A Virtual Reality Application to Make Mathematical Functions Accessible

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ABSTRACT

This paper focusses on creating accessible alternatives for the graphical representation of mathematical functions, with particular attention to students with visual impairments and children with Attention Deficit/Hyperactivity Disorder (ADHD). We design a virtual reality (VR) application which can stimulate different senses by immersing students in a virtual reality environment where they can explore the graphical representation of a function and hear how it is played. The purpose of the application is to improve immersion and inclusion in mathematics. The results of the tests show that this type of tool is useful both for learning through play and for students with different learning needs. The use of audio is an effective way to make graphs accessible, but the problem is that there are no universal design principles that apply to various graphs that encode different data types. Additionally, users with ADHD could appreciate the flexibility, speed, cost effectiveness and greater measure of independence provided by the system; however, more research is needed to justify this statement.

CCS CONCEPTS

• **Human-centered computing** → **Virtual reality**; **Accessibility systems and tools**.

KEYWORDS

accessibility, math graph, teaching mathematics

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1 INTRODUCTION

The study of the graphical representation of a given function is a topic that is usually covered in the last two years of high school. Students can use paper and pencil to plot a graph or more advanced software like GeoGebra[1] but, independently of the tool used, they perceive the graphical representation through sight and they need to pay a lot of attention to fully understand them. These two requirements cannot be taken for granted, particularly for students with disabilities.

According to the UN Convention on the Rights of Persons with Disabilities[27] a disability is a long-term physical, mental, intellectual or sensory impairments which in interaction with various barriers may limit a person full and effective participation in society on an equal basis with others. In this paper, we take into account users with visual impairments and children with *Attention-Deficit/Hyperactivity Disorder* (ADHD). Students with disabilities may face different challenges in mathematical education due to a lack of appropriate tools and materials designed to support the development of conceptual understanding in mathematics. According to Beal and Shaw [1] people with total and partial visual impairment tend to have lower results in mathematics than in other subjects due to the difficulty in having accessible resources for the representation of symbols [5]. Furthermore, the lack of adequate tools makes the student with impairments feel excluded from the rest of the class. Since maths is the basis for many other academic subjects, this problem can deeply affect the access of people with impairments to high education.

The study of mathematics is more complex for people with visual impairments since they cannot see an equation and its graphical representation, so one of the most difficult challenges [6] is to represent graphs, charts, and tables in an accessible format. An accessible alternative for students with visual impairments is auditory and tactile perception of graphs[18]: the former uses speakers or headphones and the latter thermal papers or 3D printers [15, 31] but usually their use and implementation require a lot of time.

Students with ADHD do not find difficulty in the perception of content through the senses but are not able to stay focused on an activity for a long time. For this reason, we designed a system to improve engagement in learning graphs through the stimulation of two different senses, sight and hearing, and the use of a game-like interface. Recent studies have investigated the use of serious games and gamification techniques [4, 12–14, 26] for children with ADHD. The game experience, usually linked to a pleasant moment and having fun, is useful to obtain users' attention since school and related activities are often perceived as an environment that forces students to learn. *Serious games*, i.e., games whose main purpose is different from mere entertainment, can stimulate participation, increase involvement, concentration and avoid the sensation of having particular treatment compared to others [22]: the social

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[1] https://www.geogebra.org/

component is in fact very important when working with children and teenagers.

Providing accessible didactic content to students with impairments is not only an ethical commitment, but is also regulated by the law: in Italy, the Stanca law [16] states that public administrations must promote access to high education for people with disabilities by providing appropriate didactic and training material since 2004. In the United States, Section 508 of the Workforce Rehabilitation Act [28] requires federal agencies and their contractors to make their electronic and information technology accessible to people with disabilities. The European Union had promoted the rights of all its citizens and the creation of an inclusive society for everyone. In 2000, the EU presented *eEurope2002*, a project whose aim was to guarantee the right to e-participation even to those groups of the population that had more necessities and difficulties, e. g., people with disabilities. Moreover, in 2001 the European Commission published a communication entitled *"eEurope 2002: Accessibility of Public Web Sites and their Content"* [7] to invite Member States to implement the WCAG guidelines [30] on all public web sites and intensified its commitment to disability, and a new strategy was approved [8]. Its three most important results are the European Accessibility Act [9], the Directive 2016/2102 [11] and the European standard for ICT Accessibility [10].

