A Unified Framework for Extreme Sub-daily Precipitation Frequency Analyses based on Ordinary Events

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Key Points:

- Unified methodology for metastatistical extreme value analysis of sub-daily precipitation across durations
- The simplified formulation of the method permits to analyze extremes emerging from tail of ordinary events rather than entire distribution
- Consistent definition across durations yields ordinary events which scale with the same scaling exponent of annual maxima

Abstract

The metastatistical extreme value approach proved promising in the frequency analysis of daily precipitation from ordinary events, outperforming traditional methods based on sampled extremes. However, sub-daily applications are currently restrained as it is not known if ordinary events can be consistently examined over durations, and it is not clear to what extent their entire distributions represent extremes. We propose here a unified definition of ordinary events across durations, and suggest the simplified metastatistical extreme value formulation for dealing with extremes emerging from the tail, rather than the entire distributions, of ordinary events. This unified framework provides robust estimates of extreme quantiles (<10% error on the 100-year from a 26-year long record), and allows scaling representations in which ordinary and extreme events share the scaling exponent. Future applications could improve our knowledge of sub-daily extreme precipitation and help investigating the impact of local factors and climatic forcing on their frequency.

Plain Language Summary

We propose here a unified methodology to quantify the intensity of extreme rainfall of short duration, such as events expected to occur on average once every 100 years. As opposed to alternative methods in literature, we rely on the simultaneous analysis of all every-day rainfall events, which, being much larger in number than extremes, were shown to provide improved estimates for daily rainfall. We show that, under our approach, the hypothesis of every-day and extreme events being similar enough holds also for short-duration rainfall. Application of our method to 26 years of data from an individual station reproduces analyses based on more than 150 years of observations from multiple nearby stations, with less than 10% error on the estimation of rain intensities expected to occur on average once every 100 years, which are not directly quantifiable from the 26 years of observations. The proposed methodology could help improving our knowledge of short-duration rainfall extremes, with implications for water resources and risk management, and could help investigating the impact of climate change on extreme rainfall events.

1 Introduction

The Metastatistical Extreme Value (MEV) framework was recently proposed for the frequency analyses of extremes under pre-asymptotic conditions. The method relies on the hypothesis that the extremes of the variable of interest emerge from the yearly distributions of underlying ordinary events, which are sampled a finite number of times every year (Marani and Ignaccolo, 2015). Once the cumulative distributions of the ordinary events $F(x, \boldsymbol{\theta}_j)$, where $\boldsymbol{\theta}_j$ are the distribution parameters, are known for every year $j = 1 \dots M$, the extreme values cumulative distribution can be written as: $\zeta(x) = \frac{1}{M}\sum_{j=1}^{M} F_i(x, \boldsymbol{\theta}_j)^{n_j}$, where n_j is the number of ordinary events observed in the *j-th* year (Zorzetto et al., 2016). The framework can include any class of distributions for F, and allows to consider multiple types of ordinary events (e.g., non-tropical and tropical cyclones) to derive compound extreme value distributions (Marra et al., 2019; Miniussi et al., 2020). Making use of the full available data record, MEV also largely decreases the parameter estimation uncertainty and the stochastic uncertainty related to the sampling of extremes.

It can be shown that when the inter-annual variability of the ordinary events can be neglected (i.e., $\boldsymbol{\theta}_j \approx \boldsymbol{\theta}_k, \forall j, k$) and the focus is on extreme quantiles (i.e., $F \to 1$), the inter-annual variability of the number of yearly events also becomes negligible. In these conditions, a simplified MEV formulation (SMEV), closely resembling ordinary statistics, can be written: $\zeta(x) \approx F(x, \boldsymbol{\theta})^n$, where n is here the average number of ordinary events per year (Marra et al., 2019). SMEV was originally proposed as an instantaneous limit ($M \to 0$) of MEV for the analysis of nonstationary processes. However, when tested for extreme value analyses on observational records, SMEV was found to perform similarly, even if less accurately than MEV, and to be preferable when the small number of ordinary events per year prevents to accurately estimate parameters for individual years ($n \lesssim 20$) (Schellander et al., 2019; Miniussi and Marani, 2020). Owing to the largely increased data sample used to estimate the distribution parameters (all years are used together), SMEV permits to focus on the tail of the ordinary events distribution by left-censoring the data (Marra et al., 2019). It should be noted that this is not equivalent to threshold exceedance methods as, in such cases, the information below threshold is discarded and results hold only asymptotically for thresholds tending to the upper limit of the distribution support (Davison and Smith, 1990), while SMEV describes tails which include large portions of the data and whose definition accounts for the whole data sample.

