

# **A 3D plastic-damage constitutive model for concrete failure**

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## **Abstract**

A coupled plastic-damage mathematical and numerical model to investigate the mechanical behaviour of concrete at the scale of its constituents is presented herein. The plastic-damage model combines a pressure-dependent plastic model with a damage model able to combine compressive and tensile mechanisms to describe concrete failure. Specifically, the damage model includes a stiffness recovery function for a more realistic description of the transition from tensile to compressive failure of the composite. The plastic potential is defined based on the mechanical behaviour of concrete under triaxial stress states. Along this line the model is expected to simulate the local confinement effects that involve the cement paste when surrounded by the aggregates. A new cohesive contact law has been used to characterize the Interfacial Transition Zone (ITZ) between the two, so accounting for the control of the shear stresses developed during the slipping of the two phases during failure.

After calibrating the model, a uniaxial compression test has been reproduced numerically. The experimental stress-strain curve is found to be in good agreement with the model prediction.

Moreover, a comparison with experimental results prove that the specific cohesion contact law formulation is able to efficiently describe the behaviour of ITZ, as well as the gradual decohesion process around aggregates.

**Keywords:** concrete, mesoscale, plastic-damage, ITZ, cohesive contact, elasto-plasticity, damage.

## 1 Introduction

Experimental tests carried out on concrete samples show that concrete is a complex material which exhibits highly non-linear behaviour. Namely, its mechanical response is remarkably pressure-dependent, that is, strictly dependent on the confinement stress levels. Under a low confining pressure the post-peak behaviour of concrete is characterized by a softening response accompanied by a reduction of stiffness during the unloading process and the appearance of permanent deformations.

In a mesoscopic scale, concrete can be seen as a heterogeneous medium composed by coarse aggregates, cement paste and an interface layer, between matrix and inclusions, known as the Interfacial Transition Zone (ITZ). The presence of stiffer aggregates with different shapes implies a local confinement effect (LCE) of the cement paste between aggregates. The LCE may vary considerably within the concrete sample and it is related to the relative distance between the aggregates, their stiffness and their dimensions. Strain mismatch between matrix and aggregate can result into stress concentration at the level of the ITZ, so influencing the overall strength and material performance.

Therefore, it is crucial to establish a contact formulation able to describe accurately the interaction between aggregates and the cement paste. Furthermore, the cyclic behaviour of concrete is characterised by hysteresis loops during the loading and unloading processes.

A pressure-dependent plastic model is extended to incorporate damage, aiming at reproducing the main experimental observations of the phenomenological behaviour of concrete under different confinement levels and cyclic loadings. Additionally, a cohesive contact law able to take into account the coupling between normal/peeling and shear stresses has been used to investigate the ITZ failure.

Results are provided proving the performance of the model in describing the behaviour of ordinary concrete both globally and locally at the scale of its constituents.

## 2 Methods

The cement paste has been modelled as a plastic-damage material where the plastic part [1, 2] considers a pressure-sensitive yield surface, which is extended to include a damage variable [3]. The yield surface is calculated using the unified coordinates in the Haigh-Westergaard stress space  $(\xi, \rho, \vartheta)$  as

$$f_p = 1.5 \frac{\rho^2}{(1-\omega)^2 f_c^2} + \frac{q_h(\kappa)m}{(1-\omega)f_c} \left[ \frac{\rho}{\sqrt{6}} r(\vartheta, e) + \frac{\xi}{\sqrt{3}} \right] - q_h(\kappa)q_s(\kappa) \leq 0, \quad (1)$$

where  $\kappa$  is the internal variable and  $m$  is the cohesion parameter;  $f_c$  is the strength of concrete in uniaxial compression; the function  $r(\vartheta, e)$  describes the out-of-roundness of the yield surface as a function of the eccentricity  $e$ ;  $q_h(\kappa)$  and  $q_s(\kappa)$  are the hardening and softening functions.

The yield surface has been defined using the following damage parameter

$$\omega = 1 - [1 - \omega_c(\kappa_c)][1 - s(\sigma^{tr})\omega_t(\kappa_t)], \quad (2)$$

where  $\omega_c$  and  $\omega_t$  are the damage components, taking into account compressive and tensile failure mechanisms, respectively, and  $s(\sigma^{tr})$  is the stiffness recovery function. The damage variables in tension and in compression are function of two distinct internal variables,  $\kappa_c$  and  $\kappa_t$ . A non-associated flow rule has been considered, where the plastic potential is defined in agreement with [2] as

$$g = -\frac{A}{f_c} \left( \frac{\rho}{\sqrt{q(\kappa)}} \right)^2 - B \left( \frac{\rho}{\sqrt{q(\kappa)}} \right) + \frac{\xi}{\sqrt{q(\kappa)}} = 0, \quad (3)$$

where  $q(\kappa) = q_h(\kappa)q_s(\kappa)$  is the hardening/softening function, and  $A$  and  $B$  are calibration parameters that reflect the concrete behaviour under triaxial stress states.

In order to describe the possible decohesion between the matrix and the inclusions, the ITZ has been taken into account using a cohesive contact formulation [4], combined with a failure surface represented by

$$f = a \frac{\tau^2}{f_s^2} + \frac{q_s}{f_s} (b\tau + ct_n) - q_s^2 \frac{f_p}{f_s} c \leq 0, \quad (4)$$

where  $\tau$  is the shear component in the ITZ layer;  $f_p$  is the peeling strength;  $f_s$  is the shear strength;  $q_s$  is the softening-damage parameter, and  $a, b, c$  are calibration constants.