In this paper, we proposed a solution for the creation on the fly of accessible alternatives for graphical representation of mathematical functions, with particular attention to students with visual impairments or ADHD. We aim to seek answers to the following questions:

- RQ1 Is it possible to recognize the graphical representation of a math function using spatialized sound?
- RQ2 Is the developed tool easy to use for students?
- RQ3 Do students like the developed tool?

The main contribution of this paper is the development of a game-like tool to represent mathematical functions through sound. Touchable alternatives for graphs already exist, but they usually require many hours to be printed. Therefore, they are not suitable for an educational environment where teachers do not always prepare all the examples in advance, but adapt the lesson and the examples to the students' needs. To address difficulties encountered both by students with visual impairments and ADHD, we design a virtual reality (VR) application which can stimulate different senses by immersing students into a virtual environment in which they can draw the graphical representation of a function and hear how it is played. The use of a VR headset allows to better spatialize sound according to head movements and to remove all the distractions of the external environment and helps students with ADHD to concentrate on data and graphs depicted and played by the tools. Moreover, the system looks like a game and uses the interaction device of a game, and this increases the engagement and participation of the students. However, mediating between the aspect of accessibility for students with visual impairments or ADHD, was sometimes difficult and often required to find the correct trade-off. To the best of our knowledge, this is the first attempt to address problems in graph perception and understanding for these two kinds of disabilities with a unique and engaging tool.

The paper is organised as follows: Section 2 initially discusses related works. Section 3 describes the prototype and its implementation. Section 4 presents the test with high school students, and the results are analysed in Section 5 and discussed in Section 6. Finally, conclusions are drawn in Section 7.

2 RELATED WORK

Other works in the literature have studied the difficulties encountered by people with impairments when learning mathematics. Klingenberg et al. [18] summarised current evidence-based knowledge about e-learning in mathematics among students with severe visual impairments and concluded that interactive e-learning with audio and tactile learning materials may be a useful resource for these types of students to enhance their mathematical skills.

The first system for the creation of computer-generated sound patterns of two-dimensional line graphs is described in [21]. The goal was to provide a tool to understand line graphs for people with severe visual impairment in the holistic manner used by those with sight. A continuously varying pitch is used to represent motion in the x direction. The authors discovered that mathematical concepts such as symmetry, monotonicity, and the slopes of lines could be determined quickly using sound. In addition, users with visual impairments deeply appreciated the flexibility, speed, cost effectiveness and greater measure of independence provided by the system.

The use of audio is an effective way to make graphs accessible for people with visual impairments, but the problem is that there are no universal design principles that apply to various charts that encode different data types. Wang et al. [29] conducted an exploratory experiment to assess how different auditory channels (e.g., pitch, volume) impact data and visualisation perception among people with visual impairments to define generalisable principles for sonification. They found that pitch is the most intuitive, while the number of tappings and the length of sounds yielded the most accurate perception in decoding data. Unfortunately, researchers also found that data-level perception might not directly transfer to chart-level perception as participants reflect on visual aspects of the charts while listening to audio.

Better results can be achieved if we take into consideration that people generally experience reality with more than one sense at a time, and this makes the perceptual experience much more engaging. Ramloll et al. [24] focused on line graphs with the idea that the progress of a graph can be easily understood through a tactile device enhanced with sound. They worked with preadolescent and adolescent students and observed that the construction of a tactile graph requires some experience and often takes a long time to be completed.

Yu and Brewster [31] compared a multimodal Virtual Reality interface with traditional tactile diagrams to convey information to visually impaired and blind people. They designed a force-feedback device (SensAble PHANTOM) and a device for synthesised speech and non-speech audio. Even if the potential advantages of the Virtual Reality technology are well known, this was one of the first studies on its real usability. The results show the benefits of using the multimodal approach in terms of more accurate information about the graphs obtained by users.