So far, MEV methods have been mostly used for the analysis of extreme daily precipitation relying on Weibull distributions to describe the ordinary events (e.g., Marani and Ignaccolo, 2015; Zorzetto et al., 2016; Miniussi and Marani, 2020). Results showed a number of advantages over traditional methods based on the sampled extremes: (i) rare quantiles, corresponding to return periods longer than the available data record, are estimated with significantly reduced errors; (ii) short records are sufficient to obtain robust estimates; (iii) the method is less sensitive to measurement errors typically affecting extremes (Zorzetto et al., 2016; Marra et al., 2018; Schellander et al., 2019; Miniussi and Marani, 2020; Zorzetto and Marani, 2020). Extending the applicability of MEV to sub-daily durations by means of a unified methodology that allows to examine ordinary events across durations could pose the bases for more general frameworks relying on the scaling properties of ordinary events. This could improve our understanding of extreme precipitation at the global scale and provide more accurate information for hydraulic infrastructure design and risk management.

Two knowledge gaps currently restrain the application of MEV to sub-daily durations. First, the only method so far proposed to define sub-daily ordinary events is based on the temporal autocorrelation of the individual time series and do not permit to consider multiple durations together (Marra et al., 2018). Second, the results reported above only pertain to extremes emerging from the entire yearly distributions of ordinary events. This assumption is in contrast with results showing that sub-daily precipitation is better described by more general distributions, whose tails only can be approximated by stretched-exponential (e.g., Weibull) or power-type distributions (e.g., Papalexiou et al., 2018; Papalexiou, 2018). Here, we address these gaps by (a) proposing a consistent definition of ordinary events which allows scaling representations across durations, and (b) suggesting SMEV as a viable option for dealing with extremes emerging from the tail of the ordinary events distribution, as opposed to their entire distributions. We evaluate the robustness of the proposed MEV and SMEV approaches on a study case in the southeastern Mediterranean coast for which 26 years of 10-min data are available. We rely on regional estimates of rare quantiles from 9 stations (>150 years of record in total) as a reference.

2 Methods

We propose to define ordinary events based on *storms*, i.e. consecutive wet events separated by dry hiatuses whose length is to be determined based on the climatology of the region. Ordinary events can then be computed as the maxima intensities observed within each storm using moving windows of the desired duration. In this way, ordinary events are directly related to meteorological features, and their number remains consistent across durations.

Series of 10-min precipitation data are collected for 9 quality-checked automatic stations (164-year record in total) in the south-eastern Mediterranean coast. Distance between the stations ranges between 1.5 and 70 km (27 ± 16 km) and individual records span 10 to 26 years (18.2 ± 6.1 years). Homogeneity of the region was ensured using the method based on the coefficient of L-variation recommended by Hosking and Wallis (1997) ($H < 0.25$ for all durations), so that the stations could be collectively used in a regional framework to estimate extremely low yearly exceedance probabilities. Reference quantiles are computed from the 9 stations using the regional L-moments method by Hosking and Wallis (1997) and the generalized extreme value (GEV) distribution (Fig. 1) by normalizing the annual maxima over their mean values.

