Credit constraints and the inverted-U relationship between competition and innovation

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Abstract

Empirical studies have uncovered an inverted-U relationship between product-market competition and innovation. This is inconsistent with the original Schumpeterian Model, where greater competition always reduces the profitability of innovation and thus the incentives to innovate. We show that the model can predict the inverted-U, if the innovators' talent is heterogenous, and asymmetrically observable. When competition is low and profitability is high, talented innovators are credit constrained, since untalented innovators are eager to mimic them. As competition increases and profitability decreases, untalented innovators become less eager to mimic, and talented innovators can invest more. This generates the increasing part of the relationship. When competition is high and profitability is low, credit constraints disappear, and the relationship is decreasing. Our theory generates additional specific predictions that are well born out by the existing evidence.

Keywords: Innovation, Competition, Schumpeterian Model of Growth, Asymmetric Information

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Several empirical studies have uncovered an inverted-U relationship between product-market competition and innovation.[1] This finding is inconsistent with the original Schumpeterian model (Aghion and Howitt (1992)), where stronger competition always reduces the incentives to innovate because it reduces post-innovation rents (the so-called "Schumpeterian effect"). To address this inconsistency, Aghion, Bloom, Blundell, Griffith, and Howitt (2005) have modified the original model to allow for innovation by established firms, who also care about *pre*-innovation rents. These firms are crucial to explaining the increasing part of the inverted-U, because it is only for them that competition may strengthen the incentives to innovate (by decreasing pre-innovation rents more than it decreases post-innovation rents).

While the mechanism in Aghion, Bloom, Blundell, Griffith, and Howitt (2005) is intuitive and easy to believe, we suspect that it may not fully account for the existence of an inverted-U. On the one hand, a vast majority of innovations are actually realised by either new entrants, or by established firms innovating on entirely new product lines - for both of which competition in the target market should primarily affect post-innovation rents, as in the original Schumpeterian model.[2] This casts a doubt on whether the increasing part of the inverted-U can entirely be explained by the actions of firms focused on pre-innovation rents. On the other hand, some evidence of a positive relationship between competition and innovation has also been found for start-ups,[3] which again are less likely to fit the notion of firms focused on pre-innovation rents.

In this paper, we show that the Schumpeterian model can predict the inverted U even under the original assumption that innovators focus on post-innovation rents. Only two, reasonable ingredients must be added to that effect: heterogeneous talent of innovators, and asymmetric information on talent. We start from a standard version of the model with overlapping generations and a fringe of competitive producers (as in Aghion and Howitt (2009)), and allow for innovators to be of two types (talented and untalented), and for this to be the innovators' private information. We construct a separating equilibrium in which the talented innovators signal themselves to investors by contributing their entire wage in equity, and by limiting the amount they borrow.[4] We study the comparative statics of this equilibrium, and show that the relationship between the strength of competition and the probability of innovation is first increasing and then decreasing.

More in detail, our mechanism works as follow. At low levels of competition, when post-innovation rents are high, the talented innovators would like to invest a lot. However, they cannot borrow enough at favourable conditions, since the untalented innovators are eager to mimic them (given the high profitability of innovation). They then invest less than optimal. As competition increases and post-innovation rents decrease, the untalented innovators become less eager to mimic. As this happens, the amount that the talented innovators can borrow at favourable conditions increases, leading them to invest *more*. This explains the increasing part of the curve. We call this effect the *selection effect*, because it leads to a higher weight of the talented innovators in overall investment. At high levels of competition, when post-innovation rents are low, the talented innovators would like to invest only a modest amount. Moreover, they can borrow a lot at favourable conditions, since the untalented agents are not eager to mimic them. They then invest their optimal amount, which by the Schumpeterian effect is decreasing in the strength of competition. This generates the decreasing part of the curve.

One attractive feature of our model is that it rests on reasonable assumptions. We have already argued that innovators focused on post-innovation rents are an empirically relevant group. In addition, talent heterogeneity and asymmetric information are recognised features of the market for innovation financing. Hubbard (1998) and Brown and Petersen (2009) argue that asymmetric information is likely to be a particularly severe problem for R&D-intensive firms. This is not just because of the inherent difficulty to evaluate frontier research, but also because innovative firms are reluctant to reveal their ideas to investors, reducing the quality of the signal they can make about potential projects (Lerner and Hall (2010), p. 614). Lerner and Hall (2010) and Kerr and Nanda (2015) both list asymmetric information on the quality of projects as one of several key sources of frictions in the market for innovation financing. Not only are talent heterogeneity and asymmetric information reasonable assumptions: it is also the case that the credit constraints that these frictions generate are important enough to explain macro patterns such as the inverted-U. For example, Brown and Petersen (2009) argue that most of the unprecedented 1990s R&D boom can be explained with a relaxation of the credit constraints of young R&D intensive firms.

To provide corroborating evidence in support of our mechanism, we show that the equilibrium we characterise has specific features which match the evidence well. First, the increasing relationship between competition and innovation should be more pronounced in industries where credit constraints are more prevalent. This prediction fits the finding in Aghion, Bloom, Blundell, Griffith, and Howitt (2004), who show that the inverted-U shifts to the right if one focuses on firms which are more under debt-pressure. Second, credit constraints should be more severe in industries where profits are higher (either because competition is lower, or for other reasons). Indeed, several papers in the finance literature have documented a positive relationship between credit constraints and return on equity. Interestingly, Li (2011) shows that this relationship is particularly important among R&D intensive firms, where our mechanism is also likely to be particularly important.

This paper contributes to Schumpeterian growth theory (see Aghion, Akcigit, and Howitt (2014), for a survey). It complements the main existing explanation for the inverted-U relationship between competition and innovation (Aghion, Bloom, Blundell, Griffith, and Howitt (2005)),[5] by showing that Schumpeterian theory is consistent with the inverted-U under a broader (and as discussed above, more empirically relevant) set of assumptions. Other theoretical explanations of the inverted-U focus squarely on issues of industry organization and dynamics, leaving virtually no role for financial factors.[6]

Conversely, our model puts asymmetric information in financial markets at its front and center. This simple, realistic addition to an otherwise standard model allows us to generate the "inverted-U" pattern through an intuitive mechanism.

A number of other papers in the Schumpeterian tradition have placed financial features at center stage. These articles differ from ours in their assumptions (usually and most importantly, the nature of the financial frictions they consider), as well as their subject matters and applications. For example, Diallo and Koch (2018) investigate the relationship between economic growth and bank concentration. Malamud and Zucchi (2016) study corporate cash management when firms face exogenous financing costs. Sunaga (2018) extends the standard model to deal with moral hazard in financial markets and monitoring by intermediaries.[7] Bryce Campodonico, Bonfatti, and Pisano (2016) and Plehn-Dujowich (2009) develop Schumpterian growth models with adverse selection in financing, but use them to study optimal tax policy and to quantify the reduction in the rate of growth stemming from the presence of financial frictions, respectively. Finally, Ates and Saffie (2013) study a general equilibrium endogenous growth model in which financial intermediaries screen the quality of projects from a heterogeneous population of entrepreneurs. None of these papers concerns itself with the relationship between an industry's degree of competition and its R&D outcomes, which is the main focus of the present essay.

Finally, the paper also relates to the burgeoning literature on the macroeconomic implications of financial frictions (see Brunnermeier, Eisenbach, and

Sannikov (2013)), and more specifically the branch analyzing their effects on countries' economic development (see Levine (2005)).

The paper is organised as follows. In Section I we present the baseline model. Section II introduces imperfect information in financial markets, and derives the inverted-U relationship between competition and innovation. Section III discusses the empirical validity of two predictions that are specific to our model. Finally, Section IV concludes.

I BASELINE MODEL

The baseline model is a standard Schumpeterian model with overlapping generations and a fringe of competitive producers (as in Aghion and Howitt (2009), pp. 130-32 and 90-91), which we generalise to allow for heterogeneous talent of innovators. A final good is produced competitively using labour and a continuum of intermediate goods, according to production function

$$Y_t = L^{1-\alpha} \int_0^1 A_{it}^{1-\alpha} X_{it}^{\alpha}, \tag{I.1}$$

where X_{it} is input of the latest version of intermediate i, and A_{it} is its productivity. Each intermediate is produced and sold by a monopolist, who can produce one unit of the intermediate at the cost of one unit of the final good. However, in each industry, there is also a fringe of competitive firms that can produce the intermediate at cost of $1/\kappa_i$ units of the final good per unit produced. The parameter $\kappa_i \in [\alpha, 1]$ measures the strength of competition faced by the monopolist. As will be clear below, $\kappa_i = \alpha$ denotes the case of no competition, while $\kappa_i = 1$ denotes the case of perfect competition. For simplicity, all industries have the same initial level of productivity, $A_{t-1} \equiv \int_0^1 A_{it-1} di$.[8]

Agents live for two periods, are risk neutral, and have a discount factor equal to one. There are two equally-sized cohorts alive in each period, the young and the old. The young work in the final good sector, where they earn a wage. Before turning old, one of them per industry (the "innovator ") tries to invent a new version of the intermediate good which is $\gamma > 1$ times more productive than the previous version. If successful, she invests in the production of the new version, which she sells as the monopolist when she turns old in the next period. If unsuccessful, a young agent is chosen at random to invest in the production of the previous version, and to sell it as the monopolist when he turns old. As for the old agents, there is one of them in each industry who is the current monopolist, while all others are idle consumers.

There are borrowers and lenders in this model. Borrowers include young agents undertaking an investment - be it innovation or production - which they cannot fund through the wage they have earned. The lenders are all young agents, who may want to use part of the wage they have earned to consume when they are old. While production is a risk-free activity, innovators only pay back if successful. Then, the financing of innovation is the only interesting part of the financial market. We assume that the maximum supply of credit (the total wage bill) is greater than demand, so that the risk-free interest rate is equal to the discount rate (zero).

A monopolist faces iso-elastic demand $P_{it} = \alpha \left(A_{t-1}L/X_{it}\right)^{1-\alpha}$, given which her optimal price is $1/\alpha$.[9] However, facing competition from the fringe, the monopolist is forced to charge $1/\kappa_i \leq 1/\alpha$ instead. Plugging back in the demand function, we find the optimal X_{it}, which can then be multiplied by profit per unit, $(1-\kappa_i)/\kappa_i$, to find total profits. There are (normalised by initial productivity)

$$\pi(\kappa_i) = \frac{1-\kappa_i}{\kappa_i} (\kappa_i \alpha)^{\frac{1}{1-\alpha}} L \tag{I.2}$$

in an industry that has not innovated, and $\gamma\pi(\kappa_i)$ in an industry that has. It is easy to show that π is decreasing in $\kappa_i \in [\alpha, 1]$: intuitively, the stronger is competition, the lower are the monopolist's profits.

Substituting optimal X_{it} in the production function, differentiating with respect to L, and dividing by A_{t-1}, we find the normalised wage,

$$w = (1-\alpha)(\kappa\alpha)^{\frac{\alpha}{1-\alpha}}, \tag{I.3}$$

where $\kappa \equiv \int_\alpha^1 \kappa_i di$ is average level of competition in the economy.

Innovators can be of two types, a high type (H) and a low type (L). If an innovator of type $J \in \{H, L\}$ invests a normalised amount z in research, she is successful with probability $a^J \mu(z)$, where μ is an increasing and concave function satisfying standard conditions, and $a^H > a^L$. In each generation, there is an equal share of high types and low types.

For now, we assume that an innovator's type is perfectly observable to everyone. Then, competing lenders demand interest rate $1/[a^J \mu(z)] - 1$ from type J, given which the innovator's net present value is

$$npv_i^J(z) = a^J\mu(z)\left[\gamma\pi(\kappa_i) - \frac{1}{a^J\mu(z)}(z-e)\right] - e$$
$$= a^J\mu(z)\gamma\pi(\kappa_i) - z, \tag{I.4}$$

where e is her normalised equity contribution. With perfect information, the innovator's net present value does not depend on her choice of financing, since the expected cost of both equity and external financing is equal to the risk-free interest rate.

Let $\hat{z}_i^J$ denote optimal, perfect-information investment by type J in industry i. This is the level of investment that maximises the $npv_i^J(z)$ function, and is thus implicitly defined by condition

$$a^J\mu'\left(\hat{z}_i^J\right)\gamma\pi(\kappa_i) = 1. \tag{I.5}$$

Condition I.5 clarifies that there are two (and only two) reasons why z may vary across industries. First, industries may differ in the type of their innovators, and, *ceteris paribus*, the high types always invest more than the low types. For example, if industry i only differs from industry j for having an innovator of the high type, then the two industries will invest $\hat{z}_i^H$ and $\hat{z}_j^L$, and it will be $\hat{z}_i^H > \hat{z}_i^L$. Second, industries may differ in terms of the strength of competition within them. For example, if i and j only differ in the fact that $\kappa_i > \kappa_j$, then it will be $\hat{z}_i^J < \hat{z}_j^J$: investment in the more competitive industry will be lower. The latter is the well-known *Schumpeterian effect* of competition on innovation: by reducing the profitability of innovation, stronger competition reduces the incentives to innovate. Because the Schumpeterian effect is the only effect existing at all levels of competition, the original Schumpeterian model predicts a monotonically decreasing relationship between competition and innovation.

