{\theta y1},\ u_{\theta z1}, \ldots \ldots, u_{xn},\ u_{yn},\ u_{zn},\ u_{\theta xn},\ u_{\theta yn},\ u_{\theta zn}\right]^T \tag{2}$$

The loads in $\boldsymbol{f}$ can be expressed as the sum of the known dead loads $\boldsymbol{f_D}$ and live loads $\boldsymbol{f_L}$, multiplied by an unknown scalar factor α:

$$\boldsymbol{f} = \boldsymbol{f_D} + \alpha \boldsymbol{f_L} \tag{3}$$

The three internal resultant forces (i.e., axial force n_k, shear force t_{1k} and t_{2k}) acting at each k-th point of contact can be arranged in vector $\boldsymbol{x} \in \mathbb{R}^{3m}$:

$$\boldsymbol{x} = [n_1,\ t_{11},\ t_{21}, \ldots, n_k,\ t_{1k},\ t_{2k}, \ldots, n_m,\ t_{1m},\ t_{2m}]^T \tag{4}$$

The kinematic variables $\boldsymbol{q} \in \mathbb{R}^{3m}$, that correspond to the static variables $\boldsymbol{x}$ in a virtual work sense, are the relative displacement rates at the contact points (i.e., normal displacement rates ε_{1k}, tangential displacement rates γ_{1K} and γ_{2K}) defined as:

$$\boldsymbol{q} = [\varepsilon_1,\ \gamma_{11},\ \gamma_{21}, \ldots, \varepsilon_k,\ \gamma_{1k},\ \gamma_{2k}, \ldots, \varepsilon_m,\ \gamma_{1m},\ \gamma_{2m}]^T \tag{5}$$

The compact equilibrium equation can be written as follows

$$\boldsymbol{Bx} = \boldsymbol{f_D} + \alpha \boldsymbol{f_L} \tag{6}$$

where $B \in \mathbb{R}^{6n \times 3m}$ is the equilibrium matrix.

The following constraint conditions are then specified for each contact point:

$$\begin{aligned} y_{rk} &= n_k \geq 0 \\ y_{sk} &= -\sqrt{t_{1k}^2 + t_{2k}^2} + \mu n_k \geq 0 \end{aligned} \tag{7}$$

where μ is the friction coefficient and the compressive force n_k is positive according to the convention shown in Figure 3. Equation (7) represent the failure modes by opening and sliding but the interaction of contact points considers other failure modes that may occur such as rocking and twisting.

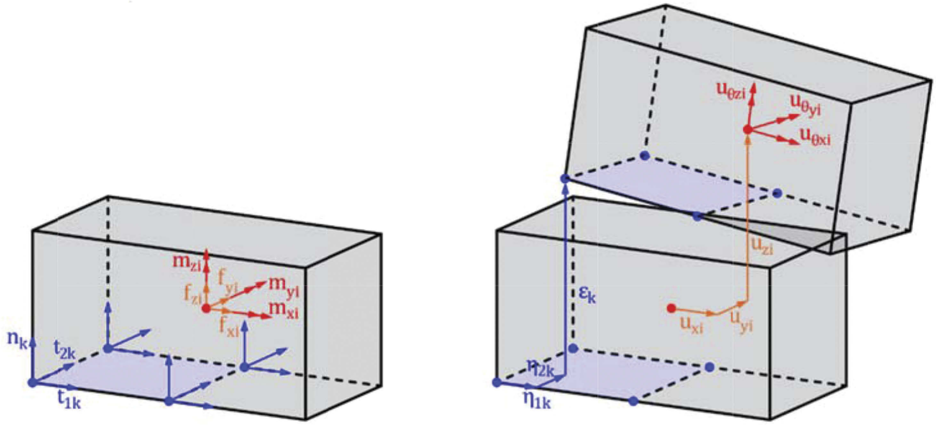

Figure 3. (a) Static and (b) kinematic variables at the block centroid i and contact point k (rearranged from Portioli et al., 2014).