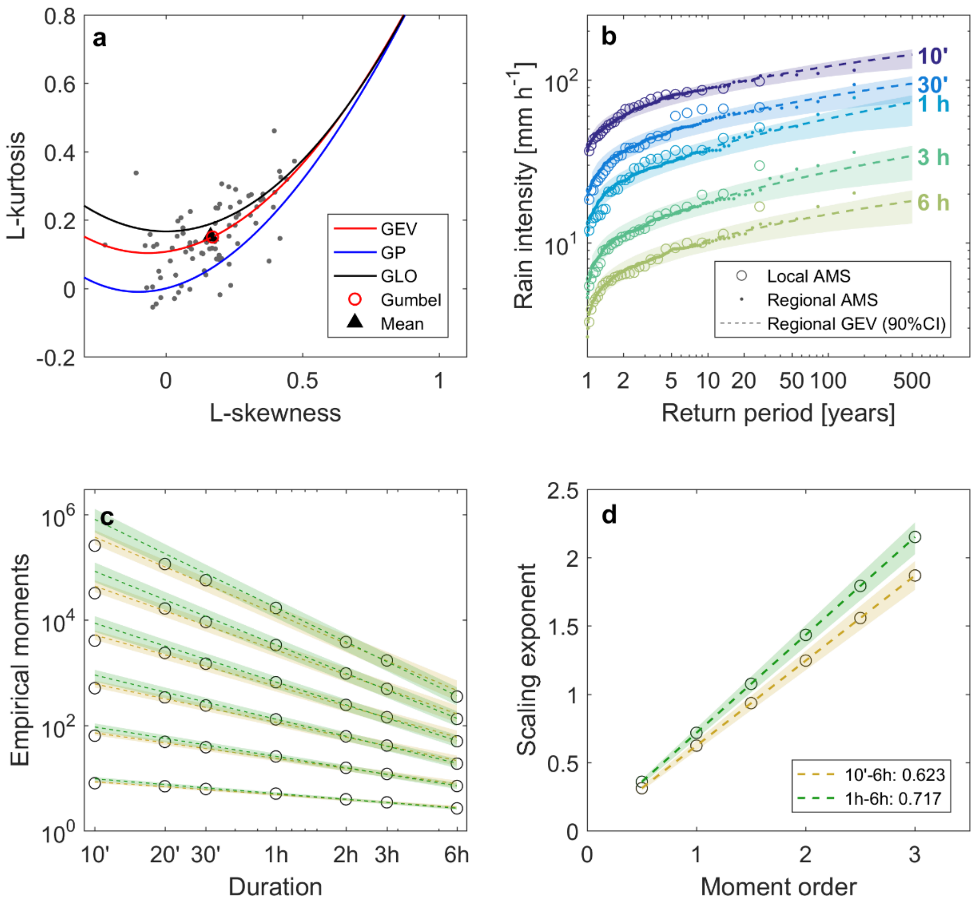

Figure 1. Regional analysis and scaling of the annual maxima. (a) L-Moments ratio diagram for all the stations and durations, the theoretical moments for generalized extreme value (GEV), generalized Pareto (GP) and generalized logistic (GLO) distributions are shown. (b) Regional GEV distributions (dashed lines) estimated using the regional L-moments framework by Hosking and Wallis (1997); shaded areas show the 90% confidence interval obtained via bootstrap with replacement among the regional annual maxima (AMS); circles show the AMS observed at Zykhron Yaaqov station (frequency is estimated using the Weibull plotting positions), while dots show the AMS from all the 9 stations rescaled to the Zykhron Yaaqov mean. (c) Raw moments of the AMS. Colored lines show the regressions obtained using the full temporal domain (10 min – 6 hours; orange, shaded area represent the 90% confidence interval from 10^3 bootstrap samples with replacement among the years) and the 1 h – 6 h interval (green); (d) Scaling exponent of the AMS obtained using the full temporal domain (10 min – 6 hours; orange) and the 1 h – 6 h interval (green).

Following previous studies in similar climates, we require at least six-dry-hour hiatuses (i.e., lower than the minimum rain amount reported in the data, 0.1 mm, over all 10-min intervals for 6 consecutive hours) to separate storms (Restrepo-Posada and Eagleson, 1982; Tarolli et al., 2012), but general applications should use a definintion based on the local climatology. Storms lasting less than 30 min (three time intervals, in our case) are removed. We focus on durations between 10-min and 6-hour (10-min, 20-min, 30-min, 1-hour, 3-hour, 6-hour), a choice driven by the temporal resolution of the data (10-min) and the dry hiatus used to separate storms (6-hour).

MEV and SMEV are here applied *at-site* focusing on the longest recording station (Zikhron Yaaqov, 26-year). Complying with previous MEV applications, ordinary events are described using Weibull distributions in the form $F(x;\lambda,\kappa) = 1 - e^{-\left(\frac{x}{\lambda}\right)^{\kappa}}$, where λ and κ are the scale and shape parameters, respectively (Zorzetto et al., 2016). Such a model was shown as appropriate for the here-examined region (Marra et al., 2019). Three MEV parameters are obtained for every year: λ_j and κ_j are estimated from all the yearly ordinary events by using the method of the L-moments (Hosking, 1990), and n_j as the number of ordinary events in the *j-th* year. Extreme quantiles are then computed by numerically inverting the MEV cumulative distribution function (Zorzetto et al., 2016; Marra et al., 2018).