Figure 1 illustrates. We will use the functional form and parameters used in this figure (and reported at the bottom of the figure) as a running example in the remainder of the paper. The three panels only differ in the value of κ_i, which is increasing from top to bottom as reported on the left of the figure. By our choice of α, κ_i is required to range between 0.4 and 1. Panel (a) then represents the extreme of no competition (the successful innovator is a true monopolist), while the second and third panels progressively increase competition. The $npv_i^J(z)$ functions are represented by the thin solid curves (all other curves should be ignored for now). Investment choices under perfect information, $\hat{z}_i^H$ and $\hat{z}_i^H$, maximise these functions, and the related payoffs are represented by solid circles. In all panels, the high types invest more than the low types, as illustrated by $\hat{z}_i^H$ being to the right of $\hat{z}_i^L$. The Schumpeterian effect is clearly visible from the figure: as we move from panel (a) through to panel (c), due to increasing competitions, the $npv_i^J(z)$ functions rotate inwards, and investment by both types decreases.

II ASYMMETRIC INFORMATION IN FINANCIAL MARKETS

We now assume that the innovator's type is the innovator's private information. Lenders must then determine the interest rate based solely on the subset of information which is observable, that is the size of the proposed investment (z) and equity contribution (e). In this section, we describe, in an intuitive way, a specific separating equilibrium, which happens to exist when parameters are as in our running example (the one drawn in Figure 1). In the Appendix, we formally derive the separating equilibrium (Section A1), we identify the parameter space such that the equilibrium exists (A2), and we show that, in that parameter space, the equilibrium has an attractive feature: its outcome is the only one that can "reasonably" realise in a Perfect Bayesian Equilibrium that survives a standard refinement criterion (A3).

II.1 DESCRIPTION OF THE EQUILIBRIUM

Refer again to our running example (Figure 3.1). To describe the equilibrium, we fix competition at the level of panel (a). Later, we will conduct comparative statics by increasing competition to the levels in panels (b) and (c).

Recall that, with perfect information, the high and low types invest $\hat{z}_i^H$ and $\hat{z}_i^L$ respectively. It is easy to show that, with imperfect information, the low types must continue to invest $\hat{z}_i^L$ at a separating equilibrium, contributing any $e \leq \hat{z}_i^L$ in equity.[10] However, the high types may now be forced to invest less than in the perfect-information case.

To see why, consider one reasonable scenario in which the high types would be able to invest $\hat{z}_i^H$: suppose that the lenders believed that anyone offering to contribute their entire labour income in equity (w) are high types. Such a belief would allow the high types to borrow $\hat{z}_i^H - w$ at their perfect-information rate, which would mean that they are able to finance any level of investment at the risk-free interest rate (in expectations). Then, their net present value would still be $npv_i^H(z)$, and they would choose to invest $\hat{z}_i^H$ as with perfect information. However for this to be an equilibrium, the low types should not want to mimic the high types. But in the example of panel (a), the low types would indeed want to mimic the high types.

To see why, note that by contributing w in equity, and borrowing $z - w$ at the high types' perfect-information rate, the low types would receive net present value

$$\begin{aligned}\widetilde{npv}_i^L(z) &\equiv a^L\mu(z)\left[\gamma\pi(\kappa_i) - \frac{1}{a^H\mu(z)}(z-w)\right] - w \\ &= a^L\mu(z)\gamma\pi(\kappa_i) - \frac{a^L}{a^H}(z-w) - w, \end{aligned} \qquad \text{(II.1)}$$

that is the dashed line in the figure. This line is higher than $npv_i^L(z)$ in the range where it is defined, because by mimicking the high types the low types can pay less than the risk-free interest rate (in expectations) on external borrowing. Then, investment gives them a higher payoff than under perfect information. Given this higher payoff, it is optimal for them to mimic the high types and propose to invest $\hat{z}_i^H$, even though this also requires to propose to contribute w in equity. In terms of the figure, this can be seen from the fact that it is $\widetilde{npv}_i^L\left(\hat{z}_i^H\right) > npv_i^L\left(\hat{z}_i^L\right)$. Since the low types would find it optimal to mimic the high types, this cannot be a separating equilibrium.

If not $\hat{z}_i^H$, what amount can the high types invest at a separating equilibrium? Suppose that the lenders had different beliefs: that even those contributing w in equity, when they invest more than z_i^{sep}, can be high or low

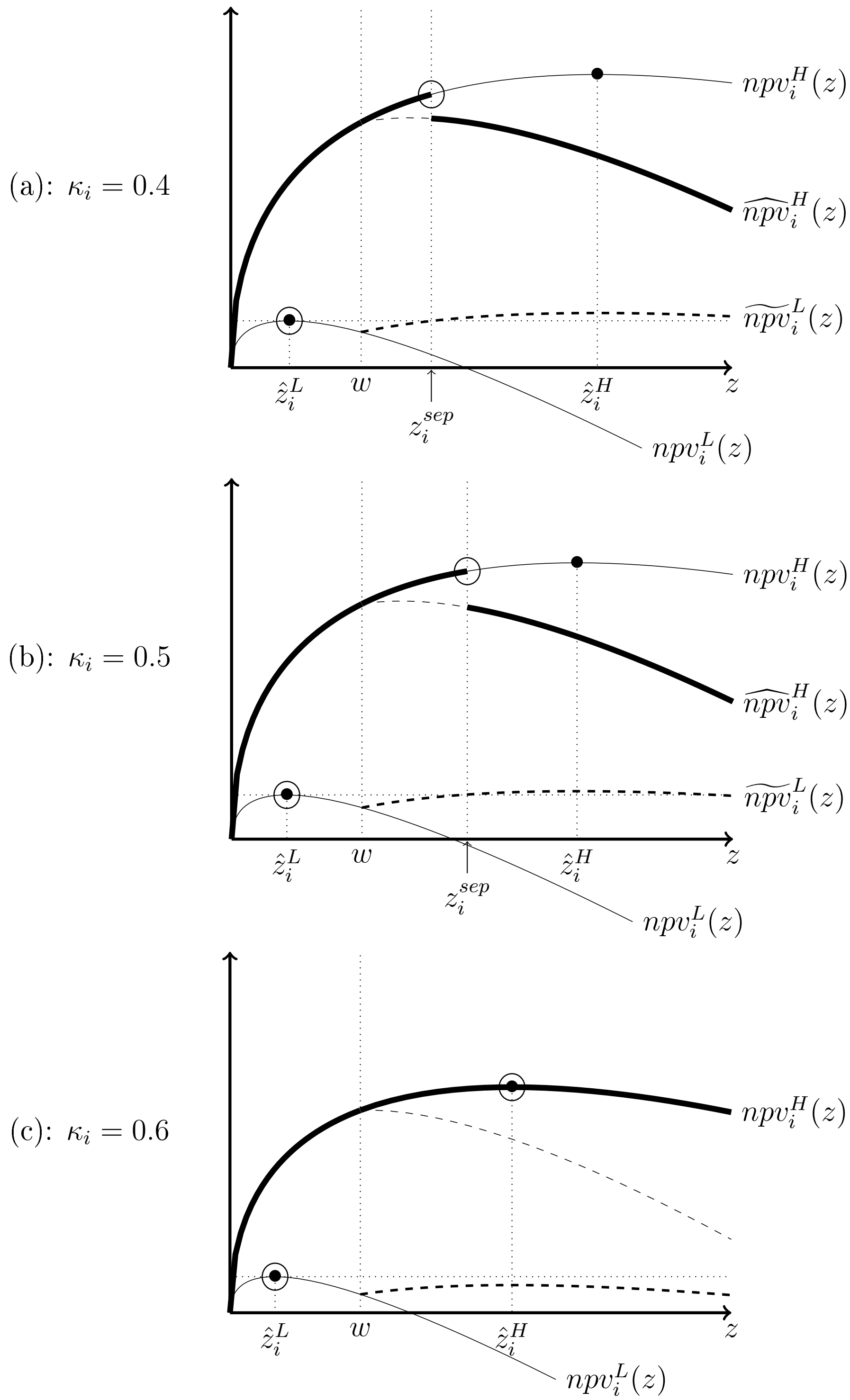


Figure 1: Illustration of the separating equilibrium. The three panels only differ by the size of κ_i, which increases from top to bottom as indicated to the left of the figure. The functional form and other parameters used are: $\alpha = 0.4$; $L = 100$; $\kappa = 0.7$; $\mu(z) = 0.22\sqrt{z}$; $a^H = 1$; $a^L = 0.4$.

types with equal probability.[11] Given these beliefs, the high types must now pay a higher-than-fair interest rate to invest more than z_i^{sep}, because they would be pooled together with the low types. In terms of the figure, their net present value is not $npv_i^H(z)$ anymore, but rather the broken solid line. This differs from $npv_i^H(z)$ to the right of z_i^{sep}, where it equals

$$\begin{aligned}\widehat{npv}_i^H(z) &\equiv a^H\mu(z)\left[\gamma\pi(\kappa_i) - \frac{1}{a\mu(z)}(z-w)\right] - w \\ &= a^H\mu(z)\gamma\pi(\kappa_i) - \frac{a^H}{a}(z-w) - w, \qquad \text{(II.2)}\end{aligned}$$

where $a = (a^H + a^L)/2$. In words, to the right of z_i^{sep}, the high types must pay the "fair" interest rate of an hypothetical average agent, $1/a\mu(z)$.

Given their new net present value function, the high types choose to invest z_i^{sep}. They are *credit constrained*, in the sense that a friction of the credit market (the non observability of talent) forces them to borrow only $z_i^{sep} - w$, which is less than they would ideally do $(\hat{z}_i^H - w)$. Crucially, the low types no longer want to mimic the high types, since the latter's conservative choice of leverage makes mimicking no more attractive then investing $\hat{z}_i^L$. In terms of the figure, it is $\widetilde{npv}_i^L(z_i^{sep}) = npv_i^L(\hat{z}_i^L)$ (of course, z_i^{sep} was chosen precisely to satisfy this indifference condition). Thus, by contributing their entire income in equity and by choosing to invest less than optimal, the high types are able to signal themselves as talented innovators to the uninformed lenders.

What we have just described is a separating Perfect Bayesian Equilibrium, since both types pay their perfect information interest rate, innovators invest optimally given the lenders' beliefs, and beliefs are correct in equilibrium. The payoffs corresponding to the investment choices at this separating equilibrium, $\hat{z}_i^L$ and z_i^{sep}, are represented by empty circles in Figure 1.

II.2 KEY COMPARATIVE STATICS

We now come to the central result of the paper, which is to show that, at the separating equilibrium just described, the relationship between product-market competition, κ_i, and the *ex-ante* probability of innovation,

$$\mu_i = \frac{1}{2} a^H \mu\left(z_i^{sep}\right) + \frac{1}{2} a^L \mu\left(\hat{z}_i^L\right),$$

is first increasing and then decreasing.

Consider again the example of Figure 1. We begin by increasing the strength of competition from the level in panel (a) to the level in panel (b). As discussed above, the curves representing $npv_i^L(z)$ and $npv_i^H(z)$ rotate inwards. Then, investment by the low types, which is the same as in the perfect information case ($\hat{z}_i^L$), still decreases by the Schumpeterian effect. However investment by the high types (z_i^{sep}) now *increases*, as is clearly visible from the figure.

To make sense of this, note that at this equilibrium the investment decisions of the high types are not driven by incentives, but rather by credit constraints. Then, the Schumpeterian effect does not apply, and what matters is the effect of competition on credit constraints. Our key result is that stronger competition reduces credit constraints, thus allowing the high types to invest more. This is because stronger competition discourages the mimickers: it makes investment less attractive for everyone, but particularly so for agents who are considering to invest more than they would normally do. Formally, recall that z_i^{sep} is what the high types must invest to make a genuine and a mimicking low type equally well off. But a fall in $\pi(\kappa_i)$ penalises the mimicker more than the genuine agent, since the mimicker invests more ($z_i^{sep} > \hat{z}_i^L$) and has thus a higher probability of innovating. To restore equality of payoffs, z_i^{sep} must then increase. In terms of figure 1, if we fixed z_i^{sep} at the level of panel (a) and decreased $\pi(\kappa_i)$ to the level of panel (b), then the value $\widetilde{npv}_i^L(z_i^{sep})$ would fall more than $npv_i^L(\hat{z}_i^L)$. For the two to remain equal, a higher z_i^{sep} is required.

In other words, stronger competition, by creating a tougher operating environment, leads to a better selection of innovators, in the sense that the high types can invest more and a greater share of available funds is allocated to them. We call this the *selection effect* of competition on innovation.

Now suppose that κ_i increases further, to the level in panel (c). While investment by the low types continues to decrease by the Schumpeterian effect, investment by the high types continues to increase by the selection effect, to the point that it is now equal to the perfect information level $\hat{z}_i^H$. Now, the high types are no longer credit constrained, because strong competition has made $\hat{z}_i^H$ low enough relative to what they can borrow. It follows that the Schumpeterian effect kicks back in for the high types as well, and any further increase in competition must now decrease investment by *both* types.

