Consensus sensor fusion to estimate the relative attitude during space capture operations

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***Abstract*—The concept of space capture refers to the capability of a spacecraft to grasp a target satellite with the ultimate goal of executing servicing operations on the captured vehicle. Much research effort is currently focused on the development of technologies that enable such complex operations by means of autonomous vehicles. Sensors and estimation technologies used to retrieve the relative pose of the two satellites in close proximity are of key importance.**

This work[1] presents an algorithm for sensor fusion based on a consensus filter that leverages on the measures provided by a set of four Time–of–Flight distance sensors. The goal is to obtain an estimate of the relative orientation of two planes, one holding the sensors, the other being a target surface. The algorithm is based on the computation of five different orientations on which the consensus filter is applied. Numerical simulations show that the estimated orientation is more accurate than the simple average of the five different orientations.

The proposed algorithm is performance is further evaluated using real measures collected from four Time–of–Flight sensors in a simple experimental setup. The experiment allows to compare the obtained estimation with the real orientation and with the output of a classic model-based Kalman filter used as a reference. The results suggests that the consensus algorithm estimates the relative attitude with a lower error, although requiring more measures compared to the Kalman filter.

I. Introduction

On–Orbit–Servicing (OOS) represents appealing mission scenarios for both the industrial and the scientific community [1]. The term OOS comprises a variety of operations involving a chaser and a client vehicle, including inspection, refuelling, maintenance and refurbishment, with the aim of extending or improving the operational life of the client satellite. A common point of OOS missions is represented by the so called close proximity operations, that culminate with the capture of the client satellite. Due to their complexity, these operations require a dedicated hardware in order to be safely executed.

Focusing on the retrieval of the relative pose, heterogeneous sensors can be employed in the same system by establishing a network of sensors. The term "sensor fusion" refers to techniques that consists in merging signals from multiple sensors, in order to retrieve an information that is more reliable or accurate than the one derived from the sensors used singularly [2].

Based on the sensor configuration, three main categories of sensor fusion can be identified [3]:

- *complementary*: sensors do not depend on each other and their signals can be mixed together in order to have a more comprehensive information of the observed property;
- *competitive*: different sensors investigate the same property;
- *cooperative*: data generated by the sensors are used to get information that would not be available from the individual measurements of a single sensor.

These configurations can be exploited at the same time since they are not mutually exclusive.

To enable space–capture operations, the authors are developing a smart interface, composed by a capture mechanism, a suite of close–range navigation sensors and a microcontroller to manage all the information. The smart interface is capable of retrieving the pose of the target by its own sensors and performing the capture by its own actuators. For the determination of the relative pose between the vehicles at very close range, the interface is equipped with two types of sensors: (i) a matrix positioning sensor [4], and (ii) four Time–of–Flight (ToF) distance sensors. The measures from the ToF sensors are used to retrieve multiple values of the orientation of the target. These information are then fused together in a competitive configuration by the application of a consensus algorithm [5] [6] [7]. The consensus algorithm employs a consensus matrix that holds the information of the variance and covariance computed between the different signals to be fused.

In the following sections, the algorithm will be presented and the approach to define the consensus matrix is described in detail. To evaluate the algorithm performance, it has been tested in two different ways: (1) numerical simulations allows to compare the algorithm outputs the simple average of the measures, and (2) an experimental set–up is used to compare the algorithm outputs with the actual relative attitude between the sensors and the target object. In the second case, in particular, the performance of the consensus estimator is compared with the output of a model–based Kalman filter.

The reminder of this paper is organized as follows: Sec. II provides a description of the ToF sensors and how they are employed; Sec. III describes the working principles of the consensus and of the model–based filters; Sec. IV presents the results obtained with numerical simulation; Sec. V presents

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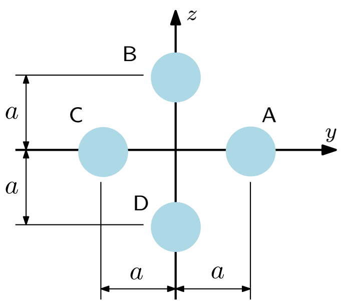

Fig. 1: Layout of the four ToF sensors (A, B, C, D) with reference frame.