In order to use linear programming techniques, the Lorentz cone, which describes the sliding failure condition, is linearized using 8 hyperplanes. The constraint conditions can be written as follows:

$$\begin{aligned} y_{rk} &= n_k \geq 0 \\ y_{s1,k} &= -t_{1k}\, cos\frac{k\pi}{4} - t_{2k}\, sin\frac{k\pi}{4} + \mu n_k \geq 0 \qquad k = 1, \ldots, 8 \end{aligned} \tag{8}$$

In general, constraint conditions are defined as:

$$\boldsymbol{y} = \boldsymbol{N^T x} \leq \boldsymbol{0} \tag{9}$$

where $\boldsymbol{y}$ is the vector of failure conditions and $\boldsymbol{N^T}$ is the matrix that collects the constraints conditions.

Both the static and kinematic approaches of limit analysis (Ferris & Tin-Loi, 2001; Portioli, et al., 2014; Zampieri, 2020; Hua & Milani, 2023) are used to identify the collapse condition of the masonry arch bridge, then solving the problem through linear programming techniques. In particular, the LP related to the static theorem is given by

$$\begin{aligned} &\text{maximize} && \alpha \\ &\textit{subject to} && \boldsymbol{Bx} - \alpha \boldsymbol{f_L} = \boldsymbol{f_D} \\ & && \boldsymbol{N^T x} \leq 0 \end{aligned} \tag{10}$$

whereas the LP arising from the kinematic theorem is

$$
\begin{array}{ll}
\text{minimize} & -\boldsymbol{f}_D^T \boldsymbol{u} \\
 & \boldsymbol{f}_L^T \boldsymbol{u} = 1 \\
\textit{subject to} & -\boldsymbol{B}^T \boldsymbol{u} + \boldsymbol{N}\, \boldsymbol{z} = 0 \\
 & \boldsymbol{z} \geq 0
\end{array}
\tag{11}
$$

where $\boldsymbol{z}$ is the vector of resultant strain rates (analogous to plastic multipliers in classical plasticity).

4 RESULTS AND DISCUSSION

In this section, the results of a rigid block analysis performed on a masonry arch bridge, expressed in terms of load multiplier and associated mechanism, are presented.

The structure under consideration is composed by two arches with a thickness of 0.55 m, a span L = 6.0 m and rise f = 2.4 m. They are connected to a central pile of height 5.0 m and two side abutments of height 3.5 m. A coefficient of friction at the interface of 0.75 is assumed.

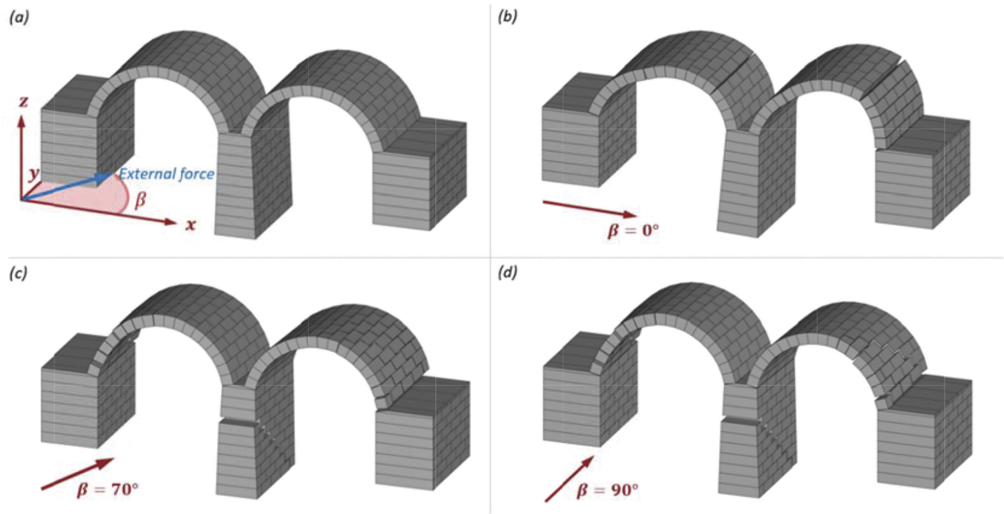

Figure 4. (a) Representation of the external horizontal force applied with variable angle β; (b) Failure mode for β = 0°(c) Failure mode for β = 70° (d) Failure mode for β = 90°.