This example suggests that, at the separating equilibrium described in the previous section, and across industries where the innovator is of a high type, one should find an increasing and then decreasing relationship between the strength of competition and innovation. This result is formally stated in

Proposition 1. *Consider any two industries i and j where the innovator is a high type, and such that competition is stronger in j than in i, $\kappa_i < \kappa_j$. At the separating equilibrium described above, there exists $\hat{\kappa} \in (\alpha, 1)$ such that, if $\alpha \leq \kappa_i < \kappa_j \leq \hat{\kappa}$, industry j has a higher probability of innovating than industry i, while if $\hat{\kappa} \leq \kappa_i < \kappa_j \leq 1$, industry j has a lower probability of innovating than industry i.*

Proof. Note that $\widetilde{npv}_i^L(z)$ is concave, and reaches a maximum at $\hat{z}_i^H$. Let

$$z_i^{sep} = \min \arg \left\{ a^L \mu(z_i^{sep}) \gamma \pi(\kappa_i) - \frac{a^L}{a^H}(z_i^{sep} - w) - w = a^L \mu\left(\hat{z}_i^L\right) \gamma \pi(\kappa_i) - \hat{z}_i^L \right\}. \tag{II.3}$$

or, if such z does not exist, then $z_i^{sep} = \hat{z}_i^H$. There are two possible cases: $z_i^{sep} < \hat{z}_i^H$ or $z_i^{sep} = \hat{z}_i^H$. Suppose $z_i^{sep} < \hat{z}_i^H$, and consider an increase in κ_i. The total differential of the equation in curly brackets in (II.3) is

$$a^L \mu'(z_i^{sep}) \gamma \pi(\kappa_i)\, dz_i^{sep} + a^L \mu(z_i^{sep}) \gamma \pi'(\kappa_i)\, d\kappa_i - \frac{a^L}{a^H} dz_i^{sep} = a^L \mu\left(\hat{z}_i^L\right) \gamma \pi'(\kappa_i)\, d\kappa_i, \tag{II.4}$$

which can be re-arranged into

$$\frac{dz_i^{sep}}{d\kappa_i} = a^H \frac{\mu\left(\hat{z}_i^L\right) - \mu\left(z_i^{sep}\right)}{a^H \mu'(z_i^{sep}) \gamma \pi(\kappa_i) - 1} \gamma \pi'(\kappa_i) > 0.$$

Since z_i^{sep} is continuously increasing in κ_i, while $\hat{z}_i^H$ is continuously decreasing and $0 \leftarrow \hat{z}_i^H$ as $\kappa_i \to 1$, there exists $\hat{\kappa} \in (\alpha, 1)$ such that, for $\kappa_i < \hat{\kappa}$, it is $z_i^{sep} < \hat{z}_i^H$, while for $\kappa_i \geq \hat{\kappa}$ it is $z_i^{sep} = \hat{z}_i^H$. In the latter range, it is $dz_i^{sep}/d\kappa_i = d\hat{z}_i^H/d\kappa_i < 0$. The result follows immediately. Note that $\hat{\kappa}$ must be the same across industries, since κ_i is the only parameter that varies across industries. □

Proposition 1 finds, for industries where the innovator is of a high type, an increasing and then decreasing relationship between competition and innovation. The region $\alpha \leq \kappa_i < \hat{\kappa}$ is where the high types invest z_i^{sep} (panels a and b in our example), while the region $\hat{\kappa} \leq \kappa_i \leq 1$ is where they invest $\hat{z}_i^H$ (panel c). The threshold $\hat{\kappa}$ is defined as the unique level of competition such that $z_i^{sep} = \hat{z}_i^H$.

One shortcoming of Proposition 1 is that it only finds an increasing and then decreasing relationship between competition and innovation across industries where the innovator is of a high type, while the relationship is decreasing across all other industries. This does not answer our initial question about the relationship between κ_i and μ_i, the *ex-ante* probability of innovation. Looking further into this, it is immediate to see that such relationship will be decreasing in the region $\hat{\kappa} \leq \kappa_i \leq 1$, where both $\hat{z}_i^H$ and $\hat{z}_i^L$ are decreasing in κ_i by the Schumpeterian effect. On the other, we are now going to show that the relationship between κ_i and μ_i will be increasing in the region $\alpha \leq \kappa_i < \hat{\kappa}$, at least for κ_i close enough to $\hat{\kappa}$. So, at least in a subset of $[\alpha, 1]$, the model predicts an increasing and then decreasing relationship between competition and the *ex-ante* probability of innovation.

This is shown formally in:

Proposition 2. *Consider any two industries i and j such that competition is stronger in j than in i, $\kappa_i < \kappa_j$. At the separating equilibrium described above, there exists $\widetilde{\kappa} \in (\alpha, \hat{\kappa})$ such that, if $\widetilde{\kappa} < \kappa_i < \kappa_j \leq \hat{\kappa}$, industry j has a higher* *ex-ante probability of innovating than industry i, while for $\hat{\kappa}_i \leq \kappa_i < \kappa_j \leq 1$, industry j has a lower ex-ante probability of innovating than industry i.*

Proof. Suppose $z_i^{sep} < \hat{z}_i^H$. It is

$$\frac{d\mu_i}{d\kappa_i} = \frac{1}{2} a^H \mu' \left(z_i^{sep}\right) \frac{dz_i^{sep}}{d\kappa_i} + \frac{1}{2} a^L \mu' \left(\hat{z}_i^L\right) \frac{\hat{z}_i^L}{d\kappa_i}.$$

The total derivative $dz_i^{sep}/d\kappa_i$ was derived in (II.3), while $d\hat{z}_i^L/d\kappa_i$ can be found by taking the total differential of (I.5) and re-arranging,

$$\mu''(\hat{z}_i^L)\gamma\pi\left(\kappa_i\right) d\hat{z}_i^L + \mu'(\hat{z}_i^L)\gamma\pi'\left(\kappa_i\right) d\kappa_i = 0$$

$$\frac{d\hat{z}_i^L}{d\kappa_i} = -\frac{\mu'(\hat{z}_i^L)\pi'\left(\kappa_i\right)}{\mu''(\hat{z}_i^L)\pi\left(\kappa_i\right)} < 0.$$

Replacing $dz_i^{sep}/d\kappa_i$ and $d\hat{z}_i^L/d\kappa_i$ into the expression for $d\mu_i/d\kappa_i$, imposing $d\mu_i/d\kappa_i > 0$, and re-arranging we obtain

$$\frac{1}{\alpha^H \mu'\left(z_i^{sep}\right)\gamma\pi\left(\kappa_i\right) - 1} > \frac{a^L \left[\mu'\left(\hat{z}_i^H\right)\right]^2}{\left[-\mu''\left(\hat{z}_i^L\right)\right]\gamma\pi\left(\kappa_i\right)\left[a^H\right]^2 \mu'\left(z_i^{sep}\right)\left[\mu\left(z_i^{sep}\right) - \mu\left(\hat{z}_i^L\right)\right]}.$$

As $\kappa_i \to \hat{\kappa}$, it is $z_i^{sep} \to \hat{z}_i^H$. As this happens, the LHS of the last inequality approaches infinity, while the RHS remains finite. Then, there exists $\widetilde{\kappa} \in (\alpha, \hat{\kappa})$ such that, for $\kappa_i \in (\widetilde{\kappa}, \hat{\kappa})$, it is $d\mu_i/d\kappa_i > 0$, while for $\kappa_i > \hat{\kappa}$ it is $d\mu_i/d\kappa_i < 0$. The result follows immediately. Note that $\widetilde{\kappa}$ must be the same across industries, since κ_i is the only parameter that varies across industries. □

To make sense of the increasing part of the curve, recall that this is driven by industries where the innovator is of a high type: in those industries, z_i^{sep} must increase as κ_i increases, to restore equality of payoffs between a genuine and a mimicking low type (given that the latter suffers more from a fall in profits). But as κ_i approaches $\hat{\kappa}$ and z_{sep} approaches $\hat{z}_i^H$, which is the maximum of the mimicker's net present value function, the gain to the mimicker from an increase in z_i^{sep} monotonically decreases to zero. It follows that, as κ_i approaches $\hat{\kappa}$, the increase in z_i^{sep} that follows from an increase in κ_i must grow unboundedly, as greater and greater increases are required to compensate the mimicker. In contrast, in industries where the innovator is of a low type, the decrease in $\hat{z}_i^L$ is always finite. In other words, as κ_i approaches $\hat{\kappa}$, the selection effect must always be stronger than the Schumpeterian effect. This also suggests that the relationship between κ_i and μ_i should be convex as we approach its peak from the left, an intuition which is confirmed by the computational exercise in the next section.

We have derived a specific separating in which the relationship between competition and innovation has an inverted-U shape. But when exactly will this equilibrium esist, and how many other plausible equilibria are there? In the Appendix, we show that our separating equilibrium exists as long as the wage is neither too high (or else the talented innovators would not need to borrow) nor too low (or else they would need to borrow so much, that they would opt for being pooled with the untalented innovators). Moreover, we argue that, in this parameter sub-space, our equilibrium outcome is the only one that can "reasonably" realise in a Perfect Bayesian Equilibrium that survives a standard refinement procedure.

We conclude this section by discussing two simplifying tricks that we have used in this paper. First, the standard Schumpeterian growth model described in Section I is an infinite-horizon model with an overlapping generation structure, and yet the signalling game described in Section II is a static game. This combination is only possible under a carefully selected set of assumptions. For example, had we assumed that an innovator can invest more than once, or that she cares about future innovators who are also more likely to succeed if the current innovator succeeds,[12] then the signalling game would have become more complicated, as the current innovator would have had to consider the future impact of her investment decisions. Second, we have only considered investment at a hypothetical period t in which initial productivity is the same across industries. But already in period $t+1$, as some industries innovate and others do not, this assumption would necessarily be invalid. Credit constraints would vary across industries, even keeping talent and competition constant. While our model can easily accommodate this additional dimension of heterogeneity (as we show in Section III), a full analysis would need to keep track of how credit constraints evolve over time. The role of these simplifications is obvious: they allow us to describe in a clearer way a mechanism that would exist even in more complicated settings.

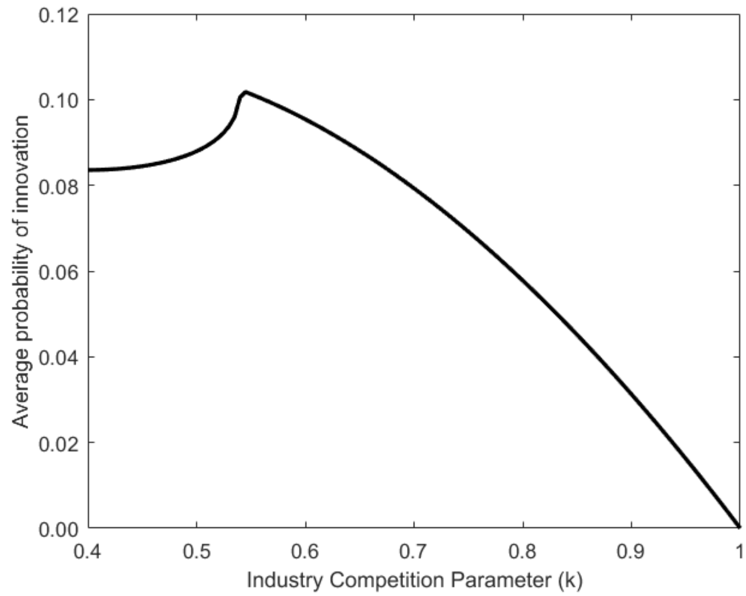
II.3 COMPUTATIONAL EXERCISE

One limitation of Proposition 2 is that it concerns itself exclusively with values of κ_i close to $\hat{\kappa}$. A computational exercise for our running example will show that our model is able to generate the inverted-U pattern for all values of $\kappa_i \in (\alpha, 1)$. Moreover, the cross-industry differences in innovation rates (across industries characterized by different levels of competition) are both statistically and economically significant.

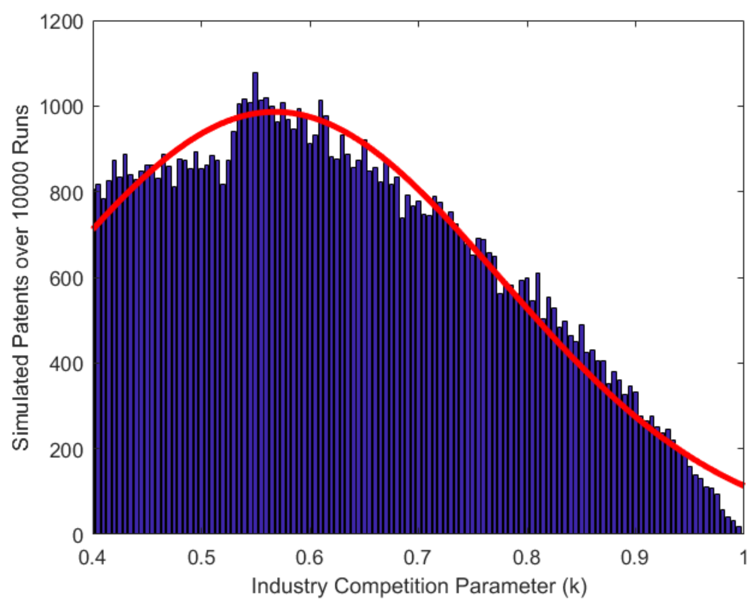
Panel (a) of Figure II. 3 plots the *ex-ante* probability of innovation for the entire economy, μ_i, for all feasible values of κ_i (which by our choice of α must range between 0.4 and 1).[13] We see that the resulting function is indeed increasing and then decreasing, for all feasible values of κ_i. As expected, the curve is convex to the left of the peak, since the marginal effect of an increase in κ_i on z_i^{sep} becomes infinitely large as we approach the peak. Consistently with our earlier discussion, the peak is reached for a level of competition between 0.5 and 0.6 (that is between panel b and c of Figure 1). More in detail, the *ex-ante* probability of innovation increases from 0.08 for $\kappa_i = 0.40$ to 0.10 for $\kappa_i = 0.54$, and then decreases to 0.00 as κ_i grows towards 1.00.