Neglecting inter-annual variability, SMEV only requires three parameters: λ and κ are estimated left-censoring the low portion of the ordinary events and using a least-squares regression in Weibull-transformed coordinates on the remaining data points (Marani and Ignaccolo, 2015), while n is the mean number of ordinary events per year ($n = \frac{\sum n_j}{M}$). It is worth noting that, when based on Weibull, the SMEV distribution becomes an exponentiated Weibull distribution (Nadarajah et al., 2013). Following previous results for the region, the ordinary events tail is here defined as the largest 25% of the ordinary events (Marra et al., 2019). It should be however noted that the definition of the tails is case-dependent and should be selected using sensitivity analyses (as in Marra et al., 2019); for example, definition of the tail as the largest 45%-20% of the ordinary events was found appropriate for the study region. Extreme quantiles are computed inverting the SMEV cumulative distribution function. In addition, in order to better evaluate the robustness of SMEV to represent the distribution of out-of-sample extremes, a second set of parameters and quantiles is derived by also censoring all the annual maxima, so to obtain results fully independent from the observed annual maxima. For all methods, confidence intervals in parameters and quantiles are computed via bootstrap with replacement (10^3 repetitions) among the years in the record (Overeem et al., 2008).

3 Results and discussion

Application of MEV and SMEV to the here defined sub-daily ordinary events provides robust estimates of extreme quantiles (Fig. 2), with the estimated MEV (blue solid line and 90% confidence interval) and SMEV (red) distributions being generally consistent with the regional reference, and only MEV slightly underestimating quantiles for 1- and 3-hour durations (the reference lies outside of the 90% confidence interval). Thanks to the focus on the ordinary events tail, SMEV provides more accurate estimates (<10% error on the 100-year quantiles for all durations, <12.5% for 500-year) than MEV (<20%) but, due to the smaller data sample used to estimate the parameters, is characterized by larger uncertainty. Quantiles obtained independently from the observed annual maxima (red dashed lines) lie within the confidence interval of SMEV, and never exceed 20% error, even for 500-year return levels. Similarly, the observed annual maxima (black dots, plotted using the Weibull plotting positions) lie within the area in which we expect to see 90% of the annual maxima *if* they were actually sampled from SMEV (shaded in grey, obtained from 10^3 random sampling from SMEV). These observations support the robustness of the SMEV approach.

