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A unified treatment of the fatigue limit of components weakened by stress raisers, subject to multiaxial mode I, II, and III loading

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Abstract

This paper discusses the dependency of the fatigue limit $\left(\Delta\sigma_{g,th}\right)$ of components on the size of stress raisers, such as cracks, defects, and U- or V-notches, under mode I, II, and III loading conditions in Constant Amplitude (CA) fatigue.

As summarized in the Atzori-Lazzarin-Meneghetti (ALM) diagram, depending on the size and acuity of the stress raiser and the material fatigue properties, three different fatigue limit regimes can be identified: (i) for short cracks, $\Delta\sigma_{g,th}$ approaches the plain fatigue limit $\Delta\sigma_0$, (ii) for long cracks or sharp notches, $\Delta\sigma_{g,th}$ is governed by the long crack fatigue propagation threshold $\Delta K_{th,LC}$, and (iii) for blunt notches, $\Delta\sigma_{g,th}$ is governed by the theoretical stress concentration factor K_t.

In this work, the ALM diagram is extended to include mode II and III loading conditions by proposing a single design curve for any stress raiser subjected to any loading condition by employing the averaged Strain Energy Density criterion. The averaged SED is a fatigue model based on Neuber's concept of elementary structural volume that considers the strain energy density averaged over a material-dependent characteristic volume as the fatigue driving force. The obtained diagram is validated against experimental data obtained from sharp V-notched steel specimens reported in the literature, showing satisfactory results.

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Keywords: Fatigue limit, defect sensitivity; notch sensitivity; V-notch; stress raiser; multiaxial fatigue; Strain Energy Density (SED); Linear Elastic Fracture Mechanics (LEFM)

1. Introduction

It is well known that the fatigue strength of a component weakened by any stress raiser depends primarily on (i) its size, (ii) its acuity, and (iii) the local loading conditions. While it is widely acknowledged that larger and sharper stress raisers have a more detrimental effect on the fatigue strength, such a relationship is far from straightforward because metallic materials may exhibit distinct behaviors depending on the abovementioned features, particularly the defect sensitivity and the notch sensitivity. Regarding the defect sensitivity, Kitagawa and Takahashi (1976) showed