Once we have computed the predicted probabilities of innovation for both high and low types, we can simulate industry-level patenting behavior, aggregating over a large number of industries for each level of competition, in order to generate a syntetic dataset that can be used to run Poisson regressions similar to those in Aghion et al. (2005). Panel (b) of Figure II. 3 shows the results of this exercise. In blue and on the background, we plot the histogram resulting from 10,000 runs of the model. We then regress the number of patents over our measure of competition, as well as this latter coefficient's square. The resulting regression curve is plotted in red over the histogram, and clearly displays the inverted-U shape found by the empirical literature. An analysis of the p-values confirms the significance of all coefficients at the 0.01 level.

This exercise suggests that, even though in theory we do not exactly find



(a)



(b)

Figure 2: panel (a) plots the economy-wide, ex-ante probability of innovation μ_i. Panel (b) reports the realised number of innovations when the model is run 10,000 times per each level of κ_i (blue bars) and a quadratic Poisson regression curve of this data (red line). The functional forms and parameters used are the same used in Figure 1.

an inverted-U relationship between competition and innovation (but rather an increasing and convex, and then decreasing and concave relationship),[14] the finding of an inverted-U by the empirical literature is consistent with the data-generating process being driven by our mechanism.

III ADDITIONAL RESULTS

We have shown that a Schumpeterian model allowing for heterogeneous talent of innovators and asymmetric information can predict the inverted-U, even under the original assumption of innovators focused on post-innovation rents. To provide corroborating evidence in support of our mechanism, we show in this section that the equilibrium we characterise has specific features which match the evidence well.

First, if our mechanism was important to explain the inverted-U relationship between product-market competition and innovation, then we should expect that, in industries in which credit constraints are more prevalent, the increasing part of the relationship should hold for a larger range of levels of competition. In other words, the peak of the inverted-U should be located more to the right. To see this formally, consider a general version of the model in which initial productivity, A_{it-1}, is allowed to vary across industries.[15] As we will show, credit constraints are more prevalent in high-productivity industries, and the peak of their inverted-U is located more to the right.

Consider first the case of perfect information. Very little changes relative to the baseline model. This is because profits are also linearly increasing in

A_{it-1}, so that, in high-productivity industries, a higher cost of investment is exactly offset by higher profits.[16] Mathematically, the $npv_i^J(z)$ functions are still as in equation (I.4), and optimal investment levels are unchanged. In terms of our example, the thin solid lines of Figure 1 are unchanged, and so are their maxima. Of course, high-productivity industries will invest more in absolute terms, *ceteris paribus*. In our example, suppose industries 1 and 2 both have an innovator of the high type, and face competition $\kappa_i = 0.4$ as in the first panel of Figure 1. Without loss of generality, assume $A_{1t-1} > A_{2t-1}$. It is $\hat{z}_1^H = \hat{Z}_1^H / A_{1t-1} = \hat{Z}_2^H / A_{2t-1} = \hat{z}_2^H$ under perfect information, which immediately implies $\hat{Z}_1^H > \hat{Z}_2^H$.

Consider now the case of imperfect information. Only credit-constrained industries are worth examining, since all other industries behave as under perfect information. credit-constrained industries are those endowed with an innovator of the high type, and located on the increasing part of the inverted-U. Across these industries, normalised investment z_i^{sep} is lower lower when A_{it-1} is higher, and credit constraints $\hat{z}_i^H - z_i^{sep}$ are tighter. This is because the normalised wage w_i (now with a subscript) is lower, and so is the amount of normalised equity that the innovator is able to contribute.[17] Intuitively, the same wage buys less innovation in high-productivity industries than in low-productivity ones. For example, consider again industries 1 and 2, which are now credit constrained by virtue of the fact that they have a low level of competition. It is $w_1 < w_2$, which implies $z_1^{sep} < z_2^{sep}$ and thus $\hat{z}_1^H - z_1^{sep} > \hat{z}_2^H - z_2^{sep}$. This can be seen using panel (a) of Figure 1 in conjunction with equation (II.1). A fall in w_i leaves the $npv_i^J(z)$ curves and $\hat{z}_i^J$ unchanged, however it shifts the origin of curve $\widetilde{npv}_i^L(z)$ to the right, and the entire curve up. Then, condition $npv_i^L\left(\hat{z}_i^L\right) = \widetilde{npv}_i^L\left(z_i^{sep}\right)$ must be reached for a lower value of z_i^{sep}.

So, credit constraints are more prevalent in high-productivity industries. But the peak of their inverted-U must then be located more to the right. For suppose that competition in industries 1 and 2 was at the level that puts industry 2 at its peak (that is the minimum level such that $z_2^{sep} = \hat{z}_2^H$). It would be $z_1^{sep} < \hat{z}_1^H$ at this point, which would imply that investment in industry 1 is still increasing in competition.

To illustrate the empirical implications of this, we repeat the computational exercise of Section II. 3 but allowing for two different levels of initial industry productivity. Industries can then be divided into two groups, high-productivity and low-productivity. At any level of competition, credit constraints are more prevalent in the first group, both at the extensive and at the intensive margin.[18] In Figure 3, we reproduce the inverted-U calculated in Figure II. 3, and overlay this with the same curve calculated separately for high- and low-productivity industries. As expected, the peak of the inverted-U is located more to the right for high-productivity industries. Their inverted-U is initially lower, to reflect the fact that tighter credit constraints are associated with lower normalised investment.

These results are consistent with a finding in Aghion, Bloom, Blundell, Griffith, and Howitt (2004).[19] They identify the 40% of firms subject to higher debt-pressure, and plot their inverted-U separately.[20] Consistently with our results, they find that their peak is located more to the right (see their figures

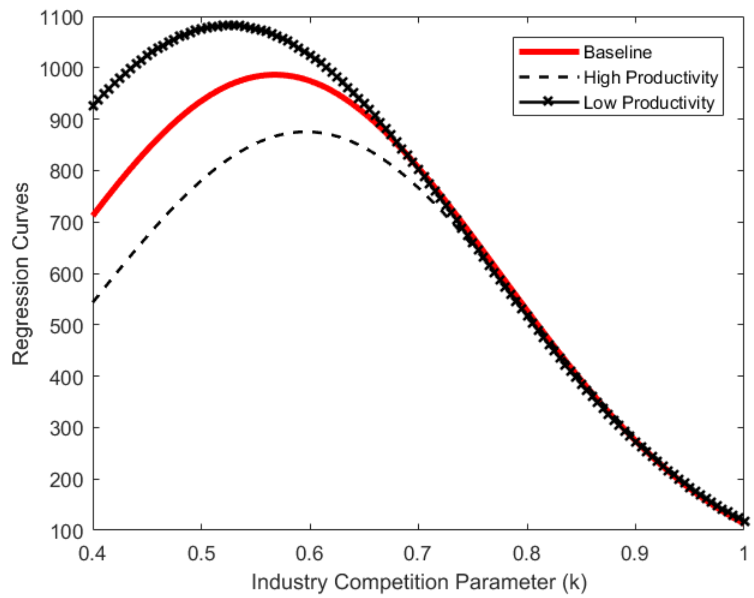


Figure 3: the peak of high-productivity industries is located more to the right. The functional form and parameters used are the same used in Figure 1. In addition, we allow for two different levels of initial industry productivity, A^H_{it-1} and A^L_{it-1}. These are equally likely to occur in the population of industries (so, $A_{t-1} = \left(A^H_{it-1} + A^L_{it-1}\right)/2$), and are such that $A_{t-1}/A^H_{it-1} = 0.82$ and $A_{t-1}/A^L_{it-1} = 1.13$.

6.6a and 6.6b). They also find that their curve is higher than for other firms, which seems at first inconsistent with our results. However, they use a *citation-weighted* patent count as a measure of innovation, while we use the simple patent count. To the extent that the patents of high-productivity industries are more likely to be cited than other patents (as would seem reasonable, given that these are more sophisticated industries that attract larger R&D investments), a trivial extension of our model would also predict a higher inverted-U for high-productivity industries.

A second prediction that is specific to our model is that in credit-constrained industries, expected profitability should be positively correlated with credit constraints. Our main comparative statics has provided an example of this, by showing that stronger competition, which is associated with lower expected profits, leads to weaker credit constraints in credit-constrained industries. To provide other examples, one could allow for cross-industry variation in the overall quality of projects (the scale of a^H and a^L, call this a) or in the opportunity for technological upgrading (γ). Lower a or γ, which are both associated with lower expected profits, would again make the $npv_i^J(z)$ curves to rotate inwards, leading to a higher z_i^{sep} and a lower $\hat{z}_i^H - z_i^{sep}$ (weaker credit constraints).

This prediction is consistent with a finding in the finance literature according to which firms with the largest returns on equity are also those which face the tighter credit constraints (see Li (2011) for a review).[21] Our interpretation of this finding is that industries with high returns are particularly attractive to "lemons", exacerbating the adverse selection problem and making it harder for high-quality projects to be financed. Interestingly, Li (2011) finds that the positive relationship between returns and credit constraints is much stronger among R&D-intensive firms than among non-R&D intensive firms. This is consistent with our interpretation, since it is precisely in R&D-intensive industries that you would expect asymmetric information to be a major issue.

IV CONCLUSIONS

We have shown that a Schumpeterian model allowing for heterogeneous talent of innovators and asymmetric information can predict the inverted-U relationship between competition and innovation, even under the original assumption of innovators focused on post-innovation rents. When competition is low and innovation is highly profitable, investment in innovation is likely to be governed by credit constraints. Then, an increase in competition may lead to a positive selection effect, increasing the rate of innovation even as it reduces the post-innovation rents. When competition is high, however, the low profitability of innovation makes it less likely for credit constraints to be important. Then, the negative impact of an increase in competition on post-innovation rents should also result in less innovation.

The main contribution of our paper is to show that an inverted-U relationship between product market competition and innovation may also emerge amongst firms focused on post-innovation rents. Given the great importance of these firms in the innovation process, this seems a desirable addition to our theoretical understanding of the inverted-U. In addition, our model has two specific predictions. First, the positive relationship between competition and innovation should be more pronounced in industries where credit constraints are more prevalent. Second, the average level of credit constraints in credit-constrained industries should be decreasing in any factor (such as stronger product market competition) that decreases expected profitability. We have argued that these predictions are consistent with, and provide an original interpretation of, existing evidence in the growth and finance literature.

One key policy implication of our work is that, at least for low levels of competition, fostering competition is a substitute for reducing asymmetric information in financial markets. Since the government is unlikely to develop an informational advantage over private investors in the market for innovation, its efforts should focus on fostering competition. The alternative explanation of how an inverted-U between innovation and competition relationship occurs, by Aghion, Bloom, Blundell, Griffith, and Howitt (2005), is based on the dynamics of step by step innovation, and relies on the varying incentives of innovators based on how far advanced they are relative to others. These dynamics are also unlikely to be structurally affected by government policy. Hence, both explanations drive toward a similar conclusion: policy should foster competition up to a point, and in particular in industries that exhibit certain properties. However, there is a clear advantage for policy to focus on asymmetric information rather than differences in technological advancement. Differences in technological advancement are practically hard to observe and must rely on unsatisfactory proxies such as patenting effort. Asymmetric information, on the other hand, leads to clear volatiliy in innovation outcomes in industries as a whole. By measuring whether that volatility become attenuated as a result of its policy efforts, the government can have a reasonable sense of whether its policy efforts are working.

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APPENDIX

In this Appendix, we first formally derive the separating equilibrium discussed in the main text (Section A1). We then identify the parameter subspace where the separating equilibrium exists (A2), and challenge the robustness of the equilibrium to a standard refinement procedure (A3). We conclude by providing more details on the computational exercise of Section II. 3 (A4).

A1. DERIVATION OF THE EQUILIBRIUM

This section is organised as follows. We begin, in Theorem 1, by showing that, if $\hat{z}_i^H|_{\kappa_i=\alpha} \leq w$, the two types must invest $\hat{z}_i^H$ and $\hat{z}_i^L$ in any Perfect Bayesian Equilibrium (PBE). Based on this result, Theorems 2-3 focus on the case $0 < w < \hat{z}_i^H|_{\kappa_i=\alpha}$.

Theorem 2 defines the threshold $\overline{\kappa}$, it establishes its properties as a function of w, and it then shows that, if $\overline{\kappa} \leq \kappa_i \leq 1$, the two types must again invest $\hat{z}_i^H$ and $\hat{z}_i^L$ in any PBE. Based on this result, the theorem further restricts the focus to the case case $\alpha \leq \kappa < \overline{\kappa}$.

Theorem 3 begins by formally defining the separating equilibrium described in Section II (points a-d). Subsequently (points 1-3), it defines the threshold $\underline{\kappa}$, and shows that the separating equilibrium exists if and only if $\underline{\kappa} \leq \kappa_i < \overline{\kappa}$. Second, it shows that if $\overline{w} \leq w < \hat{z}_i^H|_{\kappa_i=\alpha}$, it is always $z_i^{sep} = \hat{z}_i^H$ in the area where the separating equilibrium exists.