In addition to considering the self-weights of each element, calculated proportionally to the geometry of the block assuming a unit weight of 22 kN/m^3, variable horizontal loads are applied on each element, proportional to their weight force (Figure 4).

A parametric analysis is then performed, calculating the collapse multiplier and the associated failure mechanism when the angle of inclination $\beta \in [0; 90]$ of the horizontal force f_L is varied (Figure 4). For $\beta = 0°$ the variable load acts in the longitudinal direction while for $\beta = 90°$ the horizontal action is parallel to the transverse direction.

Analyzing Figure 5, an increment in collapse multipliers is observed as the angle β increases, representative of a greater resistance of the structure to transverse actions than to longitudinal ones.

The collapse mechanisms shown in Figure 4 highlight that, for values of $\beta \geq 60°$, there are transverse failures affecting the central pier and the two masonry arches. In fact, the discretization employed is functional to the failure mechanisms: the pile is a slender element in the plane, subject to failure by rocking and sliding. In contrast, transversely it has a considerable size and a discretization with more elements is made in order to capture the classical mechanisms of rocking and sliding but also diagonal shear failure, evident in Figure 4(c,d).

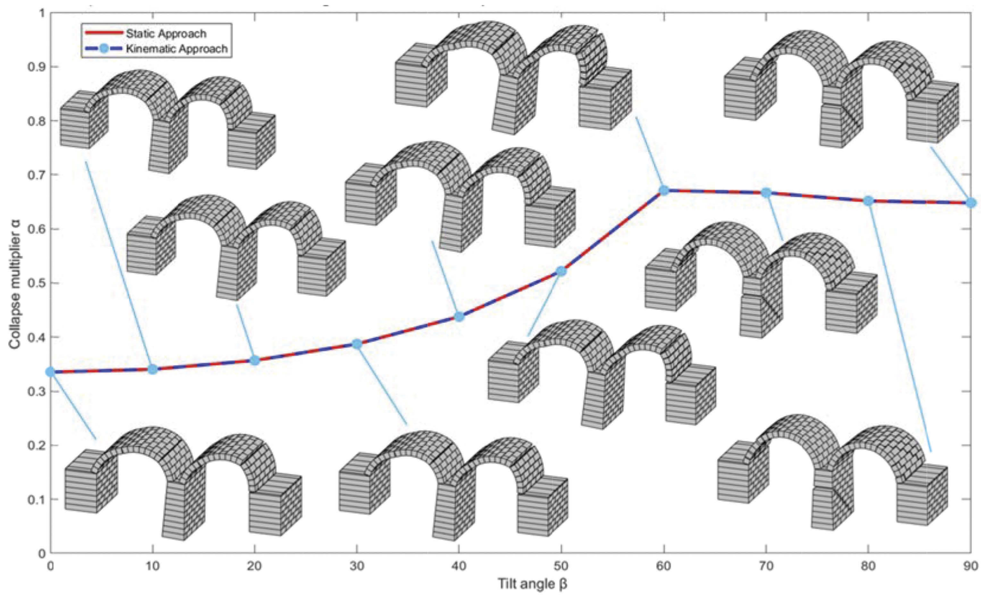

Figure 5. Collapse multiplier vs. tilt angle β of the external horizontal action.

Like most bridges in Italy, the arches are discretized with a single element on the thickness, allowing for rocking and sliding mechanisms. In contrast, multiple elements are used at depth to also capture diagonal cracking mechanisms and localized sliding.

5 CONCLUSIONS

In this paper, results on the collapse condition of masonry arch bridge subject to inclined horizontal inertial action have been presented.