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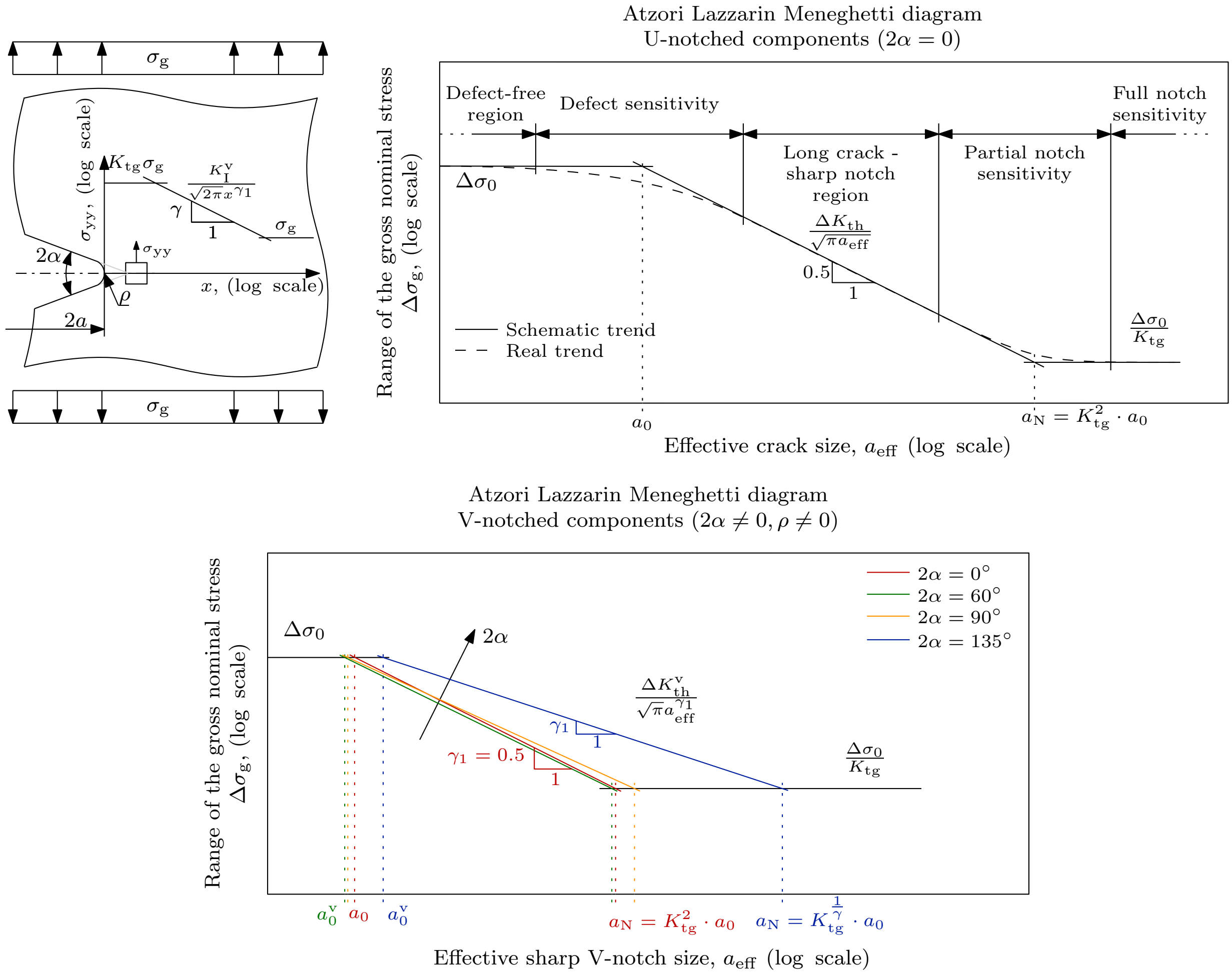

Fig. 1: Atzori-Lazzarin-Meneghetti (ALM) diagram; a) reference system, schematic linear elastic stress field at the V-notch tip; b) ALM diagram for U-notched components, c) ALM diagram for V-notched components.

the existence of a characteristic crack size below which the fatigue limit is no longer dependent on the crack (and defect) size, and it approaches that of the defect-free material. Concerning the notch sensitivity, Frost et al. (1974), by examining the effect of the notch acuity of U-notches (quantified by the theoretical stress concentration factor K_t) of constant depth under mode I loading, observed that beyond a specific value of K_t, the fatigue strength becomes independent on K_t and they behave as cracks of the same depth.

Despite originating from different phenomenological behaviors, Atzori and Lazzarin (2001) identified a connection between these seemingly different phenomena. They introduced a diagram describing both the defect and notch sensitivity, offering a unified representation for estimating the fatigue limit of centered notches and cracks under mode I loading with a unified approach independent of the crack size and notch acuity. Subsequently, in Atzori et al. (2003a), the idea was extended to real components weakened by U-notches in mode I loading by introducing the concept of the effective crack size (figure 1 b)) and later, in Atzori et al. (2005), including also open V-notches (figure 1 c)) subjected to mode I loading. In the case of V-notches, different design curves are obtained for different V-notch opening angles; however, the equations are formally identical; thus, for a given V-notch opening angle, only one curve can describe both the defect sensitivity and notch sensitivity (1 c), d)). In this paper, the ALM diagram is extended to the case of multiaxial loading. To determine the threshold conditions of components weakened by notches, cracks, or defects depending on the local multiaxial stress state, size, and acuity, the averaged Strain Energy Density model is employed. It is a phenomenological model based on Neuber's physical idea, according to which the fatigue behavior is controlled by the mean stress acting on a small but finite volume of material surrounding the notch tip, often referred to as the structural volume or control volume. It was formerly proposed for the static assessment of brittle materials and the fatigue assessment of weldments, Lazzarin and Zambardi (2001). To define the diagram, only two material parameters are required: the range of the averaged SED in threshold conditions and the size of the structural volume. This paper primarily focuses on estimating the fatigue limit of stress raisers in the long crack-sharp notch region since the defect-free and full-notch sensitivity behaviors are less challenging when performing the fatigue assessment.