Theorem 1. *If $\hat{z}_i^H|_{\kappa_i=\alpha} \leq w$, in any Perfect Bayesian Equilibrium (PBE), the two types invest respectively $\hat{z}_i^H$ and $\hat{z}_i^L$, any combination of equity and external financing being possible.*

Proof. Since the opportunity cost of equity financing is zero, the high types are always able to invest $\hat{z}_i^H$ using only personal wealth, and the minimum rate they can be offered on external financing is $1/\left[a^H\mu(z)\right]$, the high types would never select a research effort different from $\hat{z}_i^H$. Furthermore, they would never take on external financing at a rate greater than $1/\left[a^H\mu(z)\right]$. This last fact implies that a pooling equilibrium does not exist. As shown in footnote 10, at any separating equilibrium, the low types must select $\hat{z}_i^L$. Then, there only exists a separating equilibrium in which the two types invest $\hat{z}_i^H$ and $\hat{z}_i^L$ respectively. If an innovator borrows any money at such equilibrium, this must be at a rate $1/\left[a^H\mu\left(\hat{z}_i^H\right)\right]$ for the high types and $1/[a^L\mu\left(\hat{z}_i^L\right)]$ for the low types. Then, the innovator is indifferent as to the amount borrowed, and it is possible to construct an equilibrium with any combination of equity and external financing. $\square$

Theorem 2. *If* $0 < w < \hat{z}_i^H|_{\kappa_i=\alpha}$*, let*

$$\overline{\kappa} \equiv \arg\left[\hat{z}_i^H = w\right],$$

a threshold that continuously decreases from 1 *to* α *as* w *increases from* 0 *to* $\hat{z}_i^H|_{\kappa_i=\alpha}$*. Then, if* $\overline{\kappa} \leq \kappa_i \leq 1$*, in any PBE, the two types invest respectively* $\hat{z}_i^H$ *and* $\hat{z}_i^L$*, any combination of equity and external financing being possible.*

Proof. The properties of $\overline{\kappa}$ as a function of w follow from the fact that $\hat{z}_i^H$ is equal to $\hat{z}_i^H|_{\kappa_i=\alpha}$ for $\kappa_i = \alpha$, is continuously decreasing in κ, and is equal to 0 for for $\kappa_i = 1$. Then, for $w = 0$, it must be $\overline{\kappa} = 1$; $\overline{\kappa}$ must be continuously decreasing in w; and for $w = \hat{z}_i^H|_{\kappa_i=\alpha}$, it must be $\overline{\kappa} = \alpha$. The rest of the theorem can be shown in the same way as Theorem 1. □

Theorem 3. *If* $0 < w < \hat{z}_i^H|_{\kappa_i=\alpha}$ *and* $\alpha \leq \kappa_i < \overline{\kappa}$, *consider the following situation:*

a. Lenders believe that those who contribute w in equity and invest $z \in (w, z_i^{sep}]$ are high types, where z_i^{sep} is the minimum $z > w$ such that

$$\widetilde{npv}_i^L(z) = npv_i^L(\hat{z}_i^L), \tag{IV.1}$$

or, if such z does not exist, then $z_i^{sep} = \hat{z}_i^H$. They also believe that those who contribute w in equity and invest $z > z_i^{sep}$ are high and low types with equal probability. Finally, they believe that everybody else are low types.

b. Lenders offer rate $1/\left[a^H\mu(z)\right]$ to the first group, rate $1/\left[a\mu(z)\right]$ to the second, and rate $1/[a^L\mu(z)]$ to the third.

c. The low types invest $\hat{z}_i^L$ (any combination of equity and external financing being possible).

d. The high types invest z_i^{sep} (contributing w in equity).

Then, there exists $\underline{w}$ and $\overline{w}$, with $0 < \underline{w} < \overline{w} < \hat{z}_i^H|_{\kappa_i=\alpha}$, such that:

1. If $\overline{w} \leq w < \hat{z}_i^H|_{\kappa_i=\alpha}$, situation a-d is a PBE. It is $z_i^{sep} = \hat{z}_i^H$.

2. *If* $\underline{w} \leq w < \overline{w}$*, situation a-d is a PBE. There exists* $\hat{\kappa} \in (\alpha, \overline{\kappa})$ *such that it is* $z_i^{sep} < \hat{z}_i^H$ *for* $\kappa_i \in [\alpha, \hat{\kappa})$*, and* $z_i^{sep} = \hat{z}_i^H$ *for* $\kappa_i \in [\hat{\kappa}, \overline{\kappa})$.

3. *If* $0 < w < \underline{w}$*, point 2 is still true, except that there exists* $\underline{\kappa} \in (\alpha, \hat{\kappa})$ *such that situation a-d is not a PBE if* $\kappa_i \in [\alpha, \underline{\kappa})$.

Proof. **I. (Preliminary step). Situation a-d is a PBE if and only if**

$$npv_i^H(z_i^{sep}) \geq \widehat{npv}_i^H(z) \quad \forall z > z_i^{sep}. \tag{IV.2}$$

To show this, we proceed in two sub-steps. **I.i.** *If condition IV.2 does not hold, then situation a-d is not a PBE.* This follows from the fact that the high types have a profitable deviation, since they can contribute w in equity and invest some $z > z_i^{sep}$, and obtain a higher payoff. **I.ii.** *If condition (IV.2) holds, situation a-d is a PBE.* This follows from the fact that the following three facts hold true. First, for every action that borrowers could play, the lenders' action is optimal given their beliefs. Second, for actions that borrowers play in equilibrium, the lenders' beliefs are correct. Third, borrowers do not have a profitable deviation. To see the last point, let $\succsim^J$ represent type J's preferences, and let $\left(z^J, e^J\right)$ represent type J's investment profile (where e^J denotes the innovator's equity contribution). Consider first the high types. Their equilibrium action, (z_i^{sep}, w), gives payoff $npv_i^H(z_i^{sep})$. We want to show that $(z_i^{sep}, w) \succsim^H (z, e)$ for any feasible (z, e). This follows from the fact that, if $z < z_i^{sep}$, the high types can at best obtain payoff $npv_i^H(z)$. But $z < z_i^{sep} \leq \hat{z}_i^H$ implies $npv_i^H(z) < npv_i^H(z_i^{sep})$. If $z = z^{sep}$, the only way in which (z, e) may differ from (z_i^{sep}, w) is if $e < w$. But by deviating in this way, the high types are identified as low types, and receive payoff $npv_i^H(z_i^{sep}) - \left(a^H/a^L - 1\right)[z-e] < npv_i^H(z_i^{sep})$. Finally, if $z > z^{sep}$, the high types can at best obtain payoff $\widehat{npv}_i^H(z)$, but it is $\widehat{npv}_i^H(z) \leq npv_i^H(z_i^{sep})$ by condition (IV.2). Next, consider the low types. Their equilibrium action, $(\hat{z}_i^L, e^H)$, where $e^H \in \left[0, \hat{z}_i^L\right]$, gives payoff $npv_i^L(\hat{z}_i^L)$. We want to show that $(\hat{z}_i^L, e^H) \succsim^L (z,e)$ for any feasible (z,e). This follows from the fact that, if $e < w$, or if $z \leq w$, or if both conditions hold, the low types obtain payoff $npv_i^L(z) \leq npv_i^L(\hat{z}_i^L)$. If $e = w$, and $z \in (w, z_i^{sep}]$, the low types obtain payoff $\widetilde{npv}_i^L(z)$, and, by definition of z_i^{sep}, $\widetilde{npv}_i^L(z) < npv_i^L(\hat{z}_i^L)$. If $e = w$, and $z > z_i^{sep}$, the low types receive payoff $\widehat{npv}_i^L(z)$. But condition (IV.2) must hold for z. Multiplying both sides of it by a^L/a^H, we obtain

$$a^L\mu(\hat{z}_i^{sep})\gamma\pi(\kappa_i) - \frac{a^L}{a^H}\hat{z}_i^{sep} \geq a^L\mu(z)\gamma\pi - \frac{a^L}{a}[z-w] - \frac{a^L}{a^H}w,$$

which subtracting $\left[1 - \left(a^L/a^H\right)\right]w$ from both sides becomes $\widetilde{npv}_i^L(\hat{z}_i^{sep}) \geq \widehat{npv}_i^L(z)$, or, by the definition of z_i^{sep}, $npv_i^L\left(\hat{z}_i^L\right) \geq \widehat{npv}_i^L(z)$.

II. (Preliminary step). There exist $\underline{w}$ and $\overline{w}$, with $\hat{z}_i^L|_{\kappa_i=\alpha} < \underline{w} < \overline{w} < \hat{z}_i^H|_{\kappa_i=\alpha}$, such that, if $\kappa_i = \alpha$, it is $z_i^{sep} < \hat{z}_i^H$ if $w < \overline{w}$, $z_i^{sep} = \hat{z}_i^H$ otherwise; and condition (IV.2) holds if and only if $w \geq \underline{w}$. We show these two points in two separate sub-steps. **II.i.** *There exists $\overline{w}$, with $\hat{z}_i^L|_{\kappa_i=\alpha} < \overline{w} < \hat{z}_i^H|_{\kappa_i=\alpha}$, such that, if $\kappa_i = \alpha$, it is $z_i^{sep} < \hat{z}_i^H$ if $w < \overline{w}$, $z_i^{sep} = \hat{z}_i^H$ otherwise.* Suppose $\kappa_i = \alpha$. Recall the definition of z_i^{sep}, provided at part a) of the Theorem. Note that the function $\widetilde{npv}_i^L(z)$ is decreasing in w. Given $w < \hat{z}_i^H|_{\kappa_i=\alpha}$ and $\kappa_i = \alpha$, by Lemma 2, it is $w < \hat{z}_i^H$. Then, the function $\widetilde{npv}_i^L(z)$ (which is only defined for $z > w$), is concave, reaches a maximum at $\hat{z}_i^H > w$, and turns negative for z large enough. As for $npv_i^L(\hat{z}_i^L)$, it is positive and constant in both w and z. It is easy to see that, if $w = \hat{z}_i^L$, it is $\widetilde{npv}^L(\hat{z}_i^L) = npv_i^L(\hat{z}_i^L)$, implying $\widetilde{npv}^L(\hat{z}_i^H) > npv_i^L(\hat{z}_i^L)$. Furthermore, for $w \to \hat{z}_i^H$, it is $\widetilde{npv}^L(\hat{z}_i^H) \to npv_i^L(\hat{z}_i^H) < npv_i^L(\hat{z}_i^L)$. Then, there exists $\overline{w}$, with $\hat{z}_i^L|_{\kappa_i=\alpha} < \overline{w} < \hat{z}_i^H|_{\kappa_i=\alpha}$, such that, if $w < \overline{w}$, equation (IV.1) admits two solutions z_i^{sep} and $\overline{z}_i^{sep}$, with $0 < z_i^{sep} < \hat{z}_i^H < \overline{z}_i^{sep} < \infty$; if $w = \overline{w}$, it admits only one solution $z_i^{sep} = \hat{z}_i^H$; and if $w > \overline{w}$, it admits no solutions (which, by definition, still implies $z_i^{sep} = \hat{z}_i^H$). It is also the case that it is $z_i^{sep} = \hat{z}_i^L$ for $w = \hat{z}_i^L$, and $z_i^{sep} > w$ and increasing in w for $w \in \left[\hat{z}_i^L, \overline{w}\right)$. **II.ii.** *There exists $\underline{w}$, with $\hat{z}_i^L|_{\kappa_i=\alpha} < \underline{w} < \overline{w}$, such that, if $\kappa_i = \alpha$, condition (IV.2) holds if and only if $w \geq \underline{w}$.* Suppose $\kappa_i = \alpha$. The function $\widehat{npv}_i^H(z)$ is concave and maximum for $\hat{z}_i^{pool} = \arg\left[a\mu'(z)\,\gamma\pi(\kappa_i) = 1\right]$, and $\hat{z}_i^{pool} \in \left(\hat{z}_i^L, \hat{z}_i^H\right)$. Then, from results in Step II.i, there exists $\hat{w} \in \left(\hat{z}_i^L, \overline{w}\right)$ such that $z_i^{sep} \geq \hat{z}_i^{pool}$ iff $w \geq \hat{w}$. In such a case, a sufficient condition for (IV.2) to hold is $npv_i^H(z_i^{sep}) \geq \widehat{npv}_i^H(z_i^{sep})$, which is always true. If $w < \hat{w}$, a necessary and sufficient condition for (IV.2) to hold is

$$npv_i^H(z_i^{sep}) \geq \widehat{npv}_i^H(\hat{z}_i^{pool}). \tag{IV.3}$$

There exists $\underline{w}$, with $\hat{z}_i^L|_{\kappa_i=\alpha} < \underline{w} < \hat{w}$ such that (IV.3) holds iff $w \in [\underline{w}, \hat{w})$. This can be shown in two steps. First, note that expression $npv_i^H(z_i^{sep}) - \widehat{npv}_i^H(\hat{z}_i^{pool})$ is continuously increasing in w for $w \in (0, \overline{w})$. To see this, start from condition $\widetilde{npv}^L(z_i^{sep}) = npv_i^L(\hat{z}_i^L)$. Multiplying both sides by a^H/a^L and re-arranging, this can re-written as

$$a^H\mu(z_i^{sep})\gamma\pi\left(\kappa_i\right)-z_i^{sep}=a^H\mu(\hat{z}_i^L)\gamma\pi\left(\kappa_i\right)-\frac{a^H}{a^L}\hat{z}_i^L+\frac{a^H-a^L}{a^H}w, \tag{IV.4}$$

where the LHS is equal to $npv_i^H(z_i^{sep})$. Then, expression $npv_i^H(z_i^{sep})-\widehat{npv}_i^H(\hat{z}_i^{pool})$ can be written as

$$a^H\mu(\hat{z}_i^L)\gamma\pi\left(\kappa_i\right)-\frac{a^H}{a^L}\hat{z}_i^L+\frac{a^H-a^L}{a^L}w-\left[a^H\mu(\hat{z}_i^{pool})\gamma\pi\left(\kappa_i\right)-\frac{a^H}{a}\hat{z}_i^{pool}+\frac{a^H-a}{a}w\right], \tag{IV.5}$$

which is increasing in w (note that $\hat{z}_i^{pool}$ does not depend on w). Second, note that expression $npv_i^H(z_i^{sep})-\widehat{npv}_i^H(\hat{z}_i^{pool})$ is negative for $w=\hat{z}_i^L$, positive for $w=\hat{w}$. The latter follows from earlier discussion; to see the former, recall that, by Step II.i, it is $z_i^{sep}=\hat{z}_i^L$ for $w=\hat{z}_i^L$. Then, it is

$$\widehat{npv}_i^H(\hat{z}_i^{pool})>\widehat{npv}_i^H(z_i^{sep})=\widehat{npv}_i^H(w)=npv_i^H(w)=npv_i^H(z_i^{sep}).$$