Brewster [2] investigated how non-spoken audio and tactile interfaces allow better understanding of data contained in graphs and tables than a single mode interface. The author did not build a multimodal system but analysed only one single aspect at time: the experiments on the representation of graphs with sound and using Phantom as haptic device were carried out separately because the purpose was to understand the advantages and disadvantages of the single technology before combining them, in order to exploit the strengths and smooth out the weaknesses of each. The results showed that the use of non-speech audio significantly reduced workload and caused an increase in accuracy and speed of response.

Focussing on *Attention-Deficit / Hyperactivity Disorder*, many studies have proven that new technologies, e.g., virtual reality or augmented reality, are useful tools to teach concepts to children with ADHD through games. Tobar-Muñoz et al. [26] used augmented reality and the Unity framework[2] to develop and test a game in 20 students with various learning needs. The results of the tests show how this type of games is useful both for learning through play for students with different learning needs and for students without these needs. The idea of using the digital games-based approach is based on the fact that digital natives have been used to new technologies since they were children. The game is called Gremlings in my mirror and deals with the teaching of logical and mathematical skills which, according to the authors, are fundamental for children to understand mathematics. This game has been developed with a user-centred perspective, in this case on the special needs of children with ADHD.

Ou et al. [23] developed a game using virtual reality to help children with ADHD improve their cognitive skills, attention, abstract reasoning, and complex information processing. They used a HTC VIVE headset in a three-month training programme. The results showed that the children's attention, hyperactivity-impulsivity performance, and cognitive functions greatly improved, arguing that physical and mental exercises stimulate the production of dopamine, which is more scarce in children with this pathology. Furthermore, the use of games has improved adherence to rehabilitation sessions: All children were happy to play, and as soon as they got better results they wanted to play more.

Finally, a systematic review by Romero et al. [25] analysed more than 450 articles to evaluate the effectiveness of virtual reality-based interventions on cognitive deficit in children with ADHD and showed that VR-based interventions are more effective in improving attentional vigilance measures, increasing the number of correct responses, and decreasing the number of errors of omission.

3 DESCRIPTION OF THE TOOL

We designed our tool based on the analysis of previous work: it has an interface inspired by VR games to enhance the engagement of students with ADHD and supports the perception of graphical representation of maths functions through the use of sound to allow the inclusion of students with visual impairments. In particular, we follow the findings of Wang et al. [29] for the sonification of the graphs.

[2]https://unity.com/

The tool uses Meta Quest 2[3] as an interaction device: it is a VR headset, which offers an immersive and untethered VR experience. It has six-degrees-of-freedom (6DoF) tracking, allowing users to move freely in physical space, while the headset accurately tracks their movements in the virtual environment. The headset supports hand tracking, enabling users to interact with virtual objects using their own hands, but it also has two controllers that allow the user to interact with the virtual environment and perceive haptic feedback. Meta Quest 2 features integrate spatial audio speakers, providing 3D sound and eliminating the need for external headphones. The use of sound spatialization played a pivotal role in the development of the application, since the user could perceive the sound coming from a precise source inside the scene, and this helps to recognise the correct type of graph.

We developed the application using Unity 3D, a cross-platform game engine. It provides many plugins and tools to create VR games, such as:

- the Oculus Plugin, an official plugin developed by Oculus to easily integrate Meta VR headset in a project;
- the VR Camera, a preset camera that allows the handling of the head movements;
- the SDK controllers and the spatial audio integration, to realize an immersive audio experience.

The Unity engine offers many tools to design a virtual environment, such as scenes and objects primitives e. g., rectangles or spheres. The application consists of a single scene in which there is a camera, which tracks the movements of the headset, two hand-shaped objects, that represent the controllers and track their movements, and all the objects needed for the creation of the graphical representation of math functions.