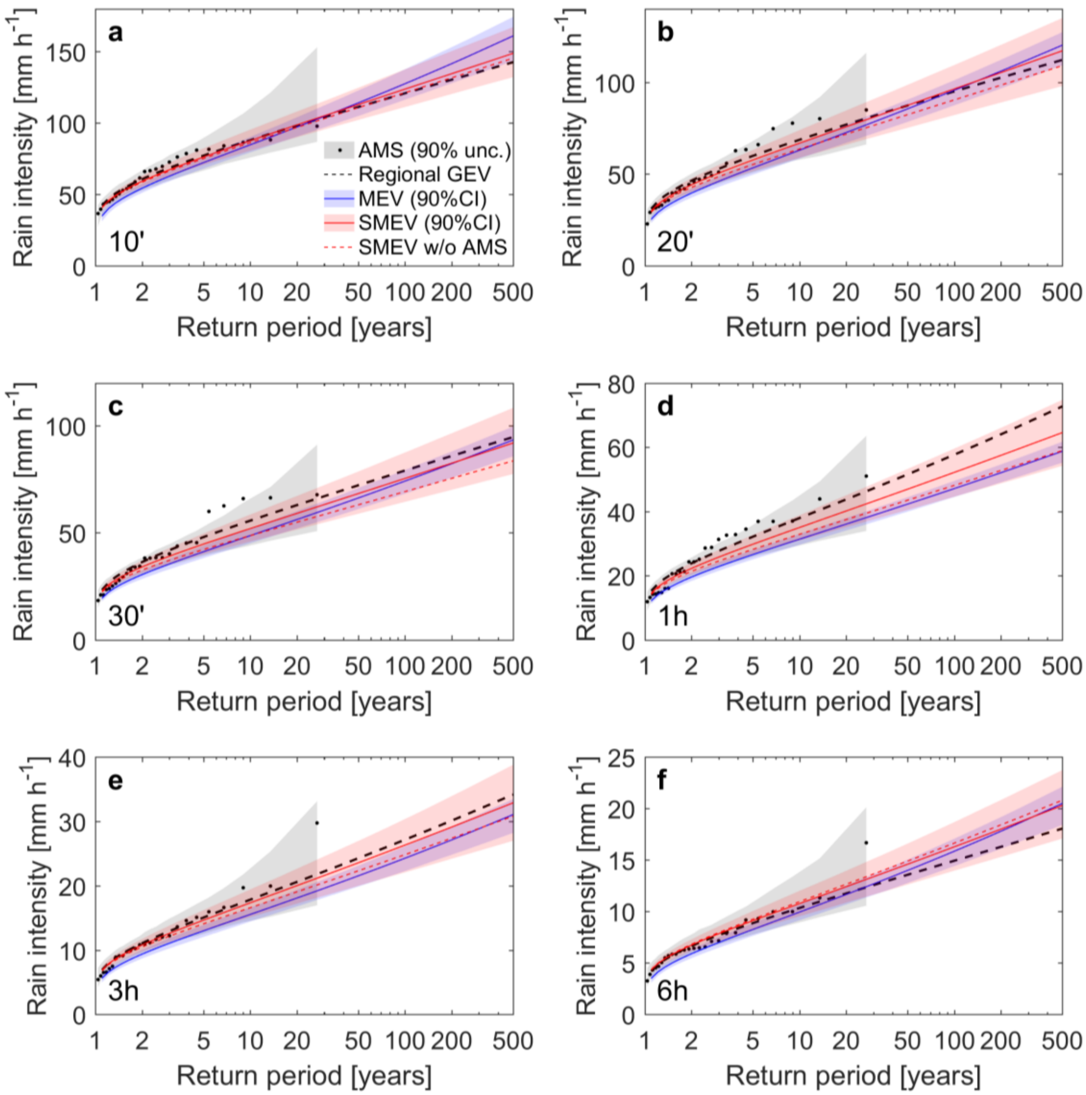

Figure 2. Quantiles estimated using MEV (blue solid line), SMEV (red solid line) for 10-min to 6-hour durations (a-f); blue and red shaded areas show the corresponding 90% confidence intervals (10^3 bootstrap samples with replacement among the years). Black dots show the observed annual maxima (AMS) and dashed black line the regional GEV estimate obtained from 9 stations in the region (reference). Shaded in grey are the areas in which 90% of the annual maxima are expected to lie *if* they were sampled from SMEV. Red dashed lines represent quantiles obtained using SMEV and censoring the annual maxima.

The scaling of MEV and SMEV parameters with duration is presented in Fig. 3, noting that blue shaded area represents the 90% inter-annual variability of MEV parameters, and red and grey areas represent the 90% confidence intervals (bootstrap among the years) of SMEV and non-zero-tail parameters, respectively. It is interesting to see a decrease of the shape parameter with increasing duration (Fig. 3b). This translates into heavier tails of the distribution of ordinary events at longer durations and, due to the consistent definition of ordinary events, of the resulting extreme value distribution. Despite seemingly contradicting previous results in which the tail of non-zero time intervals was examined (e.g., Papalexiou et al., 2018, shown here in grey), this is explained by the dramatic change in the number of non-zero time intervals per year (Fig. 3c). Using a consistent definition of the ordinary events in which their number is the same across durations, MEV and SMEV scale parameters scale with duration with the same scaling exponent of the annual maxima above 1-hour (Fig. 3a and Fig. 1). Notably, this is the temporal domain in which the simple scaling behavior of annual maxima is generally considered more robust (Burlando and Rosso, 1996; Ceresetti et al., 2010). This property suggests that extremes are indeed samples from the tail of the ordinary events, and opens MEV methods to applications in which multiple durations are used simultaneously (e.g., Burlando and Rosso, 1996; Langousis and Veneziano, 2007; Innocenti et al., 2017; Emmanouil et al., 2020). Overall, the use of consistent ordinary events across durations permits to examine the scaling behavior in all rain events, including different (if any) scaling behaviors between extreme and ordinary events.