III. Point 2 in the Theorem. Suppose $\underline{w}\leq w<\overline{w}$. **III.i.** *If* $\kappa_i\in[\alpha,\overline{\kappa})$, *situation a-d constitutes a PBE.* From Lemma 2, it is $\overline{\kappa}\in(\alpha,1)$. If $\kappa_i=\alpha$, by Step II, condition (IV.2) holds. But the condition also holds for $\kappa_i\in(\alpha,\overline{\kappa})$, which by Step I proves the result. To see this, consider two cases. First, if $z_i^{sep}\geq\hat{z}_i^{pool}$ for $\kappa_i=\alpha$, then such inequality also holds for $\kappa_i\in(\alpha,\overline{\kappa})$. This is because, $\hat{z}_i^{pool}$ is decreasing in κ, while z_i^{sep} is either increasing or equal to $\hat{z}_i^H>\hat{z}_i^{pool}$. But $z_i^{sep}>\hat{z}_i^{pool}$ implies that a sufficient condition for (IV.2) to hold is $npv_i^H(z_i^{sep})\geq\widehat{npv}_i^H(z_i^{sep})$, which is always true. Second, if $z_i^{sep}<\hat{z}_i^{pool}$

for $\kappa_i = \alpha$, there exists $\check{\kappa}$ such that this inequality also holds for $\kappa_i \in (\alpha, \check{\kappa})$, while it is $z_i^{sep} \geq \hat{z}_i^{pool}$ for $\kappa_i \in (\check{\kappa}, \overline{\kappa})$. This follows from the fact that z_i^{pool} is decreasing in κ_i, while z_i^{sep} is increasing and reaches $\hat{z}_i^H > \hat{z}_i^{pool}$ for some $\kappa_i < \overline{\kappa}$. In the first region, that condition (IV.2) follows from the fact that it does so for $\kappa_i = \alpha$, and expression (IV.5) is increasing in κ_i. In the second region, it follows from the fact that a sufficient condition for (IV.2) to hold is $npv_i^H(z_i^{sep}) \geq \widehat{npv}_i^H(z_i^{sep})$, which is always true. **III.ii.** *If $\kappa_i \in [\alpha, \overline{\kappa})$, there exists $\hat{\kappa} \in (\alpha, \overline{\kappa})$ such that it is $z_i^{sep} < \hat{z}_i^H$ for $\kappa_i \in [\alpha, \hat{\kappa})$, and $z_i^{sep} = \hat{z}_i^H$ for $\kappa_i \in [\hat{\kappa}, \overline{\kappa})$.* Given $\kappa_i < \overline{\kappa}$, by Lemma 2, it is $w < \hat{z}_i^H$. The function $\widetilde{npv}_i^L(z)$ (which is only defined for $z > w$) is concave in z, reaches a maximum at $\hat{z}_i^H > w$, and turns negative for z large enough. At the same time, given $w < \overline{w}$ and $w \geq \underline{w} > \hat{z}_i^L$, by step II.i, if $\kappa_i = \alpha$, equation (IV.1) admits two solutions z_i^{sep} and $\overline{z}_i^{sep}$, with $0 < w < z_i^{sep} < \hat{z}_i^H < \overline{z}_i^{sep} < \infty$. But note that $\hat{z}_i^H$ is decreasing in κ_i and, as shown in the proof to Proposition 1, z_i^{sep} is increasing and $\widetilde{npv}_L(z_i^{sep}) - npv_L\left(\hat{z}_i^L\right)$ is decreasing in κ_i (this can be seen by re-arranging equation II.4). Furthermore, for $\kappa \to \overline{\kappa}$, it is $w \leftarrow \hat{z}_i^H$, which by a result in step II.i implies that $\widetilde{npv}_L(z_i^{sep}) - npv_L\left(\hat{z}_i^L\right)$ converges to $\widetilde{npv}_L\left(\hat{z}_i^H\right) - npv_L\left(\hat{z}_i^L\right) < 0$. The result follows.

IV. Point 1 in the Theorem. Suppose $\overline{w} \leq w < \hat{z}_i^H|_{\kappa_i=\alpha}$. Step III.i still holds, with the simplification that, given $w > \overline{w} > \hat{w}$, by a result in Step II.ii, for $\kappa_i = \alpha$, we only need to consider the case $z_i^{sep} > \hat{z}_i^{pool}$. Step III.iii also still holds. Finally, given $w \geq \overline{w}$, by Step II, if $\kappa_i = \alpha$, it is $z_i^{sep} = \hat{z}_i^H$. Furthermore, by step III.i, $npv_i^L\left(\hat{z}_i^L\right) - \widetilde{npv}_i^L\left(\hat{z}_i^H\right)$ is decreasing in κ_i. It follows that it is $z_i^{sep} = \hat{z}_i^H$ for all $\kappa_i \in [\alpha, \overline{\kappa})$.

V. Point 3 in the Theorem. Suppose $0 < w < \underline{w}$. Steps III.ii and III.iii still hold. By Step II, if $\kappa_i = \alpha$, condition (IV.2) does not hold. Furthermore, given $w < \underline{w} < \hat{w}$, by a result in Step II.ii, if $\kappa_i = \alpha$, it is $z_i^{sep} < \hat{z}_i^{pool}$. There exists $\check{\kappa} \in (\alpha, \hat{\kappa})$ such that the last inequality also holds for $\kappa_i \in (\alpha, \check{\kappa})$, while it is $z_i^{sep} \geq \hat{z}_i^{pool}$ for $\kappa_i \in (\check{\kappa}, \overline{\kappa})$. This follows from the fact that z_i^{pool} is decreasing in κ, while z_i^{sep} is increasing and equal to $\hat{z}_i^H > \hat{z}_i^{pool}$ for $\kappa_i = \hat{\kappa}$. There then exists $\underline{\kappa} \in (\alpha, \check{\kappa})$ such that condition (IV.2) does not hold for $\kappa_i \in [\alpha, \underline{\kappa})$, while it holds for $\kappa_i \geq \underline{\kappa}$. This follows from the fact that the condition does not hold for $\kappa_i = \alpha$, that expression (IV.5) is increasing in κ_i, and that condition (IV.2) holds for $z_i^{sep} \geq \hat{z}_i^{pool}$. It follows that, by Step I, situation a-d is not a PBE if $\kappa_i \in [\alpha, \underline{\kappa})$. Otherwise, Step III.i still applies, replacing α with $\underline{\kappa}$ everywhere. □

A2. EXISTENCE OF THE EQUILIBRIUM

Figure 4 represents the $(\kappa_i, \kappa, \alpha)$ parameter space, by plotting κ_i on the vertical axis and $w = (1-\alpha)(\kappa\alpha)^{\frac{\alpha}{1-\alpha}}$ on the horizontal axis. Our comparative statics in this paper has consisted of increasing κ_i, for given w. However, we have tacitly focused on a central case ($\underline{w} < w < \overline{w}$ in the figure), while the remaining cases must also be considered.

The term $\hat{z}_i^H|_{\kappa_i=\alpha}$ represents optimal investment by the high types when the monopolist faces effectively no competition (it can charge price $1/\alpha$). It is the highest amount that the high types may ever want to invest. Then, if $w \geq \hat{z}_i^H|_{\kappa_i=\alpha}$, the high types can always finance their optimal investment purely out of equity contributions.[22] The last statement must also be true if

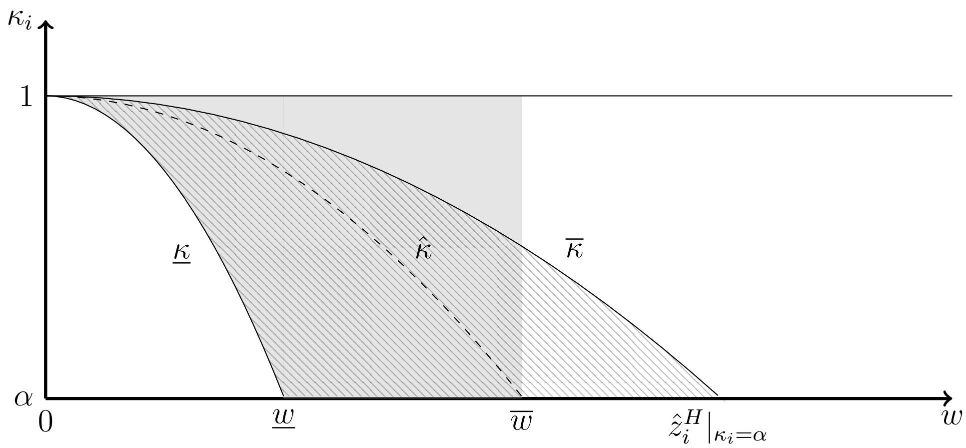

Figure 4: the striped area is where the separating equilibrium exists; the grey, shaded area is where the model can predict an inverted-U.

$0 < w < \hat{z}_i^H|_{\kappa_i=\alpha}$ and κ_i is high enough, since a high κ_i pushes $\hat{z}_i^H$ down to zero, so that it is $w \geq \hat{z}_i^H$. This second case is represented by the area $\kappa_i \geq \overline{\kappa}$ in the figure, where $\overline{\kappa}$ is the unique value of κ_i such that $w = \hat{z}_i^H$, and is intuitively decreasing in w. In both cases, the separating equilibrium does not exist, if anything because the high types would never contribute w in equity. We show in Section A1 above that, in a PBE, innovators always invest $\hat{z}_i^J$ in this area.

Consider next the area $0 < w < \hat{z}_i^H|_{\kappa_i=\alpha}$, $\kappa < \overline{\kappa}$. The separating equilibrium must also not exist if both w and κ_i are very low, that is in the area $0 < w < \underline{w}$, $\alpha \leq \kappa_i < \underline{\kappa}$ in Figure 4, (the threshold $\overline{w}$ and $\underline{\kappa}$ are derived in Section A1 above). To see why, note that $\hat{z}_i^H$ is much greater than w in this case. It follows that $\hat{z}_i^H$ must also be much greater than z_i^{sep}, or else the high types would be leveraging a lot at the separating equilibrium, and the low types would want to mimic them. In other words, there must be a large discrepancy between the high types' optimal investment, and the maximum they can invest by borrowing at their fair rate. But then the high types will prefer to pay an adverse selection premium, borrow more, and invest more. This point can be illustrated using the first panel of Figure 1: if w was very low, the maximum of the high types' net present value would not be z_i^{sep}, but rather a local maximum to the right of it.

In summary, the separating equilibrium does not exist outside of the striped area in Figure 4. We show in Section A1 above that, in the striped area, it always exists. Note that this area does not perfectly overlap with the area where the model can predict an inverted-U relationship between competition and innovation in industries where the innovators is of a high type (the shaded area). This is for two reasons. First, in the area $w \geq \overline{w}$, $\kappa_i < \overline{\kappa}$, even if the separating equilibrium exists, the threshold $\hat{\kappa}$ does not, so that Propositions 1 and 2 do not hold. Intuitively, at such high wages, the high types can always invest $\hat{z}_i^H$ at the separating equilibrium, so that only the decreasing part of the relationship obtains. Second, in the area $0 < w < \overline{w}$, consider industries i and j such that $\hat{\kappa} \leq \kappa_i < \kappa_j \leq 1$. Suppose further that $\kappa_i < \overline{\kappa} \leq \kappa_j \leq 1$, or $\overline{\kappa} \leq \kappa_i < \kappa_j \leq 1$. While strictly speaking Propositions 1 and 2 do not apply to industry j, or i and j, since these industries cannot be at the separating equilibrium, their equilibrium investment must still be $\hat{z}_i^H$ and $\hat{z}_i^L$. So, it must still be true that innovation is higher in industry i than in industry j, and the logic of Proposition 1 and 2 carries through.