To identify the collapse condition, expressed in terms of load multiplier and associated failure mechanism, rigid-block analysis has been performed, assuming the absence of tensile stresses at the interfaces and the impossibility of crushing failure. An attritive resistance at the interface has been considered, using the Mohr-Coulomb principle and an associated flow law.

A parametric analysis has been performed on the structure, varying the angle of application of the horizontal external actions, expressed as a function of the self-wight of each block. The results obtained showed that the use of a three-dimensional analysis of the structure is also capable of capturing transverse failure mechanisms, which occur on the structural elements of the masonry arch bridge for out-of-plane actions, in addition to longitudinal in-plane actions.

REFERENCES

Baggio, C. & Trovalusci, P. 1993. *Discrete models for jointed block masonry walls*. Philadelphia, Pennsylvania, pp. 939–49, Vol. 2.

Cascini, L., Gagliardo, R. & Portioli, F. 2020. LiABlock_3D: A Software tool for collapse mechanism analysis of historic masonry structures. *International Journal of Architectural Heritage*, 14(1): pp. 75–94.

Ferris, M. & Tin-Loi, F. 2001. Limit analysis of frictional block assemblies as a mathematical program with complementarity constraints. *Mechanical sciences*, 43(1): pp. 209–224.

Galassi, S. 2023. An alternative approach for limit analysis of masonry arches on moving supports in finite small displacements. *Engineering Failure Analysis*, 145.

Galassi, S. & Zampieri, P. 2023. A new automatic procedure for nonlinear analysis of masonry arches subjected to large support movements. *Engineering Structures*, 276.

Gilbert, M., Casapulla, C. & Ahmed, H. 2006. Limit analysis of masonry block structures with non-associative frictional joints using linear programming. *Computers & Structures*, 84(13-14): pp. 873–887.

Heyman, J. 1969. The safety of masonry arches. *International Journal of Mechanical Sciences*, 11(4): pp. 363–385.

Hua, Y. & Milani, G. 2023. Simple modeling of reinforced masonry arches for associated and non-associated heterogeneous limit analysis. *Computer and Structures*, 280.

Kooharian, A. 1952. Limit analysis of voussoir (segmental) and concrete arches. *ACI Journal*, 49(12): pp. 317–28.

Livesley, R. 1978. Limit analysis of structures formed from rigid blocks. *International Journal for Numerical Methods in Engineering*, 12(12): pp. 1853–1871.

Melbourne, C. & Gilbert, M. 1995. The behaviour of multiring brickwork arch bridges. *The Structural Engineer*, 73(3): pp. 39–47.

Orduña, A. & Lourenço, P. 2003. Cap Model for Limit Analysis and Strengthening of Masonry Structures. *Journal of Structural Engineering*, 129: pp. 1367–1375.

Orduña, A. & Lourenço, P. 2005. Three-dimensional limit analysis of rigid blocks assemblages. Part II: Load-path following solution procedure and validation. *International Journal of Solids and Structures*, 45(18-19): pp. 5161–5180.

Portioli, F., Casapulla, C., Gilbert, M. & Cascini, L. 2014. Limit analysis of 3D masonry block structures with non-associative frictional joints using cone programming. *Computers & Structures*, 143: pp. 108–121.

Ragni, L., Scozzese, F., Gara, F. & Tubaldi, E. 2019. *Dynamic identification and collapse assessment of Rubbianello Bridge*. Guimarães.

Scozzese, F., Ragni, L., Tubaldi, E. & Gara, F. 2019. Modal properties variation and collapse assessment of masonry arch bridges under scour action. *Engineering Structures*, 99.

Trovalusci, P. 1992. *No-tension discrete model with friction for jointed block masonry walls using interface elements*. Pittsburgh, pp. 11.73–82, Vol. 2.

Zampieri, P. 2020. Horizontal capacity of single-span masonry bridges with intrados FRCM strengthening. *Composite Structures*, 244.

Zizi, M., Cacace, D., Rouhi, J., Lourenço, P. & De Matteis, G. 2022. Automatic procedures for the safety assessment of stand-alone masonry arches. *International Journal of Architectural Heritage*, 16 (9): pp. 1306–1324.