3.1 The game-like interface

As already discussed, we designed an application that looks like a game to increase the engagement, and possible entertainment, of students with ADHD. Furthermore, the game-like environment allows for spatialization of sound, which helps students with visual impairments. When users wear the headset, they are set into the virtual environment, with a blue sky with some clouds as the background of the Cartesian plane as depicted in Figure 1. We avoid to overfill the environment in order to not create too many distractions, this is particularly important for children with ADHD.

The user can choose the type of function to be graphically drawn using the buttons on the right. On the left, there is a list of functions whose graphs have already been drawn in the Cartesian plane; the equation is associated with a dot with the colour of the curve.

Figure 2 depicts the interface that appears when the user selects one of the buttons to draw a graph: the interface covers the Cartesian plane so that the student can focus on this activity. Students can write or modify the coefficients in the function equation, draw the graph or remove the last depicted curve (the green button in the centre says "Draw the graph" and the yellow button says "Delete the last depicted curve"). Moreover, it is possible to clean all graphs in the Cartesian plane with a single interaction (the orange button says "Delete all curves").

[3]https://www.meta.com/it/quest/products/quest-2

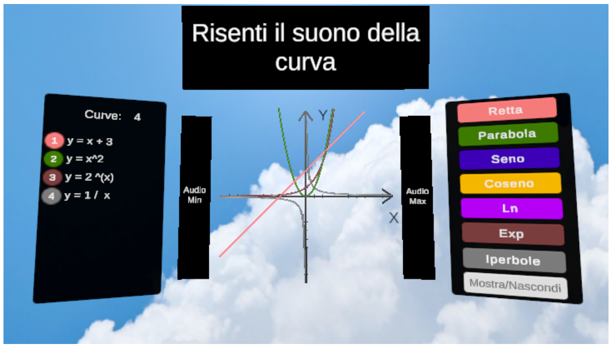

Figure 1: The scene with the Cartesian plane in which the graphs of four functions are drawn[4].

Figure 2: The central interface.

We pay particular attention to the use of colours: the main idea here is to avoid confusing the users and to allow them to easily recognise all the interface's components. For this reason, the Carthesian plane's axes, interfaces, and panels are black, to make them stand out on the blue sky background. Moreover, we choose high-contrast colours for the curves drawn in the Cartesian plane.

The interaction is facilitated by controllers that provide tactile feedback. Hand-shaped objects do not physically touch the buttons; instead, rays emitted from the objects hover over the buttons, allowing the user to click on the corresponding buttons of the right controller.

Figure 1 also shows three black panels designed to facilitate interaction for people with visual impairments. The interface described so far is, in fact, designed for students who can see. Students with visual impairments interact with the interface with the help of teachers or with a screen reader that reads the text in the interface to create a function equation and a graphical representation. Then, they can listen to the sound of the graph. The two panels which can be easily reached by moving the controllers on the left or on the right are used to hear the lowest and highest sound. The panel on the top is used to play again the sound produced by the graphical representation of the function (the last created). These panels do not ask the user to push a button, but it is sufficient to move one of the rays in their direction.

The game lets you draw seven different types of functions: straight line, parable, sine, cosine, logarithm, exponential function, and hyperbole. This choice covers a wide set of graphical representations of functions usually taught in the high schools all over the world. However, since the experiment was conducted in Italy, we checked the set against the Italian school system, but the set can be easily expanded to include other math functions.

3.2 Sound representation and other choices

The idea behind the construction of graphs is simple: they consist of a sequence of small capsule-shaped primitive objects that, one after the other, draw the whole graphical representation of the function. During the creation of capsules, a short sound is played at the point of its creation. The result is a continuous sound that represents that specific drawn graph. Let us consider, as an example, the graphical representation of the equation $y = x$, which is represented by a straight line. The sound will start playing at first from the bottom-left side of the headset, then will pass through the centre, and then it will be heard in the top-right corner. Different types of function graphical representations, such as parables or exponentials, have different types of auditory representation.