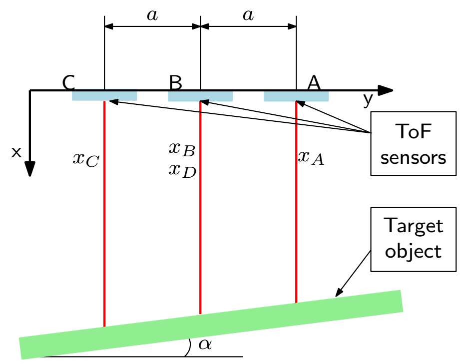

Fig. 2: Configuration of the ToF distance sensors with respect to the target surface.

the experimental setup and test results; finally in Sec. VI final conclusions are drawn.

II. Time of Flight Sensors

Time–of–Flight (ToF) distance sensors measure the time that the light takes to travel from the sensor to the object, and back after reflection. By knowing the value of the speed of light, it is possible to compute the relative distance between the sensor and the object. For the purpose of this work, the targetis a flat surface and a set of four coplanar ToF sensors is employed to retrieve the inclination of the target interface with respect to the plane of the sensors. The ToF sensors are arranged as shown in Fig. 1 to indirectly measure the rotation of the target about the z–axis, considering that the target is centred in the reference frame in the figure (the case of the rotation about the y–axis is similar). In this configuration, the measurements provided by the sensors B and D are not affected by the rotation around the z–axis.

Figure 2 depicts the layout of ToF sensors with reference to the target surface, showing the relative orientation angle α. To this aim, define x_A, x_B, x_C and x_D as the measures taken by the four sensors (A, B, C and D), and define a as the distance, measured in the y direction, of A and C from the z–axis. Five values of α can be computed by combining the measurements as follows:

$$
\begin{aligned}
\alpha_1 &= \arctan\frac{x_B - x_A}{a} & \alpha_2 &= \arctan\frac{x_D - x_A}{a} \\
\alpha_3 &= \arctan\frac{x_C - x_B}{a} & \alpha_4 &= \arctan\frac{x_C - x_D}{a} \\
\alpha_5 &= \arctan\frac{x_C - x_A}{2a}
\end{aligned}
\tag{1}
$$

A limit of ToF sensors is the considerable jitter that affects their output. In fact, their output is dispersed with a standard deviation of approximately $2\,\mathrm{mm}$, that translates into an error of $\pm 3\,\mathrm{deg}$ on planes orientation. This work presents method to reduce this error and improve the estimation of the relative attitude between the considered surfaces.

III. Estimation methods

This section presents two estimation methods applied to the problem of determining the angle α. First, the proposed consensus–based method is described, while its performances are discussed in the following sections. Additionally a model–based Kalman filter is reported, since it is used as a reference in the following to better understand the advantages provided by the consensus estimation method.

A. *Consensus filter*

A consensus filter is a way to fuse different signals of sensors, even if they have a different nature and a different logic [7]. It is an iterative filter that considers the measures of several sensors to retrieve a final estimation more reliable than the single measures.

The first step to design a consensus filter is to define the so–called consensus matrix $\mathcal{P}$ whose general element $\mathcal{P}(i,j)$ is greater than zero if sensor j has access to the state of sensor i. It must satisfy the fact that the sum of the elements of each row must be equal to one. This property guarantees that the filter has its first eigenvalue equal to 1, while all the others are lower than 1, hence it is stable and converges to a solution (due to the Gershgoring theorem).

The dynamics of the filter is described as follows:

$$\underline{v}(t+1) = \mathcal{P} \cdot \underline{v}(t) \tag{2}$$

where x is a column vector containing the values of each measure. As the equation shows, the filter might be seen as an iterative way to retrieve a final measure that is a linear combination of all the other measures.