### 3 Results

The model has been calibrated over a uniaxial compression test under loading and unloading cycles. A similar test on another sample has accounted for the validation of the model, and, specifically, the comparison between the numerical and experimental results reported in Figure 1. From the results' juxtaposition it is noted that the proposed model is able to catch accurately the post-peak behaviour of concrete and the stiffness degradation during the softening regime.

3D mesoscale models of ordinary concrete made by calcareous aggregates have been reconstructed using 3D computed tomography [3], as shown in Figure 2a). In the mesoscale models, the aggregates are considered elastic; the cement paste has been modelled using the above mentioned plastic-damage formulation, and the ITZ has been modelled using the above mentioned cohesive contact law.

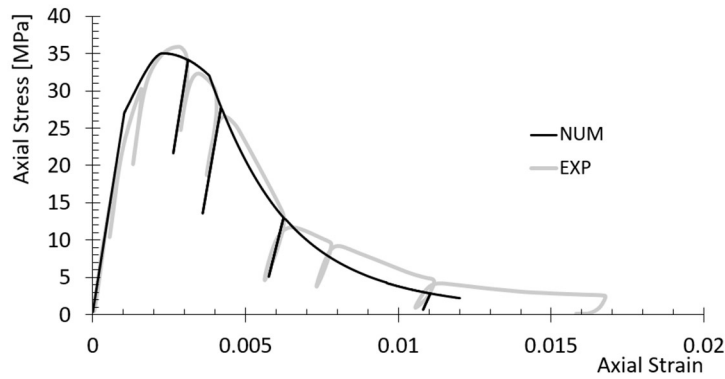


Figure 1: Comparison between numerical and experimental results.

The damage evolution inside the cubic sample during the uniaxial compression test can be observed in Figures 2b) and 2c). As expected, the damage originates at the interface between coarse aggregates and the cement paste, and then spreads through the cement paste to reach closer ITZ. On the other hand, Figure 2d) shows that the decohesion process takes place at the surface of the coarse aggregates, except for the regions where the normal vector to the aggregate surface is parallel to the direction of application of the load.

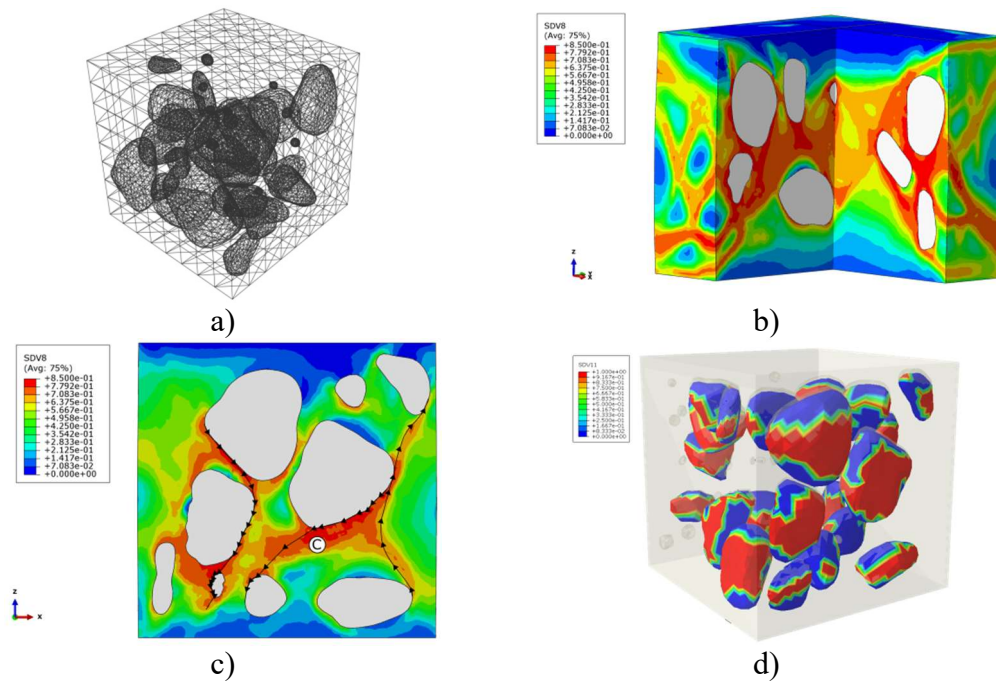


Figure 2: a) 3D sample reconstruction; b) damage contour map into the sample; c) damage distribution in a sample cross section (see cracking paths triggering from point C); d) ITZ delamination at the end of analysis (values ranging from 0 (no delamination) to 1 (complete delamination)).

## 4 Conclusions and Contributions

The plastic-damage model presented herein combines a pressure-dependent plastic model with a damage model able to combine compressive and tensile mechanisms to describe concrete failure. Specifically, the damage model includes a stiffness recovery function for a more realistic description of the transition from tensile to compressive failure. A new cohesive contact law has been used to characterize the weaker ITZ in the three-phase mesoscale model of concrete.

After the model calibration, a uniaxial compression test has been reproduced via this model. The experimental stress versus strain curve is found to be in good agreement with the model prediction.

Moreover, a comparison with experimental results prove that the new cohesion contact law formulation is able to efficiently describe the behaviour of ITZ, as well as the gradual decohesion process around aggregates.

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