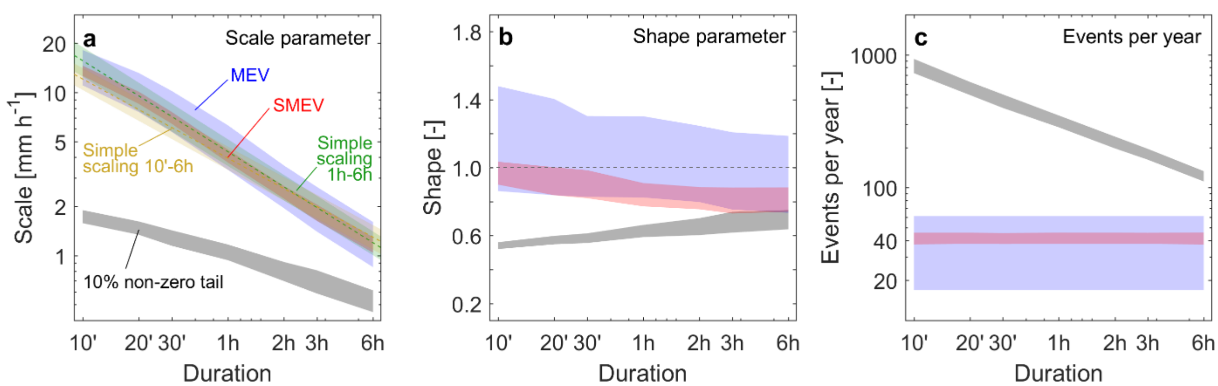

Figure 3. Scaling of MEV and SMEV parameters with duration; blue shaded area represents the 90% inter-annual variability of MEV parameters, and red and grey areas represent the 90% confidence intervals (bootstrap among the years) of SMEV and 10% non-zero tail (as in Papalexiou et al., 2018) parameters, respectively. (a) Scaling of the scale parameters with duration for MEV (blue), SMEV (red) and 10% non-zero tail (grey). Simple scaling relations from annual maxima are superimposed (10 min–6 h in orange, 1 h—6 h in in green, see Fig. 1); within the uncertainties, MEV and SMEV scale parameters share the scaling exponent with annual maxima, especially the 1 h—6 h (green); (b) Scaling of the shape

parameters with duration. The shape parameter of the non-zero tail increases with duration as reported by other studies (e.g., Papalexiou et al., 2018), while an opposite behavior is reported for the MEV and SMEV; for the case of SMEV, the decrease is larger than the parameter estimation uncertainty; (c) Scaling of the number of ordinary events per year with duration. The proposed definition makes their number independent on duration (~40 events per year in this case). The marked decrease of the number of non-zero time intervals (grey) explains the different behaviors observed for the shape parameters between non-zero tail and MEV methods.

We conclude showing further evidence for the emergence of extreme sub-daily precipitation from the tail of ordinary events. In the examined case (Fig. 4a, which confirms what observed in Fig. 2), the whole tail of ordinary events (colored dots), including annual maxima (thick black dots), are likely samples from a unique Weibull distribution whose parameters are here estimated explicitly censoring the observed annual maxima values (dashed lines, shaded areas represent 90% sampling uncertainty under the local climatology; note that 10% of the points are expected to lie outside of this area). Additionally, the robustness of SMEV in extracting information from ordinary events emerges in that the parameters estimated censoring the annual maxima (red dashed lines in Fig. 4b) are indistinguishable (i.e. within the red-shaded confidence interval) from the ones describing the whole tail (red solid line). Despite relying on an average of more than 40 ordinary events per year, the here analyzed data did not allow to separate the inter-annual variability of MEV parameters (shaded blue area in Fig. 4b) from the yearly parameter estimation uncertainty (blue dashed lines).