The concept and sonification of a graphical representation is mainly based on the study by Wang et al. [29]. In this paper, the authors have shown that the most intuitive way to represent a graph with sound is pitch with positive polarity. In practice, this corresponds to mapping every value of y in the graph with a different frequency: for example, the frequency 174.71 Hz is assigned to $y = -10$ and 3520 Hz to $y = 10$.

Moreover, we have to define which is the lowest and highest sound that a graph can play. Furthermore, Wang et al. used the timbre of different instruments (such as piano or violin) to represent different data features. For the developed application we initially chose to use a pure sine timbre, but it was immediately discarded because the result was very annoying. Then, we used the violin timbre, which has a more suave sound.

3.3 Users' interaction

Since the application uses a game-like interface, it is very different from the tools already available to draw graphical representations of mathematical functions. For this reason, the user can find difficulties when using the system for the first time or feel disoriented. Therefore, we implemented an initial tutorial to teach the student how to use the tool. As the user wears the headset, the application starts with an initial sequence of panels, which contains text instructions. In Figure 3 the instructions are in Italian since we tested the system in an Italian high school. Going through the tutorial, all the interfaces are shown and described.

After the tutorial, the user can use the tool for an interval of 5 minutes to explore the scene and become familiar with the creation of different types of function graphical representations that can be chosen in the right panel. During this period of time, the student can also change the values of the curve parameters. Moreover, it is possible to see the tutorial again if necessary. The user is encouraged to pay attention to the sound played by the different curves.

[4]The game interface is in Italian. On the top there is a button which allows the user to play the sound again. On the right, the user can choose the type of function.

Figure 3: The tutorial interface.

After 5 minutes, all the interaction widgets are disabled, and a black panel appears in front of the scene to block the view so that the student can no more see the curves but only hear the sound. Although some studies have shown that people with visual impairments cannot be completely emulated by blindfolding participants to a test because their perception is different, our aim is to test whether the use of our system helps a user to learn to distinguish a function from another only by hearing its sound.

The tool plays a graphical representation of a function and asks the user to recognise its type by selecting the correct button within one minute. Five different graphs are proposed. In case of a correct answer, the selected button is coloured green. In the event of an incorrect answer, the colour is red. The system collects data on the responses.

4 TEST

The application's testing phase was divided into two parts: the first involved five professors from Padua's University as expert users and the second involved two classes from a high school in Schio (Vicenza, Italy) as non-expert users. The participation of expert users in the application's testing allowed us to change some aspects of the application to better adapt the interface to teaching needs and to improve its accessibility. Non-expert users, on the other hand, even if they do not include people with impairments, helped us to understand whether the tool is usable by users and appropriate for improving knowledge about the graphical representation of functions. A third testing phase will involve students with visual impairments or ADHD.

4.1 Test setting

The non-expert participants in the test were 28 students from the fourth and fifth grades at the "I.T.E.T. Pasini" school in Schio (Vicenza, Italy). The experiment was carried out in the following way: in order to maximise the time available, the tutorial was removed from the tool and shown as a video played on a tablet to the participant before wearing the headset. In this way, a student can watch the video while another one can use the system, and we do not need to give them additional instructions. We use this organisation due to time limits imposed by the school and the availability of only two headsets.

After playing the tutorial, the students wore the Meta headset and began exploring and familiarising themselves with the application. Unfortunately, due to the time limits, we could not give five minutes for this activity as suggested by the expert users, but only 2,5 minutes to try to create all of the available functions. Due to a lack of time, this autonomous experience was semi-guided: we assisted participants in creating all the different function graphical representations.

The final part of the experience was the same as described in Section 3, in which the user had to decide which type of graph they were hearing, answering five questions. After using the application, users were asked to provide feedback on three topics: ease of use (RQ2), pleasantness of the experience (RQ3), and presence or absence of dizziness. The following section covers all of the findings.

5 RESULTS

In this section, we will analyse the data in order to answer some research questions presented in Section 1. The experiment described previously was administered to 28 students (11 women and 17 men) who attended the fourth and fifth grades of the "I.T.E.T. Pasini" school in Schio (Vicenza, Italy). These students are specialising in Enterprise Information Systems.