In the case of this work, the consensus filter is applied to the five orientations described in Eq. 1. The $\alpha_1, \cdots, \alpha_5$ estimates are computed N times from N sets of consecutive distance samplings $(x_A, \cdots, x_D)$. The elements $\mathcal{P}(i,j)$ of the consensus matrix $\mathcal{P}$ are related to the covariance $\mathrm{cov}(\alpha_i, \alpha_j)$ of the estimated angle α_i with respect to α_j. For each couple (α_i, α_j) of estimates, a weight is defined as the inverse of the covariance $c_{ij} = \mathrm{cov}(\alpha_i, \alpha_j)$:

$$w_{ij} = \frac{1}{c_{ij}} \tag{3}$$

The elements $\mathcal{P}(i,j)$ are computed from the weights w_{ij} and normalized so that each row of the $\mathcal{P}$ matrix is a unit vector:

$$\mathcal{P}(i,j) = \frac{w_{ij}}{\sum_{k=1}^{5} w_{ik}} \tag{4}$$

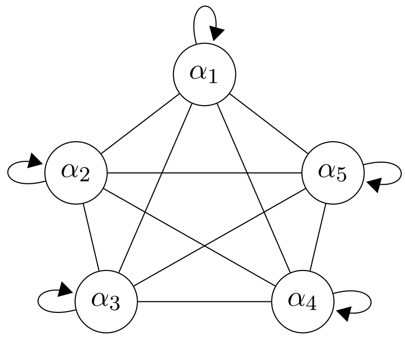

Fig. 3: The network established between the five orientation originated by the consensus matrix $\mathcal{P}$. In the network, all the orientations communicate with each other and with themselves.

The elements on the diagonal of the matrix are the inverse of the variance of α_i. The matrix is symmetric and full, meaning the network composed by the five α_i estimates is fully connected, as shown in Fig. 3. The pseudo–code that implements the consensus estimator is detailed in Alg. 1 and visually presented in Fig. 4.

Algorithm 1 Pseudo-code implementation of the consensus estimation algorithm.

Require: N measures of x_A, x_B, x_C, x_D from ToF sensors
compute N values of α_i, $i = 1 \cdots 5$
$\bar{\alpha}_i = \text{mean}(\alpha_i)$
$c_{ij} = \text{cov}\,(\alpha_i, \alpha_j)$
$w_{ij} = \dfrac{1}{c_{ij}}$
$P\,(i, j) = \dfrac{w_{ij}}{\sum_{k=1}^{5} w_{ik}}$
while $err > toll$ **do**
$\underline{\alpha}\,(k) = P \cdot \underline{\alpha}\,(k-1)$
$err = \max |\underline{\alpha}_j\,(k) - \underline{\alpha}_i\,(k)|$, $i, j = 1 \cdots 5$, $i \neq j$
$k = k + 1$
end while

B. Model–based filter

Model–based estimation methods are a common choice and the Kalman filter is widely applied. Suppose that the target is not moving except for a small oscillation around its equilibrium point, then it is possible to describe the system with the following model:

$$\mathbf{X}(t+1) = \mathbf{A} \cdot \mathbf{X}(t) + v$$
$$\mathbf{Y}(t) = \mathbf{C} \cdot \mathbf{X}(t) + w$$

leading to:

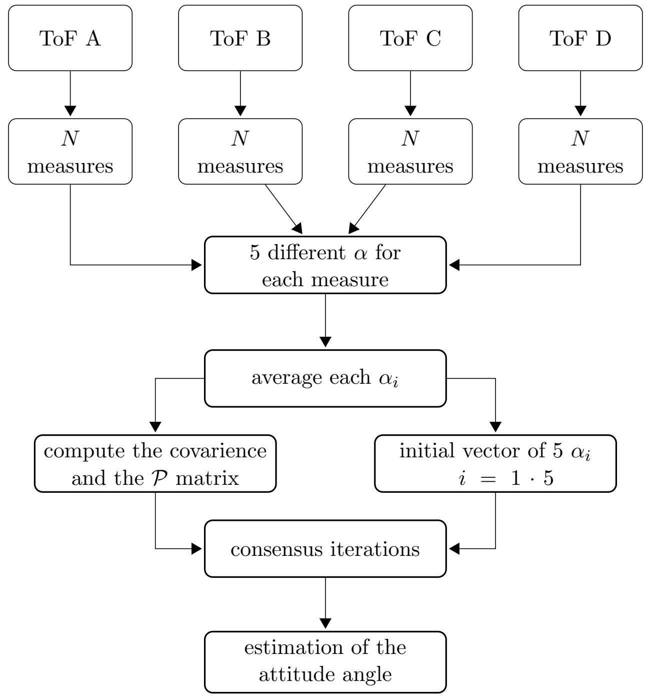

Fig. 4: Logic scheme that illustrates how the different ToF sensors and the consensus algorithm are employed to estimate the attitude orientation.

$$\begin{bmatrix} \bar{x} \\ \tan(\alpha) \\ \tan(\beta) \end{bmatrix}(t+1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \bar{x} \\ \tan(\alpha) \\ \tan(\beta) \end{bmatrix}(t) + v$$
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}(t) = \begin{bmatrix} 1 & -a & 0 \\ 1 & 0 & a \\ 1 & a & 0 \\ 1 & 0 & -a \end{bmatrix} \cdot \begin{bmatrix} \bar{x} \\ \tan(\alpha) \\ \tan(\beta) \end{bmatrix}(t) + w \tag{5}$$

where β is the rotation about the y axis, while v and w are the white Gaussian noises that affect the model and the measure respectively. The second value is related to the jitter of the ToF sensors output.

Equation 5 describes the dynamics of a common Kalman filter, whose implementation is reported in Alg. 2. This method is employed as a reference to assess the performance of the consensus method in terms of errors on the estimation on the relative attitude of the target.

IV. Simulation results

A preliminary validation of the proposed estimation method is obtained through numerical simulations. The simulations are performed as follows:

1) a series of $N = 100$ angular positions are defined between $-5\,\text{deg}$ and $5\,\text{deg}$;
2) a set of four ideal distance measurements are defined for each position;

Algorithm 2 Pseudo–code implementation of the Kalman filter algorithm.

Require: System model A, B, C
Require: Noise model v, w
Require: First estimation of the state vector x_{post}
Require: First estimation of the matrix P_{post}
Ensure: Input vector $u = 0$ (for this specific case)
loop
$x_{pre} = A \cdot x_{post} + B \cdot u$
$P_{pre} = A \cdot P_{post} \cdot A^T + Q$
$K = P_{pre} \cdot C^T \cdot \left[C \cdot P_{pre} \cdot C^T + R\right]^{-1}$
take the measure y
$err_y = y - C \cdot x_{pre}$
$x_{post} = x_{pre} + K \cdot err_y$
$P_{post} = P_{pre} - K \cdot C \cdot P_{pre}$
end loop

	average [deg]	std [deg]
α_1	0.39	0.29
α_2	0.37	0.30
α_3	0.42	0.32
α_4	0.39	0.27
α_5	0.25	0.20
filter	**0.24**	**0.18**

TABLE I: Average and standard deviation of the error on the reconstruction of the trajectory for the five orientations α_i and the filtered orientation.

3) for each ideal measurement a series of N noisy measures are defined considering a random error with a standard deviation of $2\,\mathrm{mm}$;
4) with the noisy measures, the five attitude angles α_i are computed, for each of the N measures;
5) the N orientations are used to estimate the orientation with two methods: (i) the first is based on the average of all the measurements, and (ii) the second is the consensus filter;
6) the covariance between the five $N \times 1$ vectors of α_i has been computed;
7) the elements of the consensus matrix are based on the covariance between the α_i;
8) finally the consensus problem is solved and the performances errors with respect to the nominal attitude angle are computed;
9) the process is repeated for each point on the trajectory.