Nomenclature

a	Reference dimension of a component, for example, the notch depth
ρ	Notch tip radius
γ_i	Degree of singularity of the linear elastic stress distributions in the neighborhood of sharp stress raisers, i=I,II,III relevant to opening, sliding, and tearing loading modes respectively
σ_g	Gross nominal stress
2α	V-notch opening angle
$\Delta\sigma_g$	Range of the gross nominal stress
$\Delta\sigma_{g,th}$	Threshold range of the gross nominal stress
$\Delta\sigma_0$	Plain specimen fatigue limit (in terms of stress range)
K_{tg}	Elastic stress concentration factor referred to the gross section
K_f	Fatigue reduction factor
K^V	Notch Stress Intensity Factor (NSIF); i = I,II,III relevant to opening, sliding and tearing loading modes respectively
α_{γ_i}	Geometric shape factor for a component containing a sharp V-notch, i=I,II,III relevant to opening, sliding and tearing loading modes respectively
$\Delta K^V_{th,eq}$	Equivalent notch stress intensity factor
R	Nominal stress ratio applied
R_c	Radius of the structural volume
λ_g	Biaxiality ratio referred to the gross section, defined as: $\lambda_g = \frac{\sigma_g}{\tau_g}$

2. Theoretical background

Consider a sharp V-notch as shown in figure 2 with a notch depth a and an opening angle 2α, subject to a stress range $\Delta\sigma_g$ at the gross section with a load ratio R. According to the averaged Strain Energy Density (SED) model, the averaged strain energy density, $\overline{w}$, in a material structural volume defined by a circular sector of radius R_c and unit thickness (figure 2) governs the fatigue behavior of metallic materials. According to the equations provided in Lazzarin and Zambardi (2001); Lazzarin et al. (2008), the averaged SED, $\overline{w}$ for sharp V-notches (under the hypotheses of linear elasticity, homogeneous material, and $\rho \to 0$) subjected to mode I, II and III loading can be determined as:

$$\overline{w} = c_w \cdot \sum_{i=1}^{3} \frac{e_i}{E} \left(\frac{K_i^V}{R_c^{\gamma_i}} \right)^2 \tag{1}$$

where:

- R_c is the radius of the structural volume in which the SED is averaged. It is considered a characteristic dimension of the material-microstructure-environment system, and it can be experimentally determined by comparing the fatigue limit of the plain defect-free material and the fatigue limit of a sharp notch (Lazzarin and Zambardi (2001));