To answer the research question RQ1 we collect how many times students are able to recognise the mathematical function listening to its sound. The test results showed an average score of 2.57 correct answers on 5 questions with a standard deviation of 1.23. Assuming a normal distribution, we obtain a 0.95 confidence interval of $[2.09, 3.05]$. The lower value appears quite low and corresponds to a test result of two correct answers out of the five functions tested. We must note here that it was probably necessary to have a quick review on the blackboard of the functions that would have been tested before conducting the test with all the involved students. This does not make the experiment biased, since it only serves to remind students of the names of the various functions. Let us remember that the goal of this usability test is to recognise the graph of the function without using sight. If one cannot recall the name of the recognised function, it results in an error not related to recognition but due to a lack of a common prerequisite.

An interesting aspect of this research is the average duration observed in the conducted tests. The maximum duration proposed in the test was 180 seconds. We assume to give students approximately 30 seconds to explore each function. Actually, the test results showed an average test execution time of $69s$ with a standard deviation of $30s$. Assuming a normal distribution, we obtain a 0.95 confidence interval of $[58s, 81s]$. The study of the time required to complete the test is interesting because it allows us to understand if we have provided users with an adequate amount of time to complete this task. Since the average duration is low, the time allocated to the users seems sufficient to us.

In addition to the analysis of the scores obtained by the students in recognising the functions and duration of the test, additional quantitative tests were administered to the students to gather more information about this experiment. In these three additional questions, we asked students to evaluate, after completing the test, the ease of use of the system, the presence or absence of dizziness during the use of the headset, and a comprehensive assessment of enjoyment during the test. As in the previous analyses, we estimated the confidence interval and presented the results with the aim of obtaining operational suggestions for the next test to be conducted.

The usability of the system (RQ2) and its level of acceptance were tested with a 5-point Likert questionnaire asking students to assign a value from one (very bad) to five (very good) to their experience. A high value corresponds to good experience. The test results showed an average valuation of 3.11 with a standard deviation of 0.79. Assuming a normal distribution, we obtain a 0.95 confidence interval of $[2.80, 3.41]$. This result is less positive than we expected before the test. Most likely, this is due to the students' lack of familiarity with this new technology. For many students, it was their first time wearing a virtual reality headset. Therefore, it is difficult to understand how much the result of this questionnaire is related to the software and how much is due to the limited knowledge of the hardware and the limited amount of time to familiarise with the system. For the next test, it would be useful to administer a questionnaire that allows us to stratify this question between those who have already used the VR headset and those who are wearing it for the first time. This would allow for a more precise study of the effectiveness of the interface used.

Cybersickness, also known as virtual reality sickness or simulator sickness, refers to a condition that can occur when using virtual reality (VR) or other immersive technologies. It is characterised by a range of symptoms similar to motion sickness, including nausea, dizziness, disorientation, headache, and general discomfort. Cybersickness is typically caused by a sensory mismatch between visual input and the body's vestibular (balance) system, which can occur when there is a discrepancy between what the user sees in the virtual environment and the physical movements or sensations they experience. For a survey on this critical issue, the reader can refer to [3].

The presence of dizziness was tested with a 5-point Likert questionnaire by asking students to assign a value from one (no dizziness) to five (many dizziness) to their experience. The test carried out in $N = 28$ students showed an average evaluation of 1.82 with a standard deviation of 1.19. Assuming a normal distribution, we obtain a 0.95 confidence interval of $[1.36, 2.28]$. The high standard deviation suggested that we further analyse these data. Unfortunately, we did not have enough data to stratify the analysis, so we attempted to use gender for this investigation. The size of our sample did not allow for an analysis, so we decided to assign a binary value to the presence of dizziness. We associated the presence of dizzines with a questionnaire response greater than or equal to 2, while the absence of dizzines was assigned to a questionnaire response equal to one [20]. This is the only test of its kind that we have conducted with the collected data because we seemed to observe a significant difference in the results between men and women in the data presented in Table 1. The hypothesis test conducted, therefore, has the null hypothesis that the presence of dizziness during the use of VR headsets is the same in men and women. Observing that

Table 1: Contingency Table for Yates's χ^2 Test.

