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Title: A perspective on conditional spectrum-based determination of response statistics of nonlinear systems: stationary and non-stationary cases

Authors: Pol D. Spanos[1], Beatrice Pomaro*[2,3]

[1] *L. B. Ryon Chair in Engineering, Rice University, 6100 Main Street, Houston, TX 77005-1827, USA, spanos@rice.edu*
[2] *Department of Civil, Environmental and Architectural Engineering, University of Padova, Via F. Marzolo 9, 35131 Padova, Italy, beatrice.pomaro@unipd.it*
[3] *Department of Civil and Environmental Engineering, Rice University, 6100 Main Street, Houston, TX 77005-1827, USA*

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Abstract

Despite their versatility and attractiveness, it can be argued that the Statistical Linearization [1-3] and the Stochastic Averaging [4] techniques may have limited success in determining the response power spectral density of nonlinear systems. The study is conducted by defining the resonance properties of the system in the frequency domain, in terms of the stationary power spectral density of the system displacement and velocity. This is pursued by revisiting the conditional power spectrum concept [5-7], with the assumption that the response process is ergodic and pseudo-harmonic, characterized by an amplitude and a phase. A theoretical elucidation of an existing formula for the conditional spectrum is attempted. In particular, this principle is interpreted in relation to the time averaging approximation made in the definition of the stationary probability density function of a characteristic amplitude quantity, associated with the original nonlinear system. It is shown that a proper definition of the stationary probability density of the response amplitude, along with a reasonable treatment of the distribution over the frequency domain of the amplitude contribution, lead to an improved approximation of the response stationary power spectral density function. This is confirmed by a large suite of à propos Monte Carlo simulations, both in terms of the shape and the range of the involved resonance frequencies, even at a strong degree of nonlinearity. Further, the acquired insight regarding the stationary spectral density of the nonlinear system is used to formulate a Lyapunov equation-based determination [4] of the non-stationary response statistics even in the presence of hysteretic and fractional derivative elements.

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