A3. EQUILIBRIUM REFINEMENT

In this section, we show that our equilibrium outcome is one of only two that can realise in a PBE that survives a standard refinement procedure. We begin in Section A3.1 by describing this in intuitive terms. The technical analysis is contained in Section A3.2.

A3.1 OVERVIEW

We refine beliefs using a standard, dominance-based criterion (see Mas-Colell, Whinston, and Green (1995), p. 469). Let action $a = (z, e)$ be *dominated* for type J, if there exists another action a' that gives them a strictly higher payoff, for any belief that the lenders might have in equilibrium. The refinement criterion requires that if an action is dominated for one type, but not for the other, then lenders must attach zero probability to the event that the former type undertakes that action (see below for details). We investigate the set of all possible PBE that survive this refinement in the area of existence of our separating equilibrium (the striped area in Figure 4).

This analysis leads to two main results. First, the refinement exactly dictates the beliefs that must be associated with certain actions. Most importantly, lenders must believe that only the high types would take actions of the type $\left(z \in [\hat{z}_i, z_i^{sep}), w\right)$ and $\left(z \in [\overline{z}_i^{sep}, \check{z}_i), w\right)$, where $\overline{z}_i^{sep} \geq \hat{z}_i^H$ denotes the second point at which the mimicker's payoff, $\widetilde{npv}_i^L(z)$, cuts through the payoff of the genuine low types,[23] and $\hat{z}_i \in [w, z_i^{sep})$ and $\check{z}_i > \overline{z}_i^{sep}$. This is because these actions are dominated for the low types - any action $\left(\hat{z}_i^L, e \leq \hat{z}_i^L\right)$ gives them a higher payoff, no matter what lenders believe in equilibrium - but not for the high types. The beliefs in our separating equilibrium must be changed slightly for the equilibrium to survive the refinement, however the equilibrium outcome does not change.[24]

Second, the above-described requirement on beliefs implies that the PBE *must* be a separating equilibrium in which the high types contribute w in equity, and invest either z_i^{sep} or $\overline{z}_i^{sep}$. Intuitively, these beliefs make it suboptimal for the high types to take any other action in a separating equilibrium. They also rule out the existence of a pooling equilibrium, for the same reason why our separating equilibrium exists: given the possibility to invest z_i^{sep} and be identified as high types, the high types prefer this to another action that would pool them together with the low types, even if that other action would allow them to invest more. Of course, this logic only works inside the area of existence of the separating equilibrium, where w (and thus z_i^{sep}) is high enough.

To invest $\overline{z}_i^{sep}$ gives the high types exactly the same payoff as to invest z_i^{sep}. Furthermore, the two thresholds behave in an exactly symmetric fashion. Then, $\overline{z}_i^{sep}$ is *decreasing* in κ_i. It follows that the main result of the paper needs to be qualified, since across industries where the innovator is of a high type, and invests $\overline{z}_i^{sep}$, the model still predicts a monotonic, decreasing relationship between competition and innovation. Of course, such a relationship is not due to the Schumpeterian effect, but to the effect of competition on credit constraints.

We think that, on balance, these results are good news for our theory. Most crucially, our key equilibrium outcome, z_i^{sep}, is one of only two which may realise in a refined PBE. And, while the existence of $\overline{z}_i^{sep}$ as an alternative equilibrium outcome makes it in principle harder for the model to predict an inverted-U, one may reasonably question whether such outcome will ever be observed. After all, while z_i^{sep} and $\overline{z}_i^{sep}$ give exactly the same payoffs to both lenders and borrowers, z_i^{sep} always implies a lower debt, and thus a lower expected size of default. If there was any additional cost from default, which increased with the size of the default, then z_i^{sep} would always be the preferred choice.

A3.2 TECHNICAL ANALYSIS

We closely follow the discussion in Mas-Colell, Whinston, and Green (1995), p. 469. We use to the second (and second-weakest) form of domination-based refinement discussed in the textbook (Eq. 13.AA.2). Let $J \in \overline{J} = \{H, J\}$ denote the type of the innovator. Let $a \in A = \{(z,e) : z \geq 0, 0 \leq e \leq z\}$ denote the choice of investment and equity contribution made by the innovator. Let $\pi(J|a)$ denote the probability that lenders assign to the innovator being of type J, conditional on observing action $a \in A$, and let $r \in R = \{r : r \geq 1\}$ be the interest rate that they require. Let $u(a, r, J)$ denote the expected payoff to an innovator of type J.

We will say that action a is strictly dominated for type J if there exists another action a' with

$$\min_{r \in [1/[a^H \mu(z')], 1/[a^L \mu(z')]]} u(a', r, J) > \max_{r \in [1/[a^H \mu(z)], 1/[a^L \mu(z)]]} u(a, r, J). \qquad \text{(IV.6)}$$

Define the set $\overline{J}^*(a) \subseteq \overline{J}$ as

$$\overline{J}^*(a) = \{J : \text{there is no } a' \in A \text{ satisfying } (IV.6)\}.$$

Our definition of a PBE with reasonable beliefs is as follows:

Definition 1. *A PBE has reasonable beliefs if for all $a \in A$ with $\overline{J}^*(a) \neq \varnothing$, $\mu(J|a) > 0$ only if $J \in \overline{J}^*(a)$.*

In other words, if an action is dominated for type J, and for type J only, then beliefs are said to be reasonable if and only if lenders attach a zero probability to the event that someone taking action a is of type J.

We are now ready to present our refinement result:

Theorem 4. *If $0 < w < \hat{z}_i^H|_{\kappa_i=\alpha}$ and $\underline{\kappa} < \kappa_i < \overline{\kappa}$, let z_i^{sep} and $\overline{z}_i^{sep}$ be the minimum and maximum $z > w$ such that*

$$\widetilde{npv}_i^L(z) = npv_i^L(\hat{z}_i^L), \tag{IV.7}$$

or, if such z is unique or does not exist, then $z_i^{sep} = \overline{z}_i^{sep} = \hat{z}_i^H$. Then, any PBE that has reasonable beliefs in the sense of Definition 1 is a separating equilibrium where the low types invest $\hat{z}_i^L$ (any contribution of equity and external financing being possible), and the high types invest either z_i^{sep} or $\overline{z}_i^{sep}$ (contributing w in equity).

Proof. There exist $\hat{z}_i \in [w, z_i^{sep})$ and $\check{z}_i > \overline{z}_i^{sep}$ such that, at any PBE that has reasonable beliefs in the sense of Definition 1, for any $a = (z, w)$ such that $z \in (\hat{z}_i, z_i^{sep}) \cup (\overline{z}_i^{sep}, \check{z}_i)$, lenders must believe $\mu(L|a) = 0$. To see this, note that there exists $a' = \left(\hat{z}_i^L, e\right)$, with $e \leq \hat{z}_i^L$, such that

$$\min_{r\in\left[1/\left[a^H\mu\left(\hat{z}_i^L\right)\right],1/\left[a^L\mu\left(\hat{z}_i^L\right)\right]\right]} u\left(a', r, L\right) = npv_i^L\left(\hat{z}_i^L\right) > \tag{IV.8}$$

$$\widetilde{npv}_i^L\left(z_i\right) = \max_{r\in[1/[a^H\mu(z_i)],1/[a^L\mu(z_i)]]} u\left(a, r, L\right), \tag{IV.9}$$

where the inequality follows from the definition of z_i^{sep} and $\overline{z}_i^{sep}$.

A PBE that has reasonable beliefs in the sense of Definition 1 cannot be a pooling equilibrium. To see this, proceed by contradiction. Suppose the PBE was a pooling equilibrium, and let $\left(z_i^{pool}, e_i^{pool}\right)$ be the action taken by both types in equilibrium. Distinguish two cases. If $z_i^{pool} > z_i^{sep}$, then the payoff to the high types would be

$$\begin{aligned} a^H\mu\left(z_i^{pool}\right)\gamma\pi\left(\kappa_i\right) - \frac{a^H}{a}\left(z_i^{pool} - e_i^{pool}\right) - e_i^{pool} &\leq a^H\mu\left(z_i^{pool}\right)\gamma\pi\left(\kappa_i\right) - \frac{a^H}{a}\left(z_i^{pool} - w\right) - w \\ &= \widehat{npv}_i^H(z_i^{pool}) \\ &< npv_i^H(z_i^{sep} - \epsilon), \end{aligned}$$

where ϵ is a small enough number. The last inequality follows from Theorem 3 and from continuity: since situation a-d is a PBE in this parameter subspace, it must be $npv_i^H(z_i^{sep}) \geq \widehat{npv}_i^H(z)$, and thus $npv_i^H(z_i^{sep} - \epsilon) \geq \widehat{npv}_i^H(z)$, $\forall z > z_i^{sep}$ (where the strict inequality follows from the fact that we have assumed $\kappa > \underline{\kappa}$ instead of $\kappa \geq \underline{\kappa}$). So, the high types could increase their payoff by choosing $(z_i^{sep} - \epsilon, w)$. If $z_i^{pool} = z_i^{sep}$, then the payoff to the high types would be

$$a^H \mu \left(z_i^{pool}\right) \gamma \pi \left(\kappa_i\right) - \frac{a^H}{a} \left(z_i^{pool} - e_i^{pool}\right) - e_i^{pool} < npv_i^H (z_i^{sep} - \epsilon),$$

where the inequality follows from continuity, given ϵ is low enough. Again, the high types could increase their payoff by choosing $(z_i^{sep} - \epsilon, w)$. Finally, if $0 \leq z_i^{pool} < z_i^{sep}$, then the payoff to the high types would be

$$\begin{aligned} a^H \mu \left(z_i^{pool}\right) \gamma \pi \left(\kappa_i\right) - \frac{a^H}{a} \left(z_i^{pool} - e_i^{pool}\right) - e_i^{pool} &\leq a^H \mu \left(z_i^{pool}\right) \gamma \pi \left(\kappa_i\right) - z_i^{pool} \\ &= npv_i^H (z_i^{pool}) \\ &< npv_i^H (z_i^{sep} - \epsilon), \end{aligned}$$

where the second inequality follows from the fact that $0 \leq z_i^{pool} < z_i^{sep} - \epsilon \leq \hat{z}_i^H$ for ϵ low enough. Once again, the high types could increase their payoff by choosing $(z_i^{sep} - \epsilon, w)$.

Let $a = (z, e)$ be such that either $z \in \{z_i^{sep}, \overline{z}_i^{sep}\}$ and $e < w$, or $z \in (z_i^{sep}, \overline{z}_i^{sep})$ and $e_i \leq w$. Then, at any separating equilibrium, it must be $\pi (H|a) < 1$. To see this, proceed by contradiction. Suppose it was $\pi (H|a) = 1$. Then, the low types could take action a, obtaining payoff $\widetilde{npv}_i^L (z) + \frac{a^H - a^L}{a^H} (w - e) > npv_i^L \left(\hat{z}_i^L\right)$. But since, by footnote 10, the low types must be taking action $\left(\hat{z}_i^L, e_i^L\right)$ (with $e_i^L \leq \hat{z}_i^L$) in a separating equilibrium, obtaining payoff $npv_i^L \left(\hat{z}_i^L\right)$, they would have a profitable deviation, contradicting the notion that this is a PBE.

The Theorem now follows. To see this, note that it was shown in footnote 10 that, in a separating equilibrium, the low types must be taking action

$\left(\hat{z}_i^L, e\right)$, with $e \leq \hat{z}_i^L$. As for the high types, they could not take an action (z, e) such that either $z \in \{z_i^{sep}, \overline{z}_i^{sep}\}$ and $e < w$, or $z \in (z_i^{sep}, \overline{z}_i^{sep})$ and $e \leq w$, since if they did, by step III, the lenders' beliefs would be incorrect. At the same time, they could not take an action such that $z < z_i^{sep}$, since their payoff would at best be $npv_i^H(z)$, and it would always be possible to find $\epsilon > 0$ small enough so that $z < z_i^{sep} - \epsilon$. Since $npv_i^H(z) < npv_i^H(z_i^{sep} - \epsilon)$, the high types could then increase their payoff by choosing $(z_i^{sep} - \epsilon, w)$. Finally, by a symmetric logic, the high types could not be choosing an action such that $z > \overline{z}_i^{sep}$. It follows that the high types must either take action (z_i^{sep}, w) in a PBE, or action $(\overline{z}_i^{sep}, w)$. □

A4. DETAILS ON THE COMPUTATIONAL EXERCISE

To generate panel (b) of Figure II. 3, using MATLAB, we first create a large matrix by taking 121 evenly spaced values of κ_i, going from $\alpha = 0.4$ to 1, and then stacking this vector $10,000$ times.[25] Each of the elements of the resulting matrix can be interpreted as one industry. Having κ_i as well as α, we can easily compute the average level of competition, κ, and profits, as defined by equation (I.2).[26] We assign one innovator's type to each industry using standard pseudo-random draws, where the probability of each of the two possible outcomes is fixed at 0.5.