Figure 5 shows the results of one of the numerical simulations, with the comparison between the averaged and the filtered orientations. The error of the proposed method is lower than that of the noisy measures. The average and the standard deviation of all the errors of all the orientations are reported in Tab. I.

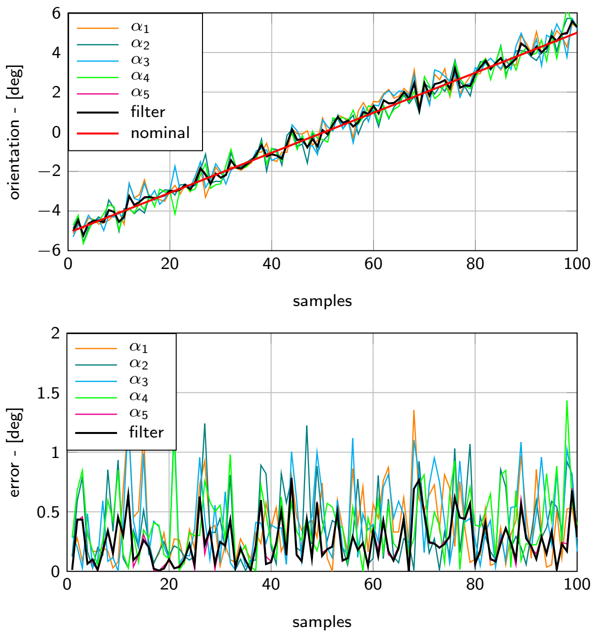

Fig. 5: Results of one numerical simulation for the five orientation α_i and the filtered orientations in terms of reconstruction of trajectory (above), and the error with respect to the nominal trajectory (below)

V. Experiment

The consensus method has been tested with measures from real ToF sensors and its results are compared with those obtained with the Kalman filter. This section describes the experiment set–up and presents the results.

A. Experimental setup

As introduced before, for the sake of simplicity, the rotation around the y axis is zero. This allows to use a planar slide to test the estimation algorithm. The slide provides 3 DoFs: two translations and one rotation. The target is mounted on the fixed part of the slide, while the plate containing the ToF sensors has been mounted on the moving part of the slide, as shown in Fig. 6.

For the experiment, eleven values of α are imposed to the slide (called nominal positions α_n). At each α_n, each ToF sensor has taken one hundred measures to have a good database for both the model–based estimation and the consensus estimation.

B. Results

The consensus filter has been employed to compute the $\alpha_1, \cdots, \alpha_5$ estimates. The process of the estimation is the following (see also Fig. 4):

1) each ToF sensor takes $N = 100$ measures;
2) for each couple of ToF sensors and for each measure, the algorithm computes the N values of α_i;
3) computation of the average $\bar{\alpha}_i$ of the N values of α_i;
4) computation of the covariance for the N couples (α_i, α_j);