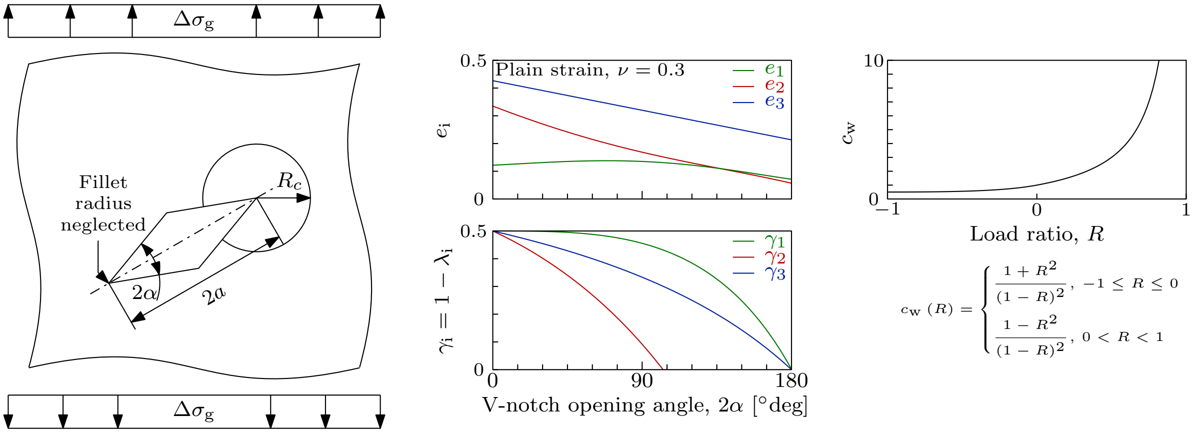

Fig. 2: SED reference system; the functions $e_i\,(2\alpha)$ and $\gamma_i\,(2\alpha)$ are approximated with the equations provided in Visentin et al. (2022).

- c_w is a coefficient accounting for the mean stress effect;
- e_i, i = I, II, III are dimensionless coefficients relevant to opening, sliding, and tearing local stresses, respectively, dependent on the V-notch opening angle 2α and on Poisson's ratio ν;
- E is the elastic modulus of the material;
- $\gamma_i = 1 - \lambda_i$, i = I, II, III are the stress singularity exponents relevant to opening, sliding, and tearing modes respectively; λ_i, i = I, II, III are the first eigenvalues of Williams' Equation for mode I, II, and III, respectively;
- K_i^v, i = I, II, III are the Notch Stress Intensity Factors (N-SIFs), relevant to opening, sliding, and tearing loading modes, respectively, and can be calculated with the engineering formula Atzori et al. (2005):

$$K_i^v = \sqrt{\pi}\alpha_{\gamma_i} \cdot a^{\gamma_i} \cdot \sigma_g, \quad i = I, II, III \tag{2}$$

Let: $\Delta\overline{w}_{th}$ be the range of averaged SED at threshold conditions in the structural volume and $\Delta\sigma_{g,th}$ be the threshold range of the gross nominal stress; by substituting Equation 2 into Equation 1, a relationship between $\Delta\overline{w}_{th}$, the stress raiser size a and the fatigue limit $\Delta\sigma_{g,th}$ can be obtained:

$$\Delta\overline{w}_{th} = c_w \cdot \sum_{i=1}^{3} \frac{e_i}{E} \left(\frac{\sqrt{\pi}\alpha_{\gamma_i} \cdot a^{\gamma_i} \cdot \Delta\sigma_{g,th}}{R_c^{\gamma_i}} \right)^2 \tag{3}$$

from which the fatigue limit in terms of the range of the nominal stress $\Delta\sigma_{g,th}$ can be determined as:

$$\Delta\sigma_{g,th} = \frac{\sqrt{\Delta\overline{w}_{th}E} \cdot R_c^{\gamma_1+\gamma_2+\gamma_3}}{\sqrt{\pi c_w}\sqrt{R_c^{2(\gamma_2+\gamma_3)}e_1\left(\alpha_{\gamma_1}a^{\gamma_1}\right)^2 + R_c^{2(\gamma_3+\gamma_1)}e_2\left(\alpha_{\gamma_2}a^{\gamma_2}\right)^2 + R_c^{2(\gamma_1+\gamma_2)}e_3\left(\alpha_{\gamma_3}a^{\gamma_3}\right)^2}} \tag{4}$$

$$= \frac{\frac{\Delta K^v_{th,eq}}{\sqrt{c_w}}}{\sqrt{\pi}\sqrt{R_c^{2(\gamma_2+\gamma_3)}e_1 \cdot a_{eff,1}^{2\gamma_1} + R_c^{2(\gamma_3+\gamma_1)}e_2 \cdot a_{eff,2}^{2\gamma_2} + R_c^{2(\gamma_1+\gamma_2)}e_3 \cdot a_{eff,3}^{2\gamma_3}}}, \quad \begin{cases} \Delta K^v_{th,eq} = \sqrt{\Delta\overline{w}_{th}E} \cdot R_c^{\gamma_1+\gamma_2+\gamma_3} \\ a_{eff,i} = \alpha_{\gamma_i}^{\frac{1}{\gamma_i}} a, \quad i = 1, 2, 3 \end{cases} \tag{5}$$

$$= \frac{\frac{\Delta K^v_{th,eq}}{\sqrt{c_w}}}{\sqrt{\pi \cdot a^v_{eq}}}, \quad a^v_{eq} = \sum_{i=1}^{3} R_c^{2(\gamma_j+\gamma_k)} \cdot e_i \cdot a_{eff,i}^{2\gamma_i}, \quad \begin{cases} i = 1, 2, 3 \\ i \neq j \neq k \end{cases} \tag{6}$$

where:

- $a_{\text{eff,i}}$, i = I,II,III are the effective crack size relevant to opening, sliding, and tearing modes as defined in Atzori et al. (2003a) and are dependent only on the local stress field and the V-notch opening angle 2α;
- a^{v}_{eq} can be considered as the length of an equivalent crack in an infinite plate, and $\Delta K^{\text{v}}_{\text{th,eq}}$ as the threshold SIF of an equivalent material, such that the fatigue limit of the equivalent crack-material system is the same of the component subjected to the local multiaxial stress field. Note that a^{v}_{eq} also depends on the material-microstructure-environment system via the structural radius and the coefficients e_{i}, i = I, II, III;

Upon normalizing for $\Delta K^{\text{v}}_{\text{th,eq}}$ and c_{w} a single design curve can summarize the fatigue behavior of any long crack or sharp notch for any local loading condition and any opening angle.

3. Experimental validation

Equation 6 is validated against the experimental data obtained by Atzori et al. (2003b, 2006) in which the normalized low carbon steel sharp V-notched cylindric specimens shown in Figure 3 a) were tested under the constant amplitude cyclic loading conditions summarized in table 1. The comparison between numerical estimate and experimental results is reported in Figure 3 b-f).

Table 1: Summary of tests performed in Atzori et al. (2003b, 2006). For multiaxial tests, Φ is the phase angle between torsional and tension loading, and λ is the biaxiality ratio, defined as the ratio between the nominal shear stress and the nominal axial stress referred to the gross section.

Specimen geometry	Loading conditions	Notch depth a, [mm]	Load ratio R	Phase angle, ϕ, [° deg]	$\lambda_{\text{g}} = \frac{\sigma_{\text{g}}}{\tau_{\text{g}}}$
V-notch	Tension	4	−1	–	–
V-notch	Torsion	4	−1	–	–
V-notch	Torsion	2	−1	–	–
V-notch	Torsion	0.5	−1	–	–
Shaft	Torsion	4	−1	–	–
V-notch	Tension + torsion	4	−1	0	1.67
V-notch	Tension + torsion	4	0	0	1.67
V-notch	Tension + torsion	4	−1	90	1.67
V-notch	Tension + torsion	4	0	90	1.67

Note that: (i) there are no failures reported below the estimation of Equation 6 for all the tested conditions; (ii) for torsion-loaded specimens, Equation 6 may seem too conservative as there are run-out tests above the estimated fatigue limit (e.g., $V-\text{notch}, a = 4\text{mm}, \text{torsion}, R = -1$); however, this could be because the stress levels tested did not reach the knee point of the S-N curves of the material, even at $5 \cdot 10^6$ cycles (Atzori et al. (2003b, 2006)), and thus they are not representative of the threshold conditions of the material under torsion loading; (iii) the estimation of Equation 6 for specimens tested under remote multiaxial loading conditions at R=0 is too conservative, a phenomena that is already well documented in the literature under local mode I loading only Atzori et al. (2003a).

4. Conclusions

In this paper, a comprehensive approach for the fatigue assessment U- and V-notches with varying notch opening angles and tip radii, subjected to local multiaxial stress fields has been introduced. Employing the averaged Strain Energy Density (SED) model, it was possible to provide a unified approach for estimating the fatigue limit of components weakened by stress raisers to changes of the stress raiser size, acuity, and local loading conditions, including combinations of mode I, II and III. Comparing the numerical predictions in an Atzori-Lazzarin-Meneghetti (ALM) diagram with the experimental results from the literature, led to satisfactory results.