Gender	Dizziness	No dizziness	Total
Male	4	13	17
Female	8	3	11
Total	12	16	28

two entries are less than 5, we performed a Yates's chi-squared test, knowing that in this case, the test tends to fail to reject the null hypothesis. However, the data obtained from our small sample allow us to reject the null hypothesis with a significance level of 0.05 ($p-$value= 0.029). Therefore, the presence of dizzines is more pronounced in women when using VR headsets. This result is consistent with the literature [19]. However, as previously highlighted, it would be interesting to have information on the previous use of VR headsets, since females usually use very few of this kind of devices, and these data could help us analyse this aspect more effectively. In the future, to analyse this problem in more detail, we will use a standardised test for the presence of dizziness, such as the Simulator Sickness Questionnaire (SSQ) [17].

Finally, to answer the RQ3 research question, the overall experience with the headset was evaluated in its entirety using a 5-point Likert questionnaire asking students to assign a value from one (very bad) to five (very good) to their experience. The results of the test conducted on the students showed an average valuation of 4.36 with a standard deviation of 0.73. Assuming a normal distribution, we obtain a 0.95 confidence interval of $[4.07, 4.64]$. These very positive data suggest that we should continue to develop the system. However, further testing will be needed on larger samples to address the issues described in this section.

6 DISCUSSION

The first observation to consider about this test is the scarcity of the sample. It would be useful to propose the experiment to a larger group of students in order to validate the results obtained in this initial phase of the research. Until today, it has not been possible due to logistical difficulties: staff should be trained on how to use the tool before going into schools to instruct students prior to the execution of the test.

Moreover, the type of school selected is important, as the results can be influenced by prior knowledge and confidence in the recognition of mathematical functions. In fact, it was probably necessary to have a quick review on the blackboard of the functions that would have been tested before conducting the test with all the involved students. The impression is that some errors did not arise from an incorrect recognition of the function but from a lack of prior knowledge of the name of the function itself. For the next test, it is suggested to ensure familiarity with the graphs of the functions before administering the test to eliminate this possible bias.

Another truly critical point in the test execution method described in this research is the interval of time given to answer the questions. Most of the students completed the required task very quickly but made several mistakes. Unfortunately, we did not save the correlation between the duration of the test and the score obtained. These would have been useful data to correlate with each other to understand how much haste and anxiety negatively influenced the final result. The anxiety caused by the time limit led the students to rush through their task as quickly as possible, but this had a negative impact on the results obtained. To overcome this problem, it is suggested that in a future implementation, students be required to explore the function for $30s$ and then be given a maximum of $10s$ to input their response. This strategy extends the duration of the test from a maximum of $180s$ to $200s$, but ensures that users utilize all available time. This could reduce the anxiety caused by the time limit and improve the quality of the collected data.

7 CONCLUSIONS

The development of a virtual reality application to draw graphs for visually impaired people and students with ADHD has great potential as a tool to improve their mathematics learning outcomes. This technology can eliminate the barriers and delays present in traditional graph study approaches, enabling students to get a visual representation of the data through touch and sound. Moreover, technology offers a more engaging approach to all students who feel a sense of immersion in a virtual world, thus enhancing the retention of learnt concepts.

The audio feedback and haptic interfaces of the application enable the students to create mental models of data representation. This is especially emphasised in people with visual impairments. Additionally, the use of a multisensory approach in the application aims at improving students' focus, attention, and cognitive abilities, especially in people with ADHD.

Despite promising findings, the study noted that the technology is still in its initial stages and more work is needed. Future studies need to expand the sample size and involve people with disabilities.

In conclusion, the innovative approach through this technology offers numerous opportunities to enhance the educational experience for students with visual impairments and ADHD. The VR graphing application is a promising tool that has the potential to transform the learning process. Further research and development should continue to explore the potential of this technology to promote inclusive and accessible education and improve the lives of students with disabilities.

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