We then want to compute the probability of successful innovation for each industry, which is characterized by a competition level-innovator's type pair. In doing so, we assume the specific equilibrium configuration described in the paper. This makes working with low-types straightforward, as they always choose $\hat{z}_i^L$, which we can compute using condition (I.5). We have computed profits already, so we need to pick values for a^L and for γ, as well as a functional form for μ. We choose $a^L = 0.4$ and $\gamma = 1.1$; as for μ, our choice of functional form is guided by the following considerations:

- As stated in the paper, μ is increasing and concave;
- we want it to be tractable and invertible, since we'll need to work with its inverse in order to compute $\hat{z}_i^L$; and
- throughout this computational exercise, we need to stay in the region of the parameter space where our model generates an inverted-U pattern.[27]

We thus choose $\mu(z) = 0.22\sqrt{z}$.[28] Once we have $\hat{z}_i^L$ for each low type industry, it's easy to compute the desired probabilities as $a^L\mu(\hat{z}_i^L)$.

Dealing with high-type innovators is trickier, as they will choose $\hat{z}_i^H$ or z_i^{sep}, following the logic described in Section I. For each high-type industry, then, we effectively need to model the behavior of both high and low type innovators. We thus start by computing $\hat{z}_i^L$ and $npv_i^L(\hat{z}_i^L)$, then we compute $\hat{z}_i^H$ using the same methodology (that is, condition (I.5), this time with $a^H = 1$). We then check for which high-type industries $\widetilde{npv}_i^L(z)$ (computed using equation (II.1)) goes above $npv_i^L(\hat{z}_i^L)$. For these industries, we pick z_i^{sep} as the equilibrium level of investment. If instead $\widetilde{npv}_i^L(z) < npv_i^L(\hat{z}_i^L)$ for all $z \geq 0$, we pick $\hat{z}_i^H$. Finally, we use the formula $a^H\mu(.)$ to compute the equilibrium probability of successful innovation.

Once we've computed the probability of successful innovation for each industry, we use it as a parameter of a binomial distribution, in order to simulate real world patenting behavior and generate a synthetic dataset. The resulting histogram, obtained by collecting into bins corresponding to the 121 values of κ_i all the patents secured by both high and low types, is plotted in blue in panel (b) of Figure II. 3.

We also use this synthetic dataset to generate the red curve overlayed onto the histogram. To do so, we follow the methodology used in Aghion, Bloom, Blundell, Griffith, and Howitt (2005) as closely as our synthetic dataset allows us to. Specifically, we compute a Poisson regression of the total number of patents on a constant (β_0), our vector of κ_i, and a vector containing κ_i^2 for each i.[29] Let the vector containing all κ_i be indicated as K, the vector containing κ_i^2 as K^2, and the corresponding regression coefficients as β_1 and β_2, respectively. The red curve is then computed as:

$$exp(\hat{\beta}_0 + \hat{\beta}_1 K + \hat{\beta}_2 K^2).$$

Notes

[1]The pattern has been observed in the US (Scherer (1967); but see Hashmi (2013) for contradicting results), the UK (Aghion, Bloom, Blundell, Griffith, and Howitt (2005)), Japan (Michiyuki and Shunsuke (2013)), the Netherlands (Polder and Veldhuizen (2012)), Sweden (but only for specific measures of competition: see Tingvall and Karpaty (2011)), France (but only for large firms: see Askenazy, Cahn, and Irac (2013)), and Switzerland (Peneder and Woerter (2014)), and for the pharmaceutical industry in a panel of 26 countries studied by Qian (2007).

[2]Akcigit and Kerr (2018) estimate a Shumpeterian model with multi-product incumbents and new entrants, using US data. They find that incumbents innovating on new product lines together with new entrants account for more than 80% of aggregate productivity growth. They argue that this finding is consistent with the empirical literature surveyed in Foster and Krizan (2001). The fact that established firms who do not reap pre-innovation rents face much the same incentives to innovate as new entrants is recognised by the literature, see for example Aghion, Harris, Howitt, and Vickers (2001), p. 469).

[3]See for example Schoonhoven and Lyman (1990) and Katila and Shane (2005).

[4] The general idea of "skin in the game as a screening device" has emerged repeatedly in the academic Finance literature. Applications to the field of entrepreneurship - and its financing - go all the way back to Leland and Pyle (1977); more recent contributions include Kaplan and Stromberg (2004), Skeie (2007), and Conti and Rothaermel (2013), among others. DeMarzo and Duffie (1999) and DeMarzo (2005) provide examples of this principle in the context of security design.

[5]For subsequent modifications of their model, see Askenazy, Cahn, and Irac (2013) and Hashmi (2013).

[6]Some of these papers focus on firms innovating on products they currently produce, thus sharing the same empirical limitations as Aghion, Bloom, Blundell, Griffith, and Howitt (2005)) (see Chernyshev (2016) and Rauch (2008)). Other papers rely on ad-hoc modifications of the standard model: Mukoyama (2003) needs imitators to play an important role alongside innovators; Scott (2009) requires firms to have a different perception of compe-

tition at different levels of competition; and Onori (2015) requires external learning from innovation to be more important than internal learning.

[7]In a related contribution, Chiu, Meh, and Wright (2017) develop a model in which "entrepreneurs" get new ideas randomly and without paying any R&D costs, but search frictions and the presence of financial intermediaries influence the process of technological tranfer. That is, their focus is on the allocation of these new blueprints to the agents who have the most talent for developing them and bringing them to market.

[8]This assumption, also made by Aghion and Howitt (2009), simplifies the model by ruling out that credit constraints are weaker in industries where initial productivity, and thus the size of investment, is lower. Please refer to Section III for the discussion of a more general version of the model in which this assumption is dropped.

[9]This demand function can be found by taking the first derivative of I.1 with respect to X_{it}.

[10]To see this, suppose the low types invested $z \neq \hat{z}_i^L$. Since this is a separating equilibrium, the low types would have to be asked an expected interest rate equal to the risk-free rate, and their payoff would have to be $npv_i^L(z)$. But, by choosing $\hat{z}_i^L$, they could have not been asked a higher expected rate (in equilibrium), and they would have thus obtained at least $npv_i^L(\hat{z}_i^L) > npv_i^L(z^L)$. It follows that z is not the low types' optimal choice: a contradiction.

[11]More precisely, lenders believe that those contributing w in equity are high types if they invest an amount lower than or equal to z_i^{sep}, and high or low types with equal probability if they invest more than z_i^{sep}. Additionally, they believe that those contributing less than w in equity are low types.

[12]For example, the future innovator could be someone who inherits the current innovator's talent, and whose payoffs are important to the current innovator (e.g. a descendant or employee whom the current innovator coaches). Then, her current innovator would worry about the reputation she establishes in period 1.

[13]We refer to Appendix A4 for a complete description of the methodology used.

[14]It is our understanding that the convexity to the immediate left of the peak, and the concavity to its right, are a general feature of the model, but we haven't shown that in a

formal proposition.

[15]We assume A_{it-1} to be uncorrelated with talent, and with the level of competition.

[16]Recall that profits are found by substituting the (constant) optimal price in the demand faced by the monopolist, $P_{it} = \alpha \left(A_{it-1}L/X_{it-1}\right)^{1-\alpha}$, by solving for the equilibrium quantity X_{it-1}, and by multiplying this by the (constant) profit per unit. Since X_{it-1} is linear in A_{it-1}, so are profits. *Normalised* profits are then as in equation (I.2), and do not depend on A_{it-1}.

[17]Since the wage is determined in the economy-wide labour market, it is a linear function of average (as opposed to industry-specific) productivity. The normalised wage is then $w_i = wA_{t-1}/A_{it-1}$.

[18]We have shown that if industries 1 and 2 are both credit constrained, then credit constraints are tighter in 1 (intensive margin). It is also possible that 1 is credit constrained while 2 is not, but not the opposite (extensive margin).

[19]We are grateful to an anonymous referee for pointing this out to us.

[20]They identify these firms as those with the highest debt-payments-to-cash-flow (D/C) ratio. In our model, the D/C ratio is $(\hat{Z}_i^H - W)/NPV_i^H\left(\hat{z}_i^H\right) = (\hat{z}_i^H - W/A_{it-1})/npv_i^H\left(\hat{z}_i^H\right)$ for non credit-constrained industries, and $(z_i^{sep} - W/A_{it-1})/npv_i^H\left(z_i^{sep}\right)$ for credit-constrained ones. While the former is increasing in A_{it-1} (since $\hat{z}_i^H$ is unchanged), the latter can be increasing or decreasing (since z_i^{sep} decreases). So, our high-productivity industries are not necessarily those with the higher D/C ratio. This is counterintuitive, as you would expect industries that require greater investment to feature both a higher D/C ratio and tighter credit constraints. This undesirable feature of the model is the result of simplifying assumptions. With a unified labour market, a higher A_{it-1} only makes it more expensive to innovate, without increasing the innovator's capacity to contribute (in other words, it only reduces w_i). This results in much tighter credit constraints, and may result in a lower D/C ratio. However if the wage was partially related to industry productivity, then credit constraints would still be tighter in high-productivity industries (and the peak of their inverted-U would still be located more to the right), and so would be the D/C ratio. To see this, consider an extreme case in which the wage increases almost linearly with A_{it-1}, so that w_i is only

marginally lower in high-productivity industries. Then, credit constraints $\hat{z}_i^H - z_i^{sep}$ would still be (marginally) tighter (since $\hat{z}_i^H$ is unchanged and z_i^{sep} still decreases), and the D/C ratio would be higher (since z_i^{sep} decreases only marginally).

[21]In the context of our model, the return on equity is $ROE_i^L = npv_i^H\left(\hat{z}^L\right)/w$ for the low types (given a risk free interest rate equal to zero), and $ROE_i^H = npv_i^H\left(z_i^{sep}\right)/w$ for the high types. It is easy to show that these are decreasing in κ_i. For the low types, it is $dROE_i^L/d\kappa_i = (1/w)\, a^L\mu\left(\hat{z}^L\right)\gamma\pi'\left(\kappa_i\right)d\kappa_i < 0$. For the high types, it must be $d\left[npv_i^H\left(z_i^{sep}\right)\right]/d\kappa_i < 0$, for suppose it was $d\left[npv_i^H\left(z_i^{sep}\right)\right]/d\kappa_i \geq 0$: multiplying by a^L/a^H and subtracting $(1-a^L/a^H)w$ on both sides, one would obtain $d\left[\widetilde{npv}^L\left(z_i^{sep}\right)\right]/d\kappa_i > 0$, which is in contradiction with the fact that, by definition of z_i^{sep} it must be $d\left[\widetilde{npv}^L\left(z_i^{sep}\right)\right]/d\kappa_i = d\left[npv^L\left(\hat{z}_i^L\right)\right]/d\kappa_i = dROE_i^L/d\kappa_i$, and $dROE_i^L/d\kappa_i < 0$.

[22]This case must be considered, as there always exist admissible values of the other parameters of the model, γ and a^J, and admissible forms of the function $\mu(.)$, such that $\hat{z}_i^H|_{\kappa_i=\alpha} \leq w$.

[23]The existence of such point can be gauged from the top panel of Figure 1. The function $\widetilde{npv}_i^L(z)$ is a parabola reaching its maximum at $\hat{z}_i^H$. It must then cut through the horizontal line passing through $npv_i^L\left(\hat{z}_i^L\right)$ twice, to the left and to the right of $\hat{z}_i^H$.

[24]Beliefs must be changed in the following way. First, lenders must believe that those taking actions of the type $\left(z \in \left[\overline{z}_i^{sep}, \check{z}_i\right), w\right)$ must be high types. Second, they must believe the same for those taking actions of the type $\left(z \in \left(\hat{z}_i, z_i^{sep}\right), w-\epsilon\right)$ or $\left(z \in \left(\overline{z}_i^{sep}, \check{z}_i\right), w-\eta\right)$, where ϵ and η are small enough numbers. No other belief must be changed. Since to invest $\overline{z}_i^{sep}$ gives the high types exactly the same payoff as to invest z_i^{sep}, it is easy to see that the outcome of the equilibrium does not change.

[25]The value of α is chosen in order to be broadly consistent with the labor share of income as recorded in the U.S. in postwar years.

[26]To compute profits we also need to assign a value to L and we choose $L = 100$.

[27]This is discussed in Appendix A2. What's particularly relevant for us at this stage is that the wage, w, needs to be between $\underline{w}$ and $\overline{w}$. The wage can be computed using equation

I.3. We can also compute closed-form solutions for both $\underline{w}$ and $\overline{w}$, namely,

$$\overline{w} = \frac{1}{1 - \frac{a^L}{a^H}}[a^L \mu(\hat{z}_i^H)\gamma\pi(\kappa_i) - \frac{a^L}{a^H}\hat{z}_i^H - npv_i^L(\hat{z}_i^H)],$$

where i is such that $\kappa_i = \alpha$, and

$$\underline{w} = \frac{1}{\frac{a^H}{a^L} - \frac{a^H}{a}}[a^H \mu(\hat{z}_i^{pool})\gamma\pi(\kappa_i) - \frac{a^H}{a}\hat{z}_i^{pool} - \frac{a^H}{a^L}(a^L\mu(\hat{z}_i^L)\gamma\pi(\kappa_i) - \hat{z}_i^L],$$

where again i is such that $\kappa_i = \alpha$ and $a = (a^H + a^L)/2$.

[28]Of course, $0.22\sqrt{z}$ goes to infinity as z grows, so there might be a concern of the resulting probabilities of successful innovation being larger than one. This doesn't happen in our simulations, where the probabilities are in fact rather small, and always smaller than 0.18.

[29]As in Aghion, Bloom, Blundell, Griffith, and Howitt (2005), these vectors are demeaned.