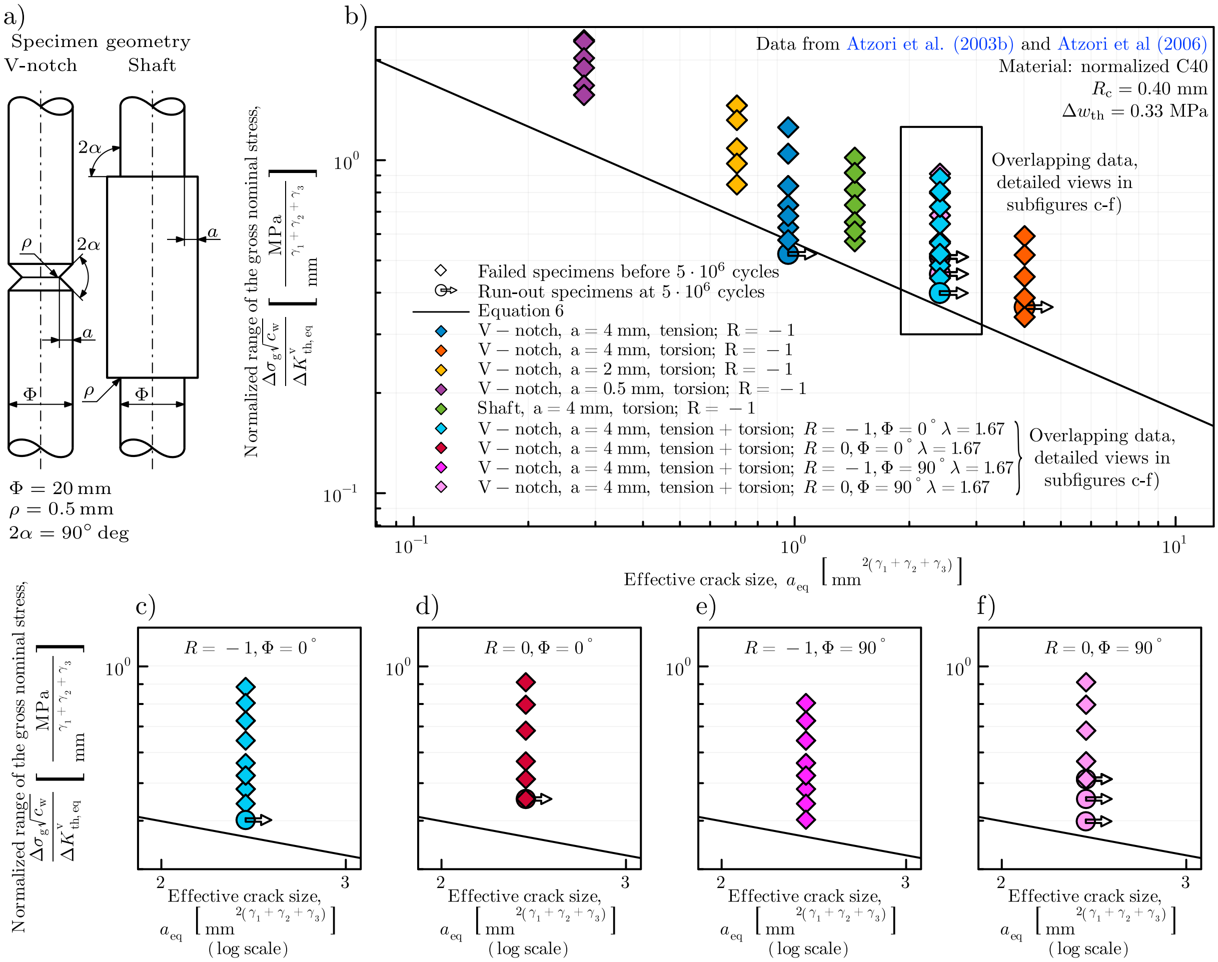

Fig. 3: Comparison between Equation 6 and experimental data from Atzori et al. (2006, 2003b); a) reference geometry for the specimens; b) Equation 6 vs experimental data; c-f) detailed views for the multiaxial loading conditions.

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