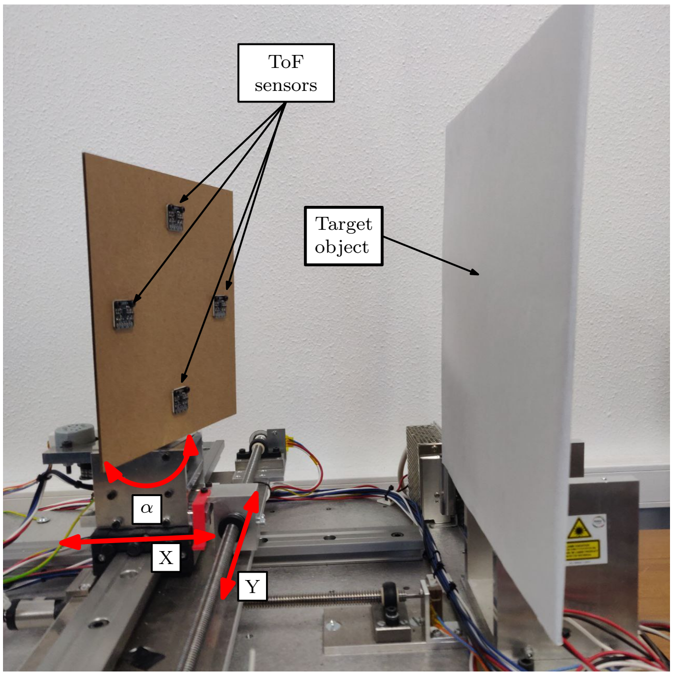

Fig. 6: Experimental set–up employed to test the estimation algorithm; the red arrows highlight the DoF of the planar slide. All the axis of motion are provided with encoders that enable to reconstruct the relative position with high accuracy.

α_n [deg]	Consensus estimation [deg]	Consensus error [deg]	Kalman estimation [deg]	Kalman error [deg]
-3.96	-3.98	0.02	-4.55	0.59
-2.88	-2.80	0.08	-2.76	0.12
-1.98	-2.03	0.06	-1.64	0.33
-1.26	-1.43	0.17	-1.68	0.42
-0.18	-0.57	0.39	-0.36	0.18
-0.09	0.16	0.25	0.40	0.49
0	0.005	0.005	0.51	0.51
1.26	1.11	0.15	0.85	0.41
1.98	2.21	0.23	2.27	0.29
3.96	3.27	0.69	3.40	0.56
4.68	4.55	0.13	4.82	0.14

TABLE II: Estimation of the relative attitude by using the consensus–based and the model–based methods. The error is taken as the absolute value of the difference between the nominal and the estimated values.

5) the average value $\bar{\alpha}_i$ is employed as the initial guess for the consensus problem;
6) repeat from point 1 until all the values of the elements of the vector are within a certain tolerance (10^{-3}deg in this case).

Tab. II reports the estimation for each α_n using respectively the consensus algorithm and with the model–based method.

By considering all the estimation errors reported in Tab. II, it is possible to evaluate a global average error and a global standard deviation of the estimation, reported in Tab. III.

The first plot of Fig. 7 shows the estimation of the different nominal orientations both for the consensus method and for the Kalman method. The second plot of the figure, shows the errors on the estimation with its average value (Tab. III).

Method	Average error [deg]	Standard deviation [deg]
Kalman	0.37	0.16
Consensus	0.20	0.20

TABLE III: Average and standard deviation of the errors of the two methods for all the different attitudes considered in the experiment.

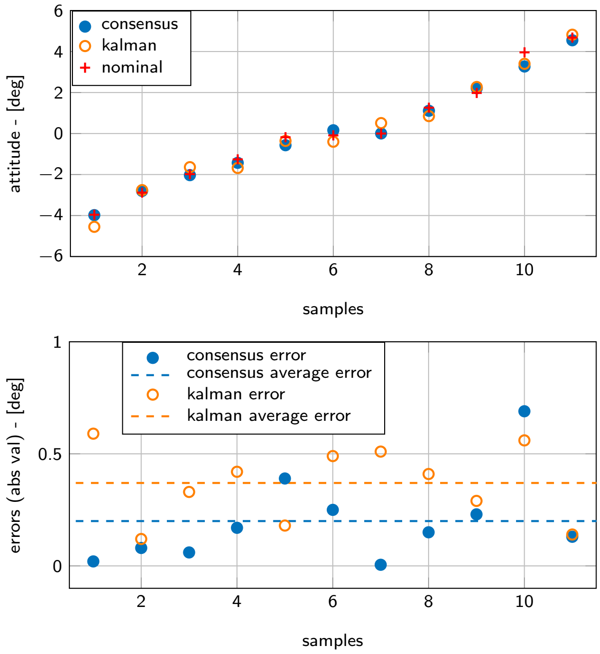

Fig. 7: For both the methods are shown the estimation with respect to the nominal position (above) and the estimation errors with the average error (below), the errors are considered as the absolute value between the estimation and the nominal orientation.