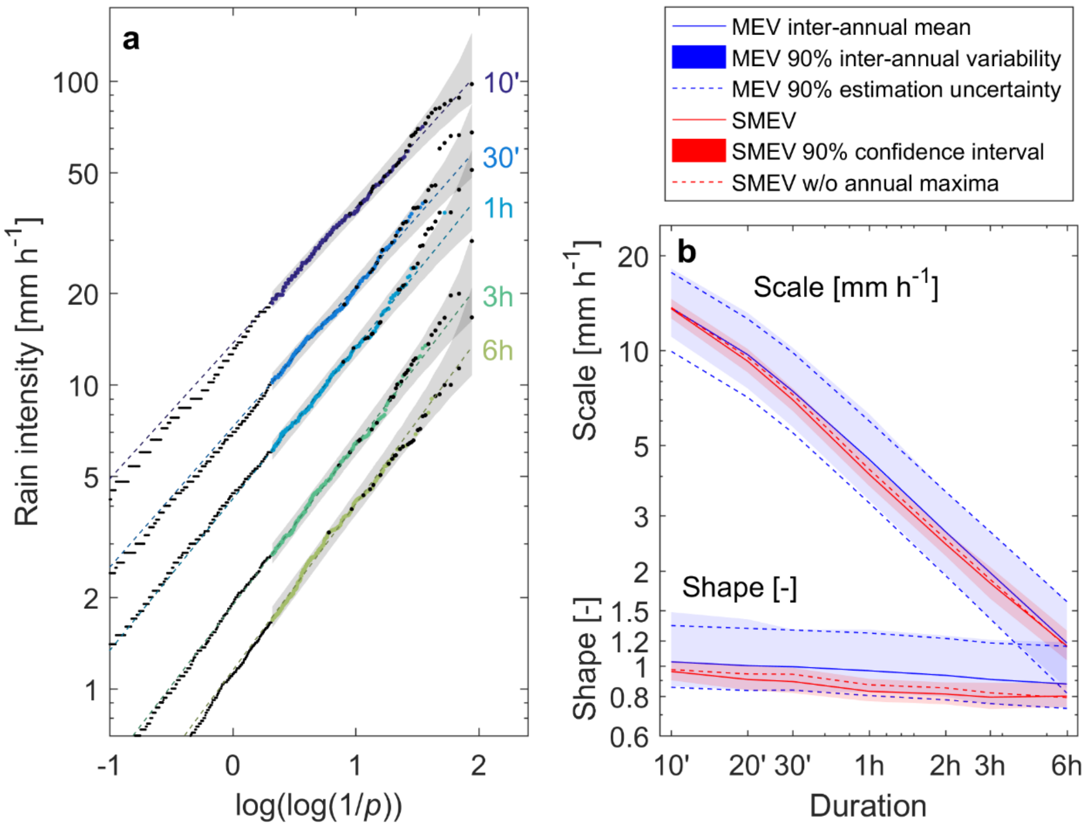

Figure 4. Robustness of MEV and SMEV assumptions. (a) Weibull plot (on the x-axis, p is the exceedance probability, i.e., $1 - F$, in our notation) showing the ordinary events for the examined durations (dots, the 25% tail used in SMEV is colored according to the duration and annual maxima are in thick black), Weibull distributions (dashed lines) whose parameters are estimated fitting the 25% tail and explicitly censoring the observed annual maxima values (thick black dots), and sampling uncertainty from the Weibull distribution (90% interval from $2 \cdot 10^3$ random samples); (b) Weibull parameters estimated using MEV and SMEV. Inter-annual variability of MEV parameters (blue shaded area) almost perfectly overlaps with parameter estimation uncertainty (blue dashed lines; 10^3 random samples from a Weibull distribution described by the mean MEV parameters). SMEV parameters computed censoring the annual maxima (red dashed lines) are within the 90% confidence interval of the SMEV parameters (10^3 bootstrap repetitions with replacement among the years).

4 Closing statement

We proposed a consistent definition of ordinary events based on independent storms to use the Metastatistical Extreme Value (MEV) framework across durations, and suggested the simplified MEV formulation (SMEV) as an option to deal with extremes emerging from the tail of the ordinary events rather than their entire distribution. This definition of ordinary events allowed to effectively use MEV methods for sub-daily extreme precipitation frequency analyses, and permitted a scaling representation in which ordinary events scale with the same exponent of the observed annual maxima. Owing to its focus on the tail of the ordinary events distribution, SMEV provided estimates of 100-year quantiles with less than 10% error (12.5% for the 500-year event) from only 26 years of data (up to ~20% for MEV). These results support the use of MEV methods for sub-daily (and sub-hourly) precipitation frequency analyses, and open to the use of analytical frameworks exploiting these methods across durations. Applications of the approach could improve our knowledge of extreme sub-daily precipitation at the global scale with important implications for hydraulic design and risk management. Additionally, the analysis of SMEV parameters could help investigating local properties of extreme precipitation that are generally masked by the stochastic uncertainties characterizing the sampling of extremes, such as their response to local and climate forcing.

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