C. Discussion

As shown in Tab. III and in Fig. 7, the consensus algorithm has a lower average on the estimation error with respect to the Kalman algorithm. However the estimation with the consensus method is based on the initial value of each α_i, hence the more precise is this value, the more the estimation is near to the real value. A proof of this is the fact that the estimation error obtained with an initial value computed by averaging 10 values of α_i is higher than the error obtained by averaging 50 values of α_i. Specifically, the error decreases by a factor of 10 from a value in the order of 10^{-1}deg to a value in the order of 10^{-2}deg.

The estimation based on the Kalman filter, instead, uses a different approach: it is iterated until it converges. Since at every iteration a new measure is required, it could occur that more than one hundred measure are taken. After the convergence, the estimation is effected by error, as a consequence of the noise of the ToF sensors, but with the lowest variance.

Based on the above results, the advantages of using the consensus estimation are:

- applicability even in the case of lack of measures, since they are necessary only for the definition of the first value of the algorithm;
- based on an iterative method that does not require any additional measure;
- possibility merge different data (even of different nature) by weighting them with their variance and covariance;
- convergence is ensured by the fact that the consensus matrix $\mathcal{P}$ has all the eigenvalues lower that 1 by definition.

VI. Conclusions

This paper presents a consensus estimation algorithm for the sensor fusion to be applied to a capture system for On–Orbit operations. The algorithm is applied to the results of the measures instead of to the measures themselves. More precisely, four Time–of–Flight sensors are employed in pairs to retrieve five different values for the orientation of a target; the algorithm is applied to these five orientations.

The consensus matrix of the filter is based on the covariance of the five orientations. In such way, the algorithm considers the fact that the measure with the lower covariance has the highest weight in the consensus process.

The numerical simulations presented in this work state that the consensus filter approach allows to reduce the error of the orientation compared to the one taking determined by taking the average of orientations

Once the numerical simulations proves the reliability of the estimation algorithm, it has been employed with measures from the ToF sensors. To this purpose, an experiment has been established with the aim to reconstruct the relative orientation between the ToF sensors and the target object by using the algorithm. To evaluate the performances of the consensus method, it has been compared with a model–based estimation model. The results of the tests showed that the consensus algorithm estimate the orientation with a lower error, but it requires more measures to be processed than the model–based algorithm.

References

[1] A. Nanjangud, P. C. Blacker, S. Bandyopadhyay, and Y. Gao, “Robotics and ai-enabled on-orbit operations with future generation of small satellites,” *Proceedings of the IEEE*, vol. 106, no. 3, pp. 429–439, 2018.

[2] J. Sasiadek, “Sensor fusion,” *Annual Reviews in Control*, vol. 26, no. 2, pp. 203–228, 2002.

[3] F. Durrant-Whyte, “Sensor models and multisensor integration,” *International Journal of Robotics Research*, vol. 7, no. 6, pp. 97–113, 1988.

[4] A. Caon, F. Branz, and A. Francesconi, “Characterization of a new positioning sensor for space capture,” in *Metrology for Aerospace*, 2021.

[5] R. Olfati-Saber and J. Shamma, “Consensus filters for sensor networks and distributed sensor fusion,” in *Proceedings of the 44th IEEE Conference on Decision and Control*, 2005, pp. 6698–6703.

[6] Olfati-Saber, “Distributed kalman filtering for sensor networks,” in *Proceedings of the 46th IEEE Conference on Decision and Control*, 2007.

[7] W. Yu and Z. Wang, “Distributed consensus filtering in sensor networks,” *IEEE transactions on system, man, and cybernetics – part B, vol.39, no.6*